Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 16
Revenue Management with Capacity Controls

(last revised March 2012)

Q16.1

a) The booking limit is capacity minus the protection level, which is 150 – 50 = 100, i.e., allow up to 100 bookings at the low fare.
b) The underage cost is C_u=200 – 120 = 80 and the overage cost is C_o = 120. The critical ratio is 80 / (120 + 80) = 0.4. From the *Std Normal Dist Function Table* we see Φ(-0.26) = 0.3974 and Φ(-0.25) = 0.4013, so choose *z* = -0.25. Evaluate *Q* = 70 – 0.25 × 29 = 63.
c) Decreases. The lower price for business travelers leads to a lower critical ratio and hence to lower protection level, i.e., it is less valuable to protect rooms for the full fare.
d) The number of unfilled rooms with a protection level of 61 is the same as expected left over inventory. Evaluate the critical ratio, $z = (61-70)/29 = -0.31$. From the *Std Normal Loss Function Table*, $L(z) = 0.5730.$ Expected lost sales is 29 × 0.5730 = 16.62 and expected left over inventory is 61 – 70 + 16.62 = 7.62. So we can expect 7.62 rooms to remain empty.
e) 70 × \$200 + (150 – 70) × \$120 = \$23,600, because on average 70 rooms are sold at the high fare and 150 – 70 = 80 are sold at the low fare.
f) 150 × \$120 = \$18,000.
g) If 50 are protected we need to determine the number of rooms that are sold at the high fare. The *z* statistic is (50 – 70) / 29 = -0.69. Expected lost sales is 29 × *L*(-0.69) = 24.22. Expected sales is 70 – 24.22 = 45.78. Revenue is then (150 – 50) × \$120 + 45.78 × \$200 = \$21,155.

Q16.2

a) The underage cost is \$120, the discount fare. The overage cost is \$325. The critical ratio is 120 / (325 + 120) = 0.2697. From the table *F*(12) = 0.2283 and *F*(13) = 0.3171, so the optimal overbook quantity is 13.
b) A reservation cannot be honored if there are 9 or fewer no-shows. *F*(9) = 0.0552, so there is a 5.5% chance the hotel will be overbooked.
c) It is fully occupied if there are 15 or fewer no-shows, which has probability *F*(15) = 0.5170.
d) Bumped customers equals 20 – number of no-shows, so it is equivalent to left over inventory. Lost sales is *L*(20) = 0.28, expected sales is 15.5 – 0.28 = 15.22 and

expected left over inventory/bumped customers = 20 – 15.22 = 4.78. Each one costs \$325, so the total cost is \$325 × 4.78 = \$1554.

Q16.3

a) First evaluate the distribution function from the density function provided in the table: $F(8)=0$, $F(9) = F(9) + 0.05 = 0.05$, $F(10) = F(9) + 0.10 = 0.15$, etc. FLet Q denote the number of slots to be protected for sale later and let D be the demand for slots at \$10,000 each. If $D > Q$, we reserved too few slots and the underage penalty is C_u = \$10,000 - \$4,000 = \$6,000. If $D < Q$, we reserved too many slots and overage penalty is C_o = \$4,000. The critical ratio is 6000 / (4000 + 6000) = 0.6. From the table we find F(13) = 0.6, so the optimal protection quantity is 13. Therefore, WAMB should sell 25-13 = 12 slots in advance.

b) The underage penalty remains the same. The overage penalty is now C_o = \$4000 - \$2500 = \$1500: setting the protection level too high before meant lost revenue on the slot, but not at least \$2500 can be gained from the slot, so the loss is only \$1500. The critical ratio is 6000 / (1500 + 6000) = 0.8. From the table, $F(15) = 0.8$, so protect 15 slots and sell 25 – 15 = 10 in advance.

c) If the booking limit is 10, there are 15 slots for last-minute sales. There will be standby messages if there are 14 or fewer last-minute sales, which has probability $F(14) = 0.70$.

d) Over overbooking means the company is hit with a \$10000 penalty, so C_o = 10000. Under overbooking means slots that could have sold for \$4000 are actually sold at the standby price of \$2500, so C_u =4000 – 2500 = 1500. The critical ratio is 1500 / (10000 + 1500) = 0.1304. From the *Poisson Distribution Function Table* with mean 9.0, $F(5) = 0.1157$ and $F(6) = 0.2068$, so the optimal overbooking quantity is 6, i.e., sell up to 31 slots.

