ADDITIONAL CASES

CASE 7.2 FARM MANAGEMENT

The Ploughman family owns and operates a 640-acre farm that has been in the family for several generations. The Ploughmans always have had to work hard to make a decent living from the farm and have had to endure some occasional difficult years. Stories about earlier generations overcoming hardships due to droughts, floods, etc., are an important part of the family history. However, the Ploughmans enjoy their self-reliant lifestyle and gain considerable satisfaction from continuing the family tradition of successfully living off the land during an era when many family farms are being abandoned or taken over by large agricultural corporations.

John Ploughman is the current manager of the farm while his wife Eunice runs the house and manages the farm's finances. John's father, Grandpa Ploughman, lives with them and still puts in many hours working on the farm. John and Eunice's older children, Frank, Phyllis, and Carl, also are given heavy chores before and after school.

The entire famiy can produce a total of 4,000 person-hours worth of labor during the winter and spring months and 4,500 person-hours during the summer and fall. If any of these person-hours are not needed, Frank, Phyllis, and Carl will use them to work on a neighboring farm for $5 per hour during the winter and spring months and $5.50 per hour during the summer and fall.

The farm supports two types of livestock: dairy cows and laying hens, as well as three crops: soybeans, corn, and wheat. (All three are cash crops, but the corn also is a feed crop for the cows and the wheat also is used for chicken feed.) The crops are harvested during the late summer and fall. During the winter months, John, Eunice, and Grandpa make a decision about the mix of livestock and crops for the coming year.

Currently, the family has just completed a particularly successful harvest which has provided an investment fund of $20,000 that can be used to purchase more livestock. (Other money is available for ongoing expenses, including the next planting of crops.) The family currently has 30 cows valued at $35,000 and 2,000 hens valued at $5,000. They wish to keep all this livestock and perhaps purchase more. Each new cow would cost $1,500, and each new hen would cost $3.

Over a year's time, the value of a herd of cows will decrease by about 10 percent and the value of a flock of hens will decrease by about 25 percent due to aging.

Each cow will require 2 acres of land for grazing and 10 person-hours of work per month, while producing a net annual cash income of $850 for the family. The corresponding figures for each hen are: no significant acreage, 0.05 person-hour per month, and an annual net cash income of $4.25. The chicken house can accommodate a maximum of 5,000 hens, and the size of the barn limits the herd to a maximum of 42 cows.

For each acre planted in each of the three crops, the following table gives the number of person-hours of work that will be required during the first and second halves of the year, as well as a rough estimate of the crop's net value (in either income or savings in purchasing feed for the livestock).

Data per acre planted

	Soybeans	Corn	Wheat
Winter and spring, person-hours	1.0	0.9	0.6
Summer and fall, person-hours	1.4	1.2	0.7
Net value	$70	$60	$40

To provide much of the feed for the livestock, John wants to plant at least 1 acre of corn for each cow in the coming year's herd and at least 0.05 acre of wheat for each hen in the coming year's flock.

John, Eunice, and Grandpa now are discussing how much acreage should be planted in each of the crops and how many cows and hens to have for the coming year. Their objective is to maximize the family's monetary worth at the end of the coming year (the *sum* of the net income from the livestock for the coming year *plus* the net value of the crops for the coming year *plus* what remains from the investment fund *plus* the value of the livestock at the end of the coming year *plus* any income from working on a neighboring farm, *minus* living expenses of $40,000 for the year).

(a) Identify verbally the components of a linear programming model for this problem.
(b) Formulate this model. (Either an algebraic or a spreadsheet formulation is acceptable.)
(c) Obtain an optimal solution and generate the additional output provided for performing postoptimality analysis (e.g., the Sensitivity Report when using Excel). What does the model predict regarding the family's monetary worth at the end of the coming year?
(d) Find the allowable range to stay optimal for the net value per acre planted for each of the three crops.

The above estimates of the net value per acre planted in each of the three crops assumes good weather conditions. Adverse weather conditions would harm the crops and greatly reduce the resulting value. The scenarios particularly feared by the family are a drought, a flood, an early frost, *both* a drought and an early frost, and *both* a flood and an early frost. The estimated net values for the year under these scenarios are shown below.

	Net Value per Acre Planted		
Scenario	**Soybeans**	**Corn**	**Wheat**
Drought	−$10	−$15	0
Flood	$15	$20	$10
Early frost	$50	$40	$30
Drought and early frost	−$15	−$20	−$10
Flood and early frost	$10	$10	$ 5

(e) Find an optimal solution under each scenario after making the necessary adjustments to the linear programming model formulated in part (*b*). In each case, what is the prediction regarding the family's monetary worth at the end of the year?
(f) For the optimal solution obtained under each of the six scenarios [including the good weather scenario considered in parts (*a*) to (*d*)], calculate what the family's monetary worth would be at the end of the year if each of the other five scenarios occur instead. In your judgment, which solution provides the best balance between yielding a large monetary worth under good weather conditions and avoiding an overly small monetary worth under adverse weather conditions.

