Solved Examples for Chapter 16

Example for Section 16.2

Consider a decision analysis problem whose payoffs (in units of thousands of dollars) are given by the following payoff table:

Alternative	State of Nature S_1	S_2
A_1	80	25
A_2	30	50
A_3	60	40
Prior probability	0.4	0.6

(a) Which alternative should be chosen under the maximin payoff criterion?

Under the maximin payoff criterion, the following template shows that alternative A3 should be chosen because 40 is the maximum in the Minimum in Row column.

	A	B	C	D	E	F	G	H	I	J
1	**Template for Maximin Payoff Criterion**									
2										
3					State of Nature			Minimum		
4		Alternative	S1	S2				in Row		
5		A1	80	25				25		
6		A2	30	50				30		
7		A3	60	40				40	Maximin	
8										
9										
10										
11										

(b) Which alternative should be chosen under the maximum likelihood criterion?

Under the maximum likelihood criterion, the following template shows that alternative A2 should be chosen because it has the maximum payoff for the state of nature that has the maximum prior probability.

Template for Maximum Likelihood Criterion

Alternative	State of Nature S1	S2	
A1	80	25	Maximum
A2	30	50	
A3	60	40	
Prior Probability	0.4	0.6	
	Maximum		

(c) Which alternative should be chosen under Bayes' decision rule?

Under the Bayes’ decision rule, the following template shows that alternative A3 should be chosen because it has the maximum expected payoff.

Template for Bayes' Decision Rule

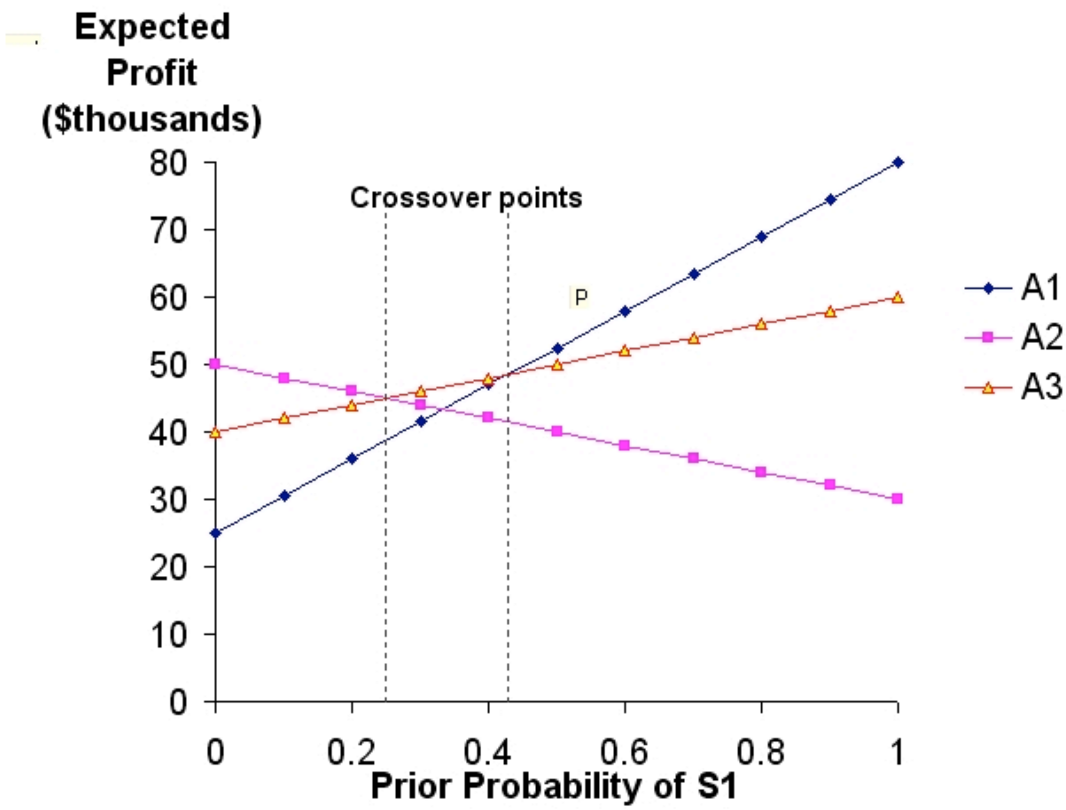
Alternative	State of Nature S1	S2	Expected Payoff	
A1	80	25	47	
A2	30	50	42	
A3	60	40	48	Maximum
Prior Probability	0.4	0.6		

