Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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Chapter 5

NUMERICAL METHODS IN HEAT CONDUCTION

SS-T-CONDUCT PROBLEMS

SS-T-CONDUCT Problem 1

5–28 Consider a large plane wall of thickness $L = 0.4$ m, thermal conductivity $k = 2.3$ W/m·K, and surface area $A = 20$ m^2. The left side of the wall is maintained at a constant temperature of 95°C, while the right side loses heat by convection to the surrounding air at $T_\infty = 15$°C with a heat transfer coefficient of $h = 18$ W/m^2·K. Assuming steady one-dimensional heat transfer and taking the nodal spacing to be 10 cm, (*a*) obtain the finite difference formulation for all nodes, (*b*) determine the nodal temperatures by solving those equations, and (*c*) evaluate the rate of heat transfer through the wall.

5–29 Repeat Prob. 5–28 using the SS-T-CONDUCT (or other) software.

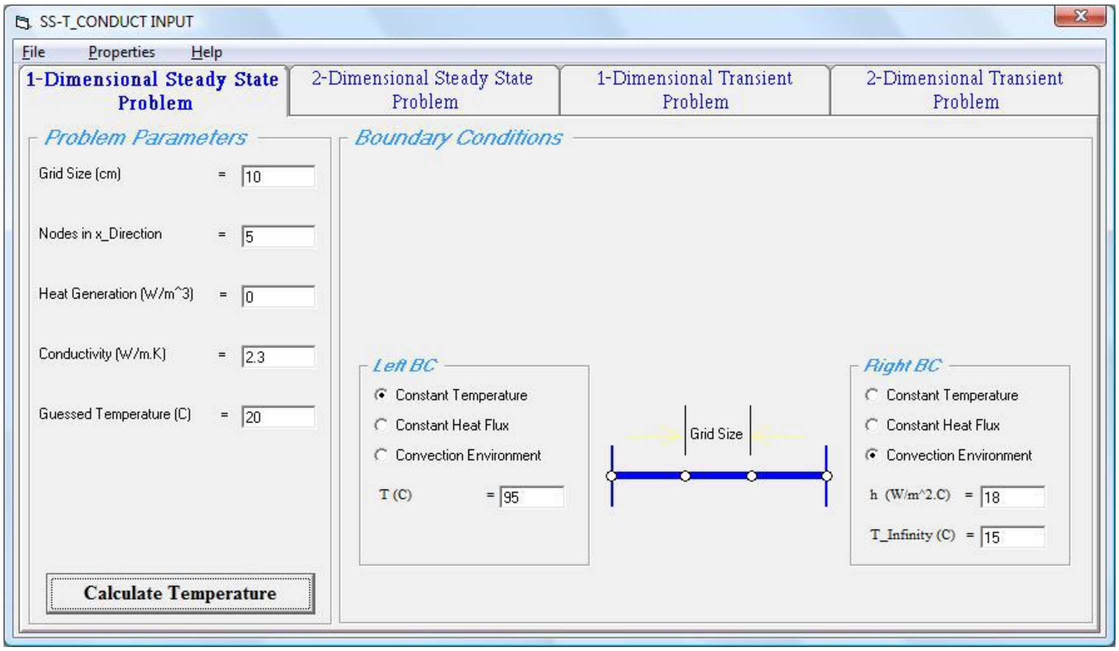
SS-T-CONDUCT Problem 1

Solution

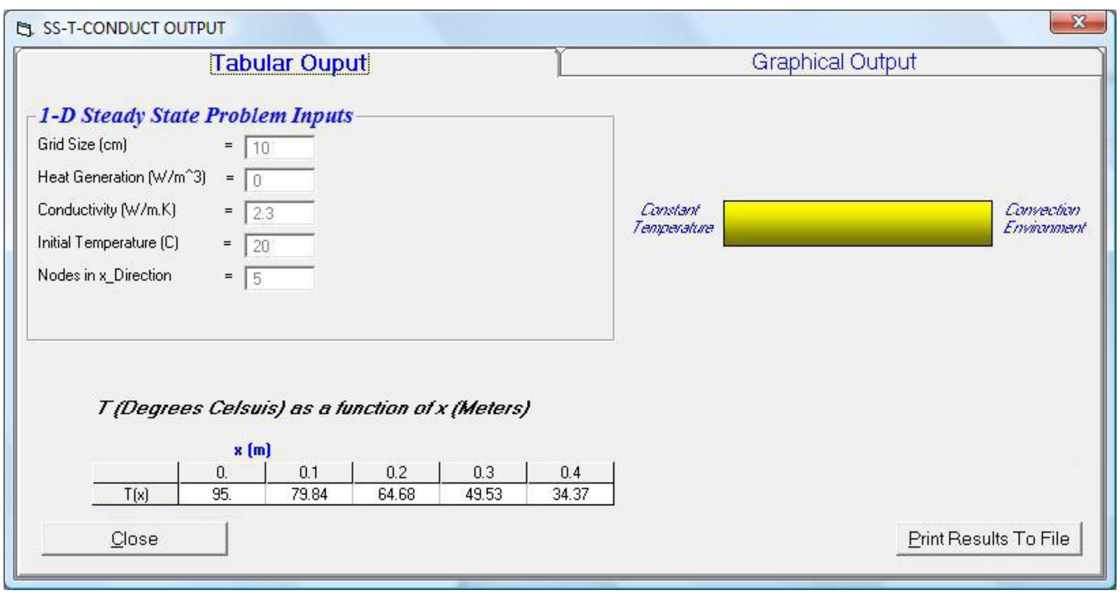
5-29 Prob. 5-28 is reconsidered. The nodal temperatures under steady conditions as well as the rate of heat transfer through the wall are to be determined.

Analysis The problem is solved using SS-T-CONDUCT, and the solution is given below.

On the SS-T-CONDUCT Input window for **1-Dimensional Steady State Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 10 cm, there are 5 nodes in the x direction.



By clicking on the **Calculate Temperature** button, the computed results are as follows.



The rate of heat transfer through the wall is simply convection heat transfer at the right surface,

$$\dot{Q}_{\text{wall}} = \dot{Q}_{\text{conv}} = hA(T_4 - T_\infty) = (18\ \text{W/m}^2.°\text{C})(20\ \text{m}^2)(34.37 - 15)°\text{C} = \mathbf{6970\ W}$$

SS-T-CONDUCT Problem 2

5–64 Consider a 5-m-long constantan block (k = 23 W/m·K) 30 cm high and 50 cm wide (Fig. 5–64 on the next page). The block is completely submerged in iced water at 0°C that is well stirred, and the heat transfer coefficient is so high that the temperatures on both sides of the block can be taken to be 0°C. The bottom surface of the bar is covered with a low-conductivity material so that heat transfer through the bottom surface is negligible. The top surface of the block is heated uniformly by a 8-kW resistance heater. Using the finite difference method with a mesh size of $\Delta x = \Delta y$ = 10 cm and taking advantage of symmetry, (*a*) obtain the finite difference formulation of this problem for steady two-dimensional heat transfer, (*b*) determine the unknown nodal temperatures by solving those equations, and (*c*) determine the rate of heat transfer from the block to the iced water.

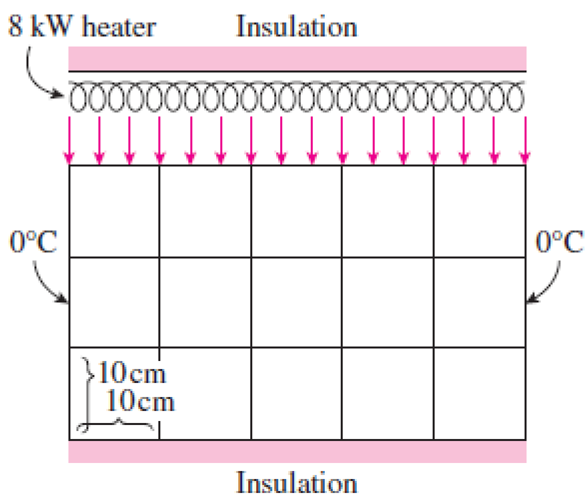


FIGURE P5–64

5–65 Repeat Prob. 5–64 using the SS-T-CONDUCT (or other) software.

SS-T-CONDUCT Problem 2

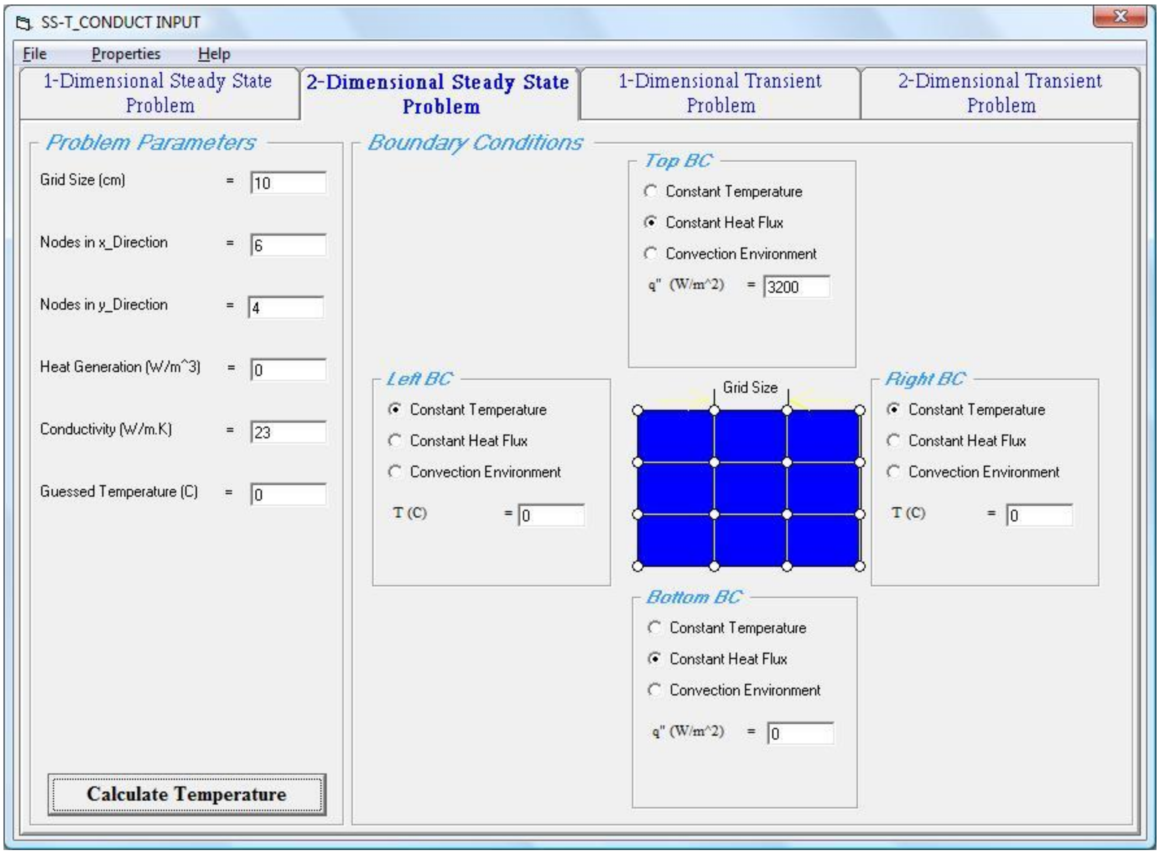
Solution

5-65 Prob. 5-64 is reconsidered. The unknown nodal temperatures as well as the rate of heat transfer to the iced water are to be determined.

Analysis The problem is solved using SS-T-CONDUCT, and the solution is given below.

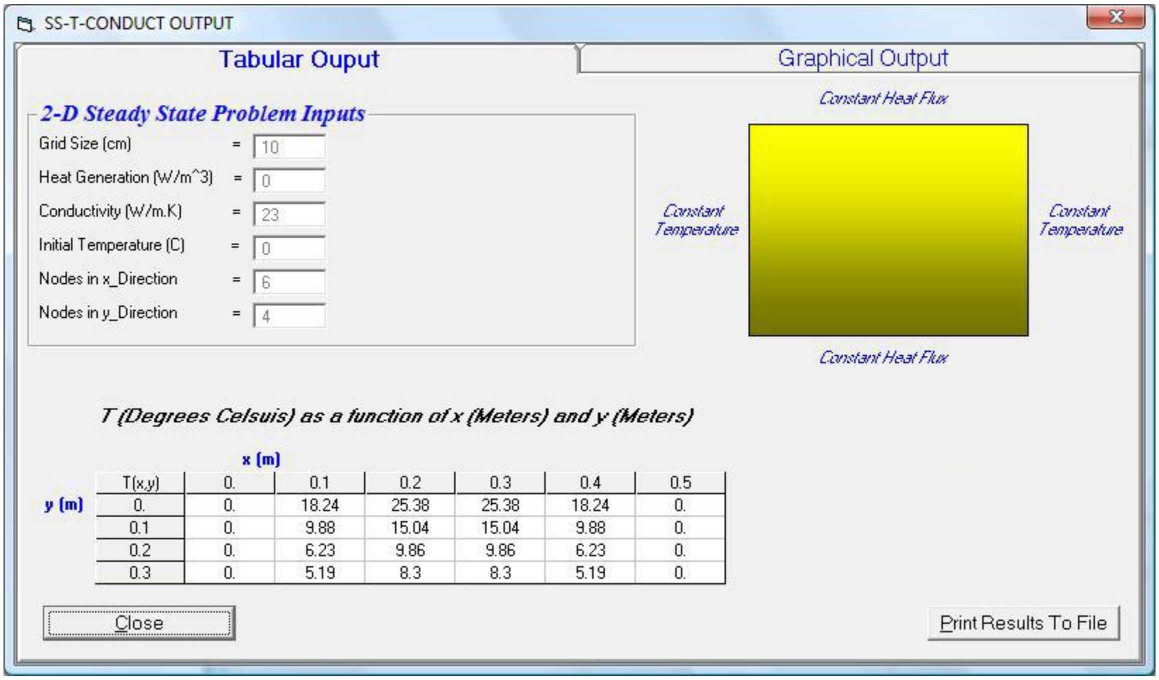
On the SS-T-CONDUCT Input window for **2-Dimensional Steady State Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. With a uniform nodal spacing of 10 cm, there are 6 nodes in the x direction and 4 nodes in the y direction. Note that on the top boundary the heat flux is

$$\dot{q}_0 = \dot{Q}_0 / A = (8000 \text{ W})/(5 \times 0.5 \text{ m}^2) = 3200 \text{ W/m}^2 .$$



