Solved Examples for Chapter 18

Example for Section 18.3

Computronics is a manufacturer of calculators, currently producing 200 per week. One component for every calculator is a liquid crystal display (LCD), which the company purchases from Displays, Inc. (DI) for $1 per LCD. Computronics management wants to avoid any shortage of LCDs, since this would disrupt production, so DI guarantees a delivery time of 1/2 week on each order. The placement of each order is estimated to require 1 hour of clerical time, with a direct cost of $15 per hour plus overhead costs of another $5 per hour. A rough estimate has been made that the annual cost of capital tied up in Computronics' inventory is 15 percent of the value (measured by purchase cost) of the inventory. Other costs associated with storing and protecting the LCDs in inventory amount to 5 cents per LCD per year.

(a) What should the order quantity and reorder point be for the LCDs? What is the corresponding total variable inventory cost per year (holding costs plus administrative costs for placing orders)?

We calculate the data needed for the basic EOQ model as follows:

Demand per year for LCD = 52(200/week) = 10,400/year.

Setup cost = direct cost + overhead cost = ($15/hr)(1 hr) + ($5/hr)(1 hr)
= $15 + $5 = $20.

Unit holding cost = 15% of the value of each LCD + 5 cents of storing and protecting cost per LCD
= 15%($1) + $0.05 = $0.20 per LCD.

Delivery time = 1/2week = 3.5 days.

Working days per year = 365 days/year.

We use the Excel template for the basic EOQ Model (shown next) and obtain the following solutions:

Optimal order quantity = 1442.

Reorder point = 99.7.

Total variable inventory cost per year = $288.44.

	A	B	C	D	E	F	G	H	I	J	K
1	**Basic EOQ Model (Analytical Version)**										
2											
3			**Data**				**Results**		**Range Name**	**Cell**	
4		D =	10400	(demand/year)		Reorder Point	99.73		AnnualHoldingCost	G7	
5		K =	$20	(setup cost)					AnnualSetupCost	G6	
6		h =	$0.20	(unit holding cost)		Annual Setup Cost	$144.22		D	C4	
7		L =	3.5	(lead time in days)		Annual Holding Cost	$144.22		h	C6	
8		WD =	365	(working days/year)		Total Variable Cost	$288.44		K	C5	
9									L	C7	
10			**Decision**						Q	C11	
11		Q =	1442	(optimal order quantity)					ReorderPoint	G4	
12									TotalVariableCost	G8	
13									WD	C8	
14											
15											

(b) Suppose the true annual cost of capital tied up in Computronics' inventory actually is 10 percent of the value of the inventory. Then what should the order quantity be? What is the difference between this order quantity and the one obtained in part (a)? What would the total variable inventory cost per year (TVC) be? How much more would TVC be if the order quantity obtained in part (a) still were used here because of the incorrect estimate of the cost of capital tied up in inventory?

If the true annual cost tied up in inventory is 10% of the value of the inventory, the unit holding cost becomes:

Unit holding cost = 10% of the value of each LCD + 5 cents of storing and protecting cost per LCD
= 10%($1) + $0.05 = $0.15 per LCD.

The new order quantity is 1665, an increase of 223 (=1665-1442) LCDs from the order quantity in part (a).

The total variable inventory cost per year is $249.80.

If the order quantity in part (a) were used, the total variable inventory cost per year is

$$20(10{,}400/1442) + 0.15(1442/2) = \$252.39,$$

an increase of $2.59 from $249.80.

	A	B	C	D	E	F	G	H	I	J	K
1	**Basic EOQ Model (Analytical Version)**										
2											
3			**Data**				**Results**		**Range Name**	**Cell**	
4		D =	10400	(demand/year)		Reorder Point	99.73		AnnualHoldingCost	G7	
5		K =	$20	(setup cost)					AnnualSetupCost	G6	
6		h =	$0.15	(unit holding cost)		Annual Setup Cost	$124.90		D	C4	
7		L =	3.5	(lead time in days)		Annual Holding Cost	$124.90		h	C6	
8		WD =	365	(working days/year)		Total Variable Cost	$249.80		K	C5	
9									L	C7	
10			**Decision**						Q	C11	
11		Q =	1665	(optimal order quantity)					ReorderPoint	G4	
12									TotalVariableCost	G8	
13									WD	C8	
14											
15											

(c) Repeat part (b) if the true annual cost of capital tied up in Computronics' inventory actually is 20 percent of the value of the inventory.