(d) Using Bayes' decision rule, do sensitivity analysis graphically with respect to the prior probabilities to determine the crossover points where the decision shifts from one alternative to another.

The graph of sensitivity analysis is shown below. From the graph, we can see that

A_2 and A_3 cross at approximately $p = 0.25$,

A_1 and A_3 cross at approximately $p = 0.43$.



(*e*) Use algebra to solve for the crossover points identified in part (*d*).

Let p be the prior probability of S_1.

For A_1: EP = 80p + 25(1-p) = 55p + 25.
For A_2: EP = 30p + 50(1-p) = -20p + 50.
For A_3: EP = 60p + 40(1-p) = 20p + 40.

A_2 and A_3 cross when $-20p + 50 = 20p + 40 \Rightarrow p = 0.25$.

A_1 and A_3 cross when $55p + 25 = 20p + 40 \Rightarrow p = 0.429$.

Therefore, the optimal policy is to choose A_2 when $p < 0.25$, choose A_3 when $0.25 \leq p < 0.429$, and choose A_1 when $p \geq 0.429$.

Example for Section 16.3

A new type of photographic film has been developed. It is packaged in sets of five sheets, where each sheet provides an instantaneous snapshot. Because this process is new, the manufacturer has attached an additional sheet to the package, so that the store may test one sheet before it sells the package of five. In promoting the film, the manufacture offers

to refund the entire purchase price of the film if one of the five is defective. This refund must be paid by the camera store, and the selling price has been fixed at \$2 if this guarantee is to be valid. The camera store may sell the film for \$1 if the preceding guarantee is replaced by one that pays \$0.20 for each defective sheet. The cost of the film to the camera store is \$0.40, and the film is not returnable. The store may choose any one of three actions:

1. Scrap the film.
2. Sell the film for \$2.
3. Sell the film for \$1.

(a) If the six states of nature correspond to 0, 1, 2, 3, 4, and 5 defective sheets in the package, complete the following payoff table.

	State of Nature					
Alternative	**0**	**1**	**2**	**3**	**4**	**5**
1	-0.40					
2	1.60		-.40			
3	0.60	0.40		0.00		

(a) The payoff table is given below.

	State of Nature					
Alternative	**0**	**1**	**2**	**3**	**4**	**5**
1	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40
2	1.60	-0.40	-0.40	-0.40	-0.40	-0.40
3	0.60	0.40	0.20	0.00	-0.20	-0.40

(b) The store has accumulated the following information on sales of 60 such packages.

Quality of	**State of Nature**					
Attached Sheet	**0**	**1**	**2**	**3**	**4**	**5**
Good	10	8	6	4	2	0
Bad	0	2	4	6	8	10
Total	10	10	10	10	10	10

These data indicate that each state of nature is equally likely, so that this prior distribution can be assumed. What is the optimal decision alternative for a package of film?

Since the data indicate that the states of nature are equally likely, the expected payoff of each alternative is calculated as follows:

$$E[\text{payoff(alternative 1)}] = -0.40.$$

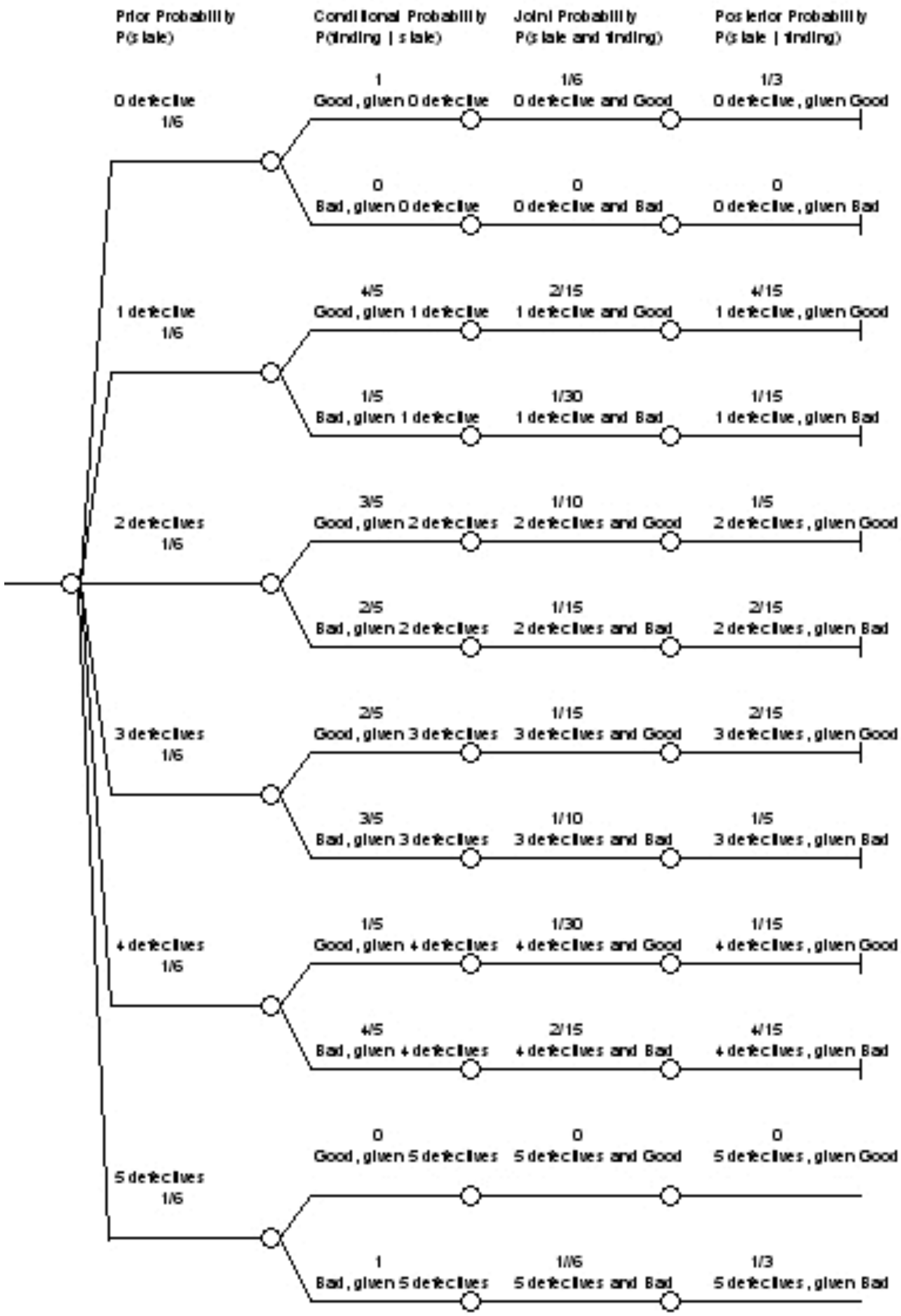
$$E[\text{payoff(alternative 2)}] = \frac{1}{6}(1.60) + \frac{5}{6}(-0.40) = -0.067.$$

$$E[\text{payoff(alternative 3)}] = \frac{1}{6}(0.60 + 0.40 + 0.20 + 0 - 0.20 - 0.40) = 0.10.$$

The optimal decision alternative is alternative 3, selling the film for $1.

(c) Now assume that the attached sheet is tested. Use a probability tree diagram to find the posterior probabilities of the state of nature for each of the two possible outcomes of this testing.

