Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

Yunus A. Cengel & Afshin J. Ghajar

McGraw-Hill, 2015

Chapter 13
RADIATION HEAT TRANSFER

(Last updated August 1, 2015)

View Factors

13-1C The view factor $F_{i\to j}$ represents the fraction of the radiation leaving surface i that strikes surface j directly. The view factor from a surface to itself is non-zero for concave surfaces.

13-2C The pair of view factors $F_{i\to j}$ and $F_{j\to i}$ are related to each other by the reciprocity rule $A_i F_{ij} = A_j F_{ji}$ where A_i is the area of the surface i and A_j is the area of the surface j. Therefore,

$$A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21}$$

13-3C The summation rule for an enclosure and is expressed as $\sum_{j=1}^{N} F_{i\to j} = 1$ where N is the number of surfaces of the enclosure. It states that the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself must be equal to unity.

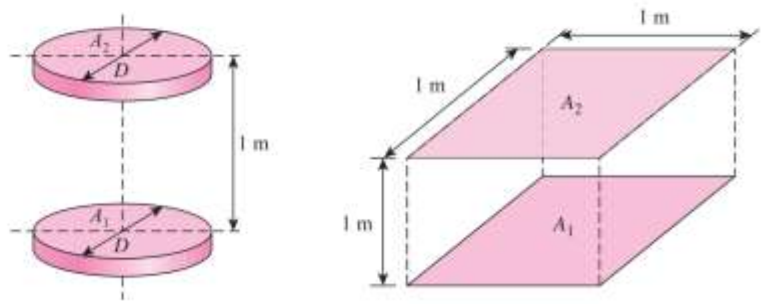
The superposition rule is stated as the view factor from a surface *i* to a surface *j* is equal to the sum of the view factors from surface *i* to the parts of surface *j*, $F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$.

13-4C The cross-string method is applicable to geometries which are very long in one direction relative to the other directions. By attaching strings between corners the Crossed-Strings Method is expressed as

$$F_{i\to j} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{string on surface } i}$$

13-5 Two coaxial parallel circular disks (D = 1 m) and two aligned parallel square plates (1 m × 1 m) have the same distance of 1 m apart. The view factors of the two geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.



Analysis For the coaxial parallel disks, from Table 13-1 with i = 1, j = 2, we have

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2}$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4(L/D)^2 = 6$$

$$F_{12,\text{disk}} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = \mathbf{0.1716}$$

For the parallel plates, from Table 13-1 with i = 1, j = 2, we have

$$\bar{X} = X/L = 1 \quad \text{and} \quad \bar{Y} = Y/L = 1$$

$$F_{12,\text{plate}} = \frac{2}{\pi \bar{X}\bar{Y}} \left\{ \ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2} \right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2} \tan^{-1} \frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} \right.$$
$$\left. + \bar{Y}(1+\bar{X}^2)^{1/2} \tan^{-1} \frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y} \right\}$$

$$F_{12,\text{plate}} = \frac{2}{\pi}\left[\ln\left(\frac{4}{3}\right)^{1/2} + 2\left(2^{1/2} \tan^{-1}\frac{1}{2^{1/2}} - \tan^{-1}(1) \right) \right] = \mathbf{0.1998}$$

The view factor of the aligned parallel square plates is greater than that of the coaxial parallel disks, $F_{12,\text{plate}} > F_{12,\text{disk}}$.

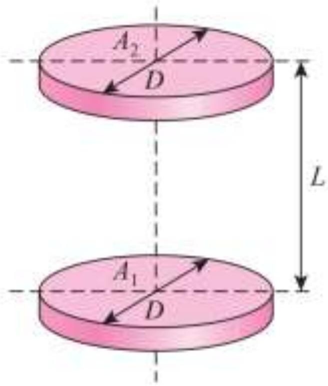
Discussion $F_{12,\text{plate}}$ is expected to be greater than $F_{12,\text{disk}}$ because the area of a 1 m × 1 m plate is greater than the area of a disk with a diameter of 1 m. At the same spacing apart, the geometry with the larger area is expected to have larger view factor.

13-6 Two coaxial parallel circular disks spaced apart at a distance L. The parameter that would increase the view factor F_{12} by a factor of 5 is to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis For coaxial parallel disks, from Table 13-1 with $r_i = r_j$, we have

$$F_{12} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L} = \frac{D}{2L}$$



So,

$$F_{12} = 1 + \frac{1 - \sqrt{4[D/(2L)]^2 + 1}}{2[D/(2L)]^2} = 1 + \frac{1 - \sqrt{(D/L)^2 + 1}}{(1/2)(D/L)^2}$$

For $F_{12} = 0.1$, and solving for D/L_1, we have

$$0.1 = 1 + \frac{1 - \sqrt{(D/L_1)^2 + 1}}{(1/2)(D/L_1)^2} \quad \rightarrow \quad \frac{D}{L_1} = 0.70273$$

To increase the view factor by a factor of five to $F_{12} = 0.5$, we have

$$0.5 = 1 + \frac{1 - \sqrt{(D/L_2)^2 + 1}}{(1/2)(D/L_2)^2} \quad \rightarrow \quad \frac{D}{L_2} = 2.8284$$

For fixed diameter of the disks, we have

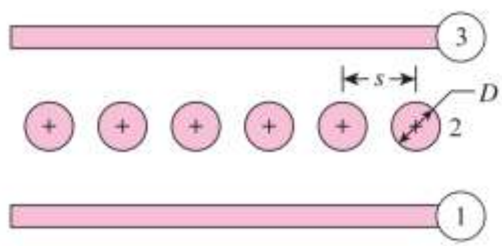
$$\frac{L_2}{L_1} = \frac{D}{L_1}\frac{L_2}{D} = \frac{0.70273}{2.8284} = \mathbf{0.2485}$$

Discussion In order to increase the view factor by a factor of five, the distance between the disks needs to be reduced to about a quarter of its initial distance, $L_2 = 0.2485L_1$.

13-7 A row of cylinders is spaced between two large parallel plates. The view factor between the plate and the row of cylinders is to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

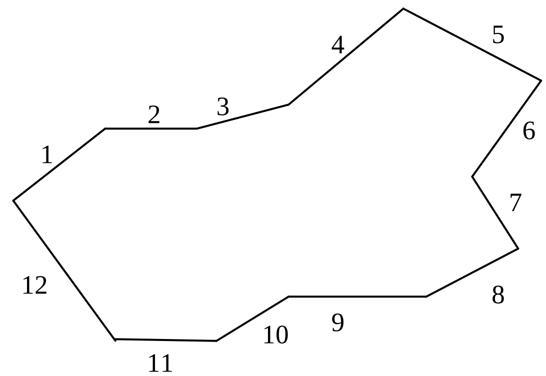
Analysis From symmetry, the view factor between the plate (top or bottom) and the row of cylinders is the same, and from Table 13-2, we get



$$\begin{aligned} F_{12} = F_{32} &= 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\} \\ &= 1 - \left[1 - \left(\frac{3.5}{5}\right)^2\right]^{0.5} + \left(\frac{3.5}{5}\right)\left\{\tan^{-1}\left[\left(\frac{5}{3.5}\right)^2 - 1\right]^{0.5}\right\} \\ &= \mathbf{0.8426} \end{aligned}$$

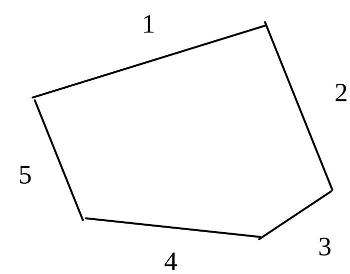
Discussion If the spacing between the cylinders is the same as the diameter ($s = D$), then the view factor would be $F_{12} = 1$. Note that the equation is only valid for $s \geq D$.

13-8 An enclosure consisting of thirteen surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



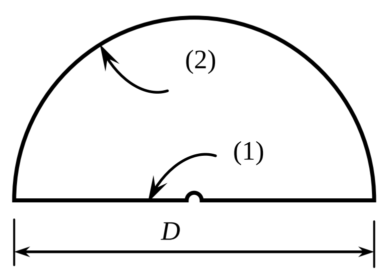
Analysis A thirteen surface enclosure (N = 13) involves $N^2 = 12^2 = \mathbf{144}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{12(12-1)}{2} = 66$ view factors directly. The remaining 144 - 66 = **78** of the view factors can be determined by the application of the reciprocity and summation rules.

13-9 An enclosure consisting of five surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



Analysis A five surface enclosure (N=5) involves $N^2 = 5^2 = \mathbf{25}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{5(5-1)}{2} = 10$ view factors directly. The remaining 25-10 = **15** of the view factors can be determined by the application of the reciprocity and summation rules.

13-10 A semispherical furnace is considered. The view factor from the dome of this furnace to its flat base is to be determined.



Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We number the surfaces as follows:

(1): circular base surface

(2): dome surface

Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{Summation rule}: \; F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$$

$$\text{reciprocity rule}: \; A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{A_2}(1) = \frac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = \frac{1}{2} = \mathbf{0.5}$$

13-11 The view factor from the conical side surface to a hole located at the center of the base of a conical enclosure is to be determined.

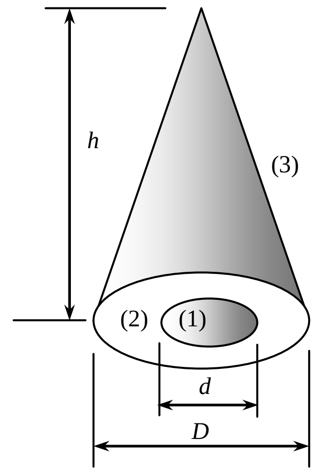
Assumptions The conical side surface is diffuse emitter and reflector.

Analysis We number different surfaces as

the hole located at the center of the base (1)

the base of conical enclosure (2)

conical side surface (3)



Surfaces 1 and 2 are flat, and they have no direct view of each other. Therefore,

$$F_{11} = F_{22} = F_{12} = F_{21} = 0$$

$$\text{summation rule: } F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1$$

$$\text{reciprocity rule: } A_1 F_{13} = A_3 F_{31} \longrightarrow \frac{\pi d^2}{4}(1) = \frac{\pi D h}{2} F_{31} \longrightarrow F_{31} = \mathbf{\frac{d^2}{2Dh}}$$

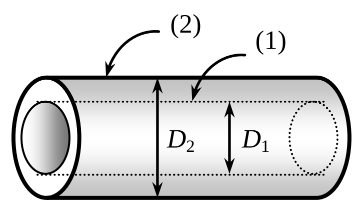
13-12 The four view factors associated with an enclosure formed by two very long concentric cylinders are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis We number different surfaces as

the outer surface of the inner cylinder (1)

the inner surface of the outer cylinder (2)



No radiation leaving surface 1 strikes itself and thus $F_{11} = \mathbf{0}$

All radiation leaving surface 1 strikes surface 2 and thus $F_{12} = \mathbf{1}$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 h}{\pi D_2 h}(1) = \mathbf{\frac{D_1}{D_2}}$$

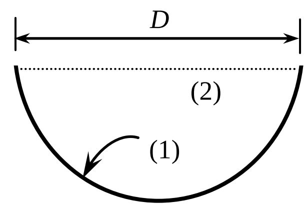
$$\text{summation rule: } F_{21} + F_{22} = 1 \longrightarrow F_{22} = 1 - F_{21} = \mathbf{1 - \frac{D_1}{D_2}}$$

13-13 View factors from the very long grooves shown in the figure to the surroundings are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis (*a*) We designate the circular dome surface by (1) and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$

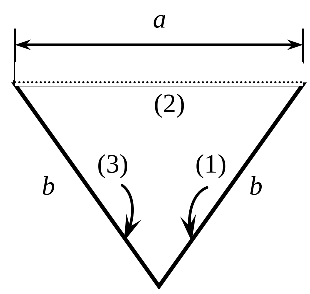


$$\text{summation rule}: \; F_{21} + F_{22} = 1 \longrightarrow F_{21} = 1$$

$$\text{reciprocity rule}: \mathrm{A}_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D}{\frac{\pi D}{2}}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) We designate the two identical surfaces of length *b* by (1) and (3), and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$



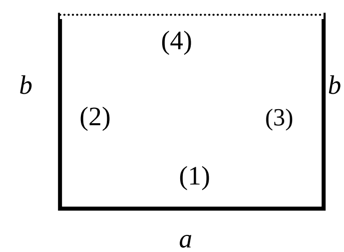
$$\text{summation rule}: \; F_{21} + F_{22} + F_{23} = 1 \longrightarrow F_{21} = F_{23} = 0.5 \quad \text{(symmetry)}$$

$$\text{summation rule}: F_{22} + F_{2\to(1+3)} = 1 \longrightarrow F_{2\to(1+3)} = 1$$

$$\text{reciprocity rule}: \mathrm{A}_2 F_{2\to(1+3)} = A_{(1+3)} F_{(1+3)\to 2}$$

$$\longrightarrow F_{(1+3)\to 2} = F_{(1+3)\to surr} = \frac{A_2}{A_{(1+3)}}(1) = \frac{\mathbf{a}}{\mathbf{2b}}$$

(*c*) We designate the bottom surface by (1), the side surfaces by (2) and (3), and the imaginary top surface by (4). Surface 4 is flat and is completely surrounded by other surfaces. Therefore, $F_{44} = 0$ and $F_{4\to(1+2+3)} = 1$.



$$\text{reciprocity rule}: \mathrm{A}_4 F_{4\to(1+2+3)} = A_{(1+2+3)} F_{(1+2+3)\to 4}$$

$$\longrightarrow F_{(1+2+3)\to 4} = F_{(1+2+3)\to surr} = \frac{A_4}{A_{(1+2+3)}}(1) = \frac{\mathbf{a}}{\mathbf{a+2b}}$$

13-14 A cylindrical enclosure is considered. (*a*) The expression for the view factor between the base and the side surface F_{13} in terms of K and (*b*) the value of the view factor F_{13} for $L = D$ are to be determined.

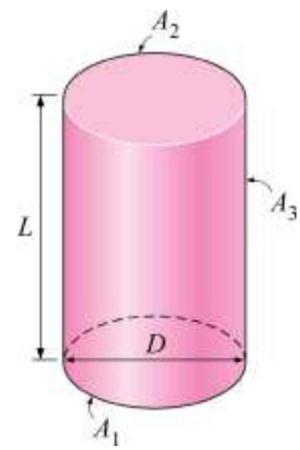
Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis (*a*) The surfaces are designated as follows: Base surface as A_1, top surface as A_2, and side surface as A_3

Applying the summation rule to A_1, we have

$$F_{11} + F_{12} + F_{13} = 1 \qquad \text{(where } F_{11} = 0\text{)}$$

or $$F_{13} = 1 - F_{12} \tag{1}$$



For coaxial parallel disks, from Table 13-1, with $i = 1$, $j = 2$,

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \tag{2}$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + \frac{4}{(D/L)^2} = 2 + 4K^2 \tag{3}$$

where

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2K}$$

Substituting Eq. (3) into Eq. (2), we get

$$\begin{aligned} F_{12} &= \frac{1}{2}\{2 + 4K^2 - [(2 + 4K^2)^2 - 4]^{1/2}\} \\ &= \frac{1}{2}[2 + 4K^2 - (16K^4 + 16K^2)^{1/2}] \\ &= \frac{1}{2}[2 + 4K^2 - 4K(K^2 + 1)^{1/2}] \\ &= 1 + 2K^2 - 2K(K^2 + 1)^{1/2} \end{aligned}$$

Substituting the above expression for F_{12} into Eq. (1) yields the expression for F_{13}:

$$F_{13} = 1 - [1 + 2K^2 - 2K(K^2 + 1)^{1/2}]$$

Hence,

$$F_{13} = 2K(K^2 + 1)^{1/2} - 2K^2$$

(*b*) The value of the view factor F_{13} for $L = D$ (i.e., $K = 1$) is

$$F_{13} = 2(1)(1^2 + 1)^{1/2} - 2(1)^2 = 2\sqrt{2} - 2 = \mathbf{0.828}$$

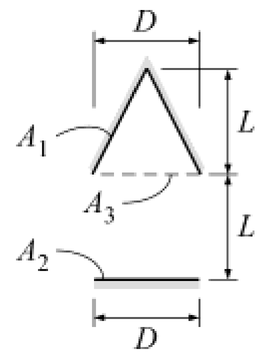
Discussion If the cylinder has a length and diameter of $L = 2D$, then from the expression for F_{13} we have

$$F_{13} = 2(2)(2^2 + 1)^{1/2} - 2(2)^2 = \mathbf{0.944}$$

13-15 A circular cone is positioned on a common axis with a circular disk, the values of F_{11} and F_{12} for specified L and D are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** The surface A_1 is treated as a single surface.

Analysis The area for A_1, A_2, and A_3 are



$$A_1 = \frac{\pi D}{2}\left[L^2 + \left(\frac{D}{2}\right)^2\right]^{1/2} = \frac{\sqrt{5}D^2\pi}{4} \quad \text{and} \quad A_2 = A_3 = \frac{\pi D^2}{4}$$

The surfaces A_1 and A_3 can be treated as an enclosure, and using the summation rule yields

$$F_{11} + F_{13} = 1$$

Applying reciprocity relation between A_1 and A_3, we have

$$A_1 F_{13} = A_3 F_{31} \quad \rightarrow \quad F_{11} = 1 - F_{13} = 1 - (A_3 / A_1) F_{31}$$

Note that

$$F_{31} + F_{33} = 1 \quad \rightarrow \quad F_{31} = 1 \quad (\text{since } F_{33} = 0)$$

Hence, the view factor F_{11} is

$$F_{11} = 1 - (A_3 / A_1) = 1 - \frac{\pi D^2}{\sqrt{5}D^2\pi} = \mathbf{0.553}$$

The radiation leaving A_2 is intercepted by A_1 and A_3 equally,

$$F_{21} = F_{23}$$

Applying reciprocity relation between A_1 and A_2, we have

$$A_1 F_{12} = A_2 F_{21} \quad \rightarrow \quad F_{12} = (A_2 / A_1) F_{21} = (A_2 / A_1) F_{23} = \frac{F_{23}}{\sqrt{5}}$$

For coaxial parallel disks, the view factor F_{23} is evaluated from Table 13-1 as

$$F_{23} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = 0.1716$$

where

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2}$$

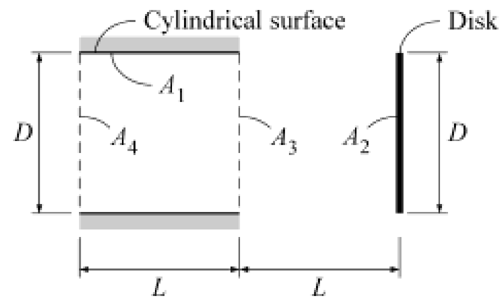
$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4 = 6$$

Hence, the view factor F_{12} is

$$F_{12} = \frac{F_{23}}{\sqrt{5}} = \frac{0.1716}{\sqrt{5}} = \mathbf{0.0767}$$

Discussion As long as $L = D$, the view factors are constant at $F_{11} = 0.553$ and $F_{12} = 0.0767$ (i.e., F_{11} and F_{12} are independent of L and D).

13-16 A cylindrical surface and a disk are oriented coaxially a distance L apart. The view factor F_{12} between them is to be determined.



Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. The summation rule for surfaces facing the disk can be expressed as

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1) and rearranging give

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i\rightarrow j} = F_{j\rightarrow i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2} \quad \text{where } R = \frac{r}{L}$$

For surface 3 (L = 2D):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{2D} = 0.25$$

and

$$F_{23} = 1 + \frac{1-\sqrt{4\times 0.25^2+1}}{2\times 0.25^2} = 0.05573$$

For surface 4 (L = 4D): $R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{4D} = 0.125$

and

$$F_{24} = 1 + \frac{1-\sqrt{4\times 0.125^2+1}}{2\times 0.125^2} = 0.01515$$

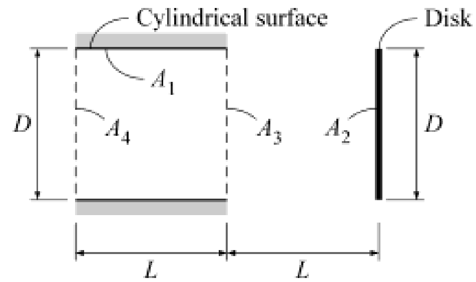
Substituting these values of F_{23} and F_{24} into Eq. (3), and noting that L = 2D, the view factor between the cylindrical surface and the disk is determined to be

$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi DL}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi D(2D)}(F_{23} - F_{24})$$
$$= \frac{1}{8}(F_{23} - F_{24}) = \frac{1}{8}(0.05573 - 0.01515) = \mathbf{0.00507}$$

Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7, but this would involve reading error.

13-17 A cylindrical surface and a disk are oriented coaxially a distance L apart. The view factor F_{12} between them is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.



Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. The summation rule for surfaces facing the disk can be expressed as

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1) and rearranging give

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i \to j} = F_{j \to i} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where } R = \frac{r}{L}$$

For surface 3 ($L = D$):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{D} = 0.5$$

and

$$F_{23} = 1 + \frac{1 - \sqrt{4 \times 0.5^2 + 1}}{2 \times 0.5^2} = 0.1716$$

For surface 4 ($L = 2D$):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{2D} = 0.25$$

and

$$F_{24} = 1 + \frac{1 - \sqrt{4 \times 0.25^2 + 1}}{2 \times 0.25^2} = 0.05573$$

Substituting these values of F_{23} and F_{24} into Eq. (3), and noting that $L = D$, the view factor between the cylindrical surface and the disk is determined to be

$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2 / 4}{\pi D L}(F_{23} - F_{24}) = \frac{1}{4}(0.1716 - 0.05573) = \mathbf{0.0290}$$

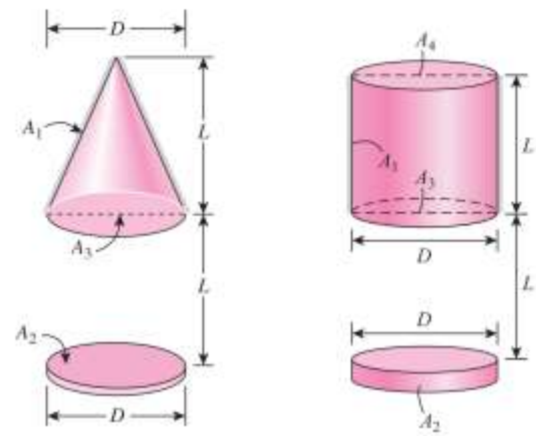
Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7, but this would involve reading error.

13-18 A circular cone is positioned on a common axis with a disk and a cylindrical surface oriented coaxially with a disk. The view factors of the two geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis For the *circular cone and disk* geometry: The area for A_1, A_2, and A_3 are

$$A_1 = \frac{\pi D}{2}\left[L^2 + \left(\frac{D}{2}\right)^2\right]^{1/2} = \frac{\sqrt{5}D^2\pi}{4} \quad \text{and} \quad A_2 = A_3 = \frac{\pi D^2}{4}$$



The radiation leaving A_2 is intercepted by A_1 and A_3 equally,

$$F_{21} = F_{23}$$

Applying reciprocity relation between A_1 and A_2, we have

$$A_1F_{12} = A_2F_{21} \quad \rightarrow \quad F_{12} = (A_2/A_1)F_{21} = (A_2/A_1)F_{23} = \frac{F_{23}}{\sqrt{5}}$$

For coaxial parallel disks, the view factor F_{23} is evaluated from Table 13-1 as

$$F_{23} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2-4)^{1/2}] = \frac{1}{2}[6 - (6^2-4)^{1/2}] = 0.1716$$

where $\quad R_1 = R_2 = R = \dfrac{D}{2L} = \dfrac{1}{2} \quad$ and $\quad S = 1 + \dfrac{1+R_2^2}{R_1^2} = 2 + \dfrac{1}{R^2} = 2 + 4 = 6$

Hence, the view factor F_{12} is

$$F_{12} = \frac{F_{23}}{\sqrt{5}} = \frac{0.1716}{\sqrt{5}} = \mathbf{0.0767} \quad \text{(circular cone and disk)}$$

For the *circular surface and disk* geometry: The end surfaces A_3 and A_4 are treated as hypothetical surfaces. From the summation rule for surfaces facing the disk and the reciprocity relation, we have

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \quad \text{and} \quad A_2F_{21} = A_1F_{12}$$

Hence

$$(A_1/A_2)F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2/A_1)(F_{23} - F_{24}) \qquad (1)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i\to j} = F_{j\to i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L}$$

For surface 3: $\quad F_{23} = 1 + \dfrac{1-\sqrt{4\times 0.5^2+1}}{2\times 0.5^2} = 0.1716 \quad$ where $\quad R = \dfrac{r}{L} = \dfrac{D/2}{L} = \dfrac{D/2}{D} = 0.5$

For surface 4: $\quad F_{24} = 1 + \dfrac{1-\sqrt{4\times 0.25^2+1}}{2\times 0.25^2} = 0.05573 \quad$ where $\quad R = \dfrac{r}{2L} = \dfrac{D/2}{2L} = \dfrac{D/2}{2D} = 0.25$

Using Eq. (1), with $L = D$, we get

$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi DL}(F_{23} - F_{24}) = \frac{1}{4}(0.1716 - 0.05573) = \mathbf{0.0290} \quad \text{(circular surface and disk)}$$

Discussion The F_{12} for the circular cone and disk geometry is greater than the F_{12} for the circular surface and disk geometry. This is expected as surface 1 of the circular cone and disk geometry is angled toward the disk, but such is not the case for the circular surface and disk geometry.

13-19 The view factors from the base of a cube to each of the other five surfaces are to be determined.

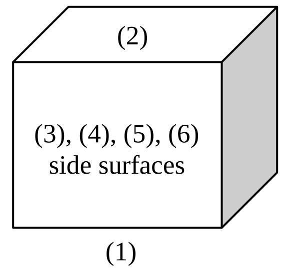
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Noting that $L_1/D = L_2/D = 1$, from Fig. 13-6 we read

$$F_{12} = 0.2$$

Because of symmetry, we have

$$F_{12} = F_{13} = F_{14} = F_{15} = F_{16} = \mathbf{0.2}$$

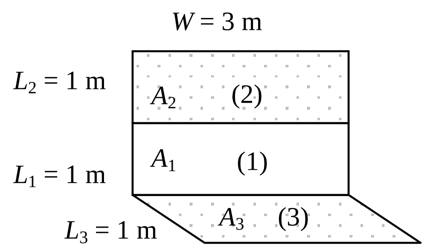


13-20 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 13-6,

$$\left.\begin{array}{l} \dfrac{L_3}{W} = \dfrac{1}{3} = 0.33 \\ \dfrac{L_1}{W} = \dfrac{1}{3} = 0.33 \end{array}\right\} F_{31} = 0.25$$



and

$$\left.\begin{array}{l} \dfrac{L_3}{W} = \dfrac{1}{3} = 0.33 \\ \dfrac{L_1 + L_2}{W} = \dfrac{2}{3} = 0.67 \end{array}\right\} F_{3\to(1+2)} = 0.32$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$\mathrm{A}_1 F_{13} = A_3 F_{31} \longrightarrow F_{13} = F_{31} = \mathbf{0.25}$$

$$F_{3\to(1+2)} = F_{31} + F_{32} \longrightarrow \quad 0.32 = 0.25 + F_{32} \longrightarrow F_{32} = 0.07$$

Finally,

$$A_2 = A_3 \longrightarrow F_{23} = F_{32} = \mathbf{0.07}$$

13-21 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

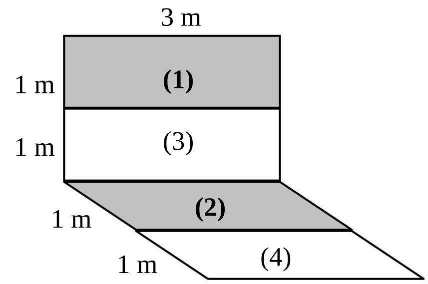
Analysis We designate the different surfaces as follows:

shaded part of perpendicular surface by (1),

bottom part of perpendicular surface by (3),

shaded part of horizontal surface by (2), and

front part of horizontal surface by (4).



(*a*) From Fig.13-6

$$\left.\begin{aligned} \frac{L_2}{W} &= \frac{1}{3} = 0.33 \\ \frac{L_1}{W} &= \frac{1}{3} = 0.33 \end{aligned}\right\} F_{23} = 0.25 \quad \text{and} \quad \left.\begin{aligned} \frac{L_2}{W} &= \frac{2}{3} = 0.67 \\ \frac{L_1}{W} &= \frac{1}{3} = 0.33 \end{aligned}\right\} F_{2\to(1+3)} = 0.32$$

$$\text{superposition rule: } F_{2\to(1+3)} = F_{21} + F_{23} \longrightarrow F_{21} = F_{2\to(1+3)} - F_{23} = 0.32 - 0.25 = 0.07$$

$$\text{reciprocity rule: } A_1 = A_2 \longrightarrow A_1F_{12} = A_2F_{21} \longrightarrow F_{12} = F_{21} = \mathbf{0.07}$$

(*b*) From Fig.13-6,

$$\left.\begin{aligned} \frac{L_2}{W} &= \frac{1}{3} = 0.33 \\ \frac{L_1}{W} &= \frac{2}{3} = 0.67 \end{aligned}\right\} F_{(4+2)\to 3} = 0.16 \quad \text{and} \quad \left.\begin{aligned} \frac{L_2}{W} &= \frac{2}{3} = 0.67 \\ \frac{L_1}{W} &= \frac{2}{3} = 0.67 \end{aligned}\right\} F_{(4+2)\to(1+3)} = 0.22$$

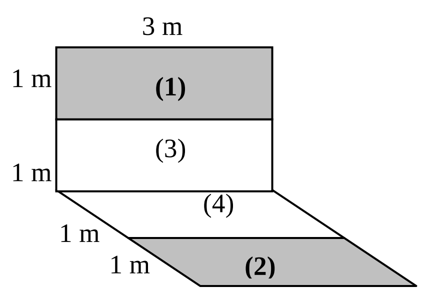
$$\text{superposition rule: } F_{(4+2)\to(1+3)} = F_{(4+2)\to 1} + F_{(4+2)\to 3} \longrightarrow F_{(4+2)\to 1} = 0.22 - 0.16 = 0.06$$

$$\text{reciprocity rule: } A_{(4+2)}F_{(4+2)\to 1} = A_1F_{1\to(4+2)}$$

$$\longrightarrow F_{1\to(4+2)} = \frac{A_{(4+2)}}{A_1}F_{(4+2)\to 1} = \frac{6}{3}(0.06) = 0.12$$

$$\text{superposition rule: } F_{1\to(4+2)} = F_{14} + F_{12}$$

$$\longrightarrow F_{14} = 0.12 - 0.06 = \mathbf{0.06}$$



since F_{12} = 0.07 (from part *a*). Note that F_{14} in part (*b*) is equivalent to F_{12} in part (*a*).

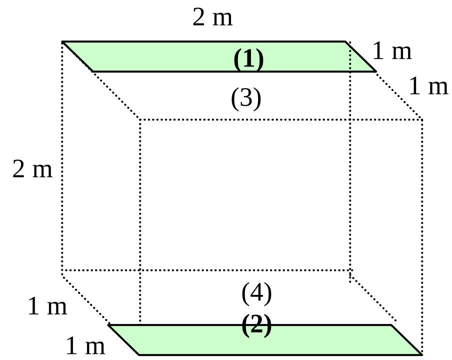
(c) We designate

shaded part of top surface by (1),

remaining part of top surface by (3),

remaining part of bottom surface by (4), and

shaded part of bottom surface by (2).



From Fig.13-5,

$$\left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{2}{2}=1\\[2mm]\dfrac{L_1}{D}=\dfrac{2}{2}=1\end{array}\right\}F_{(2+4)\to(1+3)}=0.20 \quad \text{and} \quad \left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{2}{2}=1\\[2mm]\dfrac{L_1}{D}=\dfrac{1}{2}=0.5\end{array}\right\}F_{14}=0.12$$

superposition rule: $F_{(2+4)\to(1+3)}=F_{(2+4)\to 1}+F_{(2+4)\to 3}$

symmetry rule: $F_{(2+4)\to 1}=F_{(2+4)\to 3}$

Substituting symmetry rule gives

$$F_{(2+4)\to 1}=F_{(2+4)\to 3}=\frac{F_{(2+4)\to(1+3)}}{2}=\frac{0.20}{2}=0.10$$

reciprocity rule: $A_1F_{1\to(2+4)}=A_{(2+4)}F_{(2+4)\to 1}\longrightarrow(2)F_{1\to(2+4)}=(4)(0.10)\longrightarrow F_{1\to(2+4)}=0.20$

superposition rule: $F_{1\to(2+4)}=F_{12}+F_{14}\longrightarrow 0.20=F_{12}+0.12\longrightarrow F_{12}=0.20-0.12=\mathbf{0.08}$

13-22 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined.

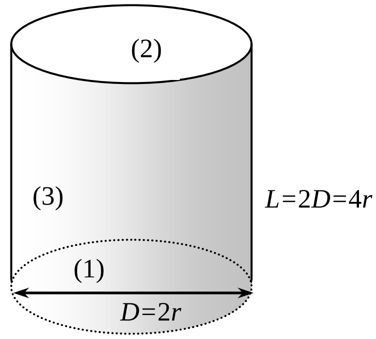
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1),

top surface by (2), and

side surface by (3).



Then from Fig. 13-7

$$\left.\begin{array}{l} \dfrac{L}{r_1}=\dfrac{4r_1}{r_1}=4 \\ \dfrac{r_2}{L}=\dfrac{r_2}{4r_2}=0.25 \end{array}\right\} F_{12}=F_{21}=0.05$$

$$\text{summation rule}: F_{11}+F_{12}+F_{13}=1$$

$$0+0.05+F_{13}=1 \longrightarrow F_{13}=0.95$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.95)=\mathbf{0.119}$$

Discussion This problem can be solved more accurately by using the view factor relation from Table 13-1 to be

$$R_1=\frac{r_1}{L}=\frac{r_1}{4r_1}=0.25$$

$$R_2=\frac{r_2}{L}=\frac{r_2}{4r_2}=0.25$$

$$S=1+\frac{1+R_2^2}{R_1^2}=1+\frac{1+0.25^2}{0.25^2}=18$$

$$F_{12}=\tfrac{1}{2}\left\{S-\left[S^2-4\left(\frac{R_2}{R_1}\right)^2\right]^{0.5}\right\}=\tfrac{1}{2}\left\{18-\left[18^2-4\left(\frac{1}{1}\right)^2\right]^{0.5}\right\}=0.056$$

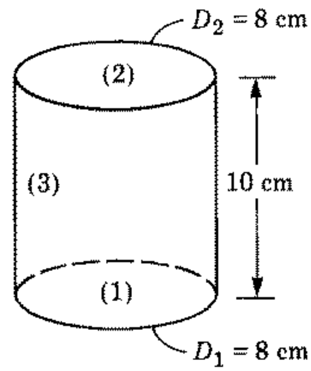
$$F_{13}=1-F_{12}=1-0.056=0.944$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.944)=\mathbf{0.118}$$

13-23 For a right circular cylinder the view factors F_{13} and F_{33} are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis Neither result is given in the book tables or figures. To obtain F_{13}, we can make use of Fig. 13-7 or Table 13-1 for the two parallel disks.



