Chapter 6: Scaled and Standard Scores: How to Change Apples Into Oranges

"An approximate answer to the right question is
worth a good deal more than the exact answer to an
approximate question." *--John Tukey*

Learning Objectives

Upon completion of this chapter, students should know

- The purpose of transforming a distribution of raw scores into scaled scores.
- The rules for transformation of a distribution of scores.
- The purpose of transforming a distribution into z scores.
- How to compute and interpret standard scores.

Key Terms

Review the following key terms from this chapter. Terms are listed in the order they appear in the chapter.

Scaled scores are scores that are adjusted via some type of scale by applying the same constant to all the scores in the distribution.

Standard scores or **z scores** are uniform values to which any raw score value can be converted. A z score indicates the number of standard deviation units a raw score is positioned either above or below the mean. $z = \frac{X - \mu}{\sigma}$ or $z = \frac{X - \bar{X}}{S}$

The **z-score distribution** is a distribution of transformed raw scores that has a mean of zero and a standard deviation of 1. The transformation of a distribution of raw scores into a distribution of z scores is done so comparisons between different distributions can be made.

Lecture and Demonstration Ideas

This chapter changes the focus of how scores are distributed and which measures best describe the distribution to the procedures of transformation to scale scores and convert raw scores to standard scores.

Transformation rules. Use transparencies 6-1 and 6-2 to demonstrate the rules of transformation. The National Mathematic Composite Scores for the 12th grade is an example of scaled math scores for different groups of students based on the number of hours they watch television and/or videos (Transparency 6-7). The standard deviations are very similar too. Ask students to evaluate this.

Flossy's and Billy's z scores. Transparency 6-3 to 6-6 may help students conceptualize z scores. A rather silly question is posed on Transparency 6-3 is designed to stimulate the reasoning process used in the transformation of raw scores to z scores. For students wanting a visualization of this process, the conceptual map on Transparency 6-8 may be useful.

Additional Assignments

The Practical Use of z-scores. Students may not be aware of the extensive use of z scores. Ask students to examine growth charts on the Center for Disease Control website (http://www.cdc.gov/nchs/about/major/nhanes/growthcharts/datafiles.htm) or the World Health Organization (http://www.ennonline.net/fex/01/fa6.html). Both websites have good examples illustrating the important use z scores. Challenge students to find other organizations that rely on the use of z scores in the statistical analyses.

References:

National Center for Education Statistics (2000) *Mathematics Assessment Composite*. Accessed 10/5/01. http://www.asbj.com/evs/01/state.html

Transparency 6-1.

Adding or Subtracting a Constant

Changes the Mean
No Change in Variation

Distance between numbers the same

Original	Add Constant	New
1	+ 5	6
2	+ 5	7
3	+ 5	8
4	+ 5	9
5	+ 5	10
$\bar{X}$ = 3 1.581	+ 5	$\bar{X}$ = 8 1.581

Distance between numbers the same

Transparency 6-2.

Multiplying or Dividing a Constant:

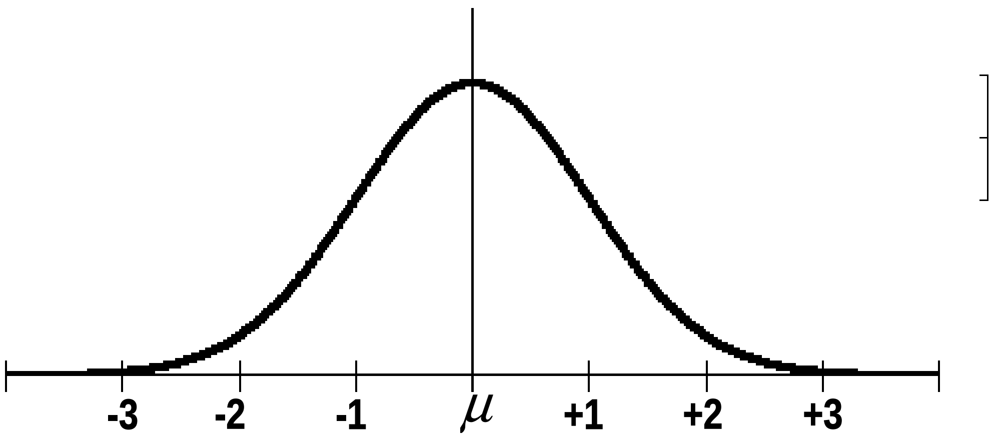
Changes the Mean
Changes the Variation

Distance between numbers is different.