If the true annual cost tied up in inventory is 20% of the value of the inventory, the unit holding cost becomes:

Unit holding cost = 20% of the value of each LCD + 5 cents of storing and protecting cost per LCD
= 20%($1) + $0.05 = $0.25 per LCD.

The new order quantity is 1290, a decrease of 152 (=1290-1442) LCDs from the order quantity in part (a).

The total variable inventory cost per year is $322.49.

If the order quantity in part (a) were used, the total variable inventory cost per year is

$$20(10{,}400/1442) + 0.25(1442/2) = \$324.50,$$

an increase of $2.01 from $322.49.

	A	B	C	D	E	F	G	H	I	J	K
1	**Basic EOQ Model (Analytical Version)**										
2											
3			**Data**				**Results**		**Range Name**	**Cell**	
4		D =	10400	(demand/year)		Reorder Point	99.73		AnnualHoldingCost	G7	
5		K =	$20	(setup cost)					AnnualSetupCost	G6	
6		h =	$0.25	(unit holding cost)		Annual Setup Cost	$161.25		D	C4	
7		L =	3.5	(lead time in days)		Annual Holding Cost	$161.25		h	C6	
8		WD =	365	(working days/year)		Total Variable Cost	$322.49		K	C5	
9									L	C7	
10			**Decision**						Q	C11	
11		Q =	1290	(optimal order quantity)					ReorderPoint	G4	
12									TotalVariableCost	G8	
13									WD	C8	
14											
15											

(d) Perform sensitivity analysis systematically on the unit holding cost by generating a table that shows what the optimal order quantity would be if the true annual cost of capital tied up in Computronics' inventory were each of the following percentages of the value of the inventory: 10, 12, 14, 16, 18, 20.

For each of these percentages for the annual cost of capital tied up in inventory, we calculate the unit holding cost and use the Excel template for the basic EOQ model to obtain the corresponding order quantity, as given in the following table.

Cost of capital	Unit holding cost	Order quantity (LCDs)
10%	$0.15	1665
12%	$0.17	1564
14%	$0.19	1480
16%	$0.21	1407
18%	$0.23	1345
20%	$0.25	1290

(e) Assuming that the rough estimate of 15 percent is correct for the cost of capital, perform sensitivity analysis on the setup cost by generating a table that shows what the optimal order quantity would be if the true number of hours of clerical time required to place each order were each of the following: 0.5, 0.75, 1, 1.25, 1.5.

For each of these hours of clerical time required to place an order, we calculate the setup cost and use the Excel template for the basic EOQ model to obtain the corresponding order quantity, as given in the following table.

Clerical Time (hrs)	Setup Cost	Order quantity (LCDs)
0.5	$10	1020
0.75	$15	1249
1	$20	1442
1.25	$25	1612
1.5	$30	1776

(f) Perform sensitivity analysis simultaneously on the unit holding cost and the setup cost by generating a table that shows the optimal order quantity for the various combinations of values considered in parts (d) and (e).

The order quantities for various combinations of the cost of capital (unit holding cost) and clerical time (setup cost) are given next.

Unit Holding Cost	Setup cost				
	$10	$15	$20	$25	$30
$0.15	1178	1442	1665	1862	2040
$0.17	1106	1355	1564	1749	1916
$0.19	1046	1281	1480	1654	1812
$0.21	995	1219	1407	1574	1724
$0.23	951	1165	1345	1504	1647
$0.25	912	1117	1290	1442	1580

Example for Section 18.4

Consider a situation where a particular product is produced and placed in in-process inventory until it is needed in a subsequent production process. No units currently are in inventory, but three units will be needed in the coming month and an additional four units will be needed in the following month. The unit production cost is the same in either month. The setup cost to produce in either month is $5,000. The holding cost for each unit left in inventory at the end of a month is $1,000.

Determine the optimal schedule that satisfies the monthly requirements by using the algorithm presented in Sec. 18.4.

