Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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Chapter 12
FUNDAMENTALS OF THERMAL RADIATION

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Electromagnetic and Thermal Radiation

12-1C Electromagnetic waves are caused by accelerated charges or changing electric currents giving rise to electric and magnetic fields. Sound waves are caused by disturbances. Electromagnetic waves can travel in vacuum, sound waves cannot.

12-2C Electromagnetic waves are characterized by their frequency v and wavelength λ. These two properties in a medium are related by $\lambda = c / v$ where c is the speed of light in that medium.

12-3C Thermal radiation is the radiation emitted as a result of vibrational and rotational motions of molecules, atoms and electrons of a substance, and it extends from about 0.1 to 100 μm in wavelength. Unlike the other forms of electromagnetic radiation, thermal radiation is emitted by bodies because of their temperature.

12-4C Microwaves in the range of 10^2 to10^5 μm are very suitable for use in cooking as they are reflected by metals, transmitted by glass and plastics and absorbed by food (especially water) molecules. Thus the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food with no conduction and convection thermal resistances involved. In conventional cooking, on the other hand, conduction and convection thermal resistances slow down the heat transfer, and thus the heating process.

12-5C Visible light is a kind of electromagnetic wave whose wavelength is between 0.40 and 0.76 μm. It differs from the other forms of electromagnetic radiation in that it triggers the sensation of seeing in the human eye.

12-6C Light (or visible) radiation consists of narrow bands of colors from violet to red. The color of a surface depends on its ability to reflect certain wavelength. For example, a surface that reflects radiation in the wavelength range 0.63-0.76 μm while absorbing the rest appears red to the eye. A surface that reflects all the light appears white while a surface that absorbs the entire light incident on it appears black. The color of a surface at room temperature is not related to the radiation it emits.

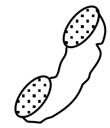
12-7C Because the snow reflects almost all of the visible and ultraviolet radiation, and the skin is exposed to radiation both from the sun and from the snow.

12-8C Infrared radiation lies between 0.76 and 100 μm whereas ultraviolet radiation lies between the wavelengths 0.01 and 0.40 μm. The human body does not emit any radiation in the ultraviolet region since bodies at room temperature emit radiation in the infrared region only.

12-9C Radiation in opaque solids is considered surface phenomena since only radiation emitted by the molecules in a very thin layer of a body at the surface can escape the solid.

12-10 A cordless telephone operates at a frequency of 8.5×10^8 Hz. The wavelength of these telephone waves is to be determined.

Analysis The wavelength of the telephone waves is

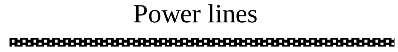


$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{8.5\times10^8 \text{ Hz(1/s)}} = 0.353\,\text{m} = \mathbf{353\,mm}$$

12-11 Electricity is generated and transmitted in power lines at a frequency of 60 Hz. The wavelength of the electromagnetic waves is to be determined.

Analysis The wavelength of the electromagnetic waves is

Power lines



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{60\,\text{Hz(1/s)}} = \mathbf{4.997\times10^6\ m}$$

12-12 The speeds of light in air, water, and glass are to be determined.

Analysis The speeds of light in air, water and glass are

Air: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1} = \mathbf{3.0\times10^8\ m/s}$$

Water: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1.33} = \mathbf{2.26\times10^8\ m/s}$$

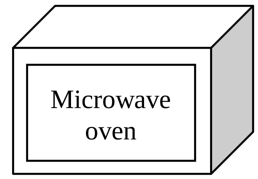
Glass: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1.5} = \mathbf{2.0\times10^8\ m/s}$$

12-13 A radio station is broadcasting radiowaves at a wavelength of 200 m. The frequency of these waves is to be determined.

Analysis The frequency of the waves is determined from

$$\lambda = \frac{c}{v} \longrightarrow v = \frac{c}{\lambda} = \frac{2.998\times10^8 \text{ m/s}}{200\,\text{m}} = \mathbf{1.499\times10^6\ Hz}$$

12-14 A microwave oven operates at a frequency of 2.2×10^9 Hz. The wavelength of these microwaves and the energy of each microwave are to be determined.



Analysis The wavelength of these microwaves is

$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{2.2\times10^9 \text{ Hz(1/s)}} = 0.136 \text{ m} = \mathbf{136\,mm}$$

Then the energy of each microwave becomes

$$e = hv = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34} \text{ Js})(2.998\times10^8 \text{ m/s})}{0.136 \text{ m}} = \mathbf{1.46\times10^{-24}\ J}$$

12-15 The photon energies of a radio wave and a γ-ray, and the photon energy ratio of the γ-ray to the radio wave are to be determined.

Assumptions **1** The medium is air and index of refraction is unity.

Properties The speed of light in a medium with a refraction index of 1 is c = 2.9979 × 10^8 m/s. The Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda}$$

The photon energy of the radio wave is

$$e_{\text{radio}} = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34} \text{ J}\cdot\text{s})(2.9979\times10^8 \text{ m/s})}{10^7 \ \mu\text{m}} = \mathbf{1.986\times10^{-26}\ J}$$

The photon energy of the γ-ray is

$$e_{\gamma\text{-ray}} = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34} \text{ J}\cdot\text{s})(2.9979\times10^8 \text{ m/s})}{10^{-7} \ \mu\text{m}} = \mathbf{1.986\times10^{-12}\ J}$$

The photon energy ratio of the γ-ray to the radio wave is

$$\frac{e_{\gamma\text{-ray}}}{e_{\text{radio}}} = \frac{\lambda_{\text{radio}}}{\lambda_{\gamma\text{-ray}}} = \frac{10^7 \ \mu\text{m}}{10^{-7} \ \mu\text{m}} = \mathbf{10^{14}}$$

Discussion There is 10^{14} times more energy in a γ-ray wave than a radio wave.

12-16 The photon energies of an electromagnetic wave in air, water, and glass are to be determined.

Assumptions **1** The refraction index of each medium is uniform.

Properties The speed of light in a vacuum is c_0 = 2.9979 × 10^8 m/s, and the Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda} = \frac{hc_0}{\lambda n}$$

Thus,

Air: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069\times10^{-34}\text{ J}\cdot\text{s})(2.9979\times10^{8}\text{ m/s})}{(0.5\times10^{-6}\text{ m})(1.0)}\left(\frac{1\,\text{eV}}{1.6022\times10^{-19}\text{ J}}\right) = \mathbf{2.48\ eV}$$

Water: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069\times10^{-34}\text{ J}\cdot\text{s})(2.9979\times10^{8}\text{ m/s})}{(0.5\times10^{-6}\text{ m})(1.33)}\left(\frac{1\,\text{eV}}{1.6022\times10^{-19}\text{ J}}\right) = \mathbf{1.86\ eV}$$

Glass: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069\times10^{-34}\text{ J}\cdot\text{s})(2.9979\times10^{8}\text{ m/s})}{(0.5\times10^{-6}\text{ m})(1.5)}\left(\frac{1\,\text{eV}}{1.6022\times10^{-19}\text{ J}}\right) = \mathbf{1.65\ eV}$$

Discussion Medium with higher index of refraction causes the speed of wave propagation to decrease. As the speed of the wave propagation decreases, so does the wave energy.

