Matching Supply with Demand: An Introduction to Operations Management

3rd Edition

Solutions to Chapter Problems

Chapter 12: Betting on Uncertain Demand: The Newsvendor Model

(last revised December 2011)

Q12.1

a) First find the *z*-statistic for 400 (Dan's blockbuster threshold): $z = (400 - 200)/80 = 2.50$. From the *Standard Normal Distribution Function Table* we see that $\Phi(2.50) = 0.9938$. So there is a 99.38% chance demand is less than 400 or fewer. It is greater than 400 with probability $1 - \Phi(2.50) = 0.0062$, i.e., there is only a 0.62% chance this is a blockbuster.

b) First find the *z*-statistic for 100 units (Dan's dog threshold): $z = (100 - 200)/80 = -1.25$. From the *Standard Normal Distribution Function Table* we see that $\Phi(-1.25) = 0.1056$. So there is a 10.56% chance demand is less than 100 or fewer, i.e., a 10.56% chance this book is a dog.

c) Demand is within 20% of the mean if it is between $1.2 \times 200 = 240$ and $0.8 \times 200 = 160$. First find the *z*-statistic for 240 units (the upper limit on that range): $z = (240 - 200)/80 = 0.5$. From the *Standard Normal Distribution Function Table* we see that $\Phi(0.5) = 0.6915$. Repeat the process for the lower limit on the range: $z = (160 - 200)/80 = -0.5$ and $\Phi(-0.5) = 0.3085$. The probability demand is between 160 and 240 is $\Phi(0.5) - \Phi(-0.50) = 0.6915 - 0.3085 = 0.3830$, i.e., 38.3%

d) The underage cost is $C_u = 20 - 12 = 8$. The salvage value is 12 – 4 = 8, because Dan can return left over books for a full refund (12) but incurs a 4 cost of shipping and handling. Thus, the overage cost is cost minus salvage value: $C_o = 12 - 8 = 4$. The critical ratio is $C_u / (C_o + C_u) =$ 8/12 = 0.6667. In the *Standard Normal Distribution Function Table* we see that $\Phi(0.43) = 0.6664$ and $\Phi(0.44) = 0.6700$, so use the round-up rule and choose $z = 0.44$. Now convert *z* into the order quantity for the actual demand distribution: $Q = \mu + z \times \sigma = 200 + 0.44 \times 80 = 235.2$.

e) We want to find a *z* such that $\Phi(z) = 0.95$. In the *Standard Normal Distribution Function Table* we see that $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$, so use the round-up rule and choose $z = 1.65$. Now convert *z* into the order quantity for the actual demand distribution: $Q = \mu + z \times \sigma = 200 + 1.65 \times 80 = 332$.

f) If the in-stock probability is 95%, then the stockout probability (which is what we are looking for) is 1 minus the in-stock, i.e., 1 – 95% = 5%.

g) Evaluate expected lost sales. The *z*-statistic for 300 units is $z = (300 - 200)/80 = 1.25$. From the *Standard Normal Loss Function Table* we see that $L(1.25) = 0.0506$. Expected lost sales is $\sigma \times L(1.25) = 4.05$. Evaluate expected

sales, then expected left over inventory and then expected profit. Expected sales is 200 – 4.05 = 195.95 and Expected left over inventory is 300 – 195.95 = 104.05 and

$$\begin{aligned} \textit{Expected profit} &= (\textit{Price - Cost}) \times \textit{Expected sales} \\ &\quad - (\textit{Cost} - \textit{Salvage value}) \times \textit{Expected left over inventory} \\ &= (20-12) \times 195.95 - (12-8) \times 104.05 \\ &= 1151.4 \end{aligned}$$

