Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 14
Service Levels and Lead Times in Supply Chains: The Order-up-to Inventory Model

(last revised March 2012)

Q14.1

a) Inventory position = inventory level + on-order = 100 + 85 = 185. Order enough to raise the inventory position to the order upto level, in this case 220 – 185 = 35 desks.
b) As in part a, inventory position = 160 + 65 = 225. Because the inventory position is above the order upto level, 220, you do not order additional inventory.
c) From the *Std Norm Dist Func Table*: $\Phi(2.05) = 0.9798$ and $\Phi(2.06) = 0.9803$, so choose z = 2.06. The lead time, l, is 2, so μ = (2 + 1)×40 = 120 and $\sigma = \sqrt{2+1} \times 20$ =34.64. $S = \mu + z \times \sigma$ =120 + 2.06 × 34.64 = 191.36.
d) The z-statistic that corresponds to S = 120 is S = (120 –120) / 34.64 = 0. Expected backorder is $\sigma \times L(0) =$ 34.64 × 0.3989 = 13.82. Expected on-hand inventory is S - μ + *Expected backorder* = 120 –120 + 13.82 = 13.82
e) From part d, on-hand inventory is 13.82 units, which equals 13.82 × \$200 = \$2764. Cost of capital is 15%, so the cost of holding inventory is 0.15 × \$2764 = \$414.6.

Q14.2

a) Mean demand over $(l+1)$ periods is 0.5 × (4+1) = 2.5 units. From the *Poisson Distribution Function Table*, with mean 2.5 we have $F(6) = 0.9858$ and $F(7) = 0.9958$, so choose S = 7 to achieve a 99% in-stock.
b) Pipeline inventory is l × *expected demand in one period* = 4 × 0.5 = 2 units. The order upto level has no influence on the pipeline inventory.
c) From the *Poisson Loss Function Table* with mean 2.5 expected backorder = $L(5) = 0.06195$. Expected on-hand inventory = 5 – 2.5 + 0.06195 = 2.56 units
d) A stockout occurs if demand is 7 or more units over $(l+1)$ periods, which is 1 minus the probability demand is 6 or fewer in that interval. From the *Poisson Distribution Function Table* with mean 2.5 we see that $F(6) = 0.9858$ and $1 - F(6) = 0.0142$, i.e., about a 1.4% chance of a stockout occurring.
e) The store is out of stock if demand is 6 or more units over $(l+1)$ periods, which is 1 minus the probability demand is 5 or fewer in that interval. From the *Poisson Distribution Function Table* with mean 2.5 we see that $F(5) = 0.9580$ and $1 - F(5) = 0.0420$, i.e., about a 4.2% chance of being out of inventory at the end of any given week.

f) The store has one or more units of inventory if demand is 5 or fewer over $(l+1)$ periods. From part g, $F(5)=0.9580$, i.e., about a 96% chance of having one or more units at the end of any given week.

g) Now the lead time is 2 periods (each period is 2 weeks and the total lead time is 4 weeks, or two periods). Demand over one period is 1.0 units. Demand over $(l+1)$ periods is (2+1) × 1 = 3.0 units. From the *Poisson Distribution Function Table* with mean 3.0 we have $F(7)=0.9881$ and $F(8)=0.9962$, so choose S = 8 to achieve a 99% in-stock.

h) Pipeline inventory is average demand over l periods = 2 × 1 = 2.0 units.

Q14.3

a) If S = 700 and the inventory position is 523 + 180 = 703, then 0 units should be ordered because the inventory position exceeds the order up-to level.

b) From the *Std Norm Dist Function Table*, $\Phi(2.32)=0.9898$ and $\Phi(2.33)=0.9901$, so choose z = 2.33. Convert to $S=\mu+z\times\sigma$ = 600 + 2.33 × 159.22 = 971.