12-17 The photon energies of violet color and red color are to be determined.

Assumptions **1** The medium is air and index of refraction is unity.

Properties The speed of light in a medium with a refraction index of 1 is c = 2.9979 × 10^8 m/s. The Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda}$$

Thus, the photon energy of each color is

Violet: $$e = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34}\text{ J}\cdot\text{s})(2.9979\times10^{8}\text{ m/s})}{(0.40\times10^{-6}\text{ m})} = \mathbf{4.966\times10^{-19}\ J}$$

Red: $$e = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34}\text{ J}\cdot\text{s})(2.9979\times10^{8}\text{ m/s})}{(0.76\times10^{-6}\text{ m})} = \mathbf{2.614\times10^{-19}\ J}$$

Discussion Violet color propagates higher level (1.9 times higher) of photon energy than red color because of its shorter wavelength. An electromagnetic wave with a shorter wavelength means that it has higher frequency, and therefore higher photon energy.

Blackbody Radiation

12-18C A blackbody is a perfect emitter and absorber of radiation. A blackbody does not actually exist. It is an idealized body that emits the maximum amount of radiation that can be emitted by a surface at a given temperature.

12-19C *Spectral blackbody emissive power* is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area and per unit wavelength about wavelength λ. The integration of the spectral blackbody emissive power over the entire wavelength spectrum gives the *total blackbody emissive power*,

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4$$

The spectral blackbody emissive power varies with wavelength, the total blackbody emissive power does not.

12-20C We defined the blackbody radiation function f_λ because the integration $\int_0^\infty E_{b\lambda}(T)d\lambda$ cannot be performed. The blackbody radiation function f_λ represents the fraction of radiation emitted from a blackbody at temperature *T* in the wavelength range from $\lambda = 0$ to λ. This function is used to determine the fraction of radiation in a wavelength range between λ_1 and λ_2.

12-21C The larger the temperature of a body, the larger the fraction of the radiation emitted in shorter wavelengths. Therefore, the body at 1500 K will emit more radiation in the shorter wavelength region. The body at 1000 K emits more radiation at 20 μm than the body at 1500 K since $\lambda T = \text{constant}$.

12-22 The maximum thermal radiation that can be emitted by a surface is to be determined.

Analysis The maximum thermal radiation that can be emitted by a surface is determined from Stefan-Boltzman law to be

$$E_b(T) = \sigma T^4 = (5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(800\ \text{K})^4 = \mathbf{23{,}200\ W/m^2}$$

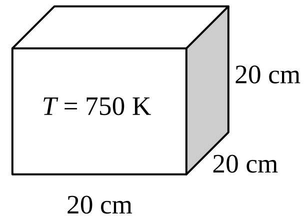
12-23 An isothermal cubical body is suspended in the air. The rate at which the cube emits radiation energy and the spectral blackbody emissive power are to be determined.

Assumptions The body behaves as a black body.

Analysis (*a*) The total blackbody emissive power is determined from Stefan-Boltzman Law to be

$$A_s = 6a^2 = 6(0.2^2) = 0.24\ \text{m}^2$$

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(750\ \text{K})^4(0.24\ \text{m}^2) = \mathbf{4306\ W}$$



(*b*) The spectral blackbody emissive power at a wavelength of 4 μm is determined from Plank's distribution law,

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = \frac{3.74177\times10^8\ \text{W}\cdot\mu\text{m}^4/\text{m}^2}{(4\ \mu\text{m})^5\left[\exp\left(\frac{1.43878\times10^4\ \mu\text{m}\cdot\text{K}}{(4\ \mu\text{m})(750\ \text{K})}\right)-1\right]}$$

$$= 3045\ \text{W/m}^2\cdot\mu\text{m}$$

$$= \mathbf{3.045\ kW/m^2\cdot\mu m}$$

12-24 The peak spectral blackbody emissive power for a match flame and moonlight is to be determined.

***Assumptions* 1** The match and the moon behave as black bodies.

Analysis Using the combination of Planck's law and Wien's displacement law, the peak spectral blackbody emissive power can be determined:

$$E_{b\lambda}(\lambda_1,\ T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T)-1]} = \frac{3.74177\times10^8}{\lambda^5[\exp(1.43878\times10^4/\lambda T)-1]}\ \text{W/m}^2\cdot\mu\text{m}$$

and

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

Then,

$$E_{b\lambda_{\text{max}}}(T) = \frac{3.74177\times10^8 T^5}{(2897.8)^5[\exp(1.43878\times10^4/2897.8)-1]}\ \text{W/m}^2\cdot\mu\text{m}$$

$$= 1.278\times10^{-11}T^5\ \text{W/m}^2\cdot\mu\text{m}$$

For a match flame (T = 1700 K), the peak spectral blackbody emissive power is

$$E_{b\lambda_{\text{max}}}(1700) = 1.278\times10^{-11}(1700)^5\ \text{W/m}^2\cdot\mu\text{m}$$

$$= \mathbf{1.81\times10^5\ W/m^2\cdot\mu m}$$

For moonlight (T = 4000 K), the peak spectral blackbody emissive power is

$$E_{b\lambda_{\text{max}}}(4000) = 1.278\times10^{-11}(4000)^5\ \text{W/m}^2\cdot\mu\text{m}$$

$$= \mathbf{1.31\times10^7\ W/m^2\cdot\mu m}$$

Discussion The peak spectral blackbody emissive power by moonlight is about 72 times higher than that by a match flame.

12-25 The blackbody temperature and the total emissive power at a given wavelength and its corresponding emissive power are to be determined.

Assumptions **1** Blackbody radiation.

Analysis (a) Using the Planck's law find the blackbody radiation

$$E_{b\lambda}(\lambda_1,\ T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T)-1]}$$

$$10^8\ \text{W/m}^3 = \frac{3.74177\times10^8\ \text{W}\cdot\mu\text{m}^4/\text{m}^2}{(0.7\times10^{-6}\ \mu\text{m})^5\left\{\exp[1.43878\times10^4\ (\mu\text{m}\cdot\text{K})/(0.7\times10^{-6}\ \mu\text{m})T\ (\text{K})]-1\right\}}$$

Solve for T

T = **1215 K**

(b) The blackbody total emitted energy at this temperature is

$$E_b(T) = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1215\ \text{K})^4 = \mathbf{123{,}563\ W/m^2}$$

