

Chapter 4: Measures of Central Tendency

Did you hear about the statistician who had his/her head in the oven and his feet in a bucket of ice? Someone asked how he/she was feeling? He/She replied, "On the average, I'm feeling fine."

- Author Unknown

Learning Objectives

Upon completion of this chapter, students should know

- The function of central tendency measures.
- How to calculate and differentiate between the mean, median, and mode.
- How to identify the skew of a distribution of scores.
- How to determine which measure of central tendency to use to represent a distribution of scores.
- How to calculate quartiles and create boxplots.

Key Terms

Measures of central tendency are scores that represent the center of the distribution. The most common measures of central tendency are the mean, median, and mode.

The **Mean** is the score that, by weight, is the center of the distribution. Although all three measures of central tendency can be called averages, the mean is the value most people associate with the average. It is also the most commonly used measure of central tendency. The mean of a sample, $\bar{X} = \frac{\sum X}{n}$, or a population, $\mu = \frac{\sum X}{N}$, is the sum of all the scores divided by the total number of scores.

Median is the middle score of any set of raw scores. When the scores in a distribution are ranked or placed in numerical order, half are above and half are below the median. If there are an odd number of scores, the median is the middle score and if there are an even number of scores, the median is the mean of the two middle scores.

$$Median_{\text{odd number of scores}} = \left[\frac{n+1}{2} \right]^{th} \text{ score} \quad Median_{\text{even number of scores}} = \frac{\left[\frac{n+2}{2} \right]^{th} \text{ score} + \left[\frac{n}{2} \right]^{th} \text{ score}}{2}$$

Mode is the most frequently occurring score in the distribution. When the scores in a distribution are ranked or placed in numerical order and one score is the most frequent, the distribution is *unimodal*. A distribution is called *bimodal* and has two modes when two different scores tie for the most frequent score. When there are more than two modes, the distribution is *multimodal*. However, if the distribution consists of only one of each score, it has n modes or as many modes as there are scores.

Skew refers to the general shape of a distribution of scores when it is graphed as a frequency polygon. There is *zero skew* when the shape of the distribution is symmetrical. It is *skewed* when most of the scores are at one end and very few at the other end of the distribution. The tails of distributions are the areas at the extreme high or low end where the data taper off to zero – where the graph approaches the abscissa. The skew is *positive* when the tail points in the positive direction and *negative* when it is pointing in the negative direction.

Lecture and Demonstration Ideas

Measures of central tendency are used to convey the most typical values in a distribution. Students are usually familiar with the term *average* as the sum of all the scores divided by the total scores. However, the notion of one or more “average” value can be confusing. Encourage students to think in terms of the value that is “most representative” of the entire distribution rather than an average value.

Introduce the topic of central tendency by reviewing the conceptual map of descriptive statistics shown on Transparency 4-11. Since descriptive statistics are represented in a visual form as graphs and numerical form as central tendency, a picture of the overall conceptual organization and the relationships between topics can be helpful to students. When students are able to visualize how certain topics are important elements of one specific concept that will form the foundation of another concept, it may help them organize and integrate new information into this organizational framework. This review will also prepare students for upcoming topic, methods of dispersion.

You may find Transparencies 4-1 to 4-4 useful aids to your lecture and demonstration of measures of central tendency.

Central Tendency Measures in Action. This demonstration allows students observe and differentiate between the three measures of central tendency. Depending on the size of the classroom, either all or a sample of students can take part in the demonstration. The demonstration takes about 25 to 30 minutes to complete in classes with 25 to 35 students. Because larger classes may need to select a random sample of 13 (or more) male and 13 (or more) female students, allow 45 minutes to complete the demonstration.

Begin the demonstration by asking the men to come to the front of the class. Arrange the students in ascending heights to demonstrate the median. If there is an odd number of students, then the height of the person(s) standing in the middle is the median height. If there is an even number of students, then the median will be halfway between the heights of the two students standing in the middle.

To demonstrate the mode, have those students that are the same height step forward. It is possible the distribution may be bimodal, multimodal, and also *n modes*.

Next, ask students in the human distribution to state their height. Have the other students compute the mean. For the sake of expediency, convert heights to inches before doing the arithmetic. Next, point out the location of the mean.

The second part of the demonstration, repeats the same procedure with women students (they may need to remove their shoes). When the demonstrations are completed, ask students to discuss which measure of central tendency was most representative of each distribution.