For

$$\frac{L}{r_1} = \frac{10}{4} = 2.5 \quad \text{and} \quad \frac{r_2}{L} = \frac{4}{10} = 0.4$$

From Fig. 13-7 or compute from item 2 of Table 13-1

$$F_{12} = 0.12$$

From the summation rule, Eq. 13-12

$$\sum_{j=1}^{N} F_{ij} = 1$$

or

$$F_{11} + F_{12} + F_{13} = 1$$

where

$$F_{11} = 0$$

$\therefore$ $\quad F_{13} = 1 - F_{12} = 1 - 0.12 = \mathbf{0.88}$

To obtain F_{33}, the summation rule (Eq. 13-12) requires

$$F_{33} + F_{31} + F_{32} = 1 = F_{33} + 2F_{31}$$

Since the cylinder has identical views from the top and bottom disks. To obtain F_{31} use the reciprocity relation (Eq. 13-10).

$$A_3F_{31} = A_1F_{13}$$

where

$$A_3 = \pi D_1 L = 80\ \pi \text{cm}^2$$

and

$$A_1 = \pi D_1^2 = 16\ \pi \text{cm}^2$$

$$F_{31} = (A_1/A_3)\ F_{13} = (16\ \pi \text{cm}^2 / 80\ \pi \text{cm}^2)\ (0.88) = 0.176$$

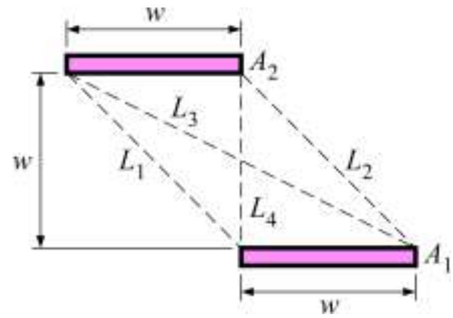
Finally, from the summation rule eq. above

$$F_{33} = 1 - 2F_{31} = 1 - 2\ (0.176) = \mathbf{0.65}$$

13-24 The expression for the view factor F_{12} of two infinitely long parallel plates is to be determined using the Hottel's crossed-strings method.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis From the Hottel's crossed-strings method, we have



$$F_{i\to j} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2\times(\text{String on surface } i)}$$

For uncrossed strings, we have

$$L_1 = L_2 = (w^2 + w^2)^{1/2} = (w^2 + w^2)^{1/2} = \sqrt{2}w$$

For crossed strings, we have

$$L_3 = (w^2 + 4w^2)^{1/2} = \sqrt{5}w \qquad \text{and} \qquad L_4 = w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

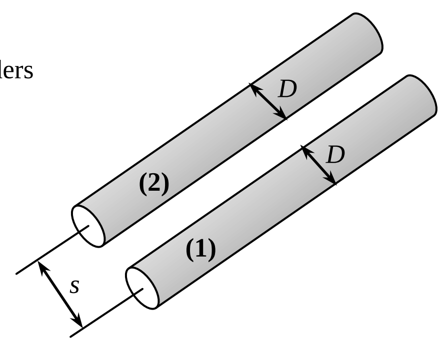
$$F_{12} = \frac{(L_3 + L_4) - (L_1 + L_2)}{2w}$$
$$= \frac{(\sqrt{5}w + w) - (\sqrt{2}w + \sqrt{2}w)}{2w}$$
$$= \mathbf{0.204}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-25 The view factor between the two infinitely long parallel cylinders located a distance *s* apart from each other is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Using the crossed-strings method, the view factor between two cylinders facing each other for $s/D > 3$ is determined to be



$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2\times \text{String on surface 1}}$$
$$= \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

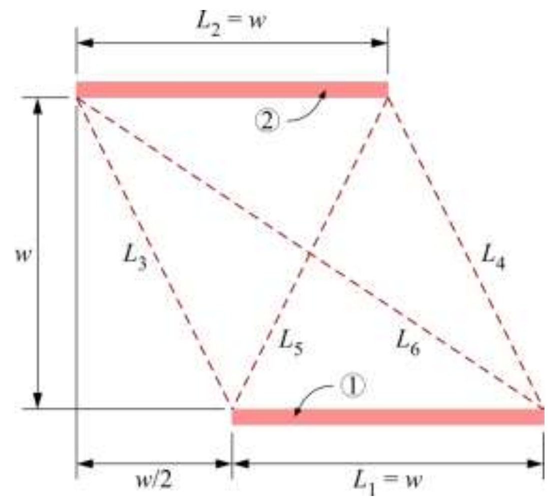
or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

13-26 The expressions for the view factors F_{12} and F_{21} of two infinitely long parallel plates are to be determined using the Hottel's crossed-strings method.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis From the Hottel's crossed-strings method, we have

$$F_{i\to j} = \frac{\Sigma(\text{Crossed strings}) - \Sigma(\text{Uncrossed strings})}{2\times(\text{String on surface } i)}$$



where,

$$L_1 = L_2 = w$$

$$L_3 = L_4 = L_5 = \sqrt{w^2 + (w/2)^2} = \frac{\sqrt{5}}{2}w$$

$$L_6 = \sqrt{w^2 + \left(\frac{3}{2}w\right)^2} = \frac{\sqrt{13}}{2}w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2w}$$
$$= \frac{(\sqrt{5}w/2 + \sqrt{13}w/2) - (\sqrt{5}w/2 + \sqrt{5}w/2)}{2w}$$
$$= \mathbf{0.3424}$$

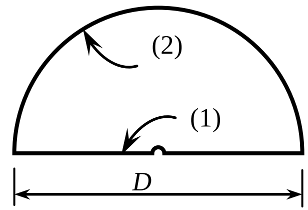
Since surface 1 and surface 2 have the same width, thus the Hottel's crossed-strings method will give

$$F_{12} = F_{21} = \mathbf{0.3424}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-27 Two view factors associated with three very long ducts with different geometries are to be determined.

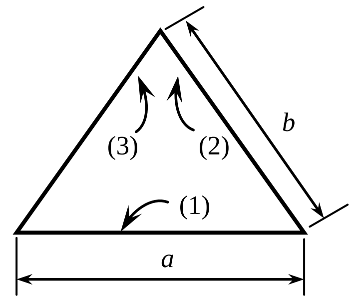
Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.



Analysis (*a*) Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{summation rule}: \; F_{11} + F_{12} = 1 \rightarrow F_{12} = \mathbf{1}$$

$$\text{reciprocity rule}: \; A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{Ds}{\left(\frac{\pi D}{2}\right)s}(1) = \frac{2}{\pi} = \mathbf{0.64}$$



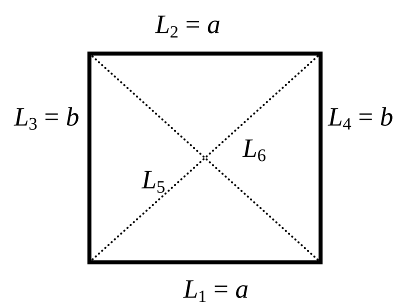
(*b*) Noting that surfaces 2 and 3 are symmetrical and thus $F_{12} = F_{13}$, the summation rule gives

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + F_{12} + F_{13} = 1 \longrightarrow F_{12} = \mathbf{0.5}$$

Also by using the equation obtained in Example 13-4,

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{a + b - b}{2a} = \frac{a}{2a} = \frac{1}{2} = \mathbf{0.5}$$

$$\text{reciprocity rule}: \; A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{a}{b}\left(\frac{1}{2}\right) = \mathbf{\frac{a}{2b}}$$



(*c*) Applying the crossed-string method gives

$$F_{12} = F_{21} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$= \frac{2\sqrt{a^2 + b^2} - 2b}{2a} = \mathbf{\frac{\sqrt{a^2 + b^2} - b}{a}}$$

Radiation Heat Transfer between Surfaces

13-28C The analysis of radiation exchange between black surfaces is relatively easy because of the absence of reflection. The rate of radiation heat transfer between two surfaces in this case is expressed as $\dot{Q} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$ where A_1 is the surface area, F_{12} is the view factor, and T_1 and T_2 are the temperatures of two surfaces.

13-29C Radiosity is the total radiation energy leaving a surface per unit time and per unit area. Radiosity includes the emitted radiation energy as well as reflected energy. Radiosity and emitted energy are equal for blackbodies since a blackbody does not reflect any radiation.

13-30C Radiation surface resistance is given as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$ and it represents the resistance of a surface to the emission of radiation. It is zero for black surfaces. The space resistance is the radiation resistance between two surfaces and is expressed as $R_{ij} = \dfrac{1}{A_i F_{ij}}$

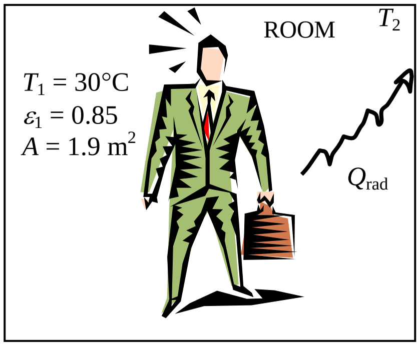
13-31C Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis, the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

13-32C The two methods used in radiation analysis are the matrix and network methods. In matrix method, equations 13-34 and 13-35 give N linear algebraic equations for the determination of the N unknown radiosities for an N -surface enclosure. Once the radiosities are available, the unknown surface temperatures and heat transfer rates can be determined from these equations respectively. This method involves the use of matrices especially when there are a large number of surfaces. Therefore this method requires some knowledge of linear algebra.

The network method involves drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the radiation problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The network method is not practical for enclosures with more than three or four surfaces due to the increased complexity of the network.

13-33 The rate of heat loss from a person by radiation in a large room whose walls are maintained at a uniform temperature is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivity of the person is given to be $\varepsilon_1 = 0.85$.

Analysis (*a*) Noting that the view factor from the person to the walls $F_{12} = 1$, the rate of heat loss from that person to the walls at a large room which are at a temperature of 300 K is

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1{}^4 - T_2{}^4) \\ &= (0.85)(1)(1.9\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\ \text{K})^4 - (300\ \text{K})^4] \\ &= \mathbf{30.1\,W}\end{aligned}$$

(*b*) When the walls are at a temperature of 280 K,

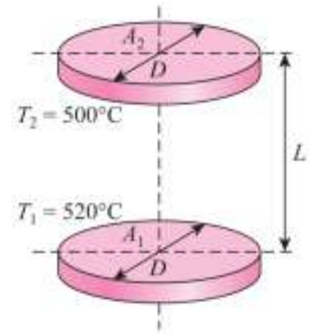
$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1{}^4 - T_2{}^4) \\ &= (0.85)(1)(1.9\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\ \text{K})^4 - (280\ \text{K})^4] \\ &= \mathbf{209\,W}\end{aligned}$$

13-34 Two coaxial parallel circular disks spaced apart at a distance of $L = D$. The radiation heat transfer coefficient between the disks is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis The net radiation heat flux between the two circular disks is

$$\dot{q}_{12} = F_{12}\sigma(T_1^4 - T_2^4)$$
$$= F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)(T_1 - T_2)$$



Also, expressed in terms of the radiation heat transfer coefficient, we have

$$\dot{q}_{12} = h_{\text{rad}}(T_1 - T_2)$$

Hence, the radiation heat transfer coefficient can be expressed as

$$h_{\text{rad}} = F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)$$

For coaxial parallel disks, from Table 13-1 with $r_i = r_j$, we have

$$F_{12} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L} = \frac{D}{2L} = 0.5$$

So,

$$F_{12} = 1 + \frac{1 - \sqrt{4(0.5)^2 + 1}}{2(0.5)^2} = 0.1716$$

Thus,

$$h_{\text{rad}} = F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)$$
$$= (0.1716)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(793 + 773)\,\text{K}\,(793^2 + 773^2)\,\text{K}^2$$
$$= \mathbf{18.69\ W/m^2 \cdot K}$$

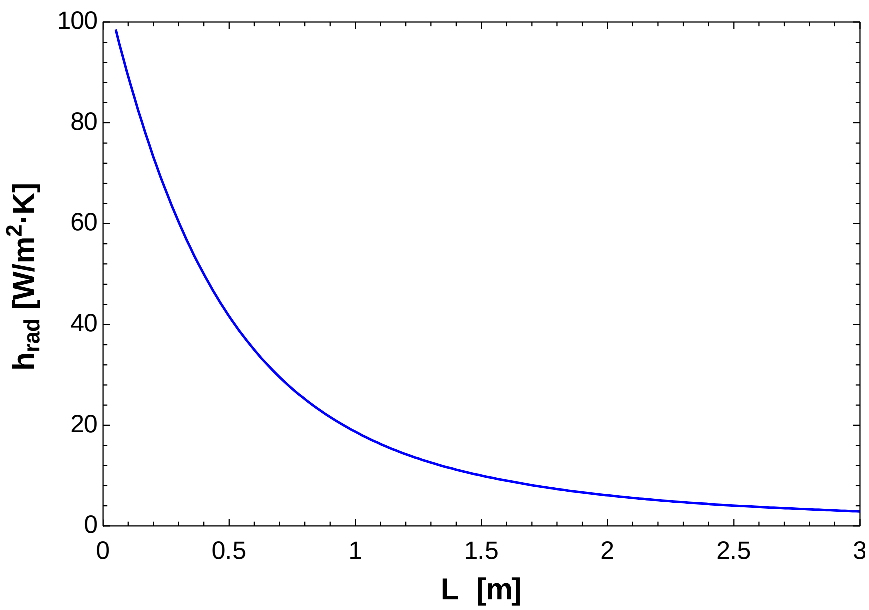
Discussion The radiation heat transfer coefficient h_{rad} is dependent upon the view factor and the temperatures of both disks.

13-35 Two coaxial parallel circular disks spaced apart at a distance L. The effect of the distance L on the radiation heat transfer coefficient between the disks is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_1=520 [C]
T_2=500 [C]
D=1 [m]