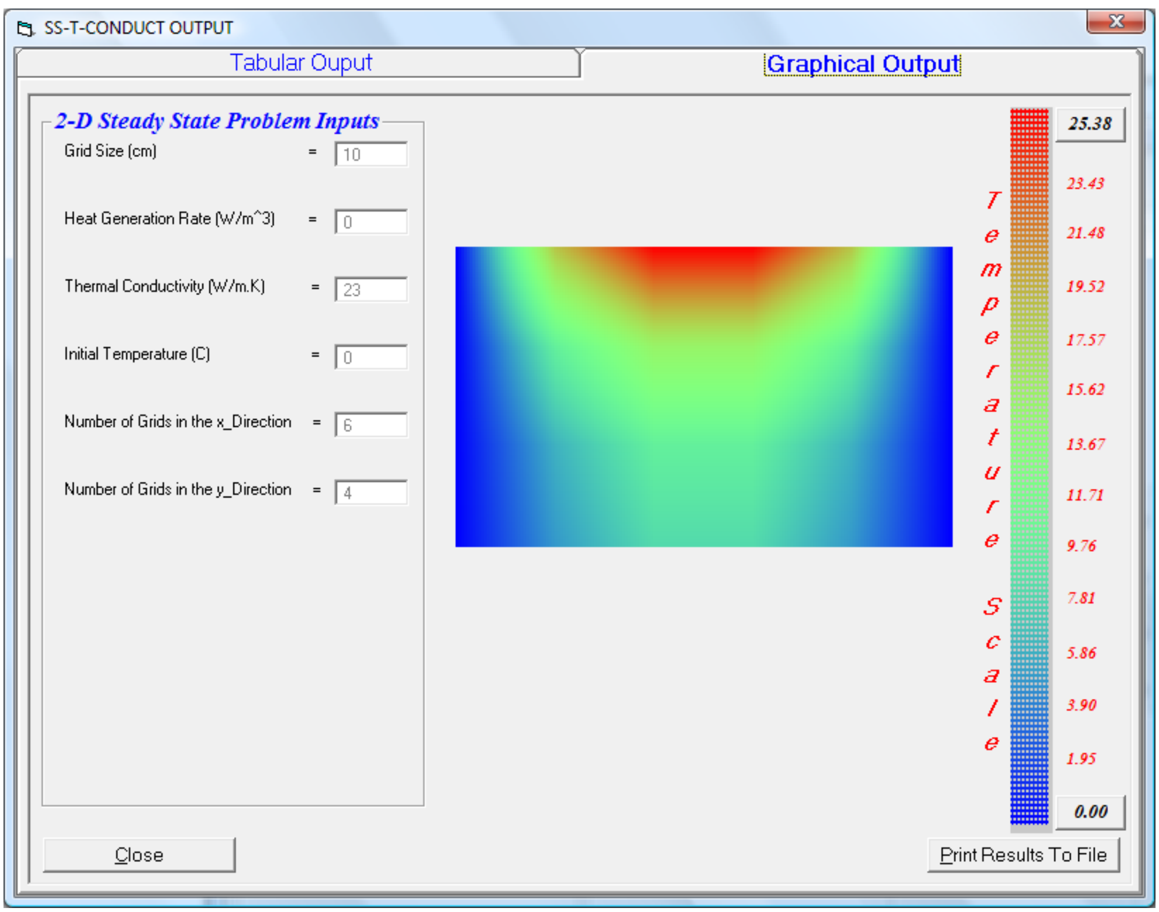
By clicking on the **Calculate Temperature** button, the computed results are as follows.

The rate of heat transfer from the block to the iced water is 8 kW since all the heat supplied to the block from the top must be equal to the heat transferred from the block. Therefore, $\dot{Q} = \mathbf{8\,kW}$.



T(x,y)	0.	0.1	0.2	0.3	0.4	0.5
0.	0.	18.24	25.38	25.38	18.24	0.
0.1	0.	9.88	15.04	15.04	9.88	0.
0.2	0.	6.23	9.86	9.86	6.23	0.
0.3	0.	5.19	8.3	8.3	5.19	0.

The temperature contour for this problem can be plotted by selecting the **Graphical Output** tab, as follows.



SS-T-CONDUCT Problem 3

5–66 Consider steady two-dimensional heat transfer in a long solid bar (k = 25 W/m·K) of square cross section (3 cm × 3 cm) with the prescribed temperatures at the top, right, bottom, and left surfaces to be 100°C, 200°C, 300°C, and 500°C, respectively. Heat is generated in the bar uniformly at a rate of $\dot{e} = 5 \times 10^6$ W/m^3. Using a uniform mesh size $\Delta x = \Delta y$ = 1 cm determine (*a*) the finite difference equations and (*b*) the nodal temperatures with the Gauss-Seidel iterative method.

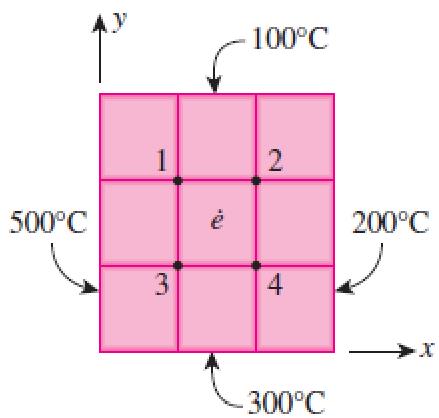


FIGURE P5–66

5–67 Repeat Prob. 5–66 using the SS-T-CONDUCT (or other) software to determine the nodal temperatures.
Answers: T_1 = 297.5°C, T_2 = 222.5°C, T_3 = 347.5°C, T_4 = 272.5°C

SS-T-CONDUCT Problem 3

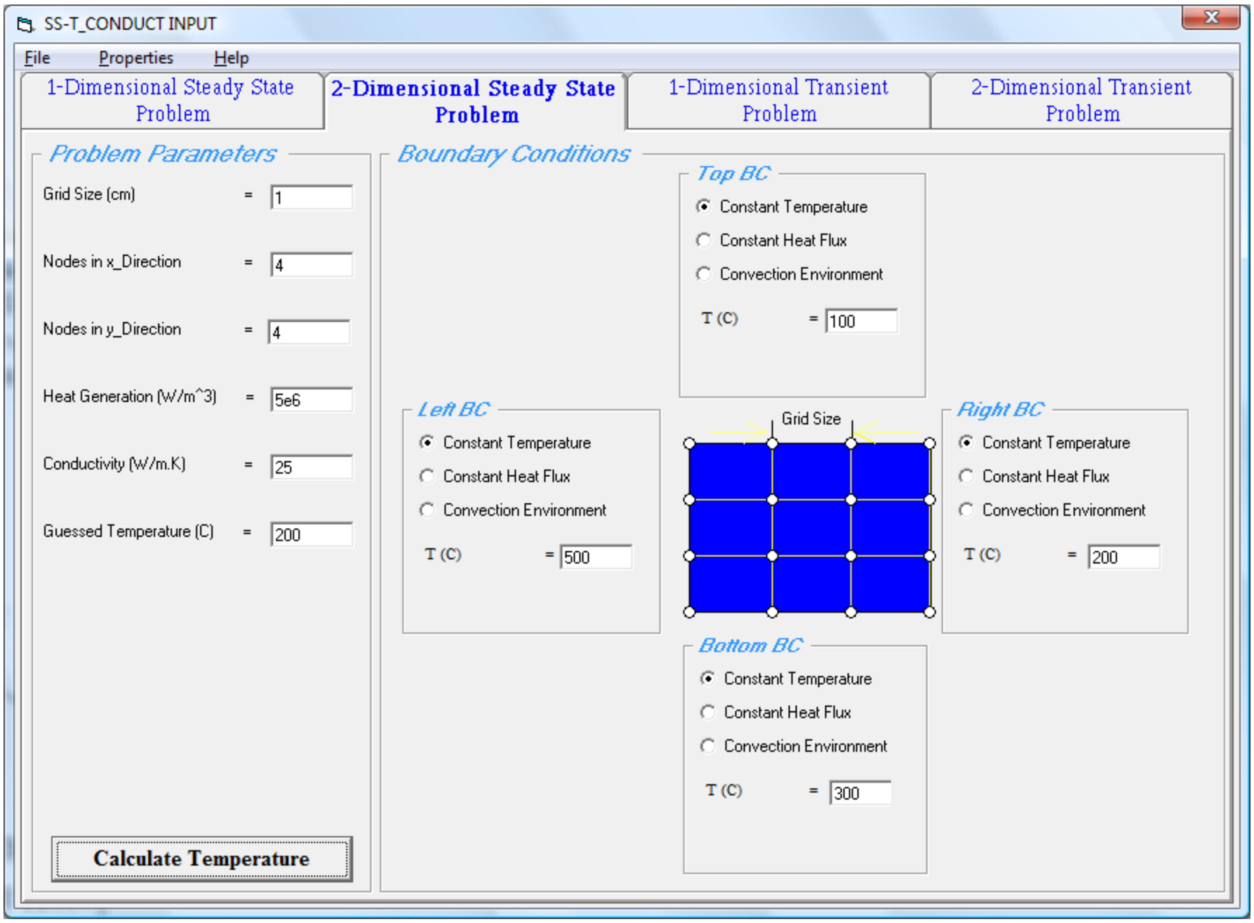
Solution

5-67 Prob. 5-66 is repeated. Using SS-T-Conduct (or other) software, the nodal temperatures are to be solved.

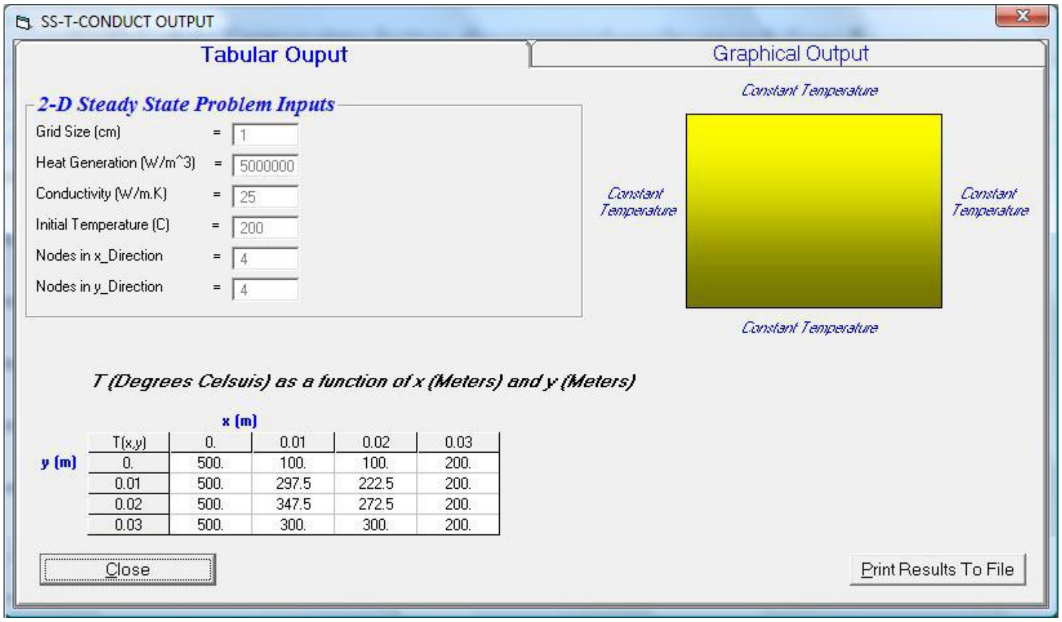
Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** The heat generation in the body is uniform.

Properties The thermal conductivity is given to be $k = 25$ W/m·K.

Analysis On the SS-T-Conduct Input window for **2-Dimensional Steady State Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 1 cm, there are 4 nodes in each the *x* and *y* directions.



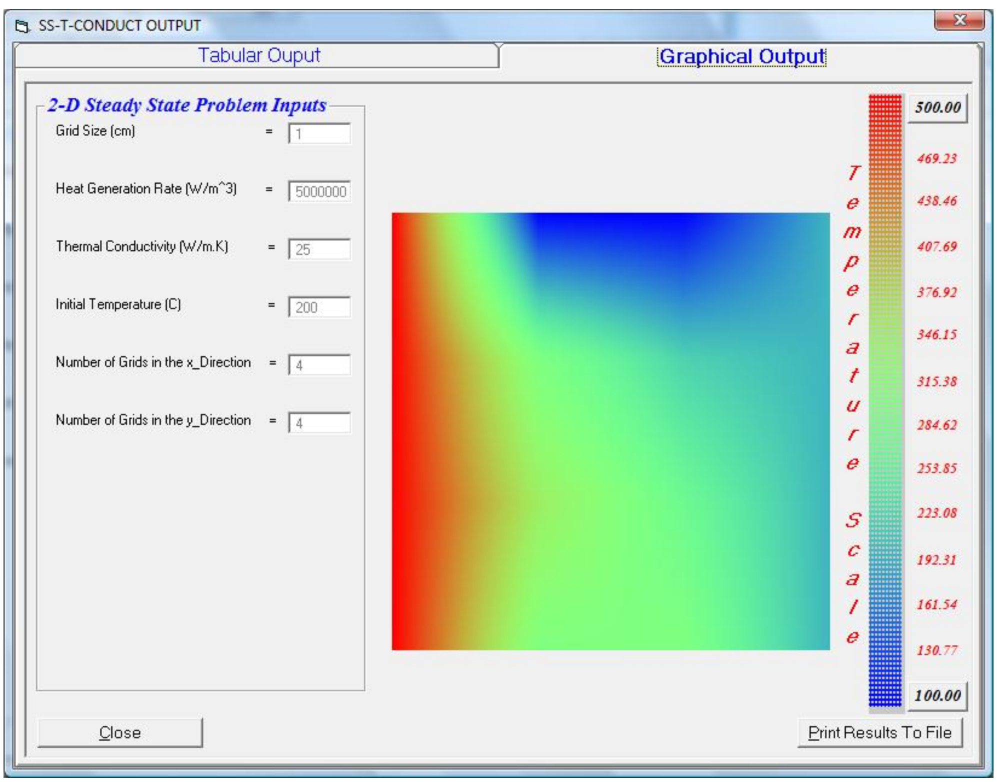
By clicking on the **Calculate Temperature** button, the computed results are as follows.



Hence, the converged nodal temperatures are

$$T_1 = \mathbf{297.5°C},\quad T_2 = \mathbf{222.5°C},\quad T_3 = \mathbf{347.5°C},\quad T_4 = \mathbf{272.5°C}$$

Discussion The temperature contour for this problem can be plotted by selecting the **Graphical Output** tab, as follows.