Q14.4

a) The critical ratio is \$25 / (\$0.5 + \$25) = 0.98039. The lead time is l = 0, so demand over $(l+1)$ periods is Poisson with mean 1.5. From the *Poisson Distribution Function Table* with mean 1.5 we see $F(3)=0.9344$ and $F(4)=0.9814$, so choose S = 4. There is currently no units on-order or on-hand, so order to raise the inventory position to 4: order 4 units.

b) The in-stock probability is the probability demand is satisfied during the week. With S = 3 the in-stock is $F(3)=0.9344$, i.e., a 93% probability.

c) Demand is not satisfied if demand is 5 or more units, which is 1 - $F(4)=0.9814$ = 1 – 0.9814 = 0.0186, or about 1.9%.

d) From the *Poisson Distribution Function Table* with mean 1.5 $F(4)=0.9814$ and $F(5)=0.9955$ so choose S = 5 to achieve a 99.5% in-stock probability.

e) If S =5, then from the *Poisson Loss Function Table* with mean 1.5 we see expected backorder $=L(5)=0.0056$. Expected on-hand inventory is S - demand over $(l+1)$ periods + expected backorder = 5 – 1.5 + 0.0056 = 3.51 units. The holding cost is 3.51 × \$0.5 = \$1.76

Q14.5

a) From the *Std Norm Dist Function Table*, $\Phi(2.43)=0.9925$, so choose z = 2.43. Convert to $S=\mu+z\times\sigma$ = 159.62 + 2.43 × 95.51 = 392.

b) The holding cost is h = 0.75 and the backorder penalty cost is 50. The critical ratio is 50 / (0.75 + 50) = 0.9852. From the *Std Norm Dist Function Table*, $\Phi(2.17)=0.9850$ and $\Phi(2.18)=0.9854$, so choose z = 2.18. Convert to $S=\mu+z\times\sigma$ = 159.62 + 2.18 × 95.51 = 368.

c) The holding cost is $h = 0.05$ and the backorder penalty cost is 5. The critical ratio is 5 / (0.05 + 5) = 0.9901. Lead time plus one demand is Poisson with mean 1 × 3 = 3. From the *Poisson Distribution Function Table*, with $\mu = 3$, $F(7) = 0.9881$ and $F(8) = 0.9962$, so S = 8 is optimal.

Q14.6

a) His inventory position is 200 + 73 = 273. His order-up-to level is 285, so he orders 285 – 273 = 12 pints.
b) From the *Normal Distribution Function Table*, $\Phi(2.32) = 0.9898$ and $\Phi(2.33) = 0.9901$, so choose z = 2.33. Convert to an order-up-to level: 200 + 2.33 × 48.08 = 312.
c) The average order quantity equals average demand during a single period. In this case average daily demand will be 100 / 7 = 14.3 pints.

Q14.7

a) $\mu = 1.5$, (demand in one period) and L = 3 (lead time). On order or pipeline inventory = $\mu \times L$ = 1.5 × 3 = 4.5.
b) Compute the probability demand over lead time plus one period exceeds 8. Demand over (l + 1) periods is Poisson with mean = 4 × μ = 4 × 1.5 = 6. From the *Poisson Distribution Function Table*, Probability$\{D > 8\}$= 1 – F(8) = 1 – 0.84724 = 0.15276.
c) We want to evaluate an order upto level that yields at least a fill rate of 90%. Fill Rate = 1 - Expected back order/Expected demand in one period. So, Expected back order = Expected demand in one period × (1 – Fill Rate) = 1.5 × (1 - .9) = 0.15. Using the *Poisson Loss Function Table* with mean = 6 (because that is the mean of l+1 periods), we find that L(9) = 0.16126 and L(10) = 0.07733. Hence, the order upto level should be S = 10.
d) Overage cost, C_o = \$0.01 (daily holding cost). Underage cost, C_u = \$6 (stockout cost). Critical Ratio = $C_u/(C_u + C_o)$ = 6/(6 + 0.01) = 0.9983. Expected demand over l + 1 days is Poisson with mean = (3 + 1) × 1.5 = 6. From the *Poisson Distribution Function Table* with mean 6, F(13) = 0.99637 and F(14) = 0.99860. Thus the order upto level should be S = 14.

