

# ***Solutions Manual***

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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## **Chapter 14**

## **MASS TRANSFER**

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## Analogy between Heat and Mass Transfer

**14-1C** The *concentration* of a commodity is defined as the amount of that commodity per unit volume. The *concentration gradient*  $dC/dx$  is defined as the change in the concentration  $C$  of a commodity per unit length in the direction of flow  $x$ . The *diffusion rate* of the commodity is expressed as

$$\dot{Q} = -k_{\text{diff}} A \frac{dC}{dx}$$

where  $A$  is the area normal to the direction of flow and  $k_{\text{diff}}$  is the *diffusion coefficient* of the medium, which is a measure of how fast a commodity diffuses in the medium.

**14-2C** Examples of different kinds of diffusion processes:

- (a) *Liquid-to-gas*: A gallon of gasoline left in an open area will eventually evaporate and diffuse into air.
- (b) *Solid-to-liquid*: A spoon of sugar in a cup of tea will eventually dissolve and move up.
- (c) *Solid-to gas*: A moth ball left in a closet will sublime and diffuse into the air.
- (d) *Gas-to-liquid*: Air dissolves in water.

**14-3C** (a) *Temperature difference* is the driving force for heat transfer, (b) *voltage difference* is the driving force for electric current flow, and (c) *concentration difference* is the driving force for mass transfer.

**14-4C** In the relation  $\dot{Q} = -kA(dT/dx)$ , the quantities  $\dot{Q}$ ,  $k$ ,  $A$ , and  $T$  represent the following in heat conduction and mass diffusion:

$\dot{Q}$  = Rate of heat transfer in heat conduction, and rate of mass transfer in mass diffusion.

$k$  = Thermal conductivity in heat conduction, and mass diffusivity in mass diffusion.

$A$  = Area normal to the direction of flow in both heat and mass transfer.

$T$  = Temperature in heat conduction, and concentration in mass diffusion.

**14-5C** *Bulk fluid flow* refers to the transportation of a fluid on a macroscopic level from one location to another in a flow section by a mover such as a fan or a pump. *Mass flow* requires the presence of two regions at different chemical compositions, and it refers to the movement of a chemical species from a high concentration region towards a lower concentration one relative to the other chemical species present in the medium. Mass transfer cannot occur in a homogeneous medium.

**14-6C** (a) *Homogenous reactions* in mass transfer represent the generation of a species within the medium. Such reactions are analogous to internal heat generation in heat transfer. (b) *Heterogeneous reactions* in mass transfer represent the generation of a species at the surface as a result of chemical reactions occurring at the surface. Such reactions are analogous to specified surface heat flux in heat transfer.

## Mass Diffusion

**14-7C** The molecular weights of CO<sub>2</sub> and N<sub>2</sub>O gases are the same (both are 44). Therefore, the mass and mole fractions of each of these two gases in a gas mixture will be the same.

**14-8C** (a) T (b) F (c) F (d) T (e) F

**14-9C** In the Fick's law of diffusion relations expressed as  $\dot{m}_{\text{diff},A} = -\rho A D_{AB} \frac{dw_A}{dx}$  and  $\dot{N}_{\text{diff},A} = -C A D_{AB} \frac{dy_A}{dx}$ , the diffusion coefficients  $D_{AB}$  are the same.

**14-10C** The mass diffusivity of a gas mixture (a) increases with increasing temperature and (a) decreases with increasing pressure.

**14-11C** In a binary ideal gas mixture of species A and B, the diffusion coefficient of A in B is equal to the diffusion coefficient of B in A. Therefore, the mass diffusivity of air in water vapor will be equal to the mass diffusivity of water vapor in air since the air and water vapor mixture can be treated as ideal gases.

**14-12C** Solids, in general, have different diffusivities in each other. At a given temperature and pressure, the mass diffusivity of copper in aluminum will not be the equal to the mass diffusivity of aluminum in copper.

**14-13C** We would carry out the hardening process of steel by carbon at high temperature since mass diffusivity increases with temperature, and thus the hardening process will be completed in a short time.

**14-14** The maximum mass fraction of calcium bicarbonate in water at 350 K is to be determined.

**Assumptions** The small amounts of gases in air are ignored, and dry air is assumed to consist of N<sub>2</sub> and O<sub>2</sub> only.

**Properties** The solubility of [Ca(HCO<sub>3</sub>)<sub>2</sub>] in 100 kg of water at 350 K is 17.88 kg (Table 14-5).

**Analysis** The maximum mass fraction is determined from

$$w_{\text{CaHCO}_3)_2} = \frac{m_{\text{CaHCO}_3)_2}}{m_{\text{total}}} = \frac{m_{\text{CaHCO}_3)_2}}{m_{\text{CaHCO}_3)_2} + m_w} = \frac{17.88 \text{ kg}}{(17.88 + 100) \text{ kg}} = \mathbf{0.152}$$

**14-15** The molar fractions of the constituents of moist air are given. The mass fractions of the constituents are to be determined.

**Assumptions** The small amounts of gases in air are ignored, and dry air is assumed to consist of  $N_2$  and  $O_2$  only.

**Properties** The molar masses of  $N_2$ ,  $O_2$ , and  $H_2O$  are 28.0, 32.0, and 18.0 kg/kmol, respectively (Table A-1)

**Analysis** The molar mass of moist air is determined to be

$$M = \sum y_i M_i = 0.78 \times 28.0 + 0.20 \times 32.0 + 0.02 \times 18 = 28.6 \text{ kg/kmol}$$

Then the mass fractions of constituent gases are determined from Eq. 14-10 to be

$$N_2 : \quad w_{N_2} = y_{N_2} \frac{M_{N_2}}{M} = (0.78) \frac{28.0}{28.6} = \mathbf{0.764}$$

$$O_2 : \quad w_{O_2} = y_{O_2} \frac{M_{O_2}}{M} = (0.20) \frac{32.0}{28.6} = \mathbf{0.224}$$

$$H_2O : \quad w_{H_2O} = y_{H_2O} \frac{M_{H_2O}}{M} = (0.02) \frac{18.0}{28.6} = \mathbf{0.012}$$

Therefore, the mass fractions of  $N_2$ ,  $O_2$ , and  $H_2O$  in dry air are 76.4%, 22.4%, and 1.2%, respectively.

Moist air
78% $N_2$
20% $O_2$
2% $H_2O$
(Mole fractions)

**14-16** The mole numbers of the constituents of a gas mixture at a specified pressure and temperature are given. The mass fractions and the partial pressures of the constituents are to be determined.

**Assumptions** The gases behave as ideal gases.

**Properties** The molar masses of  $N_2$ ,  $O_2$  and  $CO_2$  are 28, 32, and 44 kg/kmol, respectively (Table A-1)

**Analysis** When the mole fractions of a gas mixture are known, the mass fractions can be determined from

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

The apparent molar mass of the mixture is

$$M = \sum y_i M_i = 0.65 \times 28.0 + 0.20 \times 32.0 + 0.15 \times 44.0 = 31.2 \text{ kg/kmol}$$

Then the mass fractions of the gases are determined from

$$N_2 : \quad w_{N_2} = y_{N_2} \frac{M_{N_2}}{M} = (0.65) \frac{28.0}{31.2} = \mathbf{0.583} \quad (\text{or } 58.3\%)$$

$$O_2 : \quad w_{O_2} = y_{O_2} \frac{M_{O_2}}{M} = (0.20) \frac{32.0}{31.2} = \mathbf{0.205} \quad (\text{or } 20.5\%)$$

$$CO_2 : \quad w_{CO_2} = y_{CO_2} \frac{M_{CO_2}}{M_m} = (0.15) \frac{44}{31.2} = \mathbf{0.212} \quad (\text{or } 21.2\%)$$

Noting that the total pressure of the mixture is 250 kPa and the pressure fractions in an ideal gas mixture are equal to the mole fractions, the partial pressures of the individual gases become

$$P_{N_2} = y_{N_2} P = (0.65)(250 \text{ kPa}) = 162.5 \text{ kPa}$$

$$P_{O_2} = y_{O_2} P = (0.20)(250 \text{ kPa}) = \mathbf{50 \text{ kPa}}$$

$$P_{CO_2} = y_{CO_2} P = (0.15)(250 \text{ kPa}) = \mathbf{37.5 \text{ kPa}}$$

65% $N_2$
20% $O_2$
15% $CO_2$
290 K
250 kPa

**14-17E** The masses of the constituents of a gas mixture are given. The mass fractions, mole fractions, and the molar mass of the mixture are to be determined.

**Assumptions** None.

**Properties** The molar masses of  $N_2$ ,  $O_2$ , and  $CO_2$  are 28, 32, and 44 lbm/lbmol, respectively (Table A-1E)

**Analysis** (a) The total mass of the gas mixture is determined to be

$$m = \sum m_i = m_{O_2} + m_{N_2} + m_{CO_2} = 7 + 8 + 10 = 25 \text{ lbm}$$

Then the mass fractions of constituent gases are determined to be

$$N_2 : \quad w_{N_2} = \frac{m_{N_2}}{m} = \frac{8}{25} = \mathbf{0.32}$$

$$O_2 : \quad w_{O_2} = \frac{m_{O_2}}{m} = \frac{7}{25} = \mathbf{0.28}$$

$$CO_2 : \quad w_{CO_2} = \frac{m_{CO_2}}{m} = \frac{10}{25} = \mathbf{0.40}$$

7 lbm $O_2$ 8 lbm $N_2$ 10 lbm $CO_2$
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(b) To find the mole fractions, we need to determine the mole numbers of each component first,

$$N_2 : \quad N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{8 \text{ lbm}}{28 \text{ lbm/lbmol}} = \mathbf{0.286 \text{ lbmol}}$$

$$O_2 : \quad N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{7 \text{ lbm}}{32 \text{ lbm/lbmol}} = \mathbf{0.219 \text{ lbmol}}$$

$$CO_2 : \quad N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{10 \text{ lbm}}{44 \text{ lbm/lbmol}} = \mathbf{0.227 \text{ lbmol}}$$

Thus,

$$N_m = \sum N_i = N_{N_2} + N_{O_2} + N_{CO_2} = 0.286 + 0.219 + 0.227 = 0.732 \text{ lbmol}$$

Then the mole fraction of gases are determined to be

$$N_2 : \quad y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{0.286}{0.732} = \mathbf{0.391}$$

$$O_2 : \quad y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{0.219}{0.732} = \mathbf{0.299}$$

$$CO_2 : \quad y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{0.227}{0.732} = \mathbf{0.310}$$

(c) The molar mass of the mixture is determined from

$$M = \frac{m_m}{N_m} = \frac{25 \text{ lbm}}{0.732 \text{ lbmol}} = \mathbf{34.2 \text{ lbm/lbmol}}$$

**14-18** The mole fractions of the constituents of a gas mixture are given. The mass of each gas and apparent gas constant of the mixture are to be determined.

**Assumptions** None.

**Properties** The molar masses of  $H_2$  and  $N_2$  are 2.0 and 28.0 kg/kmol, respectively (Table A-1)

**Analysis** The mass of each gas is

$$H_2 : m_{H_2} = N_{H_2} M_{H_2} = (8 \text{ kmol}) \times (2 \text{ kg/kmol}) = \mathbf{16 \text{ kg}}$$

$$N_2 : m_{N_2} = N_{N_2} M_{N_2} = (2 \text{ kmol}) \times (28 \text{ kg/kmol}) = \mathbf{56 \text{ kg}}$$

The molar mass of the mixture and its apparent gas constant are determined to be

$$M = \frac{m_m}{N_m} = \frac{16 + 56 \text{ kg}}{8 + 2 \text{ kmol}} = 7.20 \text{ kg/kmol}$$

$$R = \frac{R_u}{M} = \frac{8.314 \text{ kJ/kmol} \cdot \text{K}}{7.20 \text{ kg/kmol}} = \mathbf{1.155 \text{ kJ/kg} \cdot \text{K}}$$

8 kmol $H_2$ 2 kmol $N_2$
------------------------------

**14-19** The binary diffusion coefficients of  $CO_2$  in air at various temperatures and pressures are to be determined.

**Assumptions** The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

**Properties** The binary diffusion coefficients of  $CO_2$  in air at 1 atm pressure are given in Table 14-1 to be  $0.74 \times 10^{-5}$ ,  $2.63 \times 10^{-5}$ , and  $5.37 \times 10^{-5} \text{ m}^2/\text{s}$  at temperatures of 200 K, 400 K, and 600 K, respectively.

**Analysis** Noting that the binary diffusion coefficients of gases are inversely proportional to pressure, the diffusion coefficients at given pressures are determined from

$$D_{AB}(T, P) = D_{AB}(T, 1 \text{ atm}) / P$$

where  $P$  is in atm.

$$(a) \text{ At 200 K and 1 atm: } D_{AB}(200 \text{ K}, 1 \text{ atm}) = \mathbf{0.74 \times 10^{-5} \text{ m}^2/\text{s}} \quad (\text{since } P = 1 \text{ atm}).$$

$$(b) \text{ At 400 K and 0.5 atm: } D_{AB}(400 \text{ K}, 0.5 \text{ atm}) = D_{AB}(400 \text{ K}, 1 \text{ atm}) / 0.5 = (2.63 \times 10^{-5}) / 0.5 = \mathbf{5.26 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$(c) \text{ At 600 K and 5 atm: } D_{AB}(600 \text{ K}, 5 \text{ atm}) = D_{AB}(600 \text{ K}, 1 \text{ atm}) / 5 = (5.37 \times 10^{-5}) / 5 = \mathbf{1.07 \times 10^{-5} \text{ m}^2/\text{s}}$$

**14-20** The binary diffusion coefficient of O<sub>2</sub> in N<sub>2</sub> at various temperature and pressures are to be determined.

**Assumptions** The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

**Properties** The binary diffusion coefficient of O<sub>2</sub> in N<sub>2</sub> at  $T_1 = 273 \text{ K}$  and  $P_1 = 1 \text{ atm}$  is given in Table 14-2 to be  $1.8 \times 10^{-5} \text{ m}^2/\text{s}$ .

**Analysis** Noting that the binary diffusion coefficient of gases is proportional to  $3/2$  power of temperature and inversely proportional to pressure, the diffusion coefficients at other pressures and temperatures can be determined from

$$\frac{D_{AB,1}}{D_{AB,2}} = \frac{P_2}{P_1} \left( \frac{T_1}{T_2} \right)^{3/2} \rightarrow D_{AB,2} = D_{AB,1} \frac{P_1}{P_2} \left( \frac{T_2}{T_1} \right)^{3/2}$$

$$(a) \text{ At } 200 \text{ K and } 1 \text{ atm: } D_{AB,2} = (1.8 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{1 \text{ atm}} \left( \frac{200 \text{ K}}{273 \text{ K}} \right)^{3/2} = \mathbf{1.13 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$(b) \text{ At } 400 \text{ K and } 0.5 \text{ atm: } D_{AB,2} = (1.8 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{0.5 \text{ atm}} \left( \frac{400 \text{ K}}{273 \text{ K}} \right)^{3/2} = \mathbf{6.38 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$(c) \text{ At } 600 \text{ K and } 5 \text{ atm: } D_{AB,2} = (1.8 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{5 \text{ atm}} \left( \frac{600 \text{ K}}{273 \text{ K}} \right)^{3/2} = \mathbf{1.17 \times 10^{-5} \text{ m}^2/\text{s}}$$

**14-21** The binary diffusion coefficient of (a) CO<sub>2</sub> in N<sub>2</sub>, (b) CO<sub>2</sub> in O<sub>2</sub>, and (c) CO<sub>2</sub> in H<sub>2</sub> at 320 K and 2 atm are to be determined.

**Assumptions 1** The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

**Properties** From Table 14-2, we find the following binary diffusion coefficients:

$$\text{CO}_2 \text{ in N}_2: D_{AB,1} = 1.6 \times 10^{-5} \text{ m}^2/\text{s} \quad \text{at } T_1 = 293 \text{ K and } P_1 = 1 \text{ atm}$$

$$\text{CO}_2 \text{ in O}_2: D_{AB,1} = 1.4 \times 10^{-5} \text{ m}^2/\text{s} \quad \text{at } T_1 = 273 \text{ K and } P_1 = 1 \text{ atm}$$

$$\text{CO}_2 \text{ in H}_2: D_{AB,1} = 5.5 \times 10^{-5} \text{ m}^2/\text{s} \quad \text{at } T_1 = 273 \text{ K and } P_1 = 1 \text{ atm}$$

**Analysis** Noting that the binary diffusion coefficient of gases is proportional to  $3/2$  power of temperature and inversely proportional to pressure, the diffusion coefficients at other pressures and temperatures can be determined from

$$\frac{D_{AB,1}}{D_{AB,2}} = \frac{P_2}{P_1} \left( \frac{T_1}{T_2} \right)^{3/2} \rightarrow D_{AB,2} = D_{AB,1} \frac{P_1}{P_2} \left( \frac{T_2}{T_1} \right)^{3/2}$$

(a) For CO<sub>2</sub> in N<sub>2</sub> at 320 K and 2 atm:

$$D_{AB,2} = (1.6 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{2 \text{ atm}} \left( \frac{320 \text{ K}}{293 \text{ K}} \right)^{3/2} = \mathbf{0.913 \times 10^{-5} \text{ m}^2/\text{s}}$$

(b) For CO<sub>2</sub> in O<sub>2</sub> at 320 K and 2 atm:

$$D_{AB,2} = (1.4 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{2 \text{ atm}} \left( \frac{320 \text{ K}}{273 \text{ K}} \right)^{3/2} = \mathbf{0.888 \times 10^{-5} \text{ m}^2/\text{s}}$$

(c) For CO<sub>2</sub> in H<sub>2</sub> at 320 K and 2 atm:

$$D_{AB,2} = (5.5 \times 10^{-5} \text{ m}^2/\text{s}) \frac{1 \text{ atm}}{2 \text{ atm}} \left( \frac{320 \text{ K}}{273 \text{ K}} \right)^{3/2} = \mathbf{3.49 \times 10^{-5} \text{ m}^2/\text{s}}$$

**Discussion** The binary diffusion coefficient is also known as mass diffusivity. The mass diffusivity  $D_{AB}$  in mass diffusion equation is analogous to the thermal diffusivity  $\alpha$  in heat diffusion equation.

**14-22** The diffusion coefficient of hydrogen in steel is given as a function of temperature. The diffusion coefficients at various temperatures are to be determined.

**Analysis** The diffusion coefficient of hydrogen in steel is given as

$$D_{AB} = 1.65 \times 10^{-6} \exp(-4630/T) \quad \text{m}^2/\text{s}$$

Using this relation, the diffusion coefficients at various temperatures are determined to be

$$200 \text{ K: } D_{AB} = 1.65 \times 10^{-6} \exp(-4630/200) = 1.46 \times 10^{-16} \text{ m}^2/\text{s}$$

$$500 \text{ K: } D_{AB} = 1.65 \times 10^{-6} \exp(-4630/500) = 1.57 \times 10^{-10} \text{ m}^2/\text{s}$$

$$1000 \text{ K: } D_{AB} = 1.65 \times 10^{-6} \exp(-4630/1000) = 1.61 \times 10^{-8} \text{ m}^2/\text{s}$$

$$1500 \text{ K: } D_{AB} = 1.65 \times 10^{-6} \exp(-4630/1500) = 7.53 \times 10^{-8} \text{ m}^2/\text{s}$$



**14-23** Prob. 14-22 is reconsidered. The diffusion coefficient as a function of the temperature is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

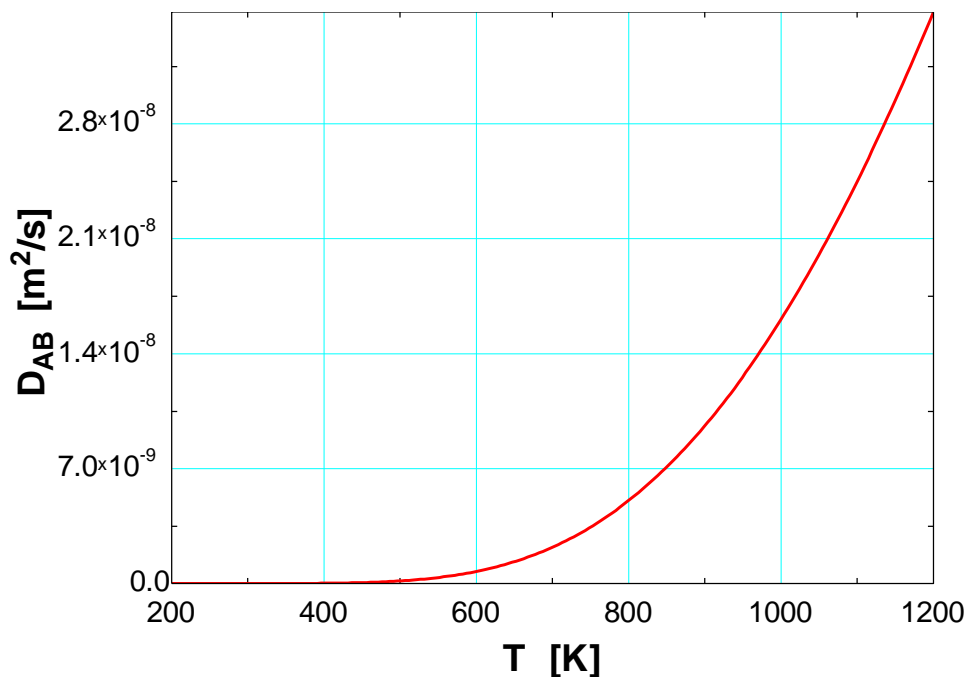
"The diffusion coefficient of hydrogen in steel as a function of temperature is given"

"ANALYSIS"

$$D_{AB} = 1.65 \text{E-}6 * \exp(-4630/T)$$

$$T = 200 \text{ [K]}$$

T [K]	D <sub>AB</sub> [m <sup>2</sup> /s]
200	1.457E-16
250	1.494E-14
300	3.272E-13
350	2.967E-12
400	1.551E-11
450	5.611E-11
500	1.570E-10
550	3.643E-10
600	7.348E-10
650	1.330E-09
700	2.213E-09
750	3.439E-09
800	5.058E-09
850	7.110E-09
900	9.622E-09
950	1.261E-08
1000	1.610E-08
1050	2.007E-08
1100	2.452E-08
1150	2.944E-08
1200	3.482E-08





## Boundary Conditions

**14-24C** Temperature is necessarily a *continuous* function, but concentration, in general, is not. Therefore, the mole fraction of water vapor in air will, in general, be different from the mole fraction of water in the lake (which is nearly 1).

**14-25C** When prescribing a boundary condition for mass transfer at a solid-gas interface, we need to specify the side of the surface (whether the solid or the gas side). This is because concentration, in general, is not a continuous function, and there may be large differences in concentrations on the gas and solid sides of the boundary. We did not do this in heat transfer because temperature is a continuous function.

**14-26C** Three boundary conditions for mass transfer (on mass basis) that correspond to specified temperature, specified heat flux, and convection boundary conditions in heat transfer are expressed as follows:

1)  $w(0) = w_0$  (specified concentration - corresponds to specified temperature)

2)  $-\rho D_{AB} \left. \frac{dw_A}{dx} \right|_{x=0} = J_{A,0}$  (specified mass flux - corresponds to specified heat flux)

3)  $j_{A,s} = -D_{AB} \left. \frac{\partial w_A}{\partial x} \right|_{x=0} = h_{\text{mass}} (w_{A,s} - w_{A,\infty})$  (mass convection - corresponds to heat convection)

**14-27C** An impermeable surface is a surface that does not allow any mass to pass through. Mathematically it is expressed (at  $x = 0$ ) as

$$\left. \frac{dw_A}{dx} \right|_{x=0} = 0$$

An impermeable surface in mass transfer corresponds to an insulated surface in heat transfer.

**14-28C** Using solubility data of a solid in a specified liquid, the mass fraction  $w$  of the solid  $A$  in the liquid at the interface at a specified temperature can be determined from

$$w_A = \frac{m_{\text{solid}}}{m_{\text{solid}} + m_{\text{liquid}}}$$

where  $m_{\text{solid}}$  is the maximum amount of solid dissolved in the liquid of mass  $m_{\text{liquid}}$  at the specified temperature.

**14-29C** Using Henry's constant data for a gas dissolved in a liquid, the mole fraction of the gas dissolved in the liquid at the interface at a specified temperature can be determined from Henry's law expressed as

$$y_{i,\text{liquid side}}(0) = \frac{P_{i,\text{gas side}}(0)}{H}$$

where  $H$  is *Henry's constant* and  $P_{i,\text{gas side}}(0)$  is the partial pressure of the gas  $i$  at the gas side of the interface. This relation is applicable for dilute solutions (gases that are weakly soluble in liquids).

**14-30C** The mole fraction of the water vapor at the surface of a lake when the temperature of the lake surface and the atmospheric pressure are specified can be determined from

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{P_{\text{sat}@T}}{P_{\text{atm}}}$$

where  $P_{\text{vapor}}$  is equal to the saturation pressure of water at the lake surface temperature.

**14-31C** The molar concentration  $C_i$  of the gas species  $i$  in the solid at the interface  $C_{i, \text{solid side}}(0)$  is proportional to the *partial pressure* of the species  $i$  in the gas  $P_{i, \text{gas side}}(0)$  on the gas side of the interface, and is determined from

$$C_{i, \text{solid side}}(0) = S \times P_{i, \text{gas side}}(0) \quad (\text{kmol/m}^3)$$

where  $S$  is the *solubility* of the gas in that solid at the specified temperature.

**14-32C** The permeability is a measure of the ability of a gas to penetrate a solid. The permeability of a gas in a solid,  $P$ , is related to the solubility of the gas by  $P = SD_{AB}$  where  $D_{AB}$  is the diffusivity of the gas in the solid.

**14-33** The mole fraction of  $\text{CO}_2$  dissolved in water at the surface of water at 300 K is to be determined.

**Assumptions** **1** Both the  $\text{CO}_2$  and water vapor are ideal gases. **2** Air at the lake surface is saturated.

**Properties** The saturation pressure of water at 300 K = 27°C is 3.60 kPa (Table A-9). The Henry's constant for  $\text{CO}_2$  in water at 300 K is 1710 bar (Table 14-6).

**Analysis** The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 27°C,

$$P_{\text{vapor}} = P_{\text{sat}@27^\circ\text{C}} = 3.60 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the partial pressure and mole fraction of dry air in the air at the surface of the lake are determined to be

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 100 - 3.60 = 96.4 \text{ kPa}$$

The partial pressure of  $\text{CO}_2$  is

$$P_{\text{CO}_2} = y_{\text{CO}_2} P_{\text{dry air}} = (0.005)(96.4) = 0.482 \text{ kPa} = 0.00482 \text{ bar}$$

$$y_{\text{CO}_2} = \frac{P_{\text{CO}_2}}{H} = \frac{0.00482 \text{ bar}}{1710 \text{ bar}} = 2.82 \times 10^{-6}$$

**14-34E** Water is sprayed into air, and the falling water droplets are collected in a container. The mass and mole fractions of air dissolved in the water are to be determined.

**Assumptions** **1** Both the air and water vapor are ideal gases. **2** Air is saturated since water is constantly sprayed into it. **3** Air is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at 80°F is 0.5073 psia (Table A-9E). Henry's constant for air dissolved in water at 80°F (300 K) is given in Table 14-6 to be  $H = 74,000$  bar. Molar masses of dry air and water are 29 and 18 lbm / lbmol, respectively (Table A-1E).

**Analysis** Noting that air is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 80°F,

$$P_{\text{vapor}} = P_{\text{sat}@80^\circ\text{F}} = 0.5073 \text{ psia}$$

Then the partial pressure of dry air becomes

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 14.3 - 0.5073 = 13.79 \text{ psia}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air, liquid side}} = \frac{P_{\text{dry air, gas side}}}{H} = \frac{13.79 \text{ psia} (1 \text{ atm} / 14.696 \text{ psia})}{74,000 \text{ bar} (1 \text{ atm} / 1.01325 \text{ bar})} = \mathbf{1.29 \times 10^{-5}}$$

which is very small, as expected. The mass and mole fractions of a mixture are related to each other by

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

where the apparent molar mass of the liquid water - air mixture is

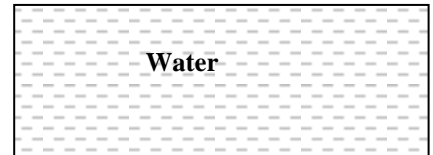
$$\begin{aligned} M_m &= \sum y_i M_i = y_{\text{liquid water}} M_{\text{water}} + y_{\text{dry air}} M_{\text{dry air}} \\ &\cong 1 \times 29.0 + 0 \times 18.0 \cong 29.0 \text{ kg/kmol} \end{aligned}$$

Then the mass fraction of dissolved air in liquid water becomes

$$w_{\text{dry air, liquid side}} = y_{\text{dry air, liquid side}} \frac{M_{\text{dry air}}}{M_m} = 1.29 \times 10^{-5} \frac{29}{29} = \mathbf{1.29 \times 10^{-5}}$$

**Discussion** The mass and mole fractions of dissolved air in this case are identical because of the very small amount of air in water.

○ ○ ○ ○  
○ ○  
○ Water droplets in air



**14-35** A glass of water is left in a room. The mole fraction of the water vapor in the air and the mole fraction of air in the water are to be determined when the water and the air are in thermal and phase equilibrium.

**Assumptions** **1** Both the air and water vapor are ideal gases. **2** Air is saturated since the humidity is 100 percent. **3** Air is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at 20°C is 2.339 kPa (Table A-9). Henry's constant for air dissolved in water at 20°C (293 K) is given in Table 14-6 to be  $H = 65,600$  bar (determined by interpolation). Molar masses of dry air and water are 29 and 18 kg/kmol, respectively (Table A-1).

**Analysis** (a) Noting that air is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 20°C,

$$P_{\text{vapor}} = P_{\text{sat}@20^\circ\text{C}} = 2.339 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air is determined to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{2.339 \text{ kPa}}{97 \text{ kPa}} = \mathbf{0.0241}$$

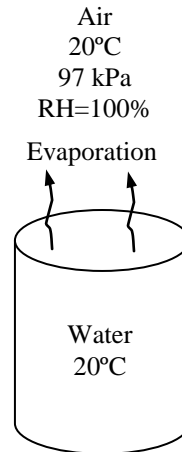
(b) Noting that the total pressure is 97 kPa, the partial pressure of dry air is

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 97 - 2.339 = 94.66 \text{ kPa} = 0.9466 \text{ bar}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air, liquid side}} = \frac{P_{\text{dry air, gas side}}}{H} = \frac{0.9466 \text{ bar}}{65,600 \text{ bar}} = \mathbf{1.44 \times 10^{-5}}$$

**Discussion** The amount of air dissolved in water is very small, as expected.



**14-36E** The mole fraction of the water vapor at the surface of a lake and the mole fraction of water in the lake are to be determined and compared.

**Assumptions** 1 Both the air and water vapor are ideal gases. 2 Air is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at 70°F is 0.3632 psia (Table A-9E). Henry's constant for air dissolved in water at 70°F (294 K) is given in Table 14-6 to be  $H = 66,800$  bar.

**Analysis** The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 70°F,

$$P_{\text{vapor}} = P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air at the surface of the lake is determined from Eq. 14-11 to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{0.3632 \text{ psia}}{13.8 \text{ psia}} = \mathbf{0.0263 \text{ (or 2.63 percent)}}$$

The partial pressure of dry air just above the lake surface is

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 13.8 - 0.3632 = 13.44 \text{ psia}$$

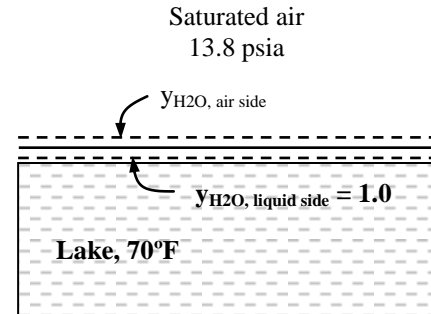
Then the mole fraction of air in the water becomes

$$y_{\text{dry air, liquid side}} = \frac{P_{\text{dry air, gas side}}}{H} = \frac{13.44 \text{ psia} (1 \text{ atm} / 14.696 \text{ psia})}{66,800 \text{ bar} (1 \text{ atm} / 1.01325 \text{ bar})} = 1.39 \times 10^{-5}$$

which is very small, as expected. Therefore, the mole fraction of water in the lake near the surface is

$$y_{\text{water, liquid side}} = 1 - y_{\text{dry air, liquid side}} = 1 - 1.39 \times 10^{-5} = \mathbf{0.99999}$$

**Discussion** The concentration of air in water just below the air-water interface is 1.39 moles per 100,000 moles. The amount of air dissolved in water will decrease with increasing depth.



**14-37** The mole fraction of the water vapor at the surface of a lake at a specified temperature is to be determined.

**Assumptions** 1 Both the air and water vapor are ideal gases. 2 Air at the lake surface is saturated.

**Properties** The saturation pressure of water at 15°C is 1.7051 kPa (Table A-9).

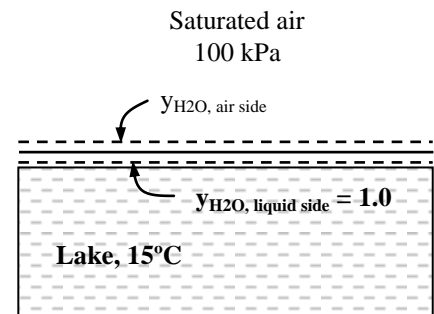
**Analysis** The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 15°C,

$$P_{\text{vapor}} = P_{\text{sat}@15^\circ\text{C}} = 1.7051 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the partial pressure and mole fraction of dry air in the air at the surface of the lake are determined to be

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 100 - 1.7051 = 98.295 \text{ kPa}$$

$$y_{\text{dry air}} = \frac{P_{\text{dry air}}}{P} = \frac{98.295 \text{ kPa}}{100 \text{ kPa}} = \mathbf{0.983 \text{ (or 98.3%)}}$$



Therefore, the mole fraction of dry air is 97.9 percent just above the air-water interface.



**14-38** Prob. 14-37 is reconsidered. The mole fraction of dry air at the surface of the lake as a function of the lake temperature is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

T=18 [C]

P\_atm=100 [kPa]

"PROPERTIES"

Fluid\$='steam\_IAPWS'

P\_sat=Pressure(Fluid\$, T=T, x=1)

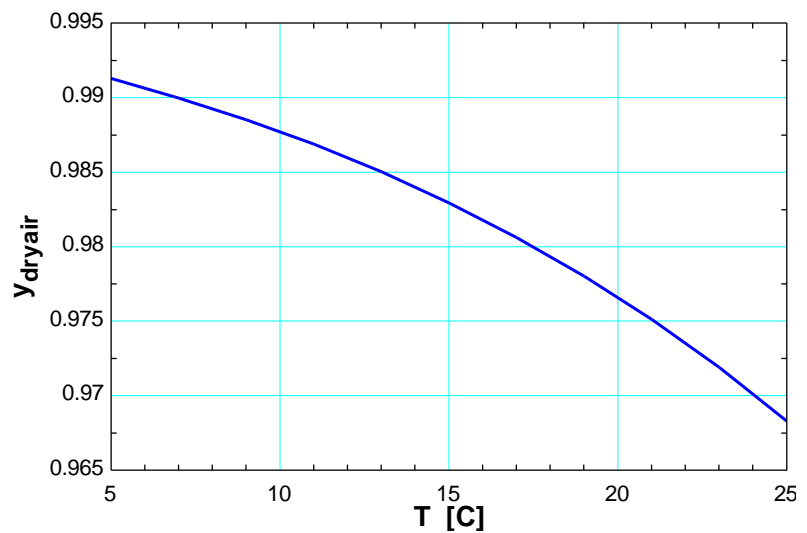
"ANALYSIS"

P\_vapor=P\_sat

P\_dryair=P\_atm-P\_vapor

y\_dryair=P\_dryair/P\_atm

T [C]	y <sub>dry air</sub>
5	0.9913
7	0.99
9	0.9885
11	0.9869
13	0.985
15	0.9829
17	0.9806
19	0.978
21	0.9751
23	0.9719
25	0.9683



**14-39** A carbonated drink in a bottle is considered. Assuming the gas space above the liquid consists of a saturated mixture of  $\text{CO}_2$  and water vapor and treating the drink as a water, determine the mole fraction of the water vapor in the  $\text{CO}_2$  gas and the mass of dissolved  $\text{CO}_2$  in a 200 ml drink are to be determined when the water and the  $\text{CO}_2$  gas are in thermal and phase equilibrium.

**Assumptions** **1** The liquid drink can be treated as water. **2** Both the  $\text{CO}_2$  and the water vapor are ideal gases. **3** The  $\text{CO}_2$  gas and water vapor in the bottle form a saturated mixture. **4** The  $\text{CO}_2$  is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at  $37^\circ\text{C}$  is 6.33 kPa (Table A-9). Henry's constant for  $\text{CO}_2$  dissolved in water at  $37^\circ\text{C}$  (310 K) is given in Table 14-6 to be  $H = 2170$  bar. Molar masses of  $\text{CO}_2$  and water are 44 and 18 kg/kmol, respectively (Table A-1).

**Analysis** (a) Noting that the  $\text{CO}_2$  gas in the bottle is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at  $37^\circ\text{C}$ ,

$$P_{\text{vapor}} = P_{\text{sat}@37^\circ\text{C}} = 6.33\text{ kPa}$$

Assuming both  $\text{CO}_2$  and vapor to be ideal gases, the mole fraction of water vapor in the  $\text{CO}_2$  gas becomes

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{6.33\text{ kPa}}{130\text{ kPa}} = \mathbf{0.0487} \quad (\text{or } 4.87\%)$$

(b) Noting that the total pressure is 130 kPa, the partial pressure of  $\text{CO}_2$  is

$$P_{\text{CO}_2 \text{ gas}} = P - P_{\text{vapor}} = 130 - 6.33 = 123.7\text{ kPa} = 1.237\text{ bar}$$

From Henry's law, the mole fraction of  $\text{CO}_2$  in the drink is determined to be

$$y_{\text{CO}_2, \text{liquid side}} = \frac{P_{\text{CO}_2, \text{gas side}}}{H} = \frac{1.237\text{ bar}}{2170\text{ bar}} = 5.70 \times 10^{-4}$$

Then the mole fraction of water in the drink becomes

$$y_{\text{water, liquid side}} = 1 - y_{\text{CO}_2, \text{liquid side}} = 1 - 5.70 \times 10^{-4} = 0.9994$$

The mass and mole fractions of a mixture are related to each other by

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

where the apparent molar mass of the drink (liquid water -  $\text{CO}_2$  mixture) is

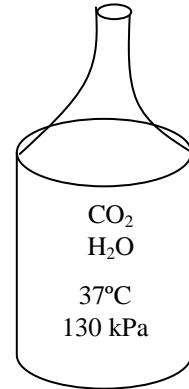
$$M_m = \sum y_i M_i = y_{\text{liquid water}} M_{\text{water}} + y_{\text{CO}_2} M_{\text{CO}_2} = 0.9994 \times 18.0 + (5.70 \times 10^{-4}) \times 44 = 18.00\text{ kg/kmol}$$

Then the mass fraction of dissolved  $\text{CO}_2$  gas in liquid water becomes

$$w_{\text{CO}_2, \text{liquid side}} = y_{\text{CO}_2, \text{liquid side}} \frac{M_{\text{CO}_2}}{M_m} = 5.70 \times 10^{-4} \frac{44}{18.00} = 0.00139$$

Therefore, the mass of dissolved  $\text{CO}_2$  in a 200 ml  $\approx$  200 g drink is

$$m_{\text{CO}_2} = w_{\text{CO}_2} m_m = 0.00139(200\text{ g}) = \mathbf{0.278\text{ g}}$$



**14-40** A rubber plate is exposed to nitrogen. The molar and mass density of nitrogen in the rubber at the interface is to be determined.

**Assumptions** Rubber and nitrogen are in thermodynamic equilibrium at the interface.

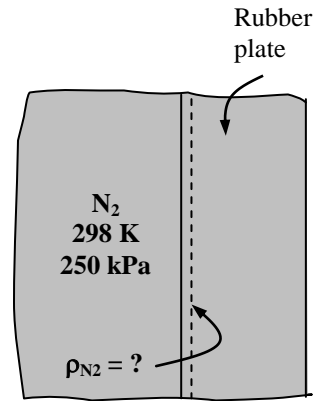
**Properties** The molar mass of nitrogen is  $M = 28.0$  kg/kmol (Table A-1). The solubility of nitrogen in rubber at 298 K is  $0.00156$  kmol/m<sup>3</sup>·bar (Table 14-7).

**Analysis** Noting that 250 kPa = 2.5 bar, the molar density of nitrogen in the rubber at the interface is determined from Eq. 14-20 to be

$$\begin{aligned} C_{\text{N}_2, \text{solidside}}(0) &= S \times P_{\text{N}_2, \text{gas side}} \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(2.5 \text{ bar}) \\ &= \mathbf{0.0039 \text{ kmol/m}^3} \end{aligned}$$

It corresponds to a mass density of

$$\begin{aligned} \rho_{\text{N}_2, \text{solidside}}(0) &= C_{\text{N}_2, \text{solidside}}(0) M_{\text{N}_2} \\ &= (0.0039 \text{ kmol/m}^3)(28 \text{ kmol/kg}) \\ &= \mathbf{0.1092 \text{ kg/m}^3} \end{aligned}$$



That is, there will be 0.0039 kmol (or 0.1092 kg) of N<sub>2</sub> gas in each m<sup>3</sup> volume of rubber adjacent to the interface.

**14-41** A rubber wall separates O<sub>2</sub> and N<sub>2</sub> gases. The molar concentrations of O<sub>2</sub> and N<sub>2</sub> in the wall are to be determined.

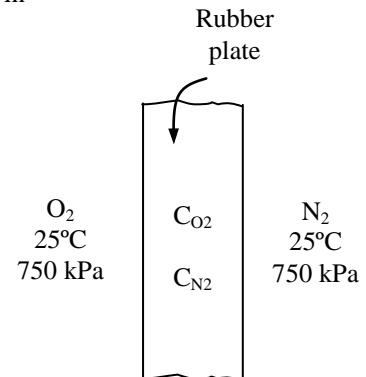
**Assumptions** The O<sub>2</sub> and N<sub>2</sub> gases are in phase equilibrium with the rubber wall.

**Properties** The molar mass of oxygen and nitrogen are 32.0 and 28.0 kg/kmol, respectively (Table A-1). The solubility of oxygen and nitrogen in rubber at 298 K are 0.00312 and 0.00156 kmol/m<sup>3</sup>·bar, respectively (Table 14-7).

**Analysis** Noting that 750 kPa = 7.5 bar, the molar densities of oxygen and nitrogen in the rubber wall are determined from Eq. 14-20 to be

$$\begin{aligned} C_{\text{O}_2, \text{solidside}}(0) &= S \times P_{\text{O}_2, \text{gas side}} \\ &= (0.00312 \text{ kmol/m}^3 \cdot \text{bar})(7.5 \text{ bar}) \\ &= \mathbf{0.0234 \text{ kmol/m}^3} \end{aligned}$$

$$\begin{aligned} C_{\text{N}_2, \text{solidside}}(0) &= S \times P_{\text{N}_2, \text{gas side}} \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(7.5 \text{ bar}) \\ &= \mathbf{0.0117 \text{ kmol/m}^3} \end{aligned}$$



That is, there will be 0.0234 kmol of O<sub>2</sub> and 0.0117 kmol of N<sub>2</sub> gas in each m<sup>3</sup> volume of the rubber wall.



**14-42** Hydrogen gas is stored in a spherical nickel vessel that is in an atmospheric air surrounding. The concentrations of hydrogen at the inner and outer surfaces are to be determined.

**Assumptions 1** Hydrogen is in thermodynamic equilibrium with the nickel wall.

**Properties** The molar mass for  $H_2$  is 2.016 kg/kmol (Table A-1). The solubility of  $H_2$  in nickel at 358 K is  $\mathcal{S} = 0.00901$  kmol/m<sup>3</sup>·bar (Table 14-7).

**Analysis** At the inner surface, there is 100%  $H_2$ , so the  $H_2$  concentration is

$$\begin{aligned} C_{H_2,1} &= \mathcal{S}_{H_2} y_{H_2} P_1 \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(1)(7.5 \text{ bar}) \\ &= \mathbf{0.0676 \text{ kmol/m}^3} \end{aligned}$$

$$\begin{aligned} \rho_{H_2,1} &= M C_{H_2,1} \\ &= (2.016 \text{ kg/kmol})(0.0676 \text{ kmol/m}^3) \\ &= \mathbf{0.136 \text{ kg/m}^3} \end{aligned}$$

At the outer surface, there is 0%  $H_2$ , so the  $H_2$  concentration is

$$C_{H_2,2} = \mathcal{S}_{H_2} y_{H_2} P_2 = 0 \quad \text{and} \quad \rho_{H_2,2} = M C_{H_2,2} = 0$$

**Discussion** The mole fraction of hydrogen in the atmosphere is extremely low (0.000055%). Thus, at the outer surface of the nickel vessel the hydrogen concentration is practically zero.

**14-43** Pure N<sub>2</sub> gas is flowing through a rubber pipe that is in an atmospheric air (79% N<sub>2</sub> & 21% O<sub>2</sub>) surrounding. The concentrations of N<sub>2</sub> and O<sub>2</sub> at the inner and outer surfaces are to be determined.

**Assumptions 1** Nitrogen and oxygen are in thermodynamic equilibrium with the rubber wall.

**Properties** The molar masses for N<sub>2</sub> and O<sub>2</sub> are 28.01 kg/kmol and 32.0 kg/kmol, respectively (Table A-1). The solubility of N<sub>2</sub> and O<sub>2</sub> in rubber at 298 K are  $\mathcal{S}_{\text{N}_2} = 0.00156 \text{ kmol/m}^3 \cdot \text{bar}$  and  $\mathcal{S}_{\text{O}_2} = 0.00312 \text{ kmol/m}^3 \cdot \text{bar}$ , respectively (Table 14-7).

**Analysis** At the inner surface, there is 100% N<sub>2</sub> and 0% O<sub>2</sub>.

For nitrogen:

$$\begin{aligned} C_{\text{N}_2,1} &= \mathcal{S}_{\text{N}_2} y_{\text{N}_2} P_1 \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(1)(2.0 \text{ bar}) \\ &= \mathbf{0.00312 \text{ kmol/m}^3} \end{aligned}$$

For oxygen:

$$\begin{aligned} C_{\text{O}_2,1} &= \mathcal{S}_{\text{O}_2} y_{\text{O}_2} P_1 \\ &= (0.00312 \text{ kmol/m}^3 \cdot \text{bar})(0)(2.0 \text{ bar}) \\ &= \mathbf{0} \end{aligned}$$

At the outer surface, there is 79% N<sub>2</sub> and 21% O<sub>2</sub>.

