Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 17
Supply Chain Coordination

(last revised March 1, 2012)

Q17.1

a) If orders are made every week, then the average order quantity equals one week's worth of demand, which is 25 cases. If at the end of the week there is one week's worth of inventory, then the average inventory is 25 / 2 + 25 = 37.5. (In this case inventory "saw-toothes" from a high of two week's worth of inventory down to one week, with an average of 1.5 weeks.) On average the inventory value is 37.5 x 9.25 = $346.9. The holding cost per year is 52 x 0.4% = 20.8%. Hence, the inventory holding cost with the first plan is 20.8% x $346.9 = $72. Purchase cost is 52 x 25 x $9.25 = $12,025. Total cost is $12,025 + $72 = $12,097.

b) Four orders are made each year, each order on average is for (52 / 4) x 25 = 325 units. Average inventory is then 325 / 2 + 25 = 187.5. The price paid per unit is $9.40 x 0.95 = $8.93. The value of that inventory is 187.5 x $8.93 = $1674. Annual holding costs are $1674 x 20.8% = $348. Purchase cost is 52 x 25 x $8.93 = $11,609. Total cost is $348 + $11,609 = $11,957.

c) P&G prefers our third plan as long as the price is higher than in the second plan, $8.93. But the retailer needs a low enough price so that its total cost with the third plan is not greater than in the second plan, $11,957 (from part b). In part a we determined that the annual holding cost with a weekly ordering plan is approximately $72. If we lower the price, the annual holding cost will be a bit lower, but $72 is a conservative approximation of the holding cost. So the retailer's purchase cost should not exceed $11,957 - $72 = $11,885. Total purchase quantity is 25 x 52 = 1300 units. So if the price is $11,885 / 1300 = $9.14, then the retailer will be slightly better off (relative to the second plan) and P&G is much better off (revenue of $12,012 instead of $11,885).

Q17.2

a) Use the newsvendor model. The overage cost is C_o = cost – salvage value = $20 - $28 /4 = $13. The underage cost is C_u = price - cost = $28 - $20 = $8. The critical ratio is 8 / (13 + 8) = 0.3810. Look up the critical ratio in the Standard Normal Distribution Function table to find the appropriate z statistic = -0.30. The optimal order quantity is $Q = \mu + z \times \sigma = 100 - 0.30 \times 42 = 87$.

b) Expected lost sales = $L(z) \times \sigma = 0.5668 \times 42 = 23.81$, where we find $L(z)$ from the Standard Normal Loss Function table and z = -0.30 (from part a). Expected sales = μ – Expected lost sales = 100 – 23.81 = 76.2. Expected left over inventory = Q –

Expected sales = 87 – 76.2 = 10.8. Profit = price x Expected sales + salvage value x Expected leftover inventory – Q x cost = \$28 x 76.2 + \$7 x 10.8 – 87 x \$20 = \$469.

c) The publisher's profit = Q x (wholesale price – cost) = 87 x (\$20 - \$7.5) = \$1087.5.

d) The underage cost remains the same because a lost sale still costs Dan the gross margin, C_u = \$8. However, the overage cost has changed because Dan can now return books to the publisher. He buys each book for \$20 and then returns leftover books for a net salvage value of \$15 - \$1 (due to the shipping cost) = \$14. So his overage cost is now C_o = cost – salvage value = \$20 - \$14 = \$6. The critical ratio is 8 / (6 + 8) = 0.5714. Look up the critical ratio in the Standard Normal Distribution Function table to find the appropriate z statistic = 0.18. The optimal order quantity is $Q = \mu + z \times \sigma = 100 + 0.18 \times 42 = 108$.

e) Expected lost sales = $L(z) \times \sigma = 0.3154 \times 42 = 13.2$, where we find $L(z)$ from the Standard Normal Loss Function table and z = 0.18 (from part d). Expected sales = μ – Expected lost sales = 100 – 13.2 = 86.8. Expected left over inventory = Q – Expected sales = 108 – 86.8 = 21.2. Profit = price x Expected sales + salvage value x Expected leftover inventory – Q x cost = \$28 x 86.8 + \$14 x 21.2 – 108 x \$20 = \$567.

f) The publisher's sales revenue is \$20 x 108 = \$2160. Production cost is \$7.5 x 108 = \$810. The publisher pays Dan \$15 x 21.2 = \$318. The publisher's total salvages revenue on returned books is \$6 x 21.2 = \$127.2. Profit is then \$2160 - \$810 -\$318 + \$127.2 = \$1159. Note that both the publisher and Dan are better off with this buy back arrangement.

g) Equation 17.1 in the text gives the buyback price that coordinates the supply chain (that is, maximizes the supply chain's profit). That buyback price is \$1 + \$28 – (\$28 - \$20) x (\$28 - \$6)/(\$28 - \$7.5) = \$20.41. Note, the publisher's buyback price is actually higher than the wholesale price because the publisher needs to subsidize Dan's shipping cost to return books: Dan's net loss on each book returned is \$20 – (20.41 – 1) = \$0.59.