Psychologists' Salaries. A good example of the differences between the mean and median is shown in a table (Transparency 4-5) constructed from a selection of data collected in a survey of the 1999 salaries in psychology (APA, 2000). In this study, questionnaires were sent to a stratified random sample of 20,000 APA members (working 35 hrs or more per week) yielding a response rate of 51% or 10,232 respondents. The mean, median, quartiles and standard deviation are shown for each salary area. Ask students to evaluate the representativeness of the mean and media. In some cases, the standard deviation is about half the size of the mean salary. This would be a good time to discuss skew (Transparencies 4-12 to 4-14) and the representativeness of the measure selected.



Instructional Video. *Against All Odds: Inside Statistics*. Program Two, "Picturing Distributions." The last ~14 minutes of this 30-minute video has a segment on TV programming and skewed distributions. Two other segments: rush hour traffic and health care costs discuss histogram bin sizes, spread, and stem and leaf plots. This video is produced by the Consortium for Mathematics and Its Applications and Chedd-Angier (1989) and available through Annenberg/CPB.

Active-Learning Activities

Violence on Television. Have students practice computing central tendency measures using data from a study conducted by the Center for Media and Public Affairs (1999). Prime-time fictional television shows

were analyzed over two weeks. Violence was defined as: "Any deliberate act of physical force or use of a weapon in an attempt to achieve a goal, further a cause, stop the action of another, act out an angry impulse, defend oneself from attack, secure material reward or merely to intimidate others (pg. 4)." The rate of violence per episode (see Transparency 4-6) was collected along with other data. The full solution is shown on Transparency 4-7.

Box Office Income. Ask students to practice computing central tendency using large numbers by computing the measures of central tendency of the average gross box office income for popular films the last ten years. Transparencies 4-8 to 4-9 include gross box office income based on the initial release of the film domestically and solutions. (Note: Re-releases and other income are not included in the figures shown).

References:

American Psychological Association (2000). *1999 salaries in psychology*. Accessed 7/3/00:
<http://research.apa.org/99salaries.html>.

At-The-Movies.com (2001). *Box Office Revenues*. Accessed: 9/28/01 <http://www.the-movie-times.com>

Center for Median and Public Affairs (1999). *Violence in popular entertainment 1998-1999*. Accessed 11/02/01: <http://www.cmpa.com/>

Transparency 4-1.

Measures of Central Tendency

Representatives of the Center of the Distribution

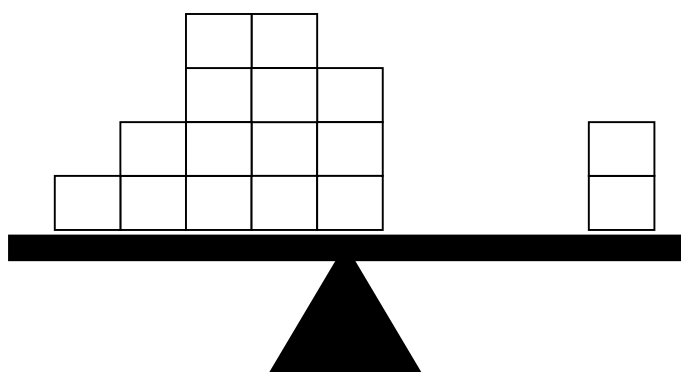
Mean the sum of all the scores in the distribution divided by
total number of scores

Median the middle score

Mode the most frequent score(s)

Transparency 4-2**Mean**

Extreme scores pull the mean in their direction.



$$\bar{X} = \frac{\sum X}{n}$$

Examples

| A | B | C |
|----------|----------|----------|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 50 | 200 |

$$\bar{X} = \frac{15}{5} = 3$$

$$\bar{X} = \frac{60}{5} = 12$$

$$\bar{X} = \frac{210}{5} = 42$$

Transparency 4-3.**Median**

Represents the distribution by its position
as the center of the scores.

| A | B | C |
|----------|----------|----------|
| 1 | 2 | 3 |
| 2 | 3 | 3 |
| 3 | 3 | 3 |
| 4 | 6 | 9 |
| 5 | 10 | 12 |



Transparency 4-4.