SS-T-CONDUCT Problem 4

5–88 A hot brass plate is having its upper surface cooled by impinging jet of air at temperature of 15°C and convection heat transfer coefficient of 220 W/m²·K. The 10-cm-thick brass plate (ρ = 8530 kg/m³, c_p = 380 J/kg·K, k = 110 W/m·K, and $\alpha = 33.9 \times 10^{-6}$ m²/s) had a uniform initial temperature of 650°C, and the lower surface of the plate is insulated. Using a uniform nodal spacing of Δx = 2.5 cm and time step of Δt = 10 s determine (*a*) the implicit finite difference equations and (*b*) the nodal temperatures of the brass plate after 10 seconds of cooling.
Answers: T_0 = 631.2°C, T_1 = 644.7°C, T_2 = 648.5°C, T_3 = 649.6°C, T_4 = 649.8°C

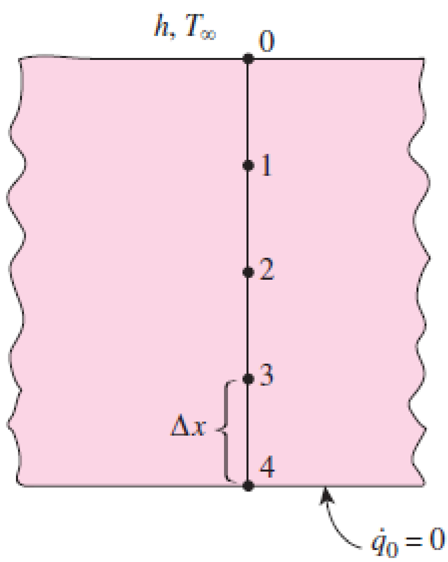


FIGURE P5–88

5–89 Repeat Prob. 5–88. Using SS-T-CONDUCT (or other) software with explicit method, plot the temperature at the surface that is being cooled by the impinging jet as a function of time as it varies from 0 to 60 minutes. How long will it take to cool the surface to 100°C? *Answer:* 3055 s

SS-T-CONDUCT Problem 4

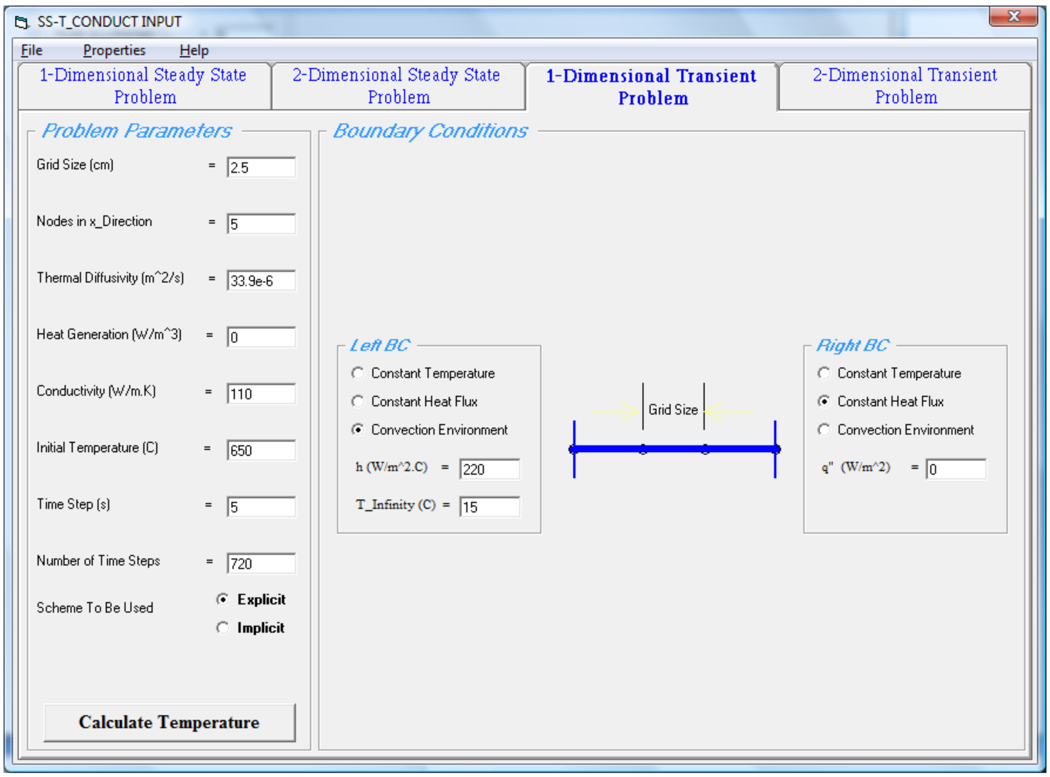
Solution

5-89 Prob. 5-88 is repeated. Using SS-T-Conduct (or other) software with explicit method, the temperature at the surface that is being cooled by the impinging jet as a function of time varying from 0 to 60 minutes is to be plotted. The duration for the surface to be cooled to 100°C is to be determined.

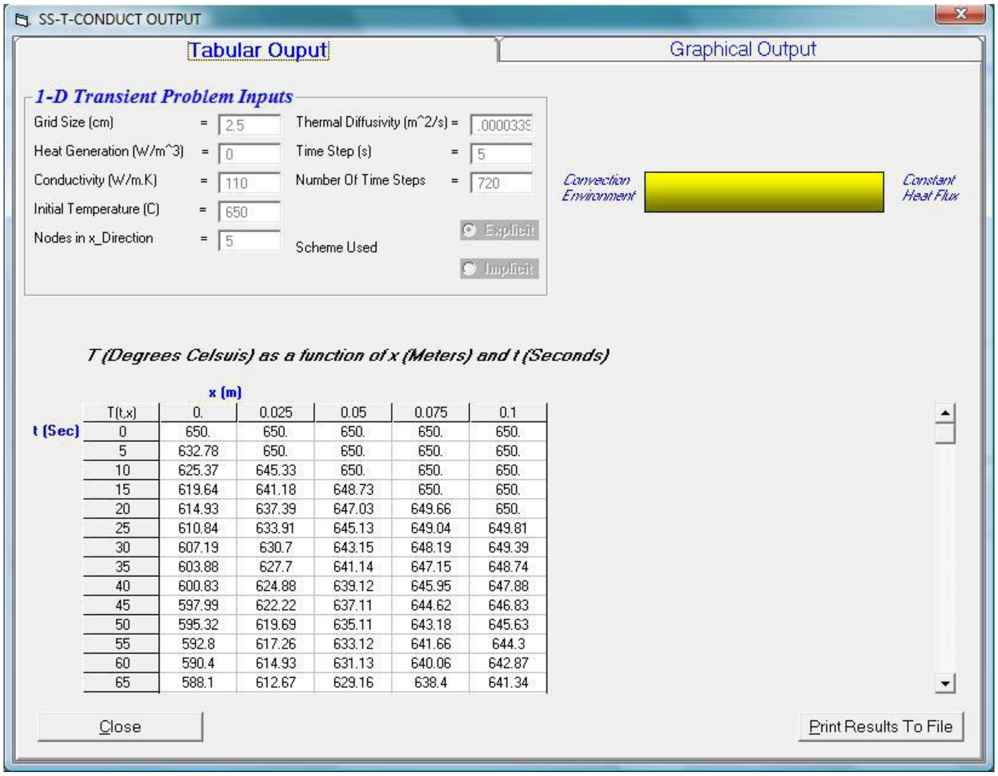
Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** There is no heat generation.

Properties The properties of the brass plate are given as $k = 110$ W/m·K and $\alpha = 33.9 \times 10^{-6}$ m^2/s.

Analysis On the SS-T-Conduct Input window for **1-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes.

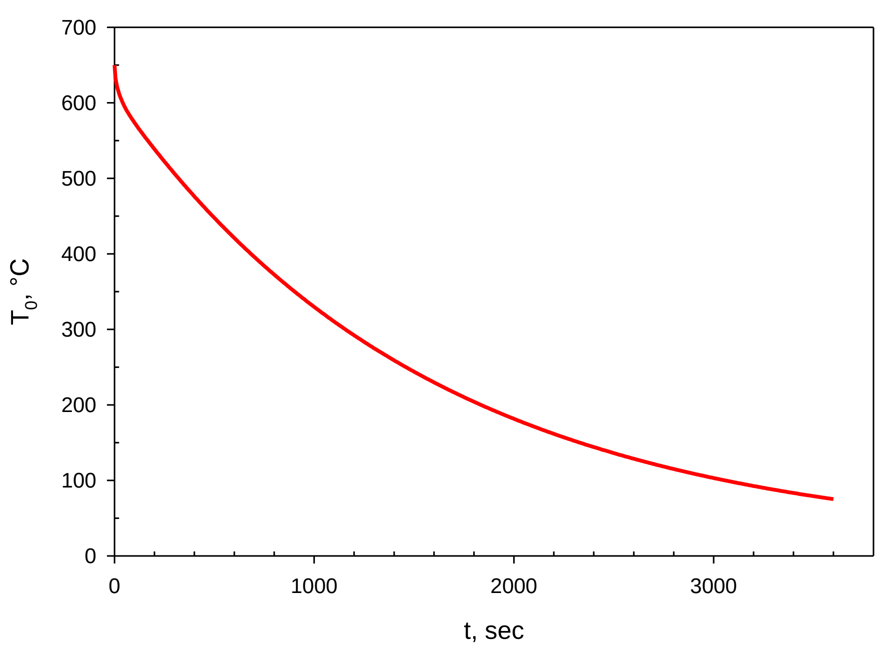


By clicking on the **Calculate Temperature** button, the computed results are as follows.



T(t,x)	0.	0.025	0.05	0.075	0.1
0	650.	650.	650.	650.	650.
5	632.78	650.	650.	650.	650.
10	625.37	645.33	650.	650.	650.
15	619.64	641.18	648.73	650.	650.
20	614.93	637.39	647.03	649.66	650.
25	610.84	633.91	645.13	649.04	649.81
30	607.19	630.7	643.15	648.19	649.39
35	603.88	627.7	641.14	647.15	648.74
40	600.83	624.88	639.12	645.95	647.88
45	597.99	622.22	637.11	644.62	646.83
50	595.32	619.69	635.11	643.18	645.63
55	592.8	617.26	633.12	641.66	644.3
60	590.4	614.93	631.13	640.06	642.87
65	588.1	612.67	629.16	638.4	641.34

The temperature at the surface as a function of time for 0 to 60 minutes is plotted as follows.



From the results computed by the SS-T-Conduct software, the surface temperature reached 100°C at t = **3055 s**.

Discussion When computing with explicit method, the time step should be chosen such that the stability criterion is satisfied. In this problem, the proper time step is $\Delta t \leq 8.779$ s.

SS-T-CONDUCT Problem 5

5–96 Consider a large uranium plate of thickness $L = 9$ cm, thermal conductivity $k = 28$ W/m·K, and thermal diffusivity $\alpha = 12.5 \times 10^{-6}$ m^2/s that is initially at a uniform temperature of 100°C. Heat is generated uniformly in the plate at a constant rate of $\dot{e} = 10^6$ W/m^3. At time $t = 0$, the left side of the plate is insulated while the other side is subjected to convection with an environment at $T_\infty = 20$°C with a heat transfer coefficient of $h = 35$ W/m^2·K. Using the explicit finite difference approach with a uniform nodal spacing of $\Delta x = 1.5$ cm, determine (*a*) the temperature distribution in the plate after 5 min and (*b*) how long it will take for steady conditions to be reached in the plate.

5–97 Repeat Prob. 5–96 using the SS-T-CONDUCT (or other) software.

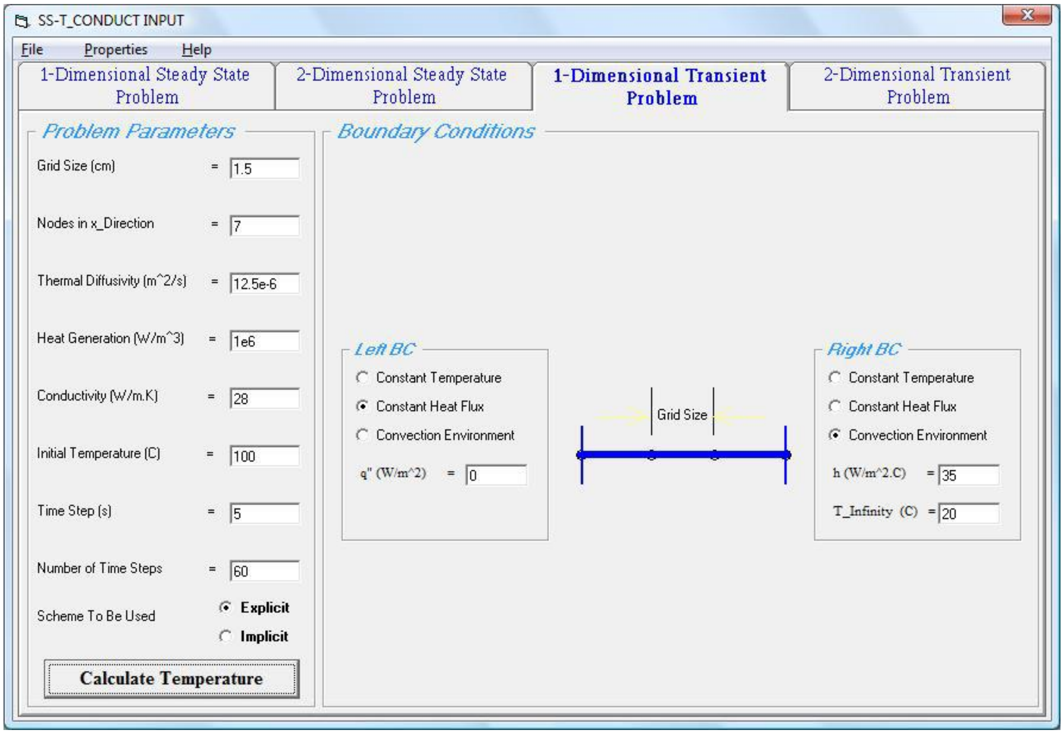
SS-T-CONDUCT Problem 5

Solution

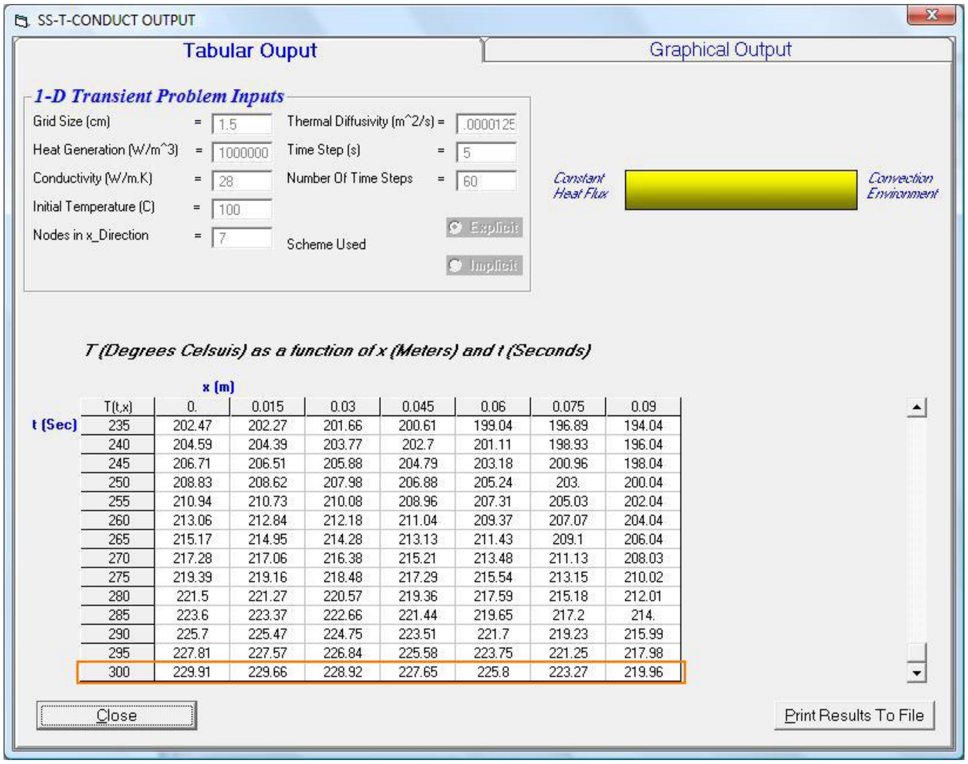
5-97 Prob. 5-96 is reconsidered. The nodal temperatures after 5 min and under steady conditions are to be determined.

Analysis The problem is solved using SS-T-CONDUCT, and the solution is given below.

(*a*) On the SS-T-CONDUCT Input window for **1-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes.