Q17.3

a) Evaluate the z-statistic for Q = 30000: z = (30000 – 24000) / 8000 = 0.75. From the Standard Normal Loss Function Table, $L(0.75)$ = 0.1312. Hence, the amount demand exceeds 30000 is 0.1312 x 8000 = 1049.6. The number of options exercised is the same as expected sales (if demand is less than Q, then sales/options exercised equals demand, but if demand is greater than Q, then sales/options exercised equals Q). Expected sales = expected demand – expected lost sales = 24000 – 1049.6 = 22950.4.

b) The expected number of displays purchased on the spot market equals the amount demand exceeds the number of options purchased, Q = 30000. From part a), the expected number of spot market purchases is 1049.6.

c) Handi's procurement cost includes the cost of buying 30000 options (30000 x \$4.5 = \$135,000) plus the cost of exercising options (22950.4 x \$3.5 = \$80,326.4) plus the cost of purchases on the spot market (1049.6 x \$9 = 9446.4), for a grand total of \$224,773.

d) Use the newsvendor model to determine the optimal number of options to purchase. If Handi buys one option too few, then Handi must make a spot market purchase of \$9. If Handi had purchased an additional option for \$4.5, then that option would have been exercised at a cost of \$3.5, for a total cost for the unit of 4.5 + 3.5 = 8. So

Handi's cost of under purchasing options is $C_u = 9 - 8 = 1$. If Handi purchases an option that is not used (over buys), the cost to Handi is $C_o = 4.5$, because there is no residual value to the option. The critical ratio is 1 / (4.5 + 1) = 0.1818. Look up in the Standard Normal Distribution Function Table and find that $\Phi(-0.91) = 0.1814$ and $\Phi(-0.90) = 0.1841$, so choose z = -0.90. The optimal number of options is then Q = 24000 – 0.90 x 8000 = 16,800.

e) Following the processes in parts a to c, the expected number of spot market purchases is 8003 = 8000 x $L(-0.91)$ = 8000 x 1.0004. Expected sales (number of options exercised) is 24000 – 8003 = 15997. The total procurement cost is then \$4.5 x 16,800 + \$3.5 x 15997 + \$9 x 8003 = \$203,618.

Q17.4

a) Cusano's discount price is (1-0.75)x 250 = 62.5. Cusano's overage cost is $C_o = 185 - 62.5 = 122.5$. Cusano's underage cost is $C_u = 250 - 185 = 65$. The critical ratio is 65 / (122.5 + 65) = 0.34667. From the Poisson Distribution Function Table $F(6)$ = 0.23051 and $F(7) = 0.35398$, so the optimal order quantity is $Q = 7$.

b) Expected lost sales, from the Poisson Loss Function Table, is $L(7) = 2.21$. Expected sales is 8.75 – 2.21 = 6.54. Expected left over inventory is 7 – 6.54 = 0.46. Expected profit is then 6.54 x 250 + 0.46 x 62.5 – 7 x 185 = 369.

c) Profit is 7 x (185 – 100) = 595.

d) The supply chain's overage cost is $C_o = 100 - 62.5 = 37.5$. The supply chain's underage cost is $C_u = 250 - 100 = 150$. The critical ratio is 150 / (37.5 + 150) = 0.8000 From the Poisson Distribution Function Table $F(10) = 0.73519$ and $F(11)$ = 0.82657, so the optimal order quantity is $Q = 11$.

e) The supply chain's underage cost remains the same, $C_u = 250 - 100 = 150$. The supply chain's overage cost is the consequence of having one grill left at the end of the season. If that grill had not been ordered, then the \$15 shipping cost could have been avoided. (One might argue that damaged grills should not be sent back to SJ, but let's assume that all grills are sent back to SJ and SJ determines whether they are damaged or not.) Fortunately, that grill may be sold next year, but if it is not sold because it is damaged, then an additional cost is incurred. There is a 45% chance it is not sold, so the expected loss on a left over grill is 100 x 0.45 = \$45: the cost of the grill times the chance the grill is disposed for no value. Hence, the total overage cost is 15 + 45 = \$60. But if left over grills were disposed at Cusano's, then the overage cost would be \$100 – \$62.5 = \$37.5. Hence, grills should NOT be sent back to SJ, they should be disposed of at Cusano's because the supply chain loses only 37.5 per grill salvaged at Cusano's while it loses 62.5 per grill sent back to SJ. Hence, the analysis is exactly as in part *d*, send 11 grills to Cusano's.