For nitrogen:

$$\begin{aligned} C_{\text{N}_2,2} &= \mathcal{S}_{\text{N}_2} y_{\text{N}_2} P_2 \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(0.79)(1.013 \text{ bar}) \\ &= \mathbf{0.00125 \text{ kmol/m}^3} \end{aligned}$$

For oxygen:

$$\begin{aligned} C_{\text{O}_2,2} &= \mathcal{S}_{\text{O}_2} y_{\text{O}_2} P_2 \\ &= (0.00312 \text{ kmol/m}^3 \cdot \text{bar})(0.21)(1.013 \text{ bar}) \\ &= \mathbf{0.000664 \text{ kmol/m}^3} \end{aligned}$$

**Discussion** The higher solubility of O<sub>2</sub> over N<sub>2</sub> in rubber means that at the same mole fraction and pressure, the concentration of O<sub>2</sub> in rubber would be higher than N<sub>2</sub>.

**14-44** A nickel wall separates  $H_2$  gas at different pressures. (a) The mass densities of  $H_2$  in the nickel wall and (b) outside the nickel wall are to be determined.

**Assumptions** 1 Nickel and hydrogen are in thermodynamic equilibrium at the interface. 2 Hydrogen an ideal gas.

**Properties** The molar mass of  $H_2$  is  $M = 2.016 \text{ kg/kmol}$  (Table A-1). The solubility of  $H_2$  in nickel at  $85^\circ\text{C} = 358\text{K}$  is  $0.00901 \text{ kmol/m}^3 \cdot \text{bar}$  (Table 14-7).

**Analysis** (a) The mass density of  $H_2$  (for 5 atm) in the nickel at the interface is determined using

$$\begin{aligned}\rho_{H_2, \text{solid side}} &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})MP_{H_2, \text{gas side}} \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(2.016 \text{ kg/kmol})(5 \text{ atm})(1.01325 \text{ bar/atm}) \\ &= \mathbf{0.0920 \text{ kg/m}^3}\end{aligned}$$

Then, the mass density of  $H_2$  (for 3 atm) in the nickel at the interface is

$$\begin{aligned}\rho_{H_2, \text{solid side}} &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})MP_{H_2, \text{gas side}} \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(2.016 \text{ kg/kmol})(3 \text{ atm})(1.01325 \text{ bar/atm}) \\ &= \mathbf{0.0552 \text{ kg/m}^3}\end{aligned}$$

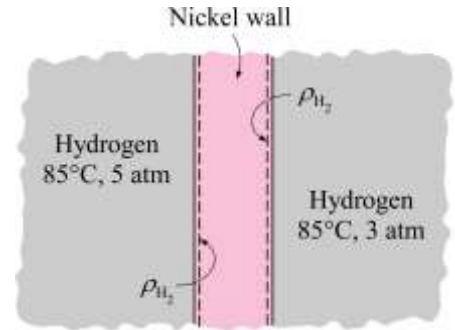
(b) The mass density of  $H_2$  (for 5 atm) outside the nickel is determined using

$$\rho_{H_2} = \frac{P_{H_2} M}{R_u T} = \frac{(5 \text{ atm})(101.325 \text{ kPa/atm})(2.016 \text{ kg/kmol})}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 85) \text{ K}} = \mathbf{0.343 \text{ kg/m}^3}$$

Then, the mass density of  $H_2$  (for 3 atm) outside the rubber is

$$\rho_{H_2} = \frac{P_{H_2} M}{R_u T} = \frac{(3 \text{ atm})(101.325 \text{ kPa/atm})(2.016 \text{ kg/kmol})}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 85) \text{ K}} = \mathbf{0.206 \text{ kg/m}^3}$$

**Discussion** Note that the densities of  $H_2$  outside the nickel wall are quite different from those in the nickel wall.



**14-45** A rubber membrane separates CO<sub>2</sub> gas at different pressures. (a) The molar densities of CO<sub>2</sub> in the membrane and (b) outside the membrane are to be determined.

**Assumptions** 1 Rubber and nitrogen are in thermodynamic equilibrium at the interface. 2 Carbon dioxide is an ideal gas.

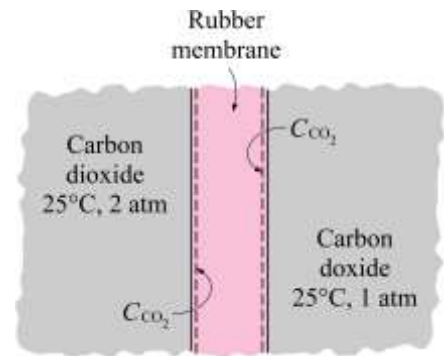
**Properties** The molar mass of CO<sub>2</sub> is  $M = 44.01$  kg/kmol (Table A-1). The solubility of CO<sub>2</sub> in rubber at 25°C = 298 K is 0.04015 kmol/m<sup>3</sup>·bar (Table 14-7).

**Analysis** (a) The molar density of CO<sub>2</sub> (for 2 atm) in the rubber at the interface is determined using

$$\begin{aligned} C_{\text{CO}_2, \text{solid side}} &= (0.04015 \text{ kmol/m}^3 \cdot \text{bar}) P_{\text{CO}_2, \text{gas side}} \\ &= (0.04015 \text{ kmol/m}^3 \cdot \text{bar})(2 \text{ atm})(1.01325 \text{ bar/atm}) \\ &= \mathbf{0.0814 \text{ kmol/m}^3} \end{aligned}$$

Then, the molar density of CO<sub>2</sub> (for 1 atm) in the rubber at the interface is

$$\begin{aligned} C_{\text{CO}_2, \text{solid side}} &= (0.04015 \text{ kmol/m}^3 \cdot \text{bar}) P_{\text{CO}_2, \text{gas side}} \\ &= (0.04015 \text{ kmol/m}^3 \cdot \text{bar})(1 \text{ atm})(1.01325 \text{ bar/atm}) \\ &= \mathbf{0.0407 \text{ kmol/m}^3} \end{aligned}$$



(b) The molar density of CO<sub>2</sub> (for 2 atm) outside the rubber is determined using

$$C_{\text{CO}_2} = \frac{P_{\text{CO}_2}}{R_u T} = \frac{(2 \text{ atm})(101.325 \text{ kPa/atm})}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = \mathbf{0.0818 \text{ kmol/m}^3}$$

Then, the molar density of CO<sub>2</sub> (for 1 atm) outside the rubber is

$$C_{\text{CO}_2} = \frac{P_{\text{CO}_2}}{R_u T} = \frac{(1 \text{ atm})(101.325 \text{ kPa/atm})}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = \mathbf{0.409 \text{ kmol/m}^3}$$

**Discussion** Due to its relatively high solubility in rubber, in comparison to O<sub>2</sub> and N<sub>2</sub> (see Table 14-7), the molar concentrations of CO<sub>2</sub> in the solid side and the gas side are almost equal.

**14-46** A nickel vessel with specified dimensions is used to contain hydrogen. The rate gas loss from the vessel and the fraction of the hydrogen lost after one year of storage are to be determined.

**Assumptions** **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the rubber plug that result in the generation or depletion of carbon dioxide.

**Properties** The binary diffusion coefficient for  $H_2$  in the nickel at 358 K is  $D_{AB} = 1.2 \times 10^{-12} \text{ m}^2/\text{s}$  (Table 14–3b). The molar mass of  $H_2$  is  $M = 2.016 \text{ kg/kmol}$  (Table A-1). The solubility of  $H_2$  in the rubber at 358 K is  $0.00901 \text{ kmol/m}^3 \cdot \text{bar}$  (Table 14-7).

**Analysis** The mass density of  $H_2$  (for 3 bar) in the nickel at the interface is determined from

$$\begin{aligned}\rho_{A,1} &= \mathcal{S} P_{A,1} \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(3 \text{ bar}) \left( \frac{2.016 \text{ kg}}{1 \text{ kmol}} \right) \\ &= 0.05449 \text{ kg/m}^3\end{aligned}$$

On the opposite side, the mass density of  $H_2$  is zero,  $\rho_{A,2} = 0$ . Then the rate of carbon dioxide gas loss through the rubber plug becomes

$$\begin{aligned}\dot{m}_{\text{diff}} &= D_{AB} A \frac{\rho_{A,1} - \rho_{A,2}}{L} = D_{AB} \left( \frac{\pi D^2}{4} \right) \frac{\rho_{A,1} - \rho_{A,2}}{L} \\ &= (1.2 \times 10^{-12} \text{ m}^2/\text{s})(1600 \times 10^{-4} \text{ m}^2) \frac{(0.05449 - 0) \text{ kg/m}^3}{0.002 \text{ m}} \\ &= \mathbf{5.23 \times 10^{-12} \text{ kg/s}} = 1.65 \times 10^{-4} \text{ kg/year}\end{aligned}$$

This corresponds to about 0.165 gram of  $H_2$  per year. The mass of  $H_2$  in the vessel is

$$m = \frac{PV}{RT} = \frac{(300 \text{ kPa})(0.005 \text{ m}^3)}{(4.124 \text{ kJ/kg} \cdot \text{K})(358 \text{ K})} = 0.001016 \text{ kg}$$

The fraction of  $H_2$  lost after one year of storage is then

$$f = \frac{1.65 \times 10^{-4} \text{ kg}}{0.001016 \text{ kg}} = \mathbf{0.162} \text{ (or 16.2\%)}$$

**Discussion** A loss of 16.2% per year may be significant, and therefore, alternatives should be considered for long-term storage.

**14-47** A nickel wall separates  $H_2$  gas at different pressures. The molar diffusion rate per unit area through the nickel wall is to be determined.

**Assumptions** **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the nickel wall that result in the generation or depletion of hydrogen.

**Properties** The binary diffusion coefficient for hydrogen in the nickel at  $85^\circ\text{C} = 358\text{ K}$  is  $D_{AB} = 1.2 \times 10^{-12}\text{ m}^2/\text{s}$  (Table 14-3b). The solubility of  $H_2$  in nickel at  $85^\circ\text{C} = 358\text{ K}$  is  $0.00901\text{ kmol}/\text{m}^3 \cdot \text{bar}$  (Table 14-7).

**Analysis** The molar density of  $H_2$  (for 5 atm) in the nickel at the interface is determined using

$$\begin{aligned} C_{A,1} &= (0.00901\text{ kmol}/\text{m}^3 \cdot \text{bar})P_{A,1} \\ &= (0.00901\text{ kmol}/\text{m}^3 \cdot \text{bar})(5\text{ atm})(1.01325\text{ bar/atm}) \\ &= 0.0456\text{ kmol}/\text{m}^3 \end{aligned}$$

Then, the molar density of  $H_2$  (for 3 atm) in the nickel at the interface is

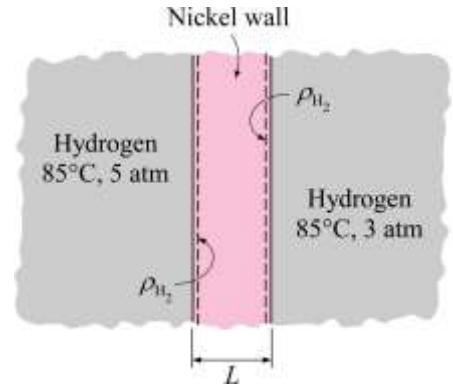
$$\begin{aligned} C_{A,2} &= (0.00901\text{ kmol}/\text{m}^3 \cdot \text{bar})P_{A,2} \\ &= (0.00901\text{ kmol}/\text{m}^3 \cdot \text{bar})(3\text{ atm})(1.01325\text{ bar/atm}) \\ &= 0.0274\text{ kmol}/\text{m}^3 \end{aligned}$$

The molar diffusion rate per unit area of hydrogen through the nickel wall can readily be determined using

$$\begin{aligned} \bar{j}_{\text{diff}} &= \frac{\dot{N}_{\text{diff}}}{A} = D_{AB} \frac{C_{A,1} - C_{A,2}}{L} \\ &= (1.2 \times 10^{-12}\text{ m}^2/\text{s}) \frac{(0.0456 - 0.0274)\text{ kmol}/\text{m}^3}{0.0001\text{ m}} \\ &= 2.18 \times 10^{-10}\text{ kmol}/\text{s} \cdot \text{m}^2 \end{aligned}$$

**Discussion** The molar mass of  $H_2$  is  $M = 2.016\text{ kg}/\text{kmol}$  (Table A-1). Hence, the mass diffusion rate per unit area of hydrogen through the nickel wall is

$$j_{\text{diff}} = (2.18 \times 10^{-10}\text{ kmol}/\text{s} \cdot \text{m}^2)(2.016\text{ kg}/\text{kmol}) = 4.40 \times 10^{-10}\text{ kg}/\text{s} \cdot \text{m}^2$$



**14-48** A dry wall separates air in a room with vapor pressure of 3 kPa from air with negligible vapor pressure in the insulation adjoining the wall. The mass diffusion rate of water vapor through the wall is to be determined.

**Assumptions** 1 Mass diffusion is steady and one-dimensional. 2 Constant properties. 3 Condensation in the wall is negligible.

**Properties** The molar mass of water vapor is  $M = 18.015 \text{ kg/kmol}$  (Table A-1).

**Analysis** The molar density of water vapor in the dry wall at the interface is determined using

$$\begin{aligned} C_{A,1} &= (0.007 \text{ kmol/m}^3 \cdot \text{bar}) P_{A,1} \\ &= (0.007 \text{ kmol/m}^3 \cdot \text{bar})(3 \text{ kPa})(0.01 \text{ bar/kPa}) \\ &= 0.00021 \text{ kmol/m}^3 \end{aligned}$$

On the opposite side, the molar density of water vapor is zero, since the vapor pressure is negligible,

$$C_{A,2} = 0$$

The molar diffusion rate of water vapor through the wall can be determined using

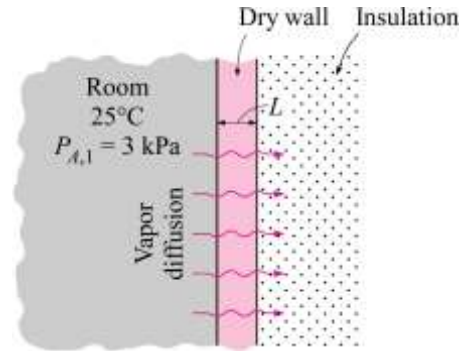
$$\begin{aligned} \dot{N}_{\text{diff}} &= D_{AB} A \frac{C_{A,1} - C_{A,2}}{L} \\ &= (0.2 \times 10^{-9} \text{ m}^2/\text{s})(3 \times 10 \text{ m}^2) \frac{(0.00021 - 0) \text{ kmol/m}^3}{0.012 \text{ m}} \\ &= 1.05 \times 10^{-10} \text{ kmol/s} \end{aligned}$$

Hence the mass diffusion rate of water vapor through the wall is

$$\dot{m}_{\text{diff}} = \dot{N}_{\text{diff}} M = (1.05 \times 10^{-10} \text{ kmol/s})(18.015 \text{ kg/kmol}) = \mathbf{1.89 \times 10^{-9} \text{ kg/s}}$$

**Discussion** At 25°C, the saturation pressure of water is 3169 Pa (from Table 14-9). With the given vapor pressure inside the room being 3 kPa, the relative humidity of the air is

$$\phi = \frac{P_v}{P_{\text{sat}}} = \frac{3000 \text{ Pa}}{3169 \text{ Pa}} = 0.947 = \mathbf{94.7\%}$$



## Steady Mass Diffusion through a Wall

**14-49C** During one-dimensional mass diffusion of species  $A$  through a plane wall, the species  $A$  content of the wall will remain constant during steady mass diffusion, but will change during transient mass diffusion.

**14-50C** The relations for steady one-dimensional heat conduction and mass diffusion through a plane wall are expressed as follows:

$$\text{Heat conduction:} \quad \dot{Q}_{\text{cond}} = -kA \frac{T_1 - T_2}{L}$$

$$\text{Mass diffusion:} \quad \dot{m}_{\text{diff},A,\text{wall}} = \rho D_{AB} A \frac{w_{A,1} - w_{A,2}}{L} = D_{AB} A \frac{\rho_{A,1} - \rho_{A,2}}{L}$$

where  $A$  is the normal area and  $L$  is the thickness of the wall, and the other variables correspond to each other as follows:

$$\text{rate of heat conduction} \quad \dot{Q}_{\text{cond}} \longleftrightarrow \dot{m}_{\text{diff},A,\text{wall}} \quad \text{rate of mass diffusion}$$

$$\text{thermal conductivity} \quad k \longleftrightarrow D_{AB} \quad \text{mass diffusivity}$$

$$\text{temperature} \quad T \longleftrightarrow \rho_A \quad \text{density of } A$$

**14-51C** (a) T, (b) F, (c) T, (d) F



**14-52** A thin plastic membrane separates hydrogen from air. The diffusion rate of hydrogen by diffusion through the membrane under steady conditions is to be determined.

**Assumptions** 1 Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations on both sides of the membrane are maintained constant. Also, there is symmetry about the center plane of the membrane. 2 There are no chemical reactions in the membrane that results in the generation or depletion of hydrogen.

**Properties** The binary diffusion coefficient of hydrogen in the plastic membrane at the operation temperature is given to be  $5.3 \times 10^{-10} \text{ m}^2/\text{s}$ . The molar mass of hydrogen is  $M = 2 \text{ kg/kmol}$  (Table A-1).

**Analysis** (a) We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the plastic membrane to be a *stationary* medium since there is no diffusion of plastic molecules ( $\dot{N}_B = 0$ ) and the concentration of the hydrogen in the membrane is extremely low ( $C_A \ll 1$ ). Then the molar flow rate of hydrogen through the membrane by diffusion per unit area is determined from

$$\begin{aligned}\bar{j}_{\text{diff}} &= \frac{\dot{N}_{\text{diff}}}{A} = D_{AB} \frac{C_{A,1} - C_{A,2}}{L} \\ &= (5.3 \times 10^{-10} \text{ m}^2/\text{s}) \frac{(0.045 - 0.002) \text{ kmol/m}^3}{2 \times 10^{-3} \text{ m}} \\ &= 1.14 \times 10^{-8} \text{ kmol/m}^2 \cdot \text{s}\end{aligned}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of hydrogen,

$$\begin{aligned}\dot{m}_{\text{diff}} &= M \bar{j}_{\text{diff}} = (2 \text{ kg/kmol})(1.14 \times 10^{-8} \text{ kmol/m}^2 \cdot \text{s}) \\ &= \mathbf{2.28 \times 10^{-8} \text{ kg/m}^2 \cdot \text{s}}\end{aligned}$$

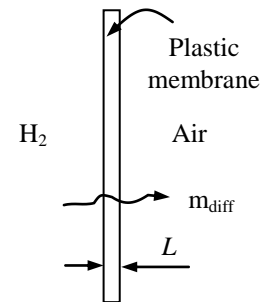
(b) Repeating the calculations for a 0.5-mm thick membrane gives


$$\begin{aligned}\bar{j}_{\text{diff}} &= \frac{\dot{N}_{\text{diff}}}{A} = D_{AB} \frac{C_{A,1} - C_{A,2}}{L} \\ &= (5.3 \times 10^{-10} \text{ m}^2/\text{s}) \frac{(0.045 - 0.002) \text{ kmol/m}^3}{0.5 \times 10^{-3} \text{ m}} \\ &= 4.56 \times 10^{-8} \text{ kmol/m}^2 \cdot \text{s}\end{aligned}$$

and

$$\dot{m}_{\text{diff}} = M \bar{j}_{\text{diff}} = (2 \text{ kg/kmol})(4.56 \times 10^{-8} \text{ kmol/m}^2 \cdot \text{s}) = \mathbf{9.12 \times 10^{-8} \text{ kg/m}^2 \cdot \text{s}}$$

The mass flow rate through the entire membrane can be determined by multiplying the mass flux value above by the membrane area.



**14-53**  A circular plastic plug with specified dimensions is used to contain ammonia inside a vessel. The diffusion rate of ammonia through plug is to be determined whether it is safe or not.

**Assumptions** **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the plug that result in the generation and depletion of ammonia.

**Properties** The diffusion coefficient of ammonia in the plug is given as  $D_{AB} = 1.3 \times 10^{-10} \text{ m}^2/\text{s}$ . The molar mass of ammonia is  $M = 17.03 \text{ kg/kmol}$  (Table A-1).

**Analysis** The ammonia concentration in the atmosphere and at the outer surface of the plug is zero,  $C_{A,2} = 0$ . Thus, the rate of ammonia diffusion through the plug is

$$\begin{aligned}\dot{N}_{\text{diff}} &= D_{AB} A \frac{C_{A,1} - C_{A,2}}{L} = D_{AB} (\pi D^2 / 4) \frac{C_{A,1} - C_{A,2}}{L} \\ &= (1.3 \times 10^{-10} \text{ m}^2/\text{s}) \frac{\pi}{4} (0.10 \text{ m})^2 \left( \frac{30 \text{ mol/L}}{0.002 \text{ m}} \right) \left( \frac{1 \text{ kmol/m}^3}{1 \text{ mol/L}} \right) \\ &= 1.532 \times 10^{-8} \text{ kmol/s}\end{aligned}$$

Thus, the ammonia mass diffusion rate is

$$\begin{aligned}\dot{m}_{\text{diff}} &= \dot{N}_{\text{diff}} M \\ &= (1.532 \times 10^{-8} \text{ kmol/s})(17.03 \text{ kg/kmol}) \\ &= \mathbf{2.61 \times 10^{-7} \text{ kg/s}} \\ &= \mathbf{0.261 \text{ mg/s}} > 0.2 \text{ mg/s}\end{aligned}$$

**Discussion** The rate of ammonia being released by diffusion through the plug is greater than the rate that the ventilation system can handle; therefore the plug cannot safely contain the ammonia inside the vessel.

The plug should be replaced with another that has lower diffusion coefficient, smaller diameter, or larger thickness.

**14-54** Natural gas with 8% hydrogen content is transported in an above ground pipeline. The highest rate of hydrogen loss through the pipe at steady conditions is to be determined.

**Assumptions** **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations inside the pipe is constant, and in the atmosphere it is negligible. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of hydrogen. **3** Both  $H_2$  and  $CH_4$  are ideal gases.

**Properties** The binary diffusion coefficient of hydrogen in the steel pipe at the operation temperature is given to be  $2.9 \times 10^{-13} \text{ m}^2/\text{s}$ . The molar masses of  $H_2$  and  $CH_4$  are 2 and 16 kg/kmol, respectively (Table A-1). The solubility of hydrogen gas in steel is given as  $w_{H_2} = 2.09 \times 10^{-4} \exp(-3950/T) P_{H_2}^{0.5}$ . The density of steel pipe is  $7854 \text{ kg/m}^3$  (Table A-3).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the steel pipe to be a *stationary* medium since there is no diffusion of steel molecules ( $\dot{N}_B = 0$ ) and the concentration of the hydrogen in the steel pipe is extremely low ( $C_A \ll 1$ ). The molar mass of the  $H_2$  and  $CH_4$  mixture in the pipe is

$$M = \sum y_i M_i = (0.08)(2) + (0.92)(16) = 14.88 \text{ kg/kmol}$$

Noting that the mole fraction of hydrogen is 0.08, the partial pressure of hydrogen is

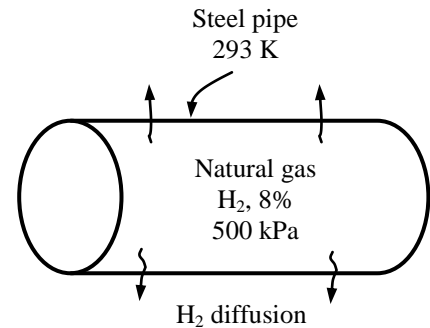
$$y_{H_2} = \frac{P_{H_2}}{P} \rightarrow P_{H_2} = (0.08)(500 \text{ kPa}) = 40 \text{ kPa} = 0.4 \text{ bar}$$

Then the mass fraction of hydrogen becomes

$$\begin{aligned} w_{H_2} &= 2.09 \times 10^{-4} \exp(-3950/T) P_{H_2}^{0.5} \\ &= 2.09 \times 10^{-4} \exp(-3950/293)(0.4)^{0.5} \\ &= 1.85 \times 10^{-10} \end{aligned}$$

The hydrogen concentration in the atmosphere is practically zero, and thus in the limiting case the hydrogen concentration at the outer surface of pipe can be taken to be zero. Then the highest rate of hydrogen loss through a 100 m long section of the pipe at steady conditions is determined to be

$$\begin{aligned} \dot{m}_{\text{diff,A,cyl}} &= 2\pi L \rho D_{AB} \frac{w_{A,1} - w_{A,2}}{\ln(r_2/r_1)} \\ &= 2\pi(100 \text{ m})(7854 \text{ kg/m}^3)(2.9 \times 10^{-13}) \frac{1.85 \times 10^{-10} - 0}{\ln(1.51/1.50)} \\ &= \mathbf{3.98 \times 10^{-14} \text{ kg/s}} \end{aligned}$$





**14-55** Prob. 14-54 is reconsidered. The highest rate of hydrogen loss as a function of the mole fraction of hydrogen in natural gas is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

thickness=0.01 [m]

$D_i=3$  [m]

$L=100$  [m]

$P=500$  [kPa]

$y_{H_2}=0.08$

$T=293$  [K]

$D_{AB}=2.9E-13$  [m<sup>2</sup>/s]

"PROPERTIES"

$MM_{H_2}=\text{molarmass}(H_2)$

$MM_{CH_4}=\text{molarmass}(CH_4)$

$R_u=8.314$  [kPa·m<sup>3</sup>/kmol·K]

$\rho=7854$  [kg/m<sup>3</sup>]

"ANALYSIS"

$MM=y_{H_2}MM_{H_2}+(1-y_{H_2})MM_{CH_4}$

$P_{H_2}=y_{H_2}P*\text{Convert}(\text{kPa}, \text{bar})$

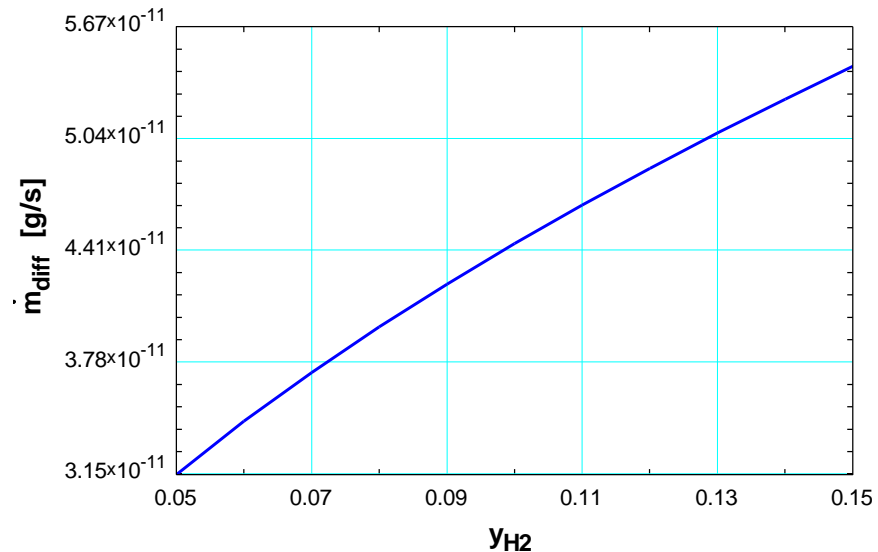
$w_{H_2}=2.09E-4*\exp(-3950/T)*P_{H_2}^{0.5}$

$\dot{m}_{diff}=2*\pi*L*\rho*D_{AB}*w_{H_2}/\ln(r_2/r_1)*\text{Convert}(\text{kg/s}, \text{g/s})$

$r_1=D_i/2$

$r_2=r_1+\text{thickness}$

$y_{H_2}$	$\dot{m}_{diff}$ [g/s]
0.05	3.144E-11
0.06	3.444E-11
0.07	3.720E-11
0.08	3.977E-11
0.09	4.218E-11
0.1	4.446E-11
0.11	4.663E-11
0.12	4.871E-11
0.13	5.070E-11
0.14	5.261E-11
0.15	5.446E-11



**14-56** Pure  $H_2$  gas is flowing through an iron pipe. The rate at which  $H_2$  leaks out by diffusion is to be determined for a known concentration at the inner surface.

**Assumptions** **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentration in the pipe and thus at the inner surface of the pipe is practically constant, and the hydrogen concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of hydrogen.

**Properties** The binary diffusion coefficient for hydrogen in the iron at  $25^\circ\text{C}$  is  $D_{AB} = 2.6 \times 10^{-13} \text{ m}^2/\text{s}$  (Table 14-3b). The molar mass of hydrogen is  $M = 2.016 \text{ kg/kmol}$  (Table A-1).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the container to be a *stationary* medium since there is no diffusion of iron molecules ( $\dot{N}_B = 0$ ) and the concentration of the hydrogen in the pipe is extremely low ( $C_A \ll 1$ ). Then the molar flow rate of hydrogen through the pipe wall by diffusion can be determined to be

$$\begin{aligned}\dot{N}_{\text{diff}} &= 2\pi L D_{AB} \frac{C_{A,1} - C_{A,2}}{\ln(r_2 / r_1)} \\ &= 2\pi(10 \text{ m})(2.6 \times 10^{-13} \text{ m}^2/\text{s}) \frac{(0.1 - 0) \text{ kmol/m}^3}{\ln(35 / 25)} \\ &= 4.855 \times 10^{-12} \text{ kmol/s}\end{aligned}$$

The mass diffusion rate of hydrogen through the pipe wall is determined by multiplying the molar flow rate with the molar mass of hydrogen,

$$\begin{aligned}\dot{m}_{\text{diff}} &= M \dot{N}_{\text{diff}} \\ &= (4.855 \times 10^{-12} \text{ kmol/s})(2.016 \text{ kg/kmol}) \\ &= \mathbf{9.79 \times 10^{-12} \text{ kg/s}}\end{aligned}$$

**Discussion** The hydrogen will leak out through the pipe wall by diffusion at a rate of  $9.79 \times 10^{-12} \text{ kg/s}$ . The leakage can be reduced by increasing the wall thickness of the pipe.



**14-57** Pure  $H_2$  gas is flowing through an iron pipe. The effect of wall thickness on the rate of  $H_2$  diffusion through the pipe wall is to be evaluated.

**Analysis** The problem is solved using EES, and the solution is given below:

"GIVEN"

$M=2.016$  [kmol/kg]

$D_1=0.025$  [m]

$D_2=D_1+2*t*1e-3$

$L=10$  [m]

$D_{AB}=2.6e-13$  [m<sup>2</sup>/s]

$C_{A_1}=0.1$  [kmol/m<sup>3</sup>]

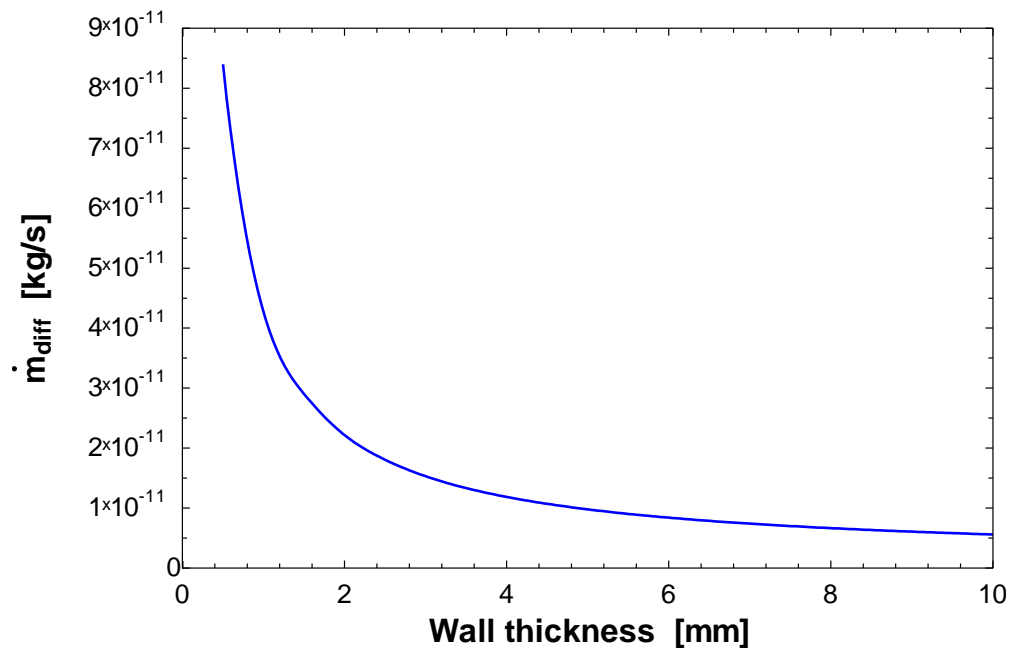
$C_{A_2}=0$  [kmol/m<sup>3</sup>]

"ANALYSIS"

$N_{dot\_diff}=2*pi*L*D_{AB}*(C_{A_1}-C_{A_2})/ln(D_2/D_1)$

$N_{dot\_diff}=m_{dot\_diff}/M$

$t$ [mm]	$\dot{m}_{diff}$ [kg/s]
0.5	8.397E-11
1.0	4.279E-11
1.5	2.906E-11
2.0	2.219E-11
2.5	1.806E-11
3.0	1.531E-11
3.5	1.334E-11
4.0	1.186E-11
4.5	1.071E-11
5.0	9.788E-12
6.0	8.401E-12
7.0	7.406E-12
8.0	6.657E-12
9.0	6.073E-12
10	5.603E-12



**Discussion** Knowing how the wall thickness affects the rate of hydrogen diffusion through the pipe wall can help engineers to design systems that minimize gas leakage from diffusion. As the wall thickness increases, the rate of mass diffusion through pipe wall decreases, at first drastically and then gradually toward zero.

**14-58** Helium gas stored inside a cylindrical Pyrex tank, and a sensor detects a leakage of the gas at  $1.8 \times 10^{-6}$  g/h. The concentration of helium at the inner surface of the Pyrex tank is to be determined.

**Assumptions** **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the tank is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the centerline. **2** There are no chemical reactions in the tank that results in the generation or depletion of helium. **3** Both ends of the cylindrical tank are impermeable.

**Properties** The binary diffusion coefficient for helium in the Pyrex tank at 20°C is  $D_{AB} = 4.5 \times 10^{-15}$  m<sup>2</sup>/s (Table 14-3b). The molar mass of helium is  $M = 4.003$  kg/kmol (Table A-1).

**Analysis** The mass diffusion rate through the tank wall is given

$$\begin{aligned}\dot{m}_{\text{diff}} &= (1.8 \times 10^{-6} \text{ g/h})(1/1000 \text{ kg/g})(1/3600 \text{ h/s}) \\ &= 5 \times 10^{-13} \text{ kg/s}\end{aligned}$$

We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the container to be a *stationary* medium since there is no diffusion of Pyrex molecules ( $\dot{N}_B = 0$ ) and the concentration of the helium in the tank is extremely low ( $C_A \ll 1$ ). Then the molar flow rate of helium through the pipe wall by diffusion can be determined to be

$$\begin{aligned}\dot{N}_{\text{diff}} &= \frac{\dot{m}_{\text{diff}}}{M} \\ &= 2\pi L D_{AB} \frac{C_{A,1} - C_{A,2}}{\ln(r_2 / r_1)}\end{aligned}$$

Solving for the concentration of helium at the inner surface  $C_{A,1}$  we have

$$\begin{aligned}C_{A,1} &= \frac{\dot{m}_{\text{diff}}}{M} \frac{\ln(r_2 / r_1)}{2\pi L D_{AB}} \\ &= \left( \frac{5 \times 10^{-13} \text{ kg/s}}{4.003 \text{ kg/kmol}} \right) \left[ \frac{\ln(12.5/12)}{2\pi(2 \text{ m})(4.5 \times 10^{-15} \text{ m}^2/\text{s})} \right] \\ &= \mathbf{0.0902 \text{ kmol/m}^3}\end{aligned}$$

**Discussion** The concentration of helium at the inner surface of the tank can be expressed in mass basis as  $\rho_{A,1} = M C_{A,1} = 0.361 \text{ kg/m}^3$ .

**14-59** Pressurized helium gas is stored in a spherical container. The diffusion rate of helium through the container is to be determined.

**Assumptions** **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the center of the container. **2** There are no chemical reactions in the pyrex shell that results in the generation or depletion of helium.

**Properties** The binary diffusion coefficient of helium in the pyrex at the specified temperature is  $4.5 \times 10^{-15} \text{ m}^2/\text{s}$  (Table 14-3b). The molar mass of helium is  $M = 4 \text{ kg/kmol}$  (Table A-1).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the container to be a *stationary* medium since there is no diffusion of pyrex molecules ( $\dot{N}_B = 0$ ) and the concentration of the helium in the container is extremely low ( $C_A \ll 1$ ). Then the molar flow rate of helium through the shell by diffusion can readily be determined from Eq. 14-28 to be

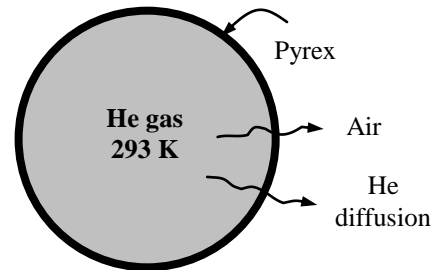
$$\begin{aligned}\dot{N}_{\text{diff}} &= 4\pi r_1 r_2 D_{AB} \frac{C_{A,1} - C_{A,2}}{r_2 - r_1} \\ &= 4\pi(1.45 \text{ m})(1.50 \text{ m})(4.5 \times 10^{-15} \text{ m}^2/\text{s}) \frac{(0.00073 - 0) \text{ kmol/m}^3}{1.50 - 1.45} \\ &= 1.796 \times 10^{-15} \text{ kmol/s}\end{aligned}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,


$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (4 \text{ kg/kmol})(1.796 \times 10^{-15} \text{ kmol/s}) = \mathbf{7.18 \times 10^{-15} \text{ kg/s}}$$

Therefore, helium will leak out of the container through the shell by diffusion at a rate of  $7.18 \times 10^{-15} \text{ kg/s}$  or  $0.00023 \text{ g/year}$ .

**Discussion** Note that the concentration of helium in the pyrex at the inner surface depends on the temperature and pressure of the helium in the tank, and can be determined as explained in the previous example. Also, the assumption of zero helium concentration in pyrex at the outer surface is reasonable since there is only a trace amount of helium in the atmosphere (0.5 parts per million by mole numbers).





**14-60**  A metal spherical vessel is used to contain hydrogen gas. The diffusion rate of hydrogen through the vessel is to be determined whether it is safe or not with respect to the room's ventilation system.

**Assumptions** **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the vessel that result in the generation and depletion of hydrogen.

**Properties** The solubility of hydrogen in the vessel is given as  $\mathcal{S}_{AB} = 0.005 \text{ kmol/m}^3 \cdot \text{bars}$ . The molar mass of hydrogen is  $M = 2.016 \text{ kg/kmol}$  (Table A-1).

**Analysis** The hydrogen concentration in the atmosphere and at the outer surface of the vessel is zero,  $C_{A,2} = 0$  (or  $P_{A,2} = 0$ ). Thus, the rate of hydrogen diffusion through the vessel is

$$\begin{aligned}\dot{N}_{\text{diff}} &= 4\pi r_1 r_2 D_{AB} \frac{C_{A,1} - C_{A,2}}{r_2 - r_1} \\ &= 4\pi r_1 r_2 D_{AB} \mathcal{S}_{AB} \frac{P_{A,1} - P_{A,2}}{r_2 - r_1} \\ &= 4\pi (2.5 \text{ m})(2.503 \text{ m})(1.5 \times 10^{-12} \text{ m}^2/\text{s})(0.005 \text{ kmol/m}^3 \cdot \text{bar}) \left( \frac{20 \text{ bar}}{2.503 \text{ m} - 2.5 \text{ m}} \right) \\ &= 3.932 \times 10^{-9} \text{ kmol/s}\end{aligned}$$

Thus, the hydrogen mass diffusion rate through the vessel is

$$\begin{aligned}\dot{m}_{\text{diff}} &= \dot{N}_{\text{diff}} M \\ &= (3.932 \times 10^{-9} \text{ kmol/s})(2.016 \text{ kg/kmol}) \\ &= \mathbf{7.93 \times 10^{-9} \text{ kg/s}} \\ &= \mathbf{7.93 \text{ }\mu\text{g/s} > 5 \text{ }\mu\text{g/s}}\end{aligned}$$

**Discussion** The rate of hydrogen leakage by diffusion through the vessel is greater than the rate that the ventilation system can handle; therefore the vessel cannot safely contain the hydrogen.

To prevent hazards from hydrogen leakage, a vessel with lower diffusion coefficient and solubility of hydrogen should be used. Or the ventilation system for the room should be upgraded.

**14-61** Helium gas is stored in a spherical fused silica container. The diffusion rate of helium through the container and the pressure drop in the tank in one week as a result of helium loss are to be determined.

**Assumptions** 1 Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the container. 2 There are no chemical reactions in the fused silica that results in the generation or depletion of helium. 3 Helium is an ideal gas. 4 The helium concentration at the inner surface of the container is at the highest possible level (the solubility).

**Properties** The solubility of helium in fused silica ( $\text{SiO}_2$ ) at 293 K and 500 kPa is  $0.00045 \text{ kmol/m}^3 \cdot \text{bar}$  (Table 14-7). The diffusivity of helium in fused silica at 293 K (actually, at 298 K) is  $4 \times 10^{-14} \text{ m}^2/\text{s}$  (Table 14-3b). The molar mass of helium is  $M = 4 \text{ kg/kmol}$  (Table A-1).

**Analysis** (a) We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the container to be a *stationary* medium since there is no diffusion of silica molecules ( $\dot{N}_B = 0$ ) and the concentration of the helium in the container is extremely low ( $C_A \ll 1$ ). The molar concentration of helium at the inner surface of the container is determined from the solubility data to be

$$C_{A,1} = S \times P_{\text{He}} = (0.00045 \text{ kmol/m}^3 \cdot \text{bar})(5 \text{ bar}) = 2.25 \times 10^{-3} \text{ kmol/m}^3 = 0.00225 \text{ kmol/m}^3$$

The helium concentration in the atmosphere and thus at the outer surface is taken to be zero since the tank is well ventilated. Then the molar flow rate of helium through the tank by diffusion becomes

$$\begin{aligned} \dot{N}_{\text{diff}} &= 4\pi r_1 r_2 D_{AB} \frac{C_{A,1} - C_{A,2}}{r_2 - r_1} \\ &= 4\pi(1 \text{ m})(1.01 \text{ m})(4 \times 10^{-14} \text{ m}^2/\text{s}) \frac{(0.00225 - 0) \text{ kmol/m}^3}{(1.01 - 1) \text{ m}} \\ &= 1.14 \times 10^{-13} \text{ kmol/s} \end{aligned}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,

$$\dot{m}_{\text{diff}} = M \dot{N}_{\text{diff}} = (4 \text{ kg/kmol})(1.14 \times 10^{-13} \text{ kmol/s}) = \mathbf{4.57 \times 10^{-13} \text{ kg/s}}$$

(b) Noting that the molar flow rate of helium is  $1.14 \times 10^{-13} \text{ kmol/s}$ , the amount of helium diffused through the shell in 1 week becomes

$$\begin{aligned} N_{\text{diff}} &= \dot{N}_{\text{diff}} \Delta t = (1.14 \times 10^{-13} \text{ kmol/s})(7 \times 24 \times 3600 \text{ s/week}) \\ &= 6.895 \times 10^{-8} \text{ kmol/week} \end{aligned}$$

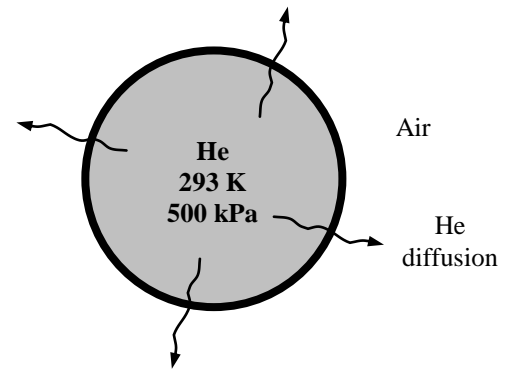
The volume of the spherical tank and the initial amount of helium gas in the tank are

$$\begin{aligned} \mathcal{V} &= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1 \text{ m})^3 = 4.189 \text{ m}^3 \\ N_{\text{initial}} &= \frac{P \mathcal{V}}{R_u T} = \frac{(500 \text{ kPa})(4.189 \text{ m}^3)}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(293 \text{ K})} = 0.85977 \text{ kmol} \end{aligned}$$

Then the number of moles of helium remaining in the tank after one week becomes

$$N_{\text{final}} = N_{\text{initial}} - N_{\text{diff}} = 0.85977 - 6.895 \times 10^{-8} \cong 0.85977 \text{ kmol}$$

which is the practically the same as the initial value. Therefore, the amount of helium that leaves the tank by diffusion is negligible, and the final pressure in the tank is the same as the initial pressure of  $P_2 = P_1 = \mathbf{500 \text{ kPa}}$ .



**14-62** Pure  $N_2$  gas is flowing through a rubber pipe. The rate at which  $N_2$  leaks out by diffusion is to be determined for the cases of vacuum and atmospheric air outside.

**Assumptions** 1 Mass diffusion is *steady* and *one-dimensional* since the nitrogen concentration in the pipe and thus at the inner surface of the pipe is practically constant, and the nitrogen concentration in the atmosphere also remains constant. Also, there is symmetry about the centerline of the pipe. 2 There are no chemical reactions in the pipe that results in the generation or depletion of nitrogen. 3 Both the nitrogen and air are ideal gases.