Q14.8

a) Mean demand over $(l+1)$ periods is $\mu = 2000$ and the standard deviation is $\sigma =$ 555. Note, is we convert daily demand into weekly demand we get the same mean (which makes sense) but a different standard deviation. Our conversion procedure assumes demand across periods is independent, hence we have data to indicate the independence assumption is not valid. Therefore, we should use the mean and standard deviation evaluated directly from the demand data for $(l+1)$ periods. Use equation 14.3. The critical ratio is 0.45 / (0.01 + 0.45) = 0.9783. From the *Std Norm Dist Function Table*, $\Phi(2.02) = 0.9783$, so choose z = 2.02. Convert to $S = \mu + z \times \sigma$ = 2000 + 2.02 × 555 = 3121.
b) The base stock level has not influence over the amount of inventory on order, which equals the lead time (4) times one period demand (400) = 1600 units.

c) Evaluate expected on-hand inventory = 2800 – 2000 + 18.65 = 818.65. (The expected backorder, 18.65, is evaluated in part b.) Annual holding cost is \$0.01 × 260 × 818.65 = \$2129.
d) From the *Std Norm Dist Function Table*, $\Phi(1.88) = 0.9699$ and $\Phi(1.89) = 0.9706$ so choose $z = 1.89$. Convert to $S = \mu + z \times \sigma$ = 2000 + 1.89 × 555 = 3049.

Q14.9

a) From the *Std Norm Dist Function Table*, $\Phi(3.08) = 0.9990$, $\Phi(3.09) = 0.9990$ and $\Phi(3.10) = 0.9990$, so choose $z = 3.10$. The lead time is 0 weeks, so $(l+1)$ periods is one week. Weekly demand has mean μ= 5 × 178 = 890 and standard deviation $\sigma = \sqrt{5} \times 45 = 100.62$. (We are not evaluating demand over 6 days because we want the demand over one period first, and then we worry about demand over $l+1$ periods.) Convert to $S = \mu + z \times \sigma$ = 890 + 3.10 × 100.62 = 1202. Now evaluated expected inventory. $L(3.10) = 0.0003$, so expected backorder is 100.62 × 0.0003 = 0.03. On-hand inventory (at the end of the week) is S - μ+ expected backorder = 1202 – 890 + 0.03 = 312.03. The average order quantity is 890, because that is the average weekly demand. (An order equals last period's demand, so the average order equals the average of last period's demand, which is just the average demand in a period.) So the average inventory is 312.03 + 890 / 2 = 757, and the weekly holding cost is 757 × \$0.08 = \$60.56. Annual holding cost is 52 × \$60.56 = \$3149.
b) An order is placed within a week if there is positive demand. With mean 890 and standard deviation 100.62, demand is greater than 0 with essentially 100% probability. If an order is placed every week, the annual ordering cost is \$58 × 52 = \$3016.

c) If orders are placed every 2 weeks, the ordering cost is cut in half, 3016 / 2 = 1508. But holding costs are potentially increased. Repeat the process in part *a* with the assumption that one period is 2 weeks and the lead time is still 0. Now one period's mean demand is 1780 and standard deviation is 142.30 ($\sqrt{10} \times 45$). z = 3.10 is still optimal, so we still have $L(3.10) = 0.0003$. The order up-to level is S = = 1780 + 3.10 × 142.30 = 2221. The expected backorder is 142.30 × 0.0003 = 0.04. On-hand inventory is 2221 – 1780 + 0.04 = 441.04. The annual holding cost is \$0.08 × (441.04 + 1780/2) × 52 = \$5,537. So the annual order cost decreases by \$1508, but the annual holding cost increases by \$5537 – \$3149 = \$2388. The holding costs dominate the ordering cost savings, so order weekly.

Q14.10

a) From the *Std Norm Dist Function Table*, $\Phi(2.24) = 0.9875$, so choose z = 2.24. Convert to $S = \mu + z \times \sigma$ = 165 + 2.24 × 51.96 = 281. But 20 bags are included in each facing, so S = 281 requires 15 facings.

b) 11 facings translates into $S = 11 \times 20 = 220$. With $S = 220$, $z = (220 - 165) / 51.96 = 1.06$. We have $L(1.06) = 0.0742$ so expected backorder is $51.96 \times 0.0742 = 3.86$. Expected on-hand inventory is $220 - 165 + 3.86 = 59$.

c) There will be an empty facing if on-hand inventory is 10 or fewer units. If $S = 220$, then there will be 10 or fewer units if demand over $(l+1)$ periods is 210 or greater, which equals 1 minus the probability demand is 209 or fewer. The z that corresponds to 209 is $z = (209 - 165) / 51.96 = 0.85$. From the *Std Norm Dist Function Table*, $\Phi(0.85) = 0.8023$. Finally, $1 - 0.8023 = 0.1977$. So there is about a 20% chance there will be an empty facing.