Q12.2

a) We need to evaluate the stock-out probability with $Q = 3$. From the Poisson Distribution Function Table, $F(3) = 0.34230$. The stock-out probability is $1 – F(3) =$ 65.8%.
b) They will need to markdown 3 or more baskets if demand is 7 or fewer. From the Poisson Distribution Function Table, $F(7) = 0.91341$, so there is a 91.3% probability this will occur.
c) First evaluate their critical ratio. The underage cost (or, cost of a lost sale), is \$55-\$32 = \$23. The overage cost (or, the cost of having a unit left in inventory) is \$32-\$20=\$12. The critical ratio is $C_u/(C_o+C_u)$ =0.6571. From the Poisson Distribution Function Table, with a mean of 4.5, we see that F(4)=0.53210 and F(5)=0.70293, so we apply the round-up rule and order 5 baskets.
d) With 4 baskets Expected Lost Sales is 1.08808, according to the Poisson Loss Function Table. Expected Sales is then 4.5 – 1.08808 = 3.4.
e) With 6 baskets Expected Lost Sales is 0.32312, according to the Poisson Loss Function Table. Expected Sales is then 4.5 – 0.32312 = 4.17688. Expected Left Over Inventory is then 6-4.17688 = 1.8.
f) From the Poisson Distribution Function Table F(6)=0.83105 and F(7)=0.91314. Hence, order 7 baskets to achieve at least a 90% in-stock probability (in fact, the in-stock probability will be 91.3%)
g) If they order 8 baskets then Expected Lost Sales is 0.06758. Expected Sales is 4.5 – 0.06758 = 4.43242. Expected Left Over Inventory is 8 – 4.43242 = 3.56758. Profit is then \$23 * 4.43242 - \$12 * 3.56758 = \$59.13.

Q12.3

a) If they purchase 40,000 units, then they need to liquate 10,000 or more units if demand is 30,000 units or lower. From the table provided, $F(30000) = 0.7852$, so there is a 78.52% chance they need to liquidate 10,000 or more units.
b) The underage cost is $C_u = 12 - 6 = 6$, the overage cost is $C_o = 6 - 2.5 = 3.5$ and the critical ratio is $6/(3.5+6) = 0.6316$. Looking in the demand forecast table we see that $F(25000) = 0.6289$ and $F(30000) = 0.7852$, so use the round-up rule and order 30,000 Elvis wigs.
c) We want to find an order quantity that generates a 90% fill rate. The target lost sales is $\mu \times (1 - \textit{Fill rate}) = 25000 \times (1 - 0.90) = 2500$. From the demand forecast table we see that $L(25000) = 3908$ and $L(30000) = 2052$, so use the round-up rule to choose $Q = 30{,}000$.

d) Evaluate expected lost sales and then the fill rate. If $Q = 30{,}000$, then expected lost sales from the table is 2,052. The fill rate is $1 - 2052/25000 = 0.9179$. Hence, the actual fill rate is about 92%.

e) We want to find a Q such that $F(Q) = 0.90$. From the demand forecast table we see that $F(35000) = 0.8894$ and $F(40000) = 0.9489$, so use the round-up rule and order 40,000 Elvis wigs. The actual in-stock probability is then 94.89%

f) Evaluate expected lost sales and then evaluate expected left over inventory. If $Q = 50{,}000$, then expected lost sales from the table is only 63 units. Expected left over inventory = $Q - \mu + \mathit{Expected\ lost\ sales} = 50000 - 25000 + 63 = 25063$.

g) A 100% in-stock probability requires an order quantity of 75,000 units. Evaluate expected lost sales: with $Q = 75{,}000$, then expected lost sales from the table is only 2 units. Evaluate expected sales, expected left over inventory and expected profit. Expected sales is expected demand minus expected loss sales = 25000 – 2 = 24998. Expected left over inventory is 75000 - 24998 = 50002

$$\begin{aligned}\mathit{Expected\ profit} &= (\mathit{Price}\text{ - }\mathit{Cost})\times \mathit{Expected\ sales}\\ &\quad -(\mathit{Cost} - \mathit{Salvage\ value})\times \mathit{Expected\ left\ over\ inventory}\\ &= (12-6)\times 24998 - (6-2.5)\times 50002\\ &= -25{,}019\end{aligned}$$

So a 100% in-stock probability is a money losing proposition.