Using the notation of Sec. 18.4, the demands for the two months are

$r_1 = 3, \qquad r_2 = 4,$

and the cost parameters for the model in Sec. 18.4 (in units of thousands of dollars) are

$K = 5, \qquad h = 1.$

Therefore, the total variable cost (which excludes the unit production cost) of an optimal policy for month 2 when that month starts with zero inventory (before producing) is

$$C_2 = C_3 + K = 0 + 5 = 5.$$

The variable cost of the policy associated with producing only what is needed for month 1 and then producing only what is needed for month 2 is

$$C_1^{(1)} = C_2 + K = 5 + 5 = 10.$$

The variable cost of the policy associated with producing enough in month 1 to meet the demand for both months is

$$C_1^{(2)} = C_3 + K + hr_2 = 0 + 5 + 1(4) = 9.$$

The minimum of the variable costs of these two policies is

$$C_1 = \min\{C_1^{(1)}, C_1^{(2)}\} = \min\{10, 9\} = 9,$$

so the optimal policy is to produce all 7 units in month 1 that are needed for both months 1 and 2.

Example for Section 18.6

Micro-Apple is a manufacturer of personal computers. It currently manufactures a single model —the MacinDOS— on an assembly line at a steady rate of 500 per week. MicroApple orders the hard drives for the MacinDOS (1 per computer) from an outside supplier at a cost of $30 each. Additional administrative costs for placing an order total $30. The annual holding cost is $6 per drive. If MicroApple stocks out of hard drives, production is halted, costing $100 per drive short. Because of the seriousness of stockouts, management wants to keep enough safety stock to prevent a shortage before the delivery arrives during 99 percent of the order cycles.

The supplier now is offering two shipping options. With option 1, the lead time would have a normal distribution with a mean of 0.5 per week and a standard deviation of 0.1 per week. For each order, the shipping cost charged to MicroApple would be $100 plus $3 per drive. With option 2, the lead time would have a uniform distribution from 1.0 weeks to 2.0 weeks. For each order, the shipping cost charged to MicroApple would be $20 plus $2 per drive.

(a) Use the stochastic continuous-review model presented in Sec. 18.6 to obtain an (R, Q) policy under each of these two shipping options.

From the problem description, the data for the continuous-review model are the following:

Unit time = 1 year.
Demand, d = (500/week)(52weeks/year) = 26,000 hard drivers per year.
Annual holding cost per hard drive, h = \$6.
Shortage cost per hard drive, p = \$100.
Service level, L = 99%.

For option 1:

Setup cost for placing and shipping an order, K = \$30 + \$100 = \$130.
The lead time has a normal distribution.
The average demand during the lead time is (500/week)(1/2 week) = 250.
The standard deviation of demand during the lead time is (500/week)(0.1week) = 50.

Using the Excel template for the stochastic continuous-review model, as shown next, we obtain the order quantity Q = 1093 and reorder point R = 366.

	A	B	C	D	E	F	G	H
1	**Template for the Stochastic Continuous-Review Model**							
2								
3			**Data**				**Results**	
4		a =	26000	(average demand/unit time)		Q =	1093	
5		K =	130	(setup cost)		R =	366	
6		h =	6	(unit holding cost)				
7		p =	100	(unit shortage cost)				
8		L =	0.99	(service level)				
9								
10								
11		**Demand During Lead Ti**						
12		Distribution =	Normal					
13		mean =	250					
14		stand. dev. =	50					
15								
16								

For option 2:

Setup cost for placing and shipping an order, K = \$30 + \$20= \$50.
The lead time has a uniform distribution.
The lower end point a = (500/week)(1 week) = 500.
The upper endpoint b = (500/per week)(2 weeks) = 1000.

Using the Excel template for the stochastic continuous-review model, as shown next, we obtain the order quantity Q = 678 and reorder point R = 995.

	A	B	C	D	E	F	G	H
1	Template for the Stochastic Continuous-Review Model							
2								
3			Data				Results	
4		a =	26000	(average demand/unit time)		Q =	678	
5		K =	50	(setup cost)		R =	995	
6		h =	6	(unit holding cost)				
7		p =	100	(unit shortage cost)				
8		L =	0.99	(service level)				
9								
10								
11		Demand During Lead Ti						
12		Distribution =	Uniform					
13		a =	500	(lower endpoint)				
14		b =	1000	(upper endpoint)				
15								
16								

(b) Show how the reorder point is calculated for each of these two policies.