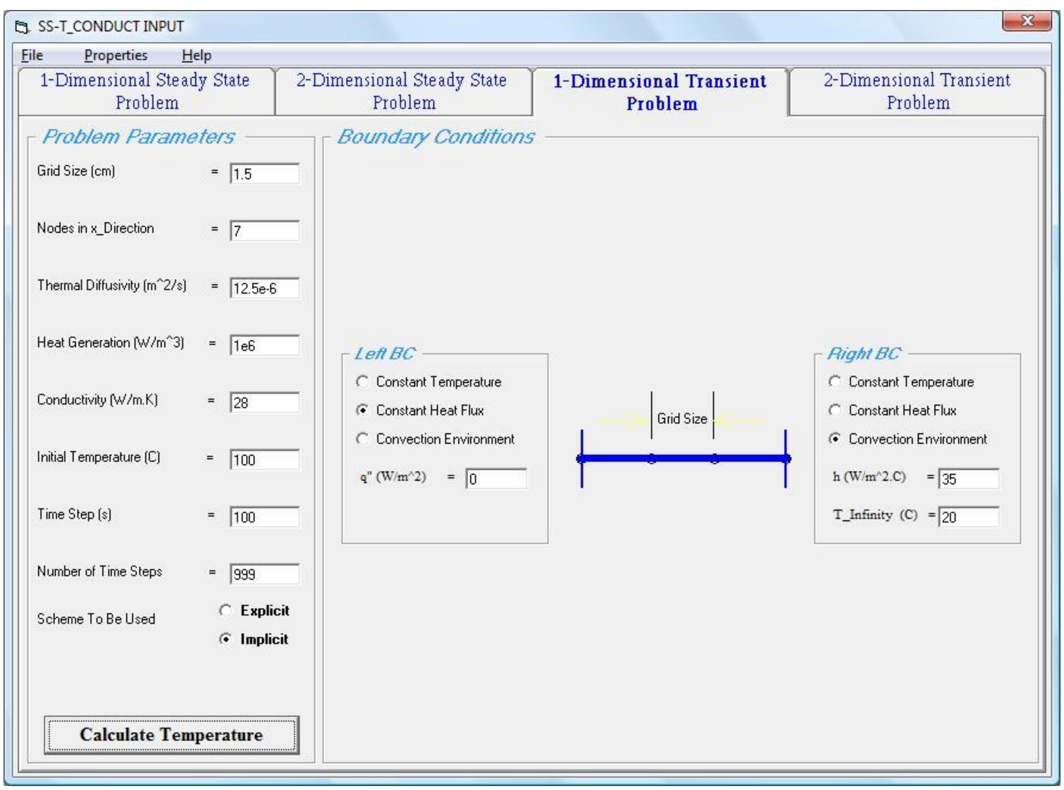
By clicking on the **Calculate Temperature** button, the computed results are as follows.



T(t,x)	0.	0.015	0.03	0.045	0.06	0.075	0.09
235	202.47	202.27	201.66	200.61	199.04	196.89	194.04
240	204.59	204.39	203.77	202.7	201.11	198.93	196.04
245	206.71	206.51	205.88	204.79	203.18	200.96	198.04
250	208.83	208.62	207.98	206.88	205.24	203.	200.04
255	210.94	210.73	210.08	208.96	207.31	205.03	202.04
260	213.06	212.84	212.18	211.04	209.37	207.07	204.04
265	215.17	214.95	214.28	213.13	211.43	209.1	206.04
270	217.28	217.06	216.38	215.21	213.48	211.13	208.03
275	219.39	219.16	218.48	217.29	215.54	213.15	210.02
280	221.5	221.27	220.57	219.36	217.59	215.18	212.01
285	223.6	223.37	222.66	221.44	219.65	217.2	214.
290	225.7	225.47	224.75	223.51	221.7	219.23	215.99
295	227.81	227.57	226.84	225.58	223.75	221.25	217.98
300	229.91	229.66	228.92	227.65	225.8	223.27	219.96

(*b*) The time needed for steady state conditions to be established is determined by using the **Implicit** method, since the number of time steps required to compute by the **Explicit** method, with a time step that does not violate the stability criteria, exceed the maximum allowable number of time steps in SS-T-COUDUCT.

On the SS-T-CONDUCT Input window for **1-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that the **Implicit** method is selected.



The nodal temperatures under steady conditions are determined to be

T_1 = **2736°C**, T_2 = **2732°C**, T_3 = **2720°C**, T_4 = **2700°C**,

T_5 = **2672°C**, T_6 = **2636°C**, and T_7 = **2591°C**

The time needed for steady state conditions to be established is about **60500 s**.

SS-T-CONDUCT Problem 6

5–100 A stainless steel plane wall (k = 15.1 W/m·K) of thickness 1 m experiences a uniform heat generation of 10000 W/m^3. The left and right sides of the wall maintain constant temperatures of 100°C and 20°C, respectively. With a uniform nodal spacing of 10 cm, use SS-T-CONDUCT (or other) software, (a) determine the nodal temperatures and (b) compare the results with analytical solution.

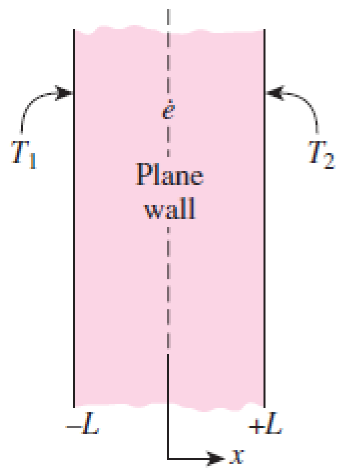


FIGURE P5–100

SS-T-CONDUCT Problem 6

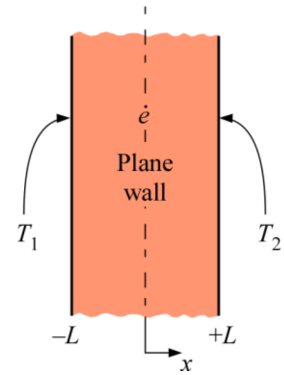
Solution

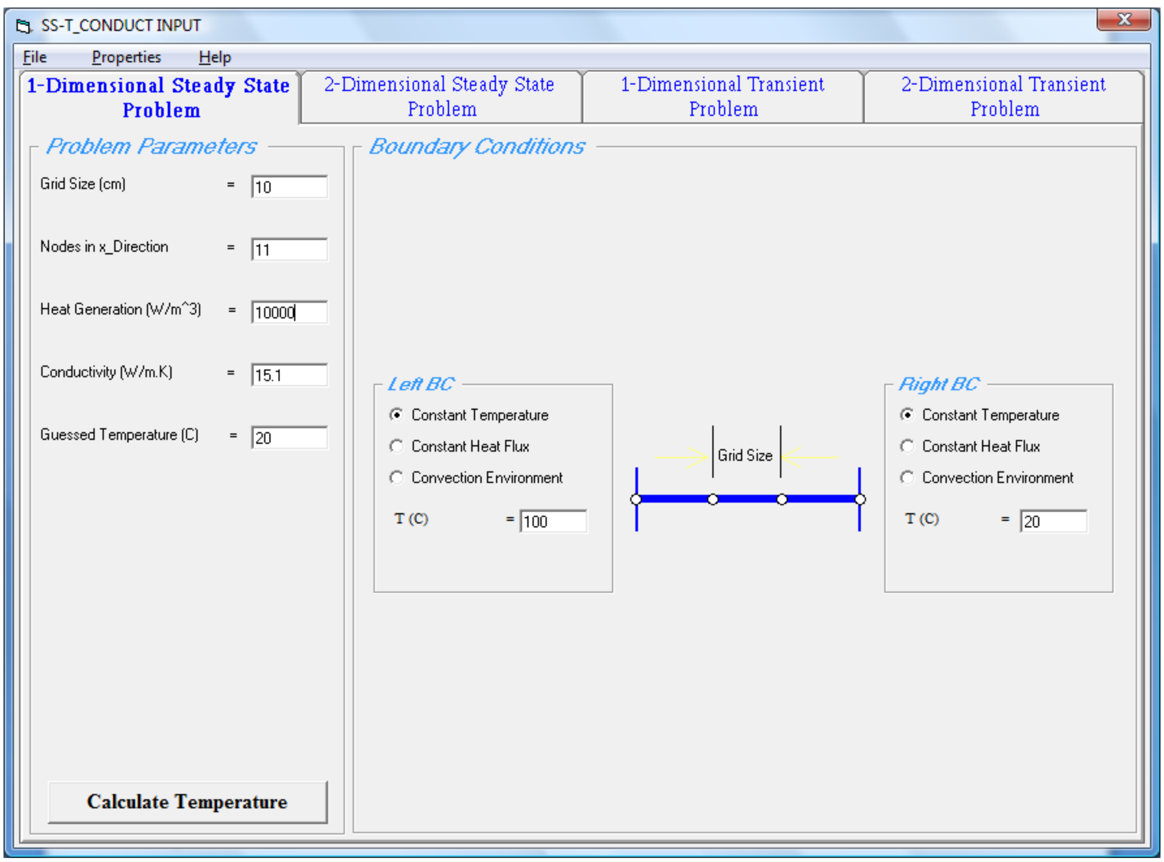
5-100 A stainless steel plane wall experiencing a uniform heat generation is subjected to constant temperatures on both side surfaces. Using SS-T-Conduct (or other) software, the nodal temperatures are to be determined, and compared with analytical solution.

Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The heat generation in the body is uniform.

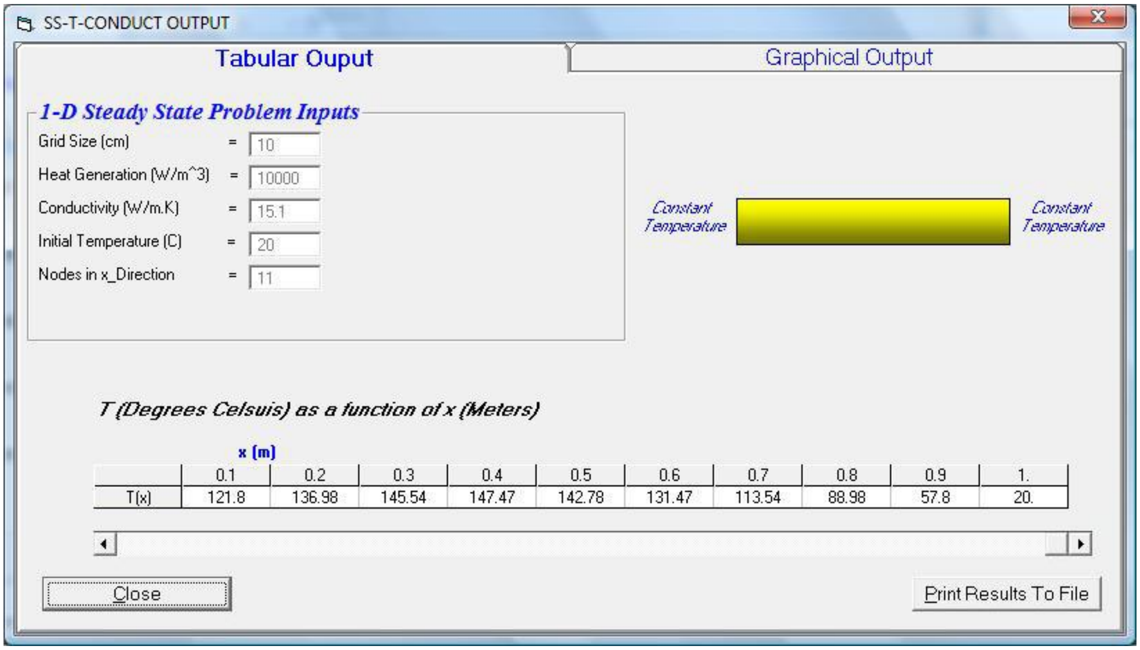
Properties The thermal conductivity is given as 15.1 W/m·K.

Analysis (*a*) On the SS-T-Conduct Input window for **1-Dimensional Steady State Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 10 cm, there are 11 nodes in the x direction.





By clicking on the **Calculate Temperature** button, the computed results are as follows.



(*b*) From Chapter 2, the temperature variation in a plane wall with uniform heat generation is given as

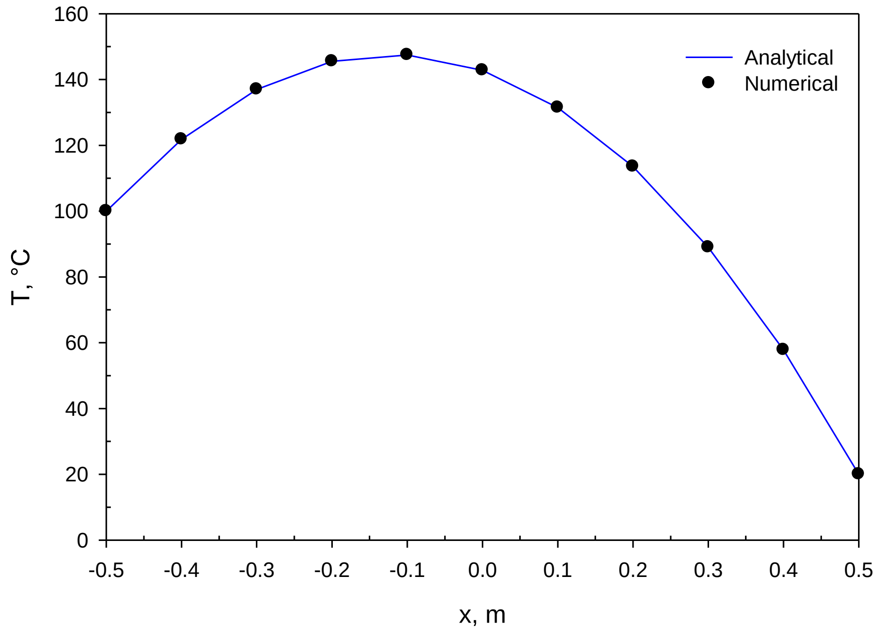
$$T(x) = \frac{\dot{e}_{gen} L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_2 - T_1}{2}\right)\frac{x}{L} + \left(\frac{T_2 + T_1}{2}\right)$$

where $\dot{e}_{gen} = 10000\ \text{W/m}^3$, $k = 15.1\ \text{W/m}\cdot\text{K}$, $L = 0.5\ \text{m}$, $T_1 = 100\,°\text{C}$, $T_2 = 20\,°\text{C}$

The nodal temperatures for analytical and numerical solutions are tabulated in the following table:

	$T(x)$,°C			$T(x)$,°C	
x, m	Analytical	Numerical	x, m	Analytical	Numerical
-0.5	100.00	100.00	0.1	131.47	131.47
-0.4	121.80	121.80	0.2	113.54	113.54
-0.3	136.98	136.98	0.3	88.98	88.98
-0.2	145.54	145.54	0.4	57.80	57.80
-0.1	147.47	147.47	0.5	20.00	20.00
0	142.78	142.78			

The comparison of the analytical and numerical solutions is shown in the following figure:



Discussion The results computed by the SS-T-Conduct software match with the analytical solution from Chapter 2. The temperature variation plot shows that the temperature profile within the wall, for the case with asymmetrical boundary conditions ($T_1 > T_2$), is not symmetric and the maximum temperature occurs to the left of the centerline.

SS-T-CONDUCT Problem 7

5–101 A hot 10-cm-thick brass plate (k = 110 W/m·K and $\alpha = 33.9 \times 10^{-6}$ m^2/s) with uniform heat generation of 3×10^6 W/m^3 and initial temperature of 550°C is having both side surfaces cooled by liquid with temperature of 15°C and convection heat transfer coefficient of 2000 W/m^2·K. With a uniform nodal spacing of Δx = 2.5 cm and using SS-T-CONDUCT (or other) software with implicit method, plot the surface temperature and the temperature at the center as a function of time as it varies from 0 to 10 minutes. What are the nodal temperatures when steady conditions are achieved?