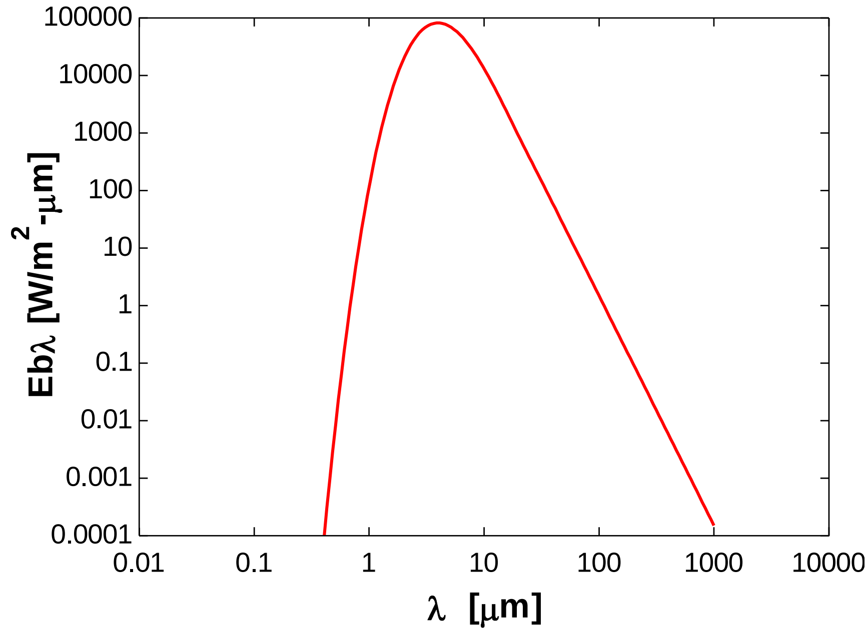
12-26 The spectral blackbody emissive power of the sun versus wavelength in the range of 0.01 μm to 1000 μm is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=5780 [K]
lambda=0.01[micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
```

λ [μm]	$E_{b,\lambda}$ [W/m^2-μm]
0.01	0
10.11	12684
20.21	846.3
30.31	170.8
40.41	54.63
50.51	22.52
60.62	10.91
70.72	5.905
80.82	3.469
90.92	2.17
…	…
…	…
909.1	0.0002198
919.2	0.0002103
929.3	0.0002013
939.4	0.0001928
949.5	0.0001847
959.6	0.000177
969.7	0.0001698
979.8	0.0001629
989.9	0.0001563
1000	0.0001501



12-27 A small body is placed inside an evacuated spherical chamber with constant surface temperature. The radiation incident on the small body surface is to be determined for (*a*) black chamber surface and (*b*) well-polished chamber surface.

Assumptions **1** The small body surface is much smaller than the chamber surface. **2** The chamber surface temperature is isothermal.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The spherical chamber with isothermal surface temperature forms a blackbody cavity regardless of the radiation properties of the chamber surface. The small body inside the chamber is too small to interfere with the blackbody nature of the cavity.

Therefore, the radiation incident on any part of the small body surface is equal to the radiation emitted by a blackbody at the surface temperature of the chamber.

(*a*) For chamber surface coated in black:

$$E_b = \sigma T_s^4$$

$$E_b = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(500 \text{ K})^4 = \mathbf{3544 \text{ W/m}^2}$$

(*b*) For a well-polished chamber surface, the radiation incident on the small body surface is **3544 W/m$^{\mathbf{2}}$**, the same as that of part (*a*), since it is independent of the radiation properties of the chamber surface.

Discussion The radiation incident on the small body surface depends only on the chamber surface temperature, and is independent of the conditions of the chamber surface.

The blackbody assumption is only valid when the surface area of the small body is much smaller than that of the chamber. This allows the radiation emitted by the chamber surface to go through multiple reflections and become diffuse.

12-28 A black ball is suspended in air. The surface temperature that is necessary to heat 10 kg of air from 20 to 30°C is to be determined.

Assumptions **1** The ball behaves as a blackbody.

Properties The specific heat of air at (20 + 30)°C / 2 = 25°C is c_v = 718 J/kg·K (Table A-1). The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis For a blackbody, the total emissive power is determined from the Stefan-Boltzmann law as

$$E_b = \sigma T_s^4$$

The heat energy release from the ball in the form of radiation is then

$$Q_{\text{rad}} = E_b A_s \Delta t = \sigma T_s^4 (\pi D^2)\Delta t$$

The energy required to heat 10 kg of air by ΔT = 10°C is

$$Q = mc_v \Delta T$$

Thus, $$mc_v \Delta T = \sigma T_s^4 (\pi D^2)\Delta t \quad \rightarrow \quad T_s = \left[\frac{mc_v \Delta T}{\sigma(\pi D^2)\Delta t}\right]^{1/4}$$

$$T_s = \left[\frac{(10\text{ kg})(718\text{ J/kg}\cdot\text{K})(10\text{ K})}{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\pi(0.25\text{ m})^2(5\times 60\text{ s})}\right]^{1/4} = 383\text{ K} = \mathbf{110°C}$$

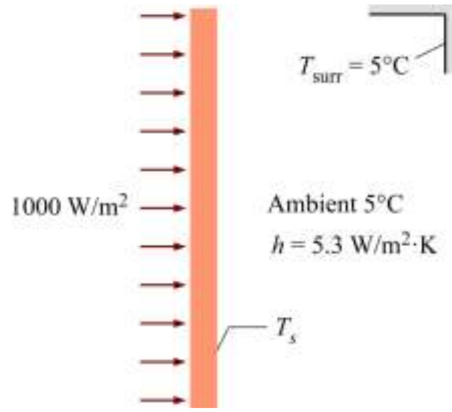
Discussion With a surface temperature of 110°C, the ball can release enough energy in the form of electromagnetic waves to heat 10 kg of the air by 10°C in 5 minutes.

12-29 A thin vertical plate, modeled as blackbody, is subjected to uniform heat flux on one side and exposed to radiation and natural convection on the other side. The plate surface temperature is to be determined.

Assumptions **1** The plate emits radiation as a blackbody. **2** Thermal properties are constant. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The heat emitted from the plate, as a blackbody, is



$$\dot{q}_{\text{emit}} = \sigma T_s^4$$

The radiation incident on the plate (blackbody) from the surroundings is

$$\dot{q}_{\text{incident}} = \sigma T_{\text{surr}}^4$$

The heat transfer from the plate to the surroundings by natural convection is

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty)$$

By performing energy balance on the plate we have

$$\dot{q} = \dot{q}_{\text{emit}} - \dot{q}_{\text{incident}} + \dot{q}_{\text{conv}}$$

$$\dot{q} = \sigma T_s^4 - \sigma T_{\text{surr}}^4 + h(T_s - T_\infty)$$

$$1000\ \text{W/m}^2 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(T_s^4 - 278^4)\ \text{K}^4 + (5.3\ \text{W/m}^2 \cdot \text{K})(T_s - 278)\ \text{K}$$

$$\rightarrow \quad T_s = 357\ \text{K} = \mathbf{84°C}$$

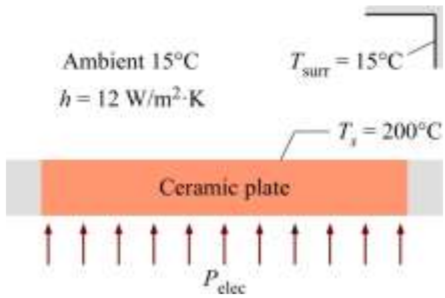
Discussion The net rate of radiation heat transfer between the plate and the surroundings is $\dot{q}_{\text{rad}} = \sigma(T_s^4 - T_{\text{surr}}^4) = 582\ \text{W/m}^2$, which is about 58% of the total heat loss from the plate. Natural convection only contributed about 42% of the total heat loss from the plate.

12-30 A circular plate that is modeled as a blackbody is heated by an electrical heater with an efficiency of 80%. The electric power required to keep the plate surface temperature at 200°C is to be determined.

Assumptions **1** The plate emits radiation as a blackbody. **2** Thermal properties are constant. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The heat emitted from the plate, as a blackbody, is



$$\dot{Q}_{emit} = \sigma A_s T_s^4 = \pi \frac{(0.30\text{ m})^4}{4}(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(473^4)\text{K}^4 = 18.06\text{ W}$$

The radiation incident on the plate (blackbody) from the surroundings is

$$\dot{Q}_{incident} = \sigma A_s T_{surr}^4 = \pi \frac{(0.30\text{ m})^4}{4}(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(288^4)\text{K}^4 = 2.48\text{ W}$$

The heat transfer from the plate to the surroundings by natural convection is

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = \pi \frac{(0.30\text{ m})^4}{4}(12\text{ W/m}^2\cdot\text{K})(200-15)\text{K} = 14.12\text{ W}$$

Thus, by performing energy balance on the plate we have

$$\eta P_{elec} = \dot{Q}_{emit} - \dot{Q}_{incident} + \dot{Q}_{conv}$$

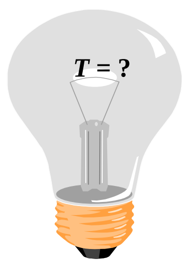
$$P_{elec} = \frac{\dot{Q}_{emit} - \dot{Q}_{incident} + \dot{Q}_{conv}}{\eta} = \frac{18.06 - 2.48 + 14.12}{0.80}\text{ W} = \mathbf{37.13\text{ W}}$$

Discussion The net rate of radiation heat transfer between the plate and the surroundings is $\dot{Q}_{rad} = \dot{Q}_{emit} - \dot{Q}_{incident} = \sigma A_s(T_s^4 - T_{surr}^4)$. Since the plate is treated as a blackbody, the emissivity is unity ($\varepsilon = 1$).

12-31 An incandescent light bulb emits 15% of its energy at wavelengths shorter than 0.8 μm. The temperature of the filament is to be determined.

Assumptions The filament behaves as a black body.

Analysis From the Table 12-2 for the fraction of the radiation, we read



$$f_\lambda = 0.15 \longrightarrow \lambda T = 2445\ \mu\text{mK}$$

For the wavelength range of $\lambda_1 = 0.0\ \mu\text{m}$ to $\lambda_2 = 0.8\ \mu\text{m}$

$$\lambda = 0.8\ \mu\text{m} \longrightarrow \lambda T = 2445\ \mu\text{mK} \longrightarrow T = \mathbf{3056\text{ K}}$$

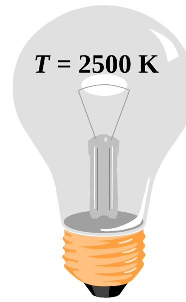
12-32 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 2500 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.053035$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = \mathbf{0.052714} \qquad \text{(or 5.2\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = \mathbf{1.16\ \mu m}$$

Discussion Note that the radiation emitted from the filament peaks in the infrared region.

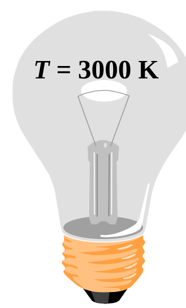
12-33 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 3000 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(3000\ \text{K}) = 1200\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.002134$$

$$\lambda_2 T = (0.76\ \mu\text{m})(3000\ \text{K}) = 2280\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.116635$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.116635 - 0.002134 = \mathbf{0.1145} \qquad \text{(or 11.5\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{3000\ \text{K}} = \mathbf{0.9659\ \mu m}$$

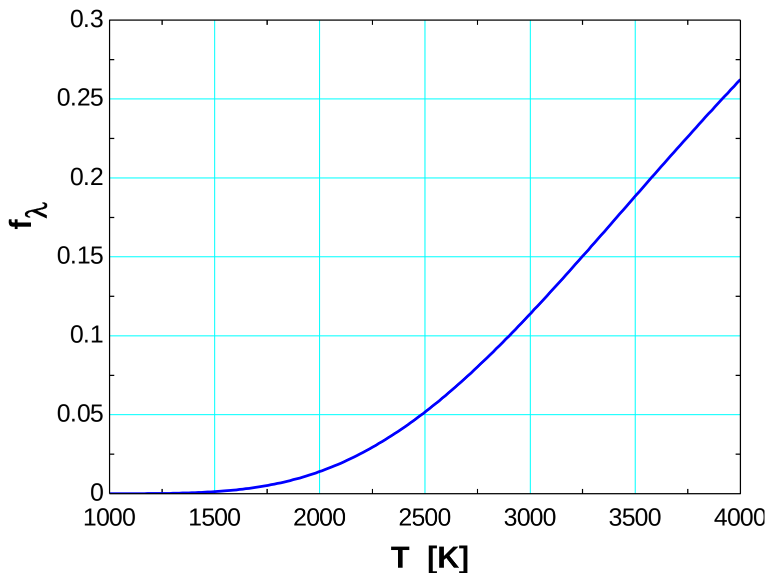
12-34 Prob. 12-33 is reconsidered. The effect of temperature on the fraction of radiation emitted in the visible range is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=3000 [K]
lambda_1=0.40 [micrometer]
lambda_2=0.76 [micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
f_lambda=integral(E_b_lambda, lambda, lambda_1, lambda_2)/E_b
E_b=sigma*T^4
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
```