Grandpa has researched what the weather conditions were in past years as far back as weather records have been kept, and obtained the following data.

Scenario	**Frequency**
Good weather	40%
Drought	20%
Flood	10%
Early frost	15%
Drought and early frost	10%
Flood and early frost	5%

With these data, the family has decided to use the following approach to making its planting and livestock decisions. Rather than the optimistic approach of assuming that good weather conditions will prevail [as done in parts (*a*) to (*d*)], the *average* net value under all weather conditions will be used for each crop (weighting the net values under the various scenarios by the frequencies in the above table).

(g) Modify the linear programming model formulated in part (*b*) to fit this new approach.
(h) Repeat part (*c*) for this modified model.
(i) Use a shadow price obtained in part (*h*) to analyze whether it would be worthwhile for the family to obtain a bank loan with a 10 percent interest rate to purchase more livestock now beyond what can be obtained with the $20,000 from the investment fund.
(j) For each of the three crops, use the postoptimality analysis information obtained in part (*h*) to identify how much latitude for error is available in estimating the net value per acre planted for that crop without changing the optimal solution. Which two net values need to be estimated most carefully? If both estimates are incorrect simultaneously, how close do the estimates need to be to guarantee that the optimal solution will not change?

This problem illustrates a kind of situation that is frequently faced by various kinds of organizations. To describe the situation in general terms, an organization faces an uncertain future where any one of a number of scenarios may unfold. Which one will occur depends on conditions that are outside the control of the organization. The organization needs to choose the levels of various activities, but the unit contribution of each activity to the overall measure of performance is greatly affected by which scenario unfolds. Under these circumstances, what is the best mix of activities?

(k) Think about specific situations outside of farm management that fit this description. Describe one.

CASE 7.3 ASSIGNING STUDENTS TO SCHOOLS (REVISITED)

Reconsider Case 4.3.

The Springfield School Board still has the policy of providing bussing for all middle school students who must travel more than approximately 1 mile. Another current policy is to allow splitting residential areas among multiple schools if this will reduce the total bussing cost. (This latter policy will be reversed in Case 12.4.) However, before adopting a bussing plan based on parts (*a*) and (*b*) of Case 4.3, the school board now wants to conduct some postoptimality analysis.

(a) If you have not already done so for parts (*a*) and (*b*) of Case 4.3, formulate and solve a linear programming model for this problem. (Either an algebraic or a spreadsheet formulation is acceptable.)

(b) Generate a sensitivity analysis report with the same software package as used in part (*a*).

One concern of the school board is the ongoing road construction in area 6. These construction projects have been delaying traffic considerably and are likely to affect the cost of bussing students from area 6, perhaps increasing them as much as 10 percent.

(c) Use the report from part (*b*) to check how much the bussing cost from area 6 to school 1 can increase (assuming no change in the costs for the other schools) before the current optimal solution would no longer be optimal. If the allowable increase is less than 10 percent, re-solve to find the new optimal solution with a 10 percent increase.

(d) Repeat part (*c*) for school 2 (assuming no change in the costs for the other schools).

(e) Now assume that the bussing cost from area 6 would increase by the same percentage for all the schools. Use the report from part (*b*) to determine how large this percentage can be before the current optimal solution might no longer be optimal. If the allowable increase is less than 10 percent, re-solve to find the new optimal solution with a 10 percent increase.

The school board has the option of adding portable classrooms to increase the capacity of one or more of the middle schools for a few years. However, this is a costly move that the board would consider only if it would significantly decrease bussing costs. Each portable classroom holds 20 students and has a leasing cost of $2,500 per year. To analyze this option, the school board decides to assume that the road construction in area 6 will wind down without significantly increasing the bussing costs from that area.

(f) For each school, use the corresponding shadow price from the report obtained in part (*b*) to determine whether it would be worthwhile to add any portable classrooms.

(g) For each school where it is worthwhile to add any portable classrooms, use the report from part (*b*) to determine how many could be added before the shadow price would no longer be valid (assuming this is the only school receiving portable classrooms).

(h) If it would be worthwhile to add portable classrooms to more than one school, use the report from part (*b*) to determine the combinations of the number to add for which the shadow prices definitely would still be valid. Then use the shadow prices to determine which of these combinations is best in terms of minimizing the total cost of bussing students and leasing portable classrooms. Re-solve to find the corresponding optimal solution for assigning students to schools.

(i) If part (*h*) was applicable, modify the best combination of portable classrooms found there by adding one more to the school with the most favorable shadow price. Find the corresponding optimal solution for assigning students to schools and generate the corresponding sensitivity analysis report. Use this information to assess whether the plan developed in part (*h*) is the best one available for minimizing the total cost of bussing students and leasing portable classrooms. If not, find the best plan.

CASE 7.4 WRITING A NONTECHNICAL MEMO

The Profit & Gambit Co. produces cleaning products for home use. This is a highly competitive market, and the company continually struggles to increase its market share. Management has decided to undertake a major new advertising campaign that will focus on the following three key products.

