

Matching Supply with Demand: An Introduction to Operations Management

Solutions to End-of-Chapter Problems

(last revised March 27, 2012; make sure to visit www.cachon-terwiesch.net for the latest updates, excel files, ppt files and other information)

Chapter 9

Q9.2. Home security

We have $m=5$, an interarrival time of $a=5$ minutes and a service time of $p=90$ minutes.

(a) We compute $r=p/a=18$; we then use the Erlang loss table to find that the probability that all 5 servers are utilized, $P_5(18)=0.7402$

(b) We compute the loss rate of the system, since this corresponds to the calls that will be handled by the police. The loss rate is 0.1480 calls per minute, which is 6395 calls per month. Since each of the calls would lead to a 500\$ charge by the police, we would have monthly payments of 3.2 million \$. What a dull business idea!

Q9.3. Video Store

We have $m=9$, $a=24/15$ hours and $p=36$ hours. $r=p/a=22.5$

(a) We use the Erlang loss table to find that all servers will be utilized with probability $P_9(22.5)=62.44\%$. That means that there will be a movie available with probability 37.56%.

(b) The flow rate is 0.235 movies per hour. Over the course of a day (and at 5\$ per movie), this brings in 28.166 \$/day revenue

(c) The loss rate is 0.39 customers per hour; since each of them gets \$1, that means a loss of \$9.36 a day.

(d) We repeat all calculations with $m=10$. We find that we get new (higher) revenues of 31.19 \$/day and new (lower) losses of 8.76 \$/day. Thus, we increase profits by \$3.62 per day. In less than 14 days, we have then recovered our investment for the new copy of the movie.

Q9.4. Gas station

20 cars want to fill in per hour. This leaves us with an interarrival time of $a=60/20$ minutes=3 minutes. The service time is $p=5$ minutes ($r=1.667$) and we have $m=6$ servers.

(a) All servers will be busy with a probability of 0.0056

(b) The flow rate is 19.89 cars per hour

Q9.5. Two work stations

(a) For both, scenario 1 and scenario 3, we obtain a flow rate of 12 units per hour (one unit every 5 minutes). Scenario 3 will not be able to run faster than this, since the bottleneck is at the first resource and has a capacity of 12 units per hour.

(b) Scenario 3 has the lowest flow time. Given that it is never above 10 minutes, but sometimes (if station B takes 3 minutes) below, the average flow time will be below 10 minutes. In scenario 1, the flow time is always exactly 10 minutes. In scenario 2, it will be longer than 10 minutes, as occasionally (when B takes 6 minutes), station A will be blocked, leading to longer than 5 minutes flow time at A. At station B, the flow time in this scenario will – on average – be 5 minutes.

Q9.6. Xtremely Fast Service

- a. $r = p/a = 5/3$. Given that number of servers, $m = 4$, the Erlang Loss Table yields a probability of 0.0624.
- b. Flow Rate = demand rate * probability that not all servers are busy * price = 1 customer / 3 minutes * (1 - 0.0624) * 60 min/hr * 8 hr/day * \$1 / call = \$150.
- c. Flow rate = demand rate * probability that all servers are busy * cost = $1/3 * (0.0624) * 60 * 8 * \$5/\text{day} = \$49.92$, which is closest to \$50.

Q9.7. Gotham City Ambulance Services

- a. $r = p/a = 6$; fraction of emergencies handled by neighboring community: 12.19%
- b. There are 3.514 emergencies served per hour, i.e. 84.2976 per day
- c. Reducing the coefficient of variation has no effect whatsoever on the number of emergencies they can attend
- d. They would have to buy 2 additional ambulances