T [K]	f_λ
1000	0.000007353
1200	0.0001032
1400	0.0006403
1600	0.002405
1800	0.006505
2000	0.01404
2200	0.02576
2400	0.04198
2600	0.06248
2800	0.08671
3000	0.1139
3200	0.143
3400	0.1732
3600	0.2036
3800	0.2336
4000	0.2623



12-35E The radiation energy emitted by the black surface per unit area (at 2060 °F) for $\lambda \geq 4$ µm is to be determined.

Assumptions **1** The surface behaves as a black body.

Analysis The blackbody radiation function corresponding to $\lambda_1 = 4$ µm is determined from Table 12-2 to be

$$\lambda_1 T = (4.0\ \mu\text{m})(2060+460)(1/1.8)\ \text{K} = 5600\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.701046$$

Then, the radiation energy emitted is determined using

$$\begin{aligned} E_{b,\ \lambda_1-\infty}(T) &= \int_{\lambda_1}^{\infty} E_{b\lambda}(\lambda, T) = \sigma T^4 f_{\lambda_1-\infty}(T) \\ &= \sigma T^4 [f_\infty(T) - f_{\lambda 1}(T)] \\ &= (0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(2520\ \text{R})^4(1-0.701046) \\ &= \mathbf{20{,}700\ Btu/h\cdot ft^2} \end{aligned}$$

Discussion The total radiation energy emitted by this black surface is simply

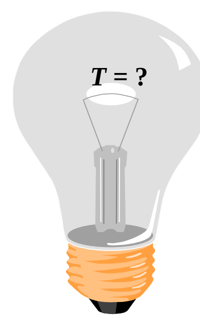
$$E_b(T) = \sigma T^4 = 69{,}100\ \text{Btu/h}\cdot\text{ft}^2$$

12-36 Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.

