Chapter 12: *t* Tests

The Lipton Company loves *t* tests. - Anonymous

Learning Objectives

Upon completion of this chapter, students should know

- The concept of sampling distribution of differences between means.
- The concept of standard error of the difference between independent and correlated sample means.
- When to use the three major types of *t* tests.
- How to compute the three major types of *t* tests.
- The influence of sample size on the *t* distribution and statistical power.
- How to interpret the results of the three major types of *t* tests.

Key Terms

The ***t* distribution** is similar to the *z* distribution in that both are symmetrical, bell-shaped sampling distributions. But, the overall shape of the *t* distribution is influenced strongly by the sample size used to generate it. For very large samples, the *t* distribution approaches the *z* distribution, but for smaller samples, the *t* distribution is flatter.

A ***t* test** is a test of the null and research hypotheses used when the research design involves two samples. It tests the difference between the means of a sample and the population, two independent samples, or two correlated samples.

Degrees of freedom are a statistical term used to denote the number of scores within any distribution that are free to vary without restriction in a sample with a fixed mean.

Sampling distribution of differences between means is a distribution generated by taking two random samples and computing the difference between their means. By doing this a great number of times, a distribution is formed. If all the samples are selected from the same population or populations with equal means, the mean of this distribution is zero. If, on the other hand, there are two independent samples from populations that have different means, then the distribution of differences will have a mean equal to the difference between the two populations.

Standard error of the difference between independent sample means is the standard deviation of the distribution of the differences between sample means that is drawn from two independent populations.

Standard error of the difference between correlated sample means is the standard deviation of the distribution of the differences between paired means using the standard deviation and correlation coefficient.

The **difference method** is used to compute the estimate of the standard error of the difference between means that uses the differences between paired scores rather than the correlation coefficient and standard deviation.

Lecture and Demonstration Aids

The *t* Test emphasizes difference between two means either between a sample mean and a population mean, two independent sample means or two means from the same sample. The difficulty students often face is choosing which *t* test to use. Once they know the appropriate test to use, the math does not seem problematic. The active-learning activity is designed to give students practice identifying the appropriate design.

Visual Summaries. Review the visual summaries as you discuss the computations of each t test. These can be used as procedural maps to help students remember each step of the analyses. Direct students' attention to the visual summary on page 313. This is a decision tree that help students select the appropriate test.

Temperature and Gender Data. Use the dataset from Shoemaker (1996) to demonstrate the independent samples t test (see Transparency 12-4). Compute a t test to determine if there temperatures differed between men and women. The solution is shown below.

Men: $X_1 = 98.105; S_1 = .699$ Women: $X_1 = 98.394; S_1 = .743$

t (128) = -2.28, p. <.05. Standard error of the difference = .1267

Instructional Video. *Against All Odds: Inside Statistics***.** Program Twenty-One, "Inference for One Mean" and Program Twenty-Two, "Comparing Two Means" are both videos you may find useful on the topic of *t* tests. The first 21 minutes of the first video discusses Gosset's t-distribution, degrees of freedom, and Nutra-Sweet's use of paired *t* tests. The initial 17 minutes on the second video covers independent

groups *t* tests relating to a welfare program and commercial products. This video is produced by the Consortium for Mathematics and Its Applications and Chedd-Angier (1989). Available through Annenberg/CPB.

Active Learning Activities

Activity 1. t Test Practice. The purpose of this activity is to give students practice identifying the appropriate t test to use in analyzing results. Distribute Handout 12-A and have students to work in small groups to answer these questions. It is also a good idea to have each group generate a research scenario for each of the three tests. This gives students additional practice from another perspective. You may want to add Identification of independent and dependent variables and potential extraneous variables to the instructions.

Additional Assignments

Assignment 1. Assign students the dataset listed below. This dataset was used by Gosset as an example in his 1908 paper, "The Probable Error of a Mean." The paper was published under the pseudonym "Student" and introduced a form that later became known as the Student's t-distribution. The Student's t-distribution that we use today was derived by R. A. Fisher (1925). The dataset includes Darwin's paired plant sizes data and the sleep differences data of Gosset's.

Handout 12-A.

t -Test Practice

Read each of the following scenarios and identify the appropriate t test. Be sure to include whether the test is one-tailed or two-tailed.

1. A research study was conducted to examine the differences between married men and married women on perceived life satisfaction. A group of 20 men (married 10 years or more) and a separate group of 20 women (married 10 years or more) completed the life satisfaction test. Scores ranged from 1 (low life satisfaction to 100 (high life satisfaction).

2. Researchers want to examine the effects of a new therapy to lower blood pressure. Participants' blood pressure was measured before the week-long therapy began and after the therapy was completed.

3. The effect of perceived control was examined by researchers among elderly patients residing in a convalescent home. Sixty participants were randomly selected to participant in the study. All patients were given a plant by the nursing staff. Half of the participants were randomly assigned to the responsibility of taking care of the plant. The other half of the patients had their plant cared for by the staff The number of health complaints was recorded for each patient for two weeks.

4. Students in Dr. Trill's night class kept falling asleep during lecture. When Dr. Trill asked the students why they could not stay awake, they said it was because they slept fewer hours than other students. Dr. Trill knew that the campus population slept 7.1 hours per night and decided to test his students' explanation. The data indicated the mean sleep for the class was 6.3 hours per night with a standard deviation of 1.8 hours.