"ANALYSIS"
R=D/(2*L)
F_12=1+(1-(4*R^2+1)^0.5)/(2*R^2)
h_rad=F_12*sigma#*((T_1+273)+(T_2+273))*((T_1+273)^2+(T_2+273)^2)
```

L [m]	F_{12}	h_{rad} [W/m^2·K]
0.05	0.9049	98.53
0.10	0.8190	89.18
0.15	0.7416	80.75
0.20	0.6721	73.18
0.25	0.6096	66.38
0.30	0.5536	60.28
0.35	0.5034	54.81
0.40	0.4584	49.91
0.45	0.4181	45.52
0.50	0.3820	41.59
0.75	0.2500	27.22
1.0	0.1716	18.68
1.5	0.09167	9.982
2.0	0.05573	6.068
2.5	0.03709	4.038
3.0	0.02633	2.867

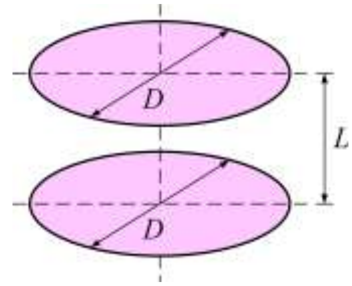


Discussion By reducing the spacing L between the parallel disks the radiation heat transfer coefficient h_{rad} increases. As L decreases below the diameter of the disks D, h_{rad} increases drastically.

13-36 Two coaxial parallel disks of equal diameter 1 m are originally placed at a distance of 1 m apart. The new distance between the disks such that there is a 75% reduction in radiation heat transfer rate from the original distance of 1 m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For coaxial parallel disks, from Table 13-1, with i = 1, j = 2,



$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \qquad (1)$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4(L/D)^2 \qquad (2)$$

where $R_1 = R_2 = R = \dfrac{D}{2L}$

Substituting Eq. (2) into Eq. (1), we get

$$\begin{aligned} F_{12} &= \frac{1}{2}(2 + 4(L/D)^2 - \{[2 + 4(L/D)^2]^2 - 4\}^{1/2}) \\ &= \frac{1}{2}\{2 + 4(L/D)^2 - [16(L/D)^4 + 16(L/D)^2]^{1/2}\} \\ &= \frac{1}{2}\{2 + 4(L/D)^2 - 4(L/D)[(L/D)^2 + 1]^{1/2}\} \end{aligned}$$

$$F_{12} = 1 + 2(L/D)^2 - 2(L/D)[(L/D)^2 + 1]^{1/2}$$

Hence, for D = 1 m we have

$$F_{12} = 1 + 2L^2 - 2L[L^2 + 1]^{1/2} \qquad (3)$$

From Eq. (3), the view factor at the original distance L = 1 m (with D = l m) is

$$F_{12,\,\text{old}} = 1 + 2(1)^2 - 2(1)[(1)^2 + 1]^{1/2} = 3 - 2\sqrt{2} = 0.1716$$

The rate of radiation heat transfer between the two surfaces is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4)$$

The percentage of reduction in radiation heat transfer rate can be expressed as

$$\%\,\text{Change} = \frac{\dot{Q}_{12,\,\text{old}} - \dot{Q}_{12,\,\text{new}}}{\dot{Q}_{12,\,\text{old}}} = \frac{F_{12,\,\text{old}} - F_{12,\,\text{new}}}{F_{12,\,\text{old}}}$$

For the two surfaces to experience 75% reduction in radiation heat transfer rate, the new view factor should be

$$F_{12,\,\text{new}} = 0.25F_{12,\,\text{old}} = 0.0429$$

Then, substituting the value of $F_{12,\,\text{new}}$ into Eq. (3), the new distance L_{new} such that the two surfaces experience 75% reduction in radiation heat transfer rate can be calculated as

$$F_{12,\,\text{new}} = 1 + 2L_{\text{new}}^2 - 2L_{\text{new}}[L_{\text{new}}^2 + 1]^{1/2}$$
$$0.0429 = 1 + 2L_{\text{new}}^2 - 2L_{\text{new}}[L_{\text{new}}^2 + 1]^{1/2}$$

Hence, $L_{\text{new}} = \mathbf{2.31\,m}$

Discussion Increasing the distance between the disks would decrease the view factor F_{12}, thereby reducing the radiation heat transfer rate $\dot{Q}_{12}$.

13-37 A row of tubes is spaced between two large parallel plates. The net radiation heat flux leaving the bottom plate is to be determined.

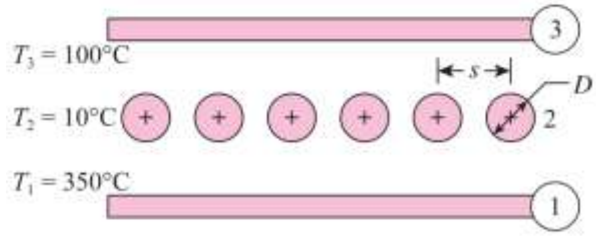
Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis From energy balance, the net radiation heat flux leaving the bottom plate (surface 1) is

$$\dot{q}_1 = \dot{q}_{12} + \dot{q}_{13} = F_{12}\sigma(T_1^4 - T_2^4) + F_{13}\sigma(T_1^4 - T_3^4)$$

With $F_{13} = 1 - F_{12}$, we have

$$\dot{q}_1 = F_{12}\sigma(T_1^4 - T_2^4) + (1 - F_{12})\sigma(T_1^4 - T_3^4)$$



The view factor between the bottom plate and the row of tubes (s = 2D), from Table 13-2, is

$$F_{12} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

$$= 1 - (1 - 0.5^2)^{0.5} + (0.5)[\tan^{-1}(2^2 - 1)^{0.5}]$$

$$= 0.6576$$

Thus,

$$\dot{q}_1 = \sigma[F_{12}(T_1^4 - T_2^4) + (1 - F_{12})(T_1^4 - T_3^4)]$$

$$= (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(0.6576)(623^4 - 283^4)\,\text{K}^4 + (1 - 0.6576)(623^4 - 373^4)\,\text{K}^4]$$

$$= \mathbf{7927 \ W/m^2}$$

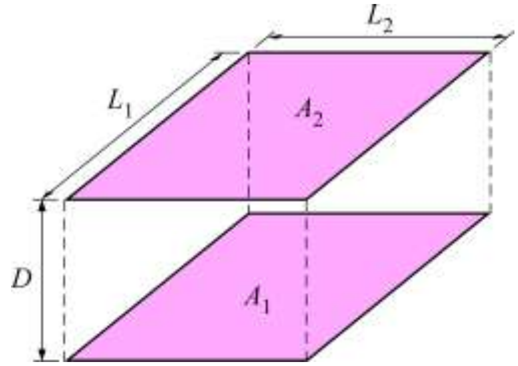
Discussion The view factor between the bottom plate and the circular tubes F_{12} is independent of the distance between them.

13-38E Two black parallel rectangles are spaced apart by a distance of 1 ft, the temperature of the bottom rectangle is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For D = 1 ft, L_1 = 3 ft and L_2 = 5 ft, we have

$$\frac{L_1}{D} = \frac{3}{1} = 3 \quad \text{and} \quad \frac{L_2}{D} = \frac{5}{1} = 5$$



From Fig. 13-5, we get

$$F_{12} \approx 0.60$$

The rate of radiation heat transfer between the two rectangles is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4) = L_1 L_2 F_{12}\sigma(T_1^4 - T_2^4)$$

Hence

$$T_1 = \left[\frac{\dot{Q}_{12}}{L_1 L_2 F_{12}\sigma} + T_2^4\right]^{1/4}$$

$$= \left[\frac{180000\ \text{Btu/h}}{(3\ \text{ft})(5\ \text{ft})(0.60)(0.1714\times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)} + (60+460)^4\ \text{R}^4\right]^{1/4}$$

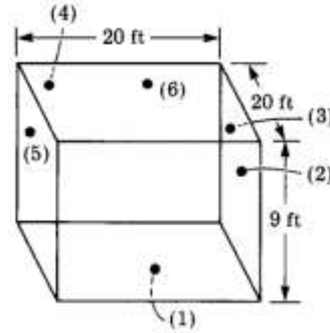
$$T_1 = \mathbf{1851\ R}$$

Discussion If T_1 and T_2 are constant, increasing the distance between the two rectangles will decrease the view factor F_{12}, thereby decreasing the radiation heat transfer rate received by the top rectangle.

13-39E For a square room with specified dimensions and floor, wall and ceiling temperatures, determine the net radiation heat transfer (a) from floor to walls and (b) from floor to ceiling.

Assumptions **1** All surfaces are assumed black.

Analysis



(a) $\dot{Q}_{1\to 2,3,4,5} = 4A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right)$

Since for a square

$F_{12} = F_{13} = F_{14} = F_{15}$

From Fig. 13-6 or Table 13-1 with

$L_1 / W = 20/20 = 1$

and

$L_2 / W = 9/20 \approx 0.45 \quad \to \quad F_{12} \approx 0.14$

$\dot{Q}_{1\to 2,3,4,5} = 4\ (20 \times 20)\ \text{ft}^2\ (0.14)\ (0.\ 1714\times 10^{-8}\ \text{Btu/h·ft·R}^4) \times [(560)^4 - (520)^4]\text{R}^4$

$\dot{Q}_{1\to 2,3,4,5} =$ **9686 Btu/h** (Floor to four walls)

(b) $\dot{Q}_{1\to 6} = A_1 F_{16}\,\sigma\left(T_1^4 - T_6^4\right)$

From Fig. 13-5 with

$$\frac{L_1}{D} = \frac{20}{9} = 2.23$$

and

$$\frac{L_2}{D} = \frac{20}{9} = 2.23 \quad \to \quad F_{16} \approx 0.45$$

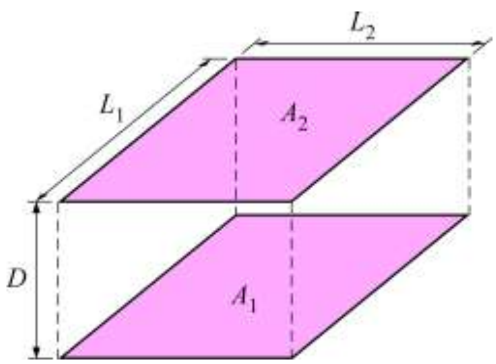
$\dot{Q}_{1\to 6} = (20 \times 20)\ \text{ft}^2\ (0.45)\ (0.\ 1714\times 10^{-8}\ \text{Btu/h·ft·R}^4) \times\ [(560)^4 - (500)^4]\text{R}^4$

$\dot{Q}_{1\to 6} =$ **11,059 Btu/h** (Floor to ceiling)

13-40 Two aligned parallel rectangles are apart by a distance of 2 m. The percentage of change in radiation heat transfer rate when the rectangles are moved apart from 2 m to 8 m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For D = 2 m, L_1 = 6 m and L_2 = 8 m, we have



$$\frac{L_1}{D} = \frac{6}{2} = 3 \qquad \text{and} \qquad \frac{L_2}{D} = \frac{8}{2} = 4$$

From Fig. 13-5, we get

$$F_{12} \approx 0.58 \qquad \text{(for } D = 2\text{m)}$$

For D = 8 m, L_1 = 6 m and L_2 = 8 m, we have

$$\frac{L_1}{D} = \frac{6}{8} = 0.75 \qquad \text{and} \qquad \frac{L_2}{D} = \frac{8}{8} = 1$$

From Fig. 13-5, we get

$$F_{12} \approx 0.165 \qquad \text{(for } D = 8\text{ m)}$$

The rate of radiation heat transfer between the two rectangles is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4)$$

Hence, the percentage of change in radiation heat transfer rate is

$$\begin{aligned}\%\,\text{Change} &= \frac{\dot{Q}_{12}(D = 2\text{ m}) - \dot{Q}_{12}(D = 8\text{ m})}{\dot{Q}_{12}(D = 2\text{ m})} = \frac{F_{12}(D = 2\text{ m}) - F_{12}(D = 8\text{ m})}{F_{12}(D = 2\text{ m})} \\ &= \frac{0.58 - 0.165}{0.58} \\ &= \mathbf{0.716\ (or\ 71.6\%)}\end{aligned}$$

Discussion By moving the distance between the two parallel rectangles from 2 m to 8 m, there is about 72% reduction in radiation heat transfer rate.

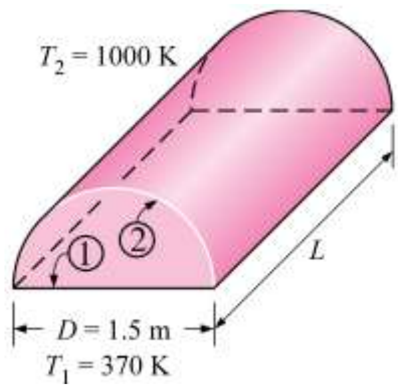
13-41 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The drying rate per unit length experienced by the base surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.

Properties The latent heat of vaporization for water is h_{fg} = 2257 kJ/kg (Table A-2)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{12} = 1 \quad \text{(where } F_{11} = 0\text{)}$$



Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$

Applying energy balance on the base surface, we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m} h_{fg}$$

Hence,

$$\dot{m} h_{fg} = A_2 F_{21} \sigma (T_2^4 - T_1^4) = \frac{\pi DL}{2} F_{21} \sigma (T_2^4 - T_1^4)$$

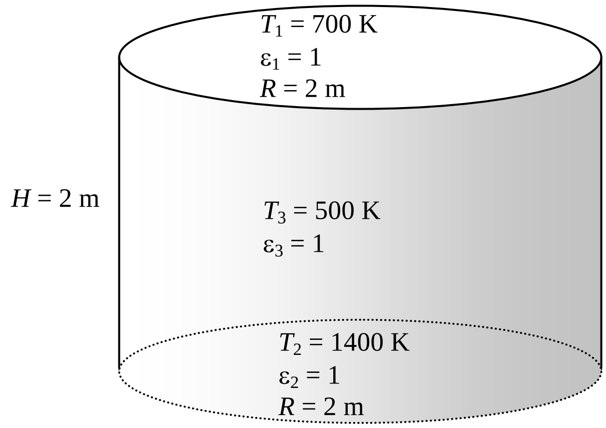
or

$$\begin{aligned}
\frac{\dot{m}}{L} &= \frac{\pi D}{2 h_{fg}} F_{21} \sigma (T_2^4 - T_1^4) = \frac{\pi D}{2 h_{fg}} \frac{2}{\pi} \sigma (T_2^4 - T_1^4) \\
&= \frac{D}{h_{fg}} \sigma (T_2^4 - T_1^4) \\
&= \frac{(1.5\ \text{m})}{(2257 \times 10^3\ \text{J/kg})} (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(1000^4 - 370^4)\ \text{K}^4 \\
&= \mathbf{0.0370\ kg/s \cdot m}
\end{aligned}$$

Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-42 The base, top, and side surfaces of a furnace of cylindrical shape are black, and are maintained at uniform temperatures. The net rate of radiation heat transfer to or from the top surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.



Properties The emissivity of all surfaces are $\varepsilon = 1$ since they are black.

Analysis We consider the top surface to be surface 1, the base surface to be surface 2 and the side surfaces to be surface 3. The cylindrical furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since all surfaces are black, the radiosities are equal to the emissive power of surfaces, and the net rate of radiation heat transfer from the top surface can be determined from

$$\dot{Q} = A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{13}\sigma(T_1^4 - T_3^4)$$

and $$A_1 = \pi R^2 = \pi(2\,\mathrm{m})^2 = 12.57\,\mathrm{m}^2$$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$ (From Figure 13-7). The view factor from the base to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$$

Substituting,

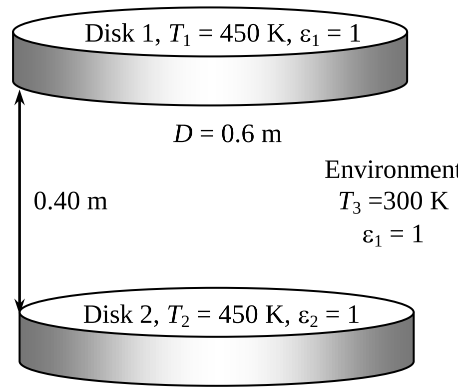
$$\begin{aligned}
\dot{Q} &= A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{13}\sigma(T_1^4 - T_3^4) \\
&= (12.57\,\mathrm{m}^2)(0.38)(5.67\times10^{-8}\ \mathrm{W/m^2.K^4})(700\,\mathrm{K}^4 - 500\,\mathrm{K}^4) \\
&\quad + (12.57\,\mathrm{m}^2)(0.62)(5.67\times10^{-8}\ \mathrm{W/m^2.K^4})(700\,\mathrm{K}^4 - 1400\,\mathrm{K}^4) \\
&= 4.810\times10^4\ \mathrm{W} - 1.591\times10^6\ \mathrm{W} = -1.543\times10^6\ \mathrm{W} = \mathbf{-1543\,kW}
\end{aligned}$$

Discussion The negative sign indicates that net heat transfer is to the top surface.

13-43 Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are ε = 1 since they are black.



Analysis Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 13-7, we read

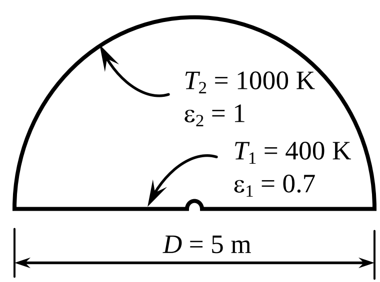
$$F_{12} = F_{21} = 0.26$$
$$F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)}$$

The net rate of radiation heat transfer from the disks into the environment then becomes

$$\begin{aligned}
\dot{Q}_3 &= \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13} \\
\dot{Q}_3 &= 2F_{13}A_1\sigma(T_1^{\,4} - T_3^{\,4}) \\
&= 2(0.74)[\pi(0.3\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(450\text{ K})^4 - (300\text{ K})^4] \\
&= \mathbf{781\,W}
\end{aligned}$$

13-44 The base and the dome of a hemispherical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Analysis The view factor is first determined from

$$F_{11} = 0 \quad \text{(flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)}$$

Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\begin{aligned}
\dot{Q}_{21} &= -\dot{Q}_{12} = -\varepsilon A_1 F_{12}\sigma(T_1^{\,4} - T_2^{\,4}) \\
&= -(0.7)[\pi(5\text{ m})^2/4](1)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (1000\text{ K})^4] \\
&= 7.594\times10^5\text{ W} \\
&= \mathbf{759\,kW}
\end{aligned}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

13-45 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The length of the dryer necessary to dry the materials at 0.1 kg/s is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.

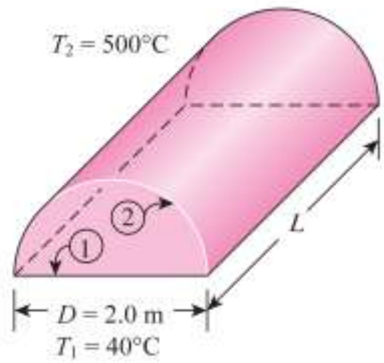
Properties The latent heat of vaporization for water is h_{fg} = 2257 kJ/kg (Table A-2)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{12} = 1 \quad \text{(with } F_{11} = 0\text{)}$$

Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$



Applying energy balance on the base (surface 1), we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m} h_{fg}$$

Hence,

$$\dot{m} h_{fg} = A_2 F_{21} \sigma (T_2^4 - T_1^4) = \frac{\pi DL}{2} \frac{2}{\pi} \sigma (T_2^4 - T_1^4)$$

which gives

$$L = \frac{\dot{m} h_{fg}}{D\sigma (T_2^4 - T_1^4)}$$

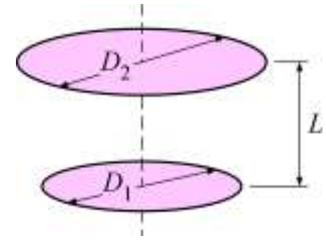
$$= \frac{(0.1\,\text{kg/s})(2257 \times 10^3 \text{ J/kg})}{(2\,\text{m})(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(773^4 - 313^4)\,\text{K}^4} = \mathbf{5.73\ m}$$

Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-46 Two parallel black disks are positioned coaxially, where the lower disk is heated electrically. The temperature of the upper disk is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For coaxial parallel disks, from Table 13-1, with i = 1, j = 2,



$$R_1 = \frac{D_1/2}{L} = \frac{0.2/2}{0.25} = 0.4 \qquad \text{and} \qquad R_2 = \frac{D_2/2}{L} = \frac{0.4/2}{0.25} = 0.8$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+(0.8)^2}{(0.4)^2} = 11.25$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{11.25 - \left[(11.25)^2 - 4\left(\frac{0.4}{0.2}\right)^2\right]^{1/2}\right\} = 0.3676$$

Then, using the summation rule,

$$F_{12} + F_{1\text{surr}} = 1 \qquad \rightarrow \qquad F_{1\text{surr}} = 1 - F_{12} = 0.6324$$

The net radiation heat transfer rate leaving the lower surface can be expressed as

$$\dot{Q}_{\text{elec}} = \dot{Q}_{12} + \dot{Q}_{1\text{surr}} = A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{1\text{surr}}\sigma(T_1^4 - T_{\text{surr}}^4)$$

$$\dot{Q}_{\text{elec}} = A_1\sigma[F_{12}(T_1^4 - T_2^4) + F_{1\text{surr}}(T_1^4 - T_{\text{surr}}^4)]$$

Hence

$$T_2 = \left[T_1^4 - \frac{\dot{Q}_{\text{elec}}}{A_1 F_{12}\sigma} + \frac{F_{1\text{surr}}}{F_{12}}(T_1^4 - T_{\text{surr}}^4)\right]^{1/4} = \left[T_1^4 - \frac{4\dot{Q}_{\text{elec}}}{\pi D_1^2 F_{12}\sigma} + \frac{F_{1\text{surr}}}{F_{12}}(T_1^4 - T_{\text{surr}}^4)\right]^{1/4}$$

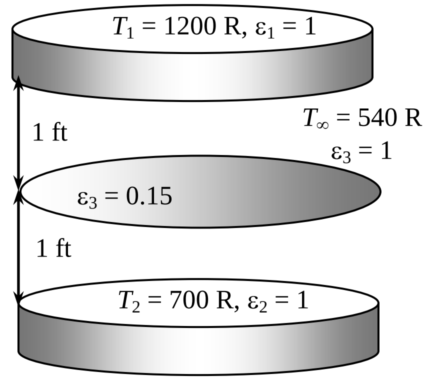
$$T_2 = \left[(500\text{ K})^4 - \frac{4(100\text{ W})}{\pi(0.2\text{ m})^2(0.3676)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)} + \frac{0.6324}{0.3676}(500^4 - 300^4)\text{ K}^4\right]^{1/4}$$

$$T_2 = \mathbf{241\ K}$$

Discussion The view factor F_{12} can also be determined using Fig. 13-7 to be

$$F_{12} \approx 0.36 \qquad \text{with} \qquad L/r_1 = 2.5 \qquad \text{and} \qquad r_2/L = 0.8$$

13-47E A radiation shield is placed between two parallel disks which are maintained at uniform temperatures. The net rate of radiation heat transfer through the shields is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 0.15$.

Analysis From Fig. 13-7 we have $F_{32} = F_{13} = 0.52$. Then $F_{34} = 1 - 0.52 = 0.48$. The disk in the middle is surrounded by black surfaces on both sides. Therefore, heat transfer between the top surface of the middle disk and its black surroundings can expressed as

$$\begin{aligned}\dot{Q}_3 &= \varepsilon A_3 \sigma[F_{31}(T_3^4 - T_1^4)] + \varepsilon A_3 \sigma[F_{32}(T_3^4 - T_2^4)] \\ &= 0.15(7.069\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.52[(T_3^4 - (1200\ \text{R})^4] + 0.48[T_3^4 - (540\ \text{R})^4]\}\end{aligned}$$

where $A_3 = \pi(3\ \text{ft})^2/4 = 7.069\ \text{ft}^2$. Similarly, for the bottom surface of the middle disk, we have

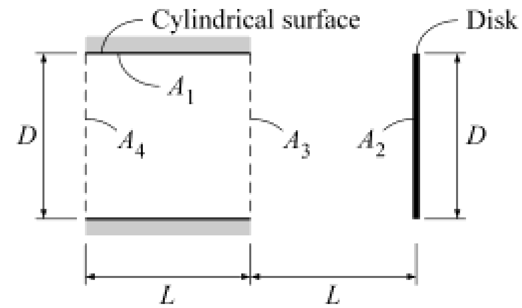
$$\begin{aligned}-\dot{Q}_3 &= \varepsilon A_3 \sigma[F_{32}(T_2^4 - T_4^4)] + \varepsilon A_3 \sigma[F_{34}(T_3^4 - T_4^4)] \\ &= 0.15(7.069\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\{0.48[(T_3^4 - (700\ \text{R})^4] + 0.52[T_3^4 - (540\ \text{R})^4]\}\end{aligned}$$

Combining the equations above, the rate of heat transfer between the disks through the radiation shield (the middle disk) is determined to be

$$\dot{Q} = \mathbf{872\ Btu/h} \quad \text{and} \quad T_3 = 894\ \text{R}$$

13-48 A hot cylindrical surface is placed coaxially with a disk at a distance L apart. The radiation heat transfer rate from the cylindrical surface to the disk is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** Outer surface of the cylinder is well insulated.



Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. Applying the summation rule, we have

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1), we get

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined by treating them as view factors for coaxial parallel disks using Table 13-1:

With $\quad R_2 = \dfrac{r_2}{L} = \dfrac{D/2}{L} = 0.5 \quad$ and $\quad R_3 = R_2 = \dfrac{r_3}{L} = \dfrac{D/2}{L} = 0.5$

We get

$$S = 1 + \frac{1 + R_3^2}{R_2^2} = 1 + \frac{1 + (0.5)^2}{(0.5)^2} = 6$$

$$F_{23} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = 0.1716$$

With $\quad R_2 = \dfrac{D/2}{2L} = 0.25 \quad$ and $\quad R_2 = R_4 = 0.25$

We get

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + (0.25)^2}{(0.25)^2} = 18$$

$$F_{24} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{D_4}{D_2}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}[18 - (18^2 - 4)^{1/2}] = 0.05573$$

Substituting the values of F_{23} and F_{24} into Eq. (3), we have

$$F_{12} = (A_2 / A_1)(F_{23} - F_{24}) = (A_2 / A_1)(0.1716 - 0.05573) = 0.1159(A_2 / A_1)$$

The rate of heat transfer by radiation is then

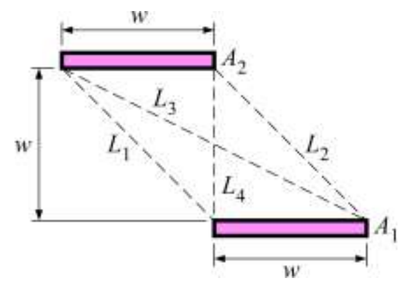
$$\begin{aligned}
\dot{Q}_{12} &= A_1 F_{12} \sigma (T_1^4 - T_2^4) \\
&= 0.1159 A_1 (A_2 / A_1) \sigma (T_1^4 - T_2^4) \\
&= 0.1159 A_2 \sigma (T_1^4 - T_2^4) \\
&= 0.1159 (\pi D^2 / 4) \sigma (T_1^4 - T_2^4) \\
&= 0.1159 \frac{\pi (0.2\text{ m})^2}{4} (5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)(1000^4 - 300^4)\text{ K}^4 \\
&= \mathbf{205\ W}
\end{aligned}$$

Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7.

13-49 The radiation heat flux between two infinitely long parallel plates of specified surface temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The surface temperatures are uniform.

Analysis From the Hottel's crossed-strings method, we have



$$F_{i\to j} = \frac{\Sigma(\text{Crossed strings}) - \Sigma(\text{Uncrossed strings})}{2\times(\text{String on surface } i)}$$

For uncrossed strings, we have

$$L_1 = L_2 = (w^2 + w^2)^{1/2} = (w^2 + w^2)^{1/2} = \sqrt{2}w$$

For crossed strings, we have

$$L_3 = (w^2 + 4w^2)^{1/2} = \sqrt{5}w \qquad \text{and} \qquad L_4 = w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$\begin{aligned} F_{12} &= \frac{(L_3 + L_4) - (L_1 + L_2)}{2w} \\ &= \frac{(\sqrt{5}w + w) - (\sqrt{2}w + \sqrt{2}w)}{2w} \\ &= 0.204 \end{aligned}$$

The radiation heat flux between the two surfaces is

$$\begin{aligned} \dot{q}_{12} &= F_{12}\sigma(T_1^4 - T_2^4) \\ &= (0.204)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(700^4 - 300^4)\ \text{K}^4 \\ &= \mathbf{2680\ W/m^2} \end{aligned}$$

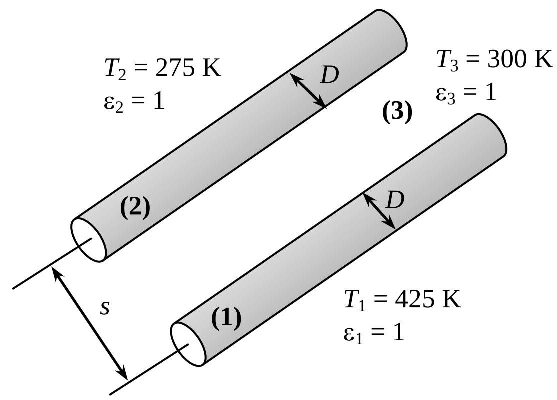
Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-50 Two long parallel cylinders are maintained at specified temperatures. The rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis We consider the hot cylinder to be surface 1, cold cylinder to be surface 2, and the surroundings to be surface 3. Using the crossed-strings method, the view factor between two cylinders facing each other is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$



or

$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D} = \frac{2\left(\sqrt{0.3^2 + 0.20^2} - 0.5\right)}{\pi(0.20)} = 0.444$$

The view factor between the hot cylinder and the surroundings is

$$F_{13} = 1 - F_{12} = 1 - 0.444 = 0.556 \text{ (summation rule)}$$

The rate of radiation heat transfer between the cylinders per meter length is

$$A = \pi DL/2 = \pi(0.20\text{ m})(1\text{ m})/2 = 0.3142\text{ m}^2$$

$$\begin{aligned}\dot{Q}_{12} &= AF_{12}\sigma(T_1^{\,4} - T_2^{\,4}) \\ &= (0.3142\text{ m}^2)(0.444)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 275^4)\text{K}^4 \\ &= \mathbf{212.8\,W}\end{aligned}$$

Note that half of the surface area of the cylinder is used, which is the only area that faces the other cylinder. The rate of radiation heat transfer between the hot cylinder and the surroundings per meter length of the cylinder is

$$A_1 = \pi DL = \pi(0.20\text{ m})(1\text{ m}) = 0.6283\text{ m}^2$$

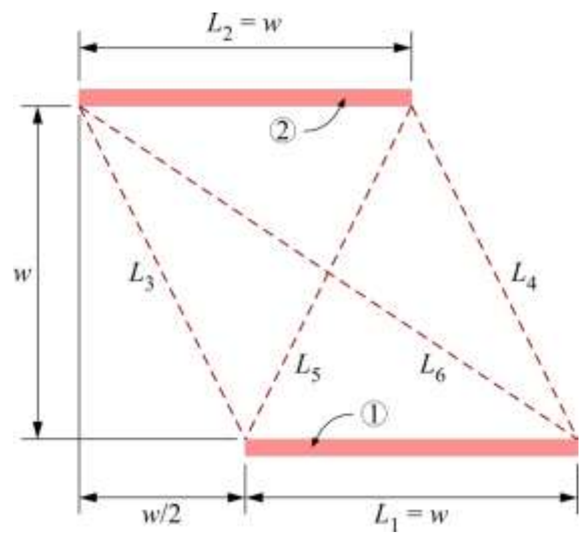
$$\begin{aligned}\dot{Q}_{13} &= A_1F_{13}\sigma(T_1^{\,4} - T_3^{\,4}) \\ &= (0.6283\text{ m}^2)(0.556)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 300^4)\text{K}^4 \\ &= \mathbf{485.8\,W}\end{aligned}$$

13-51 The radiation heat flux between two infinitely long parallel plates of specified surface temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The surface temperatures are uniform.

Analysis From the Hottel's crossed-strings method, we have

$$F_{i\to j} = \frac{\Sigma(\text{Crossed strings}) - \Sigma(\text{Uncrossed strings})}{2\times(\text{String on surface } i)}$$



where,

$$L_1 = L_2 = w$$

$$L_3 = L_4 = L_5 = \sqrt{w^2 + (w/2)^2} = \frac{\sqrt{5}}{2}w$$

$$L_6 = \sqrt{w^2 + \left(\frac{3}{2}w\right)^2} = \frac{\sqrt{13}}{2}w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$\begin{aligned} F_{12} &= \frac{(L_5 + L_6) - (L_3 + L_4)}{2w} \\ &= \frac{(\sqrt{5}w/2 + \sqrt{13}w/2) - (\sqrt{5}w/2 + \sqrt{5}w/2)}{2w} \\ &= 0.3424 \end{aligned}$$

The radiation heat flux between the two surfaces is

$$\begin{aligned} \dot{q}_{12} &= F_{12}\sigma(T_1^4 - T_2^4) \\ &= (0.3424)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4 - 323^4)\ \text{K}^4 \\ &= \mathbf{6720\ W/m^2} \end{aligned}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

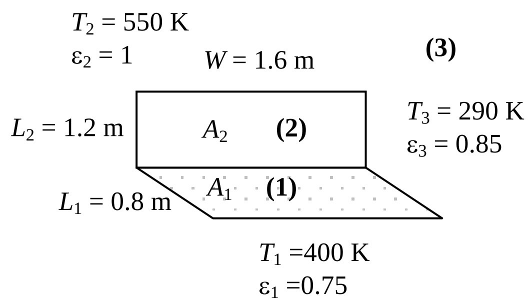
13-52 Two perpendicular rectangular surfaces with a common edge are maintained at specified temperatures. The net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the horizontal rectangle and the surroundings are $\varepsilon = 0.75$ and $\varepsilon = 0.85$, respectively.

Analysis We consider the horizontal rectangle to be surface 1, the vertical rectangle to be surface 2 and the surroundings to be surface 3. This system can be considered to be a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{array}{l}\dfrac{L_1}{W}=\dfrac{0.8}{1.6}=0.5\\[2mm]\dfrac{L_2}{W}=\dfrac{1.2}{1.6}=0.75\end{array}\right\}F_{12}=0.27\ \text{(Fig. 13-6)}$$



The surface areas are

$$A_1=(0.8\text{ m})(1.6\text{ m})=1.28\text{ m}^2$$

$$A_2=(1.2\text{ m})(1.6\text{ m})=1.92\text{ m}^2$$

$$A_3=2\times\frac{1.2\times0.8}{2}+\sqrt{0.8^2+1.2^2}\times1.6=3.268\text{ m}^2$$

Note that the surface area of the surroundings is determined assuming that surroundings forms flat surfaces at all openings to form an enclosure. Then other view factors are determined to be

$$A_1F_{12}=A_2F_{21}\longrightarrow(1.28)(0.27)=(1.92)F_{21}\longrightarrow F_{21}=0.18 \qquad \text{(reciprocity rule)}$$

$$F_{11}+F_{12}+F_{13}=1\longrightarrow0+0.27+F_{13}=1\longrightarrow F_{13}=0.73 \qquad \text{(summation rule)}$$

$$F_{21}+F_{22}+F_{23}=1\longrightarrow0.18+0+F_{23}=1\longrightarrow F_{23}=0.82 \qquad \text{(summation rule)}$$

$$A_1F_{13}=A_3F_{31}\longrightarrow(1.28)(0.73)=(3.268)F_{31}\longrightarrow F_{31}=0.29 \qquad \text{(reciprocity rule)}$$

$$A_2F_{23}=A_3F_{32}\longrightarrow(1.92)(0.82)=(3.268)F_{32}\longrightarrow F_{32}=0.48 \qquad \text{(reciprocity rule)}$$

We now apply Eq. 13-35 to each surface to determine the radiosities.

Surface 1:

$$\sigma T_1^{\,4}=J_1+\frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(400\text{ K})^4=J_1+\frac{1-0.75}{0.75}\left[0.27(J_1-J_2)+0.73(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^{\,4}=J_2\longrightarrow(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(550\text{ K})^4=J_2$$

Surface 3:

$$\sigma T_3^{\,4}=J_3+\frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(290\text{ K})^4=J_3+\frac{1-0.85}{0.85}\left[0.29(J_3-J_1)+0.48(J_3-J_2)\right]$$

Solving the above equations, we find

$$J_1=1587\text{ W/m}^2,\quad J_2=5188\text{ W/m}^2,\quad J_3=811.5\text{ W/m}^2$$

Then the net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are determined to be

$$\dot{Q}_{21}=-\dot{Q}_{12}=-A_1F_{12}(J_1-J_2)=-(1.28\text{ m}^2)(0.27)(1587-5188)\text{W/m}^2=\mathbf{1245\ W}$$

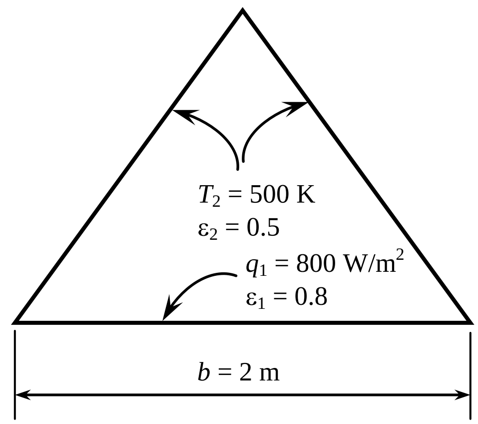
$$\dot{Q}_{13}=A_1F_{13}(J_1-J_3)=(1.28\text{ m}^2)(0.73)(1587-811.5)\text{W/m}^2=\mathbf{725\ W}$$

13-53 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** End effects are neglected.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$.

Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from



$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$800\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(T_1)^4 - (500\text{ K})^4]}{\dfrac{1-0.8}{(1\text{ m}^2)(0.8)} + \dfrac{1}{(1\text{ m}^2)(1)} + \dfrac{1-0.5}{(2\text{ m}^2)(0.5)}}$$

$$T_1 = \mathbf{543\,K}$$

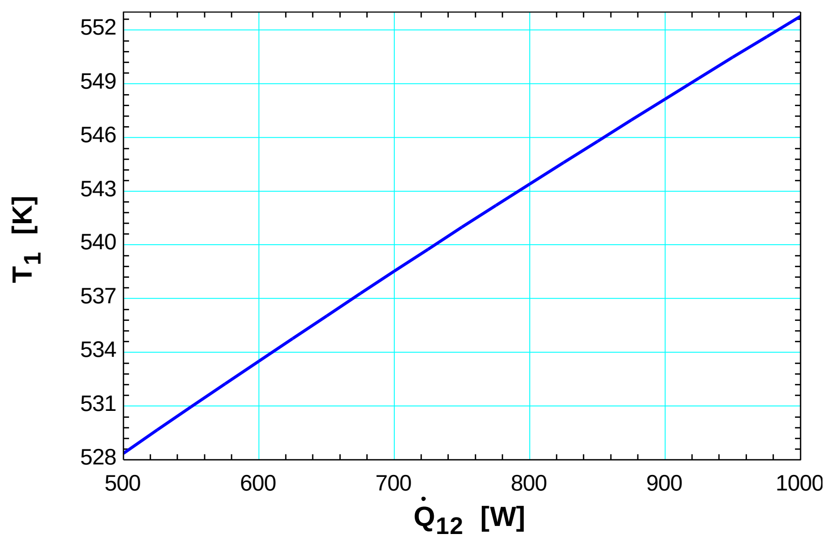
Note that $A_1 = 1\text{ m}^2$ and $A_2 = 2\text{ m}^2$.

13-54 Prob. 13-53 is reconsidered. The effects of the rate of the heat transfer at the base surface and the temperature of the side surfaces on the temperature of the base surface are to be investigated.

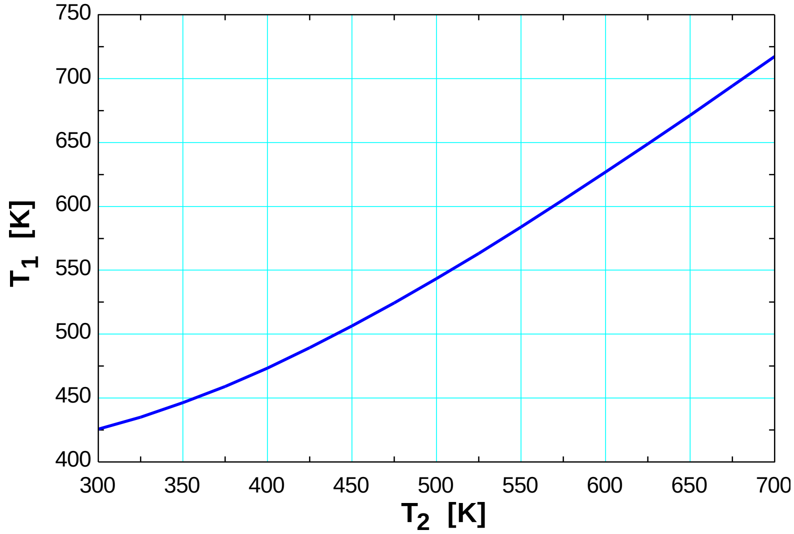
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
a=2 [m]
epsilon_1=0.8
epsilon_2=0.5
Q_dot_12=800 [W]
T_2=500 [K]
sigma=5.67E-8 [W/m^2-K^4]

"ANALYSIS"
"Consider the base surface to be surface 1, the side surfaces to be surface 2"
Q_dot_12=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_12)+(1-
epsilon_2)/(A_2*epsilon_2))
F_12=1
A_1=1 "[m^2], since rate of heat supply is given per meter square area"
A_2=2*A_1
```

$\dot{Q}_{12}$ [W]	T_1 [K]
500	528.4
525	529.7
550	531
575	532.2
600	533.5
625	534.8
650	536
675	537.3
700	538.5
725	539.8
750	541
775	542.2
800	543.4
825	544.6
850	545.8
875	547
900	548.1
925	549.3
950	550.5
975	551.6
1000	552.8



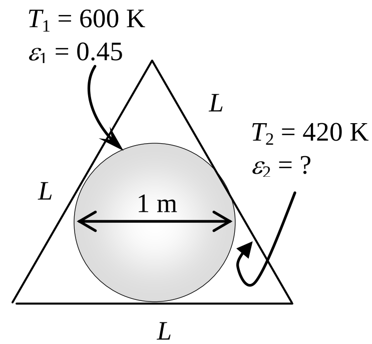
T_2 [K]	T_1 [K]
300	425.5
325	435.1
350	446.4
375	459.2
400	473.6
425	489.3
450	506.3
475	524.4
500	543.4
525	563.3
550	583.8
575	605
600	626.7
625	648.9
650	671.4
675	694.2
700	717.3



13-55 A solid sphere is placed in an evacuated equilateral triangular enclosure. The view factor from the enclosure to the sphere and the emissivity of the enclosure are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of sphere is given to be ε_1 = 0.45.



Analysis (*a*) We take the sphere to be surface 1 and the surrounding enclosure to be surface 2. The view factor from surface 2 to surface 1 is determined from reciprocity relation:

$$A_1 = \pi D^2 = \pi(1\,\text{m})^2 = 3.142\,\text{m}^2$$

$$L = D\sqrt{6} = (1\,\text{m})^2\sqrt{6} = 2.449\,\text{m}$$

$$A_2 = 4\frac{L^2\sqrt{3}}{4} = L^2\sqrt{3} = (2.449\,\text{m})^2\sqrt{3} = 10.39\,\text{m}^2$$

$$A_1F_{12} = A_2F_{21}$$

$$(3.142)(1) = (10.39)F_{21}$$

$$F_{21} = \mathbf{0.3023}$$

We note that the tetrahedron has four equal surfaces.

(b) The net rate of radiation heat transfer can be expressed for this two-surface enclosure to yield the emissivity of the enclosure:

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$3100\,\text{W} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(600\,\text{K})^4 - (420\,\text{K})^4\right]}{\dfrac{1-0.45}{(3.142\,\text{m}^2)(0.45)} + \dfrac{1}{(3.142\,\text{m}^2)(1)} + \dfrac{1-\varepsilon_2}{(5.196\,\text{m}^2)\varepsilon_2}}$$

$$\varepsilon_2 = \mathbf{0.7515}$$

13-56 A long semi-cylindrical duct with specified temperature on the side surface is considered. The temperature of the base surface for a specified heat transfer rate is to be determined.

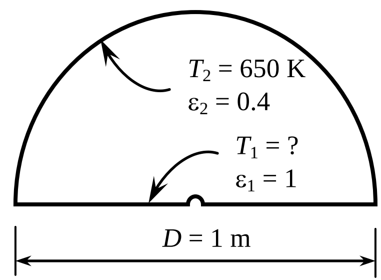
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the side surface is ε = 0.4.

Analysis We consider the base surface to be surface 1, the side surface to be surface 2. This system is a two-surface enclosure, and we consider a unit length of the duct. The surface areas and the view factor are determined as

$$A_1 = (1.0\text{ m})(1.0\text{ m}) = 1.0\text{ m}^2$$
$$A_2 = \pi DL/2 = \pi(1.0\text{ m})(1\text{ m})/2 = 1.571\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$



The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$1200\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot K^4)[T_1^{\,4} - (650\text{ K})^4]}{\dfrac{1}{(1.0\text{ m}^2)(1)} + \dfrac{1-0.4}{(1.571\text{ m}^2)(0.4)}}$$

$$T_1 = \mathbf{684.8 K}$$

13-57 A hemisphere with specified base and dome temperatures and heat transfer rate is considered. The emissivity of the dome is to be determined.

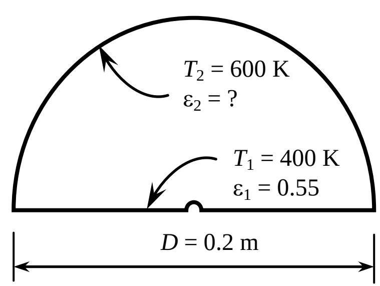
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the base surface is ε = 0.55.

Analysis We consider the base surface to be surface 1, the dome surface to be surface 2. This system is a two-surface enclosure. The surface areas and the view factor are determined as

$$A_1 = \pi D^2/4 = \pi(0.20\text{ m})^2/4 = 0.03142\text{ m}^2$$
$$A_2 = \pi D^2/2 = \pi(0.20\text{ m})^2/2 = 0.06283\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$

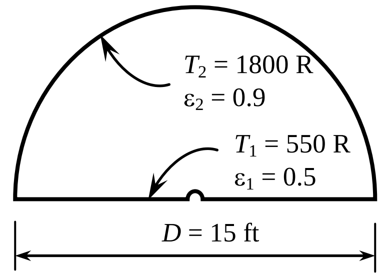


The emissivity of the dome is determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$50\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (600\text{ K})^4]}{\dfrac{1-0.55}{(0.03142\text{ m}^2)(0.55)} + \dfrac{1}{(0.03142\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(0.06283\text{ m}^2)\varepsilon_2}} \longrightarrow \varepsilon_2 = \mathbf{0.209}$$

13-58E The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$.

Analysis The view factor from the base to the dome is first determined from

$$F_{11} = 0 \text{ (flat surface)}$$

$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \text{ (summation rule)}$$

The net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = -\frac{(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(550\ \text{R})^4 - (1800\ \text{R})^4]}{\dfrac{1-0.5}{(15\ \text{ft}^2)(0.5)} + \dfrac{1}{(15\ \text{ft}^2)(1)} + \dfrac{1-0.9}{\left[\dfrac{\pi(15\ \text{ft})(1\ \text{ft})}{2}\right](0.9)}}$$

$$= \mathbf{129{,}200\ Btu/h} \text{ per ft length}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

13-59 Radiation heat transfer occurs between a sphere and a circular disk. The view factors and the net rate of radiation heat transfer for the existing and modified cases are to be determined.

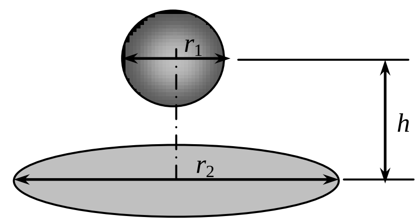
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of sphere and disk are given to be ε_1 = 0.9 and ε_2 = 0.5, respectively.

Analysis (*a*) We take the sphere to be surface 1 and the disk to be surface 2. The view factor from surface 1 to surface 2 is determined from

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.60\text{ m}}\right)^2\right]^{-0.5}\right\} = \mathbf{0.2764}$$

The view factor from surface 2 to surface 1 is determined from reciprocity relation:



$$A_1 = 4\pi r_1^2 = 4\pi(0.3\text{ m})^2 = 1.131\text{ m}^2$$
$$A_2 = \pi r_2^2 = \pi(1.2\text{ m})^2 = 4.524\text{ m}^2$$
$$A_1F_{12} = A_2F_{21}$$
$$(1.131)(0.2764) = (4.524)F_{21}$$
$$F_{21} = \mathbf{0.0691}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}+\dfrac{1}{A_1F_{12}}+\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right]}{\dfrac{1-0.9}{(1.131\text{ m}^2)(0.9)}+\dfrac{1}{(1.131\text{ m}^2)(0.2764)}+\dfrac{1-0.5}{(4.524\text{ m}^2)(0.5)}} = \mathbf{8550\text{ W}}$$

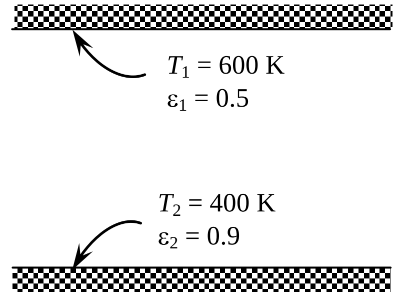
(c) The best values are $\varepsilon_1 = \varepsilon_2 = 1$ and $h = r_1 = 0.3\text{m}$. Then the view factor becomes

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.30\text{ m}}\right)^2\right]^{-0.5}\right\} = 0.3787$$

The net rate of radiation heat transfer in this case is

$$\dot{Q} = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) = (1.131\text{ m}^2)(0.3787)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right] = \mathbf{12{,}890\text{ W}}$$

13-60 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

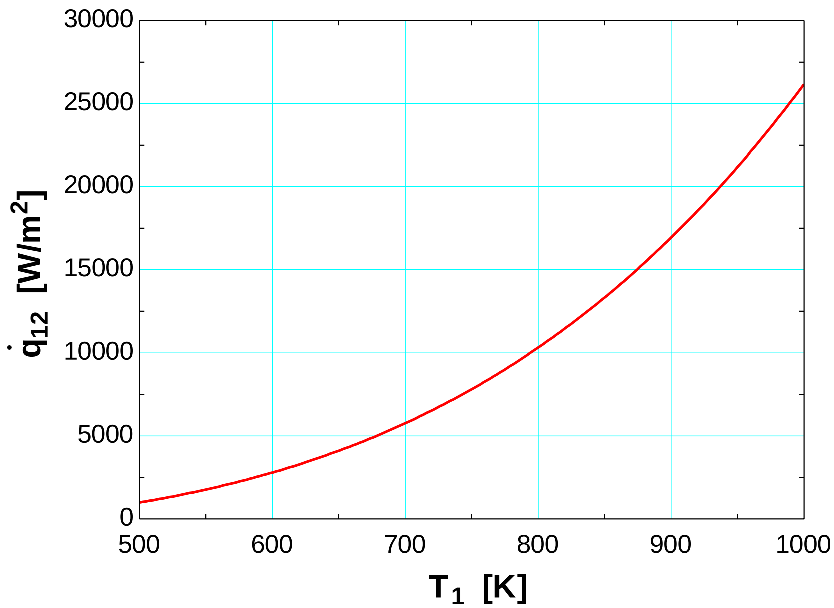
$$\frac{\dot{Q}_{12}}{A_s}=\frac{\sigma(T_1^{\,4}-T_2^{\,4})}{\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1}=\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4-(400\ \text{K})^4]}{\dfrac{1}{0.5}+\dfrac{1}{0.9}-1}=\mathbf{2793\ W/m^2}$$

13-61 Prob. 13-60 is reconsidered. The effects of the temperature and the emissivity of the hot plate on the net rate of radiation heat transfer between the plates are to be investigated.

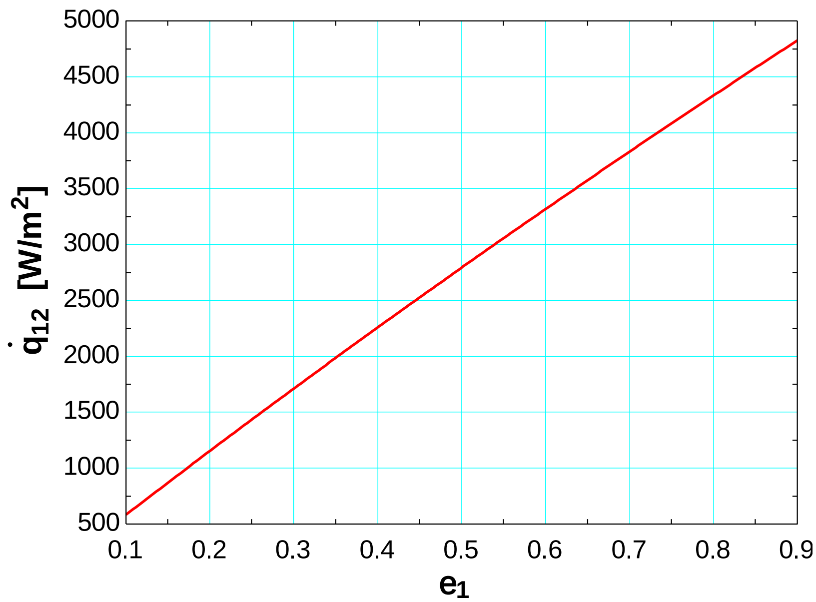
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_1=600 [K]
T_2=400 [K]
epsilon_1=0.5
epsilon_2=0.9
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_12=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

T_1 [K]	$\dot{q}_{12}$ [W/m^2]
500	991.1
525	1353
550	1770
575	2248
600	2793
625	3411
650	4107
675	4888
700	5761
725	6733
750	7810
775	9001
800	10313
825	11754
850	13332
875	15056
900	16934
925	18975
950	21188
975	23584
1000	26170



ε_1	$\dot{q}_{12}$ [W/m^2]
0.1	583.2
0.15	870
0.2	1154
0.25	1434
0.3	1712
0.35	1987
0.4	2258
0.45	2527
0.5	2793
0.55	3056
0.6	3317
0.65	3575
0.7	3830
0.75	4082
0.8	4332
0.85	4580
0.9	4825



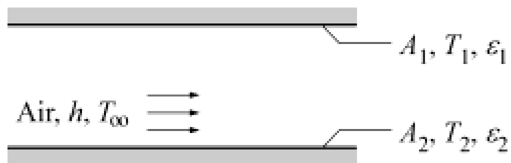
13-62 Air is flowing between two infinitely large parallel plates. The convection heat transfer coefficient associated with the air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The surface temperatures are uniform.