Assumptions The light source behaves as a black body.

Analysis The temperature of this light source is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow T = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{0.47\ \mu\text{m}} = \mathbf{6166\ K}$$



The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that $T = 6166$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(6166\ \text{K}) = 2466\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.15440$$

$$\lambda_2 T = (0.76\ \mu\text{m})(6166\ \text{K}) = 4686\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.59144$$

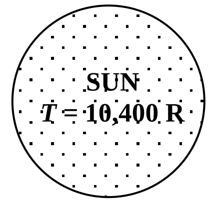
Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.59144 - 0.15440 \cong \mathbf{0.437} \qquad \text{(or 43.7\%)}$$

12-37E The sun is at an effective surface temperature of 10,400 R. The rate of infrared radiation energy emitted by the sun is to be determined.

Assumptions The sun behaves as a black body.

Analysis Noting that T = 10,400 R = 5778 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be



$$\lambda_1 T = (0.76\ \mu\text{m})(5778\ \text{K}) = 4391.3\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.547370$$

$$\lambda_2 T = (100\ \mu\text{m})(5778\ \text{K}) = 577{,}800\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 1.0$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 1.0 - 0.547 = 0.453 \qquad \text{(or 45.3\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzman Law to be

$$E_b = \sigma T^4 = (0.1714\times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(10{,}400\ \text{R})^4 = 2.005\times 10^7\ \text{Btu/h.ft}^2$$

Then,

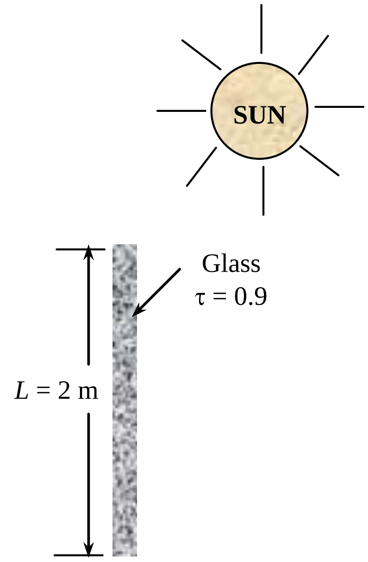
$$E_{\text{infrared}} = (0.453)E_b = (0.453)(2.005\times 10^7\ \text{Btu/h.ft}^2) = \mathbf{9.08\times 10^6\ Btu/h.ft^2}$$

12-38 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases.

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

$$A_s = 9\,\text{m}^2$$



(*a*) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(5800\,\text{K})^4(4\,\text{m}^2) = 2.567\times10^8\ \text{W}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\,\mu\text{m})(5800\,\text{K}) = 1740\,\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\,\mu\text{m})(5800\,\text{K}) = 17{,}400\,\mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$\begin{aligned} E_{\text{transmit}} &= 0.90\Delta f E_b(T) \\ &= (0.90)(0.9453)(2.567\times10^5\ \text{kW}) \\ &= \mathbf{218{,}400\ kW} \end{aligned}$$

(*b*) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\,\text{K})^4(4\,\text{m}^2) = 226{,}800\ \text{W}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\,\mu\text{m})(1000\,\text{K}) = 300\,\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0\,\mu\text{m})(1000\,\text{K}) = 3000\,\mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

$$E_{\text{transmit}} = 0.90\Delta f E_b(T) = (0.90)(0.273232)(226.8\,\text{kW}) = \mathbf{55.8\ kW}$$

12-39 The radiation energy emitted within the visible light region by daylight and candlelight is to be determined.

Assumptions **1** The sun and the candlelight behave as black bodies.

Analysis The visible range of the electromagnetic spectrum extends from λ_1 = 0.40 µm to λ_2 = 0.76 µm. For daylight (T = 5800 K), the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(5800\ \text{K}) = 2320\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_1,\,\text{daylight}} = 0.124509$$

$$\lambda_2 T = (0.76\ \mu\text{m})(5800\ \text{K}) = 4408\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_2,\,\text{daylight}} = 0.550015$$

Then the fraction of radiation emitted between these two wavelengths (for daylight) becomes

$$f_{\lambda_1-\lambda_2,\,\text{daylight}} = 0.550015 - 0.124509 = 0.4255$$

Hence, the radiation energy emitted (for daylight) is determined using

$$E_{b,\ \lambda_1-\lambda_2}(T) = \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(\lambda,\ T) = \sigma T^4 f_{\lambda_1-\lambda_2,\,\text{daylight}} = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5800\ \text{K})^4(0.4255)$$
$$= \mathbf{2.73\times10^7\ W/m^2}$$

For candlelight (T = 1800 K), the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(1800\ \text{K}) = 720\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_1,\,\text{candle}} = 0.0000096$$

$$\lambda_2 T = (0.76\ \mu\text{m})(1800\ \text{K}) = 1368\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_2,\,\text{candle}} = 0.006885$$

Then the fraction of radiation emitted between these two wavelengths (for candlelight) becomes

$$f_{\lambda_1-\lambda_2,\,\text{candle}} = 0.006885 - 0.0000096 = 0.006875$$

Hence, the radiation energy emitted (for candlelight) is determined using

$$E_{b,\ \lambda_1-\lambda_2}(T) = \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(\lambda,\ T) = \sigma T^4 f_{\lambda_1-\lambda_2,\,\text{daylight}} = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1800\ \text{K})^4(0.006875)$$
$$= \mathbf{4090\ W/m^2}$$

Discussion The total radiation energy emitted by daylight is almost 7000 times higher than that by candlelight.

12-40 The percentage of solar energy for different wavelengths assuming the sun's surface temperature is 5800 K.

Assumptions **1** Blackbody radiation.

Analysis (a) The visible range is between λ_1 = 0.40 μm and λ_2 = 0.76 μm

with

$$\lambda_1 T = (0.40\ \mu\text{m})(5800\ \text{K}) = 2320\ \mu\text{m}\cdot\text{K},\ \text{from Table 12-2} \ \rightarrow\ f_{\lambda_1} = 0.124509$$

Similarly with

$$\lambda_2 T = (0.76\ \mu\text{m})(5800\ \text{K}) = 4408\ \mu\text{m}\cdot\text{K},\ \text{from Table 12-2} \ \rightarrow\ f_{\lambda_2} = 0.550019$$

∴ The percentage of solar energy contained in the visible range is

$$f_{\lambda_2} - f_{\lambda_1} = 0.550019 - 0.124509 = 0.426 = \mathbf{42.6\%}$$

(b) The percentage of solar energy at wavelengths shorter than visible is

$$f_{\lambda_1} = 0.124509 = \mathbf{12.5\%}$$

(c) The percentage of solar energy at wavelengths longer than visible is

$$f_{\lambda_2-\infty} = 1 - f_{\lambda_2} = 1 - 0.550019 = 0.449981 \approx \mathbf{45\%}$$

Discussion Approximately 45% of solar energy is infrared, about 43% visible, and about 13% is ultraviolet.