e) The overage cost remains the same: we incur a penalty of \$10000 for each bumped customer (and we refund the \$1000 deposit of that customer too). The underage cost also remains the same. To explain, suppose they overbooked by 2 slots but there are 3 withdrawals. Because they have one empty slot, they sell it for \$2,500. Had they overbooked by one more (3 slots), then they would have collected \$4,000 on that last slot instead of the \$2,500, so the difference is C_u = \$4,000-\$2,500 = \$1500. Note, the non-refundable amount of \$1,000 is collected from the 3 withdrawals in either scenario, so it doesn't figure into the change in profit by overbooking one more unit. The critical ratio is 1500 / (10000 + 1500) = 0.1304. From the *Poisson Distribution Function Table* with mean 4.5, $F(1) = 0.0611$ and $F(2) = 0.17358$, so the optimal overbooking quantity is 2, i.e., sell up to 27 slots.

Q16.4

a) The z-statistic is (100 – 70) / 40 = 0.75. Expected lost sales is 40 × L(z) = 40 × 0.1312 = 5.248. Expected sales is 70 – 5.248 = 64.752. Expected left over inventory is 100 – 64.752 = 35.248.

b) Expected revenue is \$10000 × 64.752 = \$647,520.

c) The underage cost is \$10000 - \$6000 = \$4000, because under protecting boutique sales means a loss of \$4000 in revenue. Over protecting means a loss of \$6000 in revenue. The critical ratio is 4000 / (6000 + 4000) = 0.4. From the *Std Normal Dist Function Table* we see Φ(-0.26) = 0.3974 and Φ(-0.25) = 0.4013, so choose z = -0.25. Evaluate Q = 40 – 0.25 × 25 = 33.75. So protect 34 dresses for sales at the boutique, which means sell 100 – 34 = 66 dresses at the show.

d) If 34 dresses are sent to the boutique then expected lost sales is $\sigma \times L(z)$ = 25 × L(-0.25) = 25 × 0.5363 = 13.41. Expected sales is 40 – 13.41 = 26.59. So revenue is 26.59 × \$10000 + (100 – 34) × 6000 = \$661,900.

e) From part d, expected sales is 26.59, so expected left over inventory is 34 – 26.59 = 7.41 dresses.

Q16.5

a) The overage cost is \$800 (over overbooking means a bumped passenger, which costs \$800). The underage cost is \$475 (an empty seat). The critical ratio is 475 / (800 + 475) = 0.3725. . From the *Std Normal Dist Function Table* we see Φ(-0.33) = 0.3707 and Φ(-0.32) = 0.3745, so choose z = -0.32. Evaluate Y = 30 – 0.32 × 15 = 25.2. So the max number of reservations to accept is 200 + 25 = 225.

b) 220 – 200 = 20 seats are overbooked. The number of bumped passengers equals 20 minus the number of no-shows, which is equivalent to left over inventory with an order quantity of 20. The z-statistic is (20 – 30)/15 = –0.67. L(-0.67) = 0.8203, so lost sales is 15 × 0.8203 = 12.3. Sales is 30 – 12.3 = 17.7 and expected left over inventory is 20 – 17.7 = 2.3. If 2.3 customers are bumped, then the payout is \$800 × 2.3 =\$1840.

c) You will have bumped passengers if there are 19 or fewer no-shows. The z-statistic is (19 – 30)/15 = –0.73. $\Phi(-0.73) = 0.2317$, so there is about a 23% chance there will be bumped passengers.

Q16.6

a) The underage cost is \$675 - \$375 = \$300 and the overage cost is \$375. The critical ratio is 300 / (375 + 300) = 0.4444. All reservations are sold if the high fare demand exceeds the protection level, which has probability 1 – 0.4444 = 0.5556.

b) With Q = 85 the z statistic is (85 – 80) / 35 = 0.14 and L(0.14) = 0.3328, so expected lost sales is 35 × 0.3328 = 11.65

c) Empty seats equals 85 minus high fare demand, which is equivalent to expected left over inventory. Expected sales is 80 – 11.65 = 68.35 and expected left over inventory is 85 – 68.35 = 16.65.

d) Revenue is (200 – 85) × \$375 + 68.35 × \$675 = \$89,261.