Q12.4

a) It is within 25% of the forecast if it is greater than 750 and less than 1250. The *z*-statistic for 750 is $z = (750-1000)/600 = -0.42$ and the *z*-statistic for 1250 is $z = (1250-1000)/600 = 0.42$. From the *Standard Normal Distribution Function Table* we see that $\Phi(-0.42) = 0.3372$ and $\Phi(0.42) = 0.6628$. So there is a 33.72% chance demand is less than 750 and a 66.28% chance it is less than 1250. The chance it is between 750 and 1250 is the difference in those probabilities: $0.6628 - 0.3372 = 0.3256$.

b) The forecast is for 1000 units. Demand is greater than 40% of the forecast if demand exceeds 1400 units. Find the *z*-statistic that corresponds to 1400 units:

$$z = \frac{Q-\mu}{\sigma} = \frac{1400-1000}{600} = 0.67$$

From the *Standard Normal Distribution Function Table*, $\Phi(0.67) = 0.7486$. Therefore, there is almost a 75% probability that demand is less than 1400 units. The probability demand is greater than 1400 units is $1 - \Phi(0.67) = 0.2514$, or about 25%.

c) Find the expected profit maximizing order quantity, first identify the underage and overage costs. The underage cost is $C_u = 121 - 72 = 49$, because each lost sale costs Flextrola its gross margin. The overage cost is $C_o = 72 - 50 = 22$, because each unit of left over inventory can only be sold for 50. Now evaluate the critical ratio

$$\frac{C_u}{C_o + C_u} = \frac{49}{22+49} = 0.6901.$$

Lookup the critical ratio in the *Standard Normal Distribution Function Table*: $\Phi(0.49) = 0.6879$ and $\Phi(0.50) = 0.6915$, so choose z = 0.50. Now convert the z-statistic into an order quantity: $Q = \mu + z \times \sigma = 1000 + 0.5 \times 600 = 1300$.

d) Evaluate expected lost sales and then evaluate expected sales. If $Q = 1200$, then the corresponding z-statistic is $z = (Q - \mu)/\sigma = (1200 - 1000)/600 = 0.33$. From the *Standard Normal Distribution Loss Table* we see that $L(0.33) = 0.2555$. Expected lost sales is then $\sigma \times L(z) = 600 \times 0.2555 = 153.3$. Finally, recall that expected sales equals expected demand minus expected lost sales: expected sales = 1000 – 153.3 = 846.7.

e) Flextrola sells its left over inventory in the secondary market, which equals Q minus expected sales: 1200 – 846.7 = 353.3.

f) To evaluate the expected gross margin percentage we begin with

$$\begin{aligned} \textit{Expected revenue} &= (\textit{Price} \times \textit{Expected sales}) \\ &\quad + (\textit{Salvage value} \times \textit{Expected left over inventory}) \\ &= (121 \times 846.7) + (50 \times 353.3) \\ &= 120{,}116 \end{aligned}$$

Then we evaluate expected cost = $Q \times c = 1200 \times 72 = 86{,}400$. Finally, expected gross margin percentage = 1- 86,400 / 120,116 = 28.1%

g) Use the results from parts d and e to evaluate expected profit:

$$\begin{aligned} \textit{Expected profit} &= (\textit{Price - Cost}) \times \textit{Expected sales} \\ &\quad - (\textit{Cost} - \textit{Salvage value}) \times \textit{Expected left over inventory} \\ &= (121 - 72) \times 846.7 - (72 - 50) \times 353.3 \\ &= 33716 \end{aligned}$$

h) Solectric's expected profit is $1200 \times (72 - 52) = 24{,}000$ because units are sold to Flextrola for 72 and each unit has a production cost of 52.

i) Flextrola incurs 400 or more units of lost sales if demand exceeds the order quantity by 400 or more units, i.e., if demand is 1600 units or greater. The z-statistic that corresponds to 1600, $z = (Q - \mu)/\sigma = (1600 - 1000)/600 = 1$. In the *Standard Normal Distribution Function Table,* $\Phi(1) = 0.8413$. Demand exceeds 1600 with the probability $1 - \Phi(1) = 15.9\%$.

j) The critical ratio is 0.6901. From the graph of the distribution function we see that the probability demand is less than 1150 with the Log Normal distribution is about 0.70. Hence, the optimal order quantity with the Log Normal distribution is about 1150 units.

