Matching Supply with Demand: An Introduction to Operations Management

Solutions to End-of-Chapter Problems

(last revised March, 2012; make sure to visit www.cachon-terwiesch.net for the latest updates, excel files, ppt files and other information)

Chapter 8

Q8.2. My Law

(a) inter-arrival time: 10 emails per hour = 1 email / 6 min
a = 6 min, $CV_a = 1$
processing time: p = 5 min, CV_p = 4 min / 5 min = 0.8
waiting time = 5 min * [5/6 / (1-5/6)] * [$(1^2 + 0.8^2)/2$] = 20.5 min
total response time = waiting time + processing time = *25.5 min*

(b) emails per day = 10 emails per hour * 10 hours = *100 emails per day*

(c) idle time = (1- utilization) * 10 hours = *1.66 hours*

(d) The *average amount of time would not change*, since utilization is not dependent on the variance or standard deviation, but on the average processing and inter-arrival time.

(e) processing time: p = 5 min, CV_p = 0.5 min / 5 min = 0.1
waiting time = 5 min * [5/6 / (1-5/6)] * [$(1^2 + 0.1^2)/2$] = 12.63 min
total response time = waiting time + processing time = *17.63 min*

Q8.3. Car Rental Company

(a) We approach this problem as though the rental car is the "server". We know that a = 2.4 hours, p = 72 hours, CV_a = (2.4/2.4) = 1, CV_p = (24/72) = 0.33, and m = 50 cars. To determine the number of cars on the lot, we can look at the utilization rate of our "servers" = (1/a) / (m/p) = (1/2.4) / (50/72) = 60%. Therefore, on average 60% of the cars are in use or 30 cars, so on average *20 cars* are in the lot.

(b) We assume that the standard deviations DO NOT change. If the average demand is increased to 12 rentals per day, then a = 2 hours. If the average rental duration increases to 4 days, then p = 96 hours. These values raise the utilization rate to (1/2) / (50/96) = 96%. This means that 48 cars are rented on average. With the initial rate average revenue per day = $80*30 cars = $2400. With the proposed rate average revenue per day = $55*48 = $2640. *Therefore, the company should make the proposed changes*.

(c) Using the wait time formula, the average wait time = 0.019 hours or *1.15 minutes.*

(d) Using the wait time formula the average wait time is computed as 0.046 hours.

Q8.4. Tom Opim

(a) idle time = 1 min / 3 min * 8 hours = 160 min
pages read = 160 min * 1 page/min = *160 pages*

(b) inter-arrival time: 1 call / 3 min
$a = 3$ min, $CV_a = 3$ min / 3 min = 1
processing time: $p = 2$ min, $CV_p = 1$ min / 2 min = 0.5
waiting time = 2 min * [2/3 / (1-2/3)] * [(1^2 + 0.5^2)/2] = *2.5 min*

(c) The average total time a line is used per customer = average wait time + average processing time. In this case, the average total time per customer = 2.5 + 2 = 4.5 minutes per customer. There are an average of 20 customers per hour, so the average number of minutes per hour = 20*4.5 = 90 minutes. Thus, the total per hour charge = (90/60) * $5 = *$7.50 per hour or $60 for 8 hours*.

Another way to approach the same problem is to look at the average number of callers at any given time = average number of callers on hold at any given time (I_q) + average number of callers talking to Tom at any given time. We can calculate $I_q = R * T_q$ where the flow rate = 1 call / 3 minutes and T_q = 2.5 minutes. Thus, I_q = 0.83 calls. The average number of callers talking to Tom must be a number between 0 and 1, and is equal to Tom's utilization = 0.67. So, the average number of callers at any given time = 0.83 + 0.67 = 1.5 callers. The line charge for 8 hours = $5*8 = $40 per line. Therefore the total cost over an 8-hour shift = 1.5*40 = $60.

Q8.5. Atlantic Video

(a) inter-arrival time: 30 customers per hour = 1 customer / 2 min
$a = 2$ min, $CV_a = 1$
process time: $p = 1.7$ min, $CV_p = 3$ min / 1.7 min = 1.765
waiting time = 1.7 min * [(1.7/2)/(1-1.7/2)] * [(1^2 + 1.765^2)/2] = *19.82 min*

(b) utilization = 1.7 min / 2 min = 0.85
idle time = 0.15 * 8 hrs = 72 min / 1.5 min per sort = *48 sorts*

(c) Using R = minimum of 1/a and 1/p, we calculate R = 0.5. Thus, the average number of customers in line waiting I = R*Tq = 0.5*19.82 = 9.9 customers. In addition, an average of .85 customers (equal to the utilization u) are being served at any given time. So the average number of customers at the check-out desk = 9.9 + .85 = *10.75.*