Properties The emissivity of the upper plate is given as $\varepsilon_1 = 0.7$. The lower plate surface is black, $\varepsilon_2 = 1$.

Analysis For infinitely large parallel plates, the rate of radiation heat transfer is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$



Applying energy balance on the lower plate, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}}$$

$$hA(T_2 - T_\infty) = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$h = \frac{\sigma}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \frac{(T_1^4 - T_2^4)}{(T_2 - T_\infty)}$$

$$= \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)}{\frac{1}{0.7} + 1 - 1} \frac{(500^4 - 330^4)\ \text{K}^4}{(330 - 290)\ \text{K}}$$

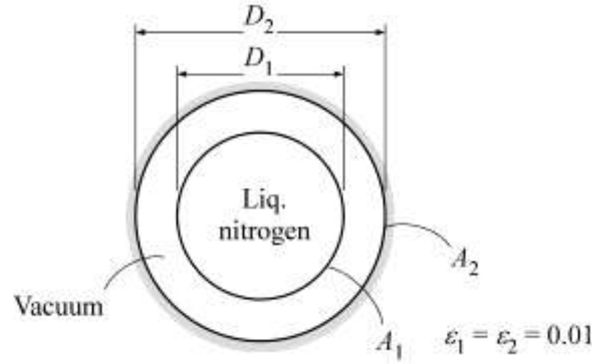
$$h = \mathbf{50.3\ W/m^2 \cdot K}$$

Discussion The calculated value of the convection heat transfer coefficient (h = 50.3 W/m^2·K) is typical for forced convection of gases (see Table 1-5)

13-63 Liquid nitrogen is stored in a spherical tank this is enclosed by a concentric spherical surface at 273 K. The rate of vaporization for the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Heat transfer by radiation only.

Properties The emissivity of the two surfaces is given as $\varepsilon_1 = \varepsilon_2 = 0.01$. The latent heat of vaporization for nitrogen is h_{fg} = 198.6 kJ/kg (Table A-2).



Analysis For concentric spheres, the rate of radiation heat transfer at the inner surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

Hence,

$$\dot{Q}_{12} = \frac{\pi(1\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(80^4 - 273^4)\,\text{K}^4}{\dfrac{1}{0.01} + \dfrac{1-0.01}{0.01}\left(\dfrac{1}{1.6}\right)^2} = -7.082\ \text{W}$$

The rate of vaporization can be determined using

$$-\dot{Q}_{12} = \dot{m}h_{fg}$$

$$\dot{m} = -\frac{\dot{Q}_{12}}{h_{fg}} = -\frac{-7.082\ \text{W}}{198.6\times10^3\ \text{J/kg}}$$

$$\dot{m} = \mathbf{3.57\times10^{-5}\ kg/s}$$

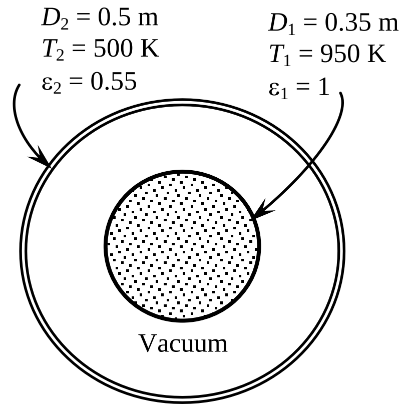
Discussion The rate of vaporization can be reduced by placing a radiation shield midway between the inner and outer spherical surfaces.

13-64 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.55$.

Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

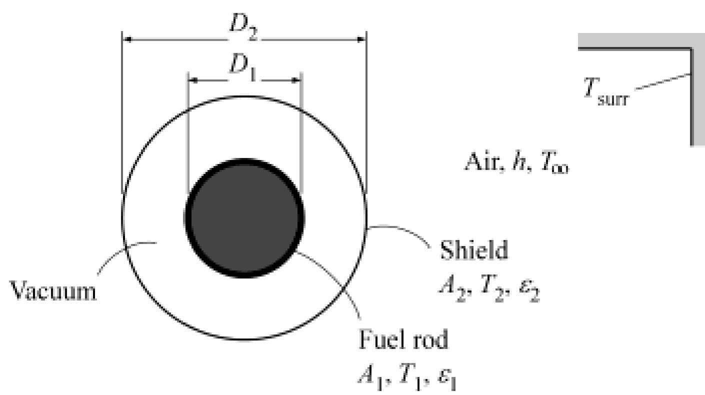
$$= \frac{[\pi(0.35\,\text{m})(1\,\text{m})](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(950\,\text{K})^4 - (500\,\text{K})^4]}{\dfrac{1}{1} + \dfrac{1-0.55}{0.55}\left(\dfrac{3.5}{5}\right)}$$

$$= 29{,}810\ \text{W} = \mathbf{29.81\,kW}$$

13-65 A long cylindrical black surface fuel rod is shielded by a concentric surface that has a uniform temperature. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The fuel rod surface is black. 3 The shield is opaque, diffuse, and gray. **4** The fuel rod and shield form an infinitely long concentric cylinder.

Properties The emissivity of the shield is given as $\varepsilon_2 = 0.05$. The fuel rod surface is black, $\varepsilon_1 = 1$.



Analysis For infinitely long concentric cylinder, the rate of radiation heat transfer at the fuel rod surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{1} + \dfrac{1-0.05}{0.05}\left(\dfrac{25}{50}\right)} = 0.09524 A_1\sigma(T_1^4 - T_2^4)$$

Applying energy balance on the shield, we have the following expression:

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_2(T_2 - T_\infty) + \varepsilon_2 A_2\sigma(T_2^4 - T_{\text{surr}}^4)$$

Hence

$$T_1^4 = \frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{A_2}{A_1}\right) + T_2^4$$

$$= \frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{D_2}{D_1}\right) + T_2^4$$

or

$$T_1 = \left[\frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{D_2}{D_1}\right) + T_2^4\right]^{1/4}$$

$$T_1 = \left[\frac{(15)(320-300) + (0.05)(5.67\times10^{-8})(320^4 - 300^4)}{0.09524(5.67\times10^{-8})}\left(\frac{50}{25}\right)\text{K}^4 + (320\text{ K})^4\right]^{1/4}$$

$$T_1 = \mathbf{594\ K}$$

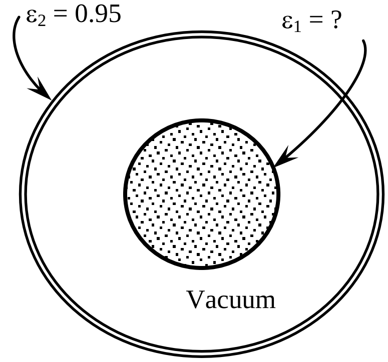
Discussion The use of absolute temperatures is necessary for calculations involving radiation heat transfer.

13-66 A long cylindrical rod coated with a new material is placed in an evacuated long cylindrical enclosure which is maintained at a uniform temperature. The emissivity of the coating on the rod is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the enclosure is given to be $\varepsilon_2 = 0.95$.

Analysis The emissivity of the coating on the rod is determined from



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

$$8\text{ W} = \frac{[\pi(0.01\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(500\text{ K})^4 - (200\text{ K})^4]}{\dfrac{1}{\varepsilon_1} + \dfrac{1-0.95}{0.95}\left(\dfrac{1}{10}\right)}$$

which gives

$\varepsilon_1 =$ **0.0738**

13-67 Liquid NH_3 flows in an insulated tube that is protected by a concentric shield. The surrounding temperature is to be determined so that the NH_3 is maintained in the liquid state.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{surr}$.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.33$.

Analysis The net rate of radiation heat transfer between the two concentric cylinders is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)}$$

Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{rad} = \varepsilon A_2 \sigma(T_{surr}^4 - T_2^4) = \varepsilon A_2 \sigma(T_\infty^4 - T_2^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{conv} = hA_2(T_\infty - T_2)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

$$\frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)} = hA_2(T_\infty - T_2) + \varepsilon A_2 \sigma(T_\infty^4 - T_2^4)$$

$$\frac{D_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)} = hD_2(T_\infty - T_2) + \varepsilon D_2 \sigma(T_\infty^4 - T_2^4)$$

Thus,

$$\frac{(0.04\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(283^4 - 273^4)\text{K}^4}{\dfrac{1}{0.33} + \dfrac{1-0.33}{0.33}\left(\dfrac{0.04\text{ m}}{0.08\text{ m}}\right)} = (3\text{ W/m}^2\cdot\text{K})(0.08\text{ m})(T_\infty - 283)\text{K}$$

$$+ (0.33)(0.08\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(T_\infty^4 - 283^4)\text{K}^4$$

The surrounding air temperature T_∞ can be solved by trial-and-error to yield T_∞ = **11.3°C**.

Discussion By monitoring the temperatures of the surrounding air and the shield surface, the vaporization of the liquid NH_3 flowing inside the tube can be prevented.

13-68 Hot fluid flowing inside a long tube and the tube is enclosed in a concentric cylindrical thin cover. The emissivity of the inside tube is to be determined so that the outer surface temperature is kept below 45°C to prevent thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{surr}$.

Properties The emissivity of outer cylindrical cover is given to be $\varepsilon_2 = 0.6$.

Analysis The net rate of radiation heat transfer between the two concentric cylinders is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{\text{rad}} = \varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{\text{conv}} = hA_2(T_2 - T_\infty)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}$$

$$\frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hA_2(T_2 - T_\infty)$$

Thus,

$$\varepsilon_1 = \left[\frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hA_2(T_2 - T_\infty)} - \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]^{-1}$$

$$= \left[\frac{D_1\sigma(T_1^{\,4} - T_2^{\,4})}{\varepsilon_2 D_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hD_2(T_2 - T_\infty)} - \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]^{-1}$$

$$= \left[\frac{(0.025\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(423^4 - 318^4)\text{ K}^4}{(0.6)(0.05\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(318^4 - 293^4)\text{ K}^4 + (8\text{ W/m}^2\cdot\text{K})(0.05\text{ m})(318-293)\text{ K}} - \frac{1-0.6}{0.6}\left(\frac{0.025\text{ m}}{0.05\text{ m}}\right)\right]^{-1}$$

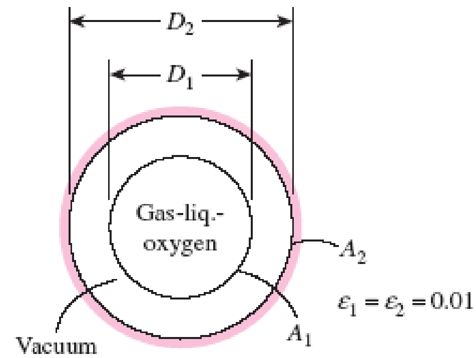
$$\varepsilon_1 = \mathbf{0.573}$$

Discussion In order to keep the outer surface at 45°C, the emissivity of the inner tube should be 0.573 or lower.

13-69 Two phase gas-liquid oxygen is stored in a spherical tank this is enclosed by a concentric spherical surface at 273 K. The heat transfer rate at the spherical tank surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Heat transfer by radiation only.

Properties The emissivity of the two surfaces is given as $\varepsilon_1 = \varepsilon_2 = 0.01$. The normal boiling point of oxygen is −183°C (Table A-2).



Analysis For concentric spheres, the rate of radiation heat transfer at the inner surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

Note that the spherical tank surface has the same temperature as the oxygen at normal boiling point, $T_1 = -183°\text{C} = 90\text{ K}$. Hence,

$$\dot{Q}_{12} = \frac{\pi(1\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(90^4 - 273^4)\,\text{K}^4}{\dfrac{1}{0.01} + \dfrac{1-0.01}{0.01}\left(\dfrac{1}{1.6}\right)^2} = \mathbf{-7.05\ W}$$

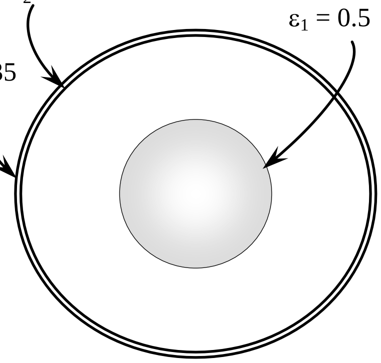
Discussion The negative value of $\dot{Q}_{12}$ indicates that heat is being added to the oxygen. As long as the oxygen is maintained in the two-phase gas-liquid state, its temperature will remain constant at the normal boiling point.

13-70 Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.7$.

Analysis The net rate of radiation heat transfer between the two spheres is



$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$= \frac{\left[\pi(0.3\,\text{m})^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(700\,\text{K})^4 - (500\,\text{K})^4\right]}{\dfrac{1}{0.5} + \dfrac{1-0.7}{0.7}\left(\dfrac{0.15\,\text{m}}{0.3\,\text{m}}\right)^2}$$

$$= \mathbf{1351\,W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2\sigma(T_2^4 - T_{surr}^4)$$
$$= (0.35)(1)[\pi(0.4\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(500\,\text{K})^4 - (30+273\,\text{K})^4]$$
$$= 539\ \text{W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 1351 - 539 = 812\ \text{W}$$

Then the convection heat transfer coefficient becomes

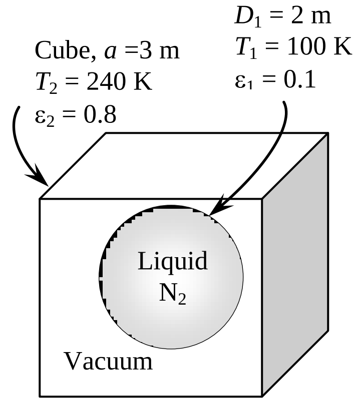
$$\dot{Q}_{conv.} = hA_2\left(T_2 - T_\infty\right)$$
$$812\ \text{W} = h\left[\pi(0.4\,\text{m})^2\right](500\,\text{K - }303\,\text{K})$$
$$h = \mathbf{8.20\,W/m^2\cdot{}^\circ C}$$

13-71 A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that $F_{12} = 1$, for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right)}$$

$$= -\frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(100\text{ K})^4 - (240\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{\pi(2\text{ m})^2}{6(3\text{ m})^2}\right]}$$

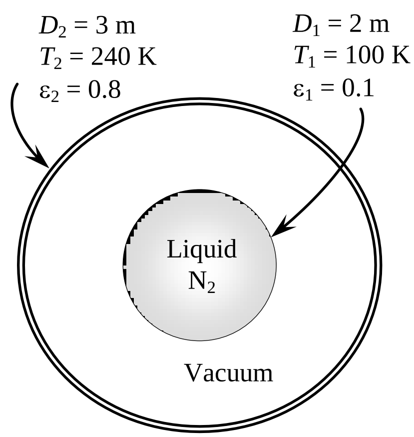
$$= \mathbf{228\ W}$$

13-72 A spherical tank filled with liquid nitrogen is kept in an evacuated spherical enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$= \frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(240\text{ K})^4 - (100\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{(1\text{ m})^2}{(1.5\text{ m})^2}\right]}$$

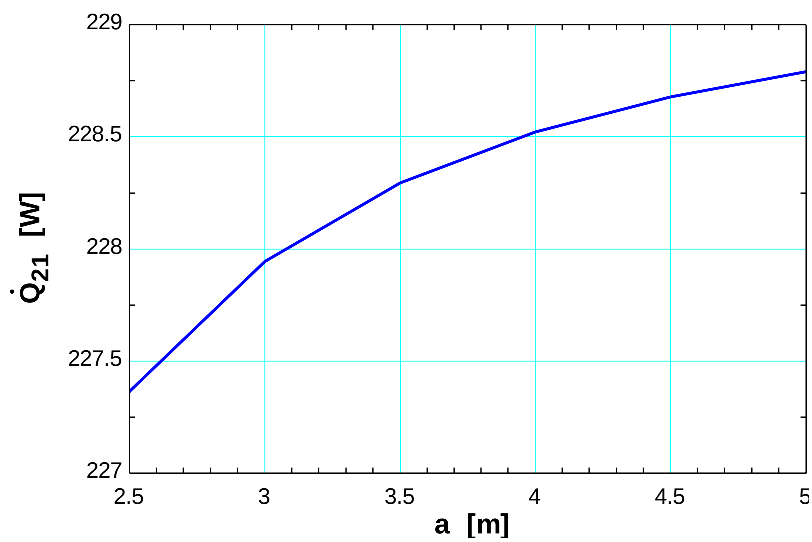
$$= \mathbf{227\ W}$$

13-73 Prob. 13-72 is reconsidered. The effects of the side length and the emissivity of the cubic enclosure, and the emissivity of the spherical tank on the net rate of radiation heat transfer are to be investigated.

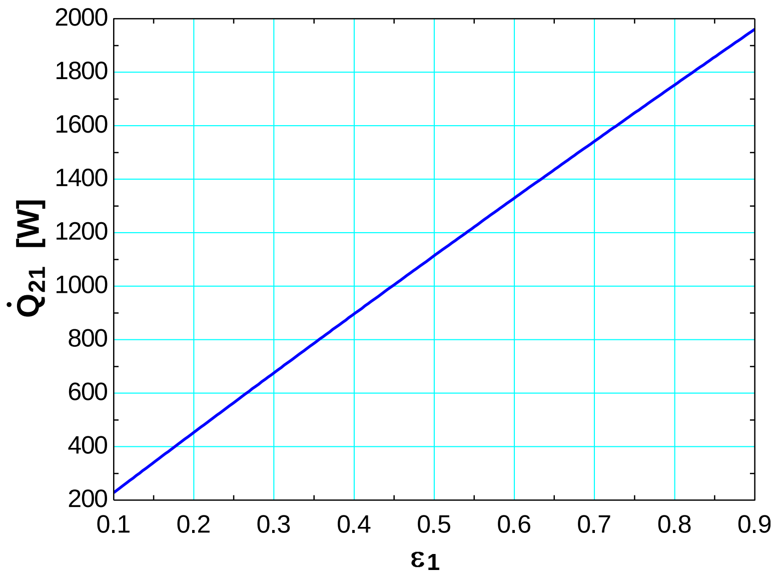
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=2 [m]
a=3 [m]
T_1=100 [K]
T_2=240 [K]
epsilon_1=0.1
epsilon_2=0.8
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the sphere to be surface 1, the surrounding cubic enclosure to be surface 2"
Q_dot_12=(A_1*sigma*(T_1^4-T_2^4))/(1/epsilon_1+(1-epsilon_2)/epsilon_2*(A_1/A_2))
Q_dot_21=-Q_dot_12
A_1=pi*D^2
A_2=6*a^2
```

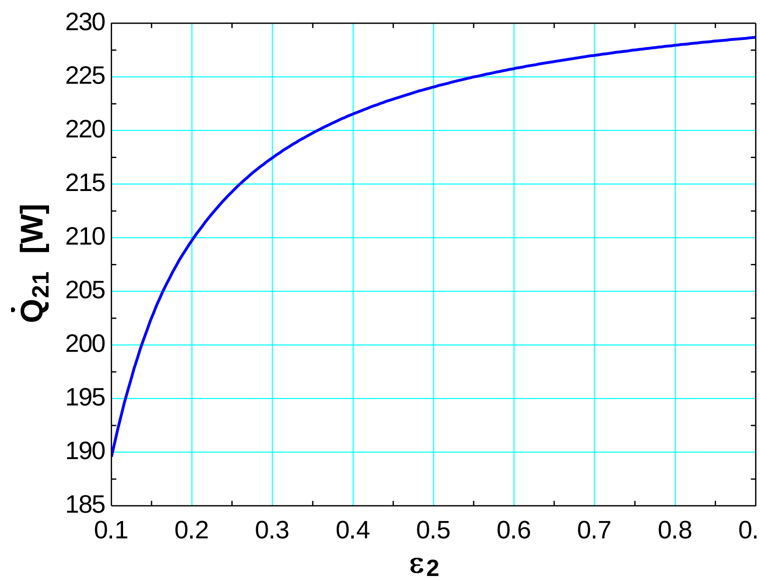
a [m]	$\dot{Q}_{21}$ [W]
2.5	227.4
2.625	227.5
2.75	227.7
2.875	227.8
3	227.9
3.125	228
3.25	228.1
3.375	228.2
3.5	228.3
3.625	228.4
3.75	228.4
3.875	228.5
4	228.5
4.125	228.6
4.25	228.6
4.375	228.6
4.5	228.7
4.625	228.7
4.75	228.7
4.875	228.8
5	228.8



ε_1	$\dot{Q}_{21}$ [W]
0.1	227.9
0.15	340.9
0.2	453.3
0.25	565
0.3	676
0.35	786.4
0.4	896.2
0.45	1005
0.5	1114
0.55	1222
0.6	1329
0.65	1436
0.7	1542
0.75	1648
0.8	1753
0.85	1857
0.9	1961



ε_2	$\dot{Q}_{21}$ [W]
0.1	189.6
0.15	202.6
0.2	209.7
0.25	214.3
0.3	217.5
0.35	219.8
0.4	221.5
0.45	222.9
0.5	224.1
0.55	225
0.6	225.8
0.65	226.4
0.7	227
0.75	227.5
0.8	227.9
0.85	228.3
0.9	228.7



13-74 PtD Cold fluid stored in a spherical tank enclosed in a concentric outer cover. The gap of the vacuumed enclosure is to be determined so that the outer surface temperature is not below the dew point.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Radiation heat transfer between the outer surface and the surrounding is negligible.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.6$.

Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2}$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{\text{conv}} = hA_2(T_2 - T_\infty)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}}$$

$$\frac{A_1\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = hA_2(T_2 - T_\infty) \quad \to \quad \frac{D_1^2\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = hD_2^2(T_2 - T_\infty)$$

Hence,

$$\begin{aligned} D_2 &= \left[\frac{D_1^2\,\varepsilon\sigma(T_1^{\ 4} - T_2^{\ 4})}{h(T_2 - T_\infty)} - (1-\varepsilon)D_1^2\right]^{0.5} \\ &= \left[\frac{(3\,\text{m})^2(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(278^4 - 283^4)\,\text{K}^4}{(3\,\text{W/m}^2\cdot\text{K})(283-286)\,\text{K}} - (1-0.6)(3\,\text{m})^2\right]^{0.5} \\ &= 3.38\,\text{m} \end{aligned}$$

Thus, the gap of the vacuumed enclosure is

$$L_{\text{gap}} = \frac{D_2 - D_1}{2} = \frac{3.38 - 3}{2}\,\text{m} = \mathbf{0.19\,m}$$

Discussion To keep the outer surface temperature above the dew point of 10°C, the vacuumed gap should be greater than 19 cm. Gap size below 19 cm will bring the outer surface temperature to below 10°C and condensation could occur to cause electrical hazards.

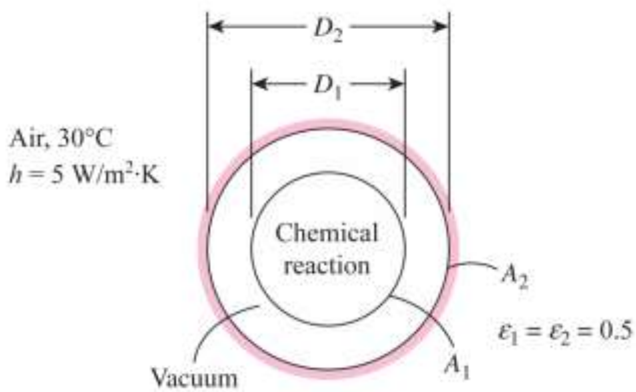
13-75 A spherical tank is filled with chemical in an exothermic reaction that heats up the surface temperature. The tank in enclosed by a concentric outer cover to prevent thermal burn hazards. The temperature of the outer cover is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{surr}$.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.5$.

Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1 \sigma (T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2}$$



Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{rad} = \varepsilon A_2 \sigma (T_2^{\ 4} - T_{surr}^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{conv} = hA_2 (T_2 - T_\infty)$$

Performing an energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{rad} + \dot{Q}_{conv}$$

$$\frac{A_1 \sigma (T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = \varepsilon A_2 \sigma (T_2^{\ 4} - T_{surr}^4) + hA_2 (T_2 - T_\infty)$$

$$\frac{D_1^2 \sigma (T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = \varepsilon D_2^2 \sigma (T_2^{\ 4} - T_{surr}^4) + hD_2^2 (T_2 - T_\infty)$$

Hence,

$$\frac{(3\,\text{m})^2 (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(393^4 - T_2^4)\,\text{K}^4}{\dfrac{1}{0.5} + \dfrac{1-0.5}{0.5}\left(\dfrac{3\,\text{m}}{3.1\,\text{m}}\right)^2} = (0.5)(3.1\,\text{m})^2 (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(T_2^4 - 303^4)\,\text{K}^4$$

$$+ (5\ \text{W/m}^2\cdot\text{K})(3.1\,\text{m})^2 (T_2 - 303)\,\text{K}$$

The outer cover temperature T_2 can be solved by trial-and-error to yield T_2 = **55.7°C** > 45°C.

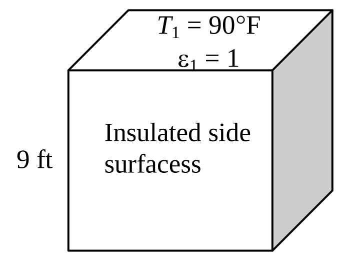
Discussion The outer cover temperature is above the safe temperature of 45°C, and that is a potential thermal burn hazard. This hazard can be alleviated by lowering the emissivity of both surfaces to 0.23 or lower. Another approach is adding radiation shields between the concentric surfaces to lower the radiation heat transfer from the inner surface to the outer surface (see section 13-5 of the text for additional information).

13-76E A room is heated by electric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are ε = 1 for the ceiling and ε = 0.8 for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from



$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90+460\ \text{R})^4 = 157\ \text{Btu/h.ft}^2$$
$$E_{b2} = \sigma T_2^{\,4} = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65+460\ \text{R})^4 = 130\ \text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\ \text{ft})^2 = 144\ \text{ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 13-5). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.8}{(144\ \text{ft}^2)(0.8)} = 0.00174\ \text{ft}^{-2}$$
$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(144\ \text{ft}^2)(0.27)} = 0.02572\ \text{ft}^{-2}$$
$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(144\ \text{ft}^2)(0.73)} = 0.009513\ \text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157-130)\ \text{Btu/h.ft}^2}{\left(\dfrac{1}{0.02572\ \text{ft}^{-2}} + \dfrac{1}{2(0.009513\ \text{ft}^{-2})}\right)^{-1} + 0.00174\ \text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

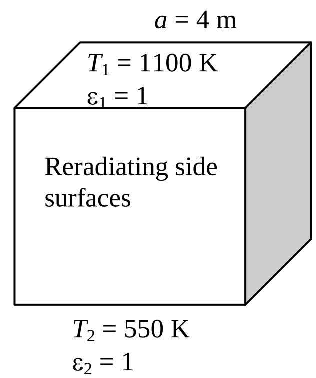
13-77 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$



where

$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(550\ \text{K})^4 = 5188\ \text{W/m}^2$$

and

$$A_1 = A_2 = (4\,\text{m})^2 = 16\,\text{m}^2$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(16\,\text{m}^2)(0.2)} = 0.3125\,\text{m}^{-2}$$

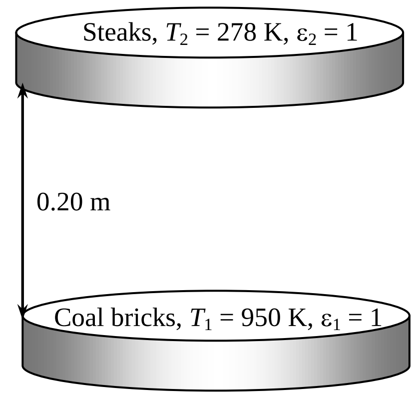
$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(16\,\text{m}^2)(0.8)} = 0.078125\,\text{m}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015 - 5188)\ \text{W/m}^2}{\left(\dfrac{1}{0.3125\,\text{m}^{-2}} + \dfrac{1}{2(0.078125\,\text{m}^{-2})}\right)^{-1}} = 7.47\times10^5\ \text{W} = \mathbf{747\,kW}$$

13-78 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.

Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 13-1),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\text{ m}}{0.20\text{ m}} = 0.75$$

$$S = 1 + \frac{1+R_j^2}{R_i^2} = \frac{1+0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2\right]^{1/2}\right\} = 0.2864$$

(It can also be determined from Fig. 13-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}\dot{Q}_{12} &= F_{12}A_1\sigma(T_1^4 - T_2^4)\\ &= (0.2864)[\pi(0.3\text{ m})^2/4](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (278\text{ K})^4]\\ &= \mathbf{928\ W}\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the coal bricks can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13}+R_{23}}\right)^{-1}}$$

where
$$E_{b1} = \sigma T_1^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(950\text{ K})^4 = 46{,}183\text{ W/m}^2$$
$$E_{b2} = \sigma T_2^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(5+273\text{ K})^4 = 339\text{ W/m}^2$$

and $$A_1 = A_2 = \frac{\pi(0.3\text{ m})^2}{4} = 0.07069\text{ m}^2$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(0.07069\text{ m}^2)(0.2864)} = 49.39\text{ m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(0.07069\text{ m}^2)(1-0.2864)} = 19.82\text{ m}^{-2}$$

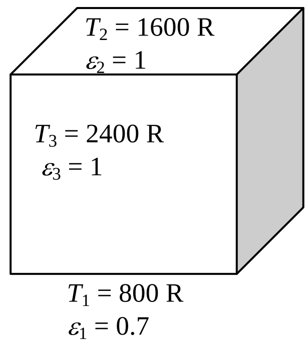
Substituting, $$\dot{Q}_{12} = \frac{(46{,}183 - 339)\text{ W/m}^2}{\left(\dfrac{1}{49.39\text{ m}^{-2}} + \dfrac{1}{2(19.82\text{ m}^{-2})}\right)^{-1}} = \mathbf{2085\ W}$$

13-79E Top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. Net radiation heat transfer rate to the base from the top and side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are given to be $\varepsilon = 0.7$ for the bottom surface and 1 for other surfaces.

Analysis We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure. The areas and blackbody emissive powers of surfaces are



$$A_1 = A_2 = (10\text{ ft})^2 = 100\text{ ft}^2 \qquad A_3 = 4(10\text{ ft})^2 = 400\text{ ft}^2$$

$$E_{b1} = \sigma T_1^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(800\text{ R})^4 = 702\text{ Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(1600\text{ R})^4 = 11{,}233\text{ Btu/h.ft}^2$$

$$E_{b3} = \sigma T_3^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(2400\text{ R})^4 = 56{,}866\text{ Btu/h.ft}^2$$

The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. From the summation rule, the view factor from the base or top to the side surfaces is

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.7}{(100\text{ ft}^2)(0.7)} = 0.004286\text{ ft}^{-2}$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(100\text{ ft}^2)(0.2)} = 0.0500\text{ ft}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{(100\text{ ft}^2)(0.8)} = 0.0125\text{ ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined from

$$\frac{E_{b1}-J_1}{R_1} + \frac{E_{b2}-J_1}{R_{12}} + \frac{E_{b3}-J_1}{R_{13}} = 0$$

Substituting,

$$\frac{702-J_1}{0.004286} + \frac{11{,}233-J_1}{0.05} + \frac{56{,}866-J_1}{0.0125} = 0 \longrightarrow J_1 = 14{,}813\text{ W/m}^2$$

(*a*) The net rate of radiation heat transfer between the base and the side surfaces is

$$\dot{Q}_{31} = \frac{E_{b3}-J_1}{R_{13}} = \frac{(56{,}866-14{,}813)\text{ Btu/h.ft}^2}{0.0125\text{ ft}^{-2}} = \mathbf{3.364\times10^6\ Btu/h}$$

(*b*) The net rate of radiation heat transfer between the base and the top surfaces is

$$\dot{Q}_{12} = \frac{J_1-E_{b2}}{R_{12}} = \frac{(14{,}813-11{,}233)\text{ Btu/h.ft}^2}{0.05\text{ ft}^{-2}} = \mathbf{7.161\times10^4\ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

$$\dot{Q}_1 = \dot{Q}_{21} + \dot{Q}_{31} = -7.161\times10^4 + 3.364\times10^6 = \mathbf{3.293\times10^6\ Btu/h}$$

Discussion The same result can be found form

$$\dot{Q}_1 = \frac{J_1-E_{b1}}{R_1} = \frac{(14{,}813-702)\text{ Btu/h.ft}^2}{0.004286\text{ ft}^{-2}} = 3.292\times10^6\text{ Btu/h}$$

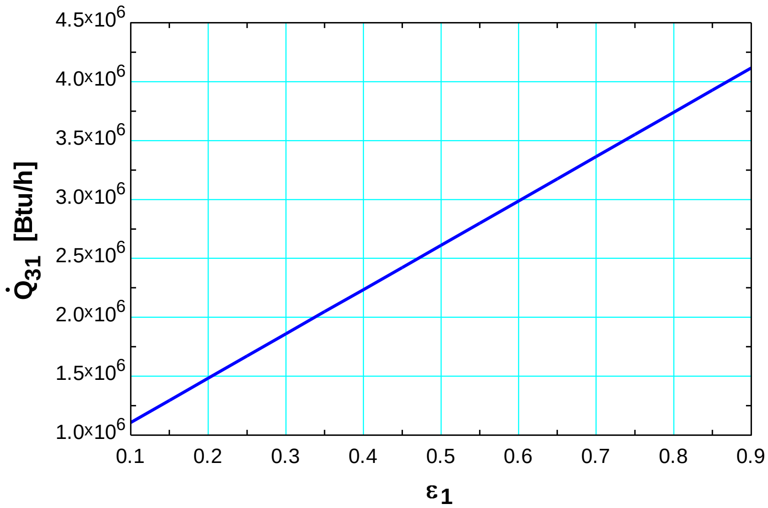
The result is the same as expected. The slight difference is due to round-off error.

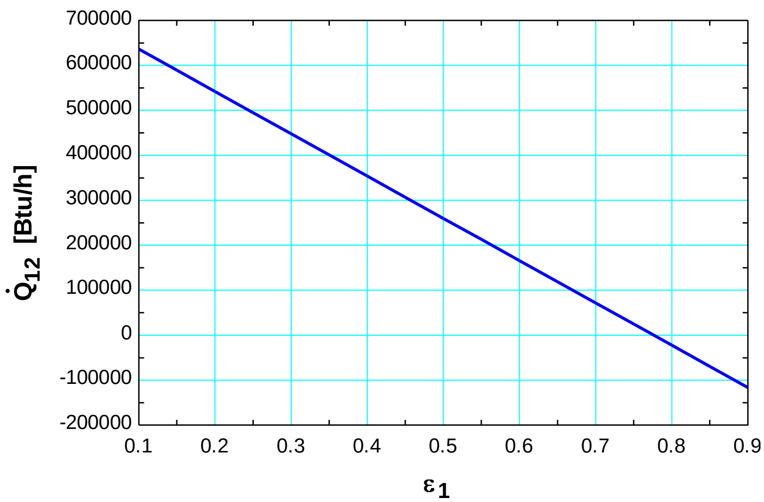
13-80E Prob. 13-79E is reconsidered. The effect of base surface emissivity on the net rates of radiation heat transfer between the base and the side surfaces, between the base and top surfaces, and to the base surface is to be investigated.

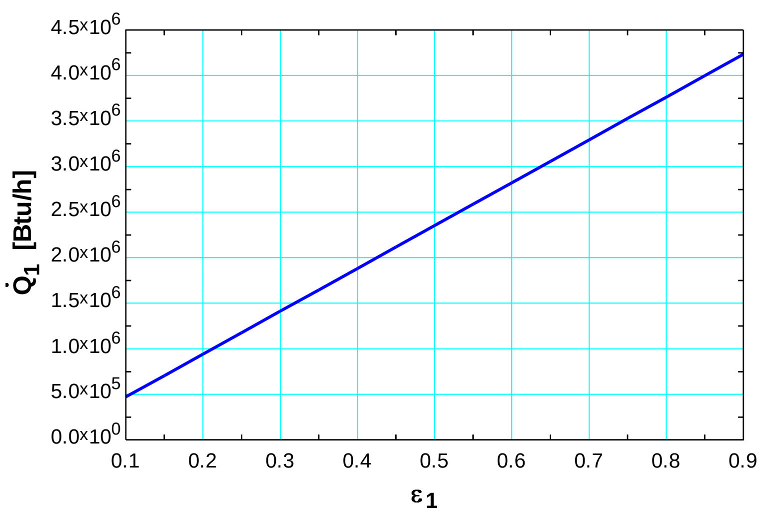
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
a=10 [ft]
epsilon_1=0.7
T_1=800 [R]
T_2=1600 [R]
T_3=2400 [R]

"ANALYSIS"
sigma=0.1714E-8 [Btu/h-ft^2-R^4] “Stefan-Boltzmann constant"
"Consider the base surface 1, the top surface 2, and the side surface 3"
E_b1=sigma*T_1^4
E_b2=sigma*T_2^4
E_b3=sigma*T_3^4
A_1=a^2
A_2=A_1
A_3=4*a^2
F_12=0.2 "view factor from the base to the top of a cube"
F_11+F_12+F_13=1 "summation rule"
F_11=0 "since the base surface is flat"
R_1=(1-epsilon_1)/(A_1*epsilon_1) "surface resistance"
R_12=1/(A_1*F_12) "space resistance"
R_13=1/(A_1*F_13) "space resistance"
(E_b1-J_1)/R_1+(E_b2-J_1)/R_12+(E_b3-J_1)/R_13=0 "J_1 : radiosity of base surface"
"(a)"
Q_dot_31=(E_b3-J_1)/R_13
"(b)"
Q_dot_12=(J_1-E_b2)/R_12
Q_dot_21=-Q_dot_12
Q_dot_1=Q_dot_21+Q_dot_31
```

ε_1	$\dot{Q}_{31}$ [Btu/h]	$\dot{Q}_{12}$ [Btu/h]	$\dot{Q}_1$ [Btu/h]
0.1	1.106E+06	636061	470376
0.15	1.295E+06	589024	705565
0.2	1.483E+06	541986	940753
0.25	1.671E+06	494948	1.176E+06
0.3	1.859E+06	447911	1.411E+06
0.35	2.047E+06	400873	1.646E+06
0.4	2.235E+06	353835	1.882E+06
0.45	2.423E+06	306798	2.117E+06
0.5	2.612E+06	259760	2.352E+06
0.55	2.800E+06	212722	2.587E+06
0.6	2.988E+06	165685	2.822E+06
0.65	3.176E+06	118647	3.057E+06
0.7	3.364E+06	71610	3.293E+06
0.75	3.552E+06	24572	3.528E+06
0.8	3.741E+06	-22466	3.763E+06
0.85	3.929E+06	-69503	3.998E+06
0.9	4.117E+06	-116541	4.233E+06


4.5×10^6
4.0×10^6
3.5×10^6
3.0×10^6
2.5×10^6
2.0×10^6
1.5×10^6
1.0×10^6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\dot{Q}_{31}$ [Btu/h]
ε_1


700000
600000
500000
400000
300000
200000
100000
0
-100000
-200000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\dot{Q}_{12}$ [Btu/h]
ε_1


4.5×10^6
4.0×10^6
3.5×10^6
3.0×10^6
2.5×10^6
2.0×10^6
1.5×10^6
1.0×10^6
5.0×10^5
0.0×10^0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\dot{Q}_1$ [Btu/h]
ε_1

Radiation Shields and the Radiation Effect

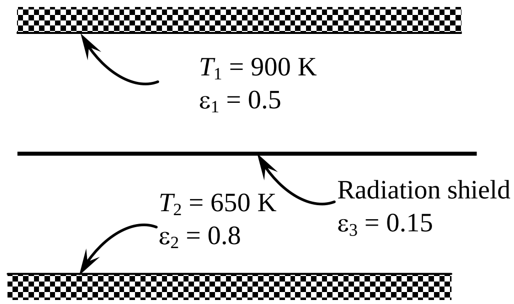
13-81C Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflectivity(low emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are known as radiation shields. Multilayer radiation shields constructed of about 20 shields per cm. thickness separated by evacuated space are commonly used in cryogenic and space applications to minimize heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect.

