Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 13: Assemble-to-order, Make-to-order, and Quick Response with Reactive Capacity

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Q13.1

a) Teddy will order from the American supplier if demand exceeds 1500 units. With for $Q = 1500$ the z-statistic is z = (1500 – 2100) / 1200 = -0.5. From the *Std Norm Dist Func Table* we see that $\Phi(-0.50) = 0.3085$, which is the probability demand is 1500 or fewer. The probability demand exceeds 1500 is $1 - \Phi(-0.50) = 0.6915$, or about 69%.

b) The supplier's expected demand equals Teddy's expected lost sales with an order quantity of 1500 parkas. From the *Std Norm Loss Func Table*, $L(-0.50) = 0.6978$. Expected lost sales is $\sigma \times L(z) = 1200 \times 0.6978$ = 837.4.

c) The overage cost is C_o=10 – 0 = 10, because left over parkas must have been purchased in the 1st order at a cost of \$10 and they have no value at the end of the season. The underage cost is C_u=15 – 10 = 5 because there is a \$5 premium on units ordered from the American vendor. The critical ratio is 5 / (10 + 5) = 0.3333. From the *Std Norm Dist Func Table* we see that $\Phi(-0.44) = 0.3300$ and $\Phi(-0.43) = 0.3336$, so choose z = -0.43. Convert to Q : Q = 2100 – 0.43 × 1200 = 1584.

d) First evaluate some performance measures. We already know that with Q = 1584 the corresponding z is –0.43. From *Std Norm Loss Func Table*, $L(-0.43) = 0.6503$. Expected lost sales is then 1200 × 0.6503 = 780.4, i.e., that is the expected order quantity to the American vendor. If the American vendor were not available, then expected sales would be 2100 – 780.4 = 1319.6. Expected left over inventory is then 1584 – 1319.6 = 264.4. Now evaluate expected profit with the American vendor option available. Expected revenue is 2100 × 22 = \$46,200. The cost of the 1st order is 1584 × 10 = \$15,840. Salvage revenue from left over inventory is 264.4 × 0 = 0. Finally, the cost of the 2nd order is 780.4 × 15 = \$11,706. Thus, profit is 46200 – 15840 – 11706 = \$18,654.

e) If Teddy only sources from the American supplier, then expected profit would be (\$22 - \$15) × 2100 = \$14,700, because expected sales would be 2100 units and the gross margin on each unit is \$7 = \$22 - \$15.

Q13.2

a) Expected sales = 1000 and the gross margin per sale is 121 – 83.5 = \$37.5. Expected profit is then 1000 × \$37.5 = \$37,500.

b) C_o = 72 – 50 = 22. C_u = 83.5 – 72 = 11.5: the premium on orders from XE is $11.5. The critical ratio is 11.5 / (22 + 11.5) = 0.3433. From the *Std Norm Dist Func Table* $\Phi(-0.41) = 0.3409$ and $\Phi(-0.40) = 0.3446$, so z = -0.40. Convert to Q = 1000 – 0.4 × 600 = 760.

c) The underage cost on an option is the change in profit if one additional option had been purchased that could be exercised. For example, if 700 options are purchased, but demand is 701, then 1 additional option could have been purchased. The cost of the option plus exercising it is $25 + $50 = $75. The cost of obtaining the unit without the option is $83.5, so purchasing the option would have saved C_u = $83.5 - $75 = $8.5. The overage cost on an option is the extra profit that could have been earned if the option were not purchased assuming it isn't needed. For example, if demand were 699, then the last option was not necessary. The cost of that unnecessary option is C_o=$25. The critical ratio is 8.5 / (25 + 8.5) = 0.2537. From the *Std Norm Dist Func Table* $\Phi(-0.67) = 0.2514$ and $\Phi(-0.66) = 0.2546$, so z = -0.66. Convert to Q = 1000 – 0.66 × 600 = 604.

d) Evaluate some performance measures. Expected number of units ordered beyond the purchased options (expected lost sales) is $\sigma \times L(-0.66)$ = 600 × 0.8128 = 487.7. Expected number of options exercised (expected sales) is 1000 – 487.7 = 512.3. Expected revenue is 1000 × $121 = $121,000. So profit is revenue, minus the cost of purchasing options (604 × $25 = $15,100), minus the cost of exercising options (512.3 × $50 = $25,615), minus the cost of units purchased without options (487.7 × $83.5 = $40723): 121000 – 15100 – 25615 – 40723 = $39,562.