**Properties** The diffusivity and solubility of nitrogen in rubber at  $25^\circ\text{C}$  are  $1.5 \times 10^{-10} \text{ m}^2/\text{s}$  and  $0.00156 \text{ kmol/m}^3 \cdot \text{bar}$ , respectively (Tables 14-3 and 14-7).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the container to be a *stationary* medium since there is no diffusion of rubber molecules ( $\dot{N}_B = 0$ ) and the concentration of the nitrogen in the container is extremely low ( $C_A \ll 1$ ). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2} P = (0.79)(100 \text{ kPa}) = 79 \text{ kPa} = 0.79 \text{ bar}$$

$$P_{O_2} = y_{O_2} P = (0.21)(100 \text{ kPa}) = 21 \text{ kPa} = 0.21 \text{ bar}$$

When solubility data is available, the molar flow rate of a gas through a solid can be determined by replacing the molar concentration by  $C_{A, \text{solid side}}(0) = S_{AB} P_{A, \text{gas side}}(0)$ . For a cylindrical pipe the molar rate of diffusion can be expressed in terms of solubility as

$$\dot{N}_{\text{diff}, A, \text{cyl}} = 2\pi L D_{AB} S_{AB} \frac{P_{A,1} - P_{A,2}}{\ln(r_2 / r_1)}$$

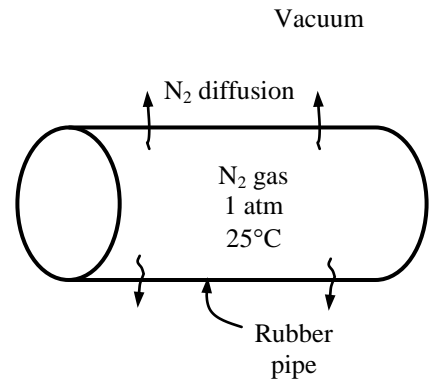
where  $S_{AB}$  is the *solubility* and  $P_{A,1}$  and  $P_{A,2}$  are the partial pressures of gas A on the two sides of the wall.

(a) The pipe is in vacuum and thus  $P_{A,2} = 0$ :

$$\begin{aligned} \dot{N}_{\text{diff}, A, \text{cyl}} &= 2\pi(10 \text{ m})(1.5 \times 10^{-10} \text{ m}^2 / \text{s})(0.00156 \text{ kmol/m}^3 \cdot \text{s} \cdot \text{bar}) \frac{(1 - 0) \text{ bar}}{\ln(0.032/0.03)} \\ &= \mathbf{2.278 \times 10^{-10} \text{ kmol/s}} \end{aligned}$$

(b) The pipe is in atmospheric air and thus  $P_{A,2} = 0.79 \text{ bar}$ :

$$\begin{aligned} \dot{N}_{\text{diff}, A, \text{cyl}} &= 2\pi(10 \text{ m})(1.5 \times 10^{-10} \text{ m}^2 / \text{s})(0.00156 \text{ kmol/m}^3 \cdot \text{s} \cdot \text{bar}) \frac{(1 - 0.79) \text{ bar}}{\ln(0.032/0.03)} \\ &= \mathbf{4.784 \times 10^{-11} \text{ kmol/s}} \end{aligned}$$



**Discussion** In the case of a vacuum environment, the diffusion rate of nitrogen from the pipe is about 5 times the rate in atmospheric air. This is expected since mass diffusion is proportional to the concentration difference.

**14-63** A balloon is filled with helium gas. The initial rates of diffusion of helium, oxygen, and nitrogen through the balloon and the mass fraction of helium that escapes during the first 5 h are to be determined.

**Assumptions** **1** The pressure of helium inside the balloon remains nearly constant. **2** Mass diffusion is *steady* for the time period considered. **3** Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. **4** There are no chemical reactions in the balloon that results in the generation or depletion of helium. **5** Both the helium and the air are ideal gases. **7** The curvature effects of the balloon are negligible so that the balloon can be treated as a plane layer.

**Properties** The permeability of rubber to helium, oxygen, and nitrogen at 25°C are given to be  $9.4 \times 10^{-13}$ ,  $7.05 \times 10^{-13}$ , and  $2.6 \times 10^{-13}$  kmol/m.s.bars, respectively. The molar mass of helium is  $M = 4$  kg/kmol and its gas constant is  $R = 2.0709$  kPa.m<sup>3</sup>/kg.K (Table A-1).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ( $\dot{N}_B = 0$ ) and the concentration of the helium in the balloon is extremely low ( $C_A \ll 1$ ). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2} P = (0.79)(100 \text{ kPa}) = 79 \text{ kPa} = 0.79 \text{ bar}$$

$$P_{O_2} = y_{O_2} P = (0.21)(100 \text{ kPa}) = 21 \text{ kPa} = 0.21 \text{ bar}$$

The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 110 kPa, the initial partial pressure of helium in the balloon is 110 kPa, and the initial partial pressures of oxygen and nitrogen are zero.

When permeability data is available, the molar flow rate of a gas through a solid wall of thickness  $L$  under steady one-dimensional conditions can be determined from Eq. 14-29,

$$\dot{N}_{\text{diff},A,\text{wall}} = P_{AB} A \frac{P_{A,1} - P_{A,2}}{L} \quad (\text{kmol/s})$$

where  $P_{AB}$  is the permeability and  $P_{A,1}$  and  $P_{A,2}$  are the partial pressures of gas A on the two sides of the wall (Note that the balloon can be treated as a plain layer since its thickness is very small compared to its diameter). Noting that the surface area of the balloon is  $A = \pi D^2 = \pi(0.15 \text{ m})^2 = 0.07069 \text{ m}^2$ , the initial rates of diffusion of helium, oxygen, and nitrogen at 25°C are determined to be

$$\dot{N}_{\text{diff},\text{He}} = P_{AB} A \frac{P_{\text{He},1} - P_{\text{He},2}}{L} = (9.4 \times 10^{-13} \text{ kmol/m.s.bar})(0.07069 \text{ m}^2) \frac{(1.1 - 0) \text{ bar}}{0.1 \times 10^{-3} \text{ m}} = \mathbf{0.7309 \times 10^{-9} \text{ kmol/s}}$$

$$\dot{N}_{\text{diff},O_2} = P_{AB} A \frac{P_{O_2,1} - P_{O_2,2}}{L} = (7.05 \times 10^{-13} \text{ kmol/m.s.bar})(0.07069 \text{ m}^2) \frac{(0 - 0.21) \text{ bar}}{0.1 \times 10^{-3} \text{ m}} = \mathbf{-0.1047 \times 10^{-9} \text{ kmol/s}}$$

$$\dot{N}_{\text{diff},N_2} = P_{AB} A \frac{P_{N_2,1} - P_{N_2,2}}{L} = (2.6 \times 10^{-13} \text{ kmol/m.s.bar})(0.07069 \text{ m}^2) \frac{(0 - 0.79) \text{ bar}}{0.1 \times 10^{-3} \text{ m}} = \mathbf{-0.1452 \times 10^{-9} \text{ kmol/s}}$$

The initial mass flow rate of helium and the amount of helium that escapes during the first 5 hours are

$$\dot{m}_{\text{diff},\text{He}} = M \dot{N}_{\text{diff},\text{He}} = (4 \text{ kg/kmol})(0.7309 \times 10^{-9} \text{ kmol/s}) = 2.924 \times 10^{-9} \text{ kg/s}$$

$$m_{\text{diff},\text{He}} = \dot{m}_{\text{diff},\text{He}} \Delta t = (2.924 \times 10^{-9} \text{ kg/s})(5 \times 3600 \text{ s}) = \mathbf{5.263 \times 10^{-5} \text{ kg} = 0.05263 \text{ g}}$$

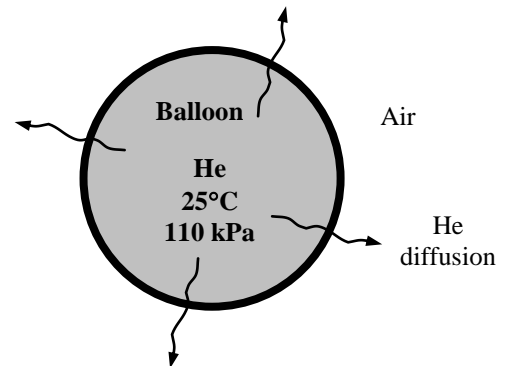
The initial mass of helium in the balloon is

$$m_{\text{initial}} = \frac{PV}{RT} = \frac{(110 \text{ kPa})[4\pi(0.075 \text{ m})^3 / 3]}{(2.077 \text{ kPa.m}^3/\text{kg.K})(298 \text{ K})} = 3.141 \times 10^{-4} \text{ kg} = 0.3141 \text{ g}$$

Therefore, the fraction of helium that escapes the balloon during the first 5 h is

$$\text{Fraction} = \frac{m_{\text{diff},\text{He}}}{m_{\text{initial}}} = \frac{0.05263 \text{ g}}{0.3141 \text{ g}} = \mathbf{0.167 \text{ (or 16.7%)}}$$

**Discussion** This is a significant amount of helium gas that escapes the balloon, and explains why the helium balloons do not last long. Also, our assumption of constant pressure for the helium in the balloon is obviously not very accurate since 16.7% of helium is lost during the process.



**14-64** A balloon is filled with helium gas. A relation for the variation of pressure in the balloon with time as a result of mass transfer through the balloon material is to be obtained, and the time it takes for the pressure in the balloon to drop from 110 to 100 kPa is to be determined.

**Assumptions** 1 The pressure of helium inside the balloon remains nearly constant. 2 Mass diffusion is *transient* since the conditions inside the balloon change with time. 3 Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. 4 There are no chemical reactions in the balloon material that results in the generation or depletion of helium. 5 Helium is an ideal gas. 6 The diffusion of air into the balloon is negligible. 7 The volume of the balloon is constant. 8 The curvature effects of the balloon are negligible so that the balloon material can be treated as a plane layer.

**Properties** The permeability of rubber to helium at 25°C is given to be  $9.4 \times 10^{-13}$  kmol/m.s.bar. The molar mass of helium is  $M = 4$  kg/kmol and its gas constant is  $R = 2.077$  kPa.m<sup>3</sup>/kg.K (Table A-1).

**Analysis** We can consider the total molar concentration to be constant ( $C = C_A + C_B \cong C_B = \text{constant}$ ), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ( $\dot{N}_B = 0$ ) and the concentration of the helium in the balloon is extremely low ( $C_A \ll 1$ ). The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 110 kPa, the initial partial pressure of helium in the balloon is 110 kPa.

When permeability data is available, the molar flow rate of a gas through a solid wall of thickness  $L$  under steady one-dimensional conditions can be determined from Eq. 14-29,

$$\dot{N}_{\text{diff,A,wall}} = P_{AB} A \frac{P_{A,1} - P_{A,2}}{L} = P_{AB} A \frac{P}{L} \quad (\text{kmol/s})$$

where  $P_{AB}$  is the permeability and  $P_{A,1}$  and  $P_{A,2}$  are the partial pressures of helium on the two sides of the wall (note that the balloon can be treated as a plain layer since its thickness very small compared to its diameter, and  $P_{A,1}$  is simply the pressure  $P$  of helium inside the balloon).

Noting that the amount of helium in the balloon can be expressed as  $N = P\mathcal{V}/R_u T$  and taking the temperature and volume to be constants,

$$N = \frac{P\mathcal{V}}{R_u T} \rightarrow \frac{dN}{dt} = \frac{\mathcal{V}}{R_u T} \frac{dP}{dt} \rightarrow \frac{dP}{dt} = \frac{R_u T}{\mathcal{V}} \frac{dN}{dt} \quad (1)$$

Conservation of mass dictates that the mass flow rate of helium from the balloon be equal to the rate of change of mass inside the balloon,

$$\frac{dN}{dt} = -\dot{N}_{\text{diff,A,wall}} = -P_{AB} A \frac{P}{L} \quad (2)$$

Substituting (2) into (1),

$$\frac{dP}{dt} = \frac{R_u T}{\mathcal{V}} \frac{dN}{dt} = -\frac{R_u T}{\mathcal{V}} P_{AB} A \frac{P}{L} = -\frac{R_u T P_{AB} A}{\mathcal{V} L} P$$

Separating the variables and integrating gives

$$\frac{dP}{P} = -\frac{R_u T P_{AB} A}{\mathcal{V} L} dt \rightarrow \ln P \Big|_{P_0}^P = -\frac{R_u T P_{AB} A}{\mathcal{V} L} t \Big|_0^t \rightarrow \ln \frac{P}{P_0} = -\frac{R_u T P_{AB} A}{\mathcal{V} L} t$$

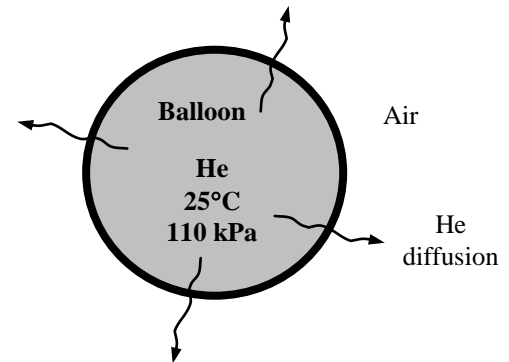
Rearranging, the desired relation for the variation of pressure in the balloon with time is determined to be

$$P = P_0 \exp\left(-\frac{R_u T P_{AB} A}{\mathcal{V} L} t\right) = P_0 \exp\left(-\frac{3R_u T P_{AB}}{rL} t\right) \quad \text{since, for a sphere, } \frac{A}{\mathcal{V}} = \frac{4\pi r^2}{4\pi r^3/3} = \frac{3}{r}$$

Then the time it takes for the pressure inside the balloon to drop from 110 kPa to 100 kPa becomes

$$\frac{100 \text{ kPa}}{110 \text{ kPa}} = \exp\left(-\frac{3(0.08314 \text{ bar} \cdot \text{m}^3 / \text{kmol} \cdot \text{K})(298 \text{ K})(9.4 \times 10^{-13} \text{ kmol/m} \cdot \text{s} \cdot \text{bar})}{(0.075 \text{ m})(0.1 \times 10^{-3} \text{ m})} t\right) \rightarrow t = 10,231 \text{ s} = \mathbf{2.84 \text{ h}}$$

Therefore, the balloon will lose 10% of its pressure in 2.84 h.



## Water Vapor Migration in Buildings

**14-65C** Excess moisture changes the *dimensions* of wood, and cyclic changes in dimensions weaken the joints, and can jeopardize the structural integrity of building components, causing “squeaking” at the minimum. Excess moisture can also cause *rotting* in woods, *mold* growth on wood surfaces, *corrosion* and *rusting* in metals, and *peeling of paint* on the interior and exterior wall surfaces.

**14-66C** The condensation or freezing of water vapor in the wall increases the thermal conductivity of the insulation material, and thus increases the rate of heat transfer through the wall. Similarly, the thermal conductivity of the soil increases with increasing amount of moisture.

**14-67C** Vapor barriers are materials that are impermeable to moisture such as sheet metals, heavy metal foils, and thick plastic layers, and they completely *eliminate* the vapor migration. Vapor retarders such as reinforced plastics or metals, thin foils, plastic films, treated papers and coated felts, on the other hand, *slow down* the flow of moisture through the structures. Vapor retarders are commonly used in residential buildings to control the vapor migration through the walls.

**14-68C** Insulations on *chilled water lines* are always wrapped with *vapor barrier jackets* to eliminate the possibility of vapor entering the insulation. This is because moisture that migrates through the insulation to the cold surface will condense and remain there indefinitely with no possibility of vaporizing and moving back to the outside.

**14-69C** A tank that contains moist air at 3 atm is located in moist air that is at 1 atm. The driving force for moisture transfer is the vapor pressure difference, and thus it is possible for the water vapor to flow into the tank from surroundings if the vapor pressure in the surroundings is greater than the vapor pressure in the tank.

**14-70C** When the temperature, total pressure, and the relative humidity are given, the vapor pressure can be determined from the psychrometric chart or the relation  $P_v = \phi P_{\text{sat}}$  where  $P_{\text{sat}}$  is the saturation (or boiling) pressure of water at the specified temperature and  $\phi$  is the relative humidity.

**14-71C** The mass flow rate of water vapor through a wall of thickness  $L$  in terms of the partial pressure of water vapor on both sides of the wall and the permeability of the wall to the water vapor can be expressed as

$$\dot{m}_{\text{diff,A,wall}} = MP_{\text{AB}} A \frac{P_{\text{A,1}} - P_{\text{A,2}}}{L}$$

where  $M$  is the molar mass of vapor,  $P_{\text{AB}}$  is the permeability,  $A$  is the normal area, and  $P_{\text{A}}$  is the partial pressure of the vapor.

**14-72** A glass of milk left on top of a counter is tightly sealed by a sheet of 0.009-mm thick aluminum foil. The drop in the level of the milk in the glass in 12 h due to vapor migration through the foil is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Mass transfer through the foil is one-dimensional. 3 The vapor permeability of the foil is constant.

**Properties** The permeance of the foil to water vapor is given to be  $2.9 \times 10^{-12} \text{ kg/s.m}^2 \cdot \text{Pa}$ . The saturation pressure of water at  $15^\circ\text{C}$  is 1705 Pa (Table 14-9). We take the density of milk to be  $1000 \text{ kg/m}^3$ .

**Analysis** The mass flow rate of water vapor through a plain layer of thickness  $L$  and normal area  $A$  is given as (Eq. 14-31)

$$\dot{m}_v = PA \frac{P_{v,1} - P_{v,2}}{L} = PA \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L} = MA(\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2})$$

where  $P$  is the vapor permeability and  $M = P/L$  is the permeance of the material,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the foil.

The diffusion area of the foil is  $A = \pi r^2 = \pi(0.06 \text{ m})^2 = 0.0113 \text{ m}^2$ .

Substituting, the mass flow rate of water vapor through the foil becomes

$$\begin{aligned} \dot{m}_v &= (2.9 \times 10^{-12} \text{ kg/s.m}^2 \cdot \text{Pa})(0.0113 \text{ m}^2)[1(1705 \text{ Pa}) - 0.5(1705 \text{ Pa})] \\ &= 2.79 \times 10^{-11} \text{ kg/s} \end{aligned}$$

Then the total amount of moisture that flows through the foil during a 12-h period becomes

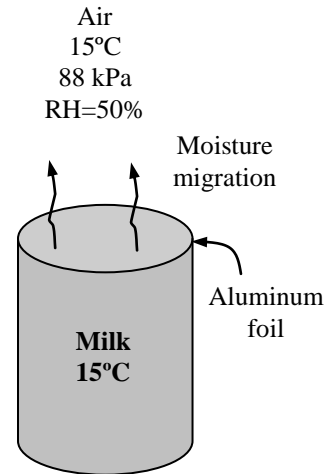
$$m_{v,12-h} = \dot{m}_v \Delta t = (2.79 \times 10^{-11} \text{ kg/s})(12 \times 3600 \text{ s}) = 1.21 \times 10^{-6} \text{ kg}$$

$$\mathcal{V} = m / \rho = (1.21 \times 10^{-6} \text{ kg}) / (1000 \text{ kg/m}^3) = 1.21 \times 10^{-9} \text{ m}^3$$

Then the drop in the level of the milk becomes

$$\Delta h = \frac{\mathcal{V}}{A} = \frac{1.21 \times 10^{-9} \text{ m}^3}{0.0113 \text{ m}^2} = 1.1 \times 10^{-7} \text{ m} = \mathbf{0.0011 \text{ mm}}$$

**Discussion** The drop in the level of the milk in 12 h is very small, and thus it is not noticeable.



**14-73** The wall of a house is made of a 20-cm thick brick. The amount of moisture flowing through the wall in 24-h is to be determined.

**Assumptions** **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant.

**Properties** The permeance of 100 mm thick wall is  $46 \times 10^{-12}$  kg/s.m<sup>2</sup>.Pa (Table 14-10). The saturation pressures of water are 3169 Pa at 25°C, and 7384 Pa at 40°C (Table 14-9).

**Analysis** The permeability of the wall is

$$P = ML = (46 \times 10^{-12} \text{ kg/s} \cdot \text{m}^2 \cdot \text{Pa})(0.10 \text{ m}) \\ = 46 \times 10^{-13} \text{ kg/s} \cdot \text{m} \cdot \text{Pa}$$

The mass flow rate of water vapor through a plain layer of thickness  $L$  and normal area  $A$  is given as (Eq. 14-31)

$$\dot{m}_v = PA \frac{P_{v,1} - P_{v,2}}{L} = PA \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L}$$

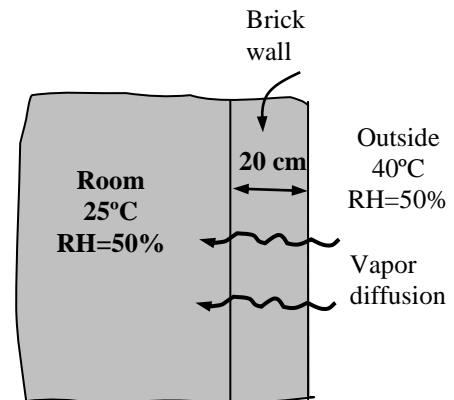
where  $P$  is the vapor permeability,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the roof. Substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (46 \times 10^{-12} \text{ kg/s} \cdot \text{m} \cdot \text{Pa}) \frac{[0.50(7384 \text{ Pa}) - 0.50(3169 \text{ Pa})]}{0.20 \text{ m}} = 4.847 \times 10^{-7} \text{ kg/s} \cdot \text{m}^2$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (4.847 \times 10^{-7} \text{ kg/s} \cdot \text{m}^2)(24 \times 3600 \text{ s}) = 0.04188 \text{ kg} = \mathbf{41.9 \text{ g}}$$

**Discussion** The moisture migration through the wall can be reduced significantly by covering the roof with a vapor barrier or vapor retarder.





**14-74** The roof of a house is made of a 20-cm thick concrete layer. The amount of water vapor that will diffuse through a 15 m × 8 m section of the roof in 24-h is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Mass transfer through the roof is one-dimensional. 3 The vapor permeability of the roof is constant.

**Properties** The permeability of the roof to water vapor is given to be  $24.7 \times 10^{-12}$  kg/s.m.Pa. The saturation pressures of water are 768 Pa at 3°C, and 3169 Pa at 25°C (Table 14-9).

**Analysis** The mass flow rate of water vapor through a plain layer of thickness  $L$  and normal area  $A$  is given as (Eq. 14-31)

$$\dot{m}_v = PA \frac{P_{v,1} - P_{v,2}}{L} = PA \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L}$$

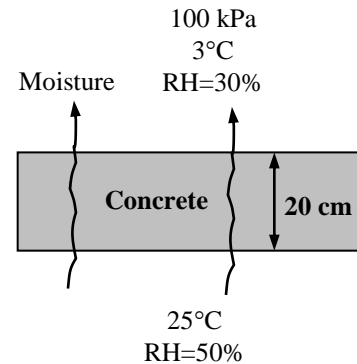
where  $P$  is the vapor permeability,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the roof. Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = (24.7 \times 10^{-12} \text{ kg/s.m.Pa})(15 \times 8 \text{ m}^2) \frac{[0.50(3169 \text{ Pa}) - 0.30(768 \text{ Pa})]}{0.20 \text{ m}} = 2.007 \times 10^{-5} \text{ kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (2.007 \times 10^{-5} \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{1.73 \text{ kg}}$$

**Discussion** The moisture migration through the roof can be reduced significantly by covering the roof with a vapor barrier or vapor retarder.





**14-75** Prob. 14-74 is reconsidered. The effects of temperature and relative humidity of air inside the house on the amount of water vapor that will migrate through the roof are to be investigated.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

$A=15 \times 8 \text{ [m}^2\text{]}$

$L=0.20 \text{ [m]}$

$T_1=25 \text{ [C]}$

$\phi_1=0.50$

$P_{\text{atm}}=100 \text{ [kPa]}$

$\text{time}=24 \times 3600 \text{ [s]}$

$T_2=3 \text{ [C]}$

$\phi_2=0.30$

$\text{Permeability}=24.7\text{E-}12 \text{ [kg/s-m-Pa]}$

"PROPERTIES"

$\text{Fluid}=\text{'steam\_IAPWS'}$

$P_{\text{sat1}}=\text{Pressure}(\text{Fluid}, T=T_1, x=1) \times \text{Convert}(\text{kPa}, \text{Pa})$

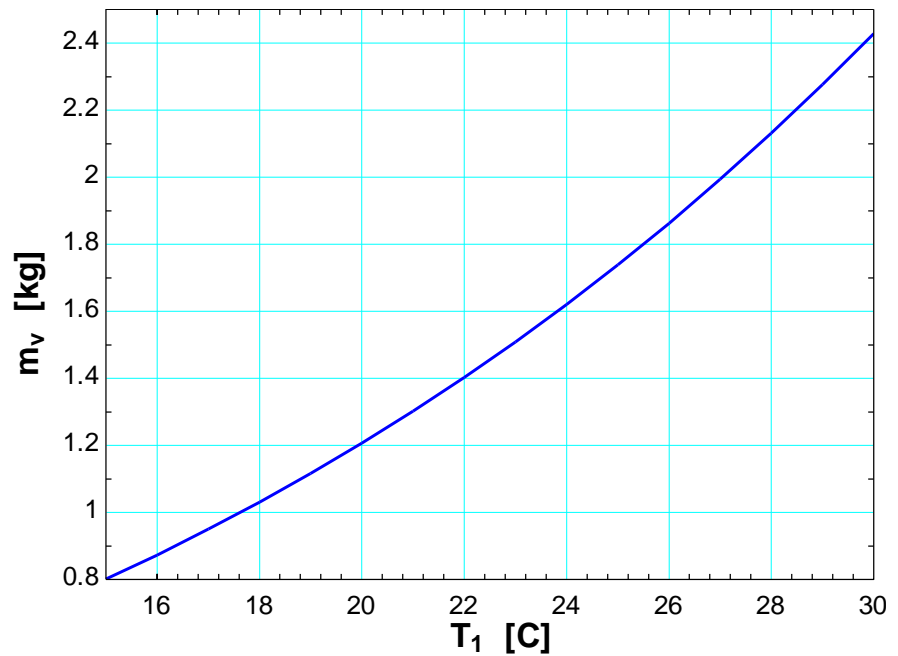
$P_{\text{sat2}}=\text{Pressure}(\text{Fluid}, T=T_2, x=1) \times \text{Convert}(\text{kPa}, \text{Pa})$

"ANALYSIS"

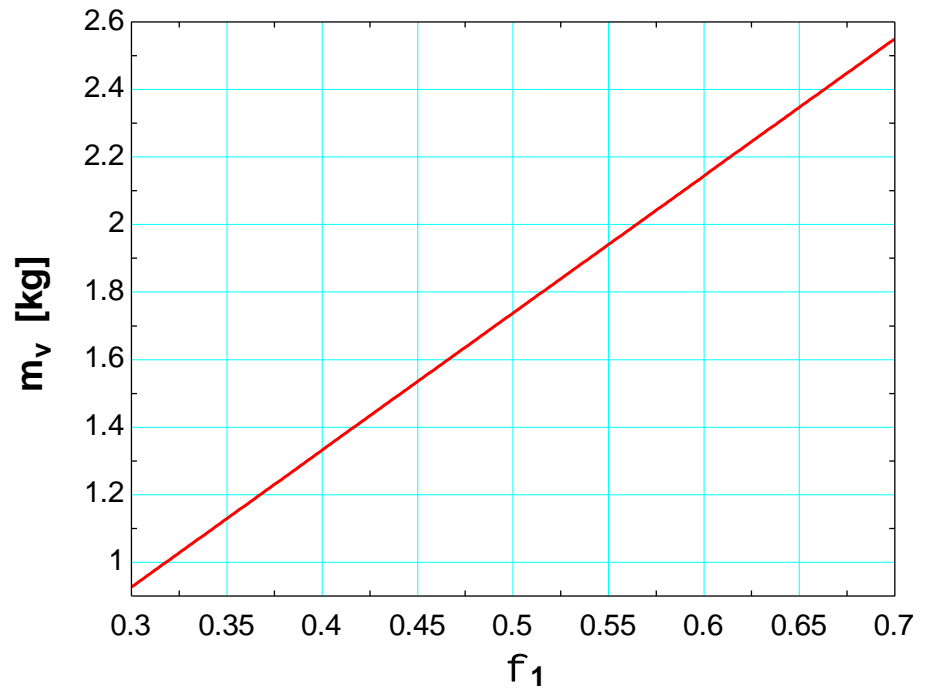
$m_{\text{dot}}_v = \text{Permeability} \times A \times (\phi_1 \times P_{\text{sat1}} - \phi_2 \times P_{\text{sat2}}) / L$

$m_v = m_{\text{dot}}_v \times \text{time}$

$T_1$ [C]	$m_v$ [kg]
15	0.8008
16	0.8732
17	0.9497
18	1.031
19	1.116
20	1.206
21	1.302
22	1.402
23	1.508
24	1.62
25	1.738
26	1.862
27	1.993
28	2.131
29	2.276
30	2.428



$\phi_1$	$m_v$ [kg]
0.3	0.9264
0.32	1.008
0.34	1.089
0.36	1.17
0.38	1.251
0.4	1.332
0.42	1.413
0.44	1.495
0.46	1.576
0.48	1.657
0.5	1.738
0.52	1.819
0.54	1.901
0.56	1.982
0.58	2.063
0.6	2.144
0.62	2.225
0.64	2.306
0.66	2.388
0.68	2.469
0.7	2.55



**14-76** The roof of a house is made of a 20-cm thick concrete layer painted with a vapor retarder paint. The amount of water vapor that will diffuse through a  $15 \text{ m} \times 8 \text{ m}$  section of the roof in 24-h is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Mass transfer through the roof is one-dimensional. 3 The vapor permeabilities of the roof and of the vapor barrier are constant.

**Properties** The permeability of concrete to water vapor and the permeance of the vapor retarder to water vapor are given to be  $24.7 \times 10^{-12} \text{ kg/s.m.Pa}$  and  $26 \times 10^{-12} \text{ kg/s.m}^2.\text{Pa}$ , respectively. The saturation pressures of water are 768 Pa at  $3^\circ\text{C}$ , and 3169 Pa at  $25^\circ\text{C}$  (Table 14-9).

**Analysis** The mass flow rate of water vapor through a two-layer plain roof of normal area  $A$  is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A \frac{P_{v,1} - P_{v,2}}{R_{v,\text{total}}} = A \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$

where  $R_{v,\text{total}}$  is the total vapor resistance of the medium,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the roof. The total vapor resistance of the roof is

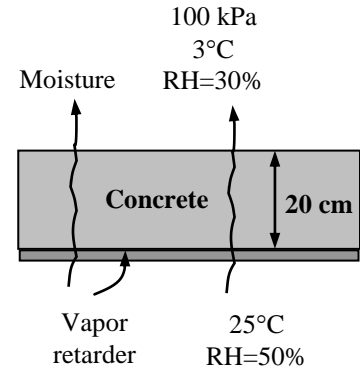
$$\begin{aligned} R_{v,\text{total}} &= R_{v,\text{roof}} + R_{v,\text{film}} \\ &= \frac{L}{P} + \frac{1}{M} \\ &= \frac{0.20 \text{ m}}{24.7 \times 10^{-12} \text{ kg/s.m.Pa}} + \frac{1}{26 \times 10^{-12} \text{ kg/s.m}^2.\text{Pa}} \\ &= 4.656 \times 10^{10} \text{ s.m}^2.\text{Pa/kg} \end{aligned}$$

Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = A \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (15 \times 8 \text{ m}^2) \frac{0.50(3169 \text{ Pa}) - 0.30(768 \text{ Pa})}{4.656 \times 10^{10} \text{ s.m}^2.\text{Pa/kg}} = 3.490 \times 10^{-6} \text{ kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (3.490 \times 10^{-6} \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{0.302 \text{ kg} = 302 \text{ g}}$$



**14-77** The inside wall of a house is finished with 9.5-mm thick gypsum wallboard. The maximum amount of water vapor that will diffuse through a  $3 \text{ m} \times 8 \text{ m}$  section of the wall in 24-h is to be determined.

**Assumptions** **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant. **4** The vapor pressure at the outer side of the wallboard is zero.

**Properties** The permeance of the 9.5 mm thick gypsum wall board to water vapor is given to be  $2.86 \times 10^{-9} \text{ kg/s.m}^2 \cdot \text{Pa}$ . (Table 14-10). The saturation pressure of water at  $20^\circ\text{C}$  is 2339 Pa (Table 14-9).

**Analysis** The mass flow rate of water vapor through a plain layer of thickness  $L$  and normal area  $A$  is given as (Eq. 14-31)

$$\begin{aligned}\dot{m}_v &= PA \frac{P_{v,1} - P_{v,2}}{L} \\ &= PA \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L} = MA(\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2})\end{aligned}$$

where  $P$  is the vapor permeability and  $M = P/L$  is the permeance of the material,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

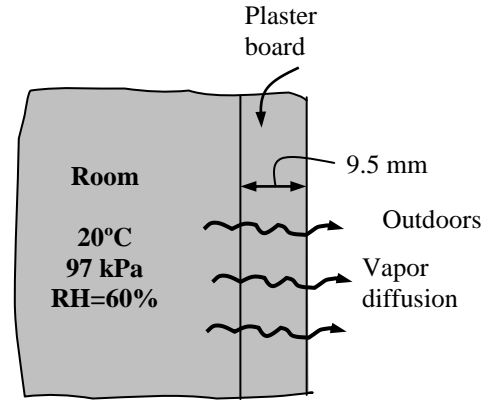
Noting that the vapor pressure at the outer side of the wallboard is zero ( $\phi_2 = 0$ ) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (2.86 \times 10^{-9} \text{ kg/s.m}^2 \cdot \text{Pa})(3 \times 8 \text{ m}^2)[0.60(2339 \text{ Pa}) - 0] = 9.63 \times 10^{-5} \text{ kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (9.63 \times 10^{-5} \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{8.32 \text{ kg}}$$

**Discussion** This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.



**14-78** The inside wall of a house is finished with 9.5-mm thick gypsum wallboard with a 0.051-mm thick polyethylene film on one side. The maximum amount of water vapor that will diffuse through a  $3 \text{ m} \times 8 \text{ m}$  section of the wall in 24-h is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Mass transfer through the wall is one-dimensional. 3 The vapor permeabilities of the wall and of the vapor barrier are constant. 4 The vapor pressure at the outer side of the wallboard is zero.

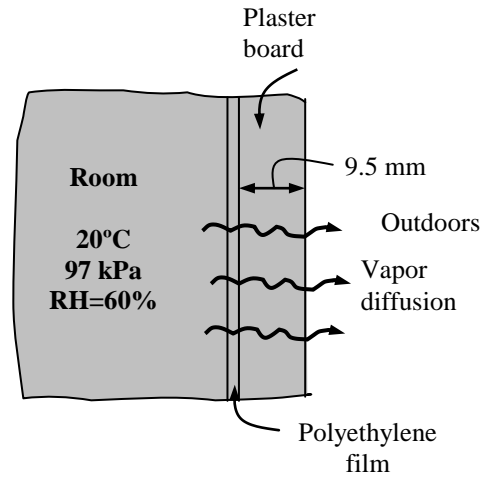
**Properties** The permeances of the 9.5 mm thick gypsum wall board and of the 0.051-mm thick polyethylene film are given to be  $2.86 \times 10^{-9}$  and  $9.1 \times 10^{-12} \text{ kg/s.m}^2 \cdot \text{Pa}$ , respectively (Table 14-10). The saturation pressure of water at  $20^\circ\text{C}$  is 2339 Pa (Table 14-9).

**Analysis** The mass flow rate of water vapor through a two-layer plain wall of normal area  $A$  is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A \frac{P_{v,1} - P_{v,2}}{R_{v,\text{total}}} = A \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$

where  $R_{v,\text{total}}$  is the total vapor resistance of the medium,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall. The total vapor resistance of the wallboard is

$$\begin{aligned} R_{v,\text{total}} &= R_{v,\text{wall}} + R_{v,\text{film}} \\ &= \frac{1}{2.86 \times 10^{-9} \text{ kg/s.m}^2 \cdot \text{Pa}} + \frac{1}{9.1 \times 10^{-12} \text{ kg/s.m}^2 \cdot \text{Pa}} \\ &= 1.10 \times 10^{11} \text{ s.m}^2 \cdot \text{Pa/kg} \end{aligned}$$



Noting that the vapor pressure at the outer side of the wallboard is zero ( $\phi_2 = 0$ ) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = A \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (3 \times 8 \text{ m}^2) \frac{0.60(2339 \text{ Pa}) - 0}{1.10 \times 10^{11} \text{ s.m}^2 \cdot \text{Pa/kg}} = 3.06 \times 10^{-7} \text{ kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (3.06 \times 10^{-7} \text{ kg/s})(24 \times 3600 \text{ s}) = \mathbf{0.0264 \text{ kg} = 26.4 \text{ g}}$$

**Discussion** This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero. Note that the vapor barrier reduced the amount of vapor migration to a negligible level.

## Transient Mass Diffusion

**14-79C** The diffusion of a solid species into another solid of finite thickness can usually be treated as a diffusion process in a semi-infinite medium regardless of the shape and thickness of the solid since the diffusion process affects a very thin layer at the surface.

**14-80C** When the density of a species  $A$  in a semi-infinite medium is known initially and at the surface, the concentration of the species  $A$  at a specified location and time can be determined from

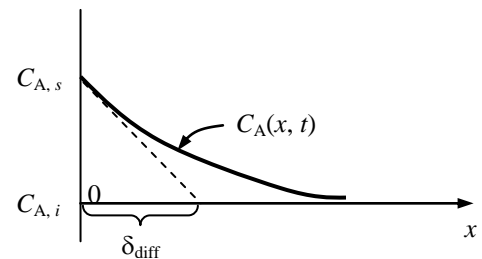
$$\frac{C_A(x, t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc} \left( \frac{x}{2\sqrt{D_{AB}t}} \right)$$

where  $C_{A,i}$  is the initial concentration of species  $A$  at time  $t = 0$ ,  $C_{A,s}$  is the concentration at the inner side of the exposed surface of the medium, and  $\text{erfc}$  is the complementary error function.

**14-81C** The penetration depth is defined as the location where the tangent to the concentration profile at the surface ( $x = 0$ ) intercepts the  $C_A = C_{A,i}$  line, and it represents the depth of diffusion at a given time. The penetration depth can be determined to be

$$\delta_{\text{diff}} = \sqrt{\pi D_{AB} t}$$

where  $D_{AB}$  is the diffusion coefficient and  $t$  is the time.



**14-82** A thick natural rubber wall is exposed to pure oxygen gas on one side of the surface. The time required for the oxygen concentration to reach 5% at  $x = 5$  mm of the concentration at the surface is to be determined.

**Assumptions** **1** The natural rubber wall can be modeled as a semi-infinite medium. **2** The  $O_2$  concentration in the rubber is initially zero. **3** The concentration of  $O_2$  at the rubber surface is constant.

**Properties** The diffusion coefficient of  $O_2$  in natural rubber at  $25^\circ\text{C} = 298\text{ K}$  is  $D_{AB} = 2.1 \times 10^{-10}\text{ m}^2/\text{s}$  (Table 14-3b).

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x, t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

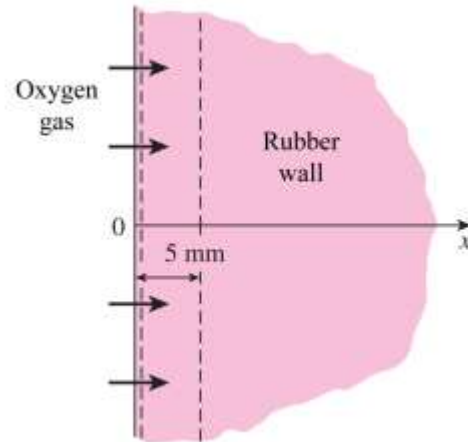
$$\begin{aligned}\frac{C_A(x, t)}{C_{A,s}} &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right) \\ 0.05 &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)\end{aligned}$$

From Table 4-4, we have  $0.05 = \text{erfc}(1.386)$ , hence

$$\frac{x}{2\sqrt{D_{AB}t}} = 1.386$$

Thus,

$$\begin{aligned}t &= \left[\frac{x}{(1.386)2}\right]^2 \frac{1}{D_{AB}} \\ &= \left[\frac{0.005\text{ m}}{(1.386)2}\right]^2 \frac{1}{2.1 \times 10^{-10}\text{ m}^2/\text{s}} = 15493\text{ s} = \mathbf{4.30\text{ h}}\end{aligned}$$



**Discussion** It takes 4 hours and 18 minutes for the  $O_2$  concentration at 5 mm from the surface to reach 5% of the  $O_2$  concentration at the surface.





**14-83** A thick natural rubber wall is exposed to pure oxygen gas on one side of the surface. The oxygen concentrations varying with the distance from the surface are to be evaluated at different time elapsed.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

$c_i = 0$

$c_s = 1$

$D_{AB} = 2.1 \times 10^{-10} \text{ [m}^2/\text{s]}$

$\text{time}_1 = 10000 \text{ [s]}$

$\text{time}_2 = 50000 \text{ [s]}$

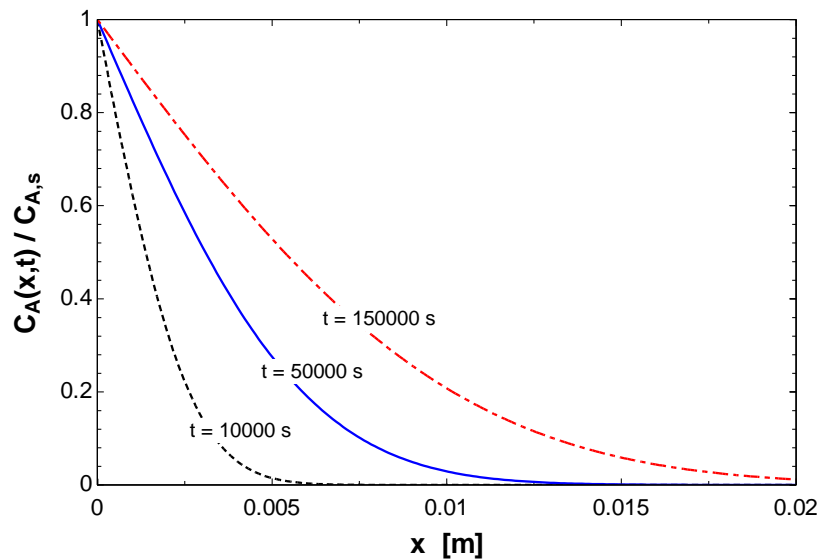
$\text{time}_3 = 150000 \text{ [s]}$

"ANALYSIS"

$c_1 = \text{Semilnf1}(c_i, c_s, D_{AB}, x, \text{time}_1)$

$c_2 = \text{Semilnf1}(c_i, c_s, D_{AB}, x, \text{time}_2)$

$c_3 = \text{Semilnf1}(c_i, c_s, D_{AB}, x, \text{time}_3)$



$x \text{ [m]}$	$C_A(x,t)/C_{A,s}$		
	$t = 10000 \text{ s}$	$50000 \text{ s}$	$150000 \text{ s}$
0	1	1	1
0.001	0.6256	0.8273	0.8997
0.002	0.3291	0.6625	0.8011
0.003	0.1432	0.5127	0.7055
0.004	0.05096	0.3827	0.6143
0.005	0.01470	0.2752	0.5287
0.006	0.003415	0.1904	0.4497
0.007	0.0006363	0.1266	0.3778
0.008	9.477E-05	0.08086	0.3135
0.009	1.125E-05	0.04953	0.2568
0.010	1.064E-06	0.02910	0.2077
0.012	4.759E-09	0.008829	0.1306
0.014	8.415E-12	0.002250	0.07776
0.016	5.847E-15	0.0004803	0.04382
0.018	1.572E-18	8.568E-05	0.02334
0.020	0	1.275E-05	0.01174

**Discussion** As the elapsed time increases the curves are approaching a linear profile that resembles the steady state profile.

**14-84** A piece of steel undergoing a decarburization process, the depth below the surface of the steel at which the concentration of carbon is reduced to 40% from its initial value as a result of the decarburization process for (a) an hour and (b) ten hours are to be determined.

**Assumptions** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape.

**Properties** The relevant properties are given in the problem statement.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.4C_{A,i} - C_{A,i}}{0 - C_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

$$0.6 = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

From Table 4-4, we have  $0.6 = \operatorname{erfc}(0.371)$ , hence

$$\frac{x}{2\sqrt{D_{AB}t}} = 0.371$$

(a) The depth of the steel after  $t = 1 \text{ h} = 3600 \text{ s}$  is

$$x = 2\sqrt{D_{AB}t} (0.371)$$

$$= 2\sqrt{(1 \times 10^{-7} \text{ cm}^2/\text{s})(3600 \text{ s})} (0.371) = \mathbf{0.0141 \text{ cm}}$$

(b) The depth of the steel after  $t = 10 \text{ h} = 36000 \text{ s}$  is

$$x = 2\sqrt{D_{AB}t} (0.371)$$

$$= 2\sqrt{(1 \times 10^{-7} \text{ cm}^2/\text{s})(36000 \text{ s})} (0.371) = \mathbf{0.0445 \text{ cm}}$$

**Discussion** The value for the complimentary error function can be determined using the EES software:

$$0.6 = \operatorname{erfc}(\eta) \rightarrow \eta = 0.3708$$

**14-85** A thick nickel wall is exposed to pure hydrogen gas on one side of the surface. The hydrogen concentration at the penetration depth, in percentage of its concentration at the surface, is to be determined.

**Assumptions** **1** The nickel wall can be modeled as a semi-infinite medium. **2** The  $H_2$  concentration in the nickel is initially zero. **3** The concentration of  $H_2$  at the rubber surface is constant.

**Properties** The diffusion coefficient of  $H_2$  in nickel at  $165^\circ\text{C} = 438\text{ K}$  is  $D_{AB} = 1.0 \times 10^{-11}\text{ m}^2/\text{s}$  (Table 14-3b).

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x, t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The penetration depth is given as

$$x = \delta_{\text{diff}} = \sqrt{\pi D_{AB}t}$$

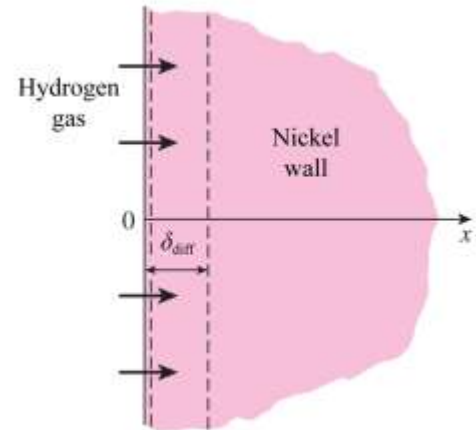
Substituting gives

$$\frac{C_A(x, t)}{C_{A,s}} = \text{erfc}\left(\frac{\sqrt{\pi D_{AB}t}}{2\sqrt{D_{AB}t}}\right) = \text{erfc}\left(\frac{\sqrt{\pi}}{2}\right) = \text{erfc}(0.8862)$$

From Table 4-4, we have  $\text{erfc}(0.8862) = 0.2101$ , hence

$$\frac{C_A(x, t)}{C_{A,s}} = 0.2101 = \mathbf{21\%}$$

**Discussion** The hydrogen concentration,  $C_A(x, t)/C_{A,s}$ , at the penetration depth is independent of  $x$  and  $t$ .