Mode

Represents the distribution by frequency.

| A | B | C |
|----------------|----------|----------|
| 1 | 1 | 1 |
| 2 | 3 | 1 |
| 3 | 3 | 1 |
| 4 | 5 | 3 |
| 5 | 10 | 5 |
| 6 | 11 | 6 |
| 7 | 12 | 7 |
| <i>n</i> modes | mode = 3 | mode = 1 |

Transparency 4-5.

Faculty Positions

Doctoral-level, 9 – 10 month Salaries for Selected Settings: 1999

| Setting & Experience | Median | Q1 | Q3 | Mean | SD | N |
|------------------------------------|--------|--------|---------|---------|--------|-----|
| University Psychology Dept. | | | | | | |
| Full-professor | 72,000 | 61,000 | 88,000 | 76,178 | 22,846 | 365 |
| Associate Professor | 50,000 | 45,000 | 56,750 | 53,000 | 26,889 | 200 |
| Assistant Professor | 41,000 | 36,000 | 45,000 | 40,757 | 7,980 | 202 |
| Other faculty position | 32,500 | 26,250 | 40,000 | 34,750 | 9,117 | 12 |
| University Education Dept. | | | | | | |
| Full-professor | 75,000 | 63,250 | 85,750 | 81,393 | 44,883 | 84 |
| Associate Professor | 50,000 | 45,000 | 57,000 | 51,228 | 10,917 | 57 |
| Assistant Professor | 43,000 | 40,000 | 46,000 | 42,577 | 7,439 | 71 |
| University Business Dept. | | | | | | |
| Full-professor | 98,000 | 83,000 | 116,000 | 106,115 | 36,296 | 26 |
| Associate Professor | 80,000 | 63,000 | 86,000 | 76,533 | 15,928 | 15 |
| Assistant Professor | 74,000 | 63,000 | 90,000 | 76,421 | 13,142 | 19 |
| University Research Center | | | | | | |
| Full-professor | 98,000 | 71,000 | 106,000 | 94,636 | 25,001 | 11 |
| Associate Professor | 55,000 | 52,000 | 61,000 | 57,833 | 14,106 | 6 |
| Assistant Professor | 37,000 | 31,000 | 41,000 | 37,000 | 8,505 | 7 |
| Other faculty position | 38,000 | 28,000 | 43,000 | 35,600 | 8,234 | 5 |

Source:

American Psychological Association (2000). *1999 Salaries in Psychology*. Accessed 7/3/00:
<http://research.apa.org/99salaries.html>.

Transparency 4-6.

| Rate of Violence in Television Series Per Episode | | | |
|---|-------------|---------------|-------------|
| Outlet | Per Episode | Outlet | Per Episode |
| ABC | 3 | TNT | 73 |
| CBC | 16 | USA | 14 |
| NBC | 2 | MTV | 3 |
| FOX | 4 | PAX | 2 |
| UPN | 8 | Basic Cable | 10 |
| WB | 7 | HBO | 31 |
| Syndicated | 35 | Showtime | 4 |
| Family | 3 | Premium Cable | 10 |
| Lifetime | 1 | S/T All Cable | 10 |
| Sci-Fi | 11 | | |

Source:

Center for Median and Public Affairs (1999). *Violence in popular entertainment 1998-1999*. Washington, D.C. <http://www.cmpa.com/>

Transparency 4-7.

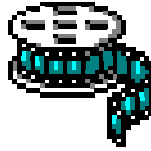
| Rate of Violence in Television Series Per Episode | |
|---|----|
| Lifetime | 1 |
| NBC | 2 |
| PAX | 2 |
| ABC | 3 |
| MTV | 3 |
| Family | 3 |
| FOX | 4 |
| Showtime | 4 |
| WB | 7 |
| UPN | 8 |
| Basic Cable | 10 |
| Premium Cable | 10 |
| S/T All Cable | 10 |
| Sci-Fi | 11 |
| USA | 14 |
| CBS | 16 |
| HBO | 31 |
| Syndicated | 35 |
| TNT | 73 |