Original	Multiply Constant	New
1	x 5	5
2	x 5	10
3	x 5	15
4	x 5	20
5	x 5	25
$\bar{X}$ = 3 1.5811	x 5	$\bar{X}$ = 15 7.906

Distance between numbers is different.

Transparency 6-3.



Prof A	**30**	**40**	**50**	**60**	**70**	**80**	**90**	
Prof B	**52**	**58**	**64**	**70**	**76**	**82**	**88**	

Prof A's History Exam $\bar{X}$ **= 60** ***S*** **= 10**
Prof B's History Exam $\bar{X}$ **= 70** ***S*** **= 6**

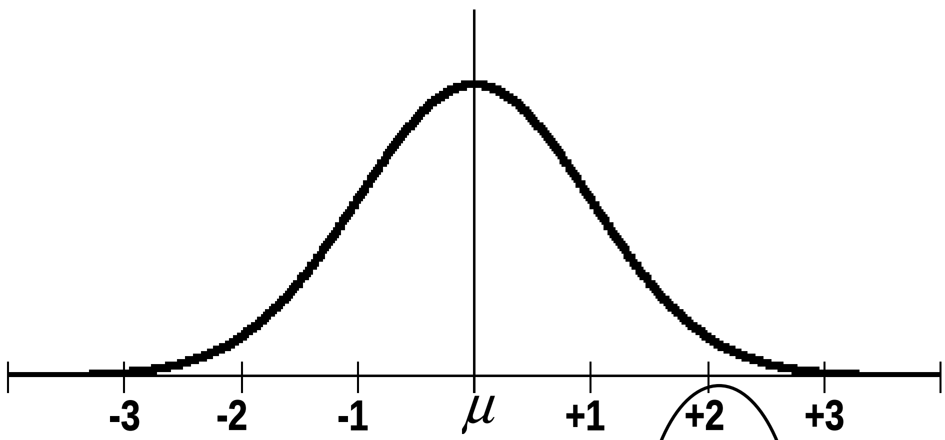
Flossy Mae is in Prof A's class and her score was 80.
Her boyfriend, Billy Bob is in Prof B's class and his score was 82.

Who did better on the exam?

Transparency 6-4.

Was your comparison based on the location of the scores relative to the mean?

Flossy Mae and Billy Bob tied!



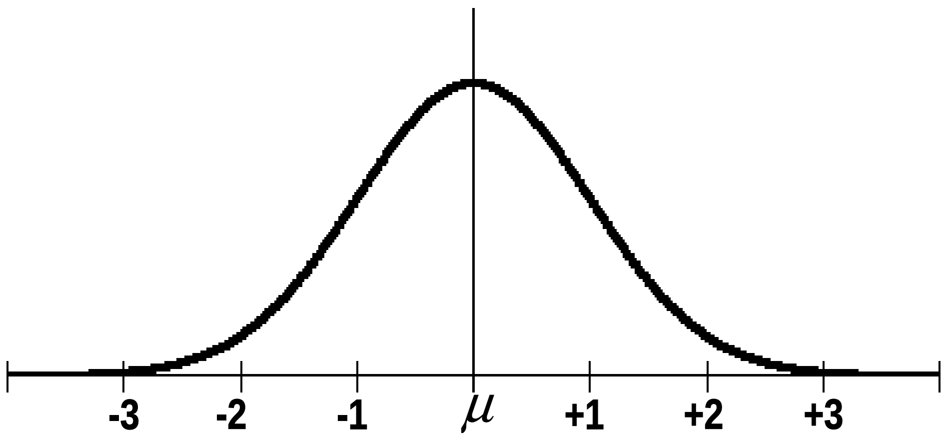
Prof A	**30**	**40**	**50**	**60**	**70**	**80**	**90**	
Prof B	**52**	**58**	**64**	**70**	**76**	**82**	**88**	

Prof A's History Exam $\bar{X}$ **= 60** ***S*** **= 10**
Prof B's History Exam $\bar{X}$ **= 70** ***S*** **= 6**

What is Flossy Mae's and Billy Bob's z-score?