**14-86** A thick nickel wall is exposed to pure hydrogen gas on one side of the surface. The hydrogen concentrations varying with time are to be evaluated at different distances from the surface.

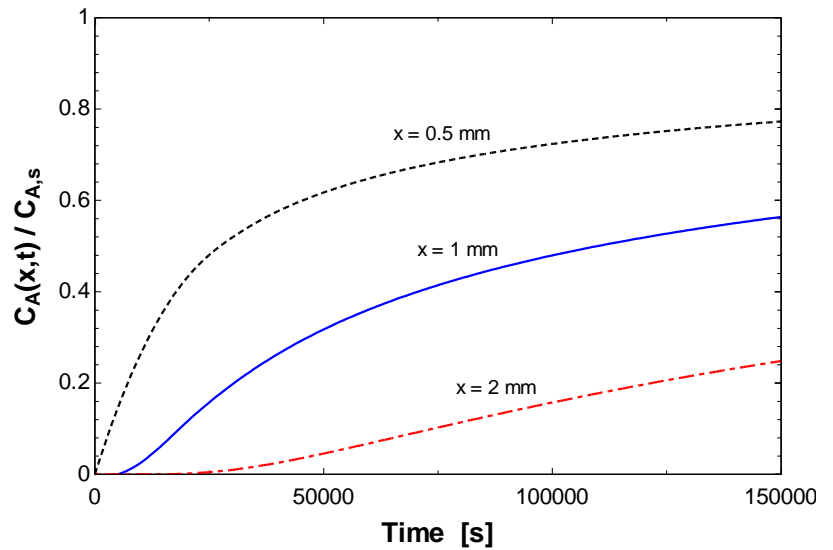
**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

$c_i = 0$   
 $c_s = 1$   
 $x_1 = 0.0005$   
 $x_2 = 0.001$   
 $x_3 = 0.002$   
 $D_{AB} = 1.0 \times 10^{-11} \text{ [m}^2/\text{s]}$   
 $k = D_{AB}$   
 $\alpha = D_{AB}$

"ANALYSIS"

$c_1 = \text{SemInf1}(c_i, c_s, D_{AB}, x_1, \text{time})$   
 $c_2 = \text{SemInf1}(c_i, c_s, D_{AB}, x_2, \text{time})$   
 $c_3 = \text{SemInf1}(c_i, c_s, D_{AB}, x_3, \text{time})$



$t \text{ [s]}$	$C_A(x,t)/C_{A,s}$		
	$x = 0.5 \text{ mm}$	$1 \text{ mm}$	$2 \text{ mm}$
0	0	0	0
10000	0.2636	0.02535	7.744E-06
20000	0.4292	0.1138	0.001565
30000	0.5186	0.1967	0.009823
40000	0.5762	0.2636	0.02535
50000	0.6171	0.3173	0.04550
60000	0.6481	0.3613	0.06789
70000	0.6726	0.3980	0.09097
80000	0.6926	0.4292	0.1138
90000	0.7094	0.4561	0.1360
100000	0.7237	0.4795	0.1573
110000	0.7360	0.5002	0.1775
120000	0.7469	0.5186	0.1967
130000	0.7565	0.5351	0.2148
140000	0.7651	0.5501	0.2320
150000	0.7728	0.5637	0.2482

**Discussion** Hydrogen concentration increases with time as it diffuses through the nickel wall.

**14-87** A long cylindrical nickel bar saturated with hydrogen is taken into an area that is free of hydrogen. The length of time for the hydrogen concentration at the center of the bar to drop by half is to be determined.

**Assumptions** **1** The bar can be treated as an infinitely long cylinder since it is very long and there is symmetry about the centerline. **2** The initial hydrogen concentration in the steel bar is uniform. **3** The hydrogen concentration at the surface remains constant at zero at all times. **4** The Fourier number is  $\tau > 0.2$  so that the one-term transient solutions are valid.

**Properties** The molar mass of hydrogen  $H_2$  is  $M = 2 \text{ kg / kmol}$  (Table A-1). The solubility of hydrogen in nickel at 358 K is  $0.00901 \text{ kmol / m}^3 \cdot \text{bar}$  (Table 14-7). The diffusion coefficient of hydrogen in nickel at 298 K is  $D_{AB} = 1.2 \times 10^{-12} \text{ m}^2/\text{s}$  (Table 14-3b).

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in an infinitely long cylinder with specified surface temperature, and thus can be solved accordingly. Noting that  $300 \text{ kPa} = 3 \text{ bar}$ , the molar density of hydrogen in the nickel bar before it is taken out of the storage room is

$$\begin{aligned} C_{H_2, \text{solid side}}(0) &= S \times P_{H_2, \text{gas side}} \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(3 \text{ bar}) \\ &= 0.027 \text{ kmol/m}^3 \end{aligned}$$

The molar concentration of hydrogen at the center of the bar can be calculated from

$$\frac{C_{H_2, o} - C_{H_2, \infty}}{C_{H_2, i} - C_{H_2, \infty}} = A_1 e^{-\lambda_1^2 \tau}$$

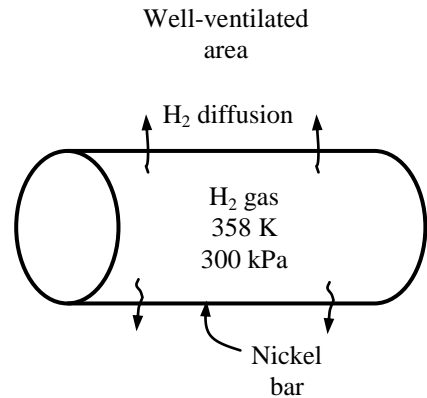
The Biot number in this case can be taken to be infinity since the bar is in a well-ventilated area during the transient case. The constants  $A_1$  and  $\lambda_1$  for the infinite Bi are determined from Table 4-2 to be 1.6021 and 2.4048, respectively. Noting that the concentration of hydrogen at the outer surface is zero, and the concentration of hydrogen at the center of the bar is one half of the initial concentration, the Fourier number,  $\tau$ , can be determined from


$$\frac{(0.027 / 2) - 0}{0.027 - 0} = 1.6021 e^{-(2.4048)^2 \tau} \longrightarrow \tau = 0.2014$$

Using the definition of the Fourier number, the time required to drop the concentration of hydrogen by half is determined to be

$$\tau = \frac{D_{AB} t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{D_{AB}} = \frac{(0.2014)(0.025)^2}{1.2 \times 10^{-12}} = 1.049 \times 10^8 \text{ s} = 1214 \text{ days} = \mathbf{3.33 \text{ years}}$$

Therefore, it will take years for this nickel bar to be free of hydrogen.



**14-88**  Chemical resistant gloves of 0.67 mm thickness are used for handling tetrachloroethylene solution. The time it will take for the solution to penetrate to the inner glove surface at 1% of the concentration at the outer surface is to be determined.

**Assumptions** **1** The glove wall can be modeled as a semi-infinite medium. **2** The solution concentration in the glove is initially zero. **3** The concentration of the solution at the outer glove surface is constant.

**Properties** The diffusion coefficient of tetrachloroethylene in the glove is given as  $D_{AB} = 3.0 \times 10^{-8} \text{ m}^2/\text{s}$ .

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x, t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

$$\begin{aligned}\frac{C_A(x, t)}{C_{A,s}} &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right) \\ 0.01 &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)\end{aligned}$$

From Table 4-4, we have  $0.01 = \text{erfc}(1.82)$ , hence

$$\frac{x}{2\sqrt{D_{AB}t}} = 1.82$$

Thus,

$$\begin{aligned}t &= \left[\frac{x}{(1.82)2}\right]^2 \frac{1}{D_{AB}} \\ &= \left[\frac{0.00067 \text{ m}}{(1.82)2}\right]^2 \frac{1}{3.0 \times 10^{-8} \text{ m}^2/\text{s}} = \mathbf{1.13 \text{ s}}\end{aligned}$$

**Discussion** It only takes the tetrachloroethylene solution about 1 second for 1% of the concentration at the exposed outer glove surface to penetrate to the inner surface. Therefore, this type of glove is not suitable for handling tetrachloroethylene solution.

Thicker gloves with lower diffusion coefficient of tetrachloroethylene should be used for the safety of the employees.

**14-89** A layer of glucose is submerged under a deep layer of water. (a) The time required for the glucose concentration at  $x = 1$  cm to reach 1% of its concentration at the glucose-water interface, and (b) the penetration depth are to be determined.

**Assumptions** **1** The water layer can be modeled as a semi-infinite medium. **2** The glucose concentration in the water is initially zero. **3** The concentration at the glucose-water interface is constant.

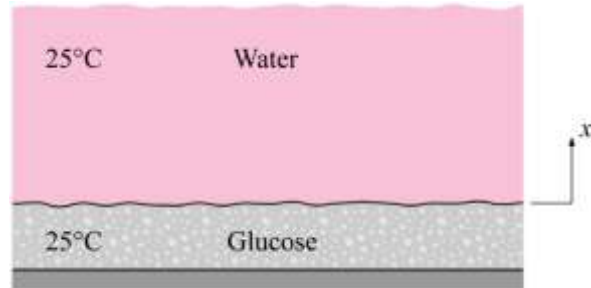
**Properties** The diffusion coefficient of glucose in water at  $25^\circ\text{C} = 298$  K is  $D_{AB} = 0.69 \times 10^{-9} \text{ m}^2/\text{s}$  (Table 14-3a).

**Analysis** (a) This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x, t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB} t}}\right)$$

Substituting the specified quantities gives

$$\begin{aligned}\frac{C_A(x, t)}{C_{A,s}} &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB} t}}\right) \\ 0.01 &= \text{erfc}\left(\frac{x}{2\sqrt{D_{AB} t}}\right)\end{aligned}$$



From Table 4-4, we have  $0.01 = \text{erfc}(1.82)$ , hence

$$\frac{x}{2\sqrt{D_{AB} t}} = 1.82$$

Thus,

$$\begin{aligned}t &= \left[\frac{x}{(1.82)2}\right]^2 \frac{1}{D_{AB}} = \left[\frac{0.01 \text{ m}}{(1.82)2}\right]^2 \frac{1}{0.69 \times 10^{-9} \text{ m}^2/\text{s}} \\ &= 10940 \text{ s} \\ &= \mathbf{3.04 \text{ h}}\end{aligned}$$

(b) The penetration depth of glucose in the water is

$$\begin{aligned}\delta_{\text{diff}} &= \sqrt{\pi D_{AB} t} \\ &= \sqrt{\pi(0.69 \times 10^{-9} \text{ m}^2/\text{s})(10940 \text{ s})} \\ &= 0.00487 \text{ m} \\ &= \mathbf{4.87 \text{ mm}}\end{aligned}$$

**Discussion** It took more than three hours for the glucose concentration in the water layer at 1 cm from the glucose-water interface to reach 1% of its concentration at the interface.

**14-90** A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 1150 K. The length of time the component should be kept in the furnace is to be determined.

**Assumptions** **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

**Properties** The relevant properties are given in the problem statement.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x, t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.0032 - 0.0012}{0.011 - 0.0012} = 0.2041 = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.2041 is determined from Table 4-4 to be 0.8980. That is,

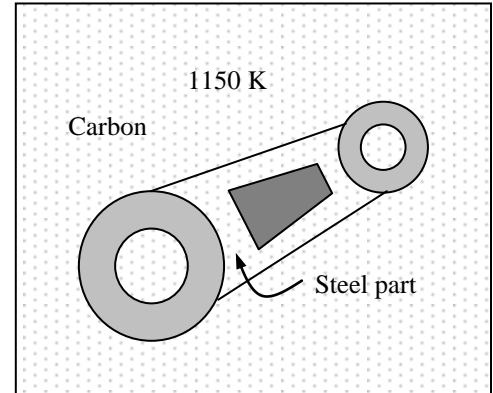
$$\frac{x}{2\sqrt{D_{AB}t}} = 0.8980$$

Then solving for the time  $t$  gives

$$t = \frac{x^2}{4D_{AB}(0.8980)^2} = \frac{(0.0007\text{ m})^2}{4 \times (7.2 \times 10^{-12}\text{ m}^2/\text{s})(0.8980)^2} = 21,098\text{ s} = \mathbf{5.86\text{ h}}$$

Therefore, the steel component must be held in the furnace for 5.86 h to achieve the desired level of hardening.

**Discussion** The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.





**14-91** A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 500 K. The length of time the component should be kept in the furnace is to be determined.

**Assumptions** **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

**Properties** The relevant properties are given in the problem statement.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x, t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.0032 - 0.0012}{0.011 - 0.0012} = 0.2041 = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.2041 is determined from Table 4-4 to be 0.8980. That is,

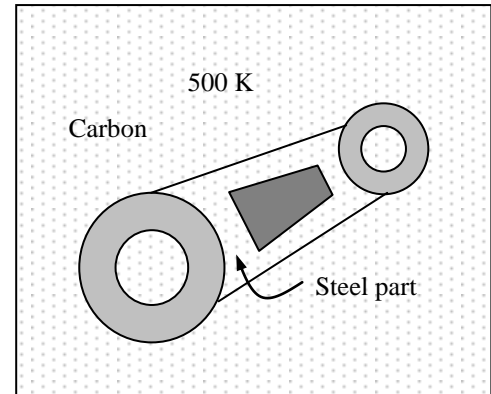
$$\frac{x}{2\sqrt{D_{AB}t}} = 0.8980$$

Solving for the time  $t$  gives

$$t = \frac{x^2}{4D_{AB}(0.8980)^2} = \frac{(0.0007\text{ m})^2}{4 \times (2.1 \times 10^{-20}\text{ m}^2/\text{s})(0.8980)^2} = 7.234 \times 10^{12}\text{ s} = 229,380\text{ years}$$

Therefore, the steel component must be held in the furnace forever to achieve the desired level of hardening.

**Discussion** The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.



**14-92** A pond is to be oxygenated by forming a tent over the water surface and filling the tent with oxygen gas. The molar concentration of oxygen at a depth of 1 cm from the surface after 24 h is to be determined.

**Assumptions** **1** The oxygen in the tent is saturated with water vapor. **2** Oxygen penetrates into a thin layer at the pond surface, and thus the pond can be modeled as a semi-infinite medium. **3** Both the water vapor and oxygen are ideal gases. **4** The initial oxygen content of the pond is zero.

**Properties** The diffusion coefficient of oxygen in water at 25°C is  $D_{AB} = 2.4 \times 10^{-9} \text{ m}^2/\text{s}$  (Table 14-3a). Henry's constant for oxygen dissolved in water at 300 K ( $\cong 25^\circ\text{C}$ ) is given in Table 14-6 to be  $H = 43,600 \text{ bar}$ . The saturation pressure of water at 25°C is 3.17 kPa (Table 14-9).

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. The vapor pressure in the tent is the saturation pressure of water at 25°C since the oxygen in the tent is saturated, and thus the partial pressure of oxygen in the tank is

$$P_{O_2} = P - P_v = 130 - 3.17 = 126.83 \text{ kPa}$$

Then the mole fraction of oxygen in the water at the pond surface becomes

$$y_{O_2, \text{liquid side}}(0) = \frac{P_{O_2, \text{gas side}}(0)}{H} = \frac{1.2683 \text{ bar}}{43,600 \text{ bar}} = 2.909 \times 10^{-5}$$

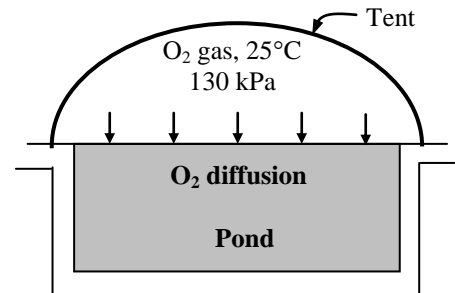
The molar concentration of oxygen at a depth of 1 cm from the surface after 12 h can be determined from

$$\frac{y_{O_2}(x, t) - y_{O_2, i}}{y_{O_2, s} - y_{O_2, i}} = \text{erfc} \left( \frac{x}{2\sqrt{D_{AB}t}} \right)$$

Substituting,

$$\frac{y_{O_2}(x, t) - 0}{2.909 \times 10^{-5} - 0} = \text{erfc} \left( \frac{0.01 \text{ m}}{2\sqrt{(2.4 \times 10^{-9} \text{ m}^2/\text{s})(24 \times 3600 \text{ s})}} \right) = \text{erfc}(0.3472) = 0.6234 \longrightarrow y_{O_2}(0.01 \text{ m}, 24 \text{ h}) = \mathbf{1.81 \times 10^{-5}}$$

Therefore, there will be 18.1 moles of oxygen per million at a depth of 1 cm from the surface in 24 h.



**14-93** A piece of steel was exposed to a carburizing atmosphere for an hour, and the percentage of mass concentration of carbon at 0.2 mm and 0.4 mm below the surface are to be determined.

**Assumptions** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape.

**Properties** The relevant properties are given in the problem statement.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. Using mass fraction for concentration since the data are given in that form, the solution can be expressed as

$$\frac{w_A(x,t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities (for  $x = 0.0002$  m and  $t = 1$  h = 3600 s) gives

$$\frac{w_A(x,t) - 0.002}{0.007 - 0.002} = \operatorname{erfc}\left(\frac{0.0002 \text{ m}}{2\sqrt{(1 \times 10^{-11} \text{ m}^2/\text{s})(3600 \text{ s})}}\right)$$

$$\frac{w_A(x,t) - 0.002}{0.007 - 0.002} = \operatorname{erfc}(0.527)$$

Thus, mass concentration of carbon at  $x = 0.2$  mm and  $t = 1$  h is

$$w_A(0.2 \text{ mm}, 1 \text{ h}) = (0.007 - 0.002)(0.4561) + 0.002 = 0.00428 = \mathbf{0.428\%}$$

Similarly, substituting the specified quantities (for  $x = 0.0004$  m and  $t = 1$  h = 3600 s) gives

$$\frac{w_A(x,t) - 0.002}{0.007 - 0.002} = \operatorname{erfc}(1.054)$$

Thus, mass concentration of carbon at  $x = 0.4$  mm and  $t = 1$  h is

$$w_A(0.4 \text{ mm}, 1 \text{ h}) = (0.007 - 0.002)(0.136) + 0.002 = 0.00268 = \mathbf{0.268\%}$$

**Discussion** The values for the complimentary error function can be determined from Table 4-4 or using the EES software:

$$z = \operatorname{erfc}(0.527) \rightarrow z = 0.4561 \quad \text{and} \quad z = \operatorname{erfc}(1.054) \rightarrow z = 0.136$$

**14-94** During an aeration process, the pond surface has its oxygen density suddenly increased to  $9 \text{ kg/m}^3$ . The oxygen density at 5 cm below the pond surface after 100 hours is to be determined.

**Assumptions** 1 The pond can be modeled as a semi-infinite medium. 2 Both air and water are stationary.

**Properties** The diffusion coefficient of oxygen in water at  $25^\circ\text{C}$  is  $D_{AB} = 2.4 \times 10^{-9} \text{ m}^2/\text{s}$  (Table 14-3a).

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{\rho_A(x, t) - \rho_{A,i}}{\rho_{A,s} - \rho_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities ( $x = 5 \text{ cm}$  and  $t = 100 \text{ h} = 360000 \text{ s}$ ) gives

$$\frac{\rho_A(x, t) - 2 \text{ kg/m}^3}{(9 \text{ kg/m}^3 - 2 \text{ kg/m}^3)} = \text{erfc}\left(\frac{0.05 \text{ m}}{2\sqrt{(2.4 \times 10^{-9} \text{ m}^2/\text{s})(360000 \text{ s})}}\right)$$

$$\frac{\rho_A(x, t) - 2 \text{ kg/m}^3}{(9 \text{ kg/m}^3 - 2 \text{ kg/m}^3)} = \text{erfc}(0.8505)$$

From Table 4-4, we have  $\text{erfc}(0.8505) = 0.229$ , hence

$$\begin{aligned}\rho_A(5 \text{ cm}, 100 \text{ h}) &= (9 \text{ kg/m}^3 - 2 \text{ kg/m}^3)(0.229) + 2 \text{ kg/m}^3 \\ &= \mathbf{3.60 \text{ kg/m}^3}\end{aligned}$$

**Discussion** After 100 h, the oxygen density at 5 cm below the pond surface is increased by 80%. This shows that mass diffusion through a stationary layer is very slow.

## Diffusion in a Moving Medium

**14-95C** The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. The average velocity of a species in a moving medium is equal to the sum of the bulk flow velocity and the diffusion velocity. Therefore, the diffusion velocity can increase or decrease the average velocity, depending on the direction of diffusion relative to the direction of bulk flow. The velocity of a species in the moving medium relative to a fixed reference point **will be zero** when the diffusion velocity of the species and the bulk flow velocity are equal in magnitude and opposite in direction.

**14-96C** The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. The **molar-average velocity** of a medium at some location is the average velocity of the molecules at that location, regardless of their mass, relative to an external reference point. If one of these velocities are zero, the other will not necessarily be zero. The mass-average and molar-average velocities of a binary mixture will be the same when the molar masses of the two constituents are equal to each other. The mass and mole fractions of each species in this case will be the same.

**14-97C** The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. It is the velocity that would be measured by a velocity sensor such as a pitot tube, a turbine device, or a hot wire anemometer inserted into the flow. The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. A **stationary medium** is a medium whose mass average velocity is zero. A **moving medium** is a medium that involves a bulk fluid motion caused by an external force.

**14-98C** (a) T, (b) T, (c) F, (d) F

**14-99C** The diffusion of a vapor through a stationary gas column is called the **Stefan flow**. The **Stefan's law** can be expressed as

$$\bar{j}_A = \dot{N}_A / A = \frac{CD_{AB}}{L} \ln \frac{1 - y_{A,L}}{1 - y_{A,o}}$$

where  $C$  is the average concentration of the mixture,  $D_{AB}$  is the diffusion coefficient of  $A$  in  $B$ ,  $L$  is the height of the gas column,  $y_{A,L}$  is the molar concentration of a species at  $x = L$ , and  $y_{A,o}$  is the molar concentration of the species  $A$  at  $x = 0$ .

**14-100** The amount of chloroform that diffuses from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The mass diffusivity of chloroform in air is to be determined.

**Assumptions** 1 Chloroform vapor and atmospheric air are ideal gases. 2 The amount of air dissolved in liquid chloroform is negligible. 3 Temperatures of air and chloroform remain constant at 25°C.

**Properties** The relevant properties are given in the problem statement.

**Analysis** The vapor pressure at the air-chloroform interface is the vapor pressure of chloroform at 25°C,  $P_{A,0} = 0.263$  atm, and the mole fraction of chloroform vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{0.263 \text{ atm}}{1 \text{ atm}} = 0.263$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{RT} = \frac{101.325 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = 0.0409 \text{ kmol/m}^3$$

The diffusion rate is given to be 222 g per 10 hours. Then the diffusion rate per unit interface area is

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{m}{MA t} = \frac{4m}{M\pi D^2 t} \\ &= \frac{4(222 \times 10^{-3} \text{ kg})}{(119.39 \text{ kg/kmol})\pi(0.05 \text{ m})^2(10 \times 3600 \text{ s})} \\ &= 2.63 \times 10^{-5} \text{ kmol/s} \cdot \text{m}^2 \end{aligned}$$

The distance between the chloroform surface and the top of the tube is initially 7 cm, and will be 7.44 cm after 10 hours. Therefore, we can take the average height to be  $(7 + 7.44)/2 = 7.22$  cm.

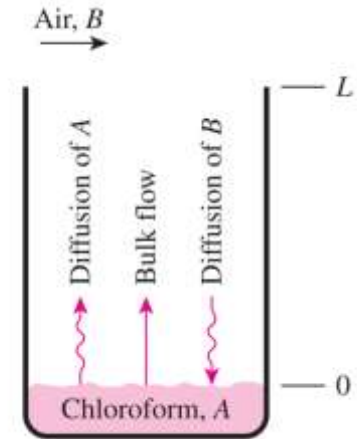
Finally, the mass diffusivity of chloroform in air is determined using

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{CD_{AB}}{L} \ln \left( \frac{1 - y_{A,L}}{1 - y_{A,0}} \right) \\ 2.63 \times 10^{-5} \text{ kmol/s} \cdot \text{m}^2 &= \frac{(0.0409 \text{ kmol/m}^3)D_{AB}}{0.0722 \text{ m}} \ln \left( \frac{1 - 0}{1 - 0.263} \right) \end{aligned}$$

which gives

$$D_{AB} = 1.52 \times 10^{-4} \text{ m}^2/\text{s}$$

**Discussion** The Stefan tube is sometimes also known as Arnold diffusion cell.



**14-101E** The amount of water that evaporates from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The diffusion coefficient of water vapor in air is to be determined.

**Assumptions** **1** Water vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid water is negligible. **3** Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 80°F.

**Properties** The saturation pressure of water at 80°F is 0.5073 psia (Table A-9E).

**Analysis** The vapor pressure at the air-water interface is the saturation pressure of water at 80°F, and the mole fraction of water vapor (species A) is determined from

$$y_{\text{vapor},o} = y_{A,o} = \frac{P_{\text{vapor},o}}{P} = \frac{0.5073 \text{ psia}}{13.8 \text{ psia}} = 0.0368$$

Dry air is blown on top of the tube and thus  $y_{\text{vapor},L} = y_{A,L} = 0$ . Also, the total molar density throughout the tube remains constant because of the constant temperature and pressure conditions, and is determined to be

$$C = \frac{P}{R_u T} = \frac{13.8 \text{ psia}}{(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(540 \text{ R})} = 0.00238 \text{ lbmol/ft}^3$$

The cross-sectional area of the valve is

$$A = \pi D^2 / 4 = \pi (1/12 \text{ ft})^2 / 4 = 5.45 \times 10^{-3} \text{ ft}^2$$

The evaporation rate is given to be 0.0025 lbm per 10 days. Then the molar flow rate of vapor is determined to be

$$\dot{N}_A = \dot{N}_{\text{vapor}} = \frac{m_{\text{vapor}}}{M_{\text{vapor}}} = \frac{0.0025 \text{ lbm}}{(10 \times 24 \times 3600 \text{ s})(18 \text{ lbm/lbmol})} = 1.61 \times 10^{-10} \text{ lbmol/s}$$

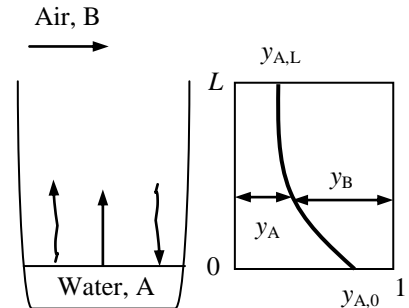
Finally, substituting the information above into Eq. 14-59 we get


$$\frac{\dot{N}_A}{A} = \frac{CD_{AB}}{L} \ln \left( \frac{1 - y_{A,L}}{1 - y_{A,o}} \right) \longrightarrow \frac{1.61 \times 10^{-10} \text{ lbmol/s}}{5.45 \times 10^{-3} \text{ ft}^2} = \frac{(0.00238 \text{ lbmol/ft}^3) D_{AB}}{10/12 \text{ ft}} \ln \left( \frac{1 - 0}{1 - 0.0368} \right)$$

It gives

$$D_{AB} = 2.76 \times 10^{-4} \text{ ft}^2/\text{s}$$

for the binary diffusion coefficient of water vapor in air at 80°F and 13.8 psia.



**14-102**  Benzene undergoes evaporation in 10 test tubes. The total evaporation rate of benzene to the surrounding air is to be determined whether or not the level of benzene could result in health risks.

**Assumptions** 1 Benzene vapor and atmospheric air are ideal gases. 2 The amount of air dissolved in liquid benzene is negligible. 3 Temperatures of air and benzene remain constant at 25°C.

**Properties** The molar mass of benzene is given as  $M = 78.11 \text{ kg/kmol}$ . The diffusion coefficient of benzene in air at 25°C is  $D_{AB} = 0.88 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2).

**Analysis** The vapor pressure at the air-benzene interface is the vapor pressure of benzene at 25°C,  $P_{A,0} = 10 \text{ kPa}$ , and the mole fraction of benzene vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{10 \text{ kPa}}{101.325 \text{ kPa}} = 0.09869$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{R_u T} = \frac{101.325 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = 0.0409 \text{ kmol/m}^3$$

Then, the diffusion rate per unit interface area for one test tube is

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{CD_{AB}}{L} \ln \left( \frac{1 - y_{A,L}}{1 - y_{A,0}} \right) \\ &= \frac{(0.0409 \text{ kmol/m}^3)(0.88 \times 10^{-5} \text{ m}^2/\text{s})}{0.01 \text{ m}} \ln \left( \frac{1 - 0}{1 - 0.09869} \right) \\ &= 3.7398 \times 10^{-6} \text{ kmol/s} \cdot \text{m}^2 \end{aligned}$$

The evaporation rate of benzene from one test tube is

$$\begin{aligned} \dot{N}_A &= \frac{\dot{m}}{M} \quad \rightarrow \quad \dot{m} = M \pi \frac{D^2}{4} \frac{\dot{N}_A}{A} \\ \dot{m} &= (78.11 \text{ kg/kmol}) \frac{\pi}{4} (0.025 \text{ m})^2 (3.7398 \times 10^{-6} \text{ kmol/s} \cdot \text{m}^2) \\ &= (1.434 \times 10^{-7} \text{ kg/s})(1000 \text{ g/kg})(3600 \text{ s/hr}) \\ &= 1.434 \times 10^{-7} \text{ kg/s} = 0.516 \text{ g/h} \end{aligned}$$

Thus, the total rate of benzene being evaporated from ten test tubes to the surrounding air is

$$\dot{m}_{\text{total}} = 10(0.516 \text{ g/h}) = \mathbf{5.16 \text{ g/h}} > 3 \text{ g/h}$$

**Discussion** The rate of benzene evaporation to the surrounding air is greater than the rate that the HVAC system can handle. That means benzene vapor will be accumulating and eventually reach a level that is hazardous to health.

To prevent health hazards from benzene exposure, the liquid benzene should be handled under a fume hood.



**14-103** The amount of ethanol that evaporates from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The mass diffusivity of ethanol in air is to be determined.

**Assumptions** 1 Ethanol vapor and atmospheric air are ideal gases. 2 The amount of air dissolved in liquid ethanol is negligible. 3 Temperatures of air and ethanol remain constant at 24°C.

**Properties** The relevant properties are given in the problem statement.

**Analysis** The vapor pressure at the air-ethanol interface is the vapor pressure of ethanol at 24°C,  $P_{A,0} = 0.0684$  atm, and the mole fraction of ethanol vapor (species A) at the interface is determined from

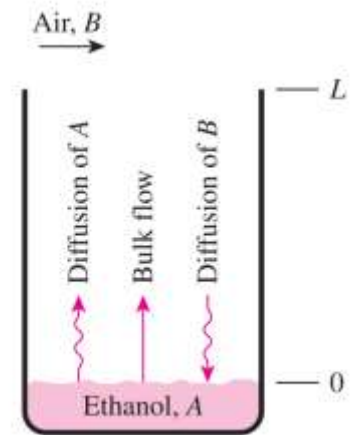
$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{0.0684 \text{ atm}}{1 \text{ atm}} = 0.0684$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{RT} = \frac{101.325 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 24) \text{ K}} = 0.04103 \text{ kmol/m}^3$$

Then the diffusion rate per unit interface area is

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{m}{MA t} = \frac{\rho V}{MA t} \\ &= \frac{(789 \text{ kg/m}^3)(0.0445 \text{ cm}^3)(10^{-6} \text{ m}^3/\text{cm}^3)}{(46 \text{ kg/kmol})(0.8 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2)(10 \times 3600 \text{ s})} \\ &= 2.65 \times 10^{-7} \text{ kmol/s} \cdot \text{m}^2 \end{aligned}$$



The distance between the chloroform surface and the top of the tube is initially 10 cm, and will be 25 cm after 10 hours. Therefore, we can take the average height to be  $(10 + 25)/2 = 17.5$  cm.

Finally, the mass diffusivity of ethanol in air is determined using

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{CD_{AB}}{L} \ln \left( \frac{1 - y_{A,L}}{1 - y_{A,0}} \right) \\ 2.65 \times 10^{-7} \text{ kmol/s} \cdot \text{m}^2 &= \frac{(0.04103 \text{ kmol/m}^3)D_{AB}}{0.175 \text{ m}} \ln \left( \frac{1 - 0}{1 - 0.0684} \right) \end{aligned}$$

which gives

$$D_{AB} = 1.60 \times 10^{-5} \text{ m}^2/\text{s}$$

**Discussion** The Stefan tube is sometimes also known as Arnold diffusion cell.



**14-104** Methanol undergoes evaporation in a vertical tube. (a) The evaporation rate of methanol is to be determined, and (b) the mole fraction of methanol vapor as a function of the tube height is to be plotted.

**Assumptions** 1 Methanol vapor and atmospheric air are ideal gases. 2 The amount of air dissolved in liquid methanol is negligible. 3 Temperatures of air and methanol remain constant at 25°C.

**Properties** The molar mass of methanol is given as  $M = 32 \text{ kg/kmol}$ . The diffusion coefficient of methanol in air is given as  $D_{AB} = 1.62 \times 10^{-5} \text{ m}^2/\text{s}$ .

**Analysis** (a) The vapor pressure at the air-methanol interface is the vapor pressure of methanol at 25°C,  $P_{A,0} = 17 \text{ kPa}$ , and the mole fraction of methanol vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{17 \text{ kPa}}{101.325 \text{ kPa}} = 0.1678$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{R_u T} = \frac{101.325 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = 0.0409 \text{ kmol/m}^3$$

Then the diffusion rate per unit interface area is

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{CD_{AB}}{L} \ln \left( \frac{1 - y_{A,L}}{1 - y_{A,0}} \right) \\ &= \frac{(0.0409 \text{ kmol/m}^3)(1.62 \times 10^{-5} \text{ m}^2/\text{s})}{0.30 \text{ m}} \ln \left( \frac{1 - 0}{1 - 0.1678} \right) \\ &= 4.0568 \times 10^{-7} \text{ kmol/s} \cdot \text{m}^2 \end{aligned}$$

The evaporation rate of the methanol in kg/h can be determined with

$$\begin{aligned} \dot{N}_A &= \frac{\dot{m}}{M} \quad \rightarrow \quad \dot{m} = M \frac{\dot{N}_A}{A} \\ \dot{m} &= (32 \text{ kg/kmol})(4.0568 \times 10^{-7} \text{ kmol/s} \cdot \text{m}^2)(0.8 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2) \\ &= 1.039 \times 10^{-9} \text{ kg/s} \\ &= \mathbf{3.74 \times 10^{-6} \text{ kg/h}} \end{aligned}$$

**Discussion** With  $\rho = 791 \text{ kg/m}^3$ , that means the evaporation rate in terms of volume is  $4.73 \times 10^{-9} \text{ m}^3/\text{h}$ .

(b) This part is solved using EES, and the solution is given below:

"GIVEN"

P\_A\_0=17 [kPa]

P=101.325 [kPa]

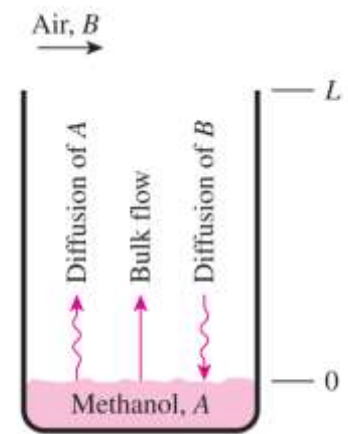
y\_A\_L=0

L=0.3 [m]

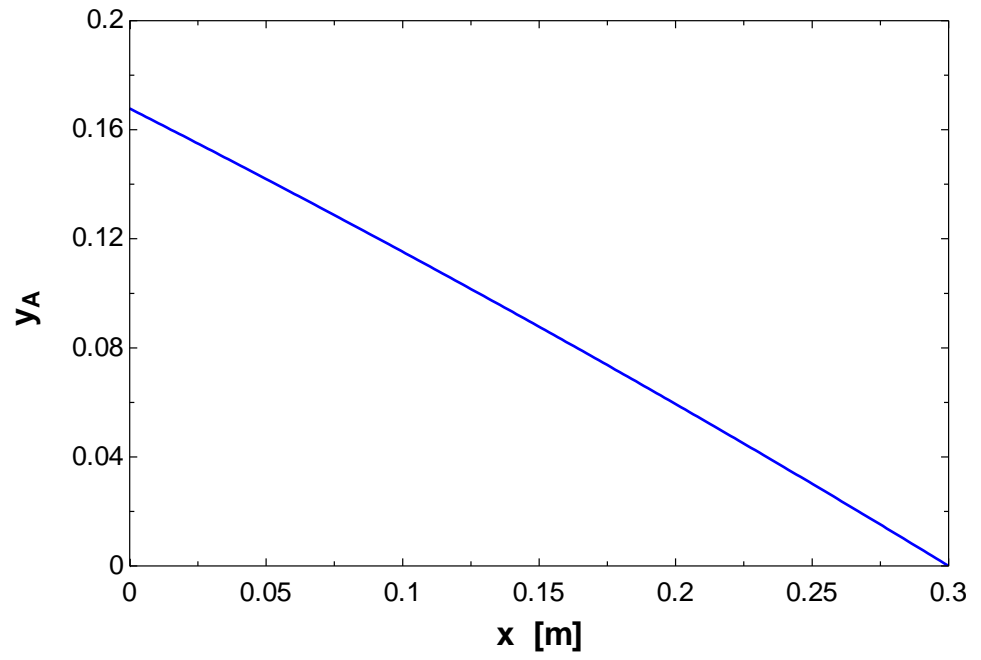
"ANALYSIS"

y\_A\_0=P\_A\_0/P

(1-y\_A)/(1-y\_A\_0)=((1-0)/(1-y\_A\_0))^(x/L)



$x$ [mm]	$y_A$
0	0.1678
0.01	0.1627
0.02	0.1575
0.03	0.1524
0.04	0.1471
0.05	0.1419
0.10	0.1152
0.15	0.08774
0.20	0.05938
0.25	0.03015
0.30	0



**14-105** A hydrogen tank is maintained at atmospheric temperature and pressure by venting to the atmosphere through the charging valve. The initial mass flow rate of hydrogen out of the tank is to be determined.

**Assumptions** 1 Steady operating conditions at initial conditions exist. 2 Hydrogen and atmospheric air are ideal gases. 3 No chemical reactions occur in the valve. 4 Air concentration in the tank and hydrogen concentration in the atmosphere are negligible so that the mole fraction of the hydrogen is 1 in the tank, and 0 in the atmosphere (we will check this assumption later).

**Properties** The molar mass of hydrogen is  $M = 2 \text{ kg/kmol}$  (Table A-1). The diffusion coefficient of hydrogen in air (or air in hydrogen) at 1 atm and  $25^\circ\text{C}$  is  $D_{AB} = 7.2 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2). However, the pressure in the tank is  $90 \text{ kPa} = 0.88 \text{ atm}$ . The diffusion coefficient at  $25^\circ\text{C}$  and  $0.88 \text{ atm}$  is determined from

$$D_{AB} = \frac{D_{AB,1 \text{ atm}}}{P (\text{in atm})} = \frac{7.2 \times 10^{-5}}{0.88} = 8.18 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant. The cross-sectional area of the valve is

$$A = \pi D^2 / 4 = \pi (0.03 \text{ m})^2 / 4 = 7.069 \times 10^{-4} \text{ m}^2$$

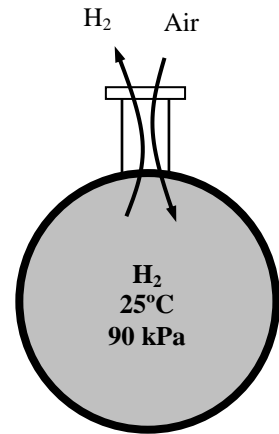
Noting that the pressure of hydrogen is  $90 \text{ kPa}$  at the bottom of the charging valve ( $x = 0$ ) and  $0 \text{ kPa}$  at the top ( $x = L$ ), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{H}_2} = \dot{N}_{\text{diff},A} &= \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(8.18 \times 10^{-5} \text{ m}^2/\text{s})(7.069 \times 10^{-4} \text{ m}^2) (90 - 0) \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(298 \text{ K}) 0.1 \text{ m}} \\ &= 2.098 \times 10^{-8} \text{ kmol/s} \end{aligned}$$

Then the mass flow rate of hydrogen becomes

$$\dot{m}_{\text{H}_2} = (\dot{N}M)_{\text{H}_2} = (2.098 \times 10^{-8} \text{ kmol/s})(2 \text{ kg/kmol}) = 4.2 \times 10^{-8} \text{ kg/s}$$

**Discussion** This is the highest mass flow rate. It will decrease during the process as air diffuses into the tank and the concentration of hydrogen in tank drops.





**14-106** Prob. 14-105 is reconsidered. The mass flow rate of hydrogen lost as a function of the diameter of the charging valve is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

thickness=0.02 [m]

T=(25+273) [K]

P\_atm=90 [kPa]

D=3 [cm]

extension=0.08 [m]

L=0.10 [m]

"PROPERTIES"

MM\_H2=Molarmass(H2)

D\_AB\_1atm=7.2E-5 [m^2/s] "from Table 14-2 of the text at 1 atm and 25 C"

D\_AB=D\_AB\_1atm\*P\_1atm/(P\_atm\*Convert(kPa, atm)) "at 90 kPa and 25 C"

P\_1atm=1 [atm]

R\_u=8.314 [kPa-m^3/kmol-K]

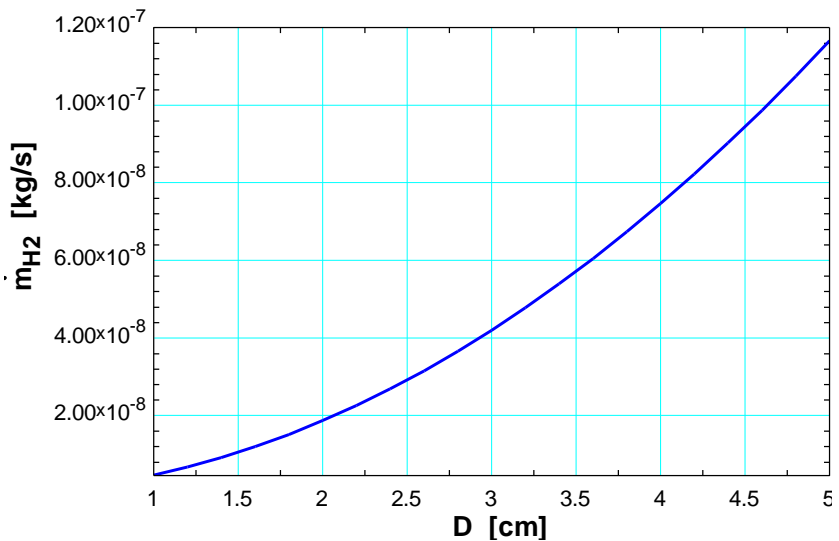
"ANALYSIS"

A=pi\*D^2/4\*Convert(cm^2, m^2)

N\_dot\_H2=(D\_AB\*A)/(R\_u\*T)\*(P\_atm-0)/L

m\_dot\_H2=N\_dot\_H2\*MM\_H2

D [cm]	$\dot{m}_{H_2}$ [kg/s]
1	4.662E-09
1.2	6.714E-09
1.4	9.138E-09
1.6	1.194E-08
1.8	1.511E-08
2	1.865E-08
2.2	2.257E-08
2.4	2.686E-08
2.6	3.152E-08
2.8	3.655E-08
3	4.196E-08
3.2	4.774E-08
3.4	5.390E-08
3.6	6.042E-08
3.8	6.732E-08
4	7.460E-08
4.2	8.224E-08
4.4	9.026E-08
4.6	9.865E-08
4.8	1.074E-07
5	1.166E-07



**14-107** A pitcher that is half filled with water is left in a room with its top open. The time it takes for the entire water in the pitcher to evaporate is to be determined.

**Assumptions** 1 Water vapor and atmospheric air are ideal gases. 2 The amount of air dissolved in liquid water is negligible. 3 Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 15°C.

**Properties** The saturation pressure of water at 15°C is 1.7051 kPa (Table A-9). The density of water in the pitcher can be taken to be 1000 kg/m<sup>3</sup>. The diffusion coefficient of water vapor in air at 15°C (= 288 K) and 87/101.325 = 0.8586 atm can be determined from

$$D_{AB} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{0.8586} = 2.716 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The flow area, which is the cross-sectional area of the pitcher, is

$$A = \pi D^2 / 4 = \pi (0.08 \text{ m})^2 / 4 = 5.026 \times 10^{-3} \text{ m}^2$$

The vapor pressure at the air-water interface is the saturation pressure of water at 15°C, which is 1.7051 kPa. The air at the top of the pitcher ( $x = L$ ) can be assumed to be dry ( $P_{A,L} = 0$ ). The distance between the water surface and the top of the pitcher is initially 15 cm, and will be 30 cm at the end of the process when all the water is evaporated. Therefore, we can take the average height of the air column above the water surface to be (15+30)/2 = 22.5 cm. Then the molar flow rate is determined from

$$\begin{aligned} \dot{N}_A &= \frac{D_{AB} A}{R_u T} \left( \frac{P_{A,o} - P_{A,L}}{L} \right) \\ &= \frac{(2.716 \times 10^{-5} \text{ m}^2/\text{s})(5.026 \times 10^{-3} \text{ m}^2) (1.7051 - 0) \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(288 \text{ K}) (0.225 \text{ m})} \\ &= 4.320 \times 10^{-10} \text{ kmol/s} \end{aligned}$$

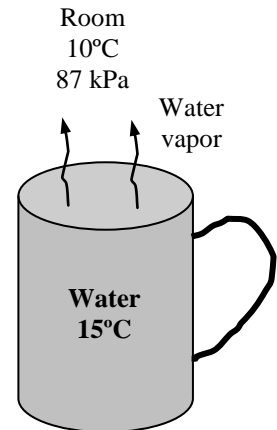
The initial mass of water in the pitcher is

$$m_{\text{water}} = \rho \frac{\pi D^2}{4} L = (1000 \text{ kg/m}^3) \frac{\pi (0.08 \text{ m})^2}{4} (0.15 \text{ m}) = 0.7540 \text{ kg}$$

Then the time required to evaporate the water completely becomes

$$\begin{aligned} \dot{N}_{\text{vapor}} &= \frac{m_{\text{vapor}}}{\Delta t \times M_{\text{vapor}}} \\ \Delta t &= \frac{m_{\text{vapor}}}{\dot{N}_{\text{vapor}} \times M_{\text{vapor}}} = \frac{0.7540 \text{ kg}}{(4.320 \times 10^{-10} \text{ kmol/s})(18 \text{ kg/kmol})} = \mathbf{9.70 \times 10^7 \text{ s}} \end{aligned}$$

which is equivalent to **1122 days**. Therefore, it will take the water in the pitcher about 3.1 years to evaporate completely.