Mean = 13.00

Median = 8

Mode = 3, 10

$$\bar{X} = \frac{\sum X}{n} = \frac{247.00}{19} = 13.00$$

Transparency 4-8.**Which is the best measure of central tendency?**

| Year | Top 10 Films – Domestic | Gross Box Office Bucks \$\$\$ |
|------|--------------------------------|-------------------------------------|
| 2000 | How The Grinch Stole Christmas | 260,031,035.00 |
| 1999 | The Phantom Menace | 429,870,576.00 |
| 1998 | Saving Private Ryan | 216,119,000.00 |
| 1997 | Titanic | 600,743,440.00 |
| 1996 | Independence Day | 306,052,958.00 |
| 1995 | Toy Story | 191,702,825.00 |
| 1994 | Forrest Gump | 329,690,974.00 |
| 1993 | Jurassic Park | 356,839,725.00 |
| 1992 | Aladdin | 217,350,219.00 |
| 1991 | Terminator 2: Judgment Day | 204,446,562.00 |
| | | 3,112,847,314.00 |

$$\bar{X} = \frac{\sum X}{n} = \frac{3,112,847,314.00}{10} = \$ 311,284,731.4$$

Transparency 4-9.

Compute the Median

| Year | Films – Domestic | Gross Box Office \$\$\$ |
|------|--------------------------------|-------------------------|
| 1997 | Titanic | 600,743,440.00 |
| 1999 | The Phantom Menace | 429,870,576.00 |
| 1993 | Jurassic Park | 356,839,725.00 |
| 1994 | Forrest Gump | 329,690,974.00 |
| 1996 | Independence Day | 306,052,958.00 |
| 2000 | How the Grinch Stole Christmas | 260,031,035.00 |
| 1992 | Aladdin | 217,350,219.00 |
| 1998 | Saving Private Ryan | 216,119,000.00 |
| 1991 | Terminator 2: Judgment Day | 204,446,562.00 |
| 1995 | Toy Story | 191,702,825.00 |



First, rank order data.

$$\begin{aligned}
 \text{Median}_{\text{even number of scores}} &= \frac{\frac{n+1}{2}^{\text{th}} \text{ score} + \frac{n+2}{2}^{\text{th}} \text{ score}}{2} \\
 &= \frac{\frac{10+1}{2}^{\text{th}} \text{ score} + \frac{10+2}{2}^{\text{th}} \text{ score}}{2} = \frac{6^{\text{th}} + 5^{\text{th}}}{2} = 5.5^{\text{th}} \text{ score}
 \end{aligned}$$

$$\$260,031,035 + \$306,052,958 = \$283,032,996.5$$

Source:

At-The-Movies.com (2001)
<http://www.the-movie-times.com>

Transparency 4-10.