A z score is a standard deviation score when the mean is zero.

Transparency 6-5.



Prof A	**30**	**40**	**50**	**60**	**70**	**80**	**90**
Prof B	**52**	**58**	**64**	**70**	**76**	**82**	**88**

Prof A's History Exam $\bar{X}$ **= 60** ***S*** **= 10**

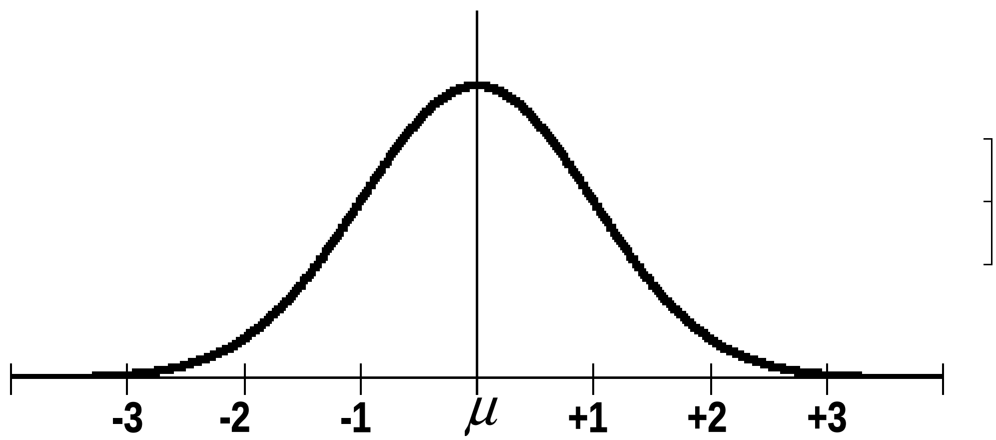
$$z = \frac{X - \bar{X}}{S}$$

Prof B's History Exam $\bar{X}$ **= 70** ***S*** **= 6**

Flossy Mae = $\frac{80-70}{10} = 1.00$ **Billy Bob =** $\frac{82-76}{6} = 1.00$

Transparency 6-6.

z scores are deviation scores



z scores use the rules of transformation to transform

scores to a distribution with

mean = 0 and standard deviation of 1

Transparency 6-7.

National (Public)/Mathematics Composite
12th Grade: 2000, 1996, 1992 and 1990

Question:

On a school day, about how many hours do you usually watch TV or videotapes outside of school hours? (self-reported response)

Average Scale Score and Standard Deviation

OVERALL PERFORMANCE

			None		1 hour or less		2 hours		3 hours		4 hours		5 hours		6 hours or more	
	Year	N	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.
Total	2000	6823	310	40	308	34	304	33	295	35	289	32	284	36	276	32
	1996	5383	312	35	313	30	305	31	300	31	292	30	289	32	278	31
	1992	5454	310	37	306	33	302	31	293	33	288	33	283	31	273	33
	1990	2521	298	38	304	35	299	34	287	35	280	32	281	30	264	34

GENDER

			None		1 hour or less		2 hours		3 hours		4 hours		5 hours		6 hours or more	
	Year	N	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.	Avg. Score	Std. Dev.
Male	2000	3294	305	43	309	36	308	35	299	37	294	31	289	39	277	33
	1996	2511	310	38	314	32	305	33	303	31	296	31	289	33	284	33
	1992	2647	305	43	307	35	303	32	296	33	291	34	289	30	278	34
	1990	1184	----	----	307	37	302	34	289	35	281	34	280	30	----	----
Female	2000	3529	314	36	308	32	299	30	291	32	284	31	279	32	275	30
	1996	2872	313	32	313	28	304	29	296	30	289	29	289	30	274	27
	1992	2807	313	33	305	31	301	30	290	32	284	31	276	30	268	30
	1990	1337	----	----	301	33	295	34	285	36	280	31	281	30	261	33

---- Sample size is insufficient to permit a reliable estimate.

Transparency 6-8.