SS-T-CONDUCT Problem 7

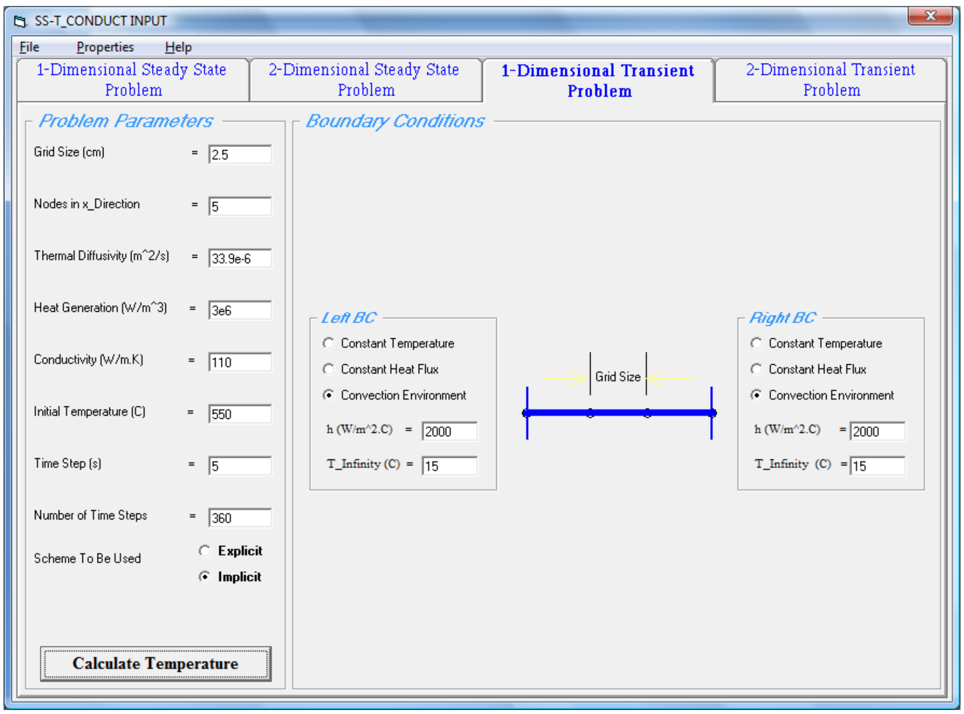
Solution

5-101 A hot 10-cm thick brass plate with uniform heat generation and both side surfaces are being cooled by liquid. Using SS-T-Conduct (or other) software with implicit method, the surface temperature and the temperature at the center as a function of time as it varies from 0 to 10 minutes are to be plotted. The nodal temperatures when steady conditions are achieved are to be determined.

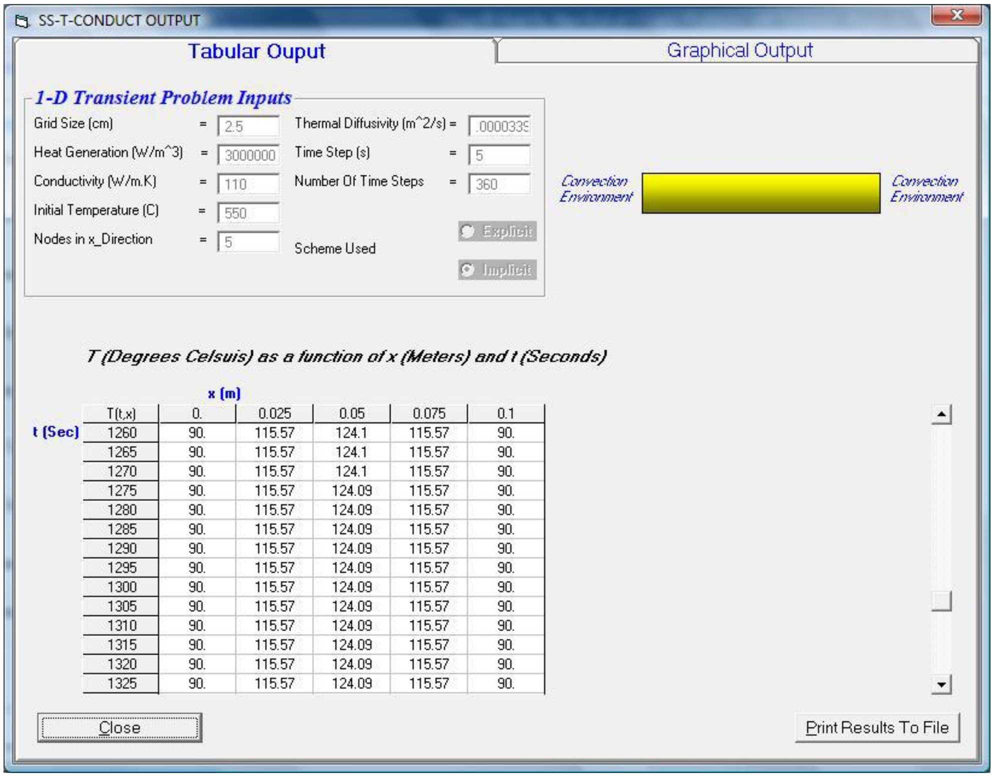
Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** Heat generation is uniform.

Properties The properties of the brass plate are given as $k = 110$ W/m·K and $\alpha = 33.9 \times 10^{-6}$ m^2/s

Analysis On the SS-T-Conduct Input window for **1-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 2.5 cm, there are 5 nodes in the x direction.

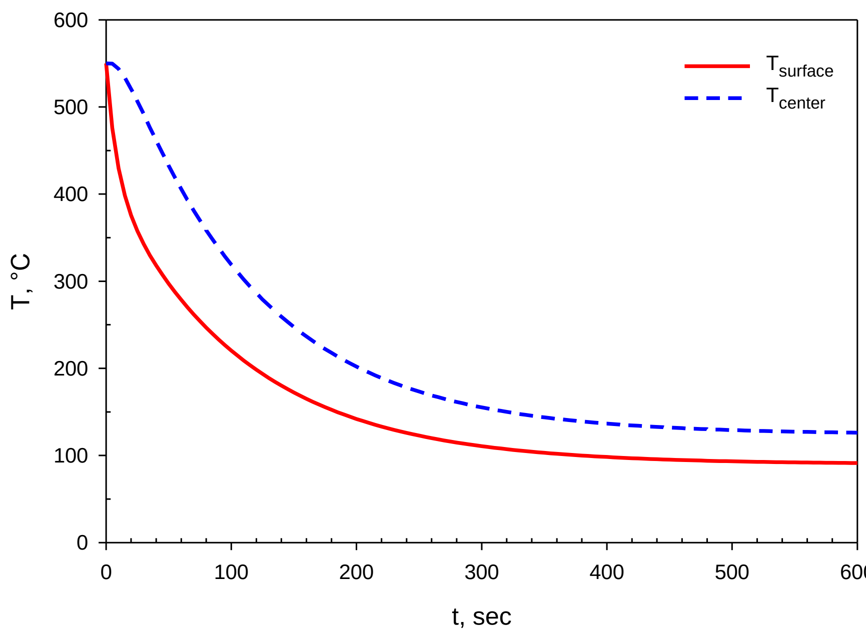


By clicking on the **Calculate Temperature** button, the computed results are as follows.



T(t,x)	0.	0.025	0.05	0.075	0.1
1260	90.	115.57	124.1	115.57	90.
1265	90.	115.57	124.1	115.57	90.
1270	90.	115.57	124.1	115.57	90.
1275	90.	115.57	124.09	115.57	90.
1280	90.	115.57	124.09	115.57	90.
1285	90.	115.57	124.09	115.57	90.
1290	90.	115.57	124.09	115.57	90.
1295	90.	115.57	124.09	115.57	90.
1300	90.	115.57	124.09	115.57	90.
1305	90.	115.57	124.09	115.57	90.
1310	90.	115.57	124.09	115.57	90.
1315	90.	115.57	124.09	115.57	90.
1320	90.	115.57	124.09	115.57	90.
1325	90.	115.57	124.09	115.57	90.

The temperatures at the surface and the center versus time are plotted as follows.



The nodal temperatures when steady conditions are achieved are

$T(0) = T(0.1\text{ m}) = \mathbf{90°C}$, $T(0.025\text{ m}) = T(0.075\text{ m}) = \mathbf{115.6°C}$, $T(0.05\text{ m}) = \mathbf{124.1°C}$,

Discussion Since both sides of the plate are exposed to the same liquid temperature and convection heat transfer coefficient, it is possible to solve half of the plane wall by treating the centerline of the plane wall as symmetry line and get the same results.

SS-T-CONDUCT Problem 8

5–105 Consider a long solid bar (k = 28 W/m·K and α = 12×10^{-6} m^2/s) of square cross section that is initially at a uniform temperature of 32°C. The cross section of the bar is 20 cm × 20 cm in size, and heat is generated in it uniformly at a rate of $\dot{e} = 8 \times 10^5$ W/m^3. All four sides of the bar are subjected to convection to the ambient air at T_∞ = 30°C with a heat transfer coefficient of h = 45 W/m^2·K. Using the explicit finite difference method with a mesh size of $\Delta x = \Delta y$ = 10 cm, determine the centerline temperature of the bar (a) after 20 min and (b) after steady conditions are established.

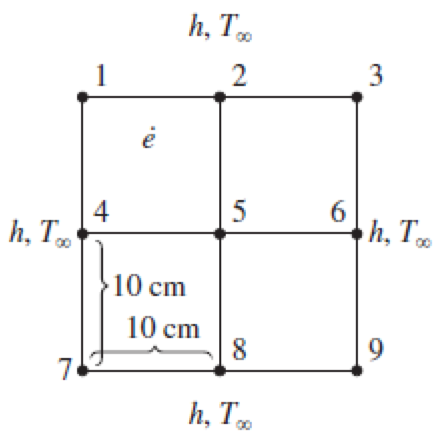


FIGURE P5–105

5–106 Repeat Prob. 5–105 using the SS-T-CONDUCT (or other) software.

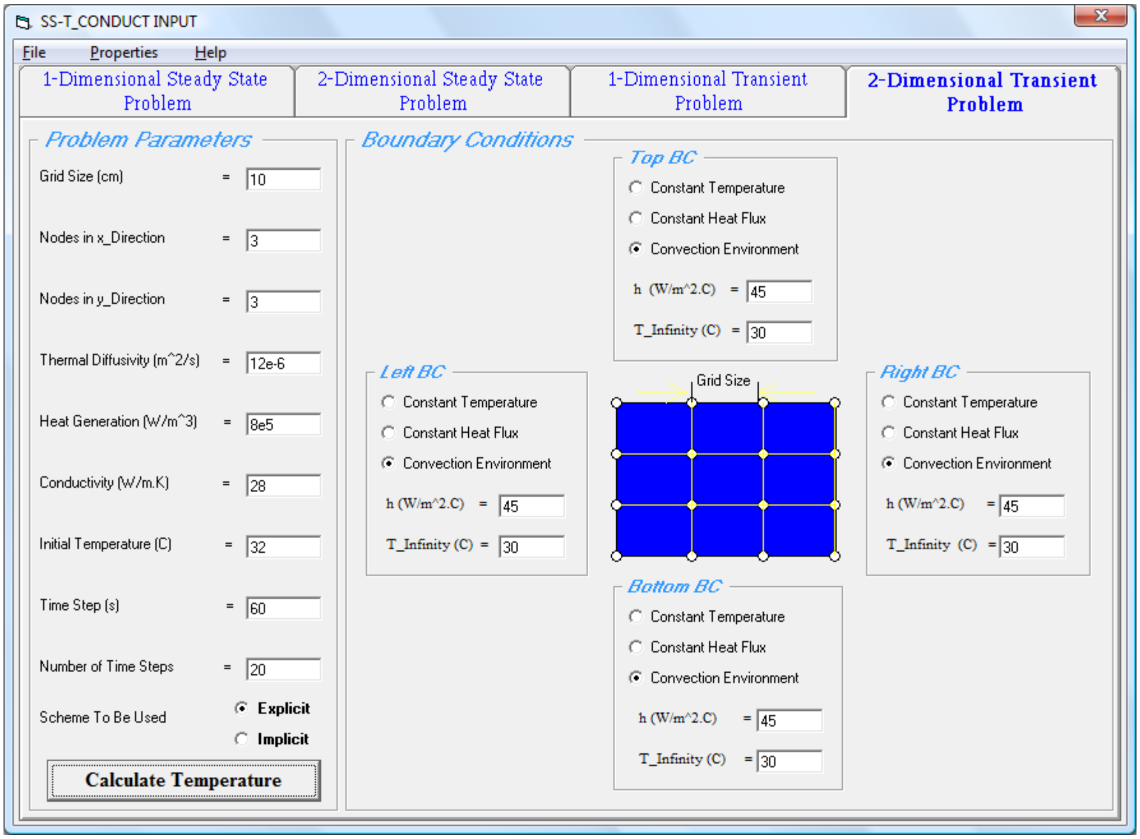
SS-T-CONDUCT Problem 8

Solution

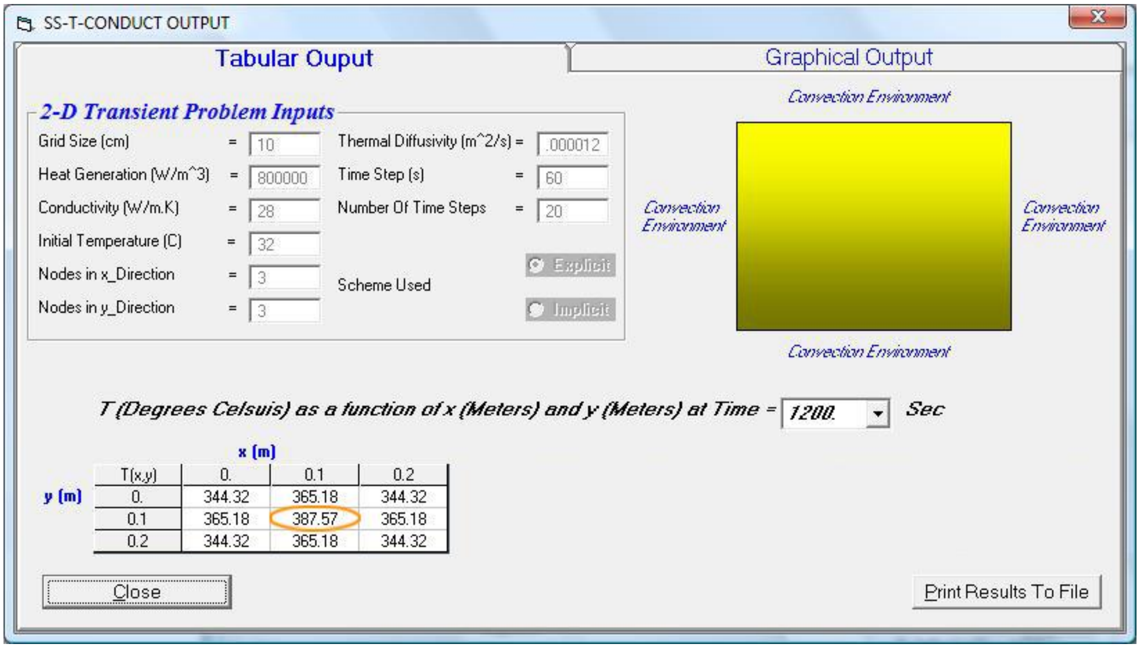
5-106 Prob. 5-105 is reconsidered. The centerline temperature of the bar after 20 min and after steady conditions are established are to be determined.

Analysis The problem is solved using SS-T-CONDUCT, and the solution is given below.