Q12.5

a) The underage cost is $C_u = 70 - 40 = 30$ and the overage cost is $C_o = 40 - 20 = 20$. The critical ratio is $C_u/(C_o + C_u) = 30/50 = 0.6$. From the *Standard Normal Distribution Function Table*, $\Phi(0.25) = 0.5987$ and $\Phi(0.26) = 0.6026$, so we choose $z = 0.26$. Convert that *z*-statistic into an order quantity $Q = \mu + z \times \sigma = 500 + 0.26 \times 200 = 552$. Note that the cost of a truckload has no impact on the profit maximizing order quantity.

b) We need to find the *z* in the *Standard Normal Distribution Function Table* such that $\Phi(z) = 0.9750$ because $\Phi(z)$ is the in-stock probability. We see that $\Phi(1.96) = 0.9750$, so we choose $z = 1.96$. Convert to $Q = \mu + z \times \sigma = 500 + 1.96 \times 200 = 892$.

c) If 725 units are ordered, then the corresponding *z*-statistic is $z = (Q - \mu)/\sigma = (725 - 500)/200 = 1.13$. We need to evaluate lost sales, expected sales and expected left over inventory before we can evaluate the expected profit. Expected lost sales with the Standard Normal is obtained from the *Standard Normal Loss Function Table*, $L(1.13) = 0.0646$. Expected lost sales is $\sigma \times L(z) = 200 \times 0.0646 = 12.9$. Expected sales is 500 – 12.9 = 487.1. Expected left over inventory is 725 – 487.1 = 237.9. Expected profit is

$$\begin{aligned} \textit{Expected profit } &= (70-40)\times 487.1 - (40-20)\times 237.9 \\ &= 9855 \end{aligned}$$

So the expected profit per sweater is 9,855. The total expected profit is five times that amount, minus 2000 times the number of truckloads required.

d) The stockout probability is the probability demand exceeds the order quantity 725, which is $1 - \Phi(1.13) = 12.9\%$.

e) If we order the expected profit maximizing order quantity for each sweater, then that equals $5 \times 552 = 2{,}760$ sweaters. With an order quantity of 552 sweaters expected lost sales is $56.5 = 200 \times L(0.26) = 200 \times 0.2824$, expected sales is 500 – 56.5 = 443.5 and expected left over inventory is 552 – 443.5 = 108.5. Expected profit per sweater is

$$\begin{aligned} \textit{Expected profit } &= (70-40)\times 443.5 - (40-20)\times 108.5 \\ &= 11{,}135 \end{aligned}$$

Because two truckloads are required, the total profit is then $5 \times 11{,}136 - 2 \times 2000 = 51{,}675$. If we order only 500 units per sweater type, then we can evaluate the expected profit per sweater to be 11,010. Total profit is then $5 \times 11{,}010 - 2000 = 53{,}050$. Therefore, we are better off just ordering one truckload with 500 sweaters of each type.

Q12.6

a) The parka sells less than half of the forecast if demand is 2100/2 = 1050 or fewer units. Normalize the quantity 1050: $z = (1050 - 2100)/1200 = -0.88$. From the *Standard Normal Distribution Function Table,* $\Phi(-0.88) = 0.1894$, which implies there is a 18.9% probability that the parka will be a dog.

b) To determine the profit maximizing order quantity, begin with the underage cost, $C_u = 22 - 10 = 12$, and the overage cost, $C_o = 10 - 0 = 10$. The critical ratio is $12/(10+12) = 0.5455$. We see from the *Standard Normal Distribution Function Table* that $\Phi(0.11) = 0.5438$ and $\Phi(0.12) = 0.5478$, so we choose $z = 0.12$. Convert that *z*-statistic back into an order quantity, $Q = \mu + z \times \sigma = 2100 + 0.12 \times 1200 = 2{,}244.$

c) To hit the target in-stock probability of 98.5%, we need to find the *z*-statistic such that $\Phi(z) = 0.9850$. We see from the *Standard Normal Distribution Function Table* that $\Phi(2.17) = 0.9850$, so we choose $z = 2.17$. Convert to *Q*: $Q = 2100 + 2.17 \times 1200 = 4704$.

d) If 3000 parkas are ordered then the corresponding *z*-statistic is $(3000 - 2100)/1200 = 0.75$. Now look up expected lost sales with the Standard Normal distribution in the *Standard Normal Loss Function Table*: $L(0.75) = 0.1312$. Convert that lost sales into the expected lost sales with the actual demand distribution: $\sigma \times L(z) = 1200 \times 0.1312 = 157.4$. Expected sales = expected demand – expected lost sales = 2100 – 157.4 = 1942.6. Expected left over inventory = 3000 – 1942.6 = 1057.4. Finally,

$$\begin{aligned} \textit{Expected profit} &= (22-10) \times 1942.6 - (10-0) \times 1057.4 \\ &= 12{,}737 \end{aligned}$$