f) The buy back price is 0.9 x 185 = 166.5. Cusano's underage remains $C_u = 250 - 185 = 65$. Cusano's overage cost is 185 – 166.5 + 15 = 33.5. The critical ratio is 65 / (33.5 + 65) = 0.65990. From the Poisson Distribution Function Table $F(9) = 0.6203$ and $F(10) = 0.7352$, so the optimal order quantity is $Q = 10$.

g) Expected lost sales is $L(10) = 0.67$. Expected sales = 8.75 – 0.67 = 8.08 and expected left over inventory is 10 – 8.08 = 1.92. Expected profit is then \$250 x 8.08 + (\$166.5 – 15) x 1.92 – 10 x 185 = \$461.

h) From part *g* we know that Cusano will return 1.92 grills on average. SJ must pay Cusano 166.5 per grill returned. SJ earns the full wholesale price on the grills that are not returned and 55% of the returned grills. So the full wholesale price is earned on 10 – 1.92 = 8.08 grills that are not returned and 0.55 x 1.92 = 1.056 grills that are returned in good condition, for a total of 8.08 + 1.056 = 9.136 grills. Profit is then \$185 x 9.136 - \$100 x 10 - \$166.5 x 1.92 = 370.48.

i) In part *e* we determined that returning grills to SJ is not optimal for the supply chain. But we can now evaluate what the supply chain's optimal decision would be assuming grills are returned to SJ. In part *e* we determined that the underage cost is C_u = 250 – 100 = 150 and the overage cost is C_o = 15 + 45 = \$60. So the critical ratio is 150 / (60 + 150) = 0.7143. From the Poisson Distribution Function Table $F(9)$ = 0.6203 and $F(10)$ = 0.7352, so the optimal order quantity is Q = 10. Equation 16.1 in the text gives us the optimal buy-back price. But to use the equation we need to know the supply chain's salvage value. If each grill costs \$100 to make and C_o = \$60, then the salvage value is \$100 – \$60 = \$40. So, from equation 16.1, the buy-back price we seek is 15 + 250 – (250 – 185)x (250 – 40) / (250 – 100) = 174. A buy-back of \$174 implies a return credit of 174/185 = 94%.

j) If Cusano does not pre-book enough, then Cusano must purchase a grill at the regular price of \$185 instead of the discount pre-book price = 0.9 x \$185 = \$166.5. Hence, Cusano's underage cost is C_u = 10% x \$185 = \$18.5. If Cusano pre-books too many grills, then the left-over grills are salvaged for \$62.5 each (see part *a*), so C_o = 166.5 – 62.5 = 104. The critical ratio is 18.5 / (104 + 18.5) = 0.15102. From the Poisson Distribution Function Table, $F(5)$ = 0.1317 and $F(6)$ = 0.23051, so the optimal pre-book quantity is 6 grills.

k) With Q = 6, expected lost sales is $L(6)$ = 2.98, expected sales is 8.75 – 2.98 = 5.77 and expected left over inventory is 6 – 5.77 = 0.23. The lost sales are not actually lost: if demand exceeds the pre-book quantity of 6 units, then Cusano purchases the needed extra grills at the regular wholesale price of \$185 and sells them for \$250. So expected profit on non pre-book grills is 2.98 x (\$250 – \$185) = \$193.7. Expected profit on pre-book grills is \$250 x 5.77 + \$62.5 x 0.23 - \$166.5 x 6 = \$457.9. Total profit is 193.7 + 457.9 = \$652.

l) SJ's profit on non-prebook grills is 2.98 x (185 – 100) = \$253.3. SJ's profit on pre-book grills is 6 x (166.5 – 100) = \$399. Total profit is then \$652.

m) The advanced purchase contract has a higher profit for all firms. However, the advanced purchase contract requires SJ to hold a sufficient stock of grills to satisfy Cusano's requests (and other retailers' requests) during the season. If that is too costly (e.g., due to expedited shipping, or additional warehouse space, or left over inventory at SJ, etc), then maybe the advanced purchase contract is not as attractive as this analysis suggests.