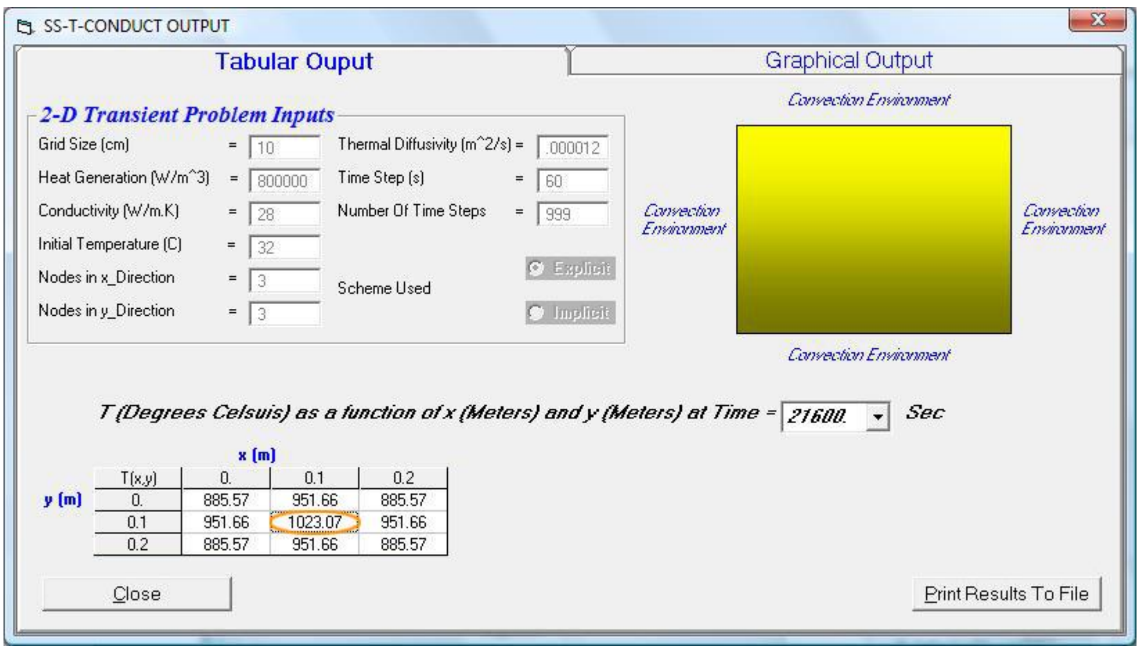
(*a*) On the SS-T-CONDUCT Input window for **2-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes.



By clicking on the **Calculate Temperature** button, the computed results are as follows.



(*b*) Steady conditions are reached in the medium after about 6 hours for which the temperature at the center node is **1023°C**.



SS-T-CONDUCT Problem 9

5–138 Consider steady two-dimensional heat transfer in a long solid bar (k = 25 W/m·K) of square cross section (2 cm × 2 cm) with heat generated in the bar uniformly at a rate of $\dot{e} = 3 \times 10^6$ W/m^3. The left and bottom surfaces maintain a constant temperature of 200°C. The top and right surfaces are subjected to convection with ambient air temperature of 100°C and heat transfer coefficient of 250 W/m^2·K. Using a uniform mesh size $\Delta x = \Delta y$ = 1 cm determine (*a*) the finite difference equations and (*b*) the nodal temperatures with the Gauss-Seidel iterative method. *Answers:* $T_1 = T_4$ = 196.4°C, T_2 = 190.3°C, T_3 = 201.2°C

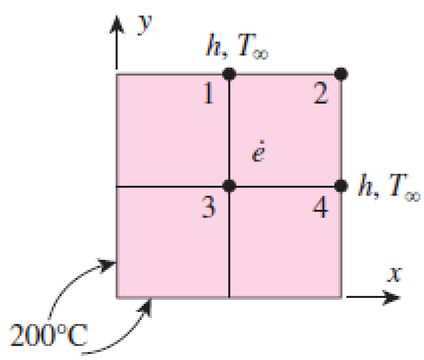


FIGURE P5–138

5–139 Repeat Prob. 5–138. Using SS-T-CONDUCT (or other) software, solve for the nodal temperatures.

SS-T-CONDUCT Problem 9

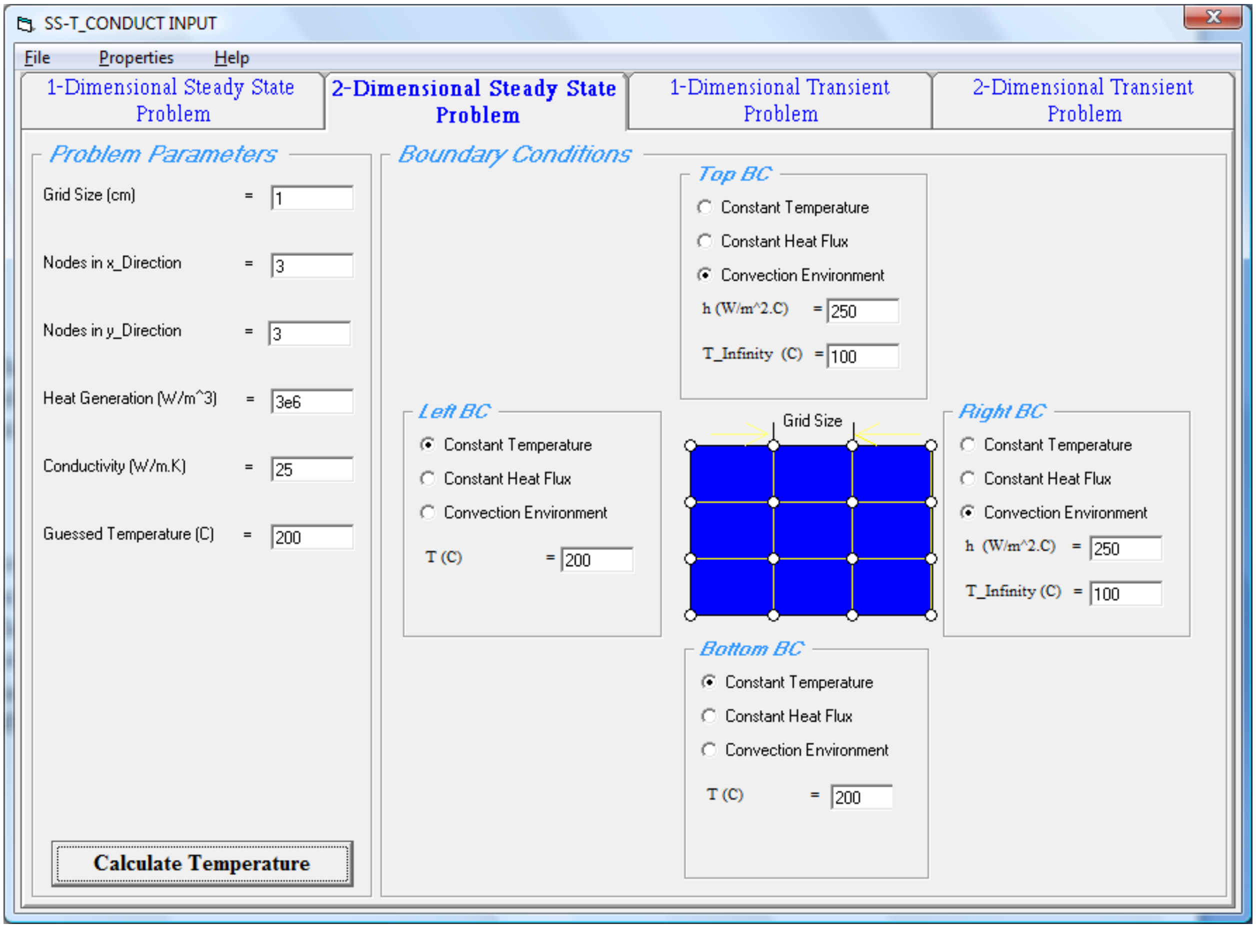
Solution

5-139 Prob. 5-138 is repeated. Using SS-T-Conduct (or other) software, the nodal temperatures are to be solved.

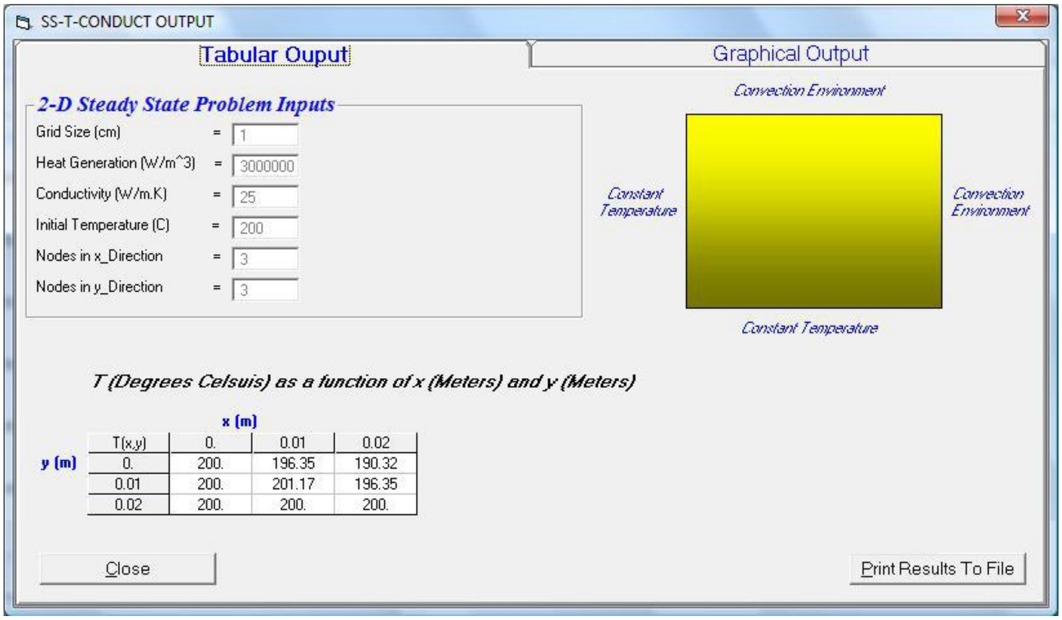
Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** The heat generation in the body is uniform.

Properties The thermal conductivity is given to be $k = 25$ W/m·K.

Analysis On the SS-T-Conduct Input window for 2-Dimensional Steady State Problem, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 1 cm, there are 3 nodes in each the x and y directions.



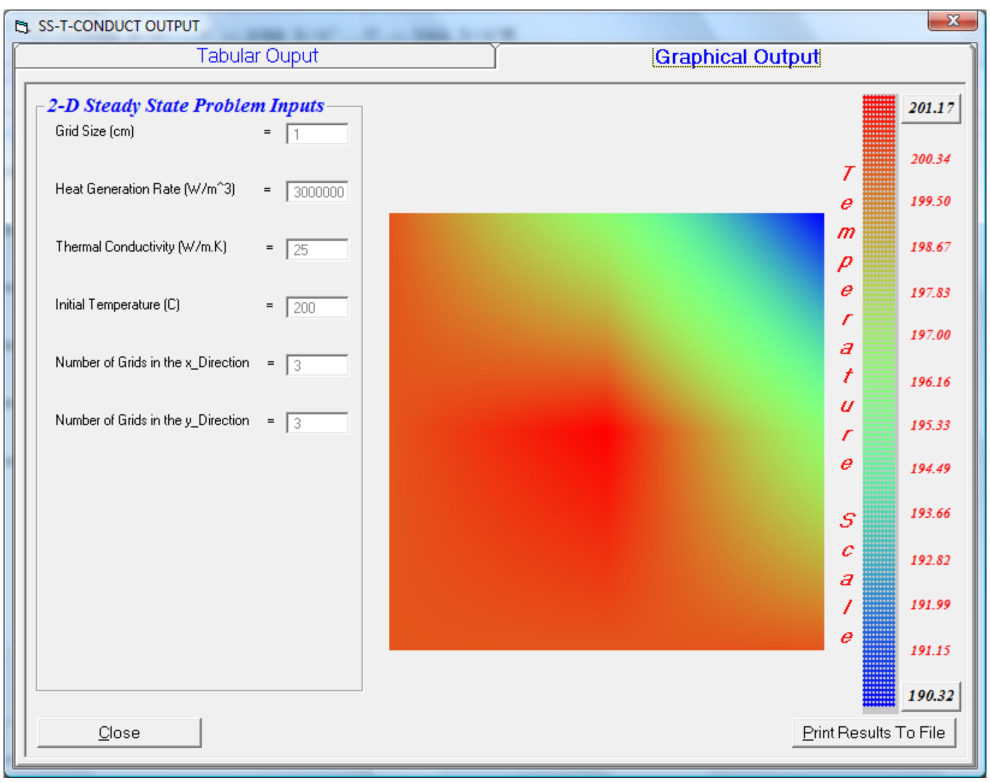
By clicking on the **Calculate Temperature** button, the computed results are as follows.



Hence, the converged nodal temperatures are

$$T_1 = T_4 = \mathbf{196.4°C}, \quad T_2 = \mathbf{190.3°C}, \quad T_3 = \mathbf{201.2°C}$$

Discussion The temperature contour for this problem can be plotted by selecting the **Graphical Output** tab, as follows.



SS-T-CONDUCT Problem 10

5–141 A long 10 cm × 20 cm rectangular cross section steel bar $k = 63.9$ W/m·K and $\alpha = 18.8 \times 10^{-6}$ m^2/s was heated to an initial temperature of 450°C. The steel bar is allowed to cool in a room with a temperature of 25°C and convection heat transfer coefficient of 25 W/m^2·K. The bottom surface of the bar is insulated while the other surfaces are exposed to

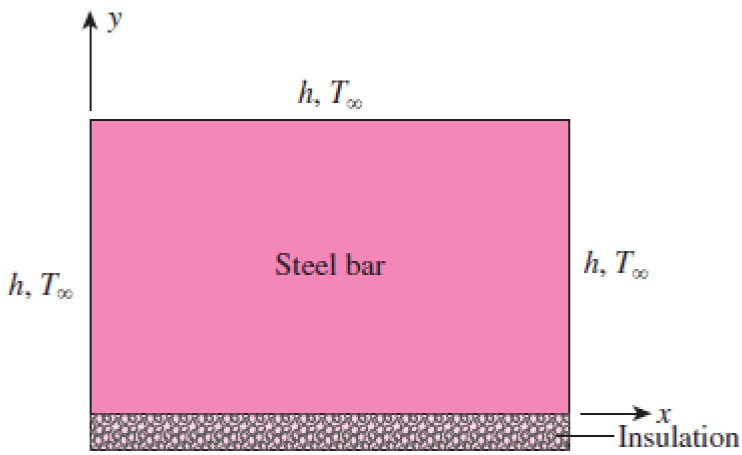


FIGURE P5–141

convection. With a uniform nodal spacing of 5 cm, use SS-T-CONDUCT (or other) software with explicit method, determine the duration required to cool the center of the bar to 100°C. *Answer:* 12,000 s