Radiation Intensity

12-41C A solid angle represents an opening in space, whereas a plain angle represents an opening in a plane. For a sphere of unit radius, the solid angle about the origin subtended by a given surface on the sphere is equal to the area of the surface. For a cicle of unit radius, the plain angle about the origin subtended by a given arc is equal to the length of the arc. The value of a solid angle associated with a sphere is 4π.

12-42C Irradiation G is the radiation flux incident on a surface from all directions. For diffusely incident radiation, irradiation on a surface is related to the intensity of incident radiation by $G = \pi I_i$ (or $G_\lambda = \pi I_{\lambda,i}$ for spectral quantities).

12-43C Radiosity J is the rate at which radiation energy leaves a unit area of a surface by emission and reflection in all directions.. For a diffusely emitting and reflecting surface, radiosity is related to the intensity of emitted and reflected radiation by $J = \pi I_{e+r}$ (or $J_\lambda = \pi I_{\lambda,e+r}$ for spectral quantities).

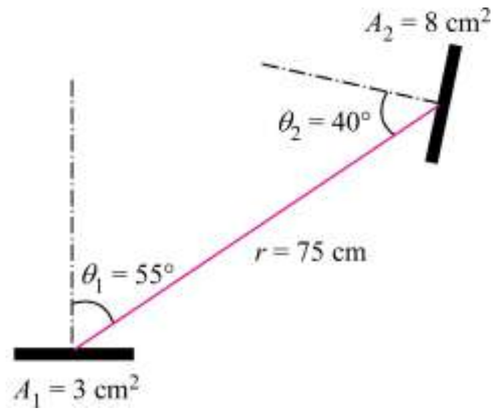
12-44C When the variation of a spectral radiation quantity with wavelength is known, the correcponding total quantity is determined by integrating that quantity with respect to wavelength from $\lambda = 0$ to $\lambda = \infty$.

12-45C The intensity of emitted radiation $I_e(\theta, \phi)$ is defined as the rate at which radiation energy $d\dot{Q}_e$ is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction. For a diffusely emitting surface, the emissive power is related to the intensity of emitted radiation by $E = \pi I_e$ (or $E_\lambda = \pi I_{\lambda,e}$ for spectral quantities).

12-46 A surface (A_2) is subjected to radiation emitted by another surface (A_1). The intensity of the radiation emitted by A_1 is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(8\,\text{cm}^2)\cos 40°}{(75\,\text{cm})^2} = 10.89\times 10^{-4}\ \text{sr}$$

The rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is given as

$$\dot{Q}_{1-2} = I_1 (A_1 \cos\theta_1)\omega_{2-1}$$

Hence, the intensity of the radiation emitted by A_1 can be determined with

$$I_1 = \frac{\dot{Q}_{1-2}}{(A_1\cos\theta_1)\omega_{1-2}} = \frac{274\times 10^{-6}\ \text{W}}{(3\times 10^{-4}\ \text{m}^2)(\cos 55°)(10.89\times 10^{-4}\ \text{sr})} = \mathbf{1460\ W/m^2\cdot sr}$$

The temperature of A_1 is determined using

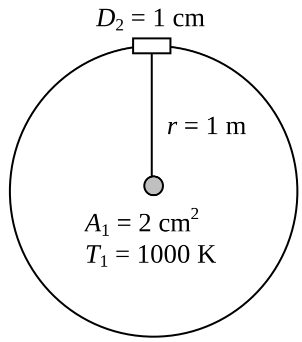
$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad \rightarrow \qquad T_1 = \left(\frac{I_1\pi}{\sigma}\right)^{1/4}$$

$$T_1 = \left[\frac{(1460\ \text{W/m}^2\cdot\text{sr})(\pi\ \text{sr})}{5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right]^{1/4} = \mathbf{533\ K}$$

Discussion If A_2 were directly above A_1 at a distance 75 cm, then θ_1 = 0° and θ_2 = 90°. That means the rate of radiation energy emitted by A_1 is $\dot{Q}_{1-2} = 0$, since $\omega_{1-2} = 0$.

12-47 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis (*a*) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\,\text{m})^2}{(1\,\text{m})^2} = 7.854\times10^{-5}\ \text{sr}$$

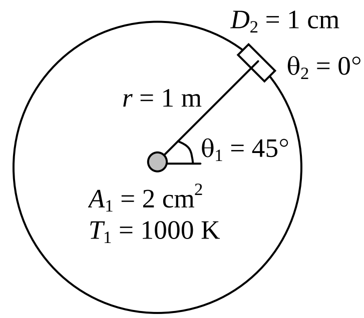
since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1000\ \text{K})^4}{\pi} = 18{,}048\ \text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\ \text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 0°\ \text{m}^2)(7.854\times10^{-5}\ \text{sr})\\ &= \mathbf{2.835\times10^{-4}\ W}\end{aligned}$$



where θ_1 = 0°. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

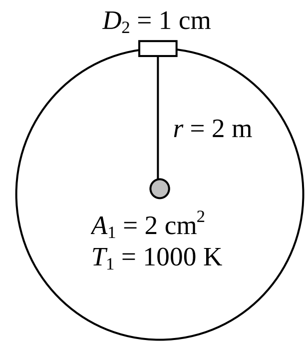
(*b*) In this orientation, θ_1 = 45° and θ_2 = 0°. Repeating the calculation we obtain the rate of radiation to be

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\ \text{W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 45°\ \text{m}^2)(7.854\times10^{-5}\ \text{sr})\\ &= \mathbf{2.005\times10^{-4}\ W}\end{aligned}$$

12-48 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (*a*) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\text{ m})^2}{(2\text{ m})^2} = 1.963\times10^{-5}\text{ sr}$$

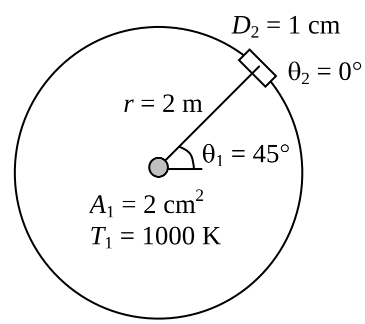
since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000\text{ K})^4}{\pi} = 18{,}048\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 0°\text{ m}^2)(1.963\times10^{-5}\text{ sr})\\ &= \mathbf{7.086\times10^{-5}\text{ W}}\end{aligned}$$



where θ_1 = 0°. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

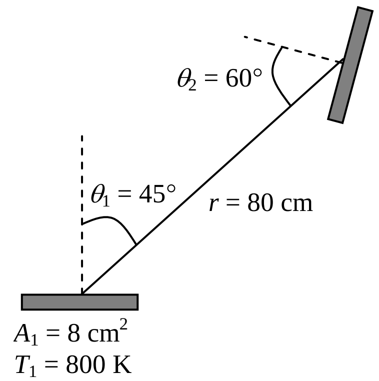
(*b*) In this orientation, θ_1 = 45° and θ_2 = 0°. Repeating the calculation we obtain the rate of radiation as

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos 45°\text{ m}^2)(1.963\times10^{-5}\text{ sr})\\ &= \mathbf{5.010\times10^{-5}\text{ W}}\end{aligned}$$

12-49 A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(8\,\text{cm}^2)\cos 60°}{(80\,\text{cm})^2} = \mathbf{6.250 \times 10^{-4}\ sr}$$

since the normal of A_2 makes 60° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\,\text{K})^4}{\pi} = 7393\ \text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (7393\ \text{W/m}^2\cdot\text{sr})(7\times10^{-4}\cos 45°\ \text{m}^2)(6.250\times10^{-4}\ \text{sr})\\ &= \mathbf{2.287\times10^{-3}\ W}\end{aligned}$$

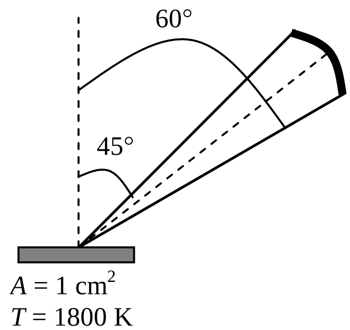
Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.287×10^{-3} W.