(d) Because only 90% of customers go through checkout, the inter-arrival time of paying customers changes: 27 customers per hour = 1 customer / 2.22 min
waiting time = 1.7 min * [(1.7/2.22)/(1-1.7/2.22)] * [(1^2 + 1.765^2)/2] = *11.38 min*

(e) The average person waits 19.81 minutes, and 30 customers arrive in one hour, so there are approximately 19.82*30=594.6 minutes of wait time per hour. This costs the store .75*594.6 = $445.95 for wait time. If 2 servers are used, we can apply the formula, and using the same methodology, calculate a cost of .75*(0.88*30) = $19.79. Finally, if 3 servers are used, the cost is .75*(0.162*30) = $3.65. Adding any more employees would not be cost effective. The most cost effective number of employees is 3.

Q8.6. Rent-a-Phone

(a) To answer this question, we must first set up the problem where we consider the phones as "servers". Thus m = 80, a = 24 hours/25 customers = 0.96 hours average interarrival time, and p = 72 hours. So, utilization = p / (m*a) = 72/(80*.96) = 94%. This means that .94*80 phones = 75 phones are in use, and *5 phones are available on average*.

(b) We are given that $CV_a = 1$ and $CV_p = 100/72 = 1.39$. Using the wait time formula, we calculate an average wait time of 9.89 hours.

(c) We first need to calculate the average number of people in the queue. We know that the flow rate R = demand rate = 1/.96 = 1.04 customers/hour. Time in the queue Tq = 9.89 hours, so Iq = Tq * R = 9.89 * 1.04 = 10.31 people in the queue on average. So we can multiply $1 * 24 hours/day * 30 days * 10.31 people in the queue = $7419.87.

(d) We can repeat the same calculations for m=81 and obtain Iq=7.379. The new expenses would be 5312, thus it would pay to buy at least one additional phone.

(e) We repeat the waiting time calculations with Cp=0 and a=24/20; we now get a utilization of u=0.75 and a waiting time of Tq=0.06166 hours

Q8.7. Webflux

a. Demand interarrival time = a = 10, service time = p = 7. Utilization = (flow rate) / (capacity) = (1/a) / (1/p) = p/a = 0.7. Time in queue = (service time) * (utilization / (1-utilization)) * $((CV_a^2 + CV_p^2) / 2)$ = 7 * (0.7 / 0.3) * ((1+1) / 2) = 16.33 days. Adding the shipping time, the answer is 17.33 days.
b. Queue length = (time in queue) / (1/ (flow rate)) = 16.33 / a = 16.33 / 10 = 1.63.
c. Number of servers = m = 2. r = (implied utilization)*(number of servers) = p/a = 7/3 = 2.333. From the Erlang Loss Table, P_m = Prob{all 2 servers are busy = all 2 DVDs are rented out} = 0.4495.

Q8.8. Security Walking Escorts

a. We know that a =5, p = 25, and m=8. Utilization = (1/a)/(m/p) = (1/5)/(8/25) = .625. Therefore, 8*(1-.625) = **3 officers available.**

b. The average wait time = 1.8 minutes. The average time to walk to the destination = 25 minutes. Therefore, the total time between calling and arriving = 25+1.8 = **e. 26.8 minutes.**
c. Again, we know that a=3.125, m=8, and p=25. We also know that if the coefficient of variation = 1 then interarrivals follow an exponential distribution. We can also calculate r = p/a = 25/3.125 = 8. So we use the Erlang Loss table to calculate $P_8(8)$ = approximately 0.2356. Since 19.2 students/hr request an escort, 19.2 x 0.2356 = 4.5 students/hour find no escort available.
d. Again, using the Erlang Loss table we see that to have .20 or less, we need m=9. Therefore we need **9 officers**

Q8.9. Mango Electronics

a. We know that a = 7 months, p = 28 months, and m = 5 months. Therefore, utilization = (1/a) / (m/p) = (1/7) / (5/28) = .80 or **d. 80%.**
b. Average wait time = 40.4 months. The average development time = 28 months. Therefore the total time = 40.4+28 = **e. 68.4 months.**
c. The patent is given for 20 years. If the entire development process = 68.4 months, or 5.7 years, then there will be **a. 14.3 years left.**

Q8.10. UPS Shipping

d. Davis' utilization does not change – he still has the same amount of time available, and the same amount of time "demanded", which is (5*10)+(4*10)+(6*5) = 120 minutes of demand and 3 hours*60 minutes = 180 minutes of time available.
e. Under Davis' supervisor's recommendation, the average wait time would decrease.