The probability tree diagram and the resulting posterior probabilities are given next.


Prior Probability
P(state)
Conditional Probability
P(finding | state)
Joint Probability
P(state and finding)
Posterior Probability
P(state | finding)
0 defective
1/6
1
Good, given 0 defective
1/6
0 defective and Good
1/3
0 defective, given Good
0
Bad, given 0 defective
0
0 defective and Bad
0
0 defective, given Bad
1 defective
1/6
4/5
Good, given 1 defective
2/15
1 defective and Good
4/15
1 defective, given Good
1/5
Bad, given 1 defective
1/30
1 defective and Bad
1/15
1 defective, given Bad
2 defectives
1/6
3/5
Good, given 2 defectives
1/10
2 defectives and Good
1/5
2 defectives, given Good
2/5
Bad, given 2 defectives
1/15
2 defectives and Bad
2/15
2 defectives, given Bad
3 defectives
1/6
2/5
Good, given 3 defectives
1/15
3 defectives and Good
2/15
3 defectives, given Good
3/5
Bad, given 3 defectives
1/10
3 defectives and Bad
1/5
3 defectives, given Bad
4 defectives
1/6
1/5
Good, given 4 defectives
1/30
4 defectives and Good
1/15
4 defectives, given Good
4/5
Bad, given 4 defectives
2/15
4 defectives and Bad
4/15
4 defectives, given Bad
5 defectives
1/6
0
Good, given 5 defectives
0
5 defectives and Good
0
5 defectives, given Good
1
Bad, given 5 defectives
1/6
5 defectives and Bad
1/3
5 defectives, given Bad

(d) What is the optimal expected payoff for a package of film if the attached sheet is tested? What is the optimal decision alternative if the sheet is good? If it is bad?

If the test is Good, then

$$E[\text{payoff(alternative 1)} \mid \text{test is Good}] = -0.40.$$

$$E[\text{payoff(alternative 2)} \mid \text{test is Good}] = \frac{1}{3}(0.60) + \frac{2}{3}(-0.40) = 0.267.$$

$$E[\text{payoff(alternative 3)} \mid \text{test is Good}] = \frac{1}{3}(0.60) + \frac{4}{15}(0.40) + \frac{1}{5}(0.20) + \frac{2}{15}(0) + \frac{1}{15}(-0.20) = 0.333.$$

If the test is Bad, then

$$E[\text{payoff(alternative 1)} \mid \text{test is Bad}] = -0.40.$$

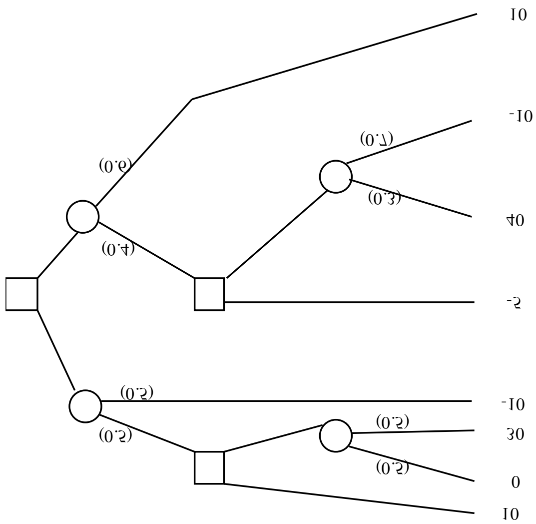
$$E[\text{payoff(alternative 2)} \mid \text{test is Bad}] = 0(0.60) + 1(-0.40) = -0.40.$$

$$E[\text{payoff(alternative 3)} \mid \text{test is Bad}] = 0 + \frac{1}{15}(0.40) + \frac{2}{15}(0.20) + \frac{1}{5}(0) + \frac{4}{15}(-0.20) + \frac{1}{3}(-0.40) = -0.133.$$