If A_2 were directly above A_1 at a distance 80 cm, θ_1 = 0° and the rate of radiation energy emitted by A_1 becomes zero.

12-50 A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.

Assumptions Surface A emits diffusely as a blackbody.

Analysis The rate of radiation emission from a surface per unit surface area in the direction (θ,ϕ) is given as



$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 60° and 45° can be expressed as

$$E = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{60} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b\frac{\pi}{4} = \frac{\sigma T^4}{\pi}\frac{\pi}{4} = \frac{\sigma T^4}{4}$$

since the blackbody radiation intensity is constant (I_b = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 60 - \sin^2 45) = \pi/4$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_e = EdA = \frac{\sigma T^4}{4}dA = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1800\ \text{K})^4}{4}(1\times10^{-4}\ \text{m}^2) = \mathbf{14.9\ W}$$

12-51 The intensity of solar radiation incident on earth's surface is given. The peak value for the intensity of incident solar radiation and the solar irradiation on earth's surface are to be determined.

Assumptions **1** The intensity is not dependent of the azimuth angle ϕ.

Analysis The intensity of solar radiation incident on earth's surface peaks at the zenith angle of θ = 0°, hence

$$I_{i,\max} = 100\cos 0° = \mathbf{100\ W/m^2\cdot sr}$$

The solar irradiation on earth's surface is

$$\begin{aligned}
G &= \int_{\text{hemisphere}} dG = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta)\cos\theta\sin\theta\, d\theta\, d\phi \\
&= \int_0^{2\pi}\int_0^{\pi/2} 100\cos^2\theta\ \sin\theta\, d\theta\, d\phi \\
&= 100(2\pi)\int_0^{\pi/2}\cos^2\theta\ \sin\theta\ d\theta \\
&= 200\pi\left[-\frac{(\cos\theta)^3}{3}\right]_0^{\pi/2} \\
&= \frac{200\pi}{3} \\
&= \mathbf{209\ W/m^2}
\end{aligned}$$

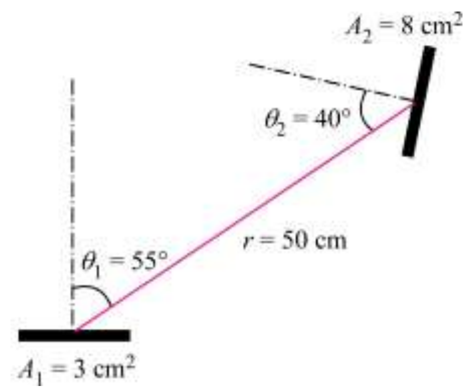
Discussion The intensity of incident solar radiation is at minimum when the sun is at the horizon, where the zenith angle is $\theta \approx 90°$.

12-52 A surface (A_2) is subjected to radiation emitted by another surface (A_1). The rate at which emitted radiation is received and the irradiation on A_2 are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(8\text{ cm}^2)\cos 40°}{(50\text{ cm})^2} = 24.51\times10^{-4}\text{ sr}$$



since the normal of A_2 makes 40° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{5.67\times10^4\text{ W/m}^2}{\pi\text{ sr}} = 18048\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18048\text{ W/m}^2\cdot\text{sr})(3\times10^{-4}\text{ m}^2)(\cos 55°)(24.51\times10^{-4}\text{ sr})\\ &= \mathbf{76.1\times10^{-4}\ W}\end{aligned}$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation on A_2.is

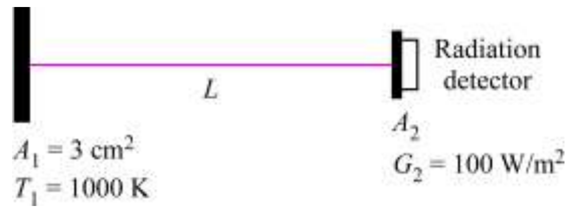
$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{76.1\times10^{-4}\text{ W}}{8\times10^{-4}\text{ m}^2} = \mathbf{9.51\ W/m^2}$$

Discussion The total rate of radiation emission from surface A_1 is $\dot{Q}_e = A_1 E_b(T_1) = 17.01$ W. Therefore, the fraction of emitted radiation that strikes A_2 is $76.1\times10^{-4} / 17.01 = 0.045$ percent.

12-53 A radiation detector (A_2) is placed normal to the direction of viewing from another surface (A_1), and is measuring a specified amount of irradiation. The distance between the radiation detector and the radiation emitting surface is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1 (A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation detector A_2.is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2} \qquad \rightarrow \qquad L = \left[\frac{\sigma T_1^4 A_1 \cos\theta_1 \cos\theta_2}{\pi G_2}\right]^{1/2}$$

Since the radiation detector is placed normal to the direction of viewing from A_1, we have $\theta_1 = \theta_2 = 0°$. Hence the distance L is

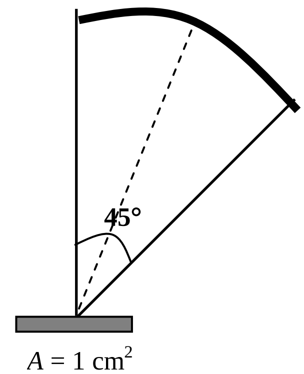
$$L = \left[\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1000\ \text{K})^4(3\times10^{-4}\ \text{m}^2)}{\pi(100\ \text{W/m}^2)}\right]^{1/2} = \mathbf{0.233\,m}$$

Discussion The solid angle subtended by A_2 is at maximum, since the radiation detector is positioned normal to the direction of viewing ($\theta_2 = 0°$).

12-54 A small surface is subjected to uniform incident radiation. The rates of radiation emission through two specified bands are to be determined.

Assumptions The intensity of incident radiation is constant.