e) The stock out probability is $1 - \Phi(z) = 1 - \Phi(0.75) = 22.7\%$

Q12.7

a) The underage cost is $C_u = 54 - 40 = 14$, and the overage cost, $C_o = 40 - 54/2 = 13$. The critical ratio is $14/(13+14) = 0.5185$. We see from the *Standard Normal Distribution Function Table* that $\Phi(0.04) = 0.5160$ and $\Phi(0.05) = 0.5199$, so we choose $z = 0.05$. Convert that *z*-statistic back into an order quantity, $Q = \mu + z \times \sigma = 400 + 0.05 \times 300 = 415.$

b) From part b, expected lost sales is 130.5. Expected sales is then 400 – 130.5 = 269.5 and expected left over inventory is 380 – 269.5 = 110.5. Expected profit is

$$\begin{aligned} \textit{Expected profit} &= (54-40) \times 269.5 - (40-27) \times 110.5 \\ &= 2{,}337 \end{aligned}$$

c) We first have to evaluate Teddy Bower's expected profit with the optimal order quantity, 415 boots. With that quantity, expected lost sales is $300 \times L(0.05) = 112.3$, expected sales is 400 – 112.3 = 287.7 and expected left over inventory is 415 – 287.7 = 127.3. Expected profit is

$$\textit{Expected profit} = (54-40)\times 287.7-(40-27)\times 127.3$$
$$= 2373$$

Given that the optimal number of boots to order is 415, if Teddy Bowers is going to get the quantity discount then they should only order 800 boots. With an order quantity of 800 the z-statistic is $z=(800-400)/300=1.33$. Expected lost sales is $300\times L(1.33)=12.81$, expected sales is 400 – 12.81 = 387.19 and expected left over inventory = 800 – 387.19 = 412.81. Due to the 10% discount, the purchase cost is now \$36. Expected profit is

$$\textit{Expected profit} = (54-36)\times 387.19-(36-27)\times 412.81$$
$$= 3254$$

Therefore, it is in Teddy Bower's interest to order 800 units to get the quantity discount.

Q12.8

a) With option 1 Land's End sales price is \$100, purchase cost is \$65 and salvage value is \$53 (because Geoff buys back unsold glasses for \$53). So the underage cost is C_u=100 – 65 =35 and the overage cost is $C_o=65-53=12$. The critical ratio is $C_u/(C_o+C_u)$= 35/47 = 0.7422. From the *Standard Normal Distribution Function Table* we see $\Phi(0.65)=0.7422$ and $\Phi(0.66)=0.7454$, so we choose $z=0.66$. The optimal order quantity is then $Q=\mu+z\times\sigma=200+0.66\times 125=282.5$.

b) With option 1 Land's End sales price is \$100, purchase cost is \$55 and the salvage value is \$0. So the underage cost is $C_u=100-55=45$ and the overage cost is $C_o=55$. The critical ratio is $C_u/(C_o+C_u)$= 45/100 = 0.4500. From the *Standard Normal Distribution Function Table* we see $\Phi(-0.13)=0.4483$ and $\Phi(-0.12)=0.4522$, so we choose $z=-0.12$. The optimal order quantity is then $Q=\mu+z\times\sigma=200-0.12\times 125=185$.

c) We need to evaluate expected profit in each case to determine which option Lands End should choose. With option 1, Geoff sells 282.5 units at \$65 for total revenue of 18,363 and production cost of $282.5\times 2=7063$. Geoff also credits Lands End for each returned sunglass, so we need to evaluate how many sunglasses Lands End will return. Expected lost sales is $125\times L(0.66)=125\times 0.1528=19.1$, expected sales is 200 – 19.1 = 180.9 and expected left over inventory is 282.5 – 180.9 = 101.6. Expected profit is then

$$\textit{Expected profit} = (100-65)\times 180.9-(65-53)\times 101.6$$
$$= 5112$$

With option 2, expected lost sales is $125\times L(-0.12)=125\times 0.4618=57.72$, expected sales is 200 – 57.72 = 142.28 and expected left over inventory is 185 – 142.28 = 42.72. Expected profit is then

$$\textit{Expected profit} = (100-55)\times 142.28-(55-0)\times 42.72$$
$$= 4053$$