**14-108** A large ammonia tank is vented to the atmosphere. The rate of loss of ammonia and the rate of air infiltration into the tank are to be determined.

**Assumptions** **1** Ammonia vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid ammonia is negligible. **3** Heat is transferred to the ammonia from the surroundings to make up for the latent heat of vaporization so that the temperature of ammonia remains constant at 25°C.

**Properties** The molar mass of ammonia is  $M = 17$  kg/kmol, and the molar mass of air is  $M = 29$  kg/kmol (Table A-1). The diffusion coefficient of ammonia in air (or air in ammonia) at 1 atm and 25°C is  $D_{AB} = 2.6 \times 10^{-5}$  m<sup>2</sup>/s (Table 14-2).

**Analysis** This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the tank and the atmosphere) remain constant. The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.015 \text{ m})^2 / 4 = 1.767 \times 10^{-4} \text{ m}^2$$

Noting that the pressure of ammonia is 1 atm = 101.3 kPa at the bottom of the tube ( $x = 0$ ) and 0 at the top ( $x = L$ ), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{ammonia}} &= \dot{N}_{\text{diff,A}} = \frac{D_{AB} A}{R_u T} \frac{P_{A,o} - P_{A,L}}{L} \\ &= \frac{(2.6 \times 10^{-5} \text{ m}^2/\text{s})(1.767 \times 10^{-4} \text{ m}^2)}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(298 \text{ K})} \frac{(101.3 - 0) \text{ kPa}}{2 \text{ m}} \\ &= \mathbf{9.39 \times 10^{-11} \text{ kmol/s}} \end{aligned}$$

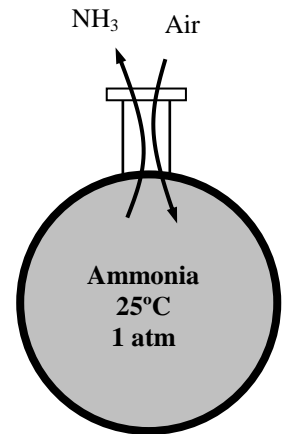
Therefore, the mass flow rate of ammonia through the tube is

$$\dot{m}_{\text{NH}_3} = (\dot{N}M)_{\text{NH}_3} = (9.39 \times 10^{-11} \text{ kmol/s})(17 \text{ kg/kmol}) = \mathbf{1.60 \times 10^{-9} \text{ kg/s}}$$

which corresponds to 0.0504 kg per year.

Note that  $\dot{N}_B = -\dot{N}_A$  during an equimolar counter diffusion process. Therefore, the molar flow rate of air into the ammonia tank is equal to the molar flow rate of ammonia out of the tank. Then the mass flow rate of air into the pipeline becomes

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-9.39 \times 10^{-11} \text{ kmol/s})(29 \text{ kg/kmol}) = \mathbf{-2.72 \times 10^{-9} \text{ kg/s}}$$



**14-109E** The pressure in a helium pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of helium and air, and the net flow velocity at the bottom of the tube are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Helium and atmospheric air are ideal gases. 3 No chemical reactions occur in the tube. 4 Air concentration in the pipeline and helium concentration in the atmosphere are negligible so that the mole fraction of the helium is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

**Properties** The diffusion coefficient of helium in air (or air in helium) at normal atmospheric conditions is  $D_{AB} = 7.75 \times 10^{-4}$  ft<sup>2</sup>/s (Table 14-2). The molar mass of helium is  $M = 4$  lbm / lbmol, and the molar mass of air is 29 lbm / lbmol (Table A-1E).

**Analysis** This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.

(a) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.25 / 12 \text{ ft})^2 / 4 = 3.409 \times 10^{-4} \text{ ft}^2$$

Noting that the pressure of helium is 14.5 psia at the bottom of the tube ( $x = 0$ ) and 0 at the top ( $x = L$ ), its molar flow rate is

$$\begin{aligned} \dot{N}_{\text{helium}} &= \dot{N}_{\text{diff,A}} = \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(7.75 \times 10^{-4} \text{ ft}^2/\text{s})(3.409 \times 10^{-4} \text{ ft}^2)}{(10.73 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R})(540 \text{ R})} \frac{(14.5 - 0) \text{ psia}}{30 \text{ ft}} \\ &= 2.204 \times 10^{-11} \text{ lbmol/s} \end{aligned}$$

Therefore, the mass flow rate of helium through the tube is

$$\dot{m}_{\text{helium}} = (\dot{N}M)_{\text{helium}} = (2.204 \times 10^{-11} \text{ lbmol/s})(4 \text{ lbm/lbmol}) = \mathbf{8.816 \times 10^{-11} \text{ lbm/s}}$$

which corresponds to 0.00278 lbm per year.

(b) Noting that  $\dot{N}_B = -\dot{N}_A$  during an equimolar counterdiffusion process, the molar flow rate of air into the helium pipeline is equal to the molar flow rate of helium. Thus the mass flow rate of air into the pipeline is

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-2.204 \times 10^{-11} \text{ lbmol/s})(29 \text{ lbm/lbmol}) = \mathbf{-6.392 \times 10^{-10} \text{ lbm/s}}$$

The mass fraction of air in helium pipeline is

$$w_{\text{air}} = \frac{|\dot{m}_{\text{air}}|}{\dot{m}_{\text{total}}} = \frac{6.392 \times 10^{-10} \text{ lbm/s}}{(5 - 8.816 \times 10^{-11} + 6.392 \times 10^{-10}) \text{ lbm/s}} = 1.278 \times 10^{-10} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(c) The net mass flow rate through the tube is

$$\dot{m}_{\text{net}} = \dot{m}_{\text{helium}} + \dot{m}_{\text{air}} = 8.816 \times 10^{-11} - 6.392 \times 10^{-10} = -5.510 \times 10^{-10} \text{ lbm/s}$$

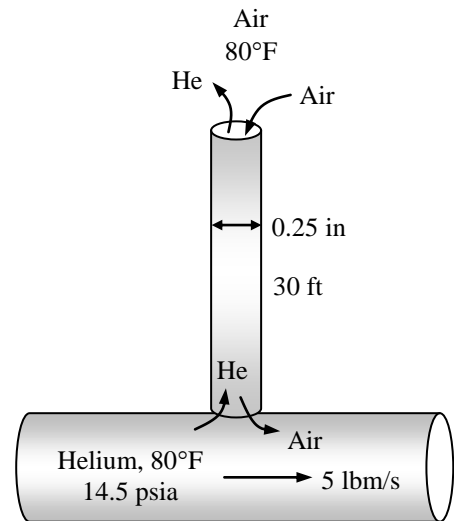
The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at  $x = 0$  can simply be taken to be the density of helium which is

$$\rho \cong \rho_{\text{helium}} = \frac{P}{RT} = \frac{14.5 \text{ psia}}{(2.681 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(540 \text{ R})} = 0.01002 \text{ lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot{m}_{\text{net}}}{\rho A} = \frac{-5.510 \times 10^{-10} \text{ lbm/s}}{(0.01002 \text{ lbm/ft}^3)(3.409 \times 10^{-4} \text{ ft}^2)} = \mathbf{-1.61 \times 10^{-4} \text{ ft/s}}$$

**Discussion** This flow rate is difficult to measure by even the most sensitive velocity measurement devices. The negative sign indicates flow in the negative  $x$  direction (towards the pipeline).





**14-110E** The pressure in a carbon dioxide pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of carbon dioxide and air, and the net flow velocity at the bottom of the tube are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Carbon dioxide and atmospheric air are ideal gases. 3 No chemical reactions occur in the tube. 4 Air concentration in the pipeline and carbon dioxide concentration in the atmosphere are negligible so that the mole fraction of the carbon dioxide is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

**Properties** The diffusion coefficient of carbon dioxide in air (or air in carbon dioxide) at normal atmospheric conditions is  $D_{AB} = 1.72 \times 10^{-4} \text{ ft}^2/\text{s}$  (Table 14-2). The molar mass of carbon dioxide is  $M = 44 \text{ lbm/lbmol}$ , and the molar mass of air is  $29 \text{ lbm/lbmol}$  (Table A-1E).

**Analysis** This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.

(a) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2 / 4 = \pi (0.25 / 12 \text{ ft})^2 / 4 = 3.409 \times 10^{-4} \text{ ft}^2$$

Noting that the pressure of carbon dioxide is 14.5 psia at the bottom of the tube ( $x = 0$ ) and 0 at the top ( $x = L$ ), its molar flow rate is determined from Eq. 14-64 to be

$$\begin{aligned} \dot{N}_{\text{CO}_2} &= \dot{N}_{\text{diff,A}} = \frac{D_{AB} A}{R_u T} \frac{P_{A,0} - P_{A,L}}{L} \\ &= \frac{(1.72 \times 10^{-4} \text{ ft}^2/\text{s})(3.409 \times 10^{-4} \text{ ft}^2)}{(10.73 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R})(540 \text{ R})} \frac{(14.5 - 0) \text{ psia}}{30 \text{ ft}} \\ &= 4.891 \times 10^{-12} \text{ lbmol/s} \end{aligned}$$

Therefore, the mass flow rate of carbon dioxide through the tube is

$$\dot{m}_{\text{CO}_2} = (\dot{N}M)_{\text{CO}_2} = (4.891 \times 10^{-12} \text{ lbmol/s})(44 \text{ lbm/lbmol}) = \mathbf{2.152 \times 10^{-10} \text{ lbm/s}}$$

which corresponds to 0.00679 lbm per year.

(b) Noting that  $\dot{N}_B = -\dot{N}_A$  during an equimolar counter diffusion process, the molar flow rate of air into the  $\text{CO}_2$  pipeline is equal to the molar flow rate of  $\text{CO}_2$ . Thus the mass flow rate of air into the pipeline is

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-4.891 \times 10^{-12} \text{ lbmol/s})(29 \text{ lbm/lbmol}) = \mathbf{-1.418 \times 10^{-10} \text{ lbm/s}}$$

The mass fraction of air in carbon dioxide pipeline is

$$w_{\text{air}} = \frac{|\dot{m}_{\text{air}}|}{\dot{m}_{\text{total}}} = \frac{1.418 \times 10^{-10} \text{ lbm/s}}{(5 + 1.418 \times 10^{-10} - 2.152 \times 10^{-10}) \text{ lbm/s}} = 2.836 \times 10^{-11} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(c) The net mass flow rate through the tube is

$$\dot{m}_{\text{net}} = \dot{m}_{\text{CO}_2} + \dot{m}_{\text{air}} = 2.152 \times 10^{-10} - 1.418 \times 10^{-10} = 7.34 \times 10^{-11} \text{ lbm/s}$$

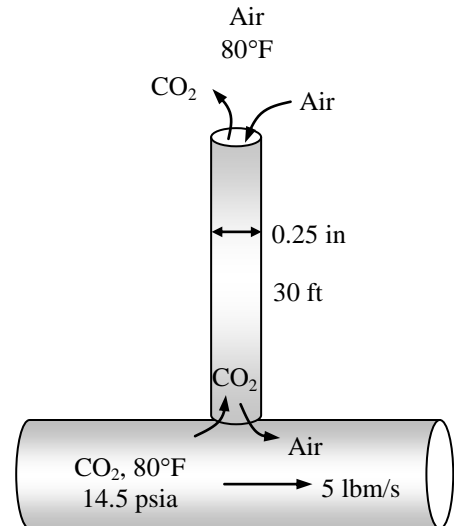
The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at  $x = 0$  can simply be taken to be the density of carbon dioxide which is

$$\rho \cong \rho_{\text{CO}_2} = \frac{P}{RT} = \frac{14.5 \text{ psia}}{(0.2438 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(540 \text{ R})} = 0.110 \text{ lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot{m}_{\text{net}}}{\rho A} = \frac{7.34 \times 10^{-11} \text{ lbm/s}}{(0.110 \text{ lbm/ft}^3)(3.409 \times 10^{-4} \text{ ft}^2)} = \mathbf{1.96 \times 10^{-6} \text{ ft/s}}$$

**Discussion** This flow rate is difficult to measure by even the most sensitive velocity measurement devices.



## Mass Convection

**14-111C** The region of the fluid near the surface in which concentration gradients exist is called the **concentration boundary layer**. In **flow over a plate**, the thickness of the concentration boundary layer  $\delta_c$  for a species  $A$  at a specified location on the surface is defined as the normal distance  $y$  from the surface at which

$$\frac{\rho_{A,s} - \rho_A}{\rho_{A,s} - \rho_\infty} = 0.99$$

where  $\rho_{A,s}$  and  $\rho_{A,\infty}$  are the densities of species  $A$  at the surface (on the fluid side) and the free stream, respectively.

**14-112C** The dimensionless **Schmidt number** is defined as the ratio of momentum diffusivity to mass diffusivity  $Sc = \nu / D_{AB}$ , and it represents the relative magnitudes of momentum and mass diffusion at molecular level in the velocity and concentration boundary layers, respectively. The Schmidt number corresponds to the *Prandtl number* in heat transfer. A Schmidt number of *unity* indicates that momentum and mass transfer by diffusion are comparable, and velocity and concentration boundary layers almost coincide with each other.

**14-113C** The dimensionless **Lewis number** is defined as the ratio of thermal diffusivity to mass diffusivity ( $Le = \alpha / D_{AB}$ ), and it represents the relative magnitudes of heat and mass diffusion at molecular level in the thermal and concentration boundary layers, respectively. A Lewis number of unity indicates that heat and mass diffuse at the same rate, and the thermal and concentration boundary layers coincide.

**14-114C** The normalized velocity, thermal, and concentration boundary layers coincide during flow over a plate when the molecular diffusivity of momentum, heat, and mass are identical. That is,  $\nu = \alpha = D_{AB}$  or  $Pr = Sc = Le = 1$ .

**14-115C** Mass convection is expressed on a mass basis in an analogous manner to heat transfer as

$$\dot{m}_{\text{conv}} = h_{\text{mass}} A_s (\rho_{A,s} - \rho_{A,\infty})$$

where  $h_{\text{mass}}$  is the average mass transfer coefficient in m/s,  $A_s$  is the surface area in  $\text{m}^2$ , and  $\rho_{A,s}$  and  $\rho_{A,\infty}$  are the densities of species  $A$  at the surface (on the fluid side) and the free stream, respectively.

**14-116C** The dimensionless **Sherwood number** is defined as  $Sh = h_{\text{mass}} L / D_{AB}$  where  $L$  is the characteristic length,  $h_{\text{mass}}$  is the mass transfer coefficient and  $D_{AB}$  is the mass diffusivity. The Sherwood number represents the effectiveness of mass convection at the surface, and serves as the dimensionless mass transfer coefficient. The Sherwood number corresponds to the *Nusselt number* in heat transfer. A Sherwood number of unity for a plain fluid layer indicates mass transfer by pure diffusion in a fluid.

**14-117C** Yes, the Grashof number evaluated using density difference instead of temperature difference can also be used in natural convection heat transfer calculations. In natural convection heat transfer, the term  $\Delta\rho/\rho$  is replaced by  $\beta\Delta T$  for convenience in calculations.

**14-118C** The relation  $f/2 = \text{Nu} = \text{Sh}$  is known as the **Reynolds analogy**. It is valid under the conditions that the Prandtl, Schmidt, and Lewis numbers are equal to unity. That is,  $\nu = \alpha = D_{AB}$  or  $\text{Pr} = \text{Sc} = \text{Le} = 1$ . Reynolds analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

**14-119C** The relation  $f/2 = \text{St} \text{Pr}^{2/3} = \text{St}_{\text{mass}} \text{Sc}^{2/3}$  is known as the **Chilton-Colburn analogy**. Here  $\text{St}$  is the Stanton number,  $\text{Pr}$  is the Prandtl number,  $\text{St}_{\text{mass}}$  is the Stanton number in mass transfer, and  $\text{Sc}$  is the Schmidt number. The relation is valid for  $0.6 < \text{Pr} < 60$  and  $0.6 < \text{Sc} < 3000$ . Its importance in engineering is that Chilton-Colburn analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

**14-120C** Using the analogy between heat and mass transfer, the mass transfer coefficient can be determined from the relations for heat transfer coefficient using the **Chilton-Colburn Analogy** expressed as

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left( \frac{\text{Sc}}{\text{Pr}} \right)^{2/3} = \rho c_p \left( \frac{\alpha}{D_{AB}} \right)^{2/3} = \rho c_p \text{Le}^{2/3}$$

Once the heat transfer coefficient  $h_{\text{heat}}$  is available, the mass transfer coefficient  $h_{\text{mass}}$  can be obtained from the relation above by substituting the values of the properties.

**14-121C** The relation  $h_{\text{heat}} = \rho c_p h_{\text{mass}}$  is the result of the Lewis number  $\text{Le} = 1$ , and is known as the **Lewis relation**. It is valid for air-water vapor mixtures in the temperature range encountered in heating and air-conditioning applications. The Lewis relation is commonly used in air-conditioning practice. It asserts that the wet-bulb and adiabatic saturation temperatures of moist air are nearly identical. The Lewis relation can be used for heat and mass transfer in turbulent flow even when the Lewis number is not unity.

**14-122C** A convection mass transfer is referred to as the **low mass flux** when the flow rate of species undergoing mass flow is low relative to the total flow rate of the liquid or gas mixture so that the mass transfer between the fluid and the surface does not affect the *flow velocity*. The evaporation of water into air from lakes, rivers, etc. can be treated as a low mass-flux process since the mass fraction of water vapor in the air in such cases is just a few percent.

**14-123** Carbon dioxide and air are separated by a flat rubber plate. The mass concentration gradient of carbon dioxide at the plate surface on the air side is to be determined.

**Assumptions** **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of CO<sub>2</sub> in the air is low. **2** There are no chemical reactions in the plate that results in the generation or depletion of CO<sub>2</sub>. **3** Mass diffusion of CO<sub>2</sub> through the plate is being convected to the air on the other side of the plate.

**Properties** The molar mass for CO<sub>2</sub> is 44.01 kg/kmol (Table A-1). The binary diffusion coefficient for CO<sub>2</sub> in air at 298 K is  $D_{AB} = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2).

**Analysis** With the CO<sub>2</sub> diffuses through the rubber plate being convected by air on the other side, we can write the mass transfer at the air-side plate surface as

$$\dot{j}_{A,\text{diff}} = \dot{j}_{A,\text{conv}} = \dot{j}_{A,s}$$

$$\dot{j}_{A,s} = -D_{AB} \left. \frac{d\rho_A}{dy} \right|_s = h_{\text{mass}} (\rho_{A,s} - \rho_{A,\infty})$$

Thus, mass concentration gradient at the air-side plate surface is

$$\begin{aligned} \left. \frac{d\rho_A}{dy} \right|_s &= -\frac{\dot{j}_{A,s}}{D_{AB}} = -\frac{\bar{j}_{A,s} M}{D_{AB}} \\ &= -\frac{4.43 \times 10^{-11} \text{ kmol/s} \cdot \text{m}^2}{1.6 \times 10^{-5} \text{ m}^2/\text{s}} (44.01 \text{ kg/kmol}) \\ &= -1.219 \times 10^{-4} \text{ kg/m}^4 \end{aligned}$$

**Discussion** The negative sign of the concentration gradient implies that mass transfers from higher concentration to lower concentration.

**14-124** A film of water on a concrete is undergoing mass convection to air. The mass fraction gradient of water at the surface is to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low. 2 Water is at the same temperature as air.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which  $\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$  and  $\rho = 1.225 \text{ kg/m}^3$  (Table A-15). The saturation pressure of water at 15°C is 1.705 kPa (Table A-9). The mass diffusivity of water vapor in air at 15° (288 K) is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{1 \text{ atm}} = 2.33 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The air at the water surface will be saturated and that the saturation pressure of water at 15°C is 1.705 kPa, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10 (molar mass of air and water is obtained from Table A-1),

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{sat}}{P} \frac{M_A}{M_{air}} = \frac{(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left( \frac{18.015 \text{ kg/kmol}}{28.97 \text{ kg/kmol}} \right) = 0.01046$$

$$w_{A,\infty} = y_{A,\infty} \frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P} \frac{M_A}{M_{air}} = \frac{(0.35)(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left( \frac{18.015 \text{ kg/kmol}}{28.97 \text{ kg/kmol}} \right) = 0.003662$$

The mass transfer conditions on the water surface is given as

$$j_{A,s} = -D_{AB} \left. \frac{dw_A}{dy} \right|_s = h_{\text{mass}} (w_{A,s} - w_{A,\infty})$$

Thus, the mass fraction gradient is

$$\begin{aligned} \left. \frac{dw_A}{dy} \right|_s &= - \frac{h_{\text{mass}} (w_{A,s} - w_{A,\infty})}{D_{AB}} \\ &= - \frac{(0.03 \text{ m/s})(0.01046 - 0.003662)}{2.33 \times 10^{-5} \text{ m}^2/\text{s}} \\ &= \mathbf{-8.75 \text{ m}^{-1}} \end{aligned}$$

**Discussion** Factors affecting the mass fraction gradient at the water surface are the mass transfer convection coefficient, mass diffusivity of water vapor in air, and the difference in concentrations at the surface and at the free stream.

**14-125** The average Reynolds number, Schmidt number, Sherwood number, and friction coefficient for (a) an evaporation process and (b) a sublimation process are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable. 2 Air and naphthalene vapors behave as ideal gases. 3 Process is isothermal.

**Properties** The mass diffusivities are

$$D_{AB} = D_{\text{H}_2\text{O}-\text{Air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(298 \text{ K})^{2.072}}{1 \text{ atm}} = 2.5 \times 10^{-5} \text{ m}^2/\text{s}$$

$$D_{AB} = D_{\text{Naph-Air}} = 0.61 \times 10^{-5} \text{ m}^2/\text{s} \quad (\text{given})$$

The kinematic viscosity of air at 298K = 25°C is  $\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15).

**Analysis** (a) For the evaporation process (water-air), we have

$$\text{Re} = \frac{VL_c}{\nu} = \frac{(2 \text{ m/s})(2 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{2.56 \times 10^5}$$

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{2.5 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{0.625}$$

$$\text{Sh} = \frac{h_{\text{mass}} L_c}{D_{AB}} = \frac{(0.015 \text{ m/s})(2 \text{ m})}{2.5 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{1200}$$

Finally, from the Chilton-Colburn analogy,

$$f = 2\text{St}_{\text{mass}} \text{Sc}^{2/3} = 2 \frac{h_{\text{mass}}}{V} \text{Sc}^{2/3} = 2 \frac{0.015 \text{ m/s}}{2 \text{ m/s}} (0.625)^{2/3} = \mathbf{0.01097}$$

(b) For the sublimation process (naphthalene-air), we have

$$\text{Re} = \frac{VL_c}{\nu} = \frac{(2 \text{ m/s})(2 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{2.56 \times 10^5}$$

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{0.61 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{2.56}$$

$$\text{Sh} = \frac{h_{\text{mass}} L_c}{D_{AB}} = \frac{(0.015 \text{ m/s})(2 \text{ m})}{0.61 \times 10^{-5} \text{ m}^2/\text{s}} = \mathbf{4920}$$

Finally, from the Chilton-Colburn analogy,

$$f = 2\text{St}_{\text{mass}} \text{Sc}^{2/3} = 2 \frac{h_{\text{mass}}}{V} \text{Sc}^{2/3} = 2 \frac{0.015 \text{ m/s}}{2 \text{ m/s}} (2.56)^{2/3} = \mathbf{0.0281}$$

**Discussion** Note that both evaporation and sublimation processes have the same Reynolds number, since in both cases the free stream fluid is air at 298 K and 1 atm.

**14-126** Air is blown over a body covered with a layer of naphthalene, and the rate of sublimation is measured. The heat transfer coefficient under the same flow conditions over the same geometry is to be determined.

**Assumptions** **1** The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases.

**Properties** The molar mass of naphthalene is 128.2 kg/kmol. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, at which  $\rho = 1.184 \text{ kg/m}^3$ ,  $c_p = 1007 \text{ J/kg} \cdot \text{K}$ , and  $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15).

**Analysis** The incoming air is free of naphthalene, and thus the mass fraction of naphthalene at free stream conditions is zero,  $w_{A,\infty} = 0$ . Noting that the vapor pressure of naphthalene at the surface is 11 Pa, the surface mass fraction is determined to be

$$w_{A,s} = \frac{P_{A,s}}{P} \left( \frac{M_A}{M_{air}} \right) = \frac{11 \text{ Pa}}{101,325 \text{ Pa}} \left( \frac{128.2 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 4.8 \times 10^{-4}$$

which confirms that the low mass flux approximation is valid. The rate of evaporation of naphthalene in this case is

$$\dot{m}_{\text{evap}} = \frac{m}{\Delta t} = \frac{0.1 \text{ kg}}{(45 \times 60 \text{ s})} = 3.703 \times 10^{-5} \text{ kg/s}$$

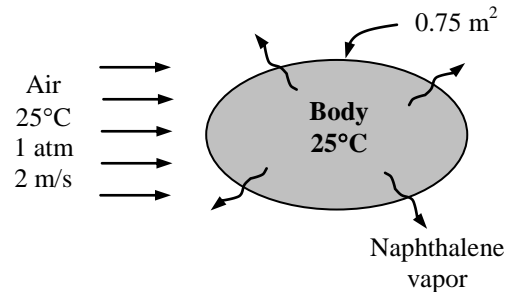
Then the mass convection coefficient becomes

$$h_{\text{mass}} = \frac{\dot{m}}{\rho A (w_{A,s} - w_{A,\infty})} = \frac{3.703 \times 10^{-5} \text{ kg/s}}{(1.184 \text{ kg/m}^3)(0.75 \text{ m}^2)(4.8 \times 10^{-4} - 0)} = 0.0869 \text{ m/s}$$

Using the analogy between heat and mass transfer, the average heat transfer coefficient is determined from Eq. 14-89 to be

$$\begin{aligned} h_{\text{heat}} &= \rho c_p h_{\text{mass}} \left( \frac{\alpha}{D_{AB}} \right)^{2/3} \\ &= (1.184 \text{ kg/m}^3)(1007 \text{ J/kg} \cdot \text{K})(0.0869 \text{ m/s}) \left( \frac{2.141 \times 10^{-5} \text{ m}^2/\text{s}}{0.61 \times 10^{-5} \text{ m}^2/\text{s}} \right)^{2/3} \\ &= 239 \text{ W/m}^2 \cdot ^\circ\text{C} \end{aligned}$$

**Discussion** Naphthalene has been commonly used in heat transfer studies to determine convection heat transfer coefficients because of the convenience it offers.



**14-127** Using a known expression for local convection heat transfer coefficient, the average mass convection coefficient over a plate is to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Water is at the same temperature as air.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, for which  $c_p = 1007 \text{ J/kg} \cdot \text{K}$ ,  $\rho = 1.184 \text{ kg/m}^3$ , and  $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The mass diffusivity of water vapor in air at 298 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O}-\text{Air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(298 \text{ K})^{2.072}}{1 \text{ atm}} = 2.5 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The average convection heat transfer coefficient can be determined for  $L = 1 \text{ m}$  as

$$\begin{aligned} h_{\text{heat}} &= \frac{1}{L} \int_0^L h_x dx = \frac{1}{L} \int_0^L (0.5 + 12x - 0.7x^3) dx \\ &= \frac{1}{L} \left[ 0.5x + 6x^2 - 0.175x^4 \right]_0^L = 0.5 + 6L - 0.175L^3 \\ &= 0.5 + 6(1 \text{ m}) - 0.175(1 \text{ m})^3 = 6.325 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

Applying the Chilton-Colburn analogy,

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left( \frac{\alpha}{D_{AB}} \right)^{2/3} \rightarrow h_{\text{mass}} = \frac{h_{\text{heat}}}{\rho c_p} \left( \frac{D_{AB}}{\alpha} \right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{\text{mass}} = \frac{6.325 \text{ W/m}^2 \cdot \text{K}}{(1.184 \text{ kg/m}^3)(1007 \text{ J/kg} \cdot \text{K})} \left( \frac{2.5 \times 10^{-5}}{2.141 \times 10^{-5}} \right)^{2/3} = 5.88 \times 10^{-3} \text{ m/s}$$

**Discussion** Using the Lewis relation, the average mass convection coefficient can be estimated to be

$$h_{\text{mass}} \cong h_{\text{heat}} / (\rho c_p) = 5.30 \times 10^{-3} \text{ m/s}$$

which is about 10% lower than  $5.88 \times 10^{-3} \text{ m/s}$ .



**14-128** Ethyl alcohol is spread over a flat table where dry air is blowing over it. The average mass transfer coefficient is to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable. 2 The critical Reynolds number for flow over a flat plate is 500,000.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm:  $\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The mass diffusivity of ethyl alcohol in air at 25°C is  $D_{AB} = 1.2 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2).

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(1 \text{ m/s})(1 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 64,020$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{1.2 \times 10^{-5} \text{ m}^2/\text{s}} = 1.3017$$

Therefore, the Sherwood number in this case is determined from Table 14-13 (for laminar flow over a flat plate with  $\text{Sc} > 0.6$ ) to be

$$\text{Sh} = 0.664 \text{Re}^{0.5} \text{Sc}^{1/3} = 0.664(64,020)^{0.5} (1.3017)^{1/3} = 183.44$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(183.44)(1.2 \times 10^{-5} \text{ m}^2/\text{s})}{1 \text{ m}} = \mathbf{0.00220 \text{ m/s}}$$

**Discussion** Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.

**14-129** A wet flat plate is dried by blowing air over it. The mass transfer coefficient is to be determined.

**Assumptions** **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 85 kPa = 85/101.325 = 0.839 atm, for which (Table A-15)

$$\nu = \nu_{1 \text{ atm}} / P(\text{atm}) = (1.47 \times 10^{-5} \text{ m}^2/\text{s}) / 0.839 \text{ atm} = 1.75 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} \\ &= 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{0.839 \text{ atm}} \\ &= 2.78 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(3 \text{ m/s})(2 \text{ m})}{1.75 \times 10^{-5} \text{ m}^2/\text{s}} = 342,857$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

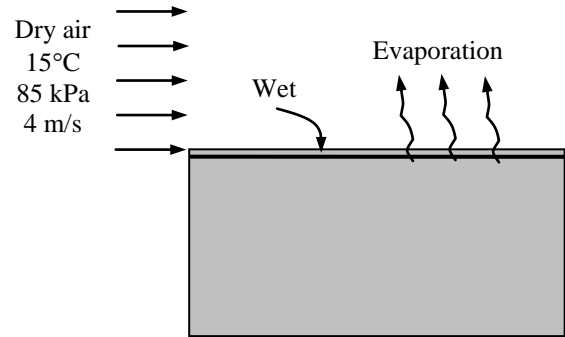
$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.75 \times 10^{-5} \text{ m}^2/\text{s}}{2.78 \times 10^{-5} \text{ m}^2/\text{s}} = 0.629$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}^{0.5} \text{Sc}^{1/3} = 0.664(342,857)^{0.5} (0.629)^{1/3} = 333.1$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{L} = \frac{(333.1)(2.78 \times 10^{-5} \text{ m}^2/\text{s})}{2 \text{ m}} = \mathbf{0.00463 \text{ m/s}}$$



**14-130** A thin slab of solid salt is being dragged through seawater. The mass convection rate of salt being dissolved in seawater is to be determined.

**Assumptions** **1** The analogy between heat and mass transfer is applicable. **2** The critical Reynolds number for flow over a flat plate is 500,000.

**Properties** The relevant properties are given in the problem statement.

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(0.6 \text{ m/s})(0.15 \text{ m})}{1.022 \times 10^{-6} \text{ m}^2/\text{s}} = 88063$$

which is less than 500,000 and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.022 \times 10^{-6} \text{ m}^2/\text{s}}{1.2 \times 10^{-9} \text{ m}^2/\text{s}} = 851.7$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}^{0.5} \text{Sc}^{1/3} = 0.664(88063)^{0.5} (851.7)^{1/3} = 1868$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(1868)(1.2 \times 10^{-9} \text{ m}^2/\text{s})}{0.15 \text{ m}} = 1.494 \times 10^{-5} \text{ m/s}$$

Hence, the mass convection rate of salt being dissolved in the seawater is

$$\begin{aligned} \dot{m}_{\text{conv}} &= h_{\text{mass}} A_s (\rho_{A,s} - \rho_{A,\infty}) \\ &= (1.494 \times 10^{-5} \text{ m/s}) 2(0.15 \times 0.15 \text{ m}^2)(35000 - 31) \text{ kg/m}^3 \\ &= \mathbf{0.0235 \text{ kg/s}} \end{aligned}$$

**Discussion** In the analysis of this problem, the mass convection from the edges of the salt slab is considered negligible. This is a reasonable assumption as the salt slab is thin and the mass convection mainly occurs on the top and bottom surfaces.

**14-131** Wet steel plates are to be dried by blowing air parallel to their surfaces. The rate of evaporation from both sides of a plate is to be determined.

**Assumptions** **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** The plates are far enough from each other so that they can be treated as flat plates. **4** The air is dry so that the amount of moisture in the air is negligible.

**Properties** The molar masses of air and water are  $M = 29$  and  $M = 18$  kg/kmol, respectively (Table A-1).

Because of low mass flux conditions, we can use dry air properties for the mixture.

The properties of the air at 1 atm and at the film temperature of  $(15 + 25) = 20^\circ\text{C}$  are (Table A-15)

$$\nu = 1.516 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\rho = 1.204 \text{ kg/m}^3$$

$$c_p = 1007 \text{ J/kg}\cdot\text{K}$$

$$\text{Pr} = 0.7309$$

The saturation pressure of water at  $15^\circ\text{C}$  is 1.705 kPa (Table A-9). The mass diffusivity of water vapor in air at  $20^\circ\text{C} = 293 \text{ K}$  is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O}-\text{air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(293 \text{ K})^{2.072}}{1 \text{ atm}} = 2.42 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The Reynolds number for flow over the flat plate is

$$\text{Re} = \frac{VL}{\nu} = \frac{(4 \text{ m/s})(0.4 \text{ m})}{1.516 \times 10^{-5} \text{ m}^2/\text{s}} = 105,540$$

which is less than 500,000, and thus the air flow is laminar over the entire plate. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.516 \times 10^{-5} \text{ m}^2/\text{s}}{2.42 \times 10^{-5} \text{ m}^2/\text{s}} = 0.626$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}_L^{0.5} \text{Sc}^{1/3} = 0.664(105,540)^{0.5} (0.626)^{1/3} = 184.5$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(184.5)(2.42 \times 10^{-5} \text{ m}^2/\text{s})}{0.4 \text{ m}} = 0.01116 \text{ m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at  $15^\circ\text{C}$  is 1.705 kPa, the mass fraction of water vapor in the air at the surface of the plate is, from Eq. 14-10,

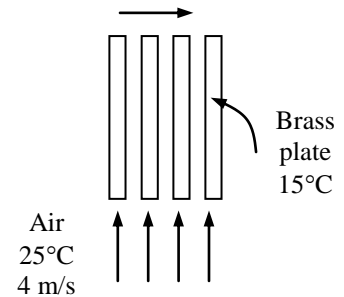
$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left( \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 0.01044$$

and  $w_{A,\infty} = 0$

Then the rate of mass transfer to the air becomes

$$\begin{aligned} \dot{m}_{\text{evap.}} &= h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}) \\ &= (0.01116 \text{ m/s})(1.204 \text{ kg/m}^3)(2 \times 0.4 \text{ m} \times 0.4 \text{ m})(0.01044 - 0) \\ &= 4.49 \times 10^{-5} \text{ kg/s} \end{aligned}$$

**Discussion** This is the upper limit for the evaporation rate since the air is assumed to be completely dry.



**14-132E** Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as the air.

**Properties** The molar masses of air and water are  $M = 29$  and  $M = 18$  lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm, for which  $\nu = 1.697 \times 10^{-4}$  ft<sup>2</sup>/s, and  $\rho = 0.0735$  lbm/ft<sup>3</sup> (Table A-15E). The saturation pressure of water at 80°F is 0.5073 psia, and the heat of vaporization is 1048 Btu/lbm. The mass diffusivity of water vapor in air at 80°F = 540 R = 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(300\text{K})^{2.072}}{1\text{atm}} = 2.54 \times 10^{-5} \text{ m}^2/\text{s} = 2.734 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** The Reynolds number for flow over the free surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10 \text{ ft/s})(15/12 \text{ ft})}{1.697 \times 10^{-4} \text{ ft}^2/\text{s}} = 73,660$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.697 \times 10^{-4} \text{ ft}^2/\text{s}}{2.734 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.6207$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}_L^{0.5} \text{Sc}^{1/3} = 0.664(73,660)^{0.5} (0.6207)^{1/3} = 153.7$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(153.7)(2.734 \times 10^{-4} \text{ ft}^2/\text{s})}{15/12 \text{ ft}} = 0.0336 \text{ ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 80°F is 0.5073 psia (= 0.0345 atm), the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(0.3)(0.5073 \text{ psia})}{14.7 \text{ psia}} \left( \frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}} \right) = 0.00643$$

$$w_{A,\infty} = y_{A,\infty} \frac{M_A}{M_{\text{air}}} = \frac{\phi P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(1.0)(0.5073 \text{ psia})}{14.7 \text{ psia}} \left( \frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}} \right) = 0.02142$$

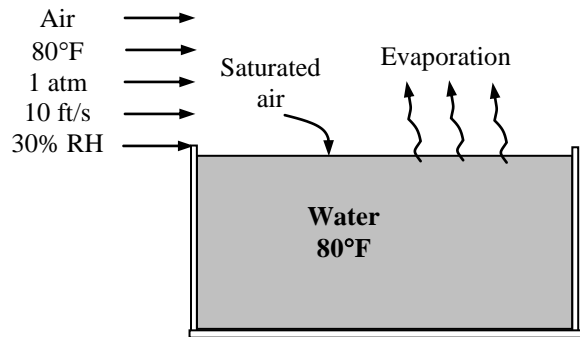
Then the rate of mass transfer to the air becomes

$$\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A_s (w_{A,s} - w_{A,\infty}) = (0.0336 \text{ ft/s})(0.074 \text{ lbm/ft}^3)(15/12 \text{ ft})^2 (0.02142 - 0.00642) = 5.83 \times 10^{-5} \text{ lbm/s}$$

Noting that the latent heat of vaporization of water at 80°F is  $h_{fg} = 1048$  Btu/lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{\text{evap}} h_{fg} = (5.83 \times 10^{-5} \text{ lbm/s})(1048 \text{ Btu/lbm}) = 0.0611 \text{ Btu/s} = 220 \text{ Btu/h}$$

**Discussion** If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.



**14-133E** Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

**Properties** The molar masses of air and water are  $M = 29$  and  $M = 18$  lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 60°F and 1 atm, for which  $\nu = 1.588 \times 10^{-4}$  ft<sup>2</sup>/s, and  $\rho = 0.07633$  lbm/ft<sup>3</sup> (Table A-15E). The saturation pressure of water at 60°F is 0.2563 psia, and the heat of vaporization is 1060 Btu/lbm. The mass diffusivity of water vapor in air at 60°F = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288.9 \text{ K})^{2.072}}{1 \text{ atm}} = 2.35 \times 10^{-5} \text{ m}^2/\text{s} = 2.53 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** The Reynolds number for flow over the free surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10 \text{ ft/s})(15/12 \text{ ft})}{1.588 \times 10^{-4} \text{ ft}^2/\text{s}} = 78,715$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.588 \times 10^{-4} \text{ ft}^2/\text{s}}{2.53 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.6277$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}_L^{0.5} \text{Sc}^{1/3} = 0.664(78,715)^{0.5} (0.6277)^{1/3} = 159.5$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{L} = \frac{(159.5)(2.53 \times 10^{-4} \text{ ft}^2/\text{s})}{15/12 \text{ ft}} = 0.0323 \text{ ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 60°F is 0.2563 psia, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(0.3)(0.2563 \text{ psia})}{14.7 \text{ psia}} \left( \frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}} \right) = 0.00325$$

$$w_{A,\infty} = y_{A,\infty} \frac{M_A}{M_{\text{air}}} = \frac{\phi P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(1.0)(0.2563 \text{ psia})}{14.7 \text{ psia}} \left( \frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}} \right) = 0.01082$$

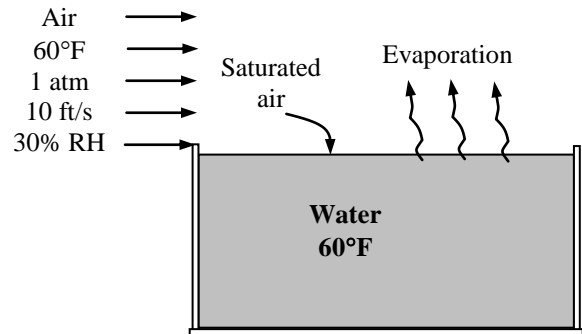
Then the rate of mass transfer to the air becomes

$$\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}) = (0.0323 \text{ ft/s}) (0.07633 \text{ lbm/ft}^3) (15/12 \text{ ft})^2 (0.01082 - 0.00325) = 2.82 \times 10^{-5} \text{ lbm/s}$$

Noting that the latent heat of vaporization of water at 60°F is  $h_{fg} = 1060$  Btu/lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{\text{evap}} h_{fg} = (2.82 \times 10^{-5} \text{ lbm/s})(1060 \text{ Btu/lbm}) = 0.0299 \text{ Btu/s} = 108 \text{ Btu/h}$$

**Discussion** If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.



**14-134** A wet concrete patio is to be dried by winds. The time it takes for the patio to dry is to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which  $\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$  and  $\rho = 1.225 \text{ kg/m}^3$  (Table A-15). The saturation pressure of water at 15°C is 1.705 kPa. The mass diffusivity of water vapor in air at 15°C = 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{1 \text{ atm}} = 2.33 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(50 \text{ km/h})(5 \text{ m})}{1.47 \times 10^{-5} \text{ m}^2/\text{s}} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 4.724 \times 10^6$$

which is more than 500,000, and thus the flow is turbulent over most of the surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47 \times 10^{-5} \text{ m}^2/\text{s}}{2.33 \times 10^{-5} \text{ m}^2/\text{s}} = 0.631$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\begin{aligned} \text{Sh} &= 0.037 \text{Re}^{0.8} \text{Sc}^{1/3} \\ &= 0.037(4.724 \times 10^6)^{0.8} (0.631)^{1/3} = 6934 \end{aligned}$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(6934)(2.33 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = 0.03231 \text{ m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 15°C is 1.705 kPa, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left( \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 0.01044$$

$$w_{A,\infty} = y_{A,\infty} \frac{M_A}{M_{\text{air}}} = \frac{\phi P_{\text{sat}}}{P} \frac{M_A}{M_{\text{air}}} = \frac{(0.35)(1.705 \text{ kPa})}{101.325 \text{ kPa}} \left( \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \right) = 0.003655$$

Then the rate of mass transfer to the air becomes

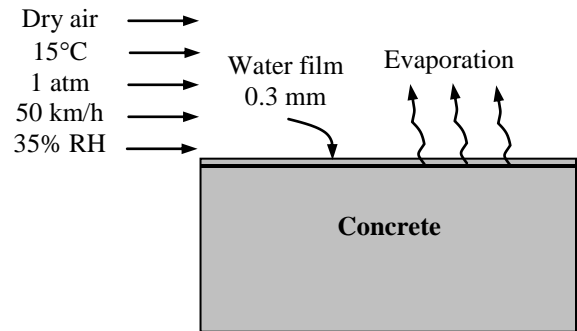
$$\begin{aligned} \dot{m}_{\text{evap.}} &= h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}) \\ &= (0.03231 \text{ m/s})(1.225 \text{ kg/m}^3)(5 \text{ m} \times 5 \text{ m})(0.01044 - 0.003655) \\ &= 0.006714 \text{ kg/s} \end{aligned}$$


The total mass of water on the concrete patio is

$$m_{\text{water}} = \rho V = (1000 \text{ kg/m}^3)(5 \text{ m} \times 5 \text{ m} \times 0.3 \times 10^{-3} \text{ m}) = 7.5 \text{ kg}$$

Then the time required to evaporate the water on the concrete patio becomes

$$\Delta t = \frac{m_{\text{water}}}{\dot{m}_{\text{evap}}} = \frac{7.5 \text{ kg}}{0.006714 \text{ kg/s}} = 1117 \text{ s} = \mathbf{18.6 \text{ min}}$$



**14-135**  Liquid benzene is spread over a highway where wind is blowing over it. The mass transfer rate by convection is to be determined whether or not residents in the vicinity should be evacuated.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable. 2 The critical Reynolds number for flow over a flat plate is 500,000.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm:  $\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $\rho = 1.184 \text{ kg/m}^3$  (Table A-15), for air:  $M = 28.97 \text{ kg/kmol}$  (Table A-1). The molar mass of benzene is given as  $M = 78.11 \text{ kg/kmol}$ , and the mass diffusivity of benzene in air at 25°C is  $D_{AB} = 0.88 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2).

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10 \text{ m/s})(10 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 6.402 \times 10^6$$

which is greater than 500,000, and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{0.88 \times 10^{-5} \text{ m}^2/\text{s}} = 1.775$$

Therefore, the Sherwood number in this case is determined from Table 14-13 (for turbulent flow over a flat plate with  $\text{Sc} > 0.6$ ) to be

$$\text{Sh} = 0.037 \text{Re}^{0.8} \text{Sc}^{1/3} = 0.037(6.402 \times 10^6)^{0.8} (1.775)^{1/3} = 12483$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(12483)(0.88 \times 10^{-5} \text{ m}^2/\text{s})}{10 \text{ m}} = 0.01099 \text{ m/s}$$

At the liquid benzene surface, the mass fraction of benzene vapor in air is

$$w_{A,s} = y_{A,s} \frac{M_A}{M} = \frac{P_{A,s}}{P} \frac{M_A}{M_{\text{air}}} = \left( \frac{10 \text{ kPa}}{101.325 \text{ kPa}} \right) \left( \frac{78.11 \text{ kg/kmol}}{28.97 \text{ kg/kmol}} \right) = 0.2661$$

At the free stream, the mass fraction of benzene vapor is zero

$$w_{A,\infty} = 0$$

Thus, the mass transfer rate of benzene to the air by convection is

$$\begin{aligned} \dot{m} &= h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}) \\ &= (0.01099 \text{ m/s})(1.184 \text{ kg/m}^3) \frac{\pi}{4} (10 \text{ m})^2 (0.2661 - 0) \\ &= 0.272 \text{ kg/s} \\ &= \mathbf{979 \text{ kg/h}} > 500 \text{ kg/h} \end{aligned}$$

**Discussion** The mass transfer rate of benzene to the air is estimated to be 979 kg/h, which is greater than 500 kg/hr. Since the mass transfer rate of benzene is greater than the safe level, the residents should be evacuated.