Q16.7

a) The high-fare protection level = Capacity – Low-fare booking limit = 100 – 50 = 50. Next, z = (Q – μ)/σ = (50 – 40)/30 = 10/30 = 0.333. L(0.333) = 0.2555. Expected lost sales = σ × L(z) = 30 × 0.2555 = 7.665. Expect sales (high fare) = μ - Expected lost sales = 40 - 7.665 = 32.335.

b) Critical Ratio = C_u/(C_u + C_o) = ($r_h - r_l$) / r_h = (80 - 0.5 × 80) / 80 = 0.5. F(0) = 0.5, i.e. z = 0. $Q = \mu + z \times \sigma = \mu$ = 60.

c) An increase in the discount leads to lower student ticket prices which increases the critical ratio. Critical Ratio = C_u/(C_u + C_o) = ($r_h - r_l$) / r_h = (80 - 0.45 × 80) / 80 = 0.55. Thus, the optimal protection level, Q will increase. This means fewer seats available for students, demand from whom is abundant and more seats that could potentially go empty if not enough full-price demand materialize. Thus, the expected number of empty seats will increase.

d) C_o = 10 * C_u. (Overbooking) Critical Ratio = C_u / (C_u + C_o) = C_u / (10 C_u + C_u) = 1/11 = 0.091. Look in the *Poisson Distribution Function Table* with a mean of 8: F(3) = 0.04238, F(4) = 0.099963, so use the round-up rule and overbook by 4 seats.

Q16.8

a) The overage cost is the cost of protecting too many rooms, which means a room goes empty that could have been sold at the discount. The incremental profit from selling that room is \$159 – \$45 = \$114 = C_o. The underage cost remains \$225 - \$159 = C_u=\$66. The critical ratio is 66 / (114 + 66) = 0.3667. From Table 16.2 we find F(24) = 0.3040 and F(25) = 0.3760, so it is optimal to protect 25 rooms.

Q16.9

a) The class size will be at least 720 if there are 30 or fewer students who decline the offer. The corresponding z statistic is (30 – 50) / 21 = -0.95 and Φ(-0.95) = 0.1711. Hence about a 17% probability the class size will be at least 720.

b) The cost of admitting too many (the overage cost) is twice the cost of under admitting, so the critical ratio is $C_u/(C_o+C_u)=C_u/(2\times C_u+C_u)$=0.3333. From the *Std Norm Distribution Function Table*, Φ(-0.44) = 0.3300 and Φ(-0.43) = 0.3336, so choose z = -0.43. The overbooking quantity is then Y = 50 – 0.43 × 21 = 40.97. Admit 720 + 41 = 761 students.

c) The cost of admitting too many (the overage cost) is five times the cost of under admitting, so the critical ratio is $C_u / (C_o+C_u) = C_u / (5 \times C_u + C_u)$ = 0.16667. From the *Std Norm Distribution Function Table*, Φ(-0.97) = 0.1660 and Φ(-0.96) = 0.1685, so choose z = -0.96. The overbooking quantity is then Y = 50 – 0.96 × 21 = 29.84. Admit 720 + 30 = 750 students.

Q16.10

a) With 58 units the z statistic is (58 – 65) / 45 = -0.16. L(-0.16) = 0.4840, so expected lost sales is 45 × 0.4840 = 21.78. Expected sales is 65 – 21.78 = 43.33. Profit is (\$2100 - \$330) × 43.33 = \$76,499.

b) With long term contracts the company sells 58 units at \$1875 each for a total profit of 58 × (\$1875 -\$330) = \$89,610.

c) The overage cost is \$1875 (over protect and lose the opportunity to collect \$1875 in revenue) while the underage cost is \$2100 - \$1875 = \$225. The critical ratio is 225 / (1875 + 225) = 0.1071. From the *Std Norm Distribution Function Table*, Φ(-1.25) =

0.1056 and Φ(-1.24) = 0.1075, so choose z = -1.24. The optimal protection level is 65 – 1.24 × 45 = 9.2. Hence, the optimal booking limit for long term contracts is 58 – 9.2 = 48.8.

d) Now the overage cost is \$1875 - \$330 = \$1545 because the incremental profit on a sale is only \$1545. The underage cost remains \$2100 - \$1875 = \$225. The critical ratio is 225 / (1545 + 225) = 0.1271. From the *Std Norm Distribution Function Table*, Φ(-1.15) = 0.1251 and Φ(-1.14) = 0.1271, so choose *z* = -1.14. The optimal protection level is 65 – 1.14 × 45 = 13.7. Hence, the optimal booking limit for long term contracts is 58 – 13.7 = 44.3.