The reorder point of option 1 is

$$R = \mu + K_L\,\sigma = 250 + 2.327(50) = 366.$$

The reorder point of option 2 is

$$R = a + L(b-a) = 500 + 0.99(1000-500) = 995.$$

(c) Determine and compare the amount of safety stock provided by these two policies.

The safety stock of option 1 = R – mean = 366 – 250 = 116.

The safety stock of option 2 = R – mean = 995 – 750 = 245.

(d) Determine and compare the average annual holding cost under these two policies.

The average annual holding cost of option 1 = $(\$6)\left(\dfrac{116+(1063+116)}{2}\right) = \$3975.$

The average annual holding cost of option 2 = $(\$6)\left(\dfrac{245+(678+245)}{2}\right) = \$3504.$

(e) Determine and compare the average annual acquisition cost (combining shipping cost and purchase price) under these two policies.

For option 1:

The annual shipping cost = K(d/Q) + ($3)d = ($130)(26,000/1093) + ($3)(26,000)
= $81,092.

The annual purchase cost = ($30)(26,000) = $780,000.

Hence, the annual acquisition cost = $81,092 + $780,000 = $861,092.

For option 2:

The annual shipping cost = K(d/Q) + ($2)d = ($50)(26,000)/678) + ($2)(26,000)
= $53,917.

The annual purchase cost = ($30)(26,000) = $780,000.

Hence, the annual acquisition cost = $53,917+ $780,000 = $833,917.

(f) Since shortages are very infrequent (and very small when they do occur), the only important costs for comparing the two shipping options are those obtained in parts (d) and (e). Add these costs for each option. Which option should be selected?

For option 1: $3,975 + $861,092 = $ 865,067.

For option 2: $3,504 + $833,917 = $ 837,421.

We should select option 2.

Example for Section 18.7

The management of Quality Airlines has decided to base its overbooking policy on the stochastic single-period model for perishable products, since this will maximize expected profit. This policy now needs to be applied to a new flight from Seattle to Atlanta. The airplane has 125 seats available for a fare of $250. However, since there commonly are a few no-shows, the airline should accept a few more than 125 reservations. On those occasions when more than 125 people arrive to take the flight, the airline will find volunteers who are willing to be put on a later flight in return for being given a certificate worth $150 toward any future travel on this airline.

Based on previous experience with similar flights, it is estimated that the relative frequency of the number of no-shows will be as shown below.

Number of No-Shows	Relative Frequency
0	5%
1	10%
2	15%
3	15%
4	15%
5	15%
6	10%
7	10%
8	5%

(a) When interpreting this problem as an inventory problem, what are the units of a perishable product being placed into inventory?

When interpreting this problem as an inventory problem, the individual seat reservations being made available for a particular flight are the units of a perishable product that is being placed into inventory.

(b) Identify the unit cost of underordering and the unit cost of overordering.

C_{under} = lost fare = \$250.

C_{over} = cost of certificate = \$150.

(c) Use the stochastic single-period model for perishable products with these costs to determine how many overbooked reservations to accept.

Service level for accepting 0 = 0.05
Service level for accepting 1 = 0.05 + 0.1 = 0.15.
Service level for accepting 2 = 0.15 + 0.15 = 0.3.
Service level for accepting 3 = 0.3 + 0.15 = 0.45.
Service level for accepting 4 = 0.45 + 0.15 = 0.6.
Service level for accepting 5 = 0.6 + 0.15 = 0.75.
Service level for accepting 6 = 0.75 + 0.1 = 0.85.
Service level for accepting 7 = 0.85 + 0.1 = 0.95.
Service level for accepting 8 = 0.95 + 0.05 = 1.

$$\text{Optimal service level} = \frac{C_{\text{under}}}{C_{\text{under}} + C_{\text{over}}} = \frac{250}{250 + 150} = 0.625.$$

The results for this model when the demand is an integer-valued random variable indicate that the smallest integer order quantity whose service level is ≥ this optimal service level should be used. Therefore, Quality Airlines should accept 5 overbooked reservations.

(d) Draw a graph of the CDF of demand to show the application of the model graphically.