13-82C The influence of radiation on heat transfer or temperature of a surface is called the radiation effect. The radiation exchange between the sensor and the surroundings may cause the thermometer to indicate a different reading for the medium temperature. To minimize the radiation effect, the sensor should be coated with a material of high reflectivity (low emissivity).

13-83C A person who feels fine in a room at a specified temperature may feel chilly in another room at the same temperature as a result of radiation effect if the walls of second room are at a considerably lower temperature. For example most people feel comfortable in a room at 22°C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5°C for some reason, the interior temperature of the room must be raised to at least 27°C to maintain the same level of comfort. Also, people sitting near the windows of a room in winter will feel colder because of the radiation exchange between the person and the cold windows.

13-84 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is

$$\dot{Q}_{12,\text{oneshield}} = \frac{\sigma(T_1^{\ 4} - T_2^{\ 4})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5} + \frac{1}{0.8} - 1\right) + \left(\frac{1}{0.15} + \frac{1}{0.15} - 1\right)}$$

$$= \mathbf{1857\ W/m^2}$$

The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\text{shield}} = \frac{\sigma(T_1^{\ 4} - T_2^{\ 4})}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5} + \frac{1}{0.8} - 1\right)} = 12{,}035\ \text{W/m}^2$$

Then the ratio of radiation heat transfer for the two cases becomes

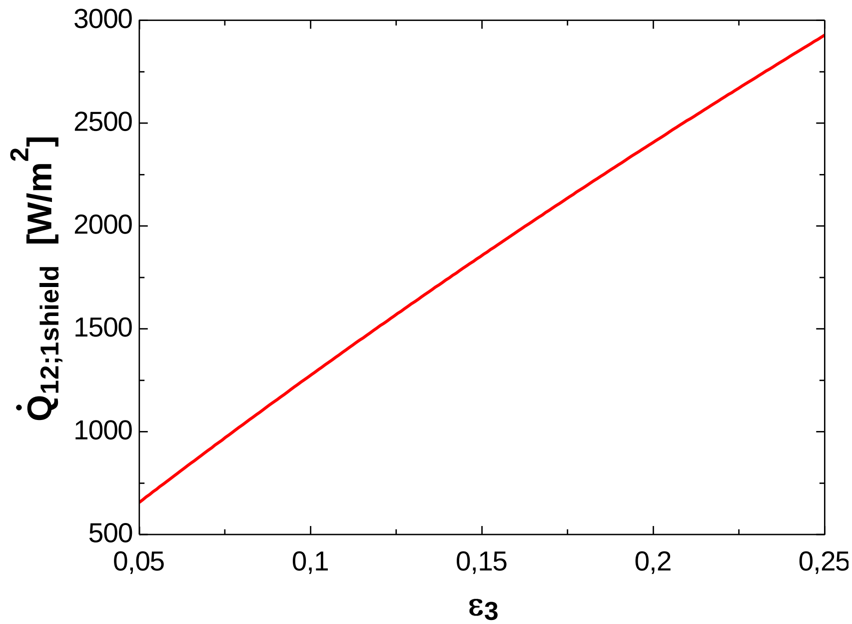
$$\frac{\dot{Q}_{12,\text{oneshield}}}{\dot{Q}_{12,\text{noshield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \mathbf{\frac{1}{6}}$$

13-85 Prob. 13-84 is reconsidered. The net rate of radiation heat transfer between the two plates as a function of the emissivity of the aluminum sheet is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
epsilon_3=0.15
T_1=900 [K]
T_2=650 [K]
epsilon_1=0.5
epsilon_2=0.8

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-1)+(1/epsilon_3+1/epsilon_3-1))
```

ε_3	$\dot{Q}_{12,1shield}$ [W/m^2]
0.05	656.5
0.06	783
0.07	908.1
0.08	1032
0.09	1154
0.1	1274
0.11	1394
0.12	1511
0.13	1628
0.14	1743
0.15	1857
0.16	1969
0.17	2081
0.18	2191
0.19	2299
0.2	2407
0.21	2513
0.22	2619
0.23	2723
0.24	2826
0.25	2928



13-86 A radiation shield is placed between two large parallel plates which are maintained at uniform temperatures. The emissivity of the radiation shield is to be determined if the radiation heat transfer between the plates is reduced to 15% of that without the radiation shield.

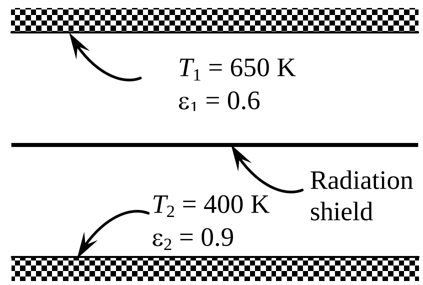
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$.

Analysis First, the net rate of radiation heat transfer between the two large parallel plates per unit area without a shield is

$$\dot{Q}_{12,\text{noshield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1} = 4877\ \text{W/m}^2$$

The radiation heat transfer in the case of one shield is



$$\begin{aligned}\dot{Q}_{12,\text{oneshield}} &= 0.15 \times \dot{Q}_{12,\text{noshield}} \\ &= 0.15 \times 4877\ \text{W/m}^2 = 731.6\ \text{W/m}^2\end{aligned}$$

Then the emissivity of the radiation shield becomes

$$\dot{Q}_{12,\text{oneshield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$731.6\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4 - (400\ \text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1\right) + \left(\dfrac{2}{\varepsilon_3} - 1\right)}$$

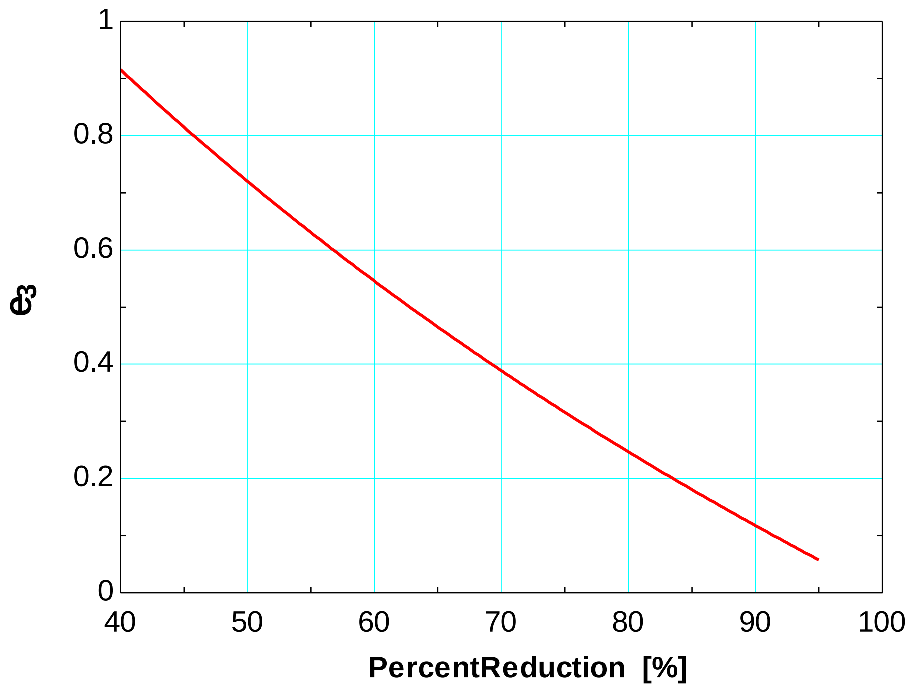
which gives $\varepsilon_3 = \mathbf{0.18}$

13-87 Prob. 13-86 is reconsidered. The effect of the percent reduction in the net rate of radiation heat transfer between the plates on the emissivity of the radiation shields is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

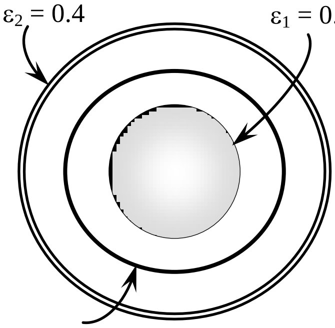
```
"GIVEN"
T_1=650 [K]
T_2=400 [K]
epsilon_1=0.6
epsilon_2=0.9
PercentReduction=85 "[%]"

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-1)+(1/epsilon_3+1/epsilon_3-1))
Q_dot_12_1shield=(1-PercentReduction/100)*Q_dot_12_NoShield
```

Percent Reduction [%]	ε_3
40	0.9153
45	0.8148
50	0.72
55	0.6304
60	0.5455
65	0.4649
70	0.3885
75	0.3158
80	0.2466
85	0.1806
90	0.1176
95	0.05751



13-88 A coaxial radiation shield is placed between two coaxial cylinders which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$. and $\varepsilon_3 = 0.2$.

Analysis The surface areas of the cylinders and the shield per unit length are

$$A_{\text{pipe,inner}} = A_1 = \pi D_1 L = \pi(0.1\,\text{m})(1\,\text{m}) = 0.314\,\text{m}^2$$

$$A_{\text{pipe,outer}} = A_2 = \pi D_2 L = \pi(0.5\,\text{m})(1\,\text{m}) = 1.571\,\text{m}^2$$

$$A_{\text{shield}} = A_3 = \pi D_3 L = \pi(0.2\,\text{m})(1\,\text{m}) = 0.628\,\text{m}^2$$

The net rate of radiation heat transfer between the two cylinders with a shield per unit length is

$$\dot{Q}_{12,\text{oneshield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{3,2}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\,\text{K})^4 - (500\,\text{K})^4]}{\dfrac{1-0.7}{(0.314)(0.7)} + \dfrac{1}{(0.314)(1)} + 2\dfrac{1-0.2}{(0.628)(0.2)} + \dfrac{1}{(0.628)(1)} + \dfrac{1-0.4}{(1.571)(0.4)}}$$

$$= \mathbf{726\,W}$$

If there was no shield,

$$\dot{Q}_{12,\text{noshield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\,\text{K})^4 - (500\,\text{K})^4]}{\dfrac{1}{0.7} + \dfrac{1-0.4}{0.4}\left(\dfrac{0.1}{0.5}\right)} = \mathbf{8329\,W}$$

Then their ratio becomes

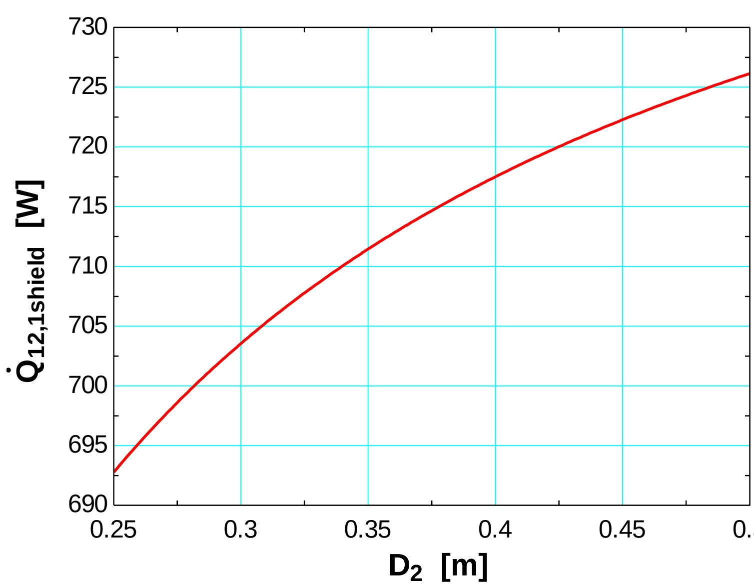
$$\frac{\dot{Q}_{12,\text{oneshield}}}{\dot{Q}_{12,\text{noshield}}} = \frac{726\ \text{W}}{8329\ \text{W}} = \mathbf{0.0872}$$

13-89 Prob. 13-88 is reconsidered. The effects of the diameter of the outer cylinder and the emissivity of the radiation shield on the net rate of radiation heat transfer between the two cylinders are to be investigated.

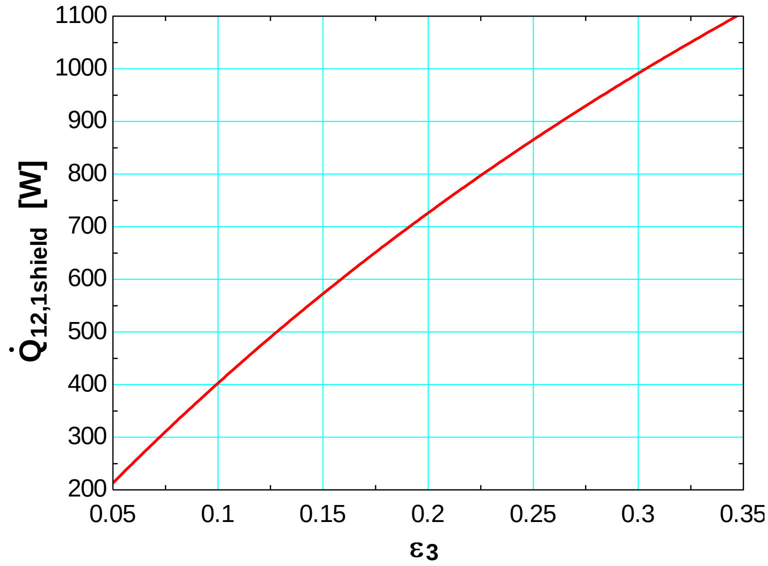
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_1=0.10 [m]
D_2=0.50 [m]
D_3=0.20 [m]
epsilon_1=0.7
epsilon_2=0.4
epsilon_3=0.2
T_1=750 [K]
T_2=500 [K]

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
L=1 [m] "a unit length of the cylinders is considered"
A_1=pi*D_1*L
A_2=pi*D_2*L
A_3=pi*D_3*L
F_13=1
F_32=1
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_13)+(1-epsilon_3)/(A_3*epsilon_3)+(1-epsilon_3)/(A_3*epsilon_3)+1/(A_3*F_32)+(1-epsilon_2)/(A_2*epsilon_2))
```

D_2 [m]	$\dot{Q}_{12,1shield}$ [W]
0.25	692.8
0.275	698.6
0.3	703.5
0.325	707.8
0.35	711.4
0.375	714.7
0.4	717.5
0.425	720
0.45	722.3
0.475	724.3
0.5	726.1



ε_3	$\dot{Q}_{12,1shield}$ [W]
0.05	213.1
0.07	291.5
0.09	366.5
0.11	438.3
0.13	507
0.15	572.9
0.17	636
0.19	696.7
0.21	755
0.23	811.1
0.25	865
0.27	917
0.29	967.1
0.31	1015
0.33	1062
0.35	1107



13-90 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

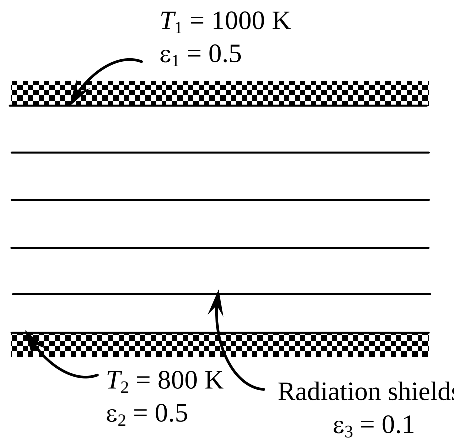
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.5$, and $\varepsilon_3 = 0.1$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\dfrac{1}{0.5} + \dfrac{1}{0.5} - 1\right)}$$

$$= 11{,}159\ \text{W/m}^2$$



The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to one-fifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_\text{shield}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\frac{1}{5}(11{,}159\ \text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\dfrac{1}{0.5} + \dfrac{1}{0.5} - 1\right) + N_\text{shield}\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)}$$

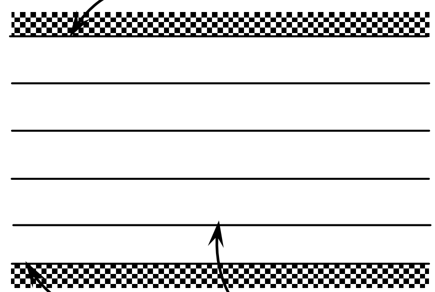
$$N_\text{shield} = 0.632 \cong \mathbf{1}$$

That is, only one sheet with a low emissivity is more than enough to reduce heat transfer to one-fifth.

13-91 Five identical thin aluminum sheets are placed between two very large parallel plates which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.1$ and $\varepsilon_3 = 0.1$.



Analysis Since the plates and the sheets have the same emissivity value, the net rate of radiation heat transfer with 5 thin aluminum shield can be determined from

$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{noshield}} = \frac{1}{N+1}\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{1}{5+1}\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (450\ \text{K})^4]}{\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)} = \mathbf{183\ W/m^2}$$

The net rate of radiation heat transfer without the shield is

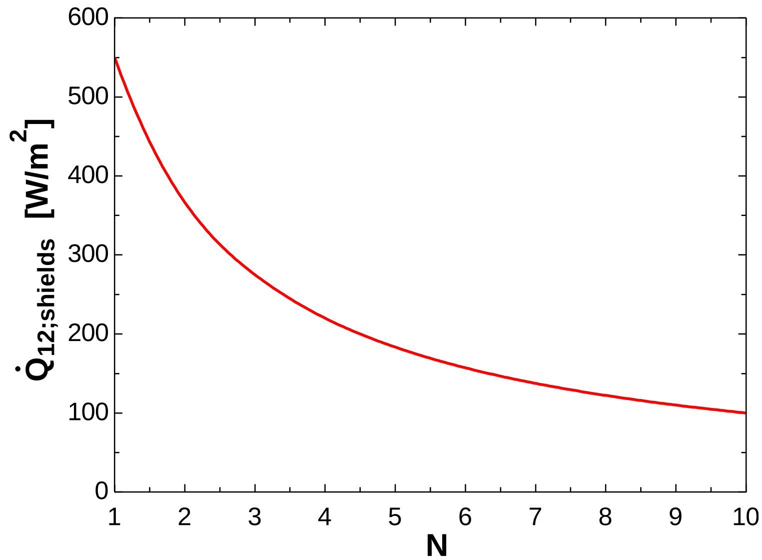
$$\dot{Q}_{12,5\,\text{shield}} = \frac{1}{N+1}\dot{Q}_{12,\,\text{noshield}} \longrightarrow \dot{Q}_{12,\,\text{noshield}} = (N+1)\dot{Q}_{12,5\,\text{shield}} = 6\times183\ \text{W} = \mathbf{1098\ W}$$

13-92 Prob. 13-91 is reconsidered. The effects of the number of the aluminum sheets and the emissivities of the plates on the net rate of radiation heat transfer between the two plates are to be investigated.

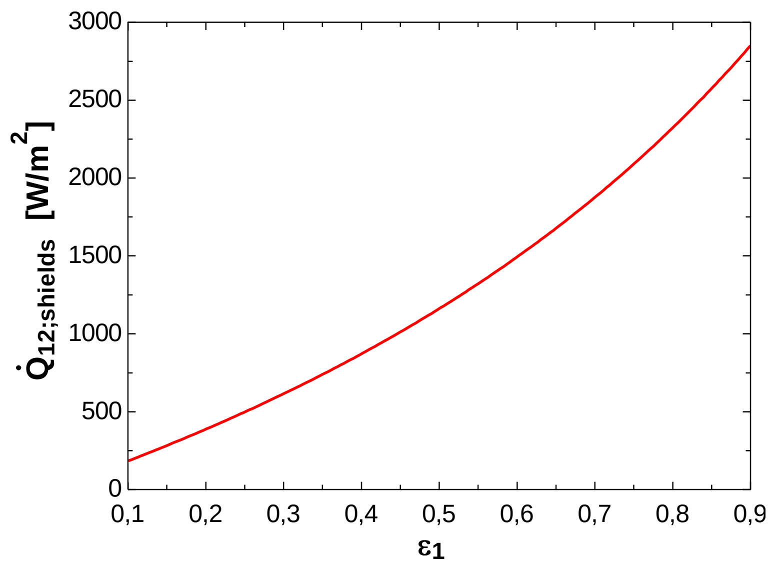
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
N=5
epsilon_3=0.1
epsilon_1=0.1
epsilon_2=epsilon_1
T_1=800 [K]
T_2=450 [K]
"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
Q_dot_12_shields=1/(N+1)*Q_dot_12_NoShield
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

N	$\dot{Q}_{12,shields}$ [W/m^2]
1	550
2	366.7
3	275
4	220
5	183.3
6	157.1
7	137.5
8	122.2
9	110
10	100



ε_1	$\dot{Q}_{12,shields}$ [W/m^2]
0.1	183.3
0.15	282.4
0.2	387
0.25	497.6
0.3	614.7
0.35	738.9
0.4	870.8
0.45	1011
0.5	1161
0.55	1321
0.6	1493
0.65	1677
0.7	1876
0.75	2090
0.8	2322
0.85	2575
0.9	2850

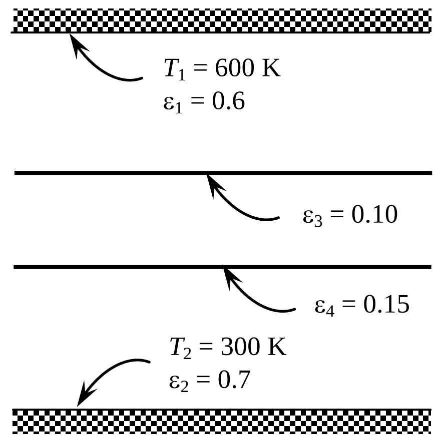


13-93 Two thin radiation shields are placed between two large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates with and without the shields, and the temperatures of radiation shields are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.10$, and $\varepsilon_4 = 0.15$.

Analysis The net rate of radiation heat transfer without the shields per unit area of the plates is



$$\dot{Q}_{12,\text{noshield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3288\ W/m^2}$$

The net rate of radiation heat transfer with two thin radiation shields per unit area of the plates is

$$\dot{Q}_{12,\text{two-shields}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_4} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.10} + \dfrac{1}{0.10} - 1\right) + \left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)}$$

$$= \mathbf{206\ W/m^2}$$

The equilibrium temperatures of the radiation shields are determined from

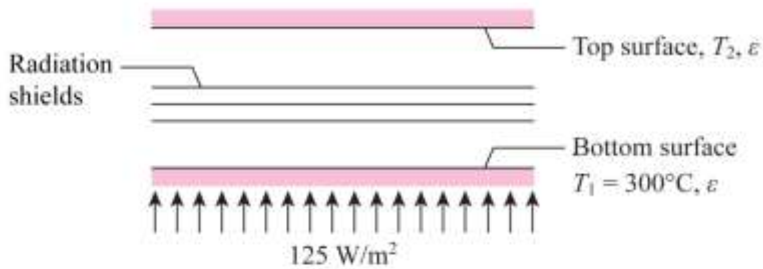
$$\dot{Q}_{13} = \frac{\sigma(T_1^{\,4} - T_3^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)} \longrightarrow 206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - T_3^{\,4}]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.10} - 1\right)} \longrightarrow T_3 = \mathbf{549\ K}$$

$$\dot{Q}_{42} = \frac{\sigma(T_4^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_2} - 1\right)} \longrightarrow 206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_4^{\,4} - (300\ \text{K})^4]}{\left(\dfrac{1}{0.15} + \dfrac{1}{0.7} - 1\right)} \longrightarrow T_4 = \mathbf{429\ K}$$

13-94 An engine cover is made of two parallel plates. The number of radiation shields necessary to keep the top plate below 150°C, to prevent fire hazards in the event of oil leakage, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of all the surfaces is given to be $\varepsilon = 0.3$.



Analysis The net radiation heat flux between the two parallel plates engine cover is

$$\dot{q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1}$$

The temperature of the top plate of the engine cover is

$$T_2 = \left[T_1^4 - \frac{\dot{q}_{12}}{\sigma}\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)\right]^{1/4}$$

$$= \left[(573\text{ K})^4 - \left(\frac{125\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right)\left(\frac{1}{0.3} + \frac{1}{0.3} - 1\right)\right]^{1/4} = 556\text{ K} = 283°\text{C} > 150°\text{C}$$

So, without radiation shield, the top plate temperature is above the safe temperature of 150°C. The number of radiation shields needed to reduce the top plate temperature to 150°C can be determined using,

$$\dot{q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{(N+1)\left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)}$$

Hence,

$$N = \frac{\sigma(T_1^4 - T_2^4)}{\dot{q}_{12}\left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)} - 1$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(573^4 - 423^4)\text{ K}^4}{(125\text{ W/m}^2)\left(\dfrac{1}{0.3} + \dfrac{1}{0.3} - 1\right)} - 1$$

$$= 5.07$$

Thus, placing 6 radiation shields will reduce the top plate to below 150°C.

Discussion In order to keep the top plate of the engine cover below 150°C, to prevent fire hazards in the event of oil leakage, the heat flux through the plates can be reduced using radiation shields. By placing 6 or more radiation shields in parallel between the two plates, the top plate temperature can be reduced to below 150°C.

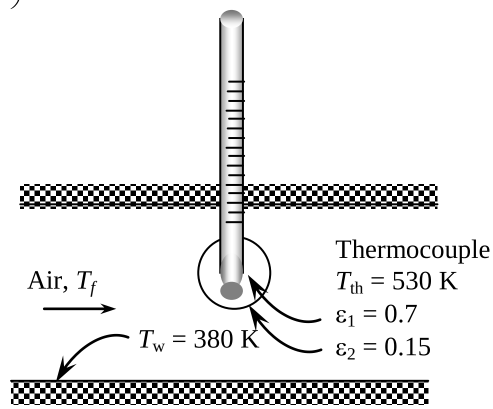
13-95 The temperature of hot gases in a duct is measured by a thermocouple. The actual temperature of the gas is to be determined, and compared with that without a radiation shield.

Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the thermocouple is given to be ε =0.7.

Analysis Assuming the area of the shield to be very close to the sensor of the thermometer, the radiation heat transfer from the sensor is determined from

$$\dot{Q}_{\text{rad, from sensor}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1}{\varepsilon_1} - 1\right) + \left(2\frac{1}{\varepsilon_2} - 1\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\ \text{K})^4 - (380\ \text{K})^4]}{\left(\frac{1}{0.7} - 1\right) + \left(2\frac{1}{0.15} - 1\right)} = 257.9\ \text{W/m}^2$$



Then the actual temperature of the gas can be determined from a heat transfer balance to be

$$\dot{q}_{\text{conv,to sensor}} = \dot{q}_{\text{conv,from sensor}}$$

$$h(T_f - T_{th}) = 257.9\ \text{W/m}^2$$

$$120\ \text{W/m}^2\cdot{}^\circ\text{C}(T_f - 530) = 257.9\ \text{W/m}^2$$

$$\longrightarrow T_f = \mathbf{532K}$$

Without the shield the temperature of the gas would be

$$\begin{aligned} T_f &= T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^{\,4} - T_w^{\,4})}{h} \\ &= 530\ \text{K} + \frac{(0.7)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(530\ \text{K})^4 - (380\ \text{K})^4]}{120\ \text{W/m}^2\cdot{}^\circ\text{C}} \\ &= \mathbf{549.2K} \end{aligned}$$

Radiation Exchange with Absorbing and Emitting Gases

13-96C A nonparticipating medium is completely transparent to thermal radiation, and thus it does not emit, absorb, or scatter radiation. A participating medium, on the other hand, emits and absorbs radiation throughout its entire volume.

13-97C Spectral transmissivity of a medium of thickness L is the ratio of the intensity of radiation leaving the medium to that entering the medium, and is expressed as $\tau_\lambda = \dfrac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L}$ and $\tau_\lambda = 1 - \alpha_\lambda$.

13-98C Gases emit and absorb radiation at a number of narrow wavelength bands. The emissivity-wavelength charts of gases typically involve various peaks and dips together with discontinuities, and show clearly the band nature of absorption and the strong nongray characteristics. This is in contrast to solids, which emit and absorb radiation over the entire spectrum.

13-99C Using Kirchhoff's law, the spectral emissivity of a medium of thickness L in terms of the spectral absorption coefficient is expressed as $\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L}$.