Q13.3

a) The underage cost is $C_u = 0.4 - 0.05 = \$0.35$: if her usage exceeds the minutes she purchases then she could have lowered her cost by $0.35 per minute if she had purchased more minutes. The overage cost is $C_o = 0.05$ because each minute purchase but not used provides no value. The critical ratio is 0.35 / (0.05 + 0.35) = 0.8749. From the *Std Norm Dist Func Table* $\Phi(1.15) = 0.8749$ and $\Phi(1.16) = 0.8770$, so z = 1.16. Convert to Q, Q = 250 + 1.16 × 24 = 278.

b) We need to evaluate the number of minutes used beyond the quantity purchased (expected lost sales). z = (240 – 250) / 24 = -0.42, $L(-0.42) = 0.6436$ and expected lost sales = 24 × 0.6436 = 15.4 minutes. Each minute costs $0.4, so the total surcharge is 15.4 × $0.4 = $6.16.

c) Find the corresponding z-statistic: z = (280 – 250) / 24 = 1.25. Now evaluate performance measures. $L(1.25) = 0.0506$ and expected lost sales = 24 × 0.0506 = 1.2 minutes, i.e., only 1.2 minutes are needed on average beyond the 280 purchased. The minutes used out of the 280 (expected sales) is 250 – 1.2 = 248.8. The unused minutes (expected left over inventory) is 280 – 248.8 = 31.2.

d) Find the corresponding z-statistic: z = (260 – 250) / 24 = 0.42. The number of minutes needed beyond the 260 is expected lost sales: $L(0.42) = 0.2236$ and expected lost sales = 24 × 0.2236 = 5.4 minutes. Total bill is 260 × 0.05 + 5.4 × 0.4 = $15.16

e) From the *Std Norm Dist Func Table* $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$, so with z = 1.65 there is a 95.05% chance the outcome of a Standard Normal is less than z. Convert to Q, Q = 250 + 1.65 × 24 = 290

f) With "Pick your minutes" the optimal number of minutes is 278. The expected bill is then \$14.46: z = (278 – 250)/24 = 1.17; L(1.17) = 0.0596; expected surcharge minutes = 24 × 0.0596 = 1.4; expected surcharge = \$0.4 × 1.4 = \$0.56; purchase cost is 278 × 0.05 = \$13.9; so the total is \$13.9 + 0.56. With "No Minimum" is total bill is \$22.5: minutes cost \$0.07 × 250 = \$17.5; plus the fixed fee, \$5. So she should stick with the original plan.

Q13.4

a) The overage cost is $C_o = \$60$ because ordering a plate for a guest that doesn't show up costs \$60. The underage cost is C_u = \$85 – 60 = \$25, because not committing to a guest that does show up costs an extra \$25. The critical ratio is 25 / (60 + 25) = 0.2941. From the table, $F(92) = 0.2727$ and $F(93) = 0.3030$, so the optimal number of guests to commit to is 93.

b) Expected number of extra guests is $L(105) = 2$, so her bill is \$60 × 105 + \$85 × 2 = \$6,470.

c) The overage cost is $C_o = \$45$ because committing to a guest that doesn't show only costs \$45. The underage cost is still C_u = \$85 – 60 = \$25, because not committing to a guest that does show up costs an extra \$25. The critical ratio is 25 / (45 + 25) = 0.3571. From the table, F(94) = 0.3333 and F(95) = 0.3636, so the optimal number of guests to commit to is 95.

d) With the original plan we need to evaluate the expected bill. Expected number of extra guests is $L(93) = 8.36$, so her bill is \$60 × 93 + \$85 × 8.36 = \$6,291. At \$70 per guest, her expected bill is \$70 × 100 = \$7000. So go with the original option contract.