**14-136** A wet flat plate is dried by blowing air over it. The mass transfer coefficient is to be determined.

**Assumptions** **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 40°C and 1 atm, for which (Table A-15)

$$\nu = 1.702 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The mass diffusivity of water vapor in air at 313 K is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} \\ &= 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(313 \text{ K})^{2.072}}{1 \text{ atm}} \\ &= 2.77 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(2.5 \text{ m/s})(5 \text{ m})}{1.702 \times 10^{-5} \text{ m}^2/\text{s}} = 734,430$$

which is greater than 500,000, and thus we have combined laminar and turbulent flow. The Schmidt number in this case is

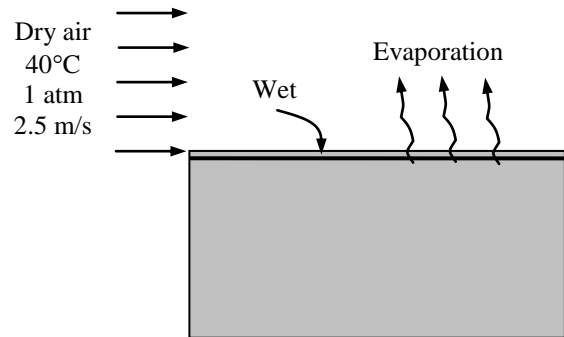
$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.702 \times 10^{-5} \text{ m}^2/\text{s}}{2.77 \times 10^{-5} \text{ m}^2/\text{s}} = 0.614$$

Therefore, the Sherwood number in this case is determined using the analogy between the heat and mass transfer to be

$$\text{Sh} = (0.037 \text{Re}^{0.8} - 871) \text{Sc}^{1/3} = (0.037 \times 734,430^{0.8} - 871)(0.614)^{1/3} = 809.8$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(809.8)(2.77 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = \mathbf{0.00449 \text{ m/s}}$$



**14-137E** A spherical naphthalene ball is suspended in a room where it is subjected to forced air flow. The average mass transfer coefficient between the naphthalene and the air is to be determined.

**Assumptions 1** The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases. **3** Both the ball and the room are at the same temperature.

**Properties** The Schmidt number of naphthalene in air at room temperature is given to be 2.35. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm from Table A-15E,

$$k = 0.01481 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

$$\mu = 1.247 \times 10^{-5} \text{ lbm/ft} \cdot \text{s}$$

$$\nu = 1.697 \times 10^{-4} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.7290$$

**Analysis** Noting that the Schmidt number for naphthalene in air is 2.35, the mass diffusivity of naphthalene in air is determined from

$$\text{Sc} = \frac{\nu}{D_{AB}} \longrightarrow D_{AB} = \frac{\nu}{\text{Sc}} = \frac{1.697 \times 10^{-4} \text{ ft}^2/\text{s}}{2.35} = 7.22 \times 10^{-5} \text{ ft}^2/\text{s}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(15 \text{ ft/s})(2/12 \text{ ft})}{(1.697 \times 10^{-4} \text{ ft}^2/\text{s})} = 14,732$$

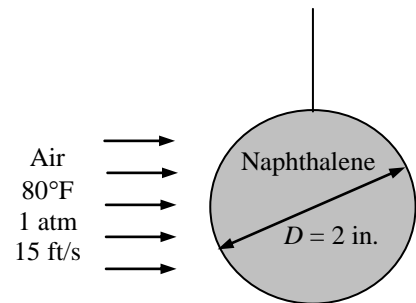
Noting that  $\mu_\infty = \mu_s$  for air in this case since the air and the ball are assumed to be at the same temperature, the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\begin{aligned} \text{Sh} &= \frac{h_{\text{mass}} D}{D_{AB}} = 2 + \left[ 0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3} \right] \text{Sc}^{0.4} \left( \frac{\mu_\infty}{\mu_s} \right)^{1/4} \\ &= 2 + \left[ 0.4(14,732)^{1/2} + 0.06(14,732)^{2/3} \right] (2.35)^{0.4} \\ &= 121 \end{aligned}$$

Then the mass transfer coefficient becomes

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(121)(7.22 \times 10^{-5} \text{ ft}^2/\text{s})}{(2/12) \text{ ft}} = \mathbf{0.0524 \text{ ft/s}}$$

**Discussion** Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.



**14-138** A raindrop is falling freely in atmospheric air. The terminal velocity of the raindrop at which the drag force equals the weight of the drop and the average mass transfer coefficient are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). 2 The raindrop is spherical in shape. 3 The reduction in the diameter of the raindrop due to evaporation when the terminal velocity is reached is negligible.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture. The properties of air at 1 atm and the free-stream temperature of 25°C (and the dynamic viscosity at the surface temperature of 9°C) are (Table A-15)

$$\begin{aligned}\rho &= 1.184 \text{ kg/m}^3 & \mu_\infty &= 1.849 \times 10^{-5} \text{ kg/m}\cdot\text{s} \\ \nu &= 1.562 \times 10^{-5} \text{ m}^2/\text{s} & \mu_{s, @ 9^\circ\text{C}} &= 1.759 \times 10^{-5} \text{ kg/m}\cdot\text{s}\end{aligned}$$

At 1 atm and the film temperature of  $(25+9)/2 = 17^\circ\text{C} = 290 \text{ K}$ , the kinematic viscosity of air is, from Table A-15,  $\nu = 1.488 \times 10^{-5} \text{ m}^2/\text{s}$ , while the mass diffusivity of water vapor in air is, Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(290 \text{ K})^{2.072}}{1 \text{ atm}} = 2.37 \times 10^{-5} \text{ m}^2/\text{s}$$

**Analysis** The weight of the raindrop before any evaporation occurs is

$$F_D = mg = \rho V g = (1000 \text{ kg/m}^3) \left[ \frac{\pi (0.003 \text{ m})^3}{6} \right] (9.8 \text{ m/s}^2) = 1.38 \times 10^{-4} \text{ N}$$

The drag force is determined from  $F_D = C_D A_N \frac{\rho u_\infty^2}{2}$  where drag coefficient  $C_D$  is to be determined using Fig. 10-20 which requires the Reynolds number. Since we do not know the velocity we cannot determine the Reynolds number. Therefore, the solution requires a trial-error approach. We choose a velocity and perform calculations to obtain the drag force. After a couple trial we choose a velocity of 8 m/s. Then the Reynolds number becomes

$$\text{Re} = \frac{VD}{\nu} = \frac{(8 \text{ m/s})(0.003 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 1536$$

The corresponding drag coefficient from Fig. 7-17 is 0.5. Then,

$$F_D = C_D A_N \frac{\rho u_\infty^2}{2} = (0.5) \left[ \frac{\pi (0.003 \text{ m})^2}{4} \right] \frac{(1.184 \text{ kg/m}^3)(8 \text{ m/s})^2}{2} = 1.34 \times 10^{-4}$$

which is sufficiently close to the value calculated before. Therefore, the terminal velocity of the raindrop is  $V = 8 \text{ m/s}$ . The Schmidt number is

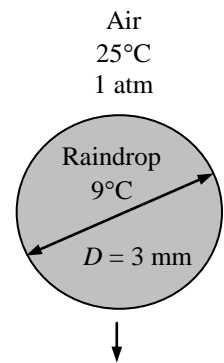
$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.488 \times 10^{-5} \text{ m}^2/\text{s}}{2.37 \times 10^{-5} \text{ m}^2/\text{s}} = 0.628$$

Then the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\begin{aligned}\text{Sh} &= \frac{h_{\text{mass}} D}{D_{AB}} = 2 + \left[ 0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3} \right] \text{Sc}^{0.4} \left( \frac{\mu_\infty}{\mu_s} \right)^{1/4} \\ &= 2 + \left[ 0.4(1536)^{1/2} + 0.06(1536)^{2/3} \right] (0.628)^{0.4} \left( \frac{1.849 \times 10^{-5}}{1.759 \times 10^{-5}} \right)^{1/4} = 21.9\end{aligned}$$

Then the mass transfer coefficient becomes

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(21.9)(2.37 \times 10^{-5} \text{ m}^2/\text{s})}{0.003 \text{ m}} = \mathbf{0.173 \text{ m/s}}$$



**14-139E** The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 540 R). 2 The flow is fully developed.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 540 R and 1 atm, for which  $\nu = 1.697 \times 10^{-4} \text{ ft}^2/\text{s}$  (Table A-15E). The mass diffusivity of water vapor in air at 540 R is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(540/1.8)^{2.072}}{1} = 2.54 \times 10^{-5} \text{ m}^2/\text{s} \\ &= 2.73 \times 10^{-4} \text{ ft}^2/\text{s} \end{aligned}$$

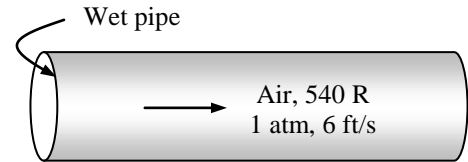
The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6 \text{ ft/s})(0.7/12 \text{ ft})}{1.697 \times 10^{-4} \text{ ft}^2/\text{s}} = 2062$$

which is less than 2300 and thus the flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are  $\text{Nu} = \text{Sh} = 3.66$ . Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(3.66)(2.73 \times 10^{-4} \text{ ft}^2/\text{s})}{0.7/12 \text{ ft}} = \mathbf{0.017 \text{ ft/s}}$$

**Discussion** The mass transfer rate (or the evaporation rate) in this case can be determined by defining logarithmic mean concentration difference in an analogous manner to the logarithmic mean temperature difference.



**14-140** The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

**Assumptions** **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which  $\nu = 1.47 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(288 \text{ K})^{2.072}}{1} = 2.332 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(3 \text{ m/s})(0.15 \text{ m})}{1.47 \times 10^{-5} \text{ m}^2/\text{s}} = 30,612$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

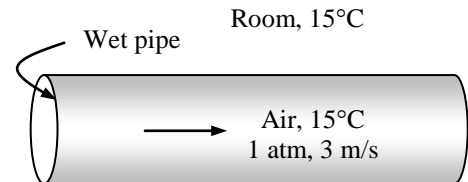
$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47 \times 10^{-5} \text{ m}^2/\text{s}}{2.332 \times 10^{-5} \text{ m}^2/\text{s}} = 0.6302$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023 \text{Re}^{0.8} \text{Sc}^{0.4} = 0.023(30,612)^{0.8} (0.6302)^{0.4} = 74.17$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(74.17)(2.332 \times 10^{-5} \text{ m}^2/\text{s})}{0.15 \text{ m}} = \mathbf{0.0115 \text{ m/s}}$$





**14-141** Prob. 14-140 is reconsidered. The mass transfer coefficient as a function of the air velocity is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

D=0.15 [m]

L=10 [m]

P=101.3 [kPa]

T=(15+273) [K]

Vel=3 [m/s]

"PROPERTIES"

Fluid\$='air'

rho=Density(Fluid\$, T=T, P=P)

mu=Viscosity(Fluid\$, T=T)

nu=mu/rho

D\_AB=1.87E-10\*T^2.072/(P\*Convert(kPa, atm)) "from the text"

"ANALYSIS"

Re=Vel\*D/nu

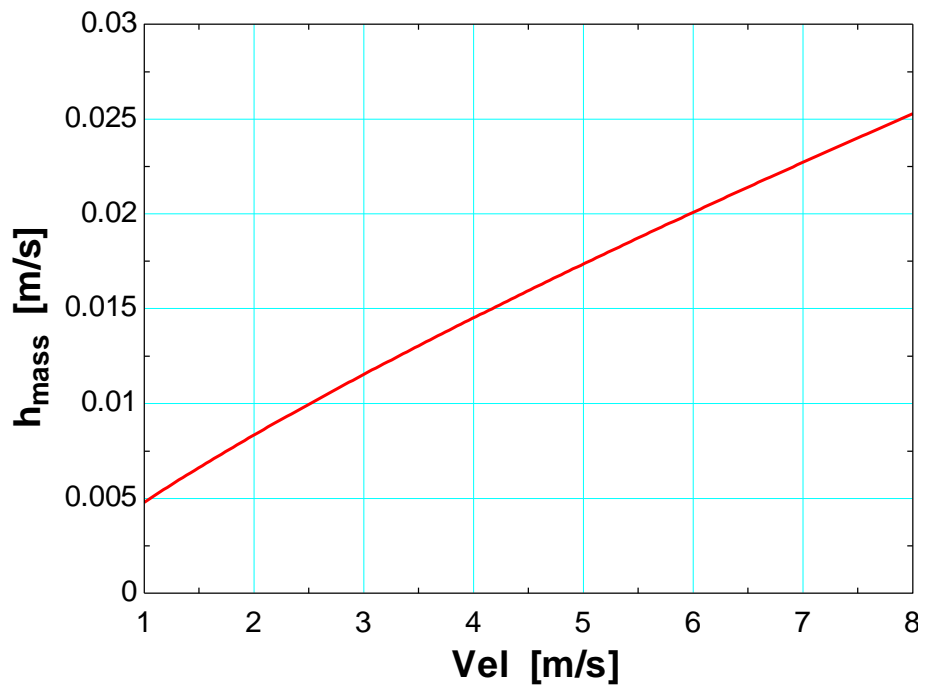
"Re is calculated to be greater than 10,000, and thus the flow is turbulent."

Sc=nu/D\_AB

Sh=0.023\*Re^0.8\*Sc^0.4

h\_mass=Sh\*D\_AB/D

Vel [m/s]	h <sub>mass</sub> [m/s]
1	0.004789
1.5	0.006624
2	0.008339
2.5	0.009968
3	0.01153
3.5	0.01305
4	0.01452
4.5	0.01595
5	0.01736
5.5	0.01873
6	0.02008
6.5	0.02141
7	0.02272
7.5	0.02401
8	0.02528



## Simultaneous Heat and Mass Transfer

**14-142C** In steady operation, the mass transfer process does not have to involve heat transfer. However, a mass transfer process that involves phase change (evaporation, sublimation, condensation, melting etc.) must involve heat transfer. For example, the evaporation of water from a lake into air (mass transfer) requires the transfer of latent heat of water at a specified temperature to the liquid water at the surface (heat transfer).

**14-143C** During evaporation from a water body to air, the latent heat of vaporization will be equal to *convection* heat transfer from the air when *conduction* from the lower parts of the water body to the surface is negligible, and temperature of the surrounding surfaces is at about the temperature of the water surface so that the *radiation* heat transfer is negligible.

**14-144C** It is possible for a shallow body of water to freeze during a cool and dry night even when the ambient air and surrounding surface temperatures never drop to 0°C. This is because when the air is not saturated ( $\phi < 100$  percent), there will be a difference between the concentration of water vapor at the water-air interface (which is always saturated) and some distance above it. Concentration difference is the driving force for mass transfer, and thus this concentration difference will drive the water into the air. But the water must vaporize first, and it must absorb the latent heat of vaporization from the water. The temperature of water near the surface must drop as a result of the sensible heat loss, possibly below the freezing point.

**14-145E** In a hot summer day, a bottle of drink is to be cooled by wrapping it in a wet cloth, and blowing air to it. The temperature of the drink in the bottle when steady conditions are reached is to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2$  which cannot be determined at this point because of the unknown surface temperature  $T_s$ . We know that  $T_s < T_\infty$  and, for the purpose of property evaluation, we take  $T_s$  to be 60°F. Then the properties of water at 60°F and the properties of dry air at the average temperature of  $(60+80)/2 = 70^\circ\text{F}$  and 1 atm are (Tables A-9E and A-15E)

Water at 60°F:  $h_{fg} = 1060 \text{ Btu/lbm}$ ,  $P_v = 0.2563 \text{ psia}$ . Also, at 80°F,  $P_{\text{sat}@80^\circ\text{F}} = 0.5073 \text{ psia}$

Dry air at 70°F:  $c_p = 0.24 \text{ Btu/lbm}\cdot^\circ\text{F}$ ,  $\alpha = 0.8093 \text{ ft}^2/\text{h} = 2.25 \times 10^{-4} \text{ ft}^2/\text{s}$

Also, the molar masses of water and air are 18 and 29 lbm/lbmol, respectively (Table A-1E), and the mass diffusivity of water vapor in air at 70°F ( $= 294.4 \text{ K}$ ) is

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(294.4 \text{ K})^{2.072}}{1 \text{ atm}} = 2.44 \times 10^{-5} \text{ m}^2/\text{s} = 2.63 \times 10^{-4} \text{ ft}^2/\text{s}$$

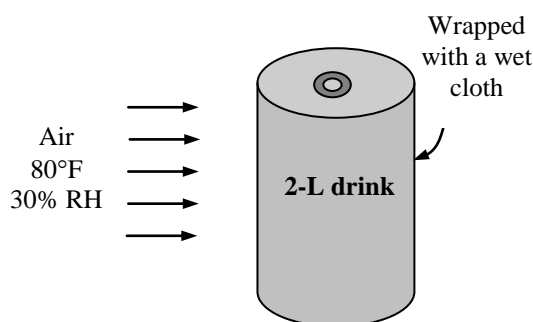
**Analysis** The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as

$$T_s = T_\infty - \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.25 \times 10^{-4} \text{ ft}^2/\text{s}}{2.63 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.856$$

Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.



The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.2563 psia). The vapor pressure of air far from the surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.3) P_{\text{sat}@80^\circ\text{F}} = (0.3)(0.5073 \text{ psia}) = 0.152 \text{ psia}$$

Noting that the atmospheric pressure is 1 atm = 14.7 psia, substituting the known quantities gives

$$T_s = 80^\circ\text{F} - \frac{1060 \text{ Btu/lbm}}{(0.24 \text{ Btu/lbm}\cdot^\circ\text{F})(0.856)^{2/3}} \left( \frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}} \right) \frac{(0.2563 - 0.152) \text{ psia}}{14.7 \text{ psia}} = \mathbf{58.4^\circ\text{F}}$$

Therefore, the temperature of the drink can be lowered to 58.4°F by this process.

**Discussion** Note that the value obtained is very close to the assumed value of 60°F for the surface temperature. Therefore, there is no need to repeat the calculations with properties at the new surface temperature of 58.4°F



**14-146** Air is blown over a jug made of porous clay to cool it by simultaneous heat and mass transfer. The temperature of the water in the jug when steady conditions are reached is to be determined.

**Assumptions** **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2$  which cannot be determined at this point because of the unknown surface temperature  $T_s$ . We know that  $T_s < T_\infty$  and, for the purpose of property evaluation, we take  $T_s$  to be 20°C. Then, the properties of water at 20°C and the properties of dry air at the average temperature of 25°C and 1 atm are (Tables A-9 and A-15)

Water at 20°C:  $h_{fg} = 2454 \text{ kJ/kg}$ ,  $P_v = 2.34 \text{ kPa}$ . Also, at 30°C,  $P_{\text{sat}@30^\circ\text{C}} = 4.246 \text{ kPa}$

Dry air at 25°C:  $c_p = 1.007 \text{ kJ/kg} \cdot ^\circ\text{C}$ ,  $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$

Also, the mass diffusivity of water vapor in air at 25°C is  $D_{\text{H}_2\text{O-air}} = 2.50 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-4), and the molar masses of water and air are 18 and 29 kg/kmol, respectively (Table A-1).

**Analysis** The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as

$$T_s = T_\infty - \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.141 \times 10^{-5} \text{ m}^2/\text{s}}{2.50 \times 10^{-5} \text{ m}^2/\text{s}} = 0.8564$$

Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.

The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (2.34 kPa). The vapor pressure of air far from the surface is determined from

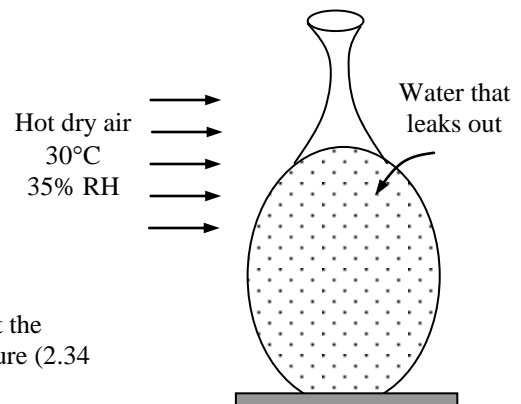
$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.35)P_{\text{sat}@30^\circ\text{C}} = (0.35)(4.246 \text{ kPa}) = 1.486 \text{ kPa}$$

Noting that the atmospheric pressure is 1 atm = 101.3 Pa, substituting the known quantities gives

$$T_s = 30^\circ\text{C} - \frac{2454 \text{ kJ/kg}}{(1.007 \text{ kJ/kg} \cdot ^\circ\text{C})(0.8564)^{2/3}} \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \frac{(2.34 - 1.486) \text{ kPa}}{101.3 \text{ kPa}} = \mathbf{15.9^\circ\text{C}}$$

Therefore, the temperature of the drink can be lowered to 15.9°C by this process.

**Discussion** The accuracy of this result can be improved by repeating the calculations with dry air properties evaluated at  $(30 + 15.9)/2 = 23.0^\circ\text{C}$  and water properties at 15.9°C.





**14-147** Prob. 14-146 is reconsidered. The water temperature as a function of the relative humidity of air is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

"GIVEN"

$P=101.3$  [kPa]

$T_{\text{infinity}}=30$  [C]

$\phi=0.35$

"PROPERTIES"

Fluid\$='steam\_IAPWS'

$h_f=\text{enthalpy}(\text{Fluid}\$, T=T_s, x=0)$

$h_g=\text{enthalpy}(\text{Fluid}\$, T=T_s, x=1)$

$h_{fg}=h_g-h_f$

$P_{\text{sat}_s}=\text{Pressure}(\text{Fluid}\$, T=T_s, x=0)$

$P_{\text{sat}_\infty}=\text{Pressure}(\text{Fluid}\$, T=T_{\text{infinity}}, x=0)$

$c_{p\_air}=\text{CP}(\text{air}, T=T_{\text{ave}})$

$T_{\text{ave}}=1/2*(T_{\text{infinity}}+T_s)$

$\alpha=2.141\text{E-}5$  [m<sup>2</sup>/s] "from the text at 25 C"

$D_{AB}=2.50\text{E-}5$  [m<sup>2</sup>/s] "from the text at 25 C"

$MM_{H_2O}=\text{molarmass}(H_2O)$

$MM_{air}=\text{molarmass}(\text{air})$

"ANALYSIS"

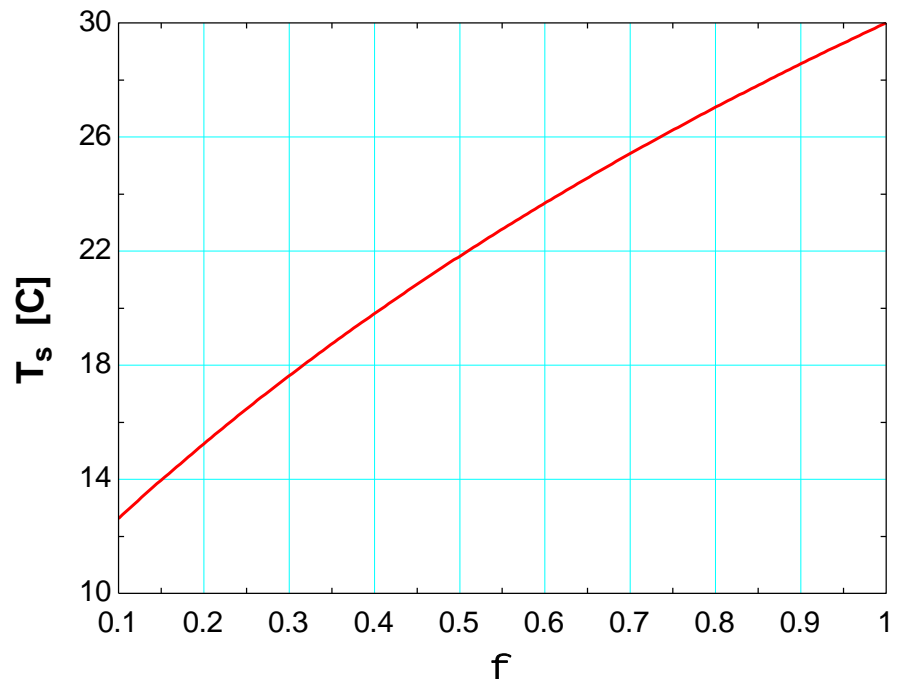
$Le=\alpha/D_{AB}$

$P_{v\_infinity}=\phi*P_{\text{sat}_\infty}$

$P_{v_s}=P_{\text{sat}_s}$

$T_s=T_{\text{infinity}}-h_{fg}/(c_{p\_air}*Le^{(2/3)})*MM_{H_2O}/MM_{air}*(P_{v_s}-P_{v\_infinity})/P$

$\phi$	$T_s$ [C]
0.1	12.63
0.15	13.97
0.2	15.24
0.25	16.46
0.3	17.63
0.35	18.74
0.4	19.81
0.45	20.83
0.5	21.82
0.55	22.77
0.6	23.68
0.65	24.57
0.7	25.42
0.75	26.24
0.8	27.04
0.85	27.81
0.9	28.56
0.95	29.29
1	30



**14-148** A soaked sponge is experiencing dry air flow over its surface. The temperature difference,  $T_\infty - T_s$ , is to be determined if it is soaked with (a) water and (b) ammonia.

**Assumptions** **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fractions of water and ammonia vapor in the air are low. **2** Air and water and ammonia vapors at specified conditions are ideal gases. **3** Radiation effects are negligible.

**Properties** The properties of air at 1 atm and 20°C (Table A-15):

$$\text{Air: } c_p = 1.007 \text{ kJ/kg}\cdot\text{K}, \quad \alpha = 2.074 \times 10^{-5} \text{ m}^2/\text{s}, \quad M = 28.97 \text{ kg/kmol (Table A-1)}$$

The properties of water at 10°C (Table A-9):

$$\text{Water: } h_{fg} = 2478 \text{ kJ/kg}, \quad P_{v,s} = 1.2276 \text{ kPa}, \quad M_v = 18.015 \text{ kg/kmol (Table A-1)}$$

The properties of ammonia at -40°C (Table A-11):

$$\text{NH}_3: \quad h_{fg} = 1389 \text{ kJ/kg}, \quad P_{v,s} = 71.66 \text{ kPa}, \quad M_v = 17.03 \text{ kg/kmol (Table A-1)}$$

**Analysis** (a) For mass transfer between water vapor and air, the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.074 \times 10^{-5} \text{ m}^2/\text{s}}{2.42 \times 10^{-5} \text{ m}^2/\text{s}} = 0.8570$$

The temperature difference ( $T_\infty - T_s$ ) can be determined from the Chilton-Colburn analogy, Eq. 14-92:

$$\begin{aligned} T_\infty - T_s &= \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P} \\ &= \frac{(2478 \text{ kJ/kg})}{(1.007 \text{ kJ/kg}\cdot\text{K})(0.8570)^{2/3}} \left( \frac{18.015}{28.97} \right) \left( \frac{1.2276 - 0}{101.325} \right) \\ &= \mathbf{20.5 \text{ K}} \end{aligned}$$

(b) For mass transfer between ammonia and air, the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.074 \times 10^{-5} \text{ m}^2/\text{s}}{2.6 \times 10^{-5} \text{ m}^2/\text{s}} = 0.7977$$

The temperature difference ( $T_\infty - T_s$ ) can be determined from the Chilton-Colburn analogy, Eq. 14-92:

$$\begin{aligned} T_\infty - T_s &= \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P} \\ &= \frac{(1389 \text{ kJ/kg})}{(1.007 \text{ kJ/kg}\cdot\text{K})(0.7977)^{2/3}} \left( \frac{17.03}{28.97} \right) \left( \frac{71.66 - 0}{101.325} \right) \\ &= \mathbf{667 \text{ K}} \end{aligned}$$

**Discussion** The much higher temperature difference ( $T_\infty - T_s$ ) experienced on the ammonia-soaked sponge means that ammonia can achieve greater evaporative cooling than water. Indeed, because of ammonia's vaporization properties, it has been used as a refrigerant.

**14-149** A water-soaked 10 cm × 10 cm square sponge is experiencing heat transfer by convection and radiation. (a) The rate of evaporation of water from the sponge and (b) the net radiation heat transfer rate are to be determined.

**Assumptions** 1 The analogy between heat and mass transfer is applicable. 2 Steady state condition exists. 3 Constant properties. 4 Water vapor behaves as ideal gas. 5 The bottom surface of the sponge is well insulated.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (20 + 30)^\circ\text{C}/2 = 25^\circ\text{C}$ . The properties of dry air at  $25^\circ\text{C}$  and 1 atm are (Table A-15)

Dry air:  $c_p = 1007 \text{ J/kg} \cdot \text{K}$ ,  $\rho = 1.184 \text{ kg/m}^3$ , and  $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$ .

Then the properties of water at  $30^\circ\text{C}$  are (Table A-9)

Water:  $\rho_{A,s} = \rho_v = 0.0304 \text{ kg/m}^3$  and  $h_{fg} = 2431 \text{ kJ/kg}$ .

The mass diffusivity of water vapor in air at  $25^\circ\text{C}$  is

$D_{AB} = 2.50 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-4).

**Analysis** (a) Applying the Chilton-Colburn analogy,

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left( \frac{\alpha}{D_{AB}} \right)^{2/3} \rightarrow h_{\text{mass}} = \frac{h_{\text{heat}}}{\rho c_p} \left( \frac{D_{AB}}{\alpha} \right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{\text{mass}} = \frac{30 \text{ W/m}^2 \cdot \text{K}}{(1.184 \text{ kg/m}^3)(1007 \text{ J/kg} \cdot \text{K})} \left( \frac{2.5 \times 10^{-5}}{2.141 \times 10^{-5}} \right)^{2/3} = 27.9 \times 10^{-3} \text{ m/s}$$

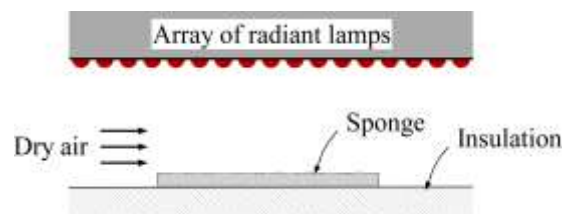
The evaporation rate is then

$$\begin{aligned} \dot{m}_v &= \dot{m}_{\text{conv}} = h_{\text{mass}} A_s (\rho_{A,s} - \rho_{A,\infty}) \\ &= (27.9 \times 10^{-3} \text{ m/s})(0.1 \text{ m})^2 (0.0304 - 0) \text{ kg/m}^3 \\ &= \mathbf{8.48 \times 10^{-6} \text{ kg/s}} \end{aligned}$$

(b) Performing energy balance on the sponge, considering the processes of evaporation, convection and radiation, we have

$$\begin{aligned} \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} &= \dot{Q}_{\text{evap}} \rightarrow \dot{Q}_{\text{rad}} = \dot{Q}_{\text{evap}} - \dot{Q}_{\text{conv}} = \dot{m}_v h_{fg} - h_{\text{heat}} A_s (T_\infty - T_s) \\ \dot{Q}_{\text{rad}} &= (8.48 \times 10^{-6} \text{ kg/s})(2431 \times 10^3 \text{ J/kg}) - (30 \text{ W/m}^2 \cdot \text{K})(0.1 \text{ m})^2 (20 - 30) \text{ K} \\ &= 20.6 \text{ W} - (-3 \text{ W}) \\ &= \mathbf{23.6 \text{ W}} \end{aligned}$$

**Discussion** Note that the heat transfer by evaporation is about 7 times larger than the heat transfer by convection. Also, for dry air flow the density of water vapor at the free stream is negligible.



**14-150** A thin layer of liquid water on a concrete surface is experiencing simultaneous heat and mass transfer. The conduction heat flux through the concrete is to be determined.

**Assumptions** 1 The analogy between heat and mass transfer is applicable. 2 Steady state condition exists. 3 Constant properties. 4 Water vapor behaves as ideal gas. 5 The bottom surface of the concrete is well insulated.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (30 + 20)^\circ\text{C}/2 = 25^\circ\text{C}$ . The properties of dry air at  $25^\circ\text{C}$  and 1 atm are (Table A-15)

Dry air:  $c_p = 1007 \text{ J/kg}\cdot\text{K}$ ,  $\rho = 1.184 \text{ kg/m}^3$ , and  $\alpha = 2.141 \times 10^{-5} \text{ m}^2/\text{s}$ .

Then the properties of water at  $20^\circ\text{C}$  are (Table A-9)

Water:  $\rho_{A,s} = \rho_v = 0.0173 \text{ kg/m}^3$  and  $h_{fg} = 2454 \text{ kJ/kg}$ .

The mass diffusivity of water vapor in air at  $25^\circ\text{C}$  is

$D_{AB} = 2.50 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-4).

**Analysis** Applying the Chilton-Colburn analogy,

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left( \frac{\alpha}{D_{AB}} \right)^{2/3} \rightarrow h_{\text{mass}} = \frac{h_{\text{heat}}}{\rho c_p} \left( \frac{D_{AB}}{\alpha} \right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{\text{mass}} = \frac{50 \text{ W/m}^2 \cdot \text{K}}{(1.184 \text{ kg/m}^3)(1007 \text{ J/kg} \cdot \text{K})} \left( \frac{2.5 \times 10^{-5}}{2.141 \times 10^{-5}} \right)^{2/3} = 46.5 \times 10^{-3} \text{ m/s}$$

The evaporation rate per unit area is then

$$\begin{aligned} j_v &= \dot{m}_{\text{conv}} / A_s = h_{\text{mass}} (\rho_{A,s} - \rho_{A,\infty}) \\ &= (46.5 \times 10^{-3} \text{ m/s})(0.0173 - 0) \text{ kg/m}^3 \\ &= 8.045 \times 10^{-4} \text{ kg/s} \cdot \text{m}^2 \end{aligned}$$

Then, the heat flux for each of the heat transfer process is

$$\text{Evaporation: } \dot{q}_{\text{evap}} = j_v h_{fg} = (8.045 \times 10^{-4} \text{ kg/s} \cdot \text{m}^2)(2454 \times 10^3 \text{ J/kg}) = 1974 \text{ W/m}^2$$

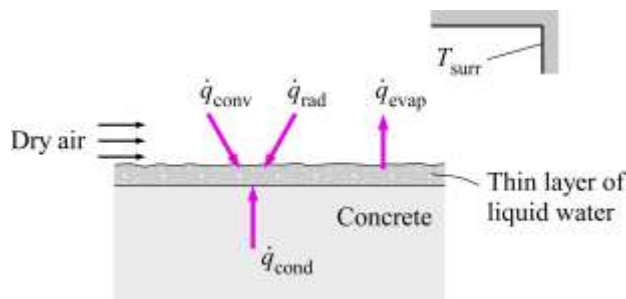
$$\text{Convection: } \dot{q}_{\text{conv}} = h_{\text{heat}} (T_\infty - T_s) = (50 \text{ W/m}^2 \cdot \text{K})(30 - 20) \text{ K} = 500 \text{ W/m}^2$$

$$\text{Radiation: } \dot{q}_{\text{rad}} = \varepsilon \sigma (T_{\text{surr}} - T_s) = (0.95)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(303^4 - 293^4) \text{ K}^4 = 57.03 \text{ W/m}^2$$

Performing energy balance on liquid water layer, considering the processes of evaporation, convection, radiation and conduction, we have

$$\begin{aligned} \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} + \dot{q}_{\text{cond}} &= \dot{q}_{\text{evap}} \\ \dot{q}_{\text{cond}} &= \dot{q}_{\text{evap}} - \dot{q}_{\text{conv}} - \dot{q}_{\text{rad}} \\ &= 1974 \text{ W} - 500 \text{ W} - 57.03 \text{ W} \\ &= \mathbf{1417 \text{ W}} \end{aligned}$$

**Discussion** The positive value of the conduction heat flux through the concrete indicates that heat flux by conduction is going into the liquid water layer.



**14-151** A person is standing outdoors in windy weather. The rates of heat loss from the head by radiation, forced convection, and evaporation are to be determined for the cases of the head being wet and dry.

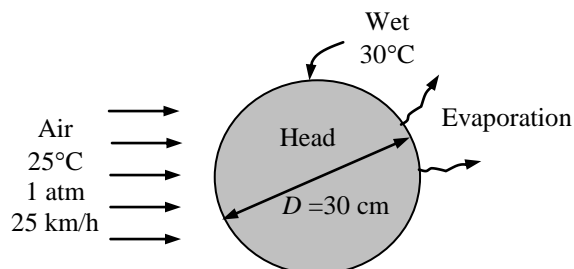
**Assumptions** **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The head can be approximated as a sphere of 30 cm diameter maintained at a uniform temperature of 30°C. **4** The surrounding surfaces are at the same temperature as the ambient air.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture. The properties of air at the free stream temperature of 25°C and 1 atm are, from Table A-15,

$$k = 0.02551 \text{ W/m} \cdot \text{C}, \quad \text{Pr} = 0.7296$$

$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \quad \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$

Also,  $\mu_s = \mu_{@30^\circ\text{C}} = 1.872 \times 10^{-5} \text{ kg/m} \cdot \text{s}.$



The mass diffusivity of water vapor in air at the average temperature of  $(25 + 30)/2 = 27.5^\circ\text{C} = 300.5 \text{ K}$  is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(300.5 \text{ K})^{2.072}}{1 \text{ atm}} = 2.55 \times 10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 25°C is  $P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$ . Properties of water at 30°C are  $h_{fg} = 2431 \text{ kJ/kg}$  and  $P_v = 4.246 \text{ kPa}$  (Table A-9).

The gas constants of dry air and water are  $R_{\text{air}} = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  and  $R_{\text{water}} = 0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  (Table A-1). Also, the emissivity of the head is given to be 0.95.

**Analysis** (a) When the head is dry, heat transfer from the head is by forced convection and radiation only. The radiation heat transfer is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)[\pi(0.3 \text{ m})^2](5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(30 + 273 \text{ K})^4 - (25 + 273 \text{ K})^4] = 8.3 \text{ W}$$

The Reynolds number for flow over the head is

$$\text{Re} = \frac{VD}{\nu} = \frac{(25/3.6 \text{ m/s})(0.3 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 133,380$$

Then the Nusselt number and the heat transfer coefficient become

$$\begin{aligned} \text{Nu} &= 2 + \left[ 0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3} \right] \text{Pr}^{0.4} \left( \frac{\mu_\infty}{\mu_s} \right)^{1/4} \\ &= 2 + \left[ 0.4(133,380)^{1/2} + 0.06(133,380)^{2/3} \right] (0.7296)^{0.4} \left( \frac{1.849 \times 10^{-5}}{1.872 \times 10^{-5}} \right)^{1/4} = 268 \end{aligned}$$

$$h = \frac{k}{D} \text{Nu} = \frac{0.02551 \text{ W/m} \cdot ^\circ\text{C}}{0.3 \text{ m}} (268) = 22.8 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the rate of convection heat transfer from the head becomes

$$\dot{Q}_{\text{conv}} = h A_s (T_s - T_\infty) = (22.8 \text{ W/m}^2 \cdot ^\circ\text{C})[\pi(0.3 \text{ m})^2](30 - 25)^\circ\text{C} = 32.2 \text{ W}$$

Therefore,

$$\dot{Q}_{\text{total,dry}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 32.2 + 8.3 = \mathbf{40.5 \text{ W}}$$

(b) When the head is wet, there is additional heat transfer mechanism by evaporation. The Schmidt number is

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{2.55 \times 10^{-5} \text{ m}^2/\text{s}} = 0.613$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\begin{aligned} Sh &= 2 + \left[ 0.4 Re^{1/2} + 0.06 Re^{2/3} \right] Sc^{0.4} \left( \frac{\mu_\infty}{\mu_s} \right)^{1/4} \\ &= 2 + \left[ 0.4(133,380)^{1/2} + 0.06(133,380)^{2/3} \right] (0.613)^{0.4} \left( \frac{1.849 \times 10^{-5}}{1.872 \times 10^{-5}} \right)^{1/4} = 250 \end{aligned}$$

$$h_{\text{mass}} = \frac{Sh D_{AB}}{L} = \frac{(250)(2.55 \times 10^{-5} \text{ m}^2/\text{s})}{0.3 \text{ m}} = 0.0213 \text{ m/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.30)P_{\text{sat}@25^\circ\text{C}} = (0.30)(3.169 \text{ kPa}) = 0.9507 \text{ kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{4.246 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(30 + 273) \text{ K}} = 0.0304 \text{ kg/m}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.9507 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 0.0069 \text{ kg/m}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0213 \text{ m/s})[\pi(0.3 \text{ m})^2](0.0304 - 0.0069) \text{ kg/m}^3 \\ &= 0.0001415 \text{ kg/s} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.0001415 \text{ kg/s})(2431 \text{ kJ/kg}) = 0.344 \text{ kW} = 344 \text{ W}$$

Then the total rate of heat loss from the wet head to the surrounding air and surfaces becomes

$$\dot{Q}_{\text{total,wet}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{evap}} = 32.2 + 8.3 + 344 = \mathbf{385 \text{ W}}$$

**Discussion** Note that the heat loss from the head can be increased by more than 9 times in this case by wetting the head and allowing heat transfer by evaporation.

**14-152** The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 30°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (20+30)/2 = 25^\circ\text{C} = 298\text{ K}$ . The properties of dry air at 298 K and 1 atm are, from Table A-15,

$$k = 0.02551\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.7296$$

$$\alpha = 2.141 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.562 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 298 K is determined from Eq. 14-15 to be

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(298\text{ K})^{2.072}}{1\text{ atm}} = 2.50 \times 10^{-5}\text{ m}^2/\text{s} \end{aligned}$$

The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$ .

Properties of water at 30°C are

$$h_{fg} = 2431\text{ kJ/kg} \text{ and } P_v = 4.246\text{ kPa} \text{ (Table A-9).}$$

The gas constants of dry air and water are  $R_{\text{air}} = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  and  $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1). The emissivity of water is 0.95 (Table A-18).

**Analysis (a)** The surface area of the pool is

$$A = (20\text{ m})(20\text{ m}) = 400\text{ m}^2$$

Heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(400\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4) [(30 + 273\text{ K})^4 - (0 + 273\text{ K})^4] = \mathbf{61,930\text{ W}}$$

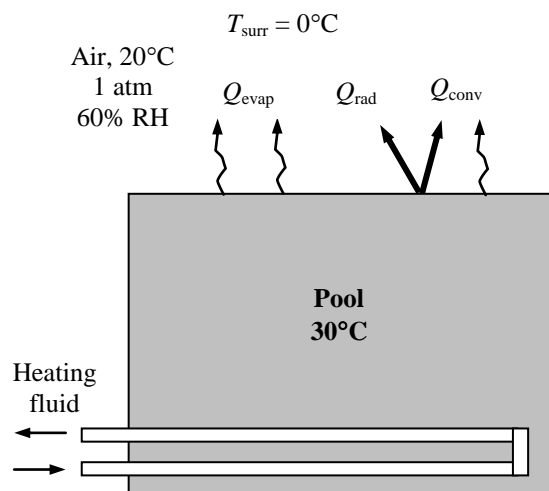
(b) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.60)P_{\text{sat}@20^\circ\text{C}} = (0.60)(2.339\text{ kPa}) = 1.40\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

*At the surface:*

$$\begin{aligned} \rho_{v,s} &= \frac{P_{v,s}}{R_v T_s} = \frac{4.246\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(30 + 273)\text{ K}} = 0.0304\text{ kg/m}^3 \\ \rho_{a,s} &= \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 4.246)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(30 + 273)\text{ K}} = 1.1164\text{ kg/m}^3 \\ \rho_s &= \rho_{v,s} + \rho_{a,s} = 0.0304 + 1.1164 = 1.1468\text{ kg/m}^3 \end{aligned}$$





Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.0104 \text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.40) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 1.1883 \text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987 \text{ kg/m}^3$$

Note that  $\rho_\infty > \rho_s$ , and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is  $p = 2(20 + 20) = 80 \text{ m}$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{400 \text{ m}^2}{80 \text{ m}} = 5 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1987 - 1.1468 \text{ kg/m}^3)(5 \text{ m})^3}{[(1.1987 + 1.1468)/2 \text{ kg/m}^3](1.562 \times 10^{-5} \text{ m}^2/\text{s})^2} = 2.224 \times 10^{11}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(2.224 \times 10^{11} \times 0.7296)^{1/3} = 818.1$$

and 
$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(818.1)(0.02551 \text{ W/m} \cdot ^\circ\text{C})}{5 \text{ m}} = 4.174 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (4.174 \text{ W/m}^2 \cdot ^\circ\text{C})(400 \text{ m}^2)(30 - 20)^\circ\text{C} = \mathbf{16,695 \text{ W}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{2.50 \times 10^{-5} \text{ m}^2/\text{s}} = 0.625$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(2.224 \times 10^{11} \times 0.625)^{1/3} = 777.0$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(777.0)(2.50 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = 0.003885 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.003885 \text{ m/s})(400 \text{ m}^2)(0.0304 - 0.0104) \text{ kg/m}^3 \\ &= 0.03108 \text{ kg/s} = 111.9 \text{ kg/h} \end{aligned}$$

and 
$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.03108 \text{ kg/s})(2,431,000 \text{ J/kg}) = \mathbf{75,555 \text{ W}}$$

Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 61,930 + 16,695 + 75,555 = \mathbf{154,180 \text{ W}}$$

Therefore, if the pool is heated electrically, a 154 kW resistance heater will be needed to make up for the heat losses from the top surface.

**14-153** The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 25°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.