13-100 An equimolar mixture of CO_2 and O_2 gases at 800 K and a total pressure of 0.5 atm is considered. The emissivity of the gas is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis Volumetric fractions are equal to pressure fractions. Therefore, the partial pressure of CO_2 is

$$P_c = y_{\mathrm{CO_2}} P = 0.5(0.5\,\text{atm}) = 0.25\,\text{atm}$$

Then,

$$P_c L = (0.25\,\text{atm})(1.2\,\text{m}) = 0.30\,\text{m}\cdot\text{atm} = 0.98\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 800 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.15$$

This is the base emissivity value at 1 atm, and it needs to be corrected for the 0.5 atm total pressure. The pressure correction factor is, from Fig. 13-37,

$$C_c = 0.90$$

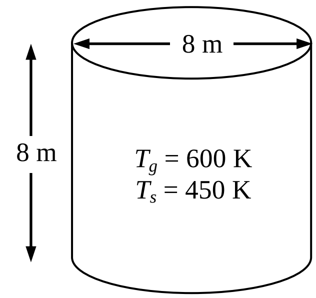
Then the effective emissivity of the gas becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} = 0.90 \times 0.15 = \mathbf{0.135}$$

13-101 A mixture of CO_2 and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 13-4



$$L = 0.60D = 0.60(8\text{ m}) = 4.8\text{ m}$$

Then,

$$P_c L = (0.15\text{ atm})(4.8\text{ m}) = 0.72\text{ m}\cdot\text{atm} = 2.36\text{ ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.16$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.15\text{ atm})(4.8\text{ m})\frac{450\text{ K}}{600\text{ K}} = 0.54\text{ m}\cdot\text{atm} = 1.77\text{ ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.14$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{600\text{ K}}{450\text{ K}}\right)^{0.65}(0.14) = 0.17$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(8\text{ m})(8\text{ m}) + 2\frac{\pi(8\text{ m})^2}{4} = 301.6\text{ m}^2$$

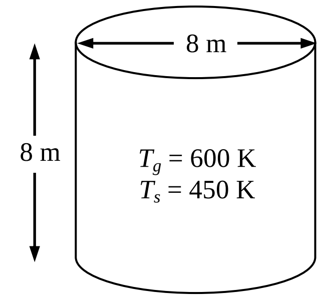
Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (301.6\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.16(600\text{ K})^4 - 0.17(450\text{ K})^4]\\ &= \mathbf{2.35\times10^5\ W}\end{aligned}$$

13-102 A mixture of H_2O and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 13-4



$$L = 0.60D = 0.60(8\text{ m}) = 4.8\text{ m}$$

Then,

$$P_w L = (0.15\,\text{atm})(4.8\,\text{m}) = 0.72\,\text{m}\cdot\text{atm} = 2.36\,\text{ft}\cdot\text{atm}$$

The emissivity of H_2O corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{w1\,\text{atm}} = 0.36$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_w L\frac{T_s}{T_g} = (0.15\,\text{atm})(4.8\,\text{m})\frac{450\,\text{K}}{600\,\text{K}} = 0.54\,\text{m}\cdot\text{atm} = 1.77\,\text{ft}\cdot\text{atm}$$

The emissivity of H_2O corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{w,1\,\text{atm}} = 0.34$$

The absorptivity of H_2O is determined from

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{600\,\text{K}}{450\,\text{K}}\right)^{0.45}(0.34) = 0.39$$

The surface area of the cylindrical surface is

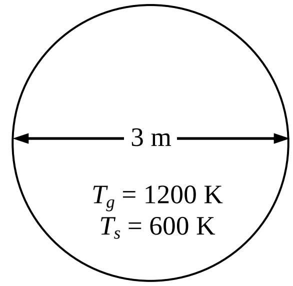
$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(8\,\text{m})(8\,\text{m}) + 2\frac{\pi(8\,\text{m})^2}{4} = 301.6\,\text{m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (301.6\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.36(600\,\text{K})^4 - 0.39(450\,\text{K})^4]\\ &= \mathbf{5.244\times10^5\ W}\end{aligned}$$

13-103 A mixture of CO_2 and N_2 gases at 1200 K and a total pressure of 1 atm are contained in a spherical furnace. The net rate of radiation heat transfer between the gas mixture and furnace walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.



Analysis The mean beam length is, from Table 13-4

$$L = 0.65D = 0.65(3 \text{ m}) = 1.95 \text{ m}$$

The mole fraction is equal to pressure fraction. Then,

$$P_c L = (0.15 \text{ atm})(1.95 \text{ m}) = 0.2925 \text{ m} \cdot \text{atm} = 0.96 \text{ ft} \cdot \text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 1200 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.16$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.15 \text{ atm})(1.95 \text{ m}) \frac{600 \text{ K}}{1200 \text{ K}} = 0.146 \text{ m} \cdot \text{atm} = 0.48 \text{ ft} \cdot \text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.11$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1200 \text{ K}}{600 \text{ K}}\right)^{0.65} (0.11) = 0.1726$$

The surface area of the sphere is

$$A_s = \pi D^2 = \pi (3 \text{ m})^2 = 28.27 \text{ m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

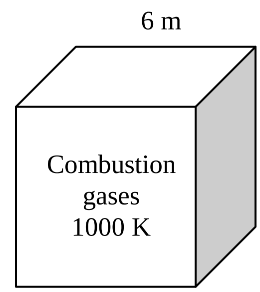
$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (28.27 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[0.16(1200 \text{ K})^4 - 0.1726(600 \text{ K})^4] \\ &= \mathbf{4.96 \times 10^5 \text{ W}}\end{aligned}$$

13-104 The temperature, pressure, and composition of a gas mixture is given. The emissivity of the mixture is to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** The emissivity determined is the mean emissivity for radiation emitted to all surfaces of the cubical enclosure.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$
$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for a cube of side length 6 m for radiation emitted to all surfaces is, from Table 13-4,

L = 0.66(6 m) = 3.96 m

Then,

$$P_c L = (0.10\,\text{atm})(3.96\,\text{m}) = 0.396\,\text{m}\cdot\text{atm} = 1.30\,\text{ft}\cdot\text{atm}$$
$$P_w L = (0.09\,\text{atm})(3.96\,\text{m}) = 0.36\,\text{m}\cdot\text{atm} = 1.18\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1000 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.16 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.24$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1000 K is, from Fig. 13-38,

$$\left.\begin{aligned} P_c L + P_w L &= 1.30 + 1.18 = 2.48 \\ \frac{P_w}{P_w + P_c} &= \frac{0.09}{0.09 + 0.10} = 0.474 \end{aligned}\right\} \Delta\varepsilon = 0.038$$

Then the effective emissivity of the combustion gases becomes

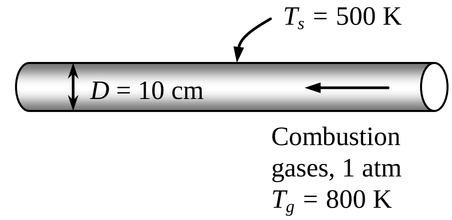
$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.16 + 1\times 0.24 - 0.038 = \mathbf{0.362}$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm.

13-105 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,



$$L = 0.95(0.10\text{ m}) = 0.095\text{ m}$$

Then,

$$P_c L = (0.12\text{ atm})(0.095\text{ m}) = 0.0114\text{ m}\cdot\text{atm} = 0.037\text{ ft}\cdot\text{atm}$$
$$P_w L = (0.18\text{ atm})(0.095\text{ m}) = 0.0171\text{ m}\cdot\text{atm} = 0.056\text{ ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 800 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\text{atm}} = 0.056 \quad\text{and}\quad \varepsilon_{w,1\text{atm}} = 0.050$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 800 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.056 = 0.093 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.18}{0.18 + 0.12} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\text{atm}} + C_w \varepsilon_{w,1\text{atm}} - \Delta\varepsilon = 1\times 0.056 + 1\times 0.050 - 0.0 = 0.106$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 500 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L\frac{T_s}{T_g} = (0.12\text{ atm})(0.095\text{ m})\frac{500\text{ K}}{800\text{ K}} = 0.007125\text{ m}\cdot\text{atm} = 0.023\text{ ft}\cdot\text{atm}$$

$$P_w L\frac{T_s}{T_g} = (0.18\text{ atm})(0.095\text{ m})\frac{500\text{ K}}{800\text{ K}} = 0.01069\text{ m}\cdot\text{atm} = 0.035\text{ ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\text{atm}} = 0.042 \quad\text{and}\quad \varepsilon_{w,1\text{atm}} = 0.050$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\text{atm}} = (1)\left(\frac{800\text{ K}}{500\text{ K}}\right)^{0.65}(0.042) = 0.057$$

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45}\varepsilon_{w,1\text{atm}} = (1)\left(\frac{800\text{ K}}{500\text{ K}}\right)^{0.45}(0.050) = 0.062$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 500 K instead of T_g = 800 K. There is no chart for 500 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and interpolate. At $P_w/(P_w + P_c) = 0.6$ and $P_cL + P_wL = 0.093$ we read $\Delta\varepsilon = 0.0$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.057 + 0.062 - 0.0 = 0.119$$

The surface area of the pipe is

$$A_s = \pi DL = \pi(0.10\text{ m})(6\text{ m}) = 1.885\text{ m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (1.885\text{ m}^2)(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.106(800\text{ K})^4 - 0.119(500\text{ K})^4]\\ &= \mathbf{3846\ W}\end{aligned}$$

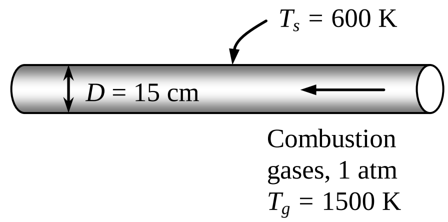
13-106 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$

$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_c L = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$

$$P_w L = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.034 + 1\times 0.016 - 0.0 = 0.05$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\,\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.031) = 0.056$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.45}(0.027) = 0.041$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1500 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c)$ = 0.6 and $P_cL + P_wL$ = 0.07 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.056 + 0.041 - 0.0 = 0.097$$

The surface area of the pipe per m length of tube is

$$A_s = \pi DL = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (0.4712\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.05(1500\,\text{K})^4 - 0.097(600\,\text{K})^4] = \mathbf{6427\,W}\end{aligned}$$

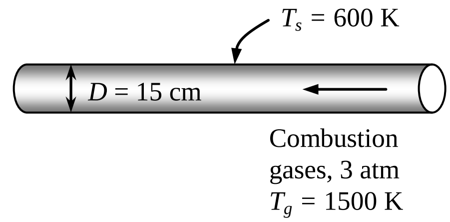
13-107 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2}P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$

$$P_w = y_{H_2O}P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_cL = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$

$$P_wL = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad\text{and}\quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

These are base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that $(P_w+P)/2$ = (0.09+3)/2 = 1.545 atm, the pressure correction factors are, from Fig. 13-37,

C_c = 1.5 and C_w = 1.8

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_cL + P_wL = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09+0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c\varepsilon_{c,1\,\text{atm}} + C_w\varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1.5\times0.034 + 1.8\times0.016 - 0.0 = 0.080$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_cL\frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\text{ K}}{1500\text{ K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\,\text{ft}\cdot\text{atm}$$

$$P_wL\frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\text{ K}}{1500\text{ K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad\text{and}\quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1.5)\left(\frac{1500\text{ K}}{600\text{ K}}\right)^{0.65}(0.031) = 0.084$$

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45}\varepsilon_{w,1\,\text{atm}} = (1.8)\left(\frac{1500\text{ K}}{600\text{ K}}\right)^{0.45}(0.027) = 0.073$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1500 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w+P_c)$ = 0.6 and $P_cL + P_wL$ = 0.07 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.084 + 0.073 - 0.0 = 0.157$$

The surface area of the pipe per m length of tube is

$$A_s = \pi DL = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\dot{Q}_{\text{net}} = A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)$$

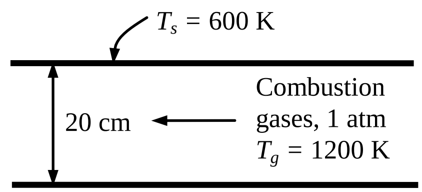
$$= (0.4712\,\text{m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.08(1500\text{ K})^4 - 0.157(600\text{ K})^4] = \mathbf{10{,}280\,W}$$

13-108 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$
$$P_w = y_{H_2O} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$



The mean beam length for this geometry is, from Table 13-4,

$$L = 3.6V/A_s = 1.8D = 1.8(0.20\text{ m}) = 0.36\text{ m}$$

where D is the distance between the plates. Then,

$$P_c L = P_w L = (0.10\,\text{atm})(0.36\,\text{m}) = 0.036\,\text{m}\cdot\text{atm} = 0.118\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1200 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.080 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.055$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1200 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.118 + 0.118 = 0.236 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.10}{0.10 + 0.10} = 0.5 \end{array}\right\} \Delta\varepsilon = 0.0025$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.080 + 1\times 0.055 - 0.0025 = 0.1325$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = P_w L \frac{T_s}{T_g} = (0.10\,\text{atm})(0.36\,\text{m})\frac{600\,\text{K}}{1200\,\text{K}} = 0.018\,\text{m}\cdot\text{atm} = 0.059\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.060 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.067$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.060) = 0.090$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.45}(0.067) = 0.092$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1200 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c) = 0.5$ and $P_cL + P_wL = 0.236$ we read $\Delta\varepsilon = 0.00125$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.090 + 0.092 - 0.00125 = 0.1808$$

Then the net rate of radiation heat transfer from the gas to each plate per unit surface area becomes

$$\begin{aligned} \dot{Q}_{\text{net}} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (1\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.1325(1200\,\text{K})^4 - 0.1808(600\,\text{K})^4] \\ &= \mathbf{1.42\times 10^4\ W} \end{aligned}$$

Special Topic: Heat Transfer from the Human Body

13-109C (*a*) Heat is lost through the skin by convection, radiation, and evaporation. (*b*) The body loses both sensible heat by convection and latent heat by evaporation from the lungs, but there is no heat transfer in the lungs by radiation.

13-110C Sensible heat is the energy associated with a temperature change. The sensible heat loss from a human body increases as (*a*) the skin temperature increases, (*b*) the environment temperature decreases, and (*c*) the air motion (and thus the convection heat transfer coefficient) increases.

13-111C Latent heat is the energy released as water vapor condenses on cold surfaces, or the energy absorbed from a warm surface as liquid water evaporates. The latent heat loss from a human body increases as (*a*) the skin wettedness increases and (*b*) the relative humidity of the environment decreases. The rate of evaporation from the body is related to the rate of latent heat loss by $\dot{Q}_{\text{latent}} = \dot{m}_{\text{vapor}} h_{fg}$ where h_{fg} is the latent heat of vaporization of water at the skin temperature.

13-112C The insulating effect of clothing is expressed in the unit **clo** with 1 clo = 0.155 m^2.°C/W = 0.880 ft^2.°F.h/Btu. Clothing serves as insulation, and thus reduces heat loss from the body by convection, radiation, and evaporation by serving as a resistance against heat flow and vapor flow. Clothing decreases heat gain from the sun by serving as a radiation shield.

13-113C Yes, roughly one-third of the metabolic heat generated by a person who is resting or doing light work is dissipated to the environment by convection, one-third by evaporation, and the remaining one-third by radiation.

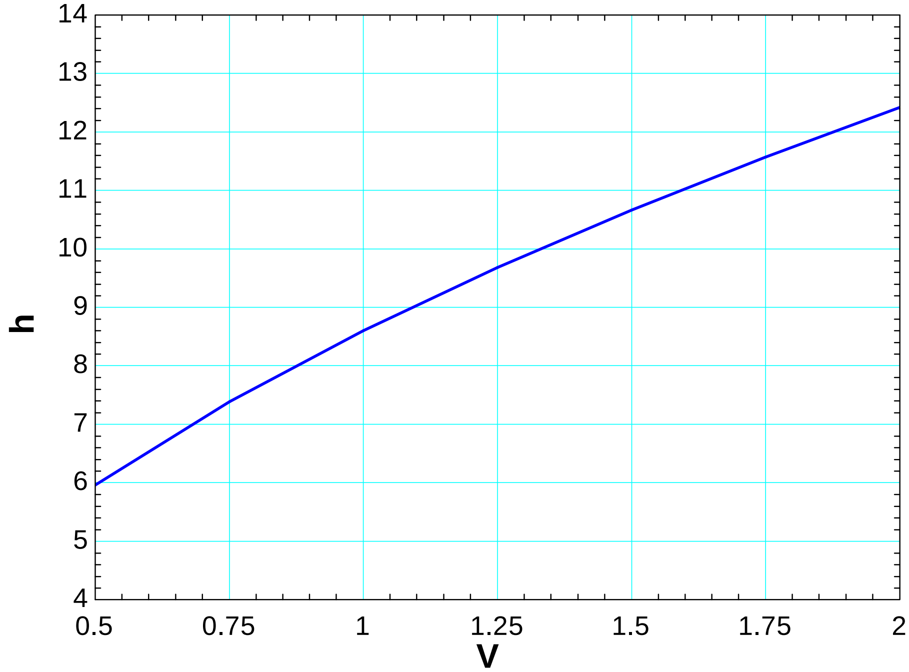
13-114C The *operative temperature* $T_{\text{operative}}$ is the average of the mean radiant and ambient temperatures weighed by their respective convection and radiation heat transfer coefficients, and is expressed as

$$T_{\text{operative}} = \frac{h_{\text{conv}} T_{\text{ambient}} + h_{\text{rad}} T_{\text{surr}}}{h_{\text{conv}} + h_{\text{rad}}} \cong \frac{T_{\text{ambient}} + T_{\text{surr}}}{2}$$

When the convection and radiation heat transfer coefficients are equal to each other, the operative temperature becomes the arithmetic average of the ambient and surrounding surface temperatures. Another environmental index used in thermal comfort analysis is the effective temperature, which combines the effects of temperature and humidity.

13-115 The convection heat transfer coefficient for a clothed person while walking in still air at a velocity of 0.5 to 2 m/s is given by $h = 8.6V^{0.53}$ where V is in m/s and h is in W/m^2.°C. The convection coefficients in that range vary from 5.96 W/m^2.°C at 0.5 m/s to 12.42 W/m^2.°C at 2 m/s. Therefore, at low velocities, the radiation and convection heat transfer coefficients are comparable in magnitude. But at high velocities, the convection coefficient is much larger than the radiation heat transfer coefficient.

Velocity, m/s	$h = 8.6V^{0.53}$ W/m^2.°C
0.50	5.96
0.75	7.38
1.00	8.60
1.25	9.68
1.50	10.66
1.75	11.57
2.00	12.42



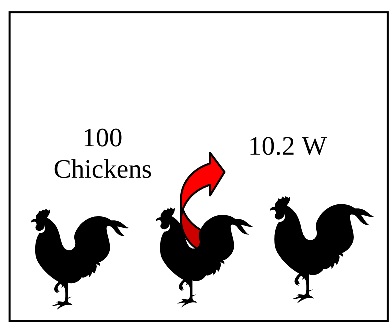
13-116 There are 100 chickens in a breeding room. The rate of total heat generation and the rate of moisture production in the room are to be determined.

Assumptions All the moisture from the chickens is condensed by the air-conditioning system.

Properties The latent heat of vaporization of water is given to be 2430 kJ/kg. The average metabolic rate of chicken during normal activity is 10.2 W (3.78 W sensible and 6.42 W latent).

Analysis The total rate of heat generation of the chickens in the breeding room is

$$\dot{Q}_{\text{gen,total}} = \dot{q}_{\text{gen,total}}(\text{No. of chickens})$$
$$= (10.2 \text{ W/chicken})(100 \text{ chickens}) = \mathbf{1020\,W}$$



The latent heat generated by the chicken and the rate of moisture production are

$$\dot{Q}_{\text{gen,latent}} = \dot{q}_{\text{gen,latent}}(\text{No. of chickens})$$
$$= (6.42 \text{ W/chicken})(100 \text{ chickens}) = 642 \text{ W}$$
$$= 0.642 \text{ kW}$$

$$\dot{m}_{\text{moisture}} = \frac{\dot{Q}_{\text{gen,latent}}}{h_{fg}} = \frac{0.642 \text{ kJ/s}}{2430 \text{ kJ/kg}} = 0.000264 \text{ kg/s} = \mathbf{0.264\,g/s}$$

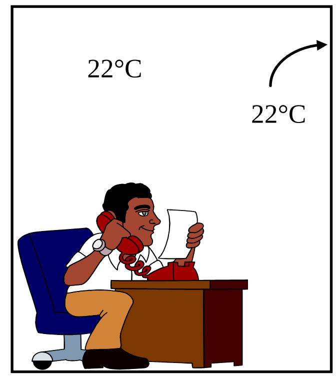
13-117 The average mean radiation temperature during a cold day drops to 18°C. The required rise in the indoor air temperature to maintain the same level of comfort in the same clothing is to be determined.

Assumptions **1** Air motion in the room is negligible. **2** The average clothing and exposed skin temperature remains the same. **3** The latent heat loss from the body remains constant. **4** Heat transfer through the lungs remain constant.

Properties The emissivity of the person is 0.95 (from Appendix tables). The convection heat transfer coefficient from the body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 3.1 W/m^2·°C (Table 13-5).

Analysis The total rate of heat transfer from the body is the sum of the rates of heat loss by convection, radiation, and evaporation,

$$\dot{Q}_{\text{body,total}} = \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}} = (\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}) + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}}$$



Noting that heat transfer from the skin by evaporation and from the lungs remains constant, the sum of the convection and radiation heat transfer from the person must remain constant.

$$\begin{aligned}\dot{Q}_{\text{sensible,old}} &= hA_s(T_s - T_{\text{air, old}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,old}}^4)\\ &= hA_s(T_s - 22) + 0.95A_s\sigma[(T_s + 273)^4 - (22+273)^4]\\ \dot{Q}_{\text{sensible,new}} &= hA_s(T_s - T_{\text{air, new}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,new}}^4)\\ &= hA_s(T_s - T_{\text{air, new}}) + 0.95A_s\sigma[(T_s + 273)^4 - (18+273)^4]\end{aligned}$$

Setting the two relations above equal to each other, canceling the surface area A_s, and simplifying gives

$$-22h - 0.95\sigma(22+273)^4 = -hT_{\text{air, new}} - 0.95\sigma(18+273)^4$$

$$3.1(T_{\text{air, new}} - 22) + 0.95\times 5.67\times 10^{-8}(291^4 - 295^4) = 0$$

Solving for the new air temperature gives

$T_{\text{air, new}}$ = **29.0°C**

Therefore, the air temperature must be raised to 29°C to counteract the increase in heat transfer by radiation.

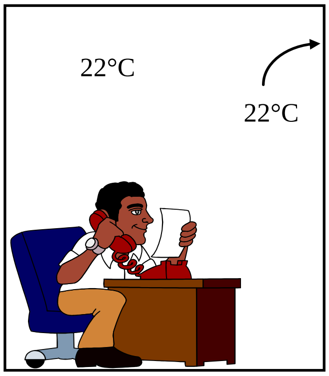
13-118 The average mean radiation temperature during a cold day drops to 10°C. The required rise in the indoor air temperature to maintain the same level of comfort in the same clothing is to be determined.

Assumptions **1** Air motion in the room is negligible. **2** The average clothing and exposed skin temperature remains the same. **3** The latent heat loss from the body remains constant. **4** Heat transfer through the lungs remain constant.

Properties The emissivity of the person is 0.95 (from Appendix tables). The convection heat transfer coefficient from the body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 3.1 W/m^2·°C (Table 13-5).

Analysis The total rate of heat transfer from the body is the sum of the rates of heat loss by convection, radiation, and evaporation,

$$\dot{Q}_{\text{body,total}} = \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}} = (\dot{Q}_{\text{conv}} + Q_{\text{rad}}) + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}}$$



Noting that heat transfer from the skin by evaporation and from the lungs remains constant, the sum of the convection and radiation heat transfer from the person must remain constant.

$$\dot{Q}_{\text{sensible,old}} = hA_s(T_s - T_{\text{air, old}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,old}}^4)$$
$$= hA_s(T_s - 22) + 0.95A_s\sigma[(T_s + 273)^4 - (22+273)^4]$$
$$\dot{Q}_{\text{sensible,new}} = hA_s(T_s - T_{\text{air, new}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,new}}^4)$$
$$= hA_s(T_s - T_{\text{air, new}}) + 0.95A_s\sigma[(T_s + 273)^4 - (10+273)^4]$$

Setting the two relations above equal to each other, canceling the surface area A_s, and simplifying gives

$$-22h - 0.95\sigma(22+273)^4 = -hT_{\text{air, new}} - 0.95\sigma(10+273)^4$$
$$3.1(T_{\text{air, new}} - 22) + 0.95\times5.67\times10^{-8}(283^4 - 295^4) = 0$$

Solving for the new air temperature gives

$T_{\text{air, new}}$ = **42.1°C**

Therefore, the air temperature must be raised to 42.11°C to counteract the increase in heat transfer by radiation.

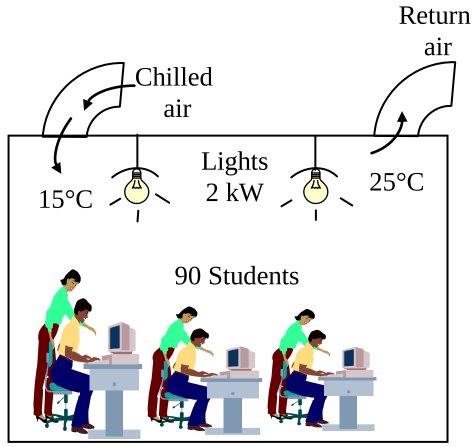
13-119 Chilled air is to cool a room by removing the heat generated in a large insulated classroom by lights and students. The required flow rate of air that needs to be supplied to the room is to be determined.

Assumptions **1** The moisture produced by the bodies leave the room as vapor without any condensing, and thus the classroom has no latent heat load. **2** Heat gain through the walls and the roof is negligible.

Properties The specific heat of air at room temperature is 1.00 kJ/kg·°C (Table A-15). The average rate of metabolic heat generation by a person sitting or doing light work is 115 W (70 W sensible, and 45 W latent).

Analysis The rate of sensible heat generation by the people in the room and the total rate of sensible internal heat generation are

$$\dot{Q}_{\text{gen,sensible}} = \dot{q}_{\text{gen,sensible}}(\text{No. of people})$$
$$= (70\text{ W/person})(90\text{ persons}) = 6300\text{ W}$$
$$\dot{Q}_{\text{total,sensible}} = \dot{Q}_{\text{gen,sensible}} + \dot{Q}_{\text{lighting}}$$
$$= 6300 + 2000 = 8300\text{ W}$$



Then the required mass flow rate of chilled air becomes

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{total,sensible}}}{c_p \Delta T}$$
$$= \frac{8.30\text{ kJ/s}}{(1.0\text{ kJ/kg}\cdot{}^\circ\text{C})(25-15)^\circ\text{C}} = \mathbf{0.830\ kg/s}$$

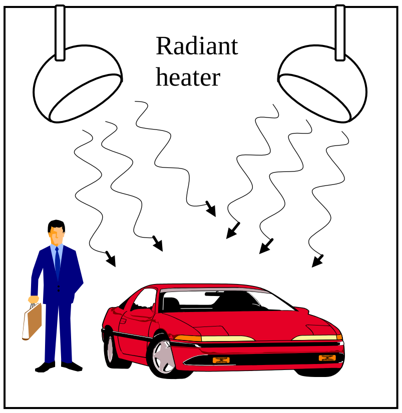
Discussion The latent heat will be removed by the air-conditioning system as the moisture condenses outside the cooling coils.

13-120 A car mechanic is working in a shop heated by radiant heaters in winter. The lowest ambient temperature the worker can work in comfortably is to be determined.

Assumptions **1** The air motion in the room is negligible, and the mechanic is standing. **2** The average clothing and exposed skin temperature of the mechanic is 33°C.

Properties The emissivity and absorptivity of the person is given to be 0.95. The convection heat transfer coefficient from a standing body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 4.0 W/m^2·°C (Table 13-5).

Analysis The equivalent thermal resistance of clothing is



$$R_{\text{cloth}} = 0.7\text{ clo} = 0.7 \times 0.155\text{ m}^2.°\text{C/W} = 0.1085\text{ m}^2.°\text{C/W}$$

Radiation from the heaters incident on the person and the rate of sensible heat generation by the person are

$$\dot{Q}_{\text{rad, incident}} = 0.05 \times \dot{Q}_{\text{rad, total}} = 0.05(4\text{ kW}) = 0.2\text{ kW} = 200\text{ W}$$

$$\dot{Q}_{\text{gen,sensible}} = 0.5 \times \dot{Q}_{\text{gen, total}} = 0.5(350\text{ W}) = 175\text{ W}$$

Under steady conditions, and energy balance on the body can be expressed as

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = 0$$

$$\dot{Q}_{\text{rad from heater}} - \dot{Q}_{\text{conv+rad from body}} + \dot{Q}_{\text{gen,sensible}} = 0$$

or

$$\alpha \dot{Q}_{\text{rad, incident}} - h_{\text{conv}} A_s (T_s - T_{\text{surr}}) - \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) + \dot{Q}_{\text{gen,sensible}} = 0$$

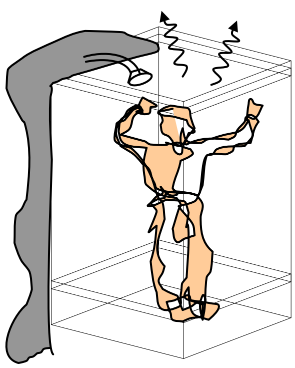
$$0.95(200\text{ W}) - (4.0\text{ W/m}^2 \cdot \text{K})(1.8\text{ m}^2)(306 - T_{\text{surr}})$$
$$-0.95(1.8\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)[(306\text{ K})^4 - T_{\text{surr}}^4) + 175\text{ W} = 0$$

Solving the equation above gives

$$T_{\text{surr}} = 284.8\text{ K} = \mathbf{11.8°C}$$

Therefore, the mechanic can work comfortably at temperatures as low as 12°C.

13-121E An average person produces 0.50 lbm of moisture while taking a shower. The contribution of showers of a family of four to the latent heat load of the air-conditioner per day is to be determined.



Assumptions All the water vapor from the shower is condensed by the air-conditioning system.

Properties The latent heat of vaporization of water is given to be 1050 Btu/lbm.

Analysis The amount of moisture produced per day is

m_{vapor} = (Moisture produced per person)(No. of persons)

= (0.5 lbm/person)(4 persons/day) = 2 lbm/day

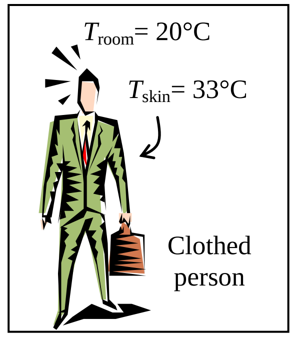
Then the latent heat load due to showers becomes

$Q_{latent} = m_{vapor} h_{fg}$ = (2 lbm/day)(1050 Btu/lbm) = **2100 Btu/day**

13-122 A man wearing summer clothes feels comfortable in a room at 20°C. The room temperature at which this man would feel thermally comfortable when unclothed is to be determined.

Assumptions **1** Steady conditions exist. **2** The latent heat loss from the person remains the same. **3** The heat transfer coefficients remain the same. **4** The air in the room is still (there are no winds or running fans). **5** The surface areas of the clothed and unclothed person are the same.

Analysis At low air velocities, the convection heat transfer coefficient for a standing man is given in Table 13-5 to be 4.0 W/m^2.°C. The radiation heat transfer coefficient at typical indoor conditions is 4.7 W/m^2.°C. Therefore, the heat transfer coefficient for a standing person for combined convection and radiation is



$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = 4.0 + 4.7 = 8.7 \text{ W/m}^2.°\text{C}$$

The thermal resistance of the clothing is given to be

$$R_{\text{cloth}} = 1.1 \text{ clo} = 1.1 \times 0.155 \text{ m}^2.°\text{C/W} = 0.171 \text{ m}^2.°\text{C/W}$$

Noting that the surface area of an average man is 1.8 m^2, the sensible heat loss from this person when clothed is determined to be

$$\dot{Q}_{\text{sensible,clothed}} = \frac{A_s(T_{\text{skin}} - T_{\text{ambient}})}{R_{\text{cloth}} + \dfrac{1}{h_{\text{combined}}}} = \frac{(1.8 \text{ m}^2)(33 - 20)°\text{C}}{0.171 \text{ m}^2.°\text{C/W} + \dfrac{1}{8.7 \text{ W/m}^2.°\text{C}}} = 82 \text{ W}$$

From heat transfer point of view, taking the clothes off is equivalent to removing the clothing insulation or setting R_{cloth} = 0. The heat transfer in this case can be expressed as

$$\dot{Q}_{\text{sensible,unclothed}} = \frac{A_s(T_{\text{skin}} - T_{\text{ambient}})}{\dfrac{1}{h_{\text{combined}}}} = \frac{(1.8 \text{ m}^2)(33 - T_{\text{ambient}})°\text{C}}{\dfrac{1}{8.7 \text{ W/m}^2.°\text{C}}}$$

To maintain thermal comfort after taking the clothes off, the skin temperature of the person and the rate of heat transfer from him must remain the same. Then setting the equation above equal to 82 W gives

$$T_{\text{ambient}} = \mathbf{27.8°C}$$

Therefore, the air temperature needs to be raised from 22 to 27.8°C to ensure that the person will feel comfortable in the room after he takes his clothes off. Note that the effect of clothing on latent heat is assumed to be negligible in the solution above. We also assumed the surface area of the clothed and unclothed person to be the same for simplicity, and these two effects should counteract each other.

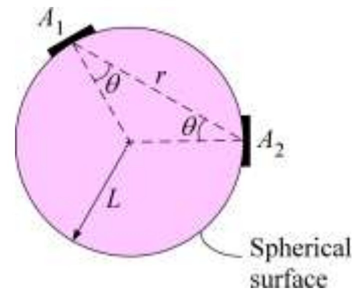
Review Problems

13-123 Two diffuse surfaces A_1 and A_2 placed at an specified orientation, (*a*) the expression for the view factor F_{12} in terms of A_2 and L, and (*b*) the value of the view factor F_{12} when A_2 = 0.02 m^2 and L = 1 m are to be determined.

Assumptions **1** The surfaces A_1 and A_2 are diffuse. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (*a*) The view factor for surfaces A_1 and A_2 can be determined using the integral

$$\begin{aligned}
F_{12} &= \frac{1}{A_1}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, dA_1 dA_2 \\
&= \frac{1}{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, A_1 dA_2 \\
&= \frac{1}{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, A_1 A_2 \\
&= \frac{\cos\theta_1 \cos\theta_2}{\pi r^2}\, A_2
\end{aligned}$$



From orientation of the two surfaces, we have

$$\theta_1 = \theta_2 \quad \rightarrow \quad \cos\theta_1 = \cos\theta_2 \qquad (1)$$

and

$$r = 2L\cos\theta_1 \qquad (2)$$

Substituting Eqs. (1) and (2) into the expression for F_{12}, we get

$$F_{12} = \frac{(\cos\theta_1)^2}{\pi(2L\cos\theta_1)^2} A_2 = \frac{A_2}{4L^2\pi}$$

(*b*) The value of the view factor F_{12} when A_2 = 0.02 m^2 and L = 1 m is

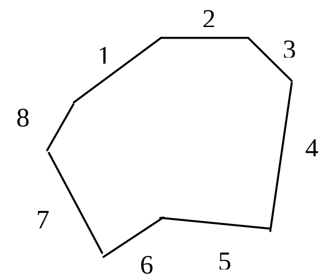
$$F_{12} = \frac{A_2}{4L^2\pi} = \frac{0.02\text{ m}^2}{4(1\text{ m})^2\pi} = \mathbf{1.59\times 10^{-3}}$$

Discussion The view factor F_{21} can simply be determined with the reciprocity relation as

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{4L^2\pi}$$

13-124 An enclosure consisting of eight surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.

Analysis An eight surface enclosure (N = 8) involves $N^2 = 8^2 = \mathbf{64}$ view factors and we need to determine $\dfrac{N(N-1)}{2} = \dfrac{8(8-1)}{2} = 28$ view factors directly. The remaining 64 - 28 = **36** of the view factors can be determined by the application of the reciprocity and summation rules.



13-125 A cylindrical enclosure is considered. (*a*) The expression for the view from the side surface to itself F_{33} in terms of K and (*b*) the value of the view factor F_{33} for $L = D$ are to be determined.