Q13.5

a) With 22 workers there is $22 \times 40 = 880$ hours of work available during the week. The z-statistic for $Q = 880$ is $(880 - 793)/111 = 0.78$. $1 - \Phi(z)$ is the probability the workload demand is greater than Q : $1 - \Phi(0.78)$ = 1 – 0.7823 = 0.2177 If there are 52 weeks in a year, then you will use overtime in $0.2177 \times 52 = 11.32$ of them.

b) With 18 workers we have 18×40 = 720 hours available per week. The z-statistic for 720 is $(720 - 793)/111 = -0.66$. The probability demand is less than 720 is $\Phi(z)$, which is $\Phi(-0.66) = 0.2546$. With 52 weeks a year we can expect to be underutilized in $0.2546 \times 52 = 13.24$ of them.

c) The overage cost is the hourly cost of labor, because if we hire workers and they are idle, then that costs us their hourly wages. The underage cost is the cost of the overtime premium: every hour of overtime costs us the premium over the regular cost of labor. Because the overtime premium is 50% of the hourly wage, the underage cost is 50% of the overage cost: $C_u = C_o / 2$. (Note, the underage cost is not 150% of

the hourly wage. Similarly, if we have a lost sale of a product then the underage cost is not the sales price but rather it is the gross margin.) Hence, the critical ratio is $C_u/(C_u+C_o) = \; = (0.5\times C_o)/(1.5\times C_o) = 1/3 = 0.3333$. From the *Std Norm Dist Func Table*, $\Phi(-0.44) = 0.3300$ and $\Phi(-0.43) = 0.3336$, so choose $z = -0.43$. Convert to Q: $Q = 793 - 0.43\times 111 = 745.27$. That translates into 745.27 / 40 = 18.6 workers.

d) The critical ratio evaluated in part c still holds even if you are using the empirical distribution function. We need to convert the given histogram, which is a density function, into a distribution function. That is done in the following table:

Hours	Observations	Cumulative observations	Percentile
480	1	1	0.0096
520	0	1	0.0096
560	1	2	0.0192
600	3	5	0.0481
640	8	13	0.1250
680	8	21	0.2019
720	12	33	0.3173
760	14	47	0.4519
800	13	60	0.5769
840	16	76	0.7308
880	11	87	0.8365
920	8	95	0.9135
960	4	99	0.9519
1000	3	102	0.9808
1040	1	103	0.9904
1080	1	104	1.0000

We are looking for 0.3333 in the percentile column. From the table we see that 31.73% of the time the workload is 720 hours and 45.19% of the time the workload is 760 hours. Based on the round up rule we should hire 19 workers so that we have 760 hours of regular time available each week.

Q13.6

a) This is a newsvendor problem because you have one opportunity to satisfy your "demand" for shillings at the favorable rates in the capital. Buying a shilling in the capital costs \$0.50 but buying one in the town costs \$1/1.6 = \$0.625. The underage cost of buying too few shillings is C_u = 0.625 – 0.50 = 0.125: each shilling that you could use but need to substitute an American dollar costs you an additional \$0.125. The overage cost of buying too few shillings is what you lose converting them back to dollars: C_o = 0.5 – 1 / 2.5 = 0.1. The critical ratio is $0.125/(0.125+0.1) = 0.5556$. From the *Std Norm Dist Func Table* we see that $\Phi(0.13) = 0.5517$ and

$\Phi(0.14) = 0.5557$, we choose $z = 0.14$. Convert the z-statistic to $Q = 400 + 0.14 \times 100 = 414$. Because you want 414 shillings, you should convert $207 dollars.

b) A 1 in 200 chance of stocking out means a 0.5% stockout probability, which in turn means a 99.5% in-stock probability. From the *Standard Normal Distribution Function Table* we see that $\Phi(2.57) = 0.9949$ and $\Phi(2.58)$ = 0.9951, so choose $z = 2.58$. Convert the z-statistic to Q = 400 + 2.58 × 100 = 658. If you need 658 shillings, then you must convert 658/2=$329.

