Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 15
Risk-Pooling Strategies to Reduce and Hedge Uncertainty

(last revised February 2012)

Q15.1

a) New standard deviation is $30x\sqrt{50} = 212$.
b) Pipeline inventory = expected demand per week × lead time = 200×50×10 = 100,000.

Q15.2

The coefficient of total demand (pooled demand) is the coefficient of the product's demand times the square root of (1+Correlation)/2. Therefore, sqrt((1-0.7)/2) × 0.6 = 0.23

Q15.3

a) Assume Fancy Paints implements the order up-to inventory model. Find the appropriate order up-to level. With a lead time of 4 weeks, the relevant demand is demand over 4+1 = 5 weeks, which is 5 x 1.25 = 6.25. From the *Poisson Distribution Function Table*, $F(10) = 0.946$ and $F(11) = 0.974$, a base stock level $S = 11$ is needed to achieve at least a 95% in-stock. On-hand inventory at the end of the week is S - 6.25 – Expected Backorder. From the *Poisson Distribution Function Loss Function Table*, the Expected Backorder is $L(11)$ =0.04673. Thus, on-hand inventory for one SKU is 11 - 6.25 + 0.04673 = 4.8 units. There are 200 SKUs, so total inventory is 200 x 4.8 = 960.
b) The standard deviation over (4 + 1) weeks is $\sigma = \sqrt{5}$ × 8 = 17.89 and $\mu = 5$ x 50 = 250. From the *Std Norm Distribution Function Table*, we see that Φ(1.64) = 0.9495 and Φ(1.65) = 0.9505, so we choose z = 1.65 to achieve the 95% in-stock probability. The base stock level is then $S = \mu + z \text{ x } \sigma$ = 250 + 1.65 x 17.89 = 279.5. From the *Std Normal Loss Function Table*, $L(1.65)$ = 0.0206. So, on-hand inventory for one product is S – 250 + Expected Backorder = 279.5 – 250 + 17.89 x 0.0206 = 29.9. There are 5 basic SKUs, so total inventory in the store is 29.9 x 5 = 149.5.
c) The original inventory investment is 960 × \$14 = \$13,440, which incurs holding cost \$13,440 × 0.20 = \$2,688. Repeat part b but now the target in-stock probability is 98%. From the *Std Norm Distribution Function Table*, we see that Φ(2.05) = 0.9798 and Φ(2.06) = 0.9803, so we choose z = 2.06 to achieve the 98% in-stock probability. The base stock level is then $S = \mu + z \text{ x } \sigma$ = 250 + 2.06 x 17.89 = 286.9. From the *Std Normal Loss Function Table*, $L(2.06)$ = 0.0072. So, on-hand inventory for one product is S – 250 + Expected Backorder = 286.9 – 250 + 17.89 x 0.0072 = 37.0.

There are 5 basic SKUs, so total inventory in the store is 37.0 x 5 = 185. With the mixing machine the total inventory investment is 185 × \$14 = \$2590. Holding cost is \$2590 × 0.2 = \$518, which is only 19% (518 / 2688) of the original inventory holding cost.

Q15.4

a) Use the newsvendor model to decide an order quantity. From the table we see that $F(3500) = 0.8480$ and $F(4000) = 0.8911$, so order 4000 for each store.
b) Evaluate expected lost sales and the expected left over inventory. Expected lost sales comes from the table, $L(4000) = 185.3$. Expected sales is $\mu - 185.3 = 2251 - 185.3 =$ 2065.7. Expected left over is Q minus expected sales, 4000 – 2065.7 = 1934.3. Across 200 stores there will be 200 × 1934.3 = 386,860 units left over.
c) The mean is 450200. The coefficient of variation of individual stores is 0.7108 = 1600 / 2251. The coefficient of variation of total demand, we are told, is ½ of that, 0.3554 = 0.7108 / 2. Hence, the standard deviation of total demand is 160001 = 450200 × 0.3554. From the *Std Norm Dist Function Table* we see $\Phi(1.03) = 0.8485$ and $\Phi(1.04) = 0.8508$, so choose z = 1.04. Covert to Q = 450200 + 1.04 × 160001 = 616601.
d) Expected lost sales = 160001 × $L(z)$ = 160001 × 0.0772 = 12,352. Expected sales = 450200 – 12,352 = 437,848. Expected left over = 616601 – 437848 = 178,753, which is only 46% of what would be left over if individual stores held their own inventory.
e) The total order quantity is 4000 × 200 = 800000. With a mean of 450200 and standard deviation of 160001 (from part c), the corresponding z is (800000 – 450200) / 160001 = 2.19. From the *Std Norm Dist Function Table* we see $\Phi(2.19) = 0.9857$, so the in-stock would be 98.57% instead of 89.11% if the inventory were held centrally.