So Land's End prefers option 1.

d) If Land's End chooses option 1 and orders 275 units, then Geoff's earns $\$65 \times 275 = \$17,875$ in initial revenue and incurs a production cost of $\$25 \times 275 = \$6875$. But Geoff also has to buy back left over glasses from Land's End. With an order quantity of 275, the z-statistic is $(275 - 200)/125 = 0.60$. Expected lost sales is then $125 \times L(0.60) = 21.09$. Expected left over inventory is $Q - \mu + \textit{Expected lost sales}$, which is $275 - 200 + 21.09 = 96.09$. So Geoff's buy back cost is $53 \times 96.09 = 5093$. Geoff's expected profit is then $17875 - 6875 - 5093 = 5907$.

Q12.9

a) We first need to evaluate the overage and underage costs. The underage cost is $C_u = 0.6 - 0.20 = 0.4$, i.e., it is the gross margin on each bagel. The overage cost is slightly more complex to evaluate. Assume Day Old bagels are sold for \$0.165 each, but only about 2/3rds of them are sold. Hence, the average salvage value received on Day Old bagels is $\$0.165 \times (2/3) = \$0.11$. The overage cost is then $C_o = 0.20 - 0.11 = 0.09$ because the average loss on each unsold bagel is 9 cents. The critical ratio is $C_u/(C_o + C_u)$ = 0.40 / 0.51 = 0.8163. In the *Standard Normal Distribution Function Table* we see that $\Phi(0.90) = 0.8159$ and $\Phi(0.91) = 0.8186$, so we choose $z = 0.91$. Convert that *z*-statistic to a quantity, $Q = \mu + z \times \sigma = 54 + 0.91 \times 21 = 73.11$. Hence, the store should have approximately 73 bagels to maximize its expected profit.

b) To ensure a 99% fill rate we need to hit the following target lost sales: $L(z) = (\mu/\sigma) \times (1 - \textit{Fill rate}) = (54/21) \times (1\text{-}0.99) = 0.0257$. In the *Standard Normal Loss Function Table* we see that $L(1.55)=$ 0.0261 and $L(1.56)=$ 0.0255, so we choose $z = 1.56$. Convert that *z*-statistic to a quantity, $Q = \mu + z \times \sigma = 54 + 1.56 \times 21 = 86.76$.

c) If there is an additional \$5 cost to a lost sales, then the underage cost is now $C_u = 0.4 + 5 = 5.4$, i.e., it is the gross margin plus the additional penalty. The critical ratio is now $C_u/(C_o + C_u) = 5.40/5.51 = 0.9836$. In the *Standard Normal Distribution Function Table* we see that $\Phi(2.13) = 0.9834$ and $\Phi(2.14) = 0.9838$, so we choose $z = 2.14$. Convert that *z*-statistic to a quantity, $Q = \mu + z \times \sigma = 54 + 2.14 \times 21 = 98.94$.

d) If 101 bagels are in stock at 3pm., then the *z*-statistic is $(101 - 54)/21 = 2.24$. Lost sales with a Standard Normal is (from the *Standard Normal Loss Function Table*) $L(2.24) = 0.0044$. Convert that lost sales into the lost sales for the actual Normal distribution = $\sigma \times L(z) = 21 \times 0.0044 = 0.0924$. Expected left over inventory = $Q - \mu + \textit{Expected lost sales}$ = 101 – 54 + 0.0924 = 47.09

Q12.10

a) The overage cost is $C_o = \$2$ because left over burritos are disposed. The underage cost is C_u = \$2.55, because not having a burrito means the gross margin is lost on a

burrito plus a soda. The critical ratio is 2.55 / (2 + 2.55) = 0.5604. From the table, $F(22) = 0.5564$ and $F(23) = 0.6374$ so the optimal number of burritos to make is 23.

b) The overage cost remains the same, $C_o = \$2$ because left over burritos still are disposed. The underage cost is now C_u = \$2 – 0.5 = \$1.5, because not having a burrito means the gross margin is lost on a burrito, but at least the gross margin on a Pop Tart is earned. The gross margin on the soda is captured either way, so it no longer figures into the analysis. The critical ratio is 1.5 / (2 + 1.5) = 0.4286. From the table, $F(20) = 0.3869$ and $F(21) = 0.4716$ so the optimal number of burritos to make is now 21.