Q13.7

a) TEC's gross margin is $110 × 0.25 = $27.5. 4101 units are produced, so total profit is 4101 × $27.5 = $112,778.

b) TEC's regular production cost is 0.75 × $110 = $82.5. TEC's expensive production cost is 2 × $82.5 = 165. TEC earns $27.5 on each of the 3263 units in O'Neill's 1st order. TEC charges 1.2 × $110 = $132 for units in the 2nd replenishment. O'Neill's expected 2nd order quantity is 437, and TEC "earns" $132 - $165 = -$33 on those units despite the premium charged of 20%. Hence, TEC's profit is $27.5 × 3263 - $33 × 437 = $75,312

c) Now TEC can produce more than 3263 units. The overage cost is $82.5 – 30 = $52.5. If a unit is not produced in the 1st production run but could be sold, TEC "earns" -$33 on that unit. If the unit were produced in the 1st production run, TEC earns $132 - $82.5 = $49.5. Hence, the underage cost is $49.5 – (-$33) = $82.5. In other words, every unit produced in the 1st production run that O'Neill eventually orders saves TEC $82.5 in profit relative to producing that unit in the 2nd production run. The critical ratio is 82.5 / (52.5 + 82.5) = 0.6111. We find that $\Phi(0.28) = 0.6103$ and $\Phi(0.29)$ = 0.6141, so choose z = 0.29. Convert the z-statistic to Q = 3192 + 0.29 × 1181 = 3534. Because that quantity is greater than O'Neill's initial order of 3263, TEC should produce 3534 units in the 1st production run.

d) If TEC produces 3534, then its expected 2nd production run is 320 units: $L(0.29)$ = 0.2706, $\sigma \times L(z) = 1181$ × 0.2706 = 320. Expected left over inventory is tricky to evaluate. Expected left over inventory with Q = 3534 is 662 units. Expected left over inventory with Q = 3263 is 508 units. Hence, if TEC produces 3534 units and O'Neill's 1st order is 3263 units, then among the 271 units (3534-3263) TEC produces above O'Neill's order, TEC can expect to have 662 – 508 = 154 remaining at the end of the season. (To explain, suppose demand is only 3200 units. Then TEC has 271 units left over, not 3534-3200 = 334, i.e., left over inventory is the amount that is left over if the order quantity is 3534 minus the amount that would be left over if the order quantity is 3263. This is probably not obvious, which is why this is labeled as a hard question.) TEC's revenue is then revenue from the 1st order $110 × 3263 = $358,930, plus revenue from the 2nd order $132 × 437 = $57,684, plus revenue from left over inventory $30 × 154 = $4620, for total revenue of $421,234. Costs include the 1st production run = 3534 × $82.5 = $291,555 and 2nd production

run costs = 320 × \$165 = \$52,800. Expected profit is then \$421,234 - \$291555 - \$52800 = \$76,878.

Q13.8

The overage cost is $C_o = \$5$ because any part remaining in inventory is charged \$5. The underage cost is C_u = \$50 - \$32 = \$18, because any emergency shipment requires an extra \$18 shipping cost. The critical ratio is 18 / (5 + 18) = 0.7826. From the table, $F(7) = 0.6728$ and $F(8) = 0.7916$, so the optimal number of parts to have on hand is 8. There are three parts in inventory, so 5 parts should be ordered and arrive on Feb 15.

Q13.9

For every car Smith sells he gets \$350 and an additional \$50 for every car sold over 5 cars. Look in the *Poisson Loss Function Table* for mean 5.5: the expected amount by which the outcome exceeds zero is $L(0)$=5.5 (same as mean) and the expected amount by which the outcome exceeds five is $L(5)$=1.178. Therefore, the expected commission is (350×5.5) + (50×1.178) = 1984.