SS-T-CONDUCT Problem 10

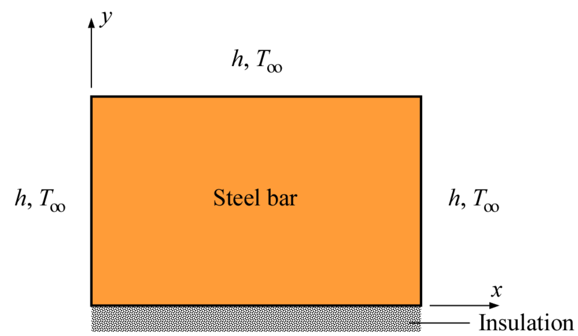
Solution

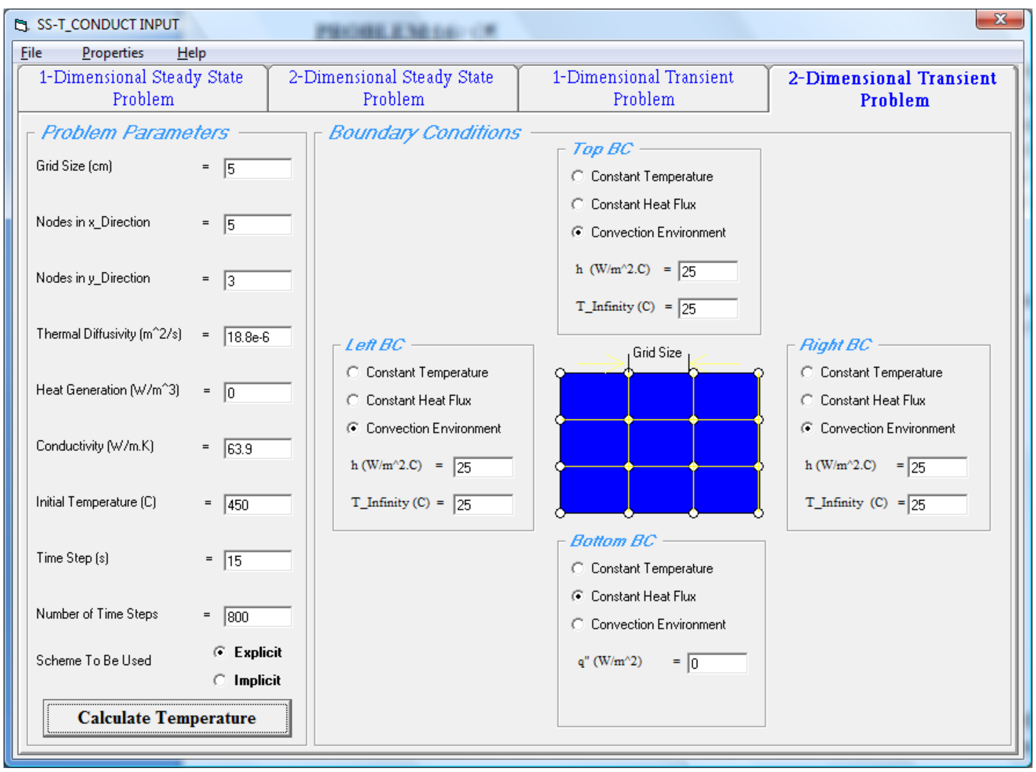
5-141 A long hot rectangular cross section steel bar is allowed to cool in a room. Using SS-T-Conduct (or other) software with explicit method, the duration required to cool the center of the bar to 100°C is to be determined.

Assumptions **1** Transient heat conduction is two-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** There is no heat generation.

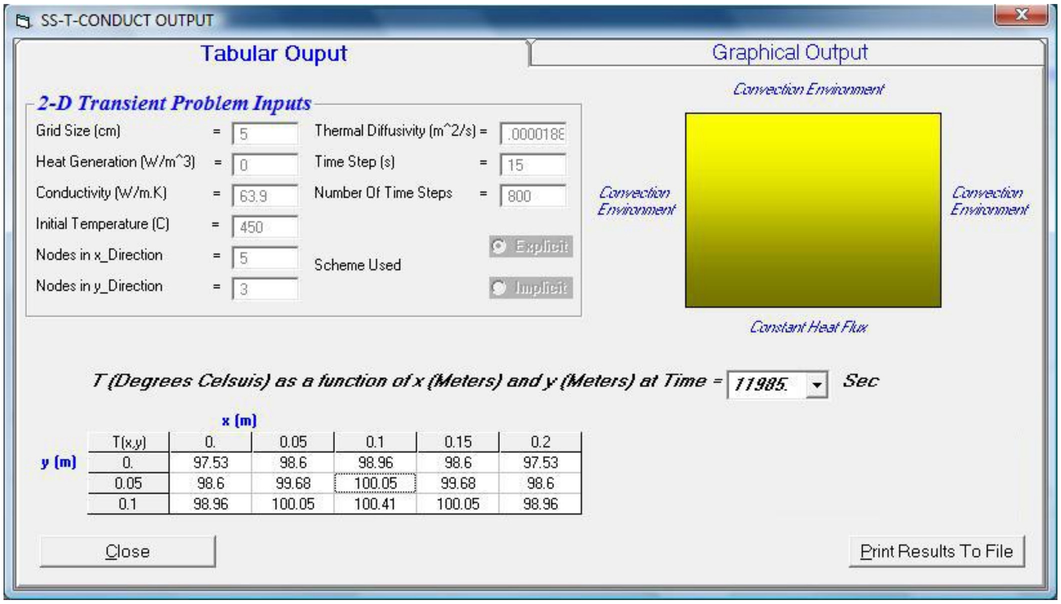
Properties The properties of the brass plate are given as $k = 63.9$ W/m·K and $\alpha = 18.8 \times 10^{-6}$ m^2/s.

Analysis On the SS-T-Conduct Input window for **2-Dimensional Transient Problem**, the problem parameters and the boundary conditions are entered into the appropriate text boxes. Note that with a uniform nodal spacing of 5 cm, there are 5 nodes in the x direction and 3 nodes in the y direction.





By clicking on the **Calculate Temperature** button, the computed results are as follows.



T(x,y)	0.	0.05	0.1	0.15	0.2
0.	97.53	98.6	98.96	98.6	97.53
0.05	98.6	99.68	100.05	99.68	98.6
0.1	98.96	100.05	100.41	100.05	98.96

From the results computed by the SS-T-Conduct software, the surface temperature reached 100°C at $t \approx$ **12000 s**.

Discussion Similar result ($t \approx 12000$ s) can also be obtained using the implicit method.

SS-T-CONDUCT Problem 11

5-64 Consider a rectangular metal block (k = 35 W/m·K) of dimensions 100 cm × 75 cm subjected to a sinusoidal temperature variation at its top surface while its bottom surface is insulated. The two sides of the metal block are exposed to a convective environment at 15°C and having a heat transfer coefficient of 50 W/m^2·K. The sinusoidal temperature distribution at the top surface is given as $100\sin(\pi x/L)$. Using a uniform mesh size of $\Delta x = \Delta y = 25cm$ determine (a) finite difference equations and (b) the nodal temperatures.

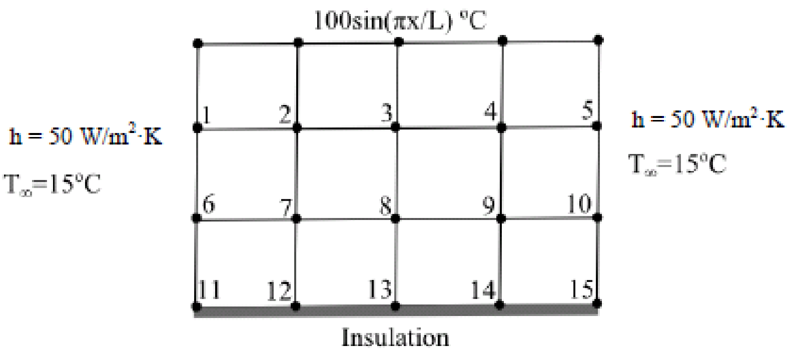


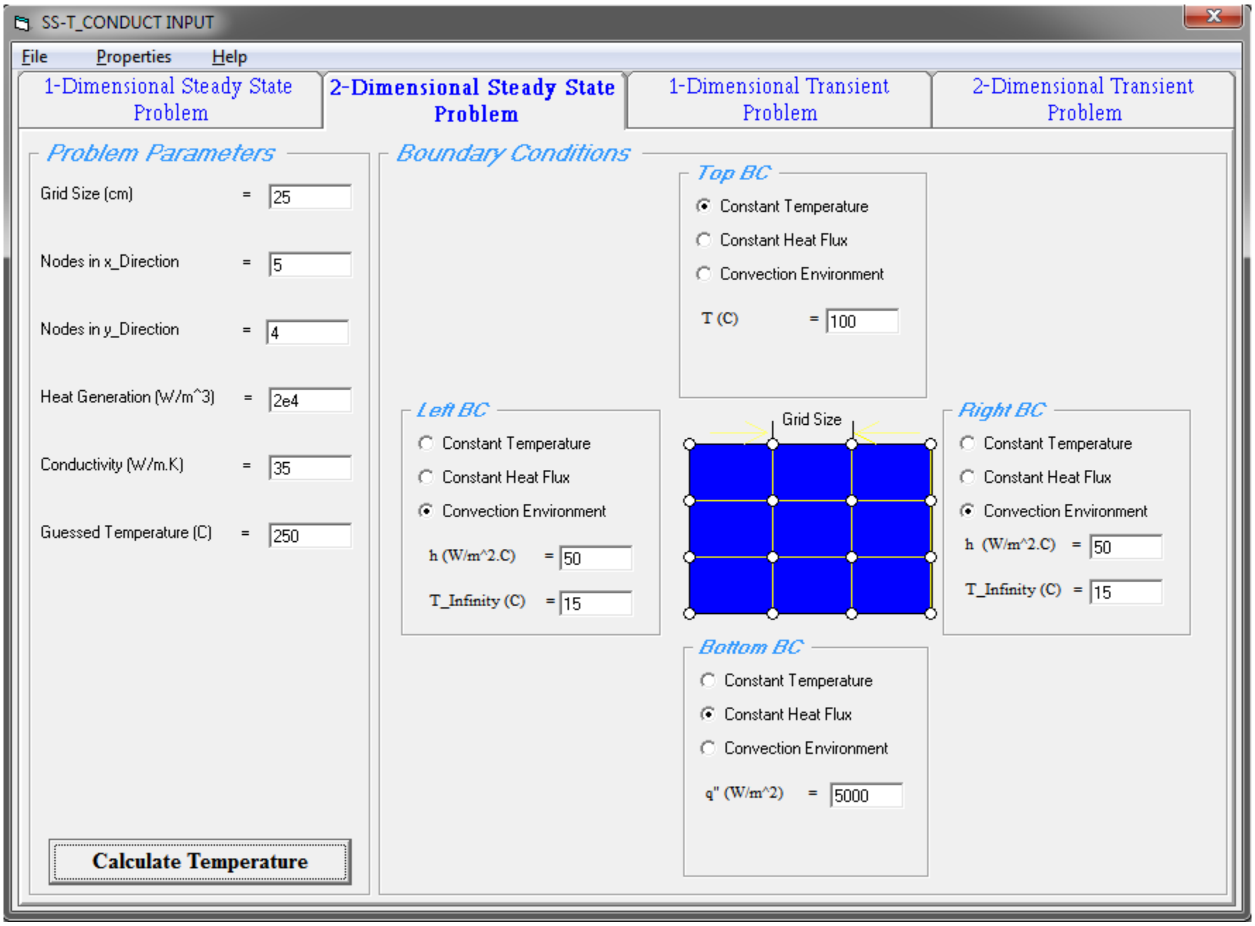
FIGURE P5-64

5-65 Solve Prob. 5-64 using SS-T-CONDUCT software for a constant temperature of 100°C on its top surface while the bottom surface is exposed to a uniform heat flux of 5000 W/m^2. For convective conditions and the mesh size same as that of Prob. 5-64 determine the nodal temperatures accounting for the internal heat generation at the rate of 2×10^4 W/m^3.

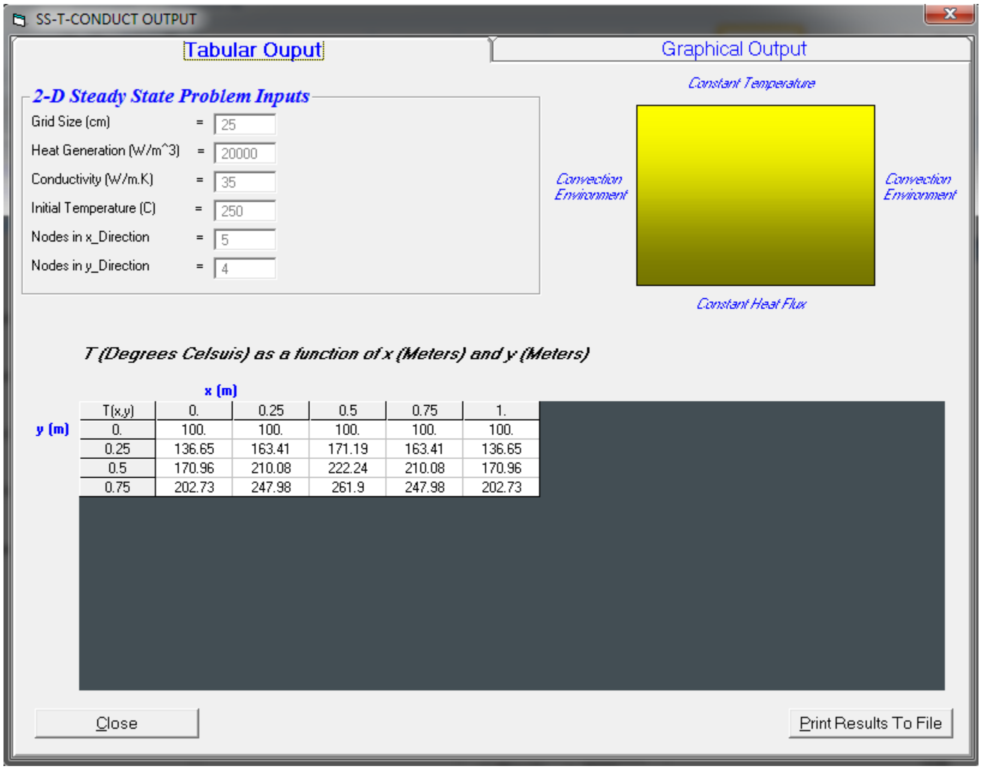
SS-T-CONDUCT Problem 11
Solution

5-65 Solving Prob. 5-64 using SS-T- CONDUCT software for different boundary conditions on the top and the bottom of the metal block with the rest of the conditions the same as Prob. 5-64.
Assumption **1** Two dimensional steady state heat generation. **2** Constant thermal properties.
Properties Thermal conductivity of the metal block is given as k=35 W/m·K.
Analysis The nodal temperatures are to be determined using SS-T-CONDUCT program.