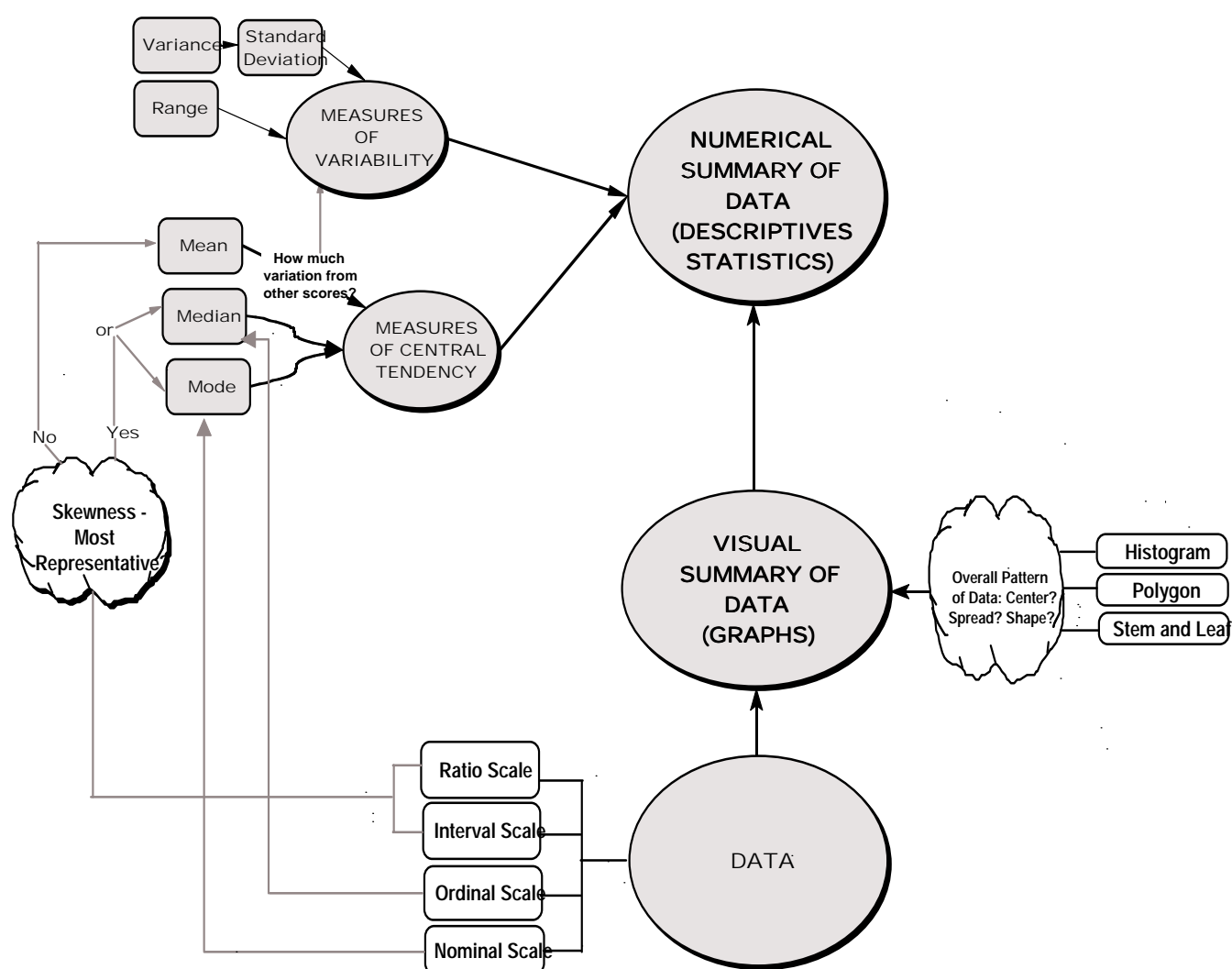
| Year | Films – Domestic | Gross Box Office Bucks |
|-------------|--------------------------------|-------------------------------|
| 1997 | Titanic | 600,743,440.00 |
| 1999 | The Phantom Menace | 429,870,576.00 |
| 1993 | Jurassic Park | 356,839,725.00 |
| 1994 | Forrest Gump | 329,690,974.00 |
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| 2000 | How the Grinch Stole Christmas | 260,031,035.00 |
| 1992 | Aladdin | 217,350,219.00 |
| 1998 | Saving Private Ryan | 216,119,000.00 |
| 1991 | Terminator 2: Judgment Day | 204,446,562.00 |
| 1995 | Toy Story | 191,702,825.00 |