Q15.5

a) With a lead time of 3 weeks, $\mu = (3 + 1) \times 5200 = 20800$ and $\sigma = \sqrt{3+1} \times 3800 =$ 7600. The target expected backorders is (5200 / 7600) × (1 – 0.999) = 0.0007. From the *Std Norm Distribution Function Table*, we see that Φ(3.10) = 0.9990, so we choose z = 3.10 to achieve the 99.9% in-stock probability. Convert to S = 20800 + 3.10 × 7600 = 44,360. Expected backorder is 7600 × 0.0003 = 2.28. Expected on-hand inventory for each product is 44,360 – 20800 + 2.28 = 23,562. The total inventory for the two is 2 x 23,562 = 47,124.
b) Weekly demand for the two products is 5200 × 2 = 10400. The standard deviation of the two products is $\sqrt{2 \times (1 - \mathit{Correlation})} \times \mathit{Standard\ deviation\ of\ one\ product}$ = $\sqrt{2 \times (1 - 0.20)} \times 3800 = 4806.66$. Lead time plus one expected demand is 10400 × 4 = 41600. Standard deviation over $(l+1)$ periods is $\sqrt{(3+1)} \times 4806.66 = 9613$. Now repeat the process in part a with the new demand parameters. Convert to S = 41,600 + 3.10 × 9,613 = 71,401. Expected backorder is 9613 × 0.0003 = 2.88. Expected on-hand inventory is 71,401 – 41,600 + 2.88 = 29,804. The inventory investment is reduced by (47,124 – 29,804) / 47,124 = 37%.

Q15.6

a) Demand over $(l+1)$ periods has mean $\mu = 1.25 \times 13 = 16.25$. Use Excel to construct the Poisson Distribution Function Table for a Poisson with mean 16.25. We find that $F(26) = 0.9910$ and $F(27) = 0.9949$, so the appropriate order up-to level is $S = 27$. From the loss function table we find $L(27) = 0.01056$. Expected on-hand is 27 – 16.25 + 0.01056 = 10.76. Annual turns is 52 × 1.25 / 10.76 = 6.04

b) Repeat the process in part a, but now the lead time is 1 week. Demand over $(l+1)$ periods has mean $\mu = 1.25 \times 2 = 2.5$. From the *Poisson Distribution Function Table* in the Appendix we find that $F(6) = 0.9858$ and $F(7) = 0.9927$, so the appropriate order up-to level is $S = 7$. From the loss function table we find $L(7) = 0.00574$. Expected on-hand is 7 – 2.5 + 0.00574 = 4.51. Annual turns is 52 × 1.25 / 4.51 = 14.4.

Q15.7

a) Demand over $(l+1)$ periods has mean $\mu = 3 \times 100 = 300$ and standard deviation $\sigma = \sqrt{3} \times 65 = 112.58$. From the *Std Norm Dist Function Table* we see $\Phi(1.88) = 0.9699$ and $\Phi(1.89) = 0.9706$, so choose $z = 1.89$. Convert to $S = 300 + 1.89 \times 112.58 = 513$. Expected backorder is $\sigma \times L(z)$, which is 112.58 × 0.0113 = 1.27. Expected on-hand inventory for each desk is 513 – 300 + 1.27 = 214. Total inventory for the two desks is 2 x 214 = 428.

b) Expected on-hand inventory is 214 units because the demand distribution, lead time and the target in-stock is the same.

c) Zero. The order up-to level of the gray bases has to be the sum of the order up-to levels of the tops. If it were lower, then there is a chance that demand is less than the order up-to level of each top but more than the order up-to level of the gray base, in which case the in-stock probability would fall below 97%. Hence, to ensure that the in-stock probability remains 97%, there must be one gray base for each top. If the in-stock probability on the tops were raised, then the number of gray bases could be reduced and still achieve a 97% in-stock probability.

Q 14.8

a) Use the newsvendor model. If the order quantity is 5 and mean demand is Poisson with mean 0.9, then expected lost sales from the table is $L(5) = 0.00039$. Expected sales is 0.9 – 0.00039 = 0.89961 and expected left over inventory is 5 – 0.89961 = 4.10. Production cost is \$269 × 5 = \$1345. Revenue from sales is \$350 × 0.89961 = \$314.86 and revenue from salvaging is \$100 × 4.10 = \$410. Profit is \$314.86 + \$410 - \$1345 = -620.

b) Expected demand is reduced by 12.5% to 0.875 × 2.0 = 1.75 units. Repeat the process in part a with the new demand parameter. Expected lost sales from the table is $L(5) = 0.01191$. Expected sales is 1.75 – 0.01191 = 1.738 and expected left over inventory is 5 – 1.738 = 3.26. Production cost is \$269 × 5 = \$1345. Revenue from sales is \$350 × 1.738 = \$608.33 and revenue from salvaging is \$100 × 3.26 = \$326.

Profit is \$608.33 + \$326 - \$1345 = -411. Hence, this product is not profitable even with only one color.

Q15.9

Option a) provides the longest chain, covering all four areas. This gives the maximum flexibility value to the firm, so that should be the chosen configuration. To see that it forms a long chain, Alice can do Regulations, as well as Bob. Bob can do Taxes, as well as Doug. Doug can do Strategy, as well as Cathy. Cathy can do Quota, as well as Alice. Hence, there is a single chain among all four consultants. The other options do not form a single chain.