- A spray prewash strain remover.
- A liquid laundry detergent.
- A powder laundry detergent.

This campaign will use both television and the print media. A commercial has been developed to run on national television that will feature the liquid detergent. The advertisement for the print media will promote all three products and will include cents-off coupons which consumers can use to purchase the products at reduced prices. The general goal is to increase the sales of each of these products (but especially the liquid detergent) over the next year by a significant percentage over the past year. Specifically, management has set the following requirements for the campaign.

- Sales of the stain remover should increase by at least 3 percent.
- Sales of the liquid detergent should increase by at least 18 percent.
- Sales of the powder detergent should increase by at least 4 percent.

The following table shows the estimated increase in sales for each *unit* of advertising in the respective outlets. (A *unit* is a standard block of advertising that Profit and Gambit commonly purchases, but fractional amounts also are allowed.) The reason for the −1 percent for the powder detergent in the Television column is that the TV commercial featuring the new liquid detergent will take away some sales from the powder detergent. The bottom row of the table shows the cost per unit of advertising for each of the two outlets.

Management's objective is to determine how much to advertise in each type of medium to meet the sales requirements at a minimum total cost.

Data for the Profit & Gambit Co. advertising-mix problem

	Increase in Sales per Unit of Advertising		
Product	**Television**	**Print Media**	**Minimum Required Increase**
Stain remover	0%	1%	3%
Liquid detergent	3%	2%	18%
Powder detergent	–1%	4%	4%
Unit cost	$1 million	$2 million	

(a) Use graphical analysis to solve this problem.

(b) Use a linear programming software package (such as the Solver, ASPE, MPL/Solvers, LINDO, or LINGO) to solve this problem.

After receiving this information, Profit & Gambit management now wants to analyze the trade-off between the total advertising cost and the resulting benefits achieved by increasing the sales of the three products. Therefore, an OR team (you) has been given the assignment to develop the information that management will need to analyze this trade-off and decide whether it should change any of its policy decisions regarding the required minimum increases in the sales of the three products. In particular, management needs detailed information about how the total advertising cost would change if it were to change any or all of these policy decisions.

(c) For each of the three products in turn, use graphical analysis to determine how much the total advertising cost would change if the required minimum increase in the sales of that product were to be increased by 1 percent (without changing the required minimum increases for the other two products).

(d) Use the linear programming software package chosen in part (*b*) to obtain the information requested in part (*c*). (Continue to use this package in the next three parts.)

(e) For each of the three products in turn, determine how the optimal solution for the model and the resulting total advertising cost would change if the required minimum increase in the sales of that product were to be systematically varied over a range of values (without changing the required minimum increases for the other two products). In each case, start the range of values at 0 percent and increase by 1 percent increments up to double the original minimum required increase. (If your software package is ASPE, its parameter analysis reports provide an efficient way of performing this part.)

(f) Use your software package to generate the usual sensitivity analysis report, and indicate how the report is able to provide the information requested in part (*c*). Also use the report to obtain the *allowable range* for the required minimum increase in the sales of each product over which the corresponding constraint's shadow price remains valid. Interpret how each of these allowable ranges relates to the results obtained in part (*e*).

(g) Suppose that all the original numbers in the rightmost column of the table for the minimum required percentage increase in sales were to be increased simultaneously by the same amount. How large can this amount be before the shadow prices provided by the sensitivity analysis report may no longer be valid?

(h) Below is the beginning of a memorandum from the operations research team to Profit & Gambit management that is intended to provide management with the information it needs to perform its trade-off analysis. Write the rest of this memorandum based on a summary of the results obtained in the preceding parts. Present your information in clear, simple terms that use the language of management. Avoid technical terms such as shadow prices, allowable ranges, and so forth.

MEMORANDUM

To: Profit & Gambit Management
From: The Operations Research Team
Subject: The trade-off between advertising expenditures and increased sales

As instructed, we have been continuing our analysis of the plans for the major new advertising campaign that will focus

on our spray prewash stain remover, liquid formulation laundry detergent, and powder laundry detergent.

Our recent report presented preliminary conclusions on how much advertising to do in the different media to meet the needed increases in sales at a minimum total cost:

Allocate $4 million to advertising on television.
Allocate $6 million to advertising in the print media.
Total advertising cost: $10 million.

We estimate that the resulting increases in sales will be

Stain remover:	3 percent increase in sales
Liquid detergent:	18 percent increase in sales
Powder detergent:	8 percent increase in sales.

You had specified that these increases should be at least 3 percent, 18 percent, and 4 percent, respectively, so we have met the minimum levels for the first two products and substantially exceeded it for the third.

However, you also indicated that your decisions on these minimum increases in sales (3 percent, 18 percent, and 4 percent) had been tentative ones, so you plan now to reevaluate these decisions to see if small changes might improve the trade-off between advertising cost and increased sales.

To assist you in reevaluating your decisions, we now have analyzed this trade-off for each of the three products. Our best estimates are the following.