5. Researchers believed time pressure negatively influences memory. Thirty participants were randomly assigned to remember a list of words either under time pressure or no time pressure. The number of nonsense words correctly recalled.was recorded.

Handout 12-B.

Gosset's Famous Dataset

Variable names: DIFF is the difference in heights (eighths inches) for 15 pairs of plants (cross-fertilized minus self-fertilized Zea mays) raised by Darwin. ADD is the additional sleep gained by patients (in hours) from the use of laevohysocyamine hydrobromide (referenced by Gosset).

DIFF	ADD
49	1.9
-67	.8
8	1.1
16	0.1
6	-0.1
23	4.4
28	5.5
41	1.6
14	4.6
29	3.4
56	
24	
75	
60	
-48	

Transparency 12-1.

Single-Sample *t* Test

$$t = \frac{\bar{X} - m}{s_{\bar{X}}}$$

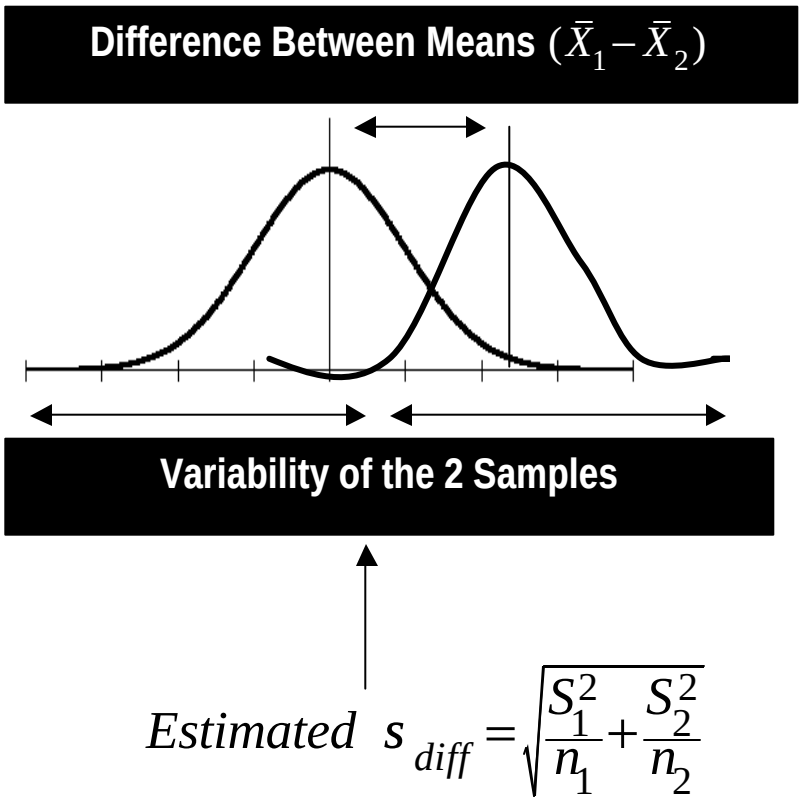
Compares a sample mean to a population mean.

If standard deviation of the population is not known, compute the standard error of the mean using the standard deviation of the sample.

Transparency 12-2.

Independent Sample Means *t* Test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{diff}}$$



$$Estimated\ s_{diff} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

Note: t will be positive if $\bar{X}_1$ is larger than $\bar{X}_2$

Transparency 12-3.

t Test for Correlated Sample

(aka Repeated Measures *t* Test, Dependent *t* Test)

$$t = \frac{\overline{D}}{est s_{diff}}$$

$$Est s_{diff} = \sqrt{\frac{\frac{\sum D^2}{n} - \overline{D^2}}{n-1}}$$

Transparency 12-4.

Temperature and Gender

96.3	97.8	98.4	99.1	98	98.5	99
96.7	97.8	98.4	99.2	98	98.6	99.1
96.9	97.8	98.4	99.3	98	98.6	99.1
97	97.9	98.5	99.4	98	98.6	99.2
97.1	97.9	98.5	99.5	98	98.6	99.2
97.1	98	98.6	96.4	98.1	98.7	99.3
97.1	98	98.6	96.7	98.2	98.7	99.4
97.2	98	98.6	96.8	98.2	98.7	99.9
97.3	98	98.6	97.2	98.2	98.7	100
97.4	98	98.6	97.2	98.2	98.7	100.8
97.4	98	98.6	97.4	98.2	98.7	
97.4	98.1	98.7	97.6	98.2	98.8	
97.4	98.1	98.7	97.7	98.3	98.8	
97.5	98.2	98.8	97.7	98.3	98.8	
97.5	98.2	98.8	97.8	98.3	98.8	
97.6	98.2	98.8	97.8	98.4	98.8	
97.6	98.2	98.9	97.8	98.4	98.8	
97.6	98.3	99	97.9	98.4	98.8	
97.7	98.3	99	97.9	98.4	98.9	
97.8	98.4	99	97.9	98.4	99	

SOURCE:

Shoemaker, A. (1996). What's normal? Temperature, gender and heart rate. *Journal of Statistics Education* 4. Accessed: 12/01.

http://www.amstat.org/publications/jse/v4n2/datasets.shoemaker.html