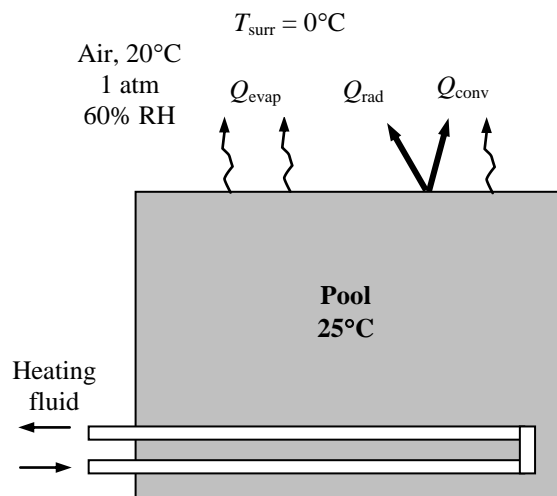
**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (20+25)/2 = 22.5^\circ\text{C} = 295.5\text{ K}$ . The properties of dry air at 22.5°C and 1 atm are, from Table A-15,

$$k = 0.02533 \text{ W/m} \cdot ^\circ\text{C}, \text{ Pr} = 0.7303$$

$$\alpha = 2.108 \times 10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.539 \times 10^{-5} \text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 295.5 K is, from Eq. 14-15,

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(295.5 \text{ K})^{2.072}}{1 \text{ atm}} \\ &= 2.46 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$



The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339 \text{ kPa}$ . Properties of water at 25°C are

$h_{fg} = 2442 \text{ kJ/kg}$  and  $P_v = 3.169 \text{ kPa}$  (Table A-9). The gas constants of dry air and water are  $R_{\text{air}} = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  and  $R_{\text{water}} = 0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  (Table A-1). The emissivity of water is 0.95 (Table A-18).

**Analysis** (a) The surface area of the pool is

$$A_s = (20 \text{ m})(20 \text{ m}) = 400 \text{ m}^2$$

Heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(400 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) \left[ (25 + 273 \text{ K})^4 - (0 + 273 \text{ K})^4 \right] = \mathbf{50,235 \text{ W}}$$

(b) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (3.169 kPa at 25°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.60)P_{\text{sat}@20^\circ\text{C}} = (0.60)(2.339 \text{ kPa}) = 1.40 \text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

*At the surface:*

$$\begin{aligned} \rho_{v,s} &= \frac{P_{v,s}}{R_v T_s} = \frac{3.169 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 0.0230 \text{ kg/m}^3 \\ \rho_{a,s} &= \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 3.169) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 1.1477 \text{ kg/m}^3 \\ \rho_s &= \rho_{v,s} + \rho_{a,s} = 0.0230 + 1.1477 = 1.1707 \text{ kg/m}^3 \end{aligned}$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.0104 \text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.40) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 1.1883 \text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987 \text{ kg/m}^3$$

Note that  $\rho_\infty > \rho_s$ , and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is  $p = 2(20 + 20) = 80 \text{ m}$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{400 \text{ m}^2}{80 \text{ m}} = 5 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}} \nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1987 - 1.1707 \text{ kg/m}^3)(5 \text{ m})^3}{[(1.1987 + 1.1707)/2 \text{ kg/m}^3](1.539 \times 10^{-5} \text{ m}^2/\text{s})^2} = 1.224 \times 10^{11}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.224 \times 10^{11} \times 0.73)^{1/3} = 670.6$$

and 
$$h_{\text{conv}} = \frac{\text{Nu} k}{L} = \frac{(670.6)(0.02533 \text{ W/m} \cdot ^\circ\text{C})}{5 \text{ m}} = 3.397 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_s - T_\infty) = (3.397 \text{ W/m}^2 \cdot ^\circ\text{C})(400 \text{ m}^2)(25 - 20)^\circ\text{C} = \mathbf{6794 \text{ W}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.539 \times 10^{-5} \text{ m}^2/\text{s}}{2.46 \times 10^{-5} \text{ m}^2/\text{s}} = 0.626$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(1.224 \times 10^{11} \times 0.626)^{1/3} = 637.1$$

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{L} = \frac{(637.1)(2.46 \times 10^{-5} \text{ m}^2/\text{s})}{5 \text{ m}} = 0.003135 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.003135 \text{ m/s})(400 \text{ m}^2)(0.0230 - 0.0104) \text{ kg/m}^3 \\ &= 0.01580 \text{ kg/s} = 56.88 \text{ kg/h} \end{aligned}$$

and 
$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.01580 \text{ kg/s})(2,442,000 \text{ J/kg}) = \mathbf{38,584 \text{ W}}$$

Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 50,235 + 6794 + 38,584 = \mathbf{95,610 \text{ W}}$$

Therefore, if the pool is heated electrically, a 95.6 kW resistance heater will be needed to make up for the heat losses from the top surface.



**14-154** Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.

**Assumptions** 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). 2 Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). 3 The entire water body and the metal container are maintained at a uniform temperature of 50°C. 4 Heat losses from the bottom surface are negligible. 5 The air motion around the bath is negligible so that there are no forced convection effects.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (25+50)/2 = 37.5^\circ\text{C}$ . The properties of dry air at 37.5°C and 1 atm are, from Table A-15,

$$k = 0.02643 \text{ W/m} \cdot ^\circ\text{C}, \quad \text{Pr} = 0.7262$$

$$\alpha = 2.312 \times 10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.679 \times 10^{-5} \text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 310.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(310.5 \text{ K})^{2.072}}{1 \text{ atm}} = 2.73 \times 10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 25°C is  $P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$ . Properties of water at 50°C are

$h_{fg} = 2383 \text{ kJ/kg}$  and  $P_v = 12.35 \text{ kPa}$  (Table A-9). The specific heat of water at the average temperature of  $(15+50)/2 = 32.5^\circ\text{C}$  is  $c_p = 4.178 \text{ kJ/kg} \cdot ^\circ\text{C}$ .

The gas constants of dry air and water are  $R_{\text{air}} = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  and  $R_{\text{water}} = 0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is given to be  $1.0 \text{ kJ/kg} \cdot ^\circ\text{C}$ .

**Analysis** (a) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150 \text{ kg/bottle})(800 \text{ bottles/min}) = 120 \text{ kg/min} = 2 \text{ kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 50°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}} c_p \Delta T = (2 \text{ kg/s})(1 \text{ kJ/kg} \cdot ^\circ\text{C})(50 - 25)^\circ\text{C} = 50,000 \text{ W}$$

The amount of water removed by the bottles is

$$\begin{aligned} \dot{m}_{\text{water, out}} &= (\text{Flow rate of bottles})(\text{Water removed per bottle}) \\ &= (800 \text{ bottles/min})(0.6 \text{ g/bottle}) = 480 \text{ g/min} = 8 \times 10^{-3} \text{ kg/s} = 28.8 \text{ kg/h} \end{aligned}$$

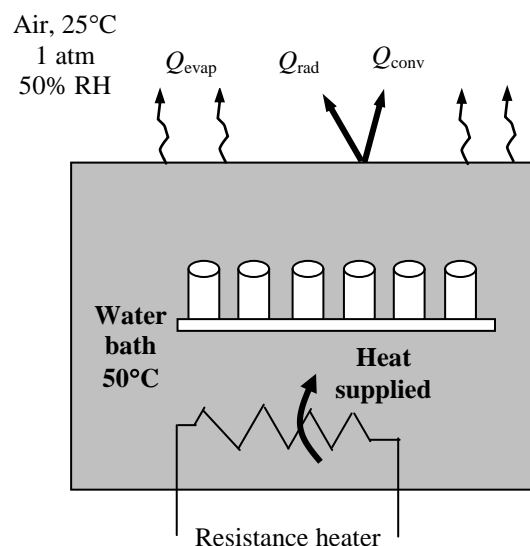
Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}} c_p \Delta T = (8 \times 10^{-3} \text{ kg/s})(4178 \text{ J/kg} \cdot ^\circ\text{C})(50 - 15)^\circ\text{C} = 1170 \text{ W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 50,000 + 1170 = 51,170 \text{ W}$$

(b) The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is



$$\dot{Q}_{\text{rad,top}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{sur}}^4) = (0.95)(8 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(50 + 273 \text{ K})^4 - (15 + 273 \text{ K})^4] = \mathbf{1726 \text{ W}}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (12.35 kPa at 50°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.50)P_{\text{sat}@25^\circ\text{C}} = (0.50)(3.169 \text{ kPa}) = 1.585 \text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

*At the surface:*

$$\begin{aligned}\rho_{v,s} &= \frac{P_{v,s}}{R_v T_s} = \frac{12.35 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(50 + 273 \text{ K})} = 0.0829 \text{ kg/m}^3 \\ \rho_{a,s} &= \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 12.35) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(50 + 273 \text{ K})} = 0.9598 \text{ kg/m}^3 \\ \rho_s &= \rho_{v,s} + \rho_{a,s} = 0.0829 + 0.9598 = 1.0427 \text{ kg/m}^3\end{aligned}$$

*Away from the surface:*

$$\begin{aligned}\rho_{v,\infty} &= \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273 \text{ K})} = 0.0115 \text{ kg/m}^3 \\ \rho_{a,\infty} &= \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.585) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273 \text{ K})} = 1.1662 \text{ kg/m}^3 \\ \rho_\infty &= \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777 \text{ kg/m}^3\end{aligned}$$

Note that  $\rho_\infty > \rho_s$ , and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is  $A_s = 2 \text{ m} \times 4 \text{ m} = 8 \text{ m}^2$  and its perimeter is  $p = 2(2 + 4) = 12 \text{ m}$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8 \text{ m}^2}{12 \text{ m}} = 0.667 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1777 - 1.0427 \text{ kg/m}^3)(0.667 \text{ m})^3}{[(1.1777 + 1.0427)/2 \text{ kg/m}^3](1.679 \times 10^{-5} \text{ m}^2/\text{s})^2} = 1.26 \times 10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.26 \times 10^9 \times 0.7262)^{1/3} = 146$$

and 
$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(146)(0.02643 \text{ W/m} \cdot ^\circ\text{C})}{0.667 \text{ m}} = 5.79 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (5.79 \text{ W/m}^2 \cdot ^\circ\text{C})(8 \text{ m}^2)(50 - 25)^\circ\text{C} = \mathbf{1158 \text{ W}}$$

Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.679 \times 10^{-5} \text{ m}^2/\text{s}}{2.73 \times 10^{-5} \text{ m}^2/\text{s}} = 0.615$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(1.27 \times 10^9 \times 0.615)^{1/3} = 138$$

$$h_{\text{mass}} = \frac{ShD_{AB}}{L} = \frac{(138)(2.73 \times 10^{-5} \text{ m}^2/\text{s})}{0.667 \text{ m}} = 0.00565 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.00565 \text{ m/s})(8 \text{ m}^2)(0.0829 - 0.0115) \text{ kg/m}^3 \\ &= 0.00324 \text{ kg/s} = 11.7 \text{ kg/h}\end{aligned}$$

and  $\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00324 \text{ kg/s})(2383 \text{ kJ/kg}) = 7.72 \text{ kW} = \mathbf{7720 \text{ W}}$

The total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 1726 + 1158 + 7720 = \mathbf{10,604 \text{ W}}$$

Therefore, if the water bath is heated electrically, a 10.6 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(c) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(1/310.5 \text{ K})(50 - 25) \text{ K}(1 \text{ m})^3}{(1.679 \times 10^{-5} \text{ m}^2/\text{s})^2} = 2.80 \times 10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(2.80 \times 10^9 \times 0.7262)^{1/3} = 127$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(127)(0.02643 \text{ W/m} \cdot ^\circ\text{C})}{1 \text{ m}} = 3.36 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}} A_s (T_s - T_\infty) = (3.36 \text{ W/m}^2 \cdot ^\circ\text{C})(12 \times 1 \text{ m}^2)(50 - 25)^\circ\text{C} = 1007 \text{ W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$$\begin{aligned}\dot{Q}_{\text{rad, side}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{sur}}^4) \\ &= (0.61)(12 \text{ m} \times 1 \text{ m})(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(50 + 273 \text{ K})^4 - (15 + 273 \text{ K})^4] = 1662 \text{ W}\end{aligned}$$

and  $\dot{Q}_{\text{total, side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1007 + 1662 = \mathbf{2669 \text{ W}}$

(d) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00324 = \mathbf{0.01124 \text{ kg/s} = 40.5 \text{ kg/h}}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 50°C is

$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}} c_p \Delta T = (0.01124 \text{ kg/s})(4178 \text{ J/kg} \cdot ^\circ\text{C})(50 - 15)^\circ\text{C} = 1644 \text{ W}$$

Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\begin{aligned}\dot{Q}_{\text{total}} &= \dot{Q}_{\text{bottle}} + (\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}})_{\text{top}} + (\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}})_{\text{side}} + \dot{Q}_{\text{makeup water}} \\ &= 51,170 + 10,604 + 2669 + 1644 = \mathbf{66,087 \text{ W}}\end{aligned}$$

Therefore, the heater must be able to supply heat at a rate of 66.1 kW to maintain steady operating conditions.

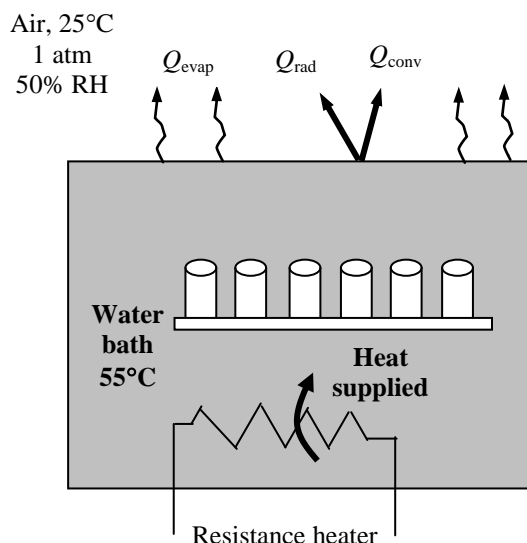
**14-155** Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.

**Assumptions 1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body and the metal container are maintained at a uniform temperature of 55°C. **4** Heat losses from the bottom surface are negligible. **5** The air motion around the bath is negligible so that there are no forced convection effects.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (25+55)/2 = 40^\circ\text{C}$ . The properties of dry air at 40°C and 1 atm are, from Table A-15,

$$k = 0.02662 \text{ W/m} \cdot ^\circ\text{C}, \quad \text{Pr} = 0.7255$$

$$\alpha = 2.346 \times 10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.700 \times 10^{-5} \text{ m}^2/\text{s}$$



The mass diffusivity of water vapor in air at the average temperature of 313 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(313 \text{ K})^{2.072}}{1 \text{ atm}} = 2.77 \times 10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 25°C is  $P_{\text{sat}@25^\circ\text{C}} = 3.169 \text{ kPa}$ . Properties of water at 55°C are

$h_{fg} = 2371 \text{ kJ/kg}$  and  $P_v = 15.76 \text{ kPa}$  (Table A-9). The specific heat of water at the average temperature of  $(15+55)/2 = 35^\circ\text{C}$  is  $c_p = 4.178 \text{ kJ/kg} \cdot ^\circ\text{C}$ .

The gas constants of dry air and water are  $R_{\text{air}} = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  and  $R_{\text{water}} = 0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$  (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is  $1.0 \text{ kJ/kg} \cdot ^\circ\text{C}$ .

**Analysis (a)** The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150 \text{ kg/bottle})(800 \text{ bottles/min}) = 120 \text{ kg/min} = 2 \text{ kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 55°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}} c_p \Delta T = (2 \text{ kg/s})(1 \text{ kJ/kg} \cdot ^\circ\text{C})(55 - 25)^\circ\text{C} = 60,000 \text{ W}$$

The amount of water removed by the bottles is

$$\begin{aligned} \dot{m}_{\text{water, out}} &= (\text{Flow rate of bottles})(\text{Water removed per bottle}) \\ &= (800 \text{ bottles/min})(0.6 \text{ g/bottle}) = 480 \text{ g/min} = 8 \times 10^{-3} \text{ kg/s} = 28.8 \text{ kg/h} \end{aligned}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}} c_p \Delta T = (8 \times 10^{-3} \text{ kg/s})(4178 \text{ J/kg} \cdot ^\circ\text{C})(55 - 15)^\circ\text{C} = 1337 \text{ W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 60,000 + 1337 = 61,337 \text{ W}$$

**(b)** The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is

$$\dot{Q}_{\text{rad, top}} = \varepsilon A \sigma (T_s^4 - T_{\text{sur}}^4) = (0.95)(8 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(55 + 273 \text{ K})^4 - (15 + 273 \text{ K})^4] = 2023 \text{ W}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (15.76 kPa at 55°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.50)P_{\text{sat}@25^\circ\text{C}} = (0.50)(3.169 \text{ kPa}) = 1.585 \text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

*At the surface:*

$$\begin{aligned}\rho_{v,s} &= \frac{P_{v,s}}{R_v T_s} = \frac{15.76 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(55 + 273 \text{ K})} = 0.1041 \text{ kg/m}^3 \\ \rho_{a,s} &= \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 15.76) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(55 + 273 \text{ K})} = 0.9090 \text{ kg/m}^3 \\ \rho_s &= \rho_{v,s} + \rho_{a,s} = 0.1041 + 0.9090 = 1.0131 \text{ kg/m}^3\end{aligned}$$

*Away from the surface:*

$$\begin{aligned}\rho_{v,\infty} &= \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(25 + 273 \text{ K})} = 0.0115 \text{ kg/m}^3 \\ \rho_{a,\infty} &= \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 1.585) \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(25 + 273 \text{ K})} = 1.1662 \text{ kg/m}^3 \\ \rho_\infty &= \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777 \text{ kg/m}^3\end{aligned}$$

Note that  $\rho_\infty > \rho_s$ , and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is  $A_s = 2 \text{ m} \times 4 \text{ m} = 8 \text{ m}^2$  and its perimeter is  $p = 2(2 + 4) = 12 \text{ m}$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8 \text{ m}^2}{12 \text{ m}} = 0.667 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1777 - 1.0131 \text{ kg/m}^3)(0.667 \text{ m})^3}{[(1.1777 + 1.0131)/2 \text{ kg/m}^3](1.702 \times 10^{-5} \text{ m}^2/\text{s})^2} = 1.52 \times 10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.52 \times 10^9 \times 0.726)^{1/3} = 155$$

and 
$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(155)(0.02662 \text{ W/m} \cdot ^\circ\text{C})}{0.667 \text{ m}} = 6.19 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (6.19 \text{ W/m}^2 \cdot ^\circ\text{C})(8 \text{ m}^2)(55 - 25)^\circ\text{C} = 1486 \text{ W}$$

Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.702 \times 10^{-5} \text{ m}^2/\text{s}}{2.77 \times 10^{-5} \text{ m}^2/\text{s}} = 0.614$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{Gr Sc})^{1/3} = 0.15(1.52 \times 10^9 \times 0.614)^{1/3} = 147$$



$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(147)(2.77 \times 10^{-5} \text{ m}^2/\text{s})}{0.667 \text{ m}} = 0.00610 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.00610 \text{ m/s})(8 \text{ m}^2)(0.1041 - 0.0115) \text{ kg/m}^3 \\ &= 0.00452 \text{ kg/s} = 16.3 \text{ kg/h}\end{aligned}$$

$$\text{and } \dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00452 \text{ kg/s})(2371 \text{ kJ/kg}) = 10.7 \text{ kW} = 10,700 \text{ W}$$

Then the total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 2023 + 1486 + 10,700 = \mathbf{14,209 \text{ W}}$$

Therefore, if the water bath is heated electrically, a 14 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(c) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(1/313 \text{ K})(55 - 25) \text{ K}(1 \text{ m})^3}{(1.702 \times 10^{-5} \text{ m}^2/\text{s})^2} = 3.25 \times 10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(3.25 \times 10^9 \times 0.7255)^{1/3} = 133$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(133)(0.02662 \text{ W/m} \cdot ^\circ\text{C})}{1 \text{ m}} = 3.54 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}} A_s (T_s - T_\infty) = (3.54 \text{ W/m}^2 \cdot ^\circ\text{C})(12 \times 1 \text{ m}^2)(55 - 25)^\circ\text{C} = 1275 \text{ W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$$\begin{aligned}\dot{Q}_{\text{rad, side}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{sur}}^4) \\ &= (0.61)(12 \text{ m} \times 1 \text{ m})(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(55 + 273 \text{ K})^4 - (15 + 273 \text{ K})^4] = 1948 \text{ W}\end{aligned}$$

$$\text{and } \dot{Q}_{\text{total, side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1275 + 1948 = \mathbf{3223 \text{ W}}$$

(d) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00452 = \mathbf{0.01252 \text{ kg/s} = 45.1 \text{ kg/h}}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 55°C is

$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}} c_p \Delta T = (0.01252 \text{ kg/s})(4178 \text{ J/kg} \cdot ^\circ\text{C})(55 - 15)^\circ\text{C} = 2092 \text{ W}$$

Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\begin{aligned}\dot{Q}_{\text{total}} &= \dot{Q}_{\text{bottle}} + (\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}})_{\text{top}} + (\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}})_{\text{side}} + \dot{Q}_{\text{makeup water}} \\ &= 61,337 + 14,209 + 3223 + 2092 = \mathbf{80,860 \text{ W}}\end{aligned}$$

Therefore, the heater must be able to supply heat at a rate of 80.9 kW to maintain steady operating conditions.

## Review Problems

**14-156** The mole fraction of the water vapor at the surface of a lake and the mole fraction of water in the lake are to be determined and compared.

**Assumptions 1** Both the air and water vapor are ideal gases. **2** Air is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at 15°C is 1.705 kPa (Table A-9).

**Analysis** The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 15°C,

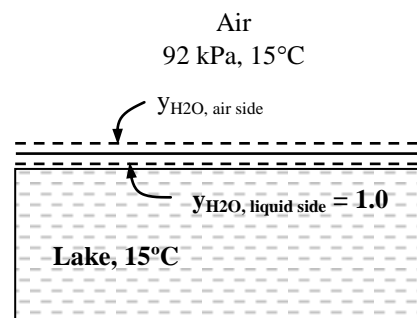
$$P_{\text{vapor}} = P_{\text{sat}@15^\circ\text{C}} = 1.705 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air at the surface of the lake is determined from Eq. 14-11 to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{1.705 \text{ kPa}}{92 \text{ kPa}} = \mathbf{0.0185} \text{ (or 18.5 percent)}$$

Water contains some dissolved air, but the amount is negligible. Therefore, we can assume the entire lake to be liquid water. Then its mole fraction becomes

$$y_{\text{water, liquid side}} \cong 1.0 \quad (\text{or 100\%})$$



**Discussion** Note that the concentration of water on a molar basis is 100 percent just beneath the air–water interface and 1.85 percent just above it, even though the air is assumed to be saturated (so this is the highest value at 15°C). Therefore, huge discontinuities can occur in the concentrations of a species across phase boundaries.

**14-157** The ideal gas relation can be expressed as  $P\mathcal{V} = NR_uT = mRT$  where  $R_u$  is the universal gas constant, whose value is the same for all gases, and  $R$  is the gas constant whose value is different for different gases. The molar and mass densities of an ideal gas mixture can be expressed as

$$P\mathcal{V} = NR_uT \rightarrow C = \frac{N}{\mathcal{V}} = \frac{P}{R_uT} = \text{constant}$$

and  $P\mathcal{V} = mRT \rightarrow \rho = \frac{m}{\mathcal{V}} = \frac{P}{RT} \neq \text{constant}$

Therefore, for an ideal gas mixture maintained at a constant temperature and pressure, the molar concentration  $C$  of the mixture remains constant but this is not necessarily the case for the density  $\rho$  of mixture.

**14-158E** The masses of the constituents of a gas mixture at a specified temperature and pressure are given. The partial pressure of each gas and the volume of the mixture are to be determined.

**Assumptions** The gas mixture and its constituents are ideal gases.

**Properties** The molar masses of  $\text{CO}_2$  and  $\text{CH}_4$  are 44 and 16 lbm/lbmol, respectively (Table A-1E)

**Analysis** The mole numbers of each gas and of the mixture are

$$\text{CO}_2 : \quad N_{\text{CO}_2} = \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} = \frac{1 \text{ lbm}}{44 \text{ lbm/lbmol}} = 0.0227 \text{ lbmol}$$

$$\text{CH}_4 : \quad N_{\text{CH}_4} = \frac{m_{\text{CH}_4}}{M_{\text{CH}_4}} = \frac{3 \text{ lbm}}{16 \text{ lbm/lbmol}} = 0.1875 \text{ lbmol}$$

$$N_{\text{total}} = N_{\text{CO}_2} + N_{\text{CH}_4} = 0.0227 + 0.1875 = 0.2102 \text{ lbmol}$$

1 lbm $\text{CO}_2$ 3 lbm $\text{CH}_4$  600 R 20 psia
--

Using the ideal gas relation for the mixture and for the constituents, the volume of the mixture and the partial pressures of the constituents are determined to be

$$\nu = \frac{NR_u T}{P} = \frac{(0.2102 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(600 \text{ R})}{20 \text{ psia}} = \mathbf{67.66 \text{ ft}^3}$$

$$P_{\text{CO}_2} = \frac{N_{\text{CO}_2} R_u T}{\nu} = \frac{(0.0227 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(600 \text{ R})}{67.66 \text{ ft}^3} = \mathbf{2.160 \text{ psia}}$$

$$P_{\text{CH}_4} = \frac{N_{\text{CH}_4} R_u T}{\nu} = \frac{(0.1875 \text{ lbmol})(10.73 \text{ psia} \cdot \text{ft}^3 / \text{lbmol} \cdot \text{R})(600 \text{ R})}{67.66 \text{ ft}^3} = \mathbf{17.84 \text{ psia}}$$

**Discussion** Note that each constituent of a gas mixture occupies the same volume (the volume of the container), and that the total pressure of a gas mixture is equal to the sum of the partial pressures of its constituents. That is,

$$P_{\text{total}} = P_{\text{CO}_2} + P_{\text{CH}_4} = 2.160 + 17.84 = 20 \text{ psia.}$$

**14-159** Dry air flows over a water body at constant pressure and temperature until it is saturated. The molar analysis of the saturated air and the density of air before and after the process are to be determined.

**Assumptions** The air and the water vapor are ideal gases.

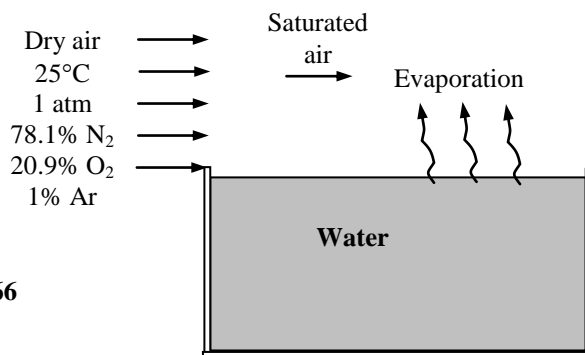
**Properties** The molar masses of  $N_2$ ,  $O_2$ , Ar, and  $H_2O$  are 28.0, 32.0, 39.95 and 18 kg / kmol, respectively (Table A-1). The molar analysis of dry air is given to be 78.1 percent  $N_2$ , 20.9 percent  $O_2$ , and 1 percent Ar. The saturation pressure of water at 25°C is 3.169 kPa (Table A-9). Also, 1 atm = 101.325 kPa.

**Analysis** (a) Noting that the total pressure remains constant at 101.32 kPa during this process, the partial pressure of air becomes

$$\begin{aligned} P &= P_{\text{air}} + P_{\text{vapor}} \\ P_{\text{air}} &= P - P_{\text{vapor}} \\ &= 101.325 - 3.169 = 98.156 \text{ kPa} \end{aligned}$$

Then the molar analysis of the saturated air becomes

$$\begin{aligned} y_{H_2O} &= \frac{P_{H_2O}}{P} = \frac{3.169}{101.325} = \mathbf{0.0313} \\ y_{N_2} &= \frac{P_{N_2}}{P} = \frac{y_{N_2, \text{dry}} P_{\text{dry air}}}{P} = \frac{0.781(98.156 \text{ kPa})}{101.325} = \mathbf{0.7566} \\ y_{O_2} &= \frac{P_{O_2}}{P} = \frac{y_{O_2, \text{dry}} P_{\text{dry air}}}{P} = \frac{0.209(98.156 \text{ kPa})}{101.325} = \mathbf{0.2025} \\ y_{Ar} &= \frac{P_{Ar}}{P} = \frac{y_{Ar, \text{dry}} P_{\text{dry air}}}{P} = \frac{0.01(98.156 \text{ kPa})}{101.325} = \mathbf{0.0097} \end{aligned}$$



(b) The molar masses of dry and saturated air are

$$\begin{aligned} M_{\text{dry air}} &= \sum y_i M_i = 0.781 \times 28.0 + 0.209 \times 32.0 + 0.01 \times 39.95 = 29.0 \text{ kg/kmol} \\ M_{\text{sat air}} &= \sum y_i M_i = 0.7566 \times 28.0 + 0.2025 \times 32.0 + 0.0097 \times 39.9 + 0.0313 \times 18 = 28.62 \text{ kg/kmol} \end{aligned}$$

Then the densities of dry and saturated air are determined from the ideal gas relation to be

$$\begin{aligned} \rho_{\text{dry air}} &= \frac{P}{(R_u / M_{\text{dry air}}) T} = \frac{101.325 \text{ kPa}}{[(8.314 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K}) / 29.0 \text{ kg/kmol}] (25 + 273) \text{ K}} = \mathbf{1.186 \text{ kg/m}^3} \\ \rho_{\text{sat air}} &= \frac{P}{(R_u / M_{\text{sat air}}) T} = \frac{101.325 \text{ kPa}}{[(8.314 \text{ kPa} \cdot \text{m}^3 / \text{kmol} \cdot \text{K}) / 28.62 \text{ kg/kmol}] (25 + 273) \text{ K}} = \mathbf{1.170 \text{ kg/m}^3} \end{aligned}$$

**Discussion** We conclude that the density of saturated air is less than that of the dry air, as expected. This is due to the molar mass of water being less than that of dry air.

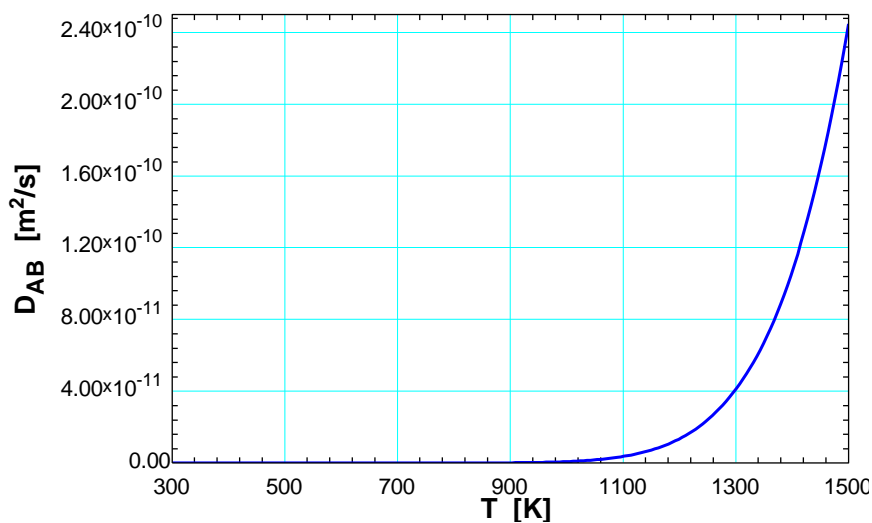


**14-160** Using the relation  $D_{AB} = 2.67 \times 10^{-5} \exp(-17,400/T)$  the diffusion coefficient of carbon in steel is to be plotted.

**Analysis** The problem is solved using EES, and the solution is given below.

$$D_{AB} = 2.67 \times 10^{-5} \exp(-17400/T)$$

$T, K$	$D_{AB}, m^2/s$
300	1.728E-30
400	3.425E-24
500	2.056E-20
600	6.792E-18
700	4.278E-16
800	9.563E-15
900	1.072E-13
1000	7.409E-13
1100	3.604E-12
1200	1.347E-11
1300	4.108E-11
1400	1.069E-10
1500	2.447E-10



**14-161** A circular pan filled with water is cooled naturally. The rate of evaporation of water, the rate of heat transfer by natural convection, and the rate of heat supply to the water needed to maintain its temperature constant are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 25°C). 2 The critical Reynolds number for flow over a flat plate is 500,000. 3 Radiation heat transfer is negligible. 4 Both air and water vapor are ideal gases.

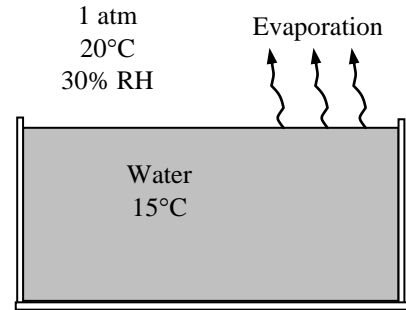
**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (15+20)/2 = 17.5^\circ\text{C} = 290.5\text{ K}$ . The properties of dry air at 17.5°C and 1 atm are, from Table A-15,

$$k = 0.02495\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.7316$$

$$\alpha = 2.042 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.493 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(290.5\text{ K})^{2.072}}{1\text{ atm}} = 2.37 \times 10^{-5}\text{ m}^2/\text{s} \end{aligned}$$



The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$ . Properties of water at 15°C are

$h_{fg} = 2466\text{ kJ/kg}$  and  $P_v = 1.7051\text{ kPa}$  (Table A-9). The specific heat of water at the average temperature of  $(15+20)/2 = 17.5^\circ\text{C}$  is  $c_p = 4.184\text{ kJ/kg}\cdot^\circ\text{C}$ . The gas constants of dry air and water are  $R_{\text{air}} = 0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  and  $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis** (a) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.30)P_{\text{sat}@20^\circ\text{C}} = (0.30)(2.339\text{ kPa}) = 0.7017\text{ kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

*At the surface:*

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15 + 273)\text{ K}} = 0.01283\text{ kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325 - 1.7051)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15 + 273)\text{ K}} = 1.2052\text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.01283 + 1.2052 = 1.21803\text{ kg/m}^3$$

*Away from the surface:*

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017\text{ kPa}}{(0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20 + 273)\text{ K}} = 0.00519\text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325 - 0.7017)\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20 + 273)\text{ K}} = 1.1966\text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.00519 + 1.1966 = 1.2018\text{ kg/m}^3$$

Note that  $\rho_\infty < \rho_s$ , and thus this corresponds to hot surface facing down. The area of the top surface of the water  $A_s = \pi r_o^2$  and its perimeter is  $p = 2\pi r_o$ . Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{\pi r_o^2}{2\pi r_o} = \frac{r_o}{2} = \frac{0.15\text{ m}}{2} = 0.075\text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.21803 - 1.2018 \text{ kg/m}^3)(0.075 \text{ m})^3}{[(1.2180 + 1.2018)/2 \text{ kg/m}^3](1.493 \times 10^{-5} \text{ m}^2/\text{s})^2} = 2.49 \times 10^5$$

Recognizing that this is a natural convection problem with cold horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be (Table 14-13)

$$\text{Nu} = 0.27(\text{Gr Pr})^{1/4} = 0.27(2.49 \times 10^5 \times 0.7316)^{1/4} = 5.58$$

and

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(5.58)(0.02495 \text{ W/m} \cdot ^\circ\text{C})}{0.075 \text{ m}} = 1.86 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_\infty - T_s) = (1.86 \text{ W/m}^2 \cdot ^\circ\text{C})[\pi(0.15 \text{ m})^2](20 - 15)^\circ\text{C} = \mathbf{0.66 \text{ W}} \quad (\text{to water})$$

(b) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.493 \times 10^{-5} \text{ m}^2/\text{s}}{2.37 \times 10^{-5} \text{ m}^2/\text{s}} = 0.630$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.27(\text{GrSc})^{1/4} = 0.27(2.53 \times 10^5 \times 0.629)^{1/4} = 5.39$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(5.39)(2.37 \times 10^{-5} \text{ m}^2/\text{s})}{0.075 \text{ m}} = 0.00170 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.00170 \text{ m/s})[\pi(0.15 \text{ m})^2](0.01283 - 0.00519) \text{ kg/m}^3 \\ &= 9.18 \times 10^{-7} \text{ kg/s} = \mathbf{0.0033 \text{ kg/h}} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (9.17 \times 10^{-7} \text{ kg/s})(2466 \text{ kJ/kg}) = 0.00226 \text{ kW} = 2.26 \text{ W}$$

(c) The net rate of heat transfer to the water needed to maintain its temperature constant at 15°C is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 2.26 + (-0.66) = \mathbf{1.6 \text{ W}}$$

**Discussion** Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

**14-162** Air is blown over a circular pan filled with water. The rate of evaporation of water, the rate of heat transfer by convection, and the rate of energy supply to the water to maintain its temperature constant are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 15°C). 2 The critical Reynolds number for flow over a flat plate is 500,000. 3 Radiation heat transfer is negligible. 4 Both air and water vapor are ideal gases.

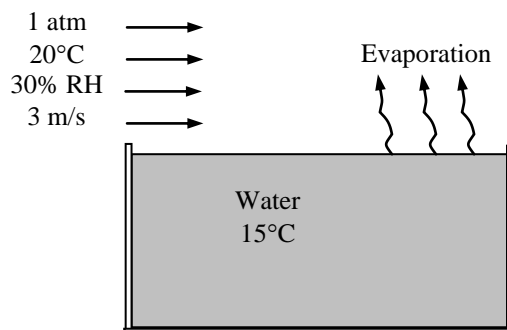
**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (15+20)/2 = 17.5^\circ\text{C} = 290.5\text{ K}$ . The properties of dry air at 17.5°C and 1 atm are, from Table A-15,

$$k = 0.02496\text{ W/m}\cdot^\circ\text{C}, \text{ Pr} = 0.7316$$

$$\alpha = 2.042 \times 10^{-5}\text{ m}^2/\text{s} \quad \nu = 1.493 \times 10^{-5}\text{ m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} \\ &= 1.87 \times 10^{-10} \frac{(290.5\text{ K})^{2.072}}{1\text{ atm}} = 2.37 \times 10^{-5}\text{ m}^2/\text{s} \end{aligned}$$



The saturation pressure of water at 20°C is  $P_{\text{sat}@20^\circ\text{C}} = 2.339\text{ kPa}$ . Properties of water at 15°C are

$h_{fg} = 2466\text{ kJ/kg}$  and  $P_v = 1.7051\text{ kPa}$  (Table A-9). Also, the gas constants of water is  $R_{\text{water}} = 0.4615\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$  (Table A-1).

**Analysis** (a) Taking the radius of the pan  $r_0 = 0.15\text{ m}$  to be the characteristic length, the Reynolds number for flow over the pan is

$$\text{Re} = \frac{VL}{\nu} = \frac{(3\text{ m/s})(0.15\text{ m})}{1.493 \times 10^{-5}\text{ m}^2/\text{s}} = 30,141$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Nusselt number and the heat transfer coefficient are

$$\text{Nu} = 0.664 \text{Re}_L^{0.5} \text{Pr}^{1/3} = 0.664(30,141)^{0.5} (0.7316)^{1/3} = 103.9$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(103.9)(0.02496\text{ W/m}\cdot^\circ\text{C})}{0.15\text{ m}} = 17.29\text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (17.29\text{ W/m}^2 \cdot ^\circ\text{C}) [\pi(0.15\text{ m})^2] (20 - 15)^\circ\text{C} = \mathbf{6.11\text{ W}} \quad (\text{to water})$$

(b) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.493 \times 10^{-5}\text{ m}^2/\text{s}}{2.37 \times 10^{-5}\text{ m}^2/\text{s}} = 0.630$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664 \text{Re}_L^{0.5} \text{Sc}^{1/3} = 0.664(30,141)^{0.5} (0.630)^{1/3} = 98.82$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(98.82)(2.37 \times 10^{-5}\text{ m}^2/\text{s})}{0.15\text{ m}} = 0.01561\text{ m/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from



$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.30)P_{\text{sat}@20^\circ\text{C}} = (0.30)(2.339 \text{ kPa}) = 0.7017 \text{ kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(15 + 273) \text{ K}} = 0.01283 \text{ kg/m}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017 \text{ kPa}}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273) \text{ K}} = 0.00519 \text{ kg/m}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.01561 \text{ m/s})[\pi(0.15 \text{ m})^2](0.01283 - 0.00519) \text{ kg/m}^3 \\ &= 8.430 \times 10^{-6} \text{ kg/s} = \mathbf{0.03035 \text{ kg/h}} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (8.430 \times 10^{-6} \text{ kg/s})(2466 \text{ kJ/kg}) = 0.02079 \text{ kW} = 20.79 \text{ W}$$

(c) The net rate of heat transfer to the water needed to maintain its temperature constant at  $15^\circ\text{C}$  is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 20.79 + (-6.11) = \mathbf{14.7 \text{ W}}$$

**Discussion** Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$ .

**14-163** Henry's law is expressed as

$$y_{i, \text{liquid side}}(0) = \frac{P_{i, \text{gas side}}(0)}{H}$$

Henry's constant  $H$  increases with temperature, and thus the fraction of gas  $i$  in the liquid  $y_{i, \text{liquid side}}$  decreases. Therefore, heating a liquid will drive off the dissolved gases in a liquid.

**14-164** A glass of water is left in a room. The mole fraction of the water vapor in the air at the water surface and far from the surface as well as the mole fraction of air in the water near the surface are to be determined when the water and the air are at the same temperature.

**Assumptions** **1** Both the air and water vapor are ideal gases. **2** Air is weakly soluble in water and thus Henry's law is applicable.

**Properties** The saturation pressure of water at 20°C is 2.339 kPa (Table A-9). Henry's constant for air dissolved in water at 20°C (293 K) is given in Table 14-6 to be  $H = 65,600$  bar. Molar masses of dry air and water are 29 and 18 kg/kmol, respectively (Table A-1).

**Analysis** (a) Noting that the relative humidity of air is 70%, the partial pressure of water vapor in the air far from the water surface will be

$$P_{v, \text{room air}} = \phi P_{\text{sat @ 20°C}} = (0.7)(2.339 \text{ kPa}) = 1.637 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the room air is

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{1.637 \text{ kPa}}{100 \text{ kPa}} = \mathbf{0.0164 \quad (\text{or } 1.64\%)}$$

(b) Noting that air at the water surface is saturated, the partial pressure of water vapor in the air near the surface will simply be the saturation pressure of water at 20°C,

$P_{v, \text{interface}} = P_{\text{sat @ 20°C}} = 2.339 \text{ kPa}$ . Then the mole fraction of water vapor in the air at the interface becomes

$$y_{v, \text{surface}} = \frac{P_{v, \text{surface}}}{P} = \frac{2.339 \text{ kPa}}{100 \text{ kPa}} = \mathbf{0.0234 \quad (\text{or } 2.34\%)}$$

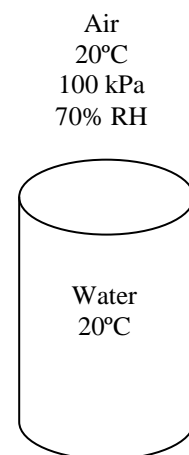
(c) Noting that the total pressure is 100 kPa, the partial pressure of dry air at the water surface is

$$P_{\text{air, surface}} = P - P_{v, \text{surface}} = 100 - 2.339 = 97.661 \text{ kPa}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air, liquid side}} = \frac{P_{\text{dry air, gas side}}}{H} = \frac{(97.661 / 101.325) \text{ bar}}{65,600 \text{ bar}} = \mathbf{1.47 \times 10^{-5} = 0.0015\%}$$

**Discussion** The water cannot remain at the room temperature when the air is not saturated. Therefore, some water will evaporate and the water temperature will drop until a balance is reached between the rate of heat transfer to the water and the rate of evaporation.



**14-165** A 0.1-mm thick soft rubber membrane separates pure O<sub>2</sub> from air. The mass flow rate of O<sub>2</sub> through the membrane per unit area and the direction of flow are to be determined.

**Assumptions** **1** Steady operating conditions exist. **2** Mass transfer through the membrane is one-dimensional. **3** The permeability of the membrane is constant.

**Properties** The mass diffusivity of oxygen in rubber at 298 K is  $D_{AB} = 2.1 \times 10^{-10} \text{ m}^2/\text{s}$  (Table 14-3b). The solubility of oxygen in rubber at 298 K is  $0.00312 \text{ kmol} / \text{m}^3 \cdot \text{bar}$  (Table 14-7). The molar mass of oxygen is  $32 \text{ kg} / \text{kmol}$  (Table A-1).

**Analysis** The molar fraction of oxygen in air is 0.21. Therefore, the partial pressure of oxygen in the air is

$$y_{\text{O}_2} = \frac{P_{\text{O}_2,2}}{P} \rightarrow P_{\text{O}_2,2} = y_{\text{O}_2} P = 0.21 \times (3 \text{ atm}) = 0.63 \text{ atm}$$

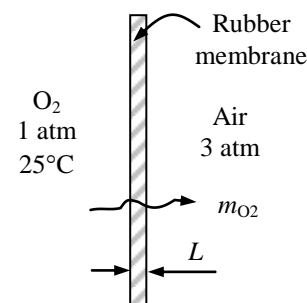
The partial pressure of oxygen on the other side is simply  $P_{\text{O}_2,1} = 1 \text{ atm}$ . Then the molar flow rate of oxygen through the membrane by diffusion can readily be determined to be

$$\begin{aligned} \dot{N}_{\text{diff,A,wall}} &= D_{AB} S \frac{P_{A,1} - P_{A,2}}{L} \\ &= (2.1 \times 10^{-10} \text{ m}^2/\text{s}) (0.00312 \text{ kmol}/\text{m}^3 \cdot \text{bar}) \frac{(1 - 0.63) \text{ atm}}{0.1 \times 10^{-3} \text{ m}} \left( \frac{1.01325 \text{ bar}}{1 \text{ atm}} \right) \\ &= 2.456 \times 10^{-9} \text{ kmol}/\text{m}^2 \cdot \text{s} \end{aligned}$$

Then the mass flow rate of oxygen gas through the membrane becomes

$$\dot{m}_{\text{diff}} = M \dot{N}_{\text{diff}} = (32 \text{ kg}/\text{kmol}) (2.456 \times 10^{-9} \text{ kmol}/\text{m}^2 \cdot \text{s}) = \mathbf{7.86 \times 10^{-8} \text{ kg}/\text{m}^2 \cdot \text{s}}$$

The positive sign indicates that the direction of the flow will be from the pure oxygen inside to the air outside.



**14-166** The walls of a house are made of 20-cm thick bricks. The maximum amount of water vapor that will diffuse through a  $3\text{ m} \times 5\text{ m}$  section of the wall in 24-h is to be determined.

**Assumptions** **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant. **4** The vapor pressure at the outer side of the wall is zero.

**Properties** The permeance of the brick wall is given to be  $23 \times 10^{-12}\text{ kg/s.m}^2\text{.Pa}$ . The saturation pressure of water at  $20^\circ\text{C}$  is 2339 Pa (Table 14-9).