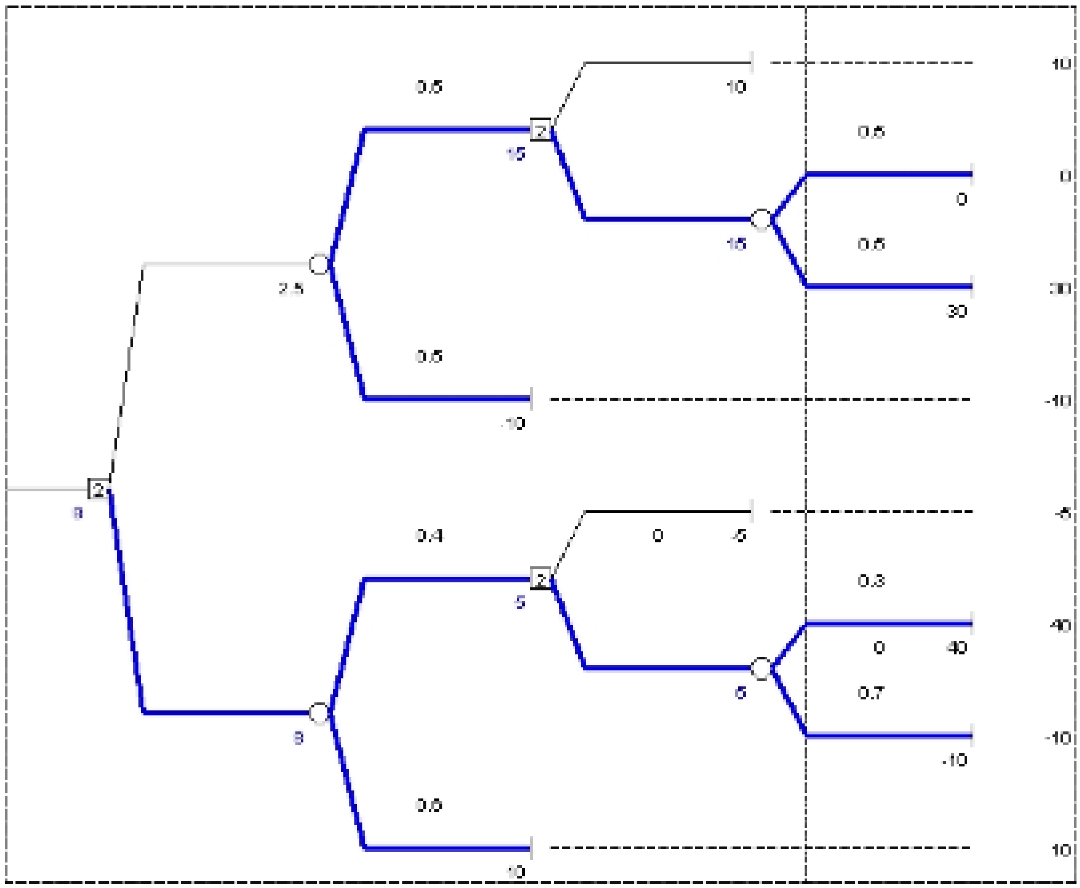
In both cases, the optimal decision alternative is alternative 3, selling the film for $1. The expected payoff is 0.1.

Example for Section 16.4

Consider the decision tree below, with the probabilities at event nodes shown in parentheses and with the payoffs at terminal points shown on the right. Analyze this decision tree to obtain the optimal policy.



This decision tree can be solved either by hand or by such software as ASPE. The optimal policy is indicated in the following decision tree by the number inside the square box at each of the three decision nodes. Since this number is 2 in each case, the optimal decision at each decision node is the one that corresponds to taking the *lower* branch out of that node. This is further emphasized in the following decision tree by having the branch to follow out of each decision node be the thicker branch. The expected payoff for this optimal policy is 8, as indicated by the 8 next to the decision node on the left that initiates the decision tree.



0.5
10
10
15
0.5
0
0
15
0.5
30
30
2.5
0.5
-10
-10
8
0.4
0
-5
-5
5
0.3
40
0
40
5
0.7
-10
-10
8
0.6
10
10