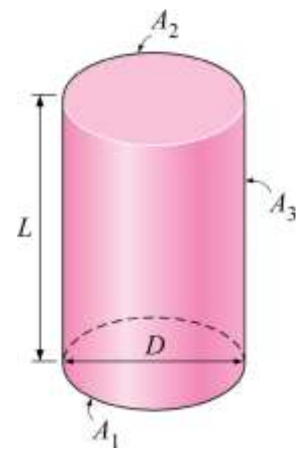
Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis (*a*) The surfaces are designated as follows: Base surface as A_1, top surface as A_2, and side surface as A_3

Applying the summation rule to A_1, we have

$$F_{11} + F_{12} + F_{13} = 1 \qquad \text{(where } F_{11} = 0\text{)}$$

or $$F_{13} = 1 - F_{12} \qquad (1)$$



For coaxial parallel disks, from Table 13-1, with $i = 1$, $j = 2$,

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \qquad (2)$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + \frac{4}{(D/L)^2} = 2 + 4K^2 \qquad (3)$$

where $$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2K}$$

Substituting Eq. (3) into Eq. (2), we get

$$\begin{aligned} F_{12} &= \frac{1}{2}\{2 + 4K^2 - [(2 + 4K^2)^2 - 4]^{1/2}\} \\ &= \frac{1}{2}[2 + 4K^2 - (16K^4 + 16K^2)^{1/2}] \\ &= \frac{1}{2}[2 + 4K^2 - 4K(K^2 + 1)^{1/2}] \\ &= 1 + 2K^2 - 2K(K^2 + 1)^{1/2} \end{aligned}$$

Substituting the above expression for F_{12} into Eq. (1) yields the expression for F_{13}:

$$F_{13} = 1 - [1 + 2K^2 - 2K(K^2 + 1)^{1/2}] = 2K(K^2 + 1)^{1/2} - 2K^2 \qquad (4)$$

Then, applying the summation rule to A_3, we have

$$F_{31} + F_{32} + F_{33} = 1 \qquad \text{(where } F_{31} = F_{32}\text{)}$$

or $$F_{33} = 1 - 2F_{31}$$

Applying reciprocity relation, we have

$$F_{33} = 1 - 2F_{13}(A_1 / A_3)$$

where $$F_{31} = F_{13}(A_1 / A_3)$$

Also, we know that

$$A_1 = \pi D^2 / 4 \qquad \text{and} \qquad A_3 = \pi DL \qquad \rightarrow \qquad \frac{A_1}{A_3} = \frac{D}{4L}$$

Hence, the expression for F_{33} becomes

$$F_{33} = 1 - 2\frac{D}{4L}F_{13} = 1 - \frac{1}{2K}F_{13} \qquad (5)$$

Finally, substituting Eq. (4) into Eq. (5) yields the expression for F_{33}:

$$F_{33} = 1 - \frac{1}{2K}[2K(K^2 + 1)^{1/2} - 2K^2]$$

Hence, $$F_{33} = 1 + K - (K^2 + 1)^{1/2}$$

(*b*) The value of the view factor F_{33} for $L = D$ (i.e., $K = 1$) is

$$F_{33} = 1 + K - (K^2 + 1)^{1/2} = 1 + 1 - (1^2 + 1)^{1/2} = 2 - \sqrt{2} = \mathbf{0.589}$$

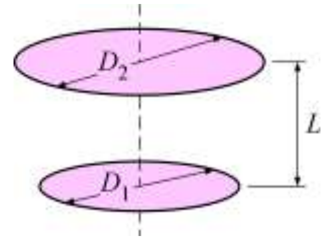
Discussion If the cylinder has a length and diameter of $L = 2D$, then from the expression for F_{33} we have

$$F_{33} = 1 + 2 - (2^2 + 1)^{1/2} = \mathbf{0.764}$$

13-126 Two parallel black disks are positioned coaxially, where the lower disk is heated electrically. The temperature of the upper disk is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** Radiation heat transfer only between the two disks. **5** No heat loss to the surrounding.

Analysis For coaxial parallel disks, from Table 13-1, with $i = 1$, $j = 2$,



$$R_1 = \frac{D_1/2}{L} = \frac{0.2/2}{0.25} = 0.4 \qquad \text{and} \qquad R_2 = \frac{D_2/2}{L} = \frac{0.4/2}{0.25} = 0.8$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+(0.8)^2}{(0.4)^2} = 11.25$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{11.25 - \left[(11.25)^2 - 4\left(\frac{0.4}{0.2}\right)^2\right]^{1/2}\right\} = 0.3676$$

The net radiation heat transfer rate leaving the lower surface can be expressed as

$$\dot{Q}_{\text{elec}} = \dot{Q}_{12} = A_1 F_{12}\sigma(T_1^4 - T_2^4) \qquad \rightarrow \qquad T_2 = \left(T_1^4 - \frac{4\dot{Q}_{\text{elec}}}{\pi D_1^2 F_{12}\sigma}\right)^{1/4}$$

$$T_2 = \left[(500\text{ K})^4 - \frac{4(20\text{ W})}{\pi(0.2\text{ m})^2(0.3676)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \mathbf{423\ K}$$

Discussion The view factor F_{12} can also be determined using Fig. 13-7 to be

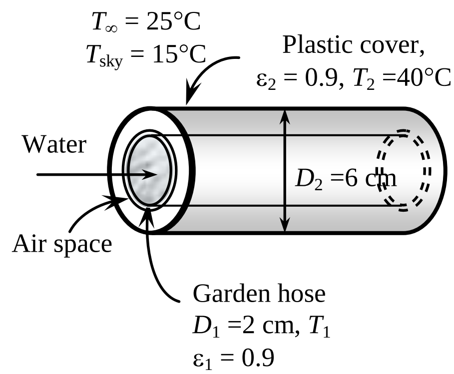
$$F_{12} \approx 0.36 \qquad \text{with} \qquad L/r_1 = 2.5 \qquad \text{and} \qquad r_2/L = 0.8$$

13-127 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+25)/2 = 32.5°C$ are (Table A-15)

$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.632\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7275$$
$$\beta = \frac{1}{(32.5+273)\text{ K}} = 0.003273 \text{ K}^{-1}$$



Analysis Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $L_c = D_{plastic} = D_2 = 0.06\text{ m}$.

$$Ra = \frac{g\beta(T_s - T_\infty)D_2^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003273\text{ K}^{-1})(40-25)\text{K}(0.06\text{ m})^3}{(1.632\times10^{-5}\text{ m}^2\text{/s})^2}(0.7275) = 2.842\times10^5$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.842\times10^5)^{1/6}}{\left[1+\left(0.559/0.7241\right)^{9/16}\right]^{8/27}}\right\}^2 = 10.30$$

$$h = \frac{k}{D_2}Nu = \frac{0.02607\text{ W/m.°C}}{0.06\text{ m}}(10.30) = 4.475\text{ W/m}^2\text{.°C}$$

$$A_{plastic} = A_2 = \pi D_2 L = \pi(0.06\text{ m})(1\text{ m}) = 0.1885\text{ m}^2$$

Then the rate of heat transfer from the outer surface by natural convection becomes

$$\dot{Q}_{conv} = hA_2(T_s - T_\infty) = (4.475\text{ W/m}^2\text{.°C})(0.1885\text{ m}^2)(40-25)\text{°C} = \mathbf{12.7\ W}$$

The rate of heat transfer by radiation from the outer surface is determined from

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_2\sigma(T_s{}^4 - T_{sky}{}^4)\\ &= (0.90)(0.1885\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(40+273\text{ K})^4 - (15+273\text{ K})^4]\\ &= \mathbf{26.1 W}\end{aligned}$$

Finally,

$$\dot{Q}_{total,loss} = 12.7 + 26.1 = 38.8\text{ W}$$

Discussion Note that heat transfer is mostly by radiation.

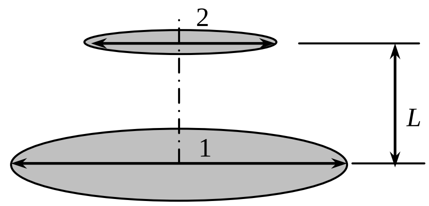
13-128 Radiation heat transfer occurs between two concentric disks. The view factors and the net rate of radiation heat transfer for two cases are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of disk 1 and 2 are given to be $\varepsilon_a = 0.6$ and $\varepsilon_b = 0.8$, respectively.

Analysis (*a*) The view factor from surface 1 to surface 2 is determined using Fig. 13-7 as

$$\frac{L}{r_1} = \frac{0.10}{0.20} = 0.5, \quad \frac{r_2}{L} = \frac{0.10}{0.10} = 1 \longrightarrow F_{12} = \mathbf{0.19}$$



Using reciprocity rule,

$$A_1 = \pi(0.2\text{ m})^2 = 0.1257\text{ m}^2$$

$$A_2 = \pi(0.1\text{ m})^2 = 0.0314\text{ m}^2$$

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{0.1257\text{ m}^2}{0.0314\text{ m}^2}(0.19) = \mathbf{0.76}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (573\text{ K})^4\right]}{\dfrac{1-0.6}{(0.1257\text{ m}^2)(0.6)} + \dfrac{1}{(0.1257\text{ m}^2)(0.19)} + \dfrac{1-0.8}{(0.0314\text{ m}^2)(0.8)}} = \mathbf{1250\,W}$$

(c) When the space between the disks is completely surrounded by a refractory surface, the net rate of radiation heat transfer can be determined from

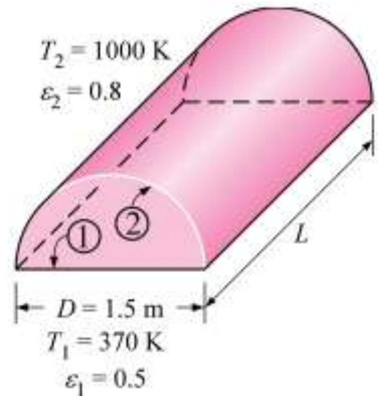
$$\dot{Q} = \frac{A_1\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{A_1 + A_2 - 2A_1F_{12}}{A_2 - A_1F_{12}^2} + \left(\dfrac{1}{\varepsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(0.1257\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (573\text{ K})^4\right]}{\dfrac{0.1257 + 0.0314 - 2(0.1257)(0.19)}{0.0314 - (0.1257)(0.19)^2} + \left(\dfrac{1}{0.6} - 1\right) + \dfrac{0.1257}{0.0314}\left(\dfrac{1}{0.8} - 1\right)} = \mathbf{1510\,W}$$

Discussion The rate of heat transfer in part (c) is 21 percent higher than that in part (b).

13-129 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The drying rate per unit length experienced by the base surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.



Properties The latent heat of vaporization for water is h_{fg} = 2257 kJ/kg (Table A-2)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{12} = 1 \quad \text{(where } F_{11} = 0\text{)}$$

Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$

Applying energy balance on the base surface, we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m} h_{fg} = \frac{\sigma(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_2 F_{21}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \quad \rightarrow \quad \dot{m} = \frac{(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{DL\varepsilon_1} + \dfrac{2}{\pi DLF_{21}} + \dfrac{2(1-\varepsilon_2)}{\pi DL\varepsilon_2}}$$

Hence

$$\begin{aligned}
\frac{\dot{m}}{L} &= \frac{D(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{2}{\pi F_{21}} + \dfrac{2(1-\varepsilon_2)}{\pi\varepsilon_2}} \\
&= \frac{D(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + 1 + \dfrac{2(1-\varepsilon_2)}{\pi\varepsilon_2}} \\
&= \frac{(1.5\text{ m})(1000^4 - 370^4)\text{ K}^4}{\dfrac{1-0.5}{0.5} + 1 + \dfrac{2(1-0.8)}{0.8\pi}} \left(\frac{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}{2257\times10^3\text{ J/kg}} \right) \\
&= \mathbf{0.0171\ kg/s \cdot m}
\end{aligned}$$

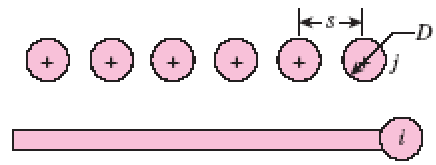
Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-130 Radiation heat transfer occurs between a tube-bank and a wall. The view factors, the net rate of radiation heat transfer, and the temperature of tube surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** The tube wall thickness and convection from the outer surface are negligible.

Properties The emissivities of the wall and tube bank are given to be $\varepsilon_i = 0.8$ and $\varepsilon_j = 0.9$, respectively.

Analysis (*a*) We take the wall to be surface *i* and the tube bank to be surface *j*. The view factor from surface *i* to surface *j* is determined from



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

$$= 1 - \left[1 - \left(\frac{1.5}{3}\right)^2\right]^{0.5} + \left(\frac{1.5}{3}\right)\left\{\tan^{-1}\left[\left(\frac{3}{1.5}\right)^2 - 1\right]^{0.5}\right\} = \mathbf{0.658}$$

The view factor from surface *j* to surface *i* is determined from reciprocity relation. Taking *s* to be the width of the wall

$$A_i F_{ij} = A_j F_{ji} \longrightarrow F_{ji} = \frac{A_i}{A_j} F_{ij} = \frac{sL}{\pi DL} F_{ij} = \frac{s}{\pi D} F_{ij} = \frac{3}{\pi(1.5)}(0.658) = \mathbf{0.419}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{q} = \frac{\sigma\left(T_i^{\,4} - T_j^{\,4}\right)}{\left(\frac{1-\varepsilon_i}{\varepsilon_i}\right)\frac{1}{A_i} + \frac{1}{A_i F_{ij}} + \left(\frac{1-\varepsilon_j}{\varepsilon_j}\right)\frac{1}{A_j}} = \frac{\sigma\left(T_i^{\,4} - T_j^{\,4}\right)}{\frac{1-\varepsilon_i}{\varepsilon_i} + \frac{1}{F_{ij}} + \left(\frac{1-\varepsilon_j}{\varepsilon_j}\right)\frac{A_i}{A_j}}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(1173\ \text{K})^4 - (333\ \text{K})^4\right]}{\frac{1-0.8}{0.8} + \frac{1}{0.658} + \left(\frac{1-0.9}{0.9}\right)\frac{(0.03\ \text{m})}{\pi(0.015\ \text{m})}} = \mathbf{57{,}900\ W/m^2}$$

(c) Under steady conditions, the rate of radiation heat transfer from the wall to the tube surface is equal to the rate of convection heat transfer from the tube wall to the fluid. Denoting T_w to be the wall temperature,

$$\dot{q}_{rad} = \dot{q}_{conv}$$

$$\frac{\sigma\left(T_i^{\,4} - T_w^{\,4}\right)}{\frac{1-\varepsilon_i}{\varepsilon_i} + \frac{1}{F_{ij}} + \left(\frac{1-\varepsilon_j}{\varepsilon_j}\right)\frac{A_i}{A_j}} = hA_j(T_w - T_j)$$

$$\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(1173\ \text{K})^4 - T_w^4\right]}{\frac{1-0.8}{0.8} + \frac{1}{0.658} + \left(\frac{1-0.9}{0.9}\right)\frac{(0.03\ \text{m})}{\pi(0.015\ \text{m})}} = (2000\ \text{W/m}^2\cdot\text{K})\left[\frac{\pi(0.015\ \text{m})}{0.03\ \text{m}}\right]\left[T_w - (40+273\ \text{K})\right]$$

Solving this equation by an equation solver such as EES, we obtain

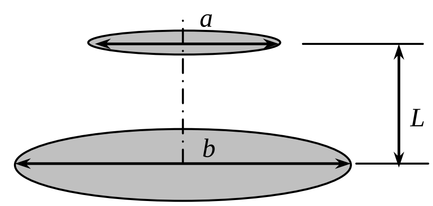
$T_w = 331.4$ K $\mathbf{= 58.4°C}$

13-131 Radiation heat transfer occurs between two parallel coaxial disks. The view factors and the rate of radiation heat transfer for the existing and modified cases are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of disk *a* and *b* are given to be $\varepsilon_a = 0.60$ and $\varepsilon_b = 0.8$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows



$$A = \frac{a}{2L} = \frac{0.20}{2(0.10)} = 1$$

$$B = \frac{b}{2L} = \frac{0.40}{2(0.10)} = 2$$

$$C = 1 + \frac{1+A^2}{B^2} = 1 + \frac{1+1^2}{2^2} = 1.5$$

$$F_{ab} = 0.5\left(\frac{B}{A}\right)^2\left\{C - \left[C^2 - 4\left(\frac{A}{B}\right)^2\right]^{0.5}\right\} = 0.5\left(\frac{2}{1}\right)^2\left\{1.5 - \left[1.5^2 - 4\left(\frac{1}{2}\right)^2\right]^{0.5}\right\} = \mathbf{0.764}$$

The view factor from surface *b* to surface *a* is determined from reciprocity relation:

$$A_a = \frac{\pi a^2}{4} = \frac{\pi(0.2\text{ m})^2}{4} = 0.0314\text{ m}^2$$

$$A_b = \frac{\pi b^2}{4} = \frac{\pi(0.4\text{ m})^2}{4} = 0.1257\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.0314)(0.764) = (0.1257)F_{ba} \longrightarrow F_{ba} = \mathbf{0.191}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a^{\,4} - T_b^{\,4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ab}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4 - (473\text{ K})^4\right]}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(0.764)} + \dfrac{1-0.8}{(0.1257\text{ m}^2)(0.8)}} = \mathbf{464\text{ W}}$$

(c) In this case we have $\varepsilon_c = 0.7,\ A_c \to \infty,\ F_{ac} = F_{bc} = 1$ and $\dot{Q}_{ac} = \dot{Q}_{cb} = \dot{Q}_{bc}$. An energy balance gives

$$\frac{\sigma\left(T_a^{\,4} - T_c^{\,4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c^{\,4} - T_b^{\,4}\right)}{\dfrac{1-\varepsilon_c}{A_c\varepsilon_c} + \dfrac{1}{A_cF_{cb}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{T_a^{\,4} - T_c^{\,4}}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + 0} = \frac{T_c^{\,4} - T_b^{\,4}}{0 + \dfrac{1}{A_bF_{bc}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(873)^4 - T_c^{\,4}}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(1)}} = \frac{T_c^{\,4} - 473^4}{\dfrac{1}{(0.1257\text{ m}^2)(1)} + \dfrac{1-0.8}{(0.1257\text{ m}^2)(0.8)}}$$

$$\longrightarrow T_c = 605\text{ K}$$

Then

$$\dot{Q}_{bc} = \dot{Q}_{ac} = \frac{\sigma\left(T_a^{\,4} - T_c^{\,4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4 - (605\text{ K})^4\right]}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(1)} + 0} = \mathbf{477\text{ W}}$$

Discussion The rate of heat transfer is higher in part (c) because the large disk c is able to collect all radiation emitted by disk a. It is not acting as a shield.

13-132 Radiation heat transfer occurs between two square parallel plates. The view factors, the rate of radiation heat transfer and the temperature of a third plate to be inserted are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

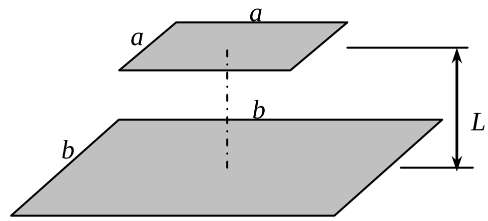
Properties The emissivities of plate a, b, and c are given to be $\varepsilon_a = 0.8$, $\varepsilon_b = 0.4$, and $\varepsilon_c = 0.1$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows

$$A = \frac{a}{L} = \frac{20}{40} = 0.5,\ \ B = \frac{b}{L} = \frac{60}{40} = 1.5$$

$$F_{ab} = \frac{1}{2A}\left\{\left[(B+A)^2+4\right]^{0.5} - \left[(B-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(1.5+0.5)^2+4\right]^{0.5} - \left[(1.5-0.5)^2+4\right]^{0.5}\right\} = \mathbf{0.592}$$

The view factor from surface b to surface a is determined from reciprocity relation:



$$A_a = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$$

$$A_b = (0.6\text{ m})(0.6\text{ m}) = 0.36\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.04)(0.592) = (0.36)F_{ba} \longrightarrow F_{ba} = \mathbf{0.0658}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a{}^4 - T_b{}^4\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_a F_{ab}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (473\text{ K})^4\right]}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.592)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}} = \mathbf{1374\ W}$$

(c) In this case we have

$$A = \frac{a}{L} = \frac{0.2\text{ m}}{0.2\text{ m}} = 1,\ \ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{ac} = \frac{1}{2A}\left\{\left[(C+A)^2+4\right]^{0.5} - \left[(C-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(10+0.5)^2+4\right]^{0.5} - \left[(10-0.5)^2+4\right]^{0.5}\right\} = 0.981$$

$$B = \frac{b}{L} = \frac{0.6\text{ m}}{0.2\text{ m}} = 3,\ \ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{bc} = \frac{1}{2A}\left\{\left[(C+B)^2+4\right]^{0.5} - \left[(C-B)^2+4\right]^{0.5}\right\}$$

$$= \frac{1}{2(3)}\left\{\left[(10+3)^2+4\right]^{0.5} - \left[(10-3)^2+4\right]^{0.5}\right\} = 0.979$$

$$A_b F_{bc} = A_c F_{cb}$$

$$(0.36)(0.979) = (4.0)F_{cb} \longrightarrow F_{ba} = 0.0881$$

An energy balance gives

$$\dot{Q}_{ac} = \dot{Q}_{cb}$$

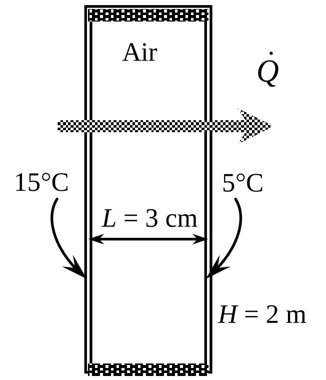
$$\frac{\sigma\left(T_a{}^4 - T_c{}^4\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_a F_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c{}^4 - T_b{}^4\right)}{\dfrac{1-\varepsilon_c}{A_c\varepsilon_c} + \dfrac{1}{A_c F_{cb}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(1073\text{ K})^4 - T_c{}^4}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.981)} + \dfrac{1-0.1}{(4\text{ m}^2)(0.1)}} = \frac{T_c{}^4 - (473\text{ K})^4}{\dfrac{1-0.1}{(4\text{ m}^2)(0.1)} + \dfrac{1}{(4\text{ m}^2)(0.0881)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}}$$

Solving the equation with an equation solver such as EES, we obtain T_c = 754 K **= 481°C**

13-133 A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.



Properties The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10°\text{C}$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$

$$\nu = \nu_{1atm}/0.3 = 1.426\times10^{-5}/0.3 = 4.753\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7336$$

$$\beta = \frac{1}{(10+273)\text{ K}} = 0.003534 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the glasses, $L_c = L = 0.03\text{ m}$

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003534\text{ K}^{-1})(15-5)\text{K}(0.03\text{ m})^3}{(4.753\times10^{-5}\text{ m}^2\text{/s})^2}(0.7336) = 3040$$

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{L}\right)^{-1/9} = 0.197(3040)^{1/4}\left(\frac{2}{0.05}\right)^{-1/9} = 0.971$$

Note that heat transfer through the air space is less than that by pure conduction as a result of partial evacuation of the space. Then the rate of heat transfer through the air space becomes

$$A_s = (2\text{ m})(5\text{ m}) = 10\text{ m}^2$$

$$\dot{Q}_{conv} = kNuA_s\frac{T_1-T_2}{L} = (0.02439\text{ W/m.°C})(0.971)(10\text{ m}^2)\frac{(15-5)°\text{C}}{0.03\text{ m}} = \mathbf{78.9\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4-T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(10\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(15+273\text{ K})^4-(5+273\text{ K})^4]}{\frac{1}{0.9}+\frac{1}{0.9}-1} = \mathbf{421 W}$$

Then the rate of total heat transfer becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 79 + 421 = \mathbf{500\ W}$$

Discussion Note that heat transfer through the window is mostly by radiation.

13-134 A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

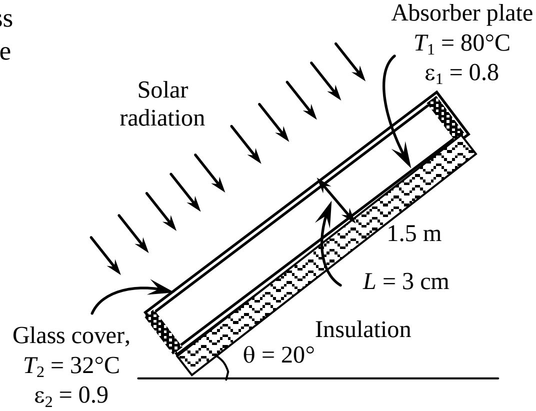
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2 = 0.8$ for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 = 56$°C are (Table A-15)

$$k = 0.02779 \text{ W/m.°C}$$

$$\nu = 1.857\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7212$$

$$\beta = \frac{1}{T_f} = \frac{1}{(56+273)\text{K}} = 0.003040 \text{ K}^{-1}$$



Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.03$ m Then,

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.03 \text{ m})^3}{(1.857\times10^{-5} \text{ m}^2\text{/s})^2}(0.7212) = 8.083\times10^4$$

$$A_s = H\times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right] + \left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18} - 1\right]^+$$

$$= 1 + 1.44\left[1 - \frac{1708}{(8.083\times10^4)\cos(20)}\right]^+\left[1 - \frac{1708[\sin(1.8\times20)]^{1.6}}{(8.083\times10^4)\cos(20)}\right] + \left[\frac{[(8.083\times10^4)\cos(20)]^{1/3}}{18} - 1\right]^+$$

$$= 3.747$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02779 \text{ W/m.°C})(3.747)(4.5 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{750 \text{ W}}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^{\,4} - T_2^{\,4})}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(80+273 \text{ K})^4 - (32+273 \text{ K})^4]}{\frac{1}{0.8} + \frac{1}{0.9} - 1} = \mathbf{1289 \text{ W}}$$

Discussion The rates of heat loss by natural convection for the horizontal and vertical cases would be as follows (Note that the Ra number remains the same):

Horizontal:

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18} - 1\right]^+ = 1 + 1.44\left[1 - \frac{1708}{8.083\times10^4}\right]^+ + \left[\frac{(8.083\times10^4)^{1/3}}{18} - 1\right]^+ = 3.812$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02779 \text{ W/m.°C})(3.812)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{1017 \text{ W}}$$

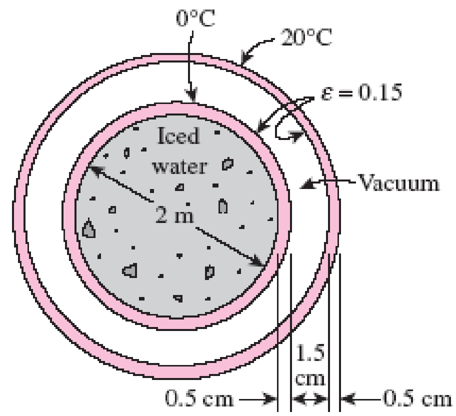
Vertical:

$$Nu = 0.42Ra^{1/4}\Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(8.083\times10^4)^{1/4}(0.7212)^{0.012}\left(\frac{2 \text{ m}}{0.03 \text{ m}}\right)^{-0.3} = 2.001$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02779 \text{ W/m.°C})(2.001)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{534 \text{ W}}$$

13-135 A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.



Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be

$$A_1 = \pi D_1^{\ 2} = \pi (2.01\,\text{m})^2 = 12.69\,\text{m}^2$$

$$\dot{Q}_{12} = \frac{A_1 \sigma (T_2^{\ 4} - T_1^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(20+273\,\text{K})^4 - (0+273\,\text{K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

$$= \mathbf{107.4\,W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074\,\text{kJ/s})(24\times3600\,\text{s}) = 9279\,\text{kJ}$$

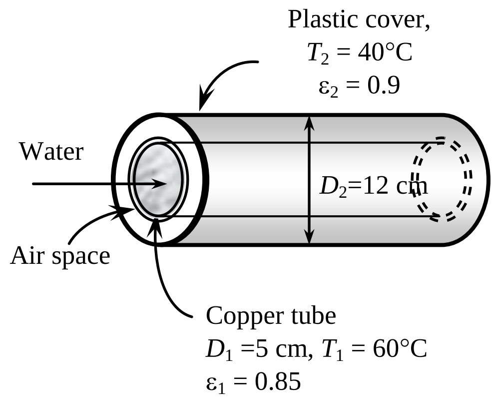
The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9279\,\text{kJ}}{333.7\,\text{kJ/kg}} = \mathbf{27.8\,kg}$$

13-136 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.85$ for the tube surface and $\varepsilon_2 = 0.9$ for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (60+40)/2 = 50°C$ are (Table A-15)



$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{(50+273)\text{ K}} = 0.003096 \text{ K}^{-1}$$

Analysis The characteristic length in this case is

$$L_c = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.12\text{ m} - 0.05\text{ m}) = 0.07\text{ m}$$

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003096\text{ K}^{-1})(60-40)\text{K}(0.035\text{ m})^3}{(1.798\times10^{-5}\text{ m}^2/\text{s})^2}(0.7228) = 5.823\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{[\ln(0.12/0.05)]^4}{(0.035\text{ m})^3\left[(0.05\text{ m})^{-3/5}+(0.12\text{ m})^{-3/5}\right]^5} = 0.1678$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02735\text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}\left[(0.1678)(5.823\times10^4)\right]^{1/4} = 0.08626\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q}_{\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.08626\text{ W/m.°C})}{\ln(0.12/0.05)}(60-40)°\text{C} = \mathbf{12.4\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)} = \frac{[\pi(0.05\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(60+273\text{ K})^4-(40+273\text{ K})^4]}{\frac{1}{0.85}+\frac{1-0.9}{0.9}\left(\frac{5}{12}\right)}$$
$$= \mathbf{19.7\ W}$$

Finally,

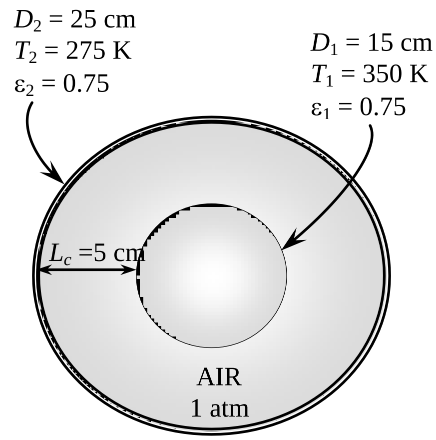
$$\dot{Q}_{total,loss} = 12.4+19.7 = 32.1\text{ W} \quad \text{(per m length)}$$

13-137 Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.75$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)

$$k = 0.02658 \text{ W/m.°C}$$
$$\nu = 1.697\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{312.5\text{ K}} = 0.0032 \text{ K}^{-1}$$



Analysis (*a*) Noting that $D_i = D_1$ and $D_o = D_2$, the characteristic length is

$$L_c = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(0.25\text{ m} - 0.15\text{ m}) = 0.05\text{ m}$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003200\text{ K}^{-1})(350-275\text{ K})(0.05\text{ m})^3}{(1.697\times10^{-5}\text{ m}^2\text{/s})^2}(0.7256) = 7.415\times10^5$$ The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05\text{ m}}{[(0.15\text{ m})(0.25\text{ m})]^4[(0.15\text{ m})^{-7/5} + (0.25\text{ m})^{-7/5}]^5} = 0.005900$$

$$\begin{aligned} k_{\text{eff}} &= 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}Ra)^{1/4} \\ &= 0.74(0.02658\text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}\left[(0.00590)(7.415\times10^5)\right]^{1/4} \\ &= 0.1315\text{ W/m.°C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) = (0.1315\text{ W/m.°C})\pi\left[\frac{(0.15\text{ m})(0.25\text{ m})}{(0.05\text{ m})}\right](350-275)\text{K} = \mathbf{23.2\ W}$$

(*b*) The rate of heat transfer by radiation is determined from

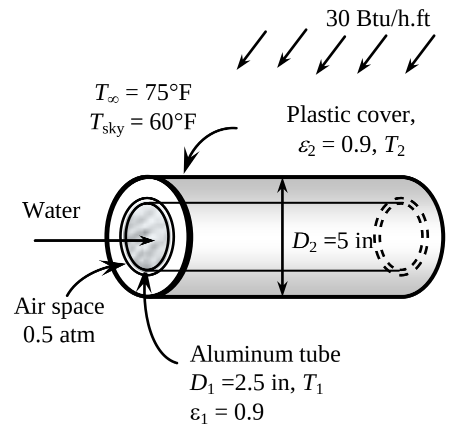
$$A_1 = \pi D_1^2 = \pi(0.15\text{ m})^2 = 0.0707\text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2} = \frac{(0.0707\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(350\text{ K})^4 - (275\text{ K})^4]}{\dfrac{1}{0.75} + \dfrac{1-0.75}{0.75}\left(\dfrac{0.15}{0.25}\right)^2} = \mathbf{25.6\ W}$$

13-138E The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 85°F, and use properties at an anticipated average temperature of (75+85)/2 =80°F (Table A-15E),



$$k = 0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

$$\nu = 1.697\times10^{-4} \text{ ft}^2/s$$

$$\text{Pr} = 0.7290$$

$$\beta = \frac{1}{T_{\text{ave}}} = \frac{1}{540\text{ R}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 0.5 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30\text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o W) = \pi(5/12\text{ ft})(1\text{ ft}) = 1.309\text{ ft}^2 \quad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, solution will require a trial-and-error approach. Assuming the glass cover temperature to be 85°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\begin{aligned}\text{Ra}_{Do} &= \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2}\text{Pr} \\ &= \frac{(32.2\text{ ft/s}^2)[1/(540\text{ R})](85-75\text{ R})(5/12\text{ ft})^3}{(1.697\times10^{-4}\text{ ft}^2/\text{s})^2}(0.7290) = 1.092\times10^6\end{aligned}$$

$$\begin{aligned}\text{Nu} &= \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.092\times10^6)^{1/6}}{\left[1+(0.559/0.7290)^{9/16}\right]^{8/27}}\right\}^2 \\ &= 14.95\end{aligned}$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.01481\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{5/12\text{ ft}}(14.95) = 0.5315\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

$$\dot{Q}_{o,\text{conv}} = h_o A_o(T_o - T_\infty) = (0.5315\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(1.309\text{ ft}^2)(85-75)°\text{F} = 6.96\text{ Btu/h}$$

Also,

$$\begin{aligned}\dot{Q}_{o,\text{rad}} &= \varepsilon_o\sigma A_o(T_o^4 - T_{\text{sky}}^4) \\ &= (0.9)(0.1714\times10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.309\text{ ft}^2)\left[(545\text{ R})^4 - (520\text{ R})^4\right] \\ &= 30.5\text{ Btu/h}\end{aligned}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\text{total}} = \dot{Q}_{o,\text{conv}} + \dot{Q}_{o,\text{rad}} = 7.0 + 30.5 = 37.5 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 85°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be 81.5°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (5 - 2.5)/2 = 1.25 \text{ in} = 1.25/12 \text{ ft}$$

Also,

$$A_i = A_{tube} = (\pi D_i W) = \pi(2.5/12 \text{ ft})(1 \text{ ft}) = 0.6545 \text{ ft}^2 \quad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 118.5°F, and thus an average temperature of (81.5+118.5)/2 = 100°F=560 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)[1/(560 \text{ R})](118.5 - 81.5 \text{ R})(1.25/12 \text{ ft})^3}{[(1.809\times10^{-4} \text{ ft}^2/\text{s})/0.5]^2}(0.726) = 1.334\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(5/2.5)]^4}{(1.25/12 \text{ ft})^3[(2.5/12 \text{ ft})^{-3/5} + (5/12 \text{ ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4}$$

$$= 0.386(0.01529 \text{ Btu/h}\cdot\text{ft}\cdot\text{°F})\left(\frac{0.726}{0.861 + 0.726}\right)(0.1466\times1.334\times10^4)^{1/4}$$

$$= 0.03227 \text{ Btu/h}\cdot\text{ft}\cdot\text{°F}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.03227 \text{ Btu/h}\cdot\text{ft}\cdot\text{°F})}{\ln(5/2.5)}(118.5 - 81.5)\text{°F} = 10.8 \text{ Btu/h}$$

Also,

$$\dot{Q}_{i,\text{rad}} = \frac{\sigma A_i(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)}$$

$$= \frac{(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.6545 \text{ ft}^2)\left[(578.5 \text{ R})^4 - (541.5 \text{ R})^4\right]}{\dfrac{1}{0.9} + \dfrac{1-0.9}{0.9}\left(\dfrac{2.5 \text{ in}}{5 \text{ in}}\right)} = 25.0 \text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\text{total}} = \dot{Q}_{i,\text{conv}} + \dot{Q}_{i,\text{rad}} = 10.8 + 25.0 = 35.8 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 118.5°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **113°F**. Therefore, the tube will reach an equilibrium temperature of 113°F when the pump fails.