**Analysis** The mass flow rate of water vapor through a plain layer of thickness  $L$  and normal area  $A$  is given by (Eq. 14-31)

$$\dot{m}_v = PA \frac{P_{v,1} - P_{v,2}}{L} = PA \frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L} = MA(\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2})$$

where  $P$  is the vapor permeability and  $M = P/L$  is the permeance of the material,  $\phi$  is the relative humidity and  $P_{\text{sat}}$  is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

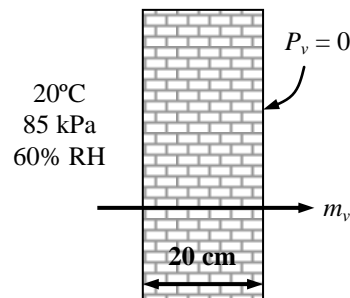
Noting that the vapor pressure at the outer side of the wallboard is zero ( $\phi_2 = 0$ ) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (23 \times 10^{-12}\text{ kg/s.m}^2\text{.Pa})(3 \times 5\text{ m}^2)[0.60(2339\text{ Pa}) - 0] = 4.842 \times 10^{-7}\text{ kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24\text{-h}} = \dot{m}_v \Delta t = (4.842 \times 10^{-7}\text{ kg/s})(24 \times 3600\text{ s}) = \mathbf{0.0418\text{ kg} = 41.8\text{ g}}$$

**Discussion** This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.



**14-167** A nickel part is put into a room filled with hydrogen. The ratio of hydrogen concentrations at the surface of the part and at a depth of 2-mm from the surface after 24 h is to be determined.

**Assumptions** 1 Hydrogen penetrates into a thin layer beneath the surface of the nickel component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. 2 The initial hydrogen concentration in the nickel part is zero.

**Properties** The molar mass of hydrogen  $H_2$  is  $M = 2 \text{ kg/kmol}$  (Table A-1). The solubility of hydrogen in nickel at 358 K ( $=85^\circ\text{C}$ ) is  $0.00901 \text{ kmol/m}^3 \cdot \text{bar}$  (Table 14-7). The mass diffusivity of hydrogen in nickel at 358 K is  $D_{AB} = 1.2 \times 10^{-12} \text{ m}^2/\text{s}$  (Table 14-3b). Also,  $1 \text{ atm} = 1.01325 \text{ bar}$ .

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x, t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The molar density of hydrogen in the nickel at the interface is determined from Eq. 14-20 to be

$$\begin{aligned} C_{H_2, \text{solid side}}(0) &= S \times P_{H_2, \text{gas side}} \\ &= (0.00901 \text{ kmol/m}^3 \cdot \text{bar})(3 \times 1.01325 \text{ bar}) \\ &= 0.0274 \text{ kmol/m}^3 \end{aligned}$$

The argument of the complementary error function is

$$\xi = \frac{x}{2\sqrt{D_{AB}t}} = \frac{2 \times 10^{-3} \text{ m}}{2\sqrt{(1.2 \times 10^{-12} \text{ m}^2/\text{s})(24 \times 3600 \text{ s})}} = 3.106$$

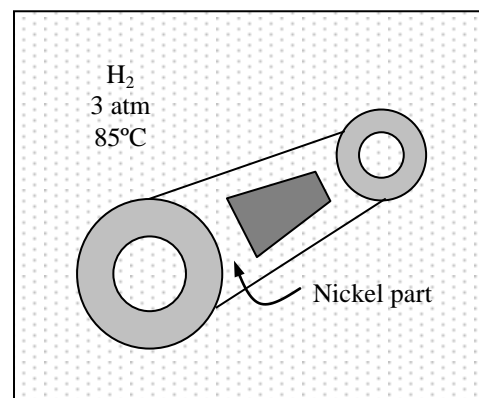
The corresponding value of the complementary error function is determined from Table 4-4 to be

$$\text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right) = \text{erfc}(3.105) = 0.000015$$

Substituting the known quantities,

$$\frac{C_A(x, t) - 0}{0.0274 - 0} = 0.000015 \rightarrow C_A(x, t) = 4.1 \times 10^{-7} \text{ kmol/m}^3$$

Therefore, the hydrogen concentration in the steel component at a depth of 2 mm in 24 h is very small.



**14-168** A tanker truck carrying liquid herbicide overturned and caused a spill over a field. The depth of the soil at which plant and insect life is likely to be affected by the spill is to be determined.

**Assumptions** The herbicide-soaked soil can be modeled as a semi-infinite medium.

**Properties** The relevant properties are given in the problem statement.

**Analysis** This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{w_A(x, t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.001 - 0}{1 - 0} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

From Table 4-4, we have  $0.001 = \operatorname{erfc}(2.33)$ , hence

$$\begin{aligned}\frac{x}{2\sqrt{D_{AB}t}} &= 2.33 \\ x &= 2\sqrt{D_{AB}t} (2.33) \\ &= 2\sqrt{(2 \times 10^{-8} \text{ m}^2/\text{s})(1800 \text{ s})} (2.33) \\ &= 0.0280 \text{ m} \\ &= \mathbf{2.8 \text{ cm}}\end{aligned}$$

**Discussion** The spill will likely affect life down to about 3 cm from the soil surface.

**14-169** An aquarium is oxygenated by forcing oxygen to the bottom of it, and letting the oxygen bubbles rise. The penetration depth of oxygen in the water during the rising time is to be determined.

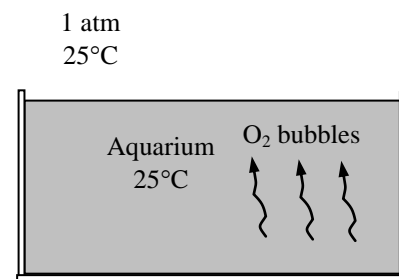
**Assumptions** **1** Convection effects in the water are negligible. **2** The pressure and temperature of the oxygen bubbles remain constant.

**Properties** The mass diffusivity of oxygen in liquid water at 298 K is  $D_{AB} = 2.4 \times 10^{-9} \text{ m}^2/\text{s}$  (Table 14-3b).

**Analysis** The penetration depth can be determined directly from its definition (Eq. 14-38) to be

$$\begin{aligned}\delta_{\text{diff}} &= \sqrt{\pi D_{AB} t} = \sqrt{\pi (2.4 \times 10^{-9} \text{ m}^2/\text{s})(4 \text{ s})} \\ &= 1.73 \times 10^{-4} \text{ m} = \mathbf{0.173 \text{ mm}}\end{aligned}$$

Therefore, oxygen will penetrate the water only a fraction of a millimeter.



**14-170** A sphere of crystalline sodium chloride (NaCl) was suspended in a stirred tank filled with water. The average mass transfer coefficient is to be determined.

**Assumptions** **1** The properties of NaCl are constant.

**Properties** The density of NaCl and its solubility in water at 20°C are given to be  $2160 \text{ kg/m}^3$  and  $320 \text{ kg/m}^3$ , respectively.

**Analysis** The initial diameter of the sphere is

$$m_1 = \rho V = \rho \frac{\pi D_1^3}{6} \longrightarrow 0.100 \text{ kg} = (2160 \text{ kg/m}^3) \frac{\pi D_1^3}{6} \longrightarrow D_1 = 0.04455 \text{ m}$$

The final diameter of the sphere is

$$m_2 = \rho V = \rho \frac{\pi D_2^3}{6} \longrightarrow (0.90)(0.100 \text{ kg}) = (2160 \text{ kg/m}^3) \frac{\pi D_2^3}{6} \longrightarrow D_2 = 0.04301 \text{ m}$$

The rate of mass change is

$$\dot{m} = \frac{m_1 - m_2}{\Delta t} = \frac{(0.100 - 0.090) \text{ kg}}{10 \times 60 \text{ s}} = 1.667 \times 10^{-5} \text{ kg/s}$$

The average surface area is

$$A_s = \frac{\pi D_1^2 + \pi D_2^2}{2} = \frac{\pi (0.04455 \text{ m})^2 + \pi (0.04301 \text{ m})^2}{2} = 6.023 \times 10^{-3} \text{ m}^2$$

The mass transfer coefficient is determined from

$$h_{\text{mass}} = \frac{\dot{m}}{A_s \Delta \rho_A} = \frac{1.667 \times 10^{-5} \text{ kg/s}}{(6.023 \times 10^{-3} \text{ m}^2)(320 - 0) \text{ kg/m}^3} = \mathbf{8.65 \times 10^{-6} \text{ m/s}}$$

**14-171** Liquid toluene evaporates into air from the open-top of a cylindrical container. The concentration of toluene at a certain location is to be determined.

**Properties** The molar mass of toluene is 92 kg/kmol. The diffusion coefficient of toluene at 25°C is given to be  $D_{AB} = 0.084 \times 10^{-4} \text{ m}^2/\text{s}$ .

**Analysis** The vapor pressure of toluene is

$$P_{A,0} = \frac{10 \text{ mmHg}}{760 \text{ mmHg}} (101,325 \text{ kPa}) = 1333 \text{ Pa}$$

The rate of evaporation can be expressed by

$$\frac{0.060 \text{ kg/day}}{24 \times 3600 \text{ s/day}} = \frac{\dot{N}_A}{A} \frac{\pi(0.2 \text{ m})^2}{4} (92 \text{ kg/kmol}) \longrightarrow \frac{\dot{N}_A}{A} = 2.403 \times 10^{-7} \text{ kmol/m}^2 \cdot \text{s}$$

The diffusion coefficient at 6.4°C is determined from

$$D_{AB} = (0.084 \times 10^{-4} \text{ m}^2/\text{s}) \left( \frac{6.4 + 273}{25 + 273} \right)^{1.5} = 7.63 \times 10^{-6} \text{ m}^2/\text{s}$$

The vapor pressure of toluene at 10 mm above the surface is determined from

$$\begin{aligned} \frac{\dot{N}_A}{A} &= \frac{D_{AB} P}{LR_u T} \ln \left( \frac{P - P_{A,L}}{P - P_{A,0}} \right) \\ 2.403 \times 10^{-7} \text{ kmol/m}^2 \cdot \text{s} &= \frac{(7.63 \times 10^{-6} \text{ m}^2/\text{s})(101,325 \text{ Pa})}{(0.010 \text{ m})(8314 \text{ Pa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(6.4 + 273 \text{ K})} \ln \left( \frac{101,325 - P_{A,L}}{101,325 - 1333} \right) \\ P_{A,L} &= 608.4 \text{ Pa} \end{aligned}$$

Then the concentration of toluene is determined to be

$$C_{A,L} = \frac{P_{A,L}}{R_u T} M = \frac{608.4 \text{ Pa}}{(8314 \text{ Pa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(6.4 + 273 \text{ K})} (92 \text{ kg/kmol}) = 0.02410 \text{ kg/m}^3 = \mathbf{24.1 \text{ g/m}^3}$$



**14-172** Liquid methanol is accidentally spilt on a 1 m × 1 m laboratory bench while a fan is providing a 20 m/s air flow parallel over the bench surface. The evaporation rate of methanol in molar basis is to be determined.

**Assumptions** 1 The analogy between heat and mass transfer is applicable. 2 The critical Reynolds number for flow over a flat plate is 500,000. 3 Methanol vapor is an ideal gas.

**Properties** The kinematic viscosity of air at 25°C and 1 atm is  $\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The diffusion coefficient of methanol vapor in air at 25°C = 298 K is  $D_{AB} = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$ .

**Analysis** The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(20 \text{ m/s})(1 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 1.2804 \times 10^6$$

which is greater than 500,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{1.6 \times 10^{-5} \text{ m}^2/\text{s}} = 0.9763$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.037 \text{Re}^{0.8} \text{Sc}^{1/3} = 0.037(1.2804 \times 10^6)^{0.8} (0.9763)^{1/3} = 2822$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(2822)(1.6 \times 10^{-5} \text{ m}^2/\text{s})}{1 \text{ m}} = 0.04515 \text{ m/s}$$

The concentration of methanol vapor at the air-methanol interface can be determined using

$$C_{A,s} = \frac{P_A}{R_u T} = \frac{4 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(273 + 25) \text{ K}} = 0.001614 \text{ kmol/m}^3$$

Hence, the evaporation rate of methanol in molar basis is

$$\begin{aligned} \dot{N}_{\text{conv}} &= h_{\text{mass}} A_s (C_{A,s} - C_{A,\infty}) \\ &= (0.04515 \text{ m/s})(1 \text{ m}^2)(0.001614 - 0) \text{ kmol/m}^3 \\ &= \mathbf{7.29 \times 10^{-5} \text{ kmol/s}} \end{aligned}$$

**Discussion** Methanol has a molar mass of  $M = 32.04 \text{ kg/kmol}$ . Hence the evaporation rate in mass basis is

$$\dot{m}_{\text{conv}} = \dot{N}_{\text{conv}} M = (7.29 \times 10^{-5} \text{ kmol/s})(32.04 \text{ kg/kmol}) = 2.33 \times 10^{-3} \text{ kg/s}$$

**14-173E** The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

**Assumptions** 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). 2 Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). 3 The water in the pool is maintained at a uniform temperature of 80°F. 4 The critical Reynolds number for flow over a flat surface is 500,000.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (70+80)/2 = 75^\circ\text{F}$ .

The properties of dry air at 75°F and 1 atm are, from Table A-15E,

$$k = 0.01469 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

$$\text{Pr} = 0.7298$$

$$\alpha = 2.288 \times 10^{-4} \text{ ft}^2/\text{s}$$

$$\nu = 0.167 \times 10^{-3} \text{ ft}^2/\text{s}$$

The saturation pressure of water at 70°F is

$$P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia. Properties of water at } 80^\circ\text{F are}$$

$h_{fg} = 1048 \text{ Btu/lbm}$  and  $P_v = 0.5073 \text{ psia}$  (Table A-9). The gas constant of water is  $R_{\text{water}} = 0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}$  (Table A-1E). The emissivity of water is 0.95 (Table A-18). The mass diffusivity of water vapor in air at the average temperature of  $75^\circ\text{F} = 535 \text{ R} = 297.2 \text{ K}$  is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(297.2 \text{ K})^{2.072}}{1 \text{ atm}} = 2.49 \times 10^{-5} \text{ m}^2/\text{s} = 2.68 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** (a) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40 \times 5280 / 3600 \text{ ft/s})(100 \text{ ft})}{0.167 \times 10^{-3} \text{ ft}^2/\text{s}} = 3.51 \times 10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

$$\text{Nu} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} = 0.037(3.51 \times 10^7)^{0.8} (0.7298)^{1/3} = 36,212$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(36,212)(0.01469 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{100 \text{ ft}} = 5.32 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

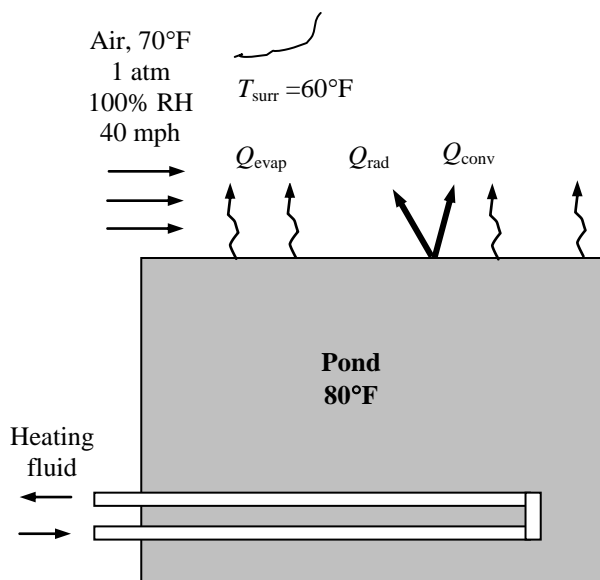
$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (5.32 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(10,000 \text{ ft}^2)(80 - 70)^\circ\text{F} = \mathbf{532,000 \text{ Btu/h}} \quad (\text{to water})$$

(b) Noting that the emissivity of water is 0.95 and the surface area of the pool is  $A_s = (100 \text{ ft})(100 \text{ ft}) = 10,000 \text{ ft}^2$ , heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(10,000 \text{ ft}^2)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)[(540 \text{ R})^4 - (520 \text{ R})^4] = \mathbf{194,000 \text{ Btu/h}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.167 \times 10^{-3} \text{ ft}^2/\text{s}}{2.68 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.623$$



Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.037 \text{Re}_L^{0.8} \text{Sc}^{1/3} = 0.037(3.51 \times 10^7)^{0.8} (0.623)^{1/3} = 34,350$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(34,350)(2.68 \times 10^{-4} \text{ ft}^2/\text{s})}{100 \text{ ft}} = 0.0921 \text{ ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ( $P_{v,s} = 0.5073$  psia at  $80^\circ\text{F}$ ). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore,  $P_{v,\infty} = P_{\text{sat}@70^\circ\text{F}} = 0.3632$  psia.

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073 \text{ psia}}{(0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(80 + 460) \text{ R}} = 0.00158 \text{ lbm/ft}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.3632 \text{ psia}}{(0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(70 + 460) \text{ R}} = 0.00115 \text{ lbm/ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0921 \text{ ft/s})(10,000 \text{ ft}^2)(0.00158 - 0.00115) \text{ lbm/ft}^3 \\ &= 0.396 \text{ lbm/s} = 1426 \text{ lbm/h} \end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (1426 \text{ lbm/h})(1048 \text{ Btu/lbm}) = \mathbf{1,494,000 \text{ Btu/h}}$$

**Discussion** All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 194,000 + 532,000 + 1,494,000 = 2,220,000 \text{ Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation

$$\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A_s (w_{A,s} - w_{A,\infty})$$

**14-174E** The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

**Assumptions** 1 The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 90°F). 2 Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). 3 The water in the pool is maintained at a uniform temperature of 90°F. 4 The critical Reynolds number for flow over a flat surface is 500,000.

**Properties** The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of  $(T_\infty + T_s)/2 = (70+90)/2 = 80^\circ\text{F}$ . The properties of dry air at 80°F and 1 atm are, from Table A-15E,

$$k = 0.01481 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

$$\text{Pr} = 0.7290$$

$$\alpha = 2.328 \times 10^{-4} \text{ ft}^2/\text{h}$$

$$\nu = 1.697 \times 10^{-4} \text{ ft}^2/\text{s}$$

The saturation pressure of water at 70°F is

$$P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia. Properties of water at } 90^\circ\text{F are}$$

$$h_{fg} = 1043 \text{ Btu/lbm and } P_v = 0.6988 \text{ psia (Table A-9).}$$

The gas constant of water is  $R_{\text{water}} = 0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}$  (Table A-1E). The emissivity of water is 0.95 (Table A-18). The mass diffusivity of water vapor in air at the average temperature of 80°F = 540 R = 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(300\text{K})^{2.072}}{1 \text{ atm}} = 2.54 \times 10^{-5} \text{ m}^2/\text{s} = 2.73 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** (a) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40 \times 5280 / 3600 \text{ ft/s})(100 \text{ ft})}{1.697 \times 10^{-4} \text{ ft}^2/\text{s}} = 3.46 \times 10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

$$\text{Nu} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} = 0.037(3.46 \times 10^7)^{0.8} (0.7290)^{1/3} = 35,785$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(35,785)(0.01481 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{100 \text{ ft}} = 5.30 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

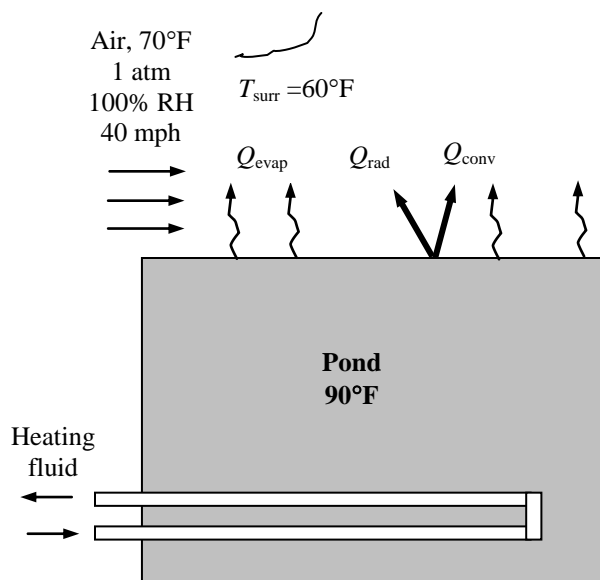
$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (5.30 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(10,000 \text{ ft}^2)(90 - 70)^\circ\text{F} = \mathbf{1,060,000 \text{ Btu/h}} \quad (\text{to water})$$

(b) Noting that the emissivity of water is 0.95 and the surface area of the pool is  $A_s = (100 \text{ ft})(100 \text{ ft}) = 10,000 \text{ ft}^2$ , heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(10,000 \text{ ft}^2)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)[(550 \text{ R})^4 - (520 \text{ R})^4] = \mathbf{299,400 \text{ Btu/h}}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.697 \times 10^{-4} \text{ ft}^2/\text{s}}{2.73 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.622$$



Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.037 \text{Re}_L^{0.8} \text{Sc}^{1/3} = 0.037(3.46 \times 10^7)^{0.8} (0.622)^{1/3} = 33,940$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(33,940)(2.73 \times 10^{-4} \text{ ft}^2/\text{s})}{100 \text{ ft}} = 0.0927 \text{ ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ( $P_{v,s} = 0.6988 \text{ psia}$  at  $90^\circ\text{F}$ ). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore,  $P_{v,\infty} = P_{\text{sat}@70^\circ\text{F}} = 0.3632 \text{ psia}$ .

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface:} \quad \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.6988 \text{ psia}}{(0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(90 + 460) \text{ R}} = 0.00213 \text{ lbm/ft}^3$$

$$\text{Away from the surface:} \quad \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.3632 \text{ psia}}{(0.5957 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(70 + 460) \text{ R}} = 0.00115 \text{ lbm/ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.0927 \text{ ft/s})(10,000 \text{ ft}^2)(0.00213 - 0.00115) \text{ lbm/ft}^3 \\ &= 0.908 \text{ lbm/s} = 3269 \text{ lbm/h} \end{aligned}$$

$$\text{and} \quad \dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (3269 \text{ lbm/h})(1043 \text{ Btu/lbm}) = \mathbf{3,410,000 \text{ Btu/h}}$$

**Discussion** All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 299,400 + 1,060,000 + 3,410,000 = 4,769,400 \text{ Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation

$$\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty}).$$

**14-175E** A swimmer extends his wet arms into the windy air outside. The rate at which water evaporates from both arms and the corresponding rate of heat transfer by evaporation are to be determined.

**Assumptions** 1 The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). 2 The arm can be modeled as a long cylinder.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of  $(40 + 80)/2 = 60^\circ\text{F}$  and 1 atm, for which  $\nu = 1.588 \times 10^{-4} \text{ ft}^2/\text{s}$ , and  $\rho = 0.07633 \text{ lbm}/\text{ft}^3$  (Table A-15E). The saturation pressure of water at 40°F is 0.1217 psia. Also, at 80°F, the saturation pressure is 0.5073 psia and the heat of vaporization is 1048 Btu/lbm (Table A-9E). The gas constant of water is  $R = 0.5957 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R}$  (Table A-1E). The mass diffusivity of water vapor in air at 60°F = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O}-\text{air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P} = 1.87 \times 10^{-10} \frac{(288.9 \text{ K})^{2.072}}{1 \text{ atm}} = 2.35 \times 10^{-5} \text{ m}^2/\text{s} = 2.53 \times 10^{-4} \text{ ft}^2/\text{s}$$

**Analysis** The Reynolds number for flow over a cylinder is

$$\text{Re} = \frac{VD}{\nu} = \frac{(20 \times 5280 / 3600 \text{ ft/s})(3/12 \text{ ft})}{1.588 \times 10^{-4} \text{ ft}^2/\text{s}} = 46,180$$

The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.588 \times 10^{-4} \text{ ft}^2/\text{s}}{2.53 \times 10^{-4} \text{ ft}^2/\text{s}} = 0.628$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined from Table 14-13 or by replacing Pr number by the Schmidt number in Eq. 7-35, the result is

$$\text{Sh} = 0.3 + \frac{0.62 \text{Re}^{0.5} \text{Sc}^{1/3}}{\left[1 + (0.4/\text{Sc})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}}{282,000}\right)^{5/8}\right]^{4/5} = 0.3 + \frac{0.62(46,180)^{0.5} (0.628)^{1/3}}{\left[1 + (0.4/0.628)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{46,180}{282,000}\right)^{5/8}\right]^{4/5} = 125$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(125)(2.53 \times 10^{-4} \text{ ft}^2/\text{s})}{3/12 \text{ ft}} = 0.1265 \text{ ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.5073 psia at 80°F). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.50)P_{\text{sat}@40^\circ\text{F}} = (0.50)(0.1217 \text{ psia}) = 0.0609 \text{ psia}$$

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

$$\text{At the surface: } \rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073 \text{ psia}}{(0.5957 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(80 + 460) \text{ R}} = 0.00158 \text{ lbm}/\text{ft}^3$$

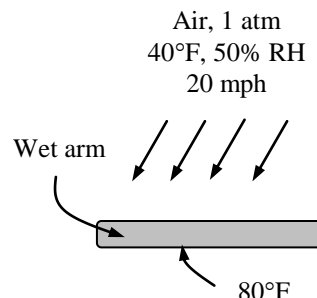
$$\text{Away from the surface: } \rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.0609 \text{ psia}}{(0.5957 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(40 + 460) \text{ R}} = 0.000204 \text{ lbm}/\text{ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned} \dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.1265 \text{ ft/s})[2 \times \pi(3/12 \text{ ft})(2 \text{ ft})](0.00158 - 0.000204) \text{ lbm}/\text{ft}^3 \\ &= 5.47 \times 10^{-4} \text{ lbm/s} = \mathbf{1.97 \text{ lbm/h}} \end{aligned}$$

$$\text{and } \dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (5.47 \times 10^{-4} \text{ lbm/s})(1048 \text{ Btu/lbm}) = \mathbf{0.573 \text{ Btu/s}}$$

**Discussion** The rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation  $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$ .



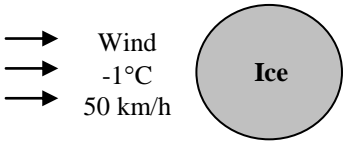
**14-176** A sphere of ice is exposed to wind. The ice evaporation rate is to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

**Properties** The properties are given in problem statement.

**Analysis** The Reynolds and Schmidt numbers are

$$\text{Re} = \frac{VD}{\nu} = \frac{(50/3.6 \text{ m/s})(0.05 \text{ m})}{1.32 \times 10^{-7} \text{ m}^2/\text{s}} = 5.26 \times 10^6$$

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.32 \times 10^{-7} \text{ m}^2/\text{s}}{2.5 \times 10^{-5} \text{ m}^2/\text{s}} = 5.28 \times 10^{-3}$$


The Sherwood number is

$$\text{Sh} = \left[ 4 + 1.21(\text{Re} \text{Sc})^{2/3} \right]^{0.5} = \left[ 4 + 1.21 \left[ (5.26 \times 10^6)(5.28 \times 10^{-3}) \right]^{2/3} \right]^{0.5} = 33.4$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh} D_{AB}}{D} = \frac{(33.4)(2.5 \times 10^{-5} \text{ m}^2/\text{s})}{0.05 \text{ m}} = 0.0167 \text{ m/s}$$

The evaporation rate is determined as follows:

$$\begin{aligned} \dot{N} &= h_{\text{mass}} \Delta C = h_{\text{mass}} \left( \frac{P_{A,0}}{R_u T} - \frac{P_{A,L}}{R_u T} \right) = h_{\text{mass}} \frac{P_v}{R_u T} (1 - 0.10) \\ &= (0.0167 \text{ m/s}) \frac{0.56 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{kg})(272 \text{ K})} (1 - 0.10) \\ &= 3.72 \times 10^{-6} \text{ kmol/m}^2 \cdot \text{s} \\ \dot{m}_{\text{evap}} &= \dot{N} M A = (3.72 \times 10^{-6} \text{ kmol/m}^2 \cdot \text{s})(18 \text{ kg/kmol}) \left[ \pi (0.05 \text{ m})^2 \right] \\ &= 5.26 \times 10^{-7} \text{ kg/s} = \mathbf{1.89 \text{ g/h}} \end{aligned}$$

**14-177** Benzene-free air flows in a tube whose inner surface is coated with pure benzene. The average mass transfer coefficient, the molar concentration of benzene in the outlet air, and the evaporation rate of benzene are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable. **2** The flow is fully developed.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, for which  $\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The mass diffusivity of benzene in air at 298 K is  $D_{AB} = 0.88 \times 10^{-5} \text{ m}^2/\text{s}$  (Table 14-2). The molar mass of benzene is 78 kg/kmol.

**Analysis** (a) The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5 \text{ m/s})(0.05 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 16,005$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562 \times 10^{-5} \text{ m}^2/\text{s}}{0.88 \times 10^{-5} \text{ m}^2/\text{s}} = 1.775$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023 \text{Re}^{0.8} \text{Sc}^{0.4} = 0.023(16,005)^{0.8} (1.775)^{0.4} = 66.8$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(66.8)(0.88 \times 10^{-5} \text{ m}^2/\text{s})}{0.05 \text{ m}} = \mathbf{0.0118 \text{ m/s}}$$

(b) The molar concentration of benzene in the outlet air is determined as follows

$$C_s = \frac{P_v}{R_u T} = \frac{13 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(25 + 273 \text{ K})} = 5.25 \times 10^{-3} \text{ kmol/m}^3$$

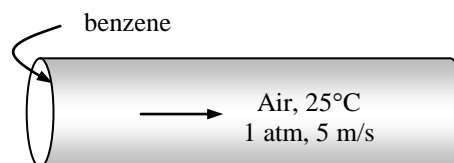
$$VA_c(C_{\text{out}} - C_{\text{in}}) = h_{\text{mass}} A \left[ \frac{(C_s - C_{\text{in}}) - (C_s - C_{\text{out}})}{\ln \left( \frac{C_s - C_{\text{in}}}{C_s - C_{\text{out}}} \right)} \right]$$

$$(5)(\pi \times 0.05^2 / 4)(C_{\text{out}} - 0) = (0.0118)(\pi \times 0.05 \times 6) \left[ \frac{(5.25 \times 10^{-3} - 0) - (5.25 \times 10^{-3} - C_{\text{out}})}{\ln \left( \frac{5.25 \times 10^{-3} - 0}{5.25 \times 10^{-3} - C_{\text{out}}} \right)} \right]$$

$$\longrightarrow C_{\text{out}} = \mathbf{3.56 \times 10^{-3} \text{ kmol/m}^3}$$

(c) The evaporation rate of benzene is determined from

$$\begin{aligned} \dot{m}_{\text{evap}} &= MVA_c(C_{\text{out}} - C_{\text{in}}) \\ &= (78 \text{ kg/kmol})(5 \text{ m/s}) \frac{\pi \times (0.05 \text{ m})^2}{4} (3.56 \times 10^{-3} \text{ kmol/m}^3 - 0) \\ &= 2.73 \times 10^{-3} \text{ kg/s} = \mathbf{9.81 \text{ kg/h}} \end{aligned}$$





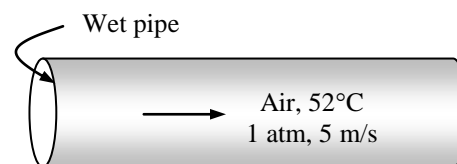
**14-178** The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The average mass transfer coefficient, log-mean driving force for mass transfer (in molar concentration units, the evaporation rate, and the tube length are to be determined.

**Assumptions 1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 325 K). **2** The flow is fully developed.

**Properties** Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 52°C and 1 atm, for which  $\nu = 1.818 \times 10^{-5} \text{ m}^2/\text{s}$  (Table A-15). The mass diffusivity of water vapor in air at  $52+273 = 325 \text{ K}$  is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10} \frac{T^{2.072}}{P}$$

$$= 1.87 \times 10^{-10} \frac{(325 \text{ K})^{2.072}}{1} = 3.00 \times 10^{-5} \text{ m}^2/\text{s}$$



**Analysis** (a) The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5 \text{ m/s})(0.05 \text{ m})}{1.818 \times 10^{-5} \text{ m}^2/\text{s}} = 13,750$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.818 \times 10^{-5} \text{ m}^2/\text{s}}{3.00 \times 10^{-5} \text{ m}^2/\text{s}} = 0.606$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023 \text{Re}^{0.8} \text{Sc}^{0.4} = 0.023(13,750)^{0.8} (0.606)^{0.4} = 38.49$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(38.49)(3.00 \times 10^{-5} \text{ m}^2/\text{s})}{0.05 \text{ m}} = \mathbf{0.02309 \text{ m/s}}$$

(b) The log-mean driving force for mass transfer (in molar concentration units) is determined as follows

$$C_w = \frac{P_v}{R_u T} = \frac{13.6 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(325 \text{ K})} = 5.03 \times 10^{-3} \text{ kmol/m}^3$$

$$C_{in} = \frac{P_v}{R_u T} = \frac{0.1 \times 13.6 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(325 \text{ K})} = 5.03 \times 10^{-4} \text{ kmol/m}^3$$

$$C_{out} = \frac{P_v}{R_u T} = \frac{10.0 \text{ kPa}}{(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(325 \text{ K})} = 3.70 \times 10^{-3} \text{ kmol/m}^3$$

$$\Delta C = \frac{(C_w - C_{in}) - (C_w - C_{out})}{\ln\left(\frac{C_w - C_{in}}{C_w - C_{out}}\right)}$$

$$= \frac{(5.03 \times 10^{-3} - 5.03 \times 10^{-4}) - (5.03 \times 10^{-3} - 3.70 \times 10^{-3})}{\ln\left(\frac{5.03 \times 10^{-3} - 5.03 \times 10^{-4}}{5.03 \times 10^{-3} - 3.70 \times 10^{-3}}\right)} = \mathbf{2.61 \times 10^{-3} \text{ kmol/m}^3}$$

(c) The evaporation rate is determined from

$$\dot{m}_{\text{evap}} = MVA_c(C_{out} - C_{in})$$

$$= (18 \text{ kg/kmol})(5 \text{ m/s}) \frac{\pi(0.05 \text{ m})^2}{4} (3.70 \times 10^{-3} - 5.03 \times 10^{-4}) \text{ kmol/m}^3$$

$$= 5.65 \times 10^{-4} \text{ kg/s} = \mathbf{2.03 \text{ kg/h}}$$

(d) The tube length is determined from

$$\frac{\dot{m}_{\text{evap}}}{M} = h_{\text{mass}} A \Delta C \rightarrow \frac{5.65 \times 10^{-4} \text{ kg/s}}{18 \text{ kg/kmol}} = (0.02309 \text{ m/s}) \pi (0.05 \text{ m}) L (2.61 \times 10^{-3} \text{ kmol/m}^3) \rightarrow L = \mathbf{3.32 \text{ m}}$$

## Fundamentals of Engineering (FE) Exam Problems

**14-179** The basic equation describing the diffusion of one medium through another stationary medium is

- (a)  $j_A = -CD_{AB} \frac{d(C_A / C)}{dx}$       (b)  $j_A = -D_{AB} \frac{d(C_A / C)}{dx}$   
 (c)  $j_A = -k \frac{d(C_A / C)}{dx}$       (d)  $j_A = -k \frac{dT}{dx}$       e) None of them

*Answer* (a)  $j_A = -CD_{AB} \frac{d(C_A / C)}{dx}$

**14-180** For the absorption of a gas, like carbon dioxide, into a liquid, like water, Henry's law states that partial pressure of the gas is proportional to the mole fraction of the gas in the liquid-gas solution with the constant of proportionality being Henry's constant. A bottle of soda pop (CO<sub>2</sub>-H<sub>2</sub>O) at room temperature has a Henry's constant of 17,100 kPa. If the pressure in this bottle is 120 kPa and the partial pressure of the water vapor in the gas volume at the top of the bottle is neglected, the concentration of the CO<sub>2</sub> in the liquid H<sub>2</sub>O is

- (a) 0.003 mol-CO<sub>2</sub>/mol      (b) 0.007 mol-CO<sub>2</sub>/mol      (c) 0.013 mol-CO<sub>2</sub>/mol  
 (d) 0.022 mol-CO<sub>2</sub>/mol      (e) 0.047 mol-CO<sub>2</sub>/mol

*Answer* (b) 0.007 mol-CO<sub>2</sub>/mol

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

H=17.1 [MPa]  
 P=0.120 [MPa]  
 y=P/H

**14-181** A rubber object is in contact with nitrogen ( $N_2$ ) at 298 K and 250 kPa. The solubility of nitrogen gas in rubber is  $0.00156 \text{ kmol/m}^3 \cdot \text{bar}$ . The mass density of nitrogen at the interface is

- (a)  $0.049 \text{ kg/m}^3$       (b)  $0.064 \text{ kg/m}^3$       (c)  $0.077 \text{ kg/m}^3$       (d)  $0.092 \text{ kg/m}^3$       (e)  $0.109 \text{ kg/m}^3$

*Answer* (e)  $0.109 \text{ kg/m}^3$

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=298 [K]
P_N2_gasside=250 [kPa]*Convert(kPa, bar)
S=0.00156 [kmol/m^3-bar] "Table 14-7"
C_N2_solidside=S*P_N2_gasside
M_N2=MolarMass(N2)
rho_N2_solidside=C_N2_solidside*M_N2
```

**14-182** Nitrogen gas at high pressure and 298 K is contained in a  $2\text{-m} \times 2\text{-m} \times 2\text{-m}$  cubical container made of natural rubber whose walls are 4 cm thick. The concentration of nitrogen in the rubber at the inner and outer surfaces are  $0.067 \text{ kg/m}^3$  and  $0.009 \text{ kg/m}^3$ , respectively. The diffusion coefficient of nitrogen through rubber is  $1.5 \times 10^{-10} \text{ m}^2/\text{s}$ . The mass flow rate of nitrogen by diffusion through the cubical container is

- (a)  $8.24 \times 10^{-10} \text{ kg/s}$       (b)  $1.35 \times 10^{-10} \text{ kg/s}$       (c)  $5.22 \times 10^{-9} \text{ kg/s}$       (d)  $9.71 \times 10^{-9} \text{ kg/s}$       (e)  $3.58 \times 10^{-8} \text{ kg/s}$

*Answer* (c)  $5.22 \times 10^{-9} \text{ kg/s}$

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=2 [m]
L=0.04 [m]
rho_A_1=0.067 [kg/m^3]
rho_A_2=0.009 [kg/m^3]
A=6*s^2
D_AB=1.5E-10 [m^2/s] "Table 14-3b"
m_dot_diff=D_AB*A*(rho_A_1-rho_A_2)/L
```

**14-183** A recent attempt to circumnavigate the world in a balloon used a helium filled balloon whose volume was  $7240 \text{ m}^3$  and surface area was  $1800 \text{ m}^2$ . The skin of this balloon is 2 mm thick and is made of a material whose helium diffusion coefficient is  $1 \times 10^{-9} \text{ m}^2/\text{s}$ . The molar concentration of the helium at the inner surface of the balloon skin is  $0.2 \text{ kmol/m}^3$  and the molar concentration at the outer surface is extremely small. The rate at which helium is lost from this balloon is

- (a) 0.26 kg/h      (b) 1.5 kg/h      (c) 2.6 kg/h      (d) 3.8 kg/h      (e) 5.2 kg/h

*Answer* (c) 2.6 kg/h

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```

Ci=0.2 [kmol/m^3]
Co=0 [kmol/m^3]
D=1E-9 [m^2/s]
L=0.002 [m]
M=4 [kg/kmol]
A=1800 [m^2]
Ndot=D*A*(Ci-Co)/L
Mdot=Ndot*M*3600

```

**14-184** Carbon at 1273 K is contained in a 7-cm-inner-diameter cylinder made of iron whose thickness is 1.2 mm. The concentration of carbon in the iron at the inner surface is  $0.5 \text{ kg/m}^3$  and the concentration of carbon in the iron at the outer surface is negligible. The diffusion coefficient of carbon through iron is  $3 \times 10^{-11} \text{ m}^2/\text{s}$ . The mass flow rate carbon by diffusion through the cylinder shell per unit length of the cylinder is

- (a)  $2.8 \times 10^{-9} \text{ kg/s}$       (b)  $5.4 \times 10^{-9} \text{ kg/s}$       (c)  $8.8 \times 10^{-9} \text{ kg/s}$       (d)  $1.6 \times 10^{-8} \text{ kg/s}$       (e)  $5.2 \times 10^{-8} \text{ kg/s}$

*Answer* (a)  $2.8 \times 10^{-9} \text{ kg/s}$

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```

T=1273 [K]
D_i=0.07 [m]
D_o=D_i+2*0.0012 [m]
rho_A_1=0.5 [kg/m^3]
rho_A_2=0 [kg/m^3]
D_AB=3.0E-11 [m^2/s] "Table 14-3b"
r_1=D_i/2
r_2=D_o/2
L=1 [m]
m_dot_diff=2*pi*L*D_AB*(rho_A_1-rho_A_2)/ln(r_2/r_1)

```

**14-185** The surface of an iron component is to be hardened by carbon. The diffusion coefficient of carbon in iron at 1000°C is given to be  $3 \times 10^{-11} \text{ m}^2/\text{s}$ . If the penetration depth of carbon in iron is desired to be 1.0 mm, the hardening process must take at least

- (a) 1.10 h                      (b) 1.47 h                      (c) 1.86 h                      (d) 2.50 h                      (e) 2.95 h

*Answer* (e) 2.95 h

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D_AB=3E-11 [m^2/s]
delta_diff=1E-3 [m]
delta_diff=sqrt(pi*D_AB*t)
t_hour=t*Convert(s, h)
```

**14-186** A natural gas (methane,  $\text{CH}_4$ ) storage facility uses 3 cm diameter by 6 m long vent tubes on its storage tanks to keep the pressure in these tanks at atmospheric value. If the diffusion coefficient for methane in air is  $0.2 \times 10^{-4} \text{ m}^2/\text{s}$  and the temperature of the tank and environment is 300 K, the rate at which natural gas is lost from a tank through one vent tube is

- (a)  $13 \times 10^{-5} \text{ kg/day}$     (b)  $3.2 \times 10^{-5} \text{ kg/day}$     (c)  $8.7 \times 10^{-5} \text{ kg/day}$     (d)  $5.3 \times 10^{-5} \text{ kg/day}$     (e)  $0.12 \times 10^{-5} \text{ kg/day}$

*Answer* (a)  $13 \times 10^{-5} \text{ kg/day}$

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.03 [m]
L=6 [m]
D_AB=0.2E-4 [m^2/s]
P=101 [kPa]
T=300 [K]
M=16 [kg/kmol]
A=pi*d^2/4
Ndot=(D_AB*A/(R#*T))*(P/L)
Mdot=Ndot*M*Convert(day, s)
```

**14-187** Saturated water vapor at 25°C ( $P_{\text{sat}} = 3.17 \text{ kPa}$ ) flows in a pipe that passes through air at 25°C with a relative humidity of 40 percent. The vapor is vented to the atmosphere through a 7-mm-internal diameter tube that extends 10 m into the air. The diffusion coefficient of vapor through air is  $2.5 \times 10^{-5} \text{ m}^2/\text{s}$ . The amount of water vapor lost to the atmosphere through this individual tube by diffusion is

- (a)  $1.02 \times 10^{-6} \text{ kg}$       (b)  $1.37 \times 10^{-6} \text{ kg}$       (c)  $2.28 \times 10^{-6} \text{ kg}$       (d)  $4.13 \times 10^{-6} \text{ kg}$       (e)  $6.07 \times 10^{-6} \text{ kg}$

*Answer* (b)  $1.37 \times 10^{-6} \text{ kg}$

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=25 [C]
phi=0.40
D=0.007 [m]
L=10 [m]
D_AB=2.5E-5 [m^2/s] "Table 14-2"
P_A_0=pressure(steam_iapws, x=1, T=T) "pressure of vapor at x=0"
P_A_L=phi*P_A_0 "pressure of vapor at x=L=10 m"
A=pi*D^2/4
R_u=8.314[kPa-m^3/kmol-K]
N_dot_vapor=(D_AB*A)/(R_u*T)*(P_A_0-P_A_L)/L
MM=MolarMass(H2O)
m_dot_vapor=N_dot_vapor*MM
time=24*3600 [s]
m_vapor=m_dot_vapor*time
```

"Some Wrong Solutions with Common Mistakes"

```
W_P_A_L=0 "Taking the vapor pressure at air side zero"
W_N_dot_vapor=(D_AB*A)/(R_u*T)*(P_A_0-W_P_A_L)/L
W_m_dot_vapor=W_N_dot_vapor*MM
W_m_vapor=W_m_dot_vapor*time
```

**14-188** When the \_\_\_\_ is unity, one can expect the momentum and mass transfer by diffusion to be the same.

- (a) Grashof      (b) Reynolds      (c) Lewis      (d) Schmidt      (e) Sherwood

*Answer* (d) Schmidt

**14-189** Air flows in a 4-cm-diameter wet pipe at 20°C and 1 atm with an average velocity of 4 m/s in order to dry the surface. The Nusselt number in this case can be determined from  $Nu = 0.023 Re^{0.8} Pr^{0.4}$  where  $Re = 10,550$  and  $Pr = 0.731$ . Also, the diffusion coefficient of water vapor in air is  $2.42 \times 10^{-5} \text{ m}^2/\text{s}$ . Using the analogy between heat and mass transfer, the mass transfer coefficient inside the pipe for fully developed flow becomes

- (a) 0.0918 m/s      (b) 0.0408 m/s      (c) 0.0366 m/s      (d) 0.0203 m/s      (e) 0.0022 m/s

*Answer* (d) 0.0203 m/s

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
T=20[C]+273 [K]
P=1 [atm]
V=4 [m/s]
Re=10550
Pr=0.731
D_AB=2.42E-5 [m^2/s]
Nus=0.023*Re^0.8*Pr^0.4 "Table 14-13"
Sh=Nus
h_mass=(Sh*D_AB)/D
"Some Wrong Solutions with Common Mistakes"
W_Sh=3.66 "Considering laminar flow"
W_h_mass=(W_Sh*D_AB)/D
```

**14-190** Air flows through a wet pipe at 298 K and 1 atm, and the diffusion coefficient of water vapor in air is  $2.5 \times 10^{-5} \text{ m}^2/\text{s}$ . If the heat transfer coefficient is determined to be  $35 \text{ W/m}^2 \cdot ^\circ\text{C}$ , the mass transfer coefficient is

- (a) 0.0326 m/s      (b) 0.0387 m/s      (c) 0.0517 m/s      (d) 0.0583 m/s      (e) 0.0707 m/s

*Answer* (a) 0.0326 m/s

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=298 [K]
P=1 [atm]
h_heat=35 [W/m^2-C]
D_AB=2.5E-5 [m^2/s] "Table 14-2"
rho=1.184 [kg/m^3]
c_p=1007 [J/kg-C]
alpha=2.141E-5 [m^2/s]
h_heat=h_mass*rho*c_p*(alpha/D_AB)^(2/3)
```

## 14-191 .... 14-194 Design and Essay Problems