Mean = \$311,284,731.4

Median = \$283,032,996.5

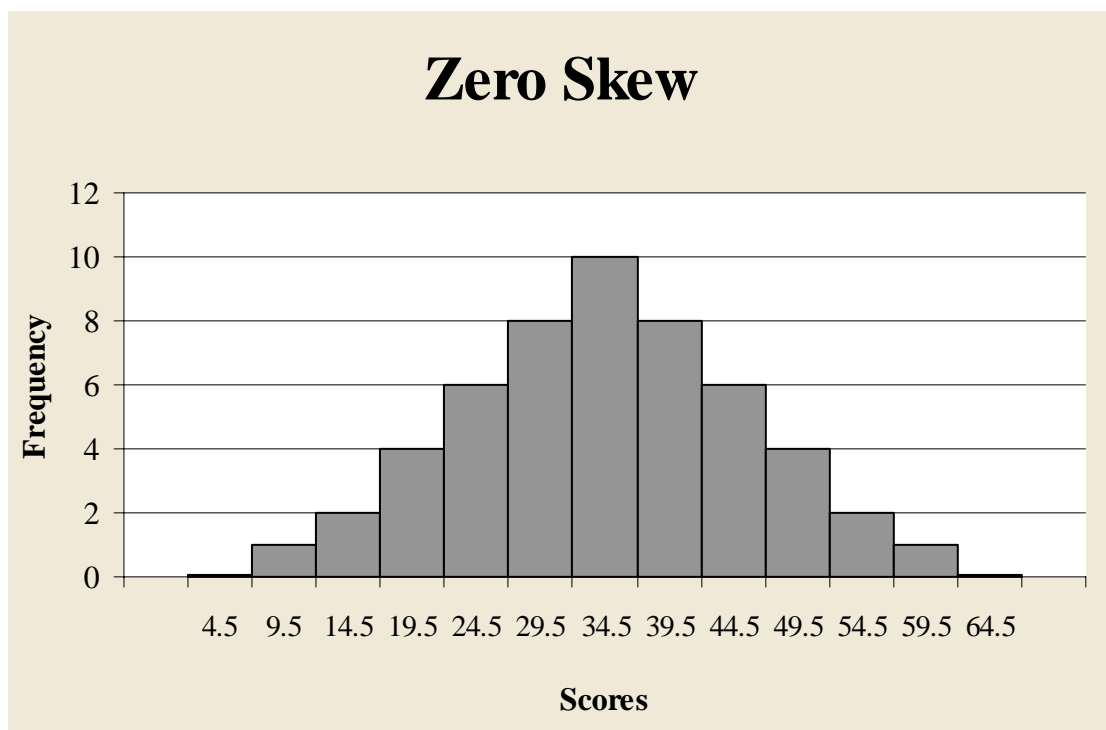
Mode = *n* modes

Concept Map



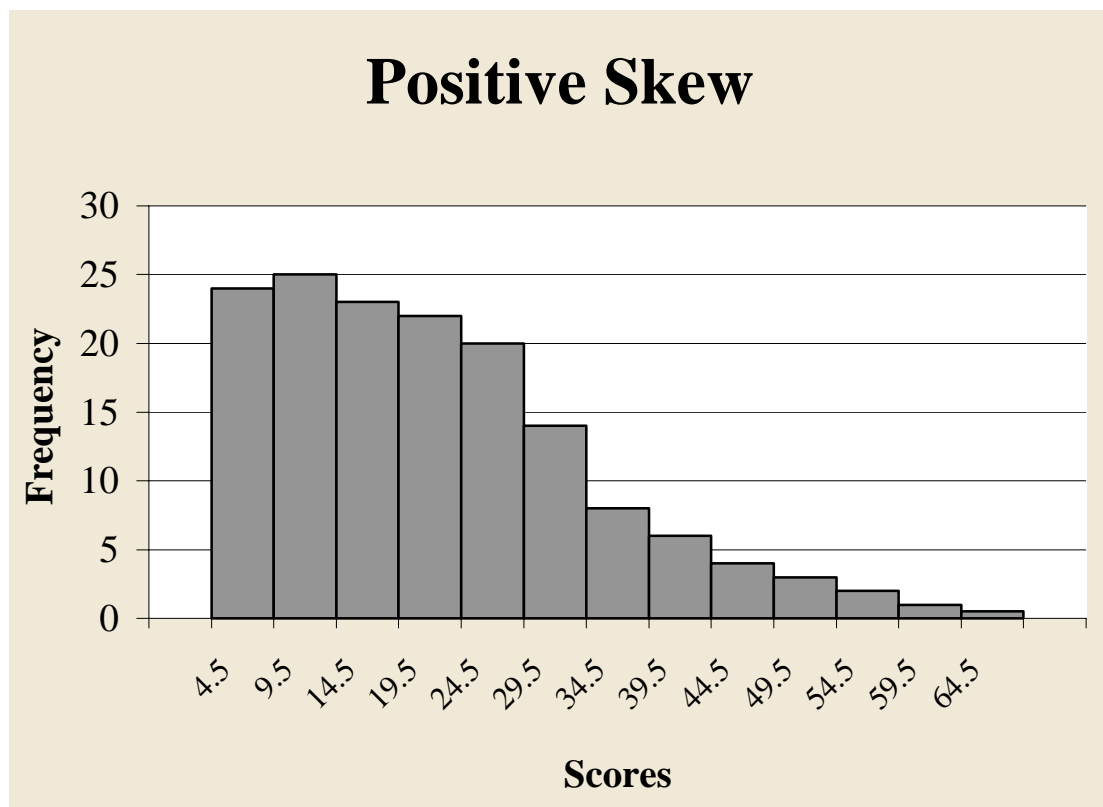
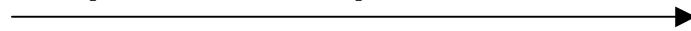
Transparency 4-12

Mean = Median = Mode



Transparency 4-13.

Tail points in the positive direction.



Transparency 4-14.

Tail points in the negative direction.