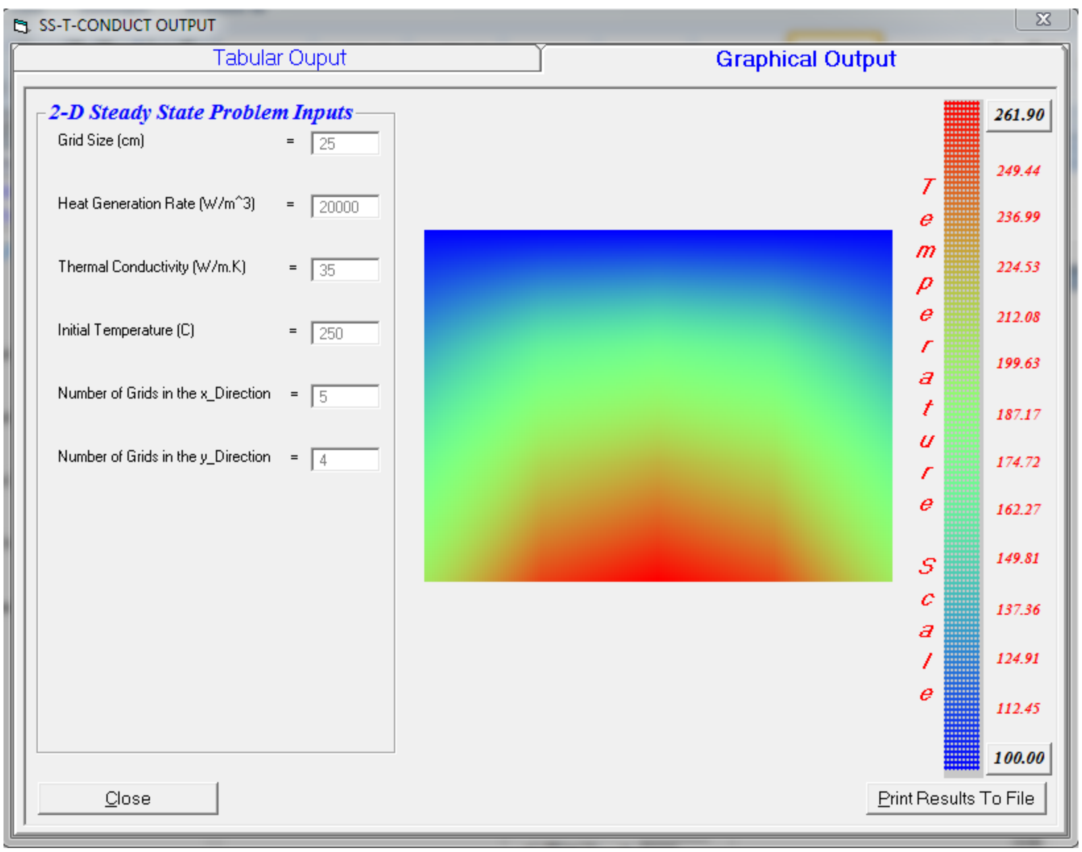
On the SS-T-CONDUCT program input window for 2-Dimensional Steady State Problem, input the required parameters in the appropriate text box. With a uniform node spacing of 25 cm, there are 5 nodes in x direction and 4 nodes in y direction.



As shown in the tabular output below, the results are
T_1 = **136.65°C,** T_2 = **163.41°C,** T_3 = **171.19°C,** T_4 = **163.41°C,** T_5 = **136.65°C,**
T_6 = **170.96°C,** T_7 = **210.08°C,** T_8 = **222.24°C,** T_9 = **210.08°C,** T_{10} = **170.96°C,**
T_{11} = **202.73°C,** T_{12} = **247.98°C,** T_{13} = **261.9°C,** T_{14} = **247.98°C,** T_{15} = **202.73°C.**



T(x,y)	0.	0.25	0.5	0.75	1.
0.	100.	100.	100.	100.	100.
0.25	136.65	163.41	171.19	163.41	136.65
0.5	170.96	210.08	222.24	210.08	170.96
0.75	202.73	247.98	261.9	247.98	202.73



Discussion The temperature distribution is symmetric about the centerline.

SS-T-CONDUCT Problem 12

5-128 Quench hardening is a mechanical process in which the ferrous metals or alloys are first heated and then quickly cooled down to improve their physical properties and avoid phase transformation. Consider a 40 cm × 20 cm block of copper alloy (k = 120 W/m·K, α = 3.91 × 10^{-6} m^2/s) being heated uniformly until it reaches a temperature of 800°C. It is then suddenly immersed into the water bath maintained at 15°C with h = 100 $W/m^2\cdot K$ for quenching process. However, the upper surface of the metal is not submerged in the water and is exposed to air at 15°C with a convective heat transfer coefficient of 10 $W/m^2\cdot K$. Using an explicit finite difference formulation, calculate the temperature distribution in the copper alloy block after 10 min have elapsed using Δt = 10 s and a uniform mesh size of $\Delta x = \Delta y$ = 10 cm.

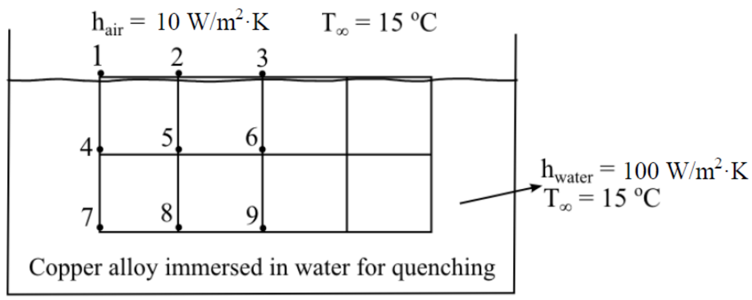


FIGURE P5–128

5-129 Solve problem Prob. 5-128 using SS-T-CONDUCT software for two separate quenching process with two different metal blocks, stainless steel (k = 15.1 W/m·K and α = 3.91 × 10^{-6} m^2/s) and carbon steel (k = 60.5 W/m·K and α = 17.7 × 10^{-6} m^2/s). Assume each metal block during the quench hardening process is completely immersed in the fluid bath. Consider three different quenching mediums of air, water and oil with convective heat transfer coefficients of 10 $W/m^2\cdot K$, 100 $W/m^2\cdot K$ and 1000 $W/m^2\cdot K$, respectively. Find the temperature distribution in each block after 10 minutes. Discuss your results. Use implicit method to find temperature distribution.

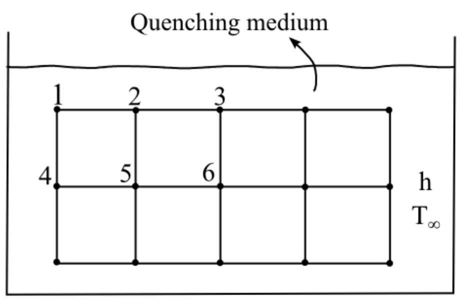
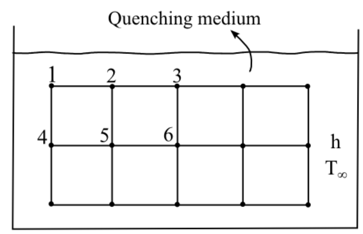
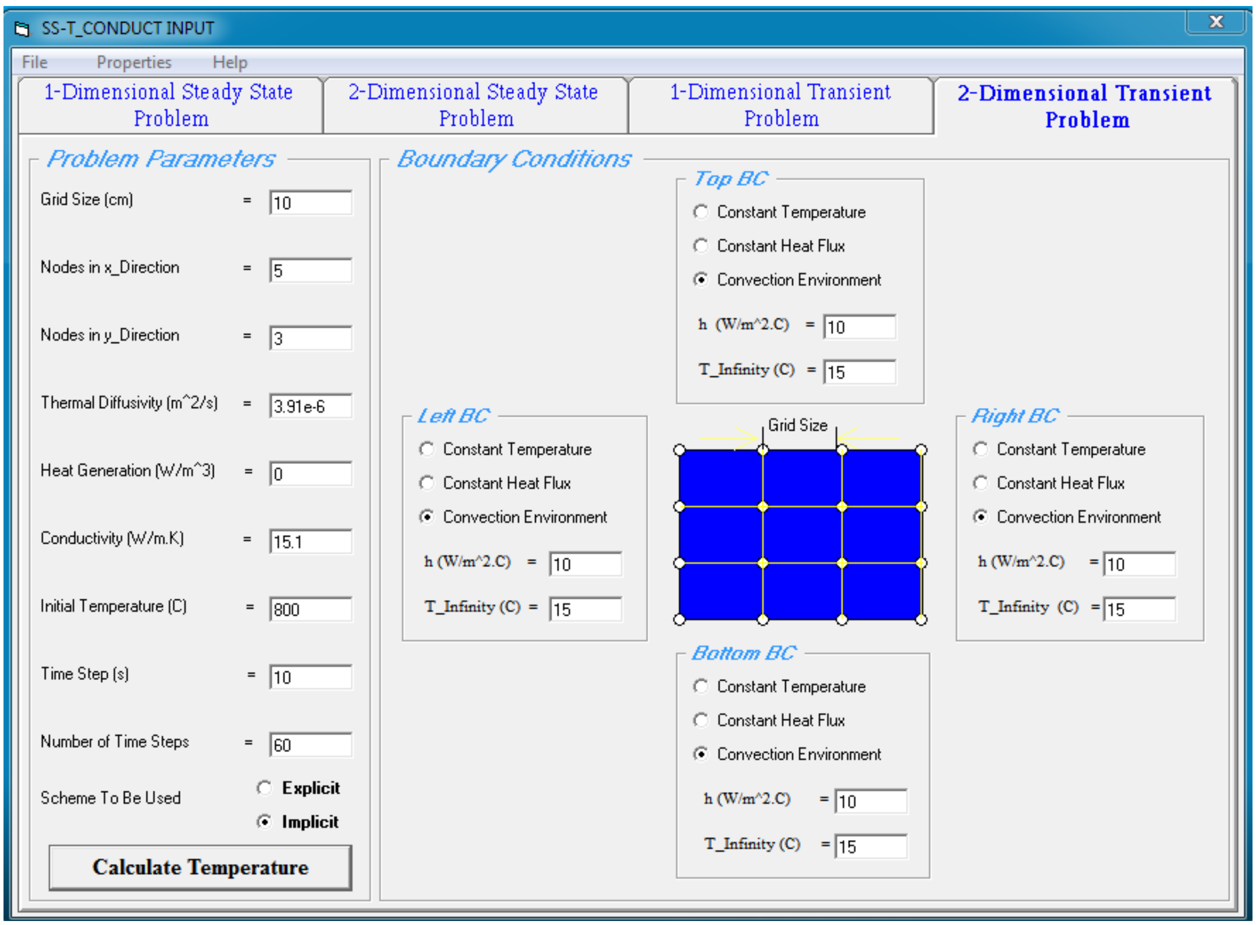


FIGURE P5–129

SS-T-CONDUCT Problem 12
Solution

5-129 The problem is solved by SS-T-CONDUCT program using implicit method and the solution is given below. On the SS-T-CONDUCT program input window for 2-Dimensional Transient Problem, input the required parameters in the appropriate text box. With a uniform node spacing of 10 cm, there are 5 nodes in x direction and 3 nodes in y direction. Since there is symmetry in the geometry we are interested only in the temperatures at nodes 1, 2, 3, 4, 5 and 6. A sample input window interface for the quenching of stainless steel in air as quenching medium is shown below. For other conditions similar steps have to be followed.





It can be seen from the 'Tabular Output' window shown below that the temperature distribution is symmetric about the center line in x and y direction.

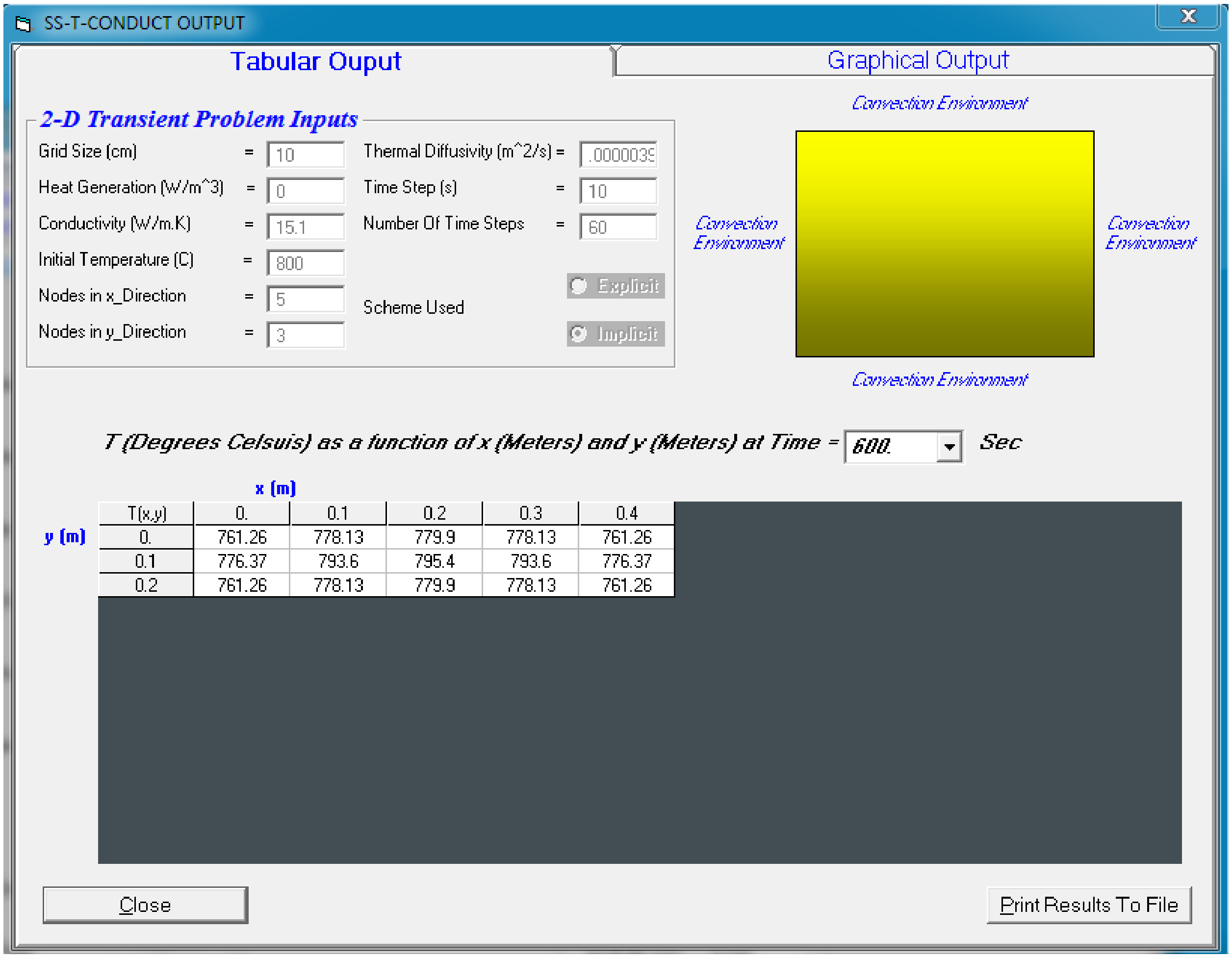


Table 1 Temperature distribution in the stainless steel block for three different quenching mediums after 10 min.

	Air	Water	Oil
Node	h = 10 W/m^2·K	h = 100 W/m^2·K	h = 1000 W/m^2·K
1	761.26	492.6	29.36
2	778.13	610.55	104.28
3	779.9	623.33	115.12
4	776.37	598.11	97.93
5	793.6	742.34	528.4
6	795.4	757.99	590.11

Table 2 Temperature distribution in the carbon steel block for three different quenching mediums after 10 min.

	Air	Water	Oil
Node	h = 10 W/m^2·K	h = 100 W/m^2·K	h = 1000 W/m^2·K
1	769.5	549.05	56.21
2	778.12	611.6	115.01
3	780.53	629.7	134.6
4	775.72	594.2	102.04
5	784.35	662.1	226.24
6	786.78	681.7	267.63

From the temperature distribution at nodes 1, 2, 3, 4, 5 and 6 it is evident that for a fixed thermal conductivity, increase in the heat transfer coefficients causes large temperature gradients within the metal blocks. For both stainless steel and carbon steel with air as quenching medium, the temperature distribution within the metal block is relatively uniform and thus the metal block may be assumed to approximate a lumped system. In case of oil as quenching medium, large temperature gradients exist with the metal block boundary and the center line.

For a constant rate of heat flux the temperature gradient is inversely proportional to the thermal conductivity. As seen in above tables, the temperature gradients in case of carbon steel (k = 60.5 W/m·K) are less than that in case of stainless steel (k = 15.1 W/m·K).

Discussion: Although the quenching process requires quick cooling of the geometry under consideration, the choice of the quenching medium should also depend upon the allowable temperature gradient within the body. Use of the quenching medium with very high heat transfer coefficients for low thermal conductivity objects may cause thermal stresses to develop in them which may result in thermal deformation.