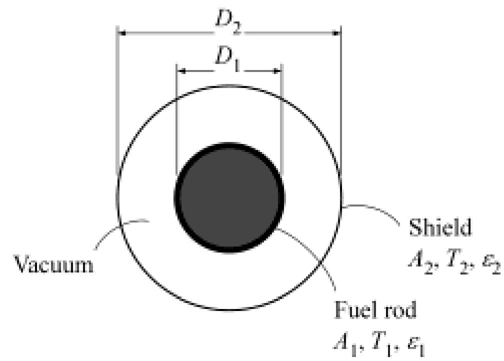
13-139 A long cylindrical black surface fuel rod is shielded by a concentric surface that has a uniform temperature. The fuel rod generates 0.5 MW/m^3 of heat per unit length. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The fuel rod surface is black. 3 The shield is opaque, diffuse, and gray. **4** The fuel rod and shield formed an infinitely long concentric cylinder.

Properties The emissivity of the shield is given as $\varepsilon_2 = 0.05$. The fuel rod surface is black, $\varepsilon_1 = 1$.

Analysis The heat generation of the fuel rod is

$$\dot{e}_{\rm gen} = \frac{\dot{E}_{\rm gen}}{V} = \frac{4\dot{E}_{\rm gen}}{\pi D_1^2 L} \qquad \rightarrow \qquad \dot{E}_{\rm gen} = \dot{e}_{\rm gen} V = \dot{e}_{\rm gen}\frac{\pi D_1^2 L}{4}$$



Hence, the total heat generation rate per unit length (L = 1 m) by the fuel rod is

$$\dot{E}_{\rm gen} = \dot{e}_{\rm gen}\frac{\pi D_1^2 L}{4} = (0.5\times 10^6 \text{ W/m}^3)\frac{\pi(0.025\text{ m})^2(1\text{ m})}{4} = 245.4 \text{ W}$$

For infinitely long concentric cylinder, the rate of radiation heat transfer at the fuel rod surface is (from Table 13-3),

$$\dot{E}_{\rm gen} = \dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

$$T_1 = \left\{\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\frac{\dot{E}_{\rm gen}}{A_1\sigma} + T_2^4\right\}^{1/4} = \left\{\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\frac{\dot{E}_{\rm gen}}{\pi D_1 L\sigma} + T_2^4\right\}^{1/4}$$

$$= \left\{\left[\frac{1}{1} + \frac{1-0.05}{0.05}\left(\frac{25}{50}\right)\right]\frac{245.4\text{ W}}{\pi(0.025\text{ m})(1\text{ m})(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)} + (320\text{ K})^4\right\}^{1/4}$$

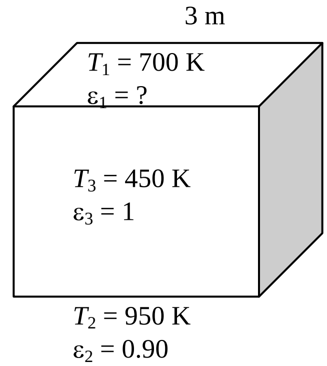
$$T_1 = \mathbf{876\ K}$$

Discussion The use of absolute temperatures is necessary for calculations involving radiation heat transfer.

13-140 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is from Fig. 13-5 $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \qquad F_{23} = F_{13} = 0.80, \qquad F_{31} = F_{32} = 0.20$$

We now apply Eq. 13-35 to each surface

Surface 1:

$$\sigma T_1^{\,4} = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K})^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[0.20(J_1 - J_2) + 0.80(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^{\,4} = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(950\ \text{K})^4 = J_2 + \frac{1-0.90}{0.90}\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^{\,4} = J_3$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(450\ \text{K})^4 = J_3$$

We now apply Eq. 13-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (9\ \text{m}^2)\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11{,}736\ \text{W/m}^2, \quad J_2 = 41{,}985\ \text{W/m}^2, \quad J_3 = 2325\ \text{W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3\ \text{m})^2 = 9\ \text{m}^2$$

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (9\ \text{m}^2)(0.20)(41{,}985 - 11{,}736)\text{W/m}^2 = \mathbf{54.4\,kW}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9\ \text{m}^2) = 36\ \text{m}^2$$

$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (9\ \text{m}^2)(0.8)(41{,}985 - 2325)\text{W/m}^2 = \mathbf{285.6\ kW}$$

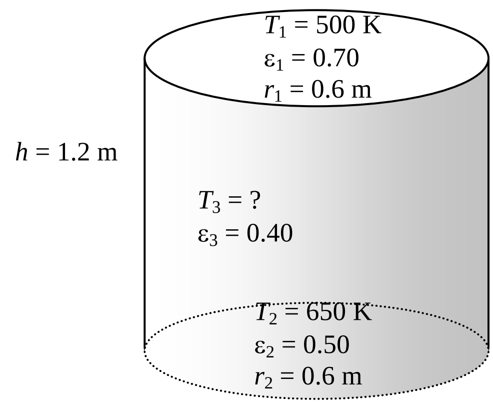
Discussion The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

13-141 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the top, bottom, and side surfaces are 0.70, 0.50, and 0.40, respectively.

Analysis We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from



$$\left.\begin{aligned}\frac{L}{r} &= \frac{1.2}{0.6} = 2\\ \frac{r}{L} &= \frac{0.6}{1.2} = 0.5\end{aligned}\right\} F_{12} = 0.17 \quad \text{(Fig. 13-7)}$$

The surface areas are

$$A_1 = A_2 = \pi D^2/4 = \pi(1.2\text{ m})^2/4 = 1.131\text{ m}^2$$

$$A_3 = \pi DL = \pi(1.2\text{ m})(1.2\text{ m}) = 4.524\text{ m}^2$$

Then other view factors are determined to be

$$F_{12} = F_{21} = 0.17$$

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + 0.17 + F_{13} = 1 \longrightarrow F_{13} = 0.83 \quad \text{(summation rule)}, \quad F_{23} = F_{13} = 0.83$$

$$A_1F_{13} = A_3F_{31} \longrightarrow (1.131)(0.83) = (4.524)F_{31} \longrightarrow F_{31} = 0.21 \quad \text{(reciprocity rule)}, \quad F_{32} = F_{31} = 0.21$$

We now apply Eq. 13-35 to each surface

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_1 + \frac{1-0.70}{0.70}\left[0.17(J_1 - J_2) + 0.83(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4 = J_2 + \frac{1-0.50}{0.50}\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3 - J_1) + F_{32}(J_3 - J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)T_3^4 = J_3 + \frac{1-0.40}{0.40}\left[0.21(J_3 - J_1) + 0.21(J_3 - J_2)\right]$$

We now apply Eq. 13-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (1.131\text{ m}^2)\left[0.17(J_2 - J_1) + 0.83(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$T_3 = \mathbf{631\ K}, \quad J_1 = 4974\text{ W/m}^2, \quad J_2 = 8883\text{ W/m}^2, \quad J_3 = 8193\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21} = A_2F_{21}(J_2 - J_1) = (1.131\text{ m}^2)(0.17)(8883 - 4974)\text{W/m}^2 = \mathbf{751.6\ W}$$

The rate of heat transfer between the bottom and the side surface is

$$\dot{Q}_{23} = A_2F_{23}(J_2 - J_3) = (1.131\text{ m}^2)(0.83)(8883 - 8193)\text{W/m}^2 = \mathbf{648.0\ W}$$

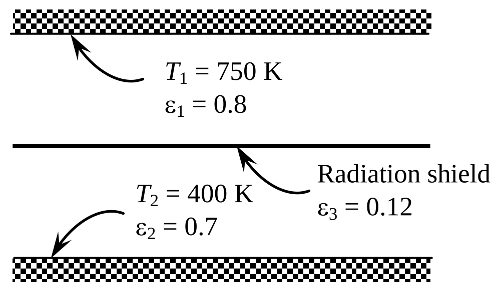
Discussion The sum of these two heat transfer rates are 751.6 + 644 = 1395.6 W, which is practically equal to 1400 W heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

13-142 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates and the temperature of the radiation shield are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.7$, and $\varepsilon_3 = 0.12$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is



$$\dot{Q}_{12,\text{oneshield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - (400\ \text{K})^4]}{\left(\dfrac{1}{0.8} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.12} + \dfrac{1}{0.12} - 1\right)}$$

$$= \mathbf{951\ W/m^2}$$

The equilibrium temperature of the radiation shield is determined from

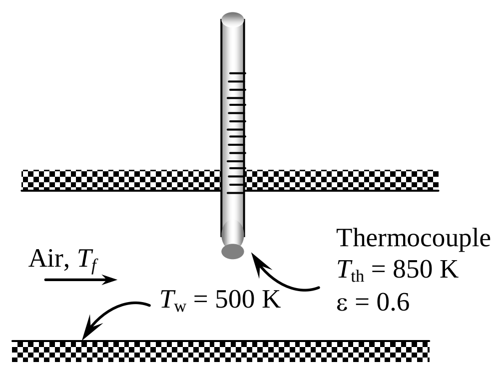
$$\dot{Q}_{13} = \frac{\sigma(T_1^{\,4} - T_3^{\,4})}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)} \longrightarrow 951\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - T_3^{\,4}]}{\left(\dfrac{1}{0.8} + \dfrac{1}{0.12} - 1\right)} \longrightarrow T_3 = \mathbf{644\ K}$$

13-143 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of thermocouple is given to be ε=0.6.

Analysis The actual temperature of the air can be determined from



$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^{\,4} - T_w^{\,4})}{h}$$

$$= 850\ \text{K} + \frac{(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(850\ \text{K})^4 - (500\ \text{K})^4]}{60\ \text{W/m}^2\cdot{}^\circ\text{C}}$$

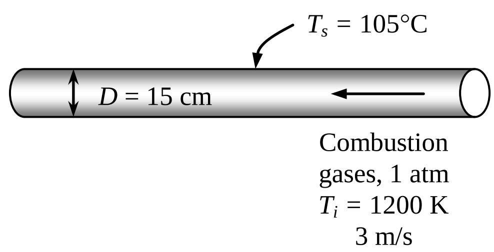
$$= \mathbf{1111 K}$$

13-144 Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

Properties The properties of air at 1200 K = 927°C and 1 atm are (Table A-15)

$$\rho = 0.2944\ \text{kg/m}^3$$
$$k = 0.07574\ \text{W/m.°C}$$
$$\nu = 1.586\times 10^{-4}\ \text{m}^2/\text{s}$$
$$c_p = 1173\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7221$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(3\ \text{m/s})(0.15\ \text{m})}{1.586\times 10^{-4}\ \text{m}^2/\text{s}} = 2837$$

which is a little higher than 2300, and thus we assume laminar flow. The Nusselt number in this case is

$$Nu = \frac{hD_h}{k} = 3.66$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.07574\ \text{W/m.°C}}{0.15\ \text{m}}(3.66) = 1.848\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho V A_c = (0.2944\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01561\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 105 - (105 - 927)e^{-\frac{(1.848)(2.827)}{(0.01561)(1173)}} = 723.0\text{°C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}c_p(T_i - T_e) = (0.01561\ \text{kg/s})(1173\ \text{J/kg.°C})(927 - 723)\text{°C} = \mathbf{3735\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 13-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,

$$P_c L = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_w L = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the average gas temperature of T_g=(T_g+T_g)/2 = (927+723)/2 = 825°C = 1098 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.045$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1100 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.075 = 0.112 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.16}{0.16 + 0.08} = 0.67 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.055 + 1\times 0.045 - 0.0 = 0.100$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 105°C = 378 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\ \text{atm})(0.1425\ \text{m})\frac{378\ \text{K}}{1098\ \text{K}} = 0.00392\ \text{m}\cdot\text{atm} = 0.013\ \text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\ \text{atm})(0.1425\ \text{m})\frac{378\ \text{K}}{1098\ \text{K}} = 0.00785\ \text{m}\cdot\text{atm} = 0.026\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 378 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.032 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.049$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1098\ \text{K}}{378\ \text{K}}\right)^{0.65}(0.032) = 0.0640$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1098\ \text{K}}{378\ \text{K}}\right)^{0.45}(0.049) = 0.0792$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 378 K instead of T_g = 1098 K. We use the chart for 400 K. At $P_w/(P_w + P_c)$ = 0.67 and $P_cL + P_wL$ = 0.112 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0640 + 0.0792 - 0.0 = 0.143$$

The emissivity of the inner surfaces of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= \frac{0.9+1}{2}(2.827\ \text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.100(1098\ \text{K})^4 - 0.143(378\ \text{K})^4]\\ &= \mathbf{21{,}690\ W}\end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

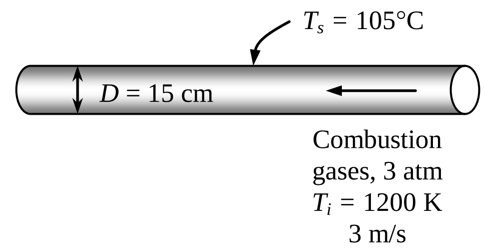
$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = \dot{m}_{\text{evap}} h_{fg} \longrightarrow \dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}}{h_{fg}} = \frac{(3735 + 21{,}690)\ \text{W}}{2257\times 10^3\ \text{J/kg}} = \mathbf{0.0113\ kg/s}$$

13-145 Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

Properties The properties of air at 1200 K = 927°C and 3 atm are (Table A-15)

$$\rho = 0.2944\ \text{kg/m}^3$$
$$k = 0.07574\ \text{W/m.°C}$$
$$\nu = (1.586\times10^{-4}\ \text{m}^2/\text{s})/3 = 0.5287\times10^{-4}\ \text{m}^2/\text{s}$$
$$c_p = 1173\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7221$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(3\ \text{m/s})(0.15\ \text{m})}{0.5287\times10^{-4}\ \text{m}^2/\text{s}} = 8511$$

which is greater than 2300 and close to 10,000. We assume the flow to be turbulent. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\ \text{Pr}^{0.3} = 0.023(8511)^{0.8}(0.7221)^{0.3} = 29.06$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.07574\ \text{W/m.°C}}{0.15\ \text{m}}(29.06) = 14.67\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho VA_c = (0.2944\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01561\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 105 - (105 - 927)e^{-\frac{(14.67)(2.827)}{(0.01561)(1173)}} = 190.4\text{°C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}c_p(T_i - T_e) = (0.01561\ \text{kg/s})(1173\ \text{J/kg.°C})(927 - 190.4)\text{°C} = \mathbf{13{,}490\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 13-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,

$$P_cL = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_wL = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the average gas temperature of T_g=(T_g+T_g)/2 = (927+190)/2 = 559°C = 832 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.062$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that (P_w+P)/2 = (0.16+3)/2 = 1.58 atm, the pressure correction factors are, from Fig. 13-37,

$C_c = 1.5$ and $C_w = 1.8$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g = 832$ K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.075 = 0.112 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.16}{0.16 + 0.08} = 0.67 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\mathrm{atm}} + C_w \varepsilon_{w,1\,\mathrm{atm}} - \Delta\varepsilon = 1.5 \times 0.055 + 1.8 \times 0.062 - 0.0 = 0.194$$

For a source temperature of $T_s = 105°\mathrm{C} = 378$ K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\,\mathrm{atm})(0.1425\,\mathrm{m})\frac{378\,\mathrm{K}}{832\,\mathrm{K}} = 0.00518\,\mathrm{m \cdot atm} = 0.017\,\mathrm{ft \cdot atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\,\mathrm{atm})(0.1425\,\mathrm{m})\frac{378\,\mathrm{K}}{832\,\mathrm{K}} = 0.0104\,\mathrm{m \cdot atm} = 0.034\,\mathrm{ft \cdot atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of $T_s = 378$ K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\mathrm{atm}} = 0.037 \quad \text{and} \quad \varepsilon_{w,1\,\mathrm{atm}} = 0.062$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\mathrm{atm}} = (1.5)\left(\frac{832\,\mathrm{K}}{378\,\mathrm{K}}\right)^{0.65}(0.037) = 0.0927$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\mathrm{atm}} = (1.8)\left(\frac{832\,\mathrm{K}}{378\,\mathrm{K}}\right)^{0.45}(0.062) = 0.1592$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s = 378$ K instead of $T_g = 832$ K. We use the chart for 400 K. At $P_w/(P_w + P_c) = 0.67$ and $P_c L + P_w L = 0.112$ we read $\Delta\varepsilon = 0.0$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0927 + 0.1592 - 0.0 = 0.252$$

The emissivity of the inner surfaces of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned} \dot{Q}_{\mathrm{rad}} &= \frac{\varepsilon_s + 1}{2} A_s \sigma (\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= \frac{0.9 + 1}{2}(2.827\,\mathrm{m}^2)(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})[0.194(832\,\mathrm{K})^4 - 0.252(378\,\mathrm{K})^4] \\ &= \mathbf{13{,}370\,W} \end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

$$\dot{Q}_{\mathrm{conv}} + \dot{Q}_{\mathrm{rad}} = \dot{m}_{\mathrm{evap}} h_{fg} \longrightarrow \dot{m}_{\mathrm{evap}} = \frac{\dot{Q}_{\mathrm{conv}} + \dot{Q}_{\mathrm{rad}}}{h_{fg}} = \frac{(13{,}490 + 13{,}370)\ \mathrm{W}}{2257 \times 10^3\ \mathrm{J/kg}} = \mathbf{0.0119\,kg/s}$$

Fundamentals of Engineering (FE) Exam Problems

13-146 Consider an infinitely long three-sided triangular enclosure with side lengths 2 cm, 3, cm, and 4 cm. The view factor from the 2 cm side to the 4 cm side is

(a) 0.25 (b) 0.50 (c) 0.64 (d) 0.75 (e) 0.87

Answer (d) 0.75

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
w1=2 [cm]
w2=3 [cm]
w3=4 [cm]
F_13=(w1+w3-w2)/(2*w1) "from Table 13-2"

"Some Wrong Solutions with Common Mistakes"
W_F_13=(w1+w2-w3)/(2*w1) "Using incorrect form of the equation"
```

13-147 Consider a 15-cm-diameter sphere placed within a cubical enclosure with a side length of 15 cm. The view factor from any of the square cube surface to the sphere is

(a) 0.09 (b) 0.26 (c) 0.52 (d) 0.78 (e) 1

Answer (c) 0.52

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.15 [m]
s=0.15 [m]
A1=pi*D^2
A2=6*s^2
F_12=1
A1*F_12=A2*F_21 "Reciprocity relation"

"Some Wrong Solutions with Common Mistakes"
W_F_21=F_21/6 "Dividing the result by 6"
```

13-148 A 70-cm-diameter flat black disk is placed in the center of the top surface of a 1 m × 1 m × 1 m black box. The view factor from the entire interior surface of the box to the interior surface of the disk is

(a) 0.077 (b) 0.144 (c) 0.356 (d) 0.220 (e) 1.0

Answer (a) 0.077

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.70 [m]
A1=pi*d^2/4 [m^2]
A2=5*1*1 [m^2]
F12=1
F21=A1*F12/A2
```

13-149 Consider two concentric spheres with diameters 12 cm and 18 cm, forming an enclosure. The view factor from the inner surface of the outer sphere to the inner sphere is

(a) 0 (b) 0.18 (c) 0.44 (d) 0.56 (e) 0.67

Answer (c) 0.44

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.12 [m]
D2=0.18 [m]
A1=pi*D1^2
A2=pi*D2^2
F_12=1
A1*F_12=A2*F_21 "Reciprocity relation"

"Some Wrong Solutions with Common Mistakes"
W1_F_21=F_12 "Using F_12 as the answer"
D1*F_12=D2*W2_F_21 "Using diameters instead of areas"
W3_F_21=1-F_21 "Evaluation of F_22"
```

13-150 The number of view factors that need to be evaluated directly for a 10-surface enclosure is

(a) 1 (b) 10 (c) 22 (d) 34 (e) 45

Answer (e) 45

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
N=10
n_viewfactors=1/2*N*(N-1)
```

13-151 Consider two concentric spheres forming an enclosure with diameters 12 and 18 cm and surface temperatures 300 and 500 K, respectively. Assuming that the surfaces are black, the net radiation exchange between the two spheres is

(a) 21 W (b) 140 W (c) 160 W (d) 1275 W (e) 3084 W

Answer (b) 140 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.12 [m]
D2=0.18 [m]
T1=300 [K]
T2=500 [K]
sigma=5.67E-8 [W/m^2-K^4]
A1=pi*D1^2
F_12=1
Q_dot=A1*F_12*sigma*(T2^4-T1^4)

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=F_12*sigma*(T2^4-T1^4) "Ignoring surface area"
W2_Q_dot=A1*F_12*sigma*T1^4 "Emissive power of inner surface"
W3_Q_dot=A1*F_12*sigma*T2^4 "Emissive power of outer surface"
```

13-152 Consider a vertical 2-m-diameter cylindrical furnace whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at 400 K, 600 K, and 900 K, respectively. If the view factor from the base surface to the top surface is 0.2, the net radiation heat transfer between the base and the side surfaces is

(a) 22.5 kW (b) 38.6 kW (c) 60.7 kW (d) 89.8 kW (e) 151 kW

Answer (d) 89.8 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=2 [m]
T1=400 [K]
T2=600 [K]
T3=900 [K]
F_12=0.2
A1=pi*D^2/4
A2=A1
F_13=1-F_12
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_13=A1*F_13*sigma*(T1^4-T3^4)

"Some Wrong Solutions with Common Mistakes"
W_Q_dot_13=A1*F_12*sigma*(T1^4-T3^4) "Using the view factor between the base and top surfaces"
```

13-153 Consider a vertical 2-m-diameter cylindrical furnace whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at 400 K, 600 K, and 900 K, respectively. If the view factor from the base surface to the top surface is 0.2, the net radiation heat transfer from the bottom surface is

(a) -93.6 kW (b) -86.1 kW (c) 0 kW (d) 86.1 kW (e) 93.6 kW

Answer (a) -93.6 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=2 [m]
T1=400 [K]
T2=600 [K]
T3=900 [K]
A1=pi*D^2/4
A2=A1
F_12=0.2
F_13=1-F_12
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12=A1*F_13*sigma*(T1^4-T2^4)
Q_dot_13=A1*F_13*sigma*(T1^4-T3^4)
Q_dot_1=Q_dot_12+Q_dot_13

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot_1=-Q_dot_1 "Using wrong sign"
W2_Q_dot_1=Q_dot_12-Q_dot_13 "Substracting heat transfer terms"
W3_Q_dot_1=Q_dot_13-Q_dot_12 "Substracting heat transfer terms"
```

13-154 A solar flux of 1400 W/m^2 directly strikes a space vehicle surface which has a solar absortivity of 0.4 and thermal emissivity of 0.6. The equilibrium temperature of this surface in space at 0 K is

(a) 300 K (b) 360 K (c) 410 K (d) 467 K (e) 510 K

Answer (b) 360 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
a=0.4
e=0.6
Q=1400 [W/m^2]
a*Q=e*sigma#*T^4
```

13-155 A 70-cm-diameter flat black disk is placed at the center of the ceiling a 1 m × 1 m × 1 m black box. If the temperature of the box is 620°C and the temperature of the disk is 27°C, the rate of heat transfer by radiation between the interior of the box and the disk is

(a) 2 kW (b) 5 kW (c) 8 kW (d) 11 kW (e) 14 kW

Answer (e) 14 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.7 [m]
A1=pi*d^2/4 [m^2]
A2=5*1*1 [m^2]
F12=1
T2=893 [K]
T1=300 [K]
F21=A1*F12/A2
Q=A2*F21*sigma#*(T2^4-T1^4)
```

13-156 Consider two infinitely long concentric cylinders with diameters 20 and 25 cm. The inner surface is maintained at 700 K and has an emissivity of 0.40 while the outer surface is black. If the rate of radiation heat transfer from the inner surface to the outer surface is 2400 W per unit area of the inner surface, the temperature of the outer surface is

(a) 605 K (b) 538 K (c) 517 K (d) 451 K (e) 415 K

Answer (a) 605 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.20 [m]
D2=0.25 [m]
T1=700 [K]
epsilon_1=0.40
epsilon_2=1
Q_dot_12=2400 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12=epsilon_1*sigma*(T1^4-T2^4)

"Some Wrong Solutions with Common Mistakes"
Q_dot_12=(sigma*(T1^4-W1_T2^4))/((1/epsilon_1)+(1-epsilon_1)/epsilon_1*D1/D2) "Incorrect equation"
A1=pi*D1*1[m] "Finding the area for a unit length of the inner cylinder"
Q_dot_12=A1*epsilon_1*sigma*(T1^4-W2_T2^4)
```

13-157 Consider a surface at 0ºC that may be assumed to be a blackbody in an environment at 25ºC. If 300 W/m^2 radiation is incident on the surface, the radiosity of this black surface is

(a) 0 W/m^2 (b) 15 W/m^2 (c) 132 W/m^2 (d) 300 W/m^2 (e) 315 W/m^2

Answer (e) 315 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=0 [C]
T_infinity=25 [C]
G=300 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
J=sigma*(T+273)^4 "J=E_b for a blackbody"

"Some Wrong Solutions with Common Mistakes"
W1_J=sigma*T^4 "Using C unit for temperature"
W2_J=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
W3_J=G "Using the incident radiation as the answer"
W4_J=J-G "Finding the difference between the emissive power and incident radiation"
```

13-158 Consider a gray and opaque surface at 0ºC in an environment at 25ºC. The surface has an emissivity of 0.8. If the radiation incident on the surface is 240 W/m^2, the radiosity of the surface is

(a) 38 W/m^2 (b) 132 W/m^2 (c) 240 W/m^2 (d) 300 W/m^2 (e) 315 W/m^2

Answer (d) 300 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=0 [C]
T_infinity=25 [C]
epsilon=0.80
G=240 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
J=epsilon*sigma*(T+273)^4+(1-epsilon)*G

"Some Wrong Solutions with Common Mistakes"
W1_J=sigma*(T+273)^4 "Radiosity for a black surface"
W2_J=epsilon*sigma*T^4+(1-epsilon)*G "Using C unit for temperature"
W3_J=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
W4_J=G "Using the incident radiation as the answer"
```

13-159 Consider a 3-m × 3-m × 3-m cubical furnace. The base surface is black and has a temperature of 400 K. The radiosities for the top and side surfaces are calculated to be 7500 W/m^2 and 3200 W/m^2, respectively. If the temperature of the side surfaces is 485 K, the emissivity of the side surfaces is

(a) 0.37 (b) 0.55 (c) 0.63 (d) 0.80 (e) 0.89

Answer (e) 0.89

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=3 [m]
T1=400 [K]
epsilon_1=1
J2=7500 [W/m^2]
J3=3200 [W/m^2]
T3=485 [K]
sigma=5.67E-8 [W/m^2-K^4]
F_31=0.2
F_32=F_31
J1=sigma*T1^4
sigma*T3^4=J3+(1-epsilon_3)/epsilon_3*(F_31*(J3-J1)+F_32*(J3-J2))
```

13-160 The base surface of a cubical furnace with a side length of 3 m has an emissivity of 0.80 and is maintained at 500 K. If the top and side surfaces also have an emissivity of 0.80 and are maintained at 900 K, the net rate of radiation heat transfer from the top and side surfaces to the bottom surface is

(a) 194 kW (b) 233 kW (c) 288 kW (d) 312 kW (e) 242 kW

Answer (b) 233 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Sigma=5.67
T1=500
T2=900
A1=3*3
A2=5*A1
Eps1=0.8
Eps2=0.8
F12=1
Q=sigma*((T1/100)^4-(T2/100)^4)/(((1-Eps1)/(A1*Eps1)+1/(A1*F12)+(1-Eps2)/(A2*Eps2)))

"Some Wrong Solutions with Common Mistakes:"
W1_Q=A1*Eps1*sigma*((T1/100)^4-(T2/100)^4)
```

13-161 Two grey surfaces that form an enclosure exchange heat with one another by thermal radiation. Surface 1 has a temperature of 400 K, an area of 0.2 m^2, and a total emissivity of 0.4. Surface 2 has a temperature of 600 K, an area of 0.3 m^2, and a total emissivity of 0.6. If the view factor F_{12} is 0.3, the rate of radiation heat transfer between the two surfaces is

(a) 135 W (b) 223 W (c) 296 W (d) 342 W (e) 422 W

Answer (b) 223 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A1=0.2 [m^2]
T1=400 [K]
e1=0.4
A2=0.3 [m^2]
T2=600 [K]
e2=0.6
F12=0.3
R1=(1-e1)/(A1*e1)
R2=1/(A1*F12)
R3=(1-e2)/(A2*e2)
Q=sigma#*(T2^4-T1^4)/(R1+R2+R3)
```

13-162 The surfaces of a two-surface enclosure exchange heat with one another by thermal radiation. Surface 1 has a temperature of 400 K, an area of 0.2 m^2, and a total emissivity of 0.4. Surface 2 is black, has a temperature of 600 K, and an area of 0.3 m^2. If the view factor F_{12} is 0.3, the rate of radiation heat transfer between the two surfaces is

(a) 87 W (b) 135 W (c) 244 W (d) 342 W (e) 386 W

Answer (c) 244 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A1=0.2 [m^2]
T1=400 [K]
e1=0.4
A2=0.3 [m^2]
T2=600 [K]
F12=0.3
R1=(1-e1)/(A1*e1)
R2=1/(A1*F12)
Q=sigma#*(T2^4-T1^4)/(R1+R2)
```

13-163 Two concentric spheres are maintained at uniform temperatures T_1 = 45ºC and T_2 = 280ºC and have emissivities ε_1 = 0.25 and ε_2 = 0.7, respectively. If the ratio of the diameters is D_1/D_2 = 0.30, the net rate of radiation heat transfer between the two spheres per unit surface area of the inner sphere is

(a) 86 W/m^2 (b) 1169 W/m^2 (c) 1181 W/m^2 (d) 2510 W/m^2 (e) 3306 W/m^2

Answer (b) 1169 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=45 [C]
T2=280 [C]
epsilon_1=0.25
epsilon_2=0.70
D1\D2=0.30
sigma=5.67E-8 [W/m^2-K^4]
Q_dot=((sigma*((T2+273)^4-(T1+273)^4)))/((1/epsilon_1)+(1-epsilon_2)/epsilon_2*D1\D2^2)

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=((sigma*(T2^4-T1^4)))/((1/epsilon_1)+(1-epsilon_2)/epsilon_2*D1\D2^2) "Using C unit for temperature"
W2_Q_dot=epsilon_1*sigma*((T2+273)^4-(T1+273)^4) "The equation when the outer sphere is black"
W3_Q_dot=epsilon_2*sigma*((T2+273)^4-(T1+273)^4) "The equation when the inner sphere is black"
```

13-164 Consider a 3-m × 3-m × 3-m cubical furnace. The base surface of the furnace is black and has a temperature of 400 K. The radiosities for the top and side surfaces are calculated to be 7500 W/m^2 and 3200 W/m^2, respectively. The net rate of radiation heat transfer to the bottom surface is

(a) 2.61 kW (b) 8.27 kW (c) 14.7 kW (d) 23.5 kW (e) 141 kW

Answer (d) 23.5 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=3 [m]
T1=400 [K]
epsilon_1=1
J2=7500 [W/m^2]
J3=3200 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
A1=s^2
F_12=0.2
F_13=0.8
J1=sigma*T1^4
Q_dot_1=A1*(F_12*(J1-J2)+F_13*(J1-J3))

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot_1=(F_12*(J1-J2)+F_13*(J1-J3)) "Not multiplying with area"
W2_A1=6*s^2 "Using total area"
W2_Q_dot_1=W2_A1*(F_12*(J1-J2)+F_13*(J1-J3))
```

13-165 Two very large parallel plates are maintained at uniform temperatures T_1 = 750 K and T_2 = 500 K and have emissivities ε_1 = 0.85 and ε_2 = 0.7, respectively. If a thin aluminum sheet with the same emissivity on both sides is to be placed between the plates in order to reduce the net rate of radiation heat transfer between the plates by 90 percent, the emissivity of the aluminum sheet must be

(a) 0.07 (b) 0.10 (c) 0.13 (d) 0.16 (e) 0.19

Answer (c) 0.13

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=750 [K]
T2=500 [K]
epsilon_1=0.85
epsilon_2=0.70
f=0.9
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_noshield=(sigma*(T1^4-T2^4))/((1/epsilon_1)+(1/epsilon_2)-1)
Q_dot_1shield=(1-f)*Q_dot_noshield
Q_dot_1shield=(sigma*(T1^4-T2^4))/((1/epsilon_1)+(1/epsilon_2)-1+(1/epsilon_3)+(1/epsilon_3)-1)
```

13-166 ….. 13-168 Design and Essay Problems