Analysis (*a*) The rate at which radiation is incident on a surface per unit surface area in the direction (θ,ϕ) is given as



$$dG = \frac{d\dot{Q}_i}{dA} = I_i(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 0° and 45° can be expressed as

$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{45} I_i(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_i\ \frac{\pi}{2}$$

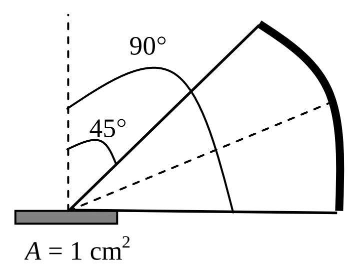
since the incident radiation is constant (I_i = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 45 - \sin^2 0) = \pi/2$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_{i,1} = G_1 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$

(*b*) Similarly, the total rate of radiation emission through the band between 45° and 90° can be expressed as



$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{90} I_i(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_i\ \frac{\pi}{2}$$

since

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta$$
$$= \pi(\sin^2 90 - \sin^2 45) = \pi/2$$

and

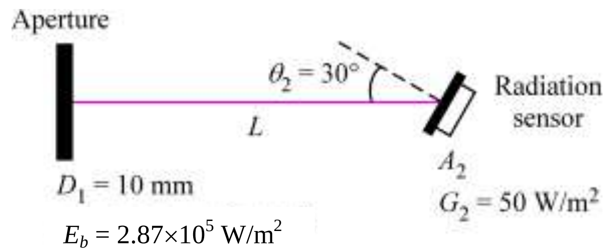
$$\dot{Q}_{i,2} = G_2 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$

Discussion Note that the viewing area for the band 0° - 45° is much smaller, but the radiation energy incident through it is equal to the energy streaming through the remaining area.

12-55 A radiation sensor is placed with a 30° tilt off the normal direction of viewing from an aperture through which radiation is emitted as a blackbody. The distance between the sensor and the aperture is to be determined.

Assumptions **1** The aperture emits diffusely as a blackbody. **2** Both aperture and sensor can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both aperture (A_1) and radiation sensor (A_2) as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

where $A_1 = \pi D_1^2 / 4$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation detector A_2.is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{E_b(A_1\cos\theta_1)(A_2\cos\theta_2)}{\pi A_2 L^2} \quad \rightarrow \quad L = \left[\frac{E_b \pi D_1^2 \cos\theta_1 \cos\theta_2}{4\pi G_2}\right]^{1/2}$$

With $\theta_1 = 0°$ and $\theta_2 = 30°$, the distance between the sensor and the aperture is

$$L = \left[\frac{(2.87\times10^5\ \text{W/m}^2)(0.01\,\text{m})^2 \cos 30°}{4(50\ \text{W/m}^2)}\right]^{1/2} = \mathbf{0.353\,m}$$

Discussion If the radiation sensor is positioned normal to the direction of viewing ($\theta_2 = 0°$) with $L = 0.353$ m, it would measure an irradiation of $G_2 = 58$ W/m^2.

12-56 A radiometer is placed normal to the direction of viewing from a circular plate (blackbody) and is measuring a specified amount of irradiation. The temperature of the plate is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the plate A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2} \quad \rightarrow \quad T_1 = \left[\frac{G_2 \pi L^2}{\sigma(\pi D_1^2/4)(\cos\theta_1)(\cos\theta_2)}\right]^{1/4}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have temperature of the plate as

$$T_1 = \left[\frac{(85\ \text{W/m}^2)(0.50\ \text{m})^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.02\ \text{m})^2/4}\right]^{1/4} = \mathbf{1391\ K}$$

Discussion If the temperature of the plate is maintained constant while the distance between the plate and the radiometer decreases, then the irradiation detected by the radiometer would increase.

12-57 A radiometer is used to monitor the surface temperature of an engine to prevent fire hazard in the event of an oil leakage. The engine surface temperature is to be determined from the irradiation measured by the radiometer.

Assumptions **1** The engine surface emits diffusely as a blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Engine surface temperature is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The solid angle subtended by the radiometer when viewed from the engine surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted by the target surface of the engine A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} \quad \rightarrow \quad T_1 = \left(\frac{G_2 \pi L^2}{\sigma A_1}\right)^{1/4}$$

The engine surface temperature is

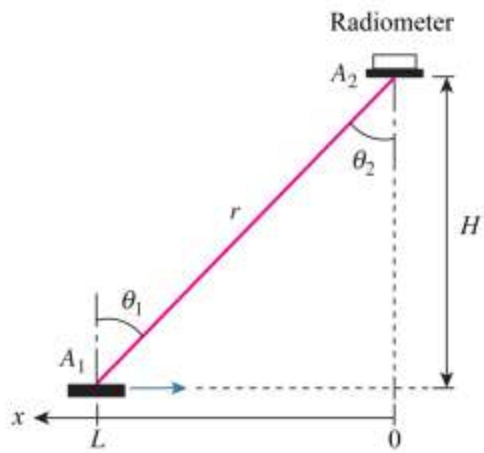
$$T_1 = \left[\frac{(0.1\ \text{W/m}^2)(1\ \text{m})^2 \pi}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1\times10^{-4}\ \text{m}^2)}\right]^{1/4} = 485\ \text{K} = \mathbf{212°C} > 180°\text{C}$$

Discussion The irradiation measured by the radiometer indicates that the engine surface temperature is higher than 180°C. Therefore there is a risk of fire hazard in the event of an oil leakage.

12-58 A radiometer is used to measure the position of an approaching hot object. The position of the object when the irradiation on the radiometer is 80% corresponding to the object position of $x = 0$ is to be determined.

Assumptions **1** The approaching object emits diffusely as a blackbody. **2** Both object and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the approaching object A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{A_2 H}{r^2 r} = \frac{A_2 H}{(H^2 + x^2)^{3/2}} \qquad (1)$$

Note that

$$r = (H^2 + x^2)^{1/2}$$

and

$$\cos\theta_1 = \cos\theta_2 = \frac{H}{r} = \frac{H}{(H^2 + x^2)^{1/2}}$$

Then, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is

$$\dot{Q}_{1-2} = I_1 (A_1 \cos\theta_1)\omega_{2-1} = I_1 A_1 \frac{H}{(H^2 + x^2)^{1/2}} \omega_{2-1} \qquad (2)$$

Substituting Eq. (1) into (2) yields

$$\dot{Q}_{1-2} = I_1 A_1 A_2 \frac{H^2}{(H^2 + x^2)^2}$$

Hence, the irradiation measured by the radiometer A_2 is

$$G_{2,x} = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1 A_1 H^2}{(H^2 + x^2)^2}$$

Hence, the position L at which the sensor is measuring 80% of the irradiation corresponding to the position of the object directly under the radiometer at $x = 0$ can be determined as

$$\frac{G_{2,L}}{G_{2,0}} = 0.80 = \left[\frac{I_1 A_1 H^2}{(H^2 + L^2)^2}\right]\left[\frac{(H^2)^2}{I_1 A_1 H^2}\right] = \left(\frac{H^2}{H^2 + L^2}\right)^2$$

$$L = \left(\frac{H^2}{0.80^{0.5}} - H^2\right)^{0.5} = \left[\frac{(0.5\,\text{m})^2}{0.80^{0.5}} - (0.5\,\text{m})^2\right]^{0.5} = \mathbf{0.172\,m}$$

Discussion Knowing the relationship of $G_{2,L}$ / $G_{2,0}$ with the position of the approaching object, engineers can use it to implement specific treatment, such as spray painting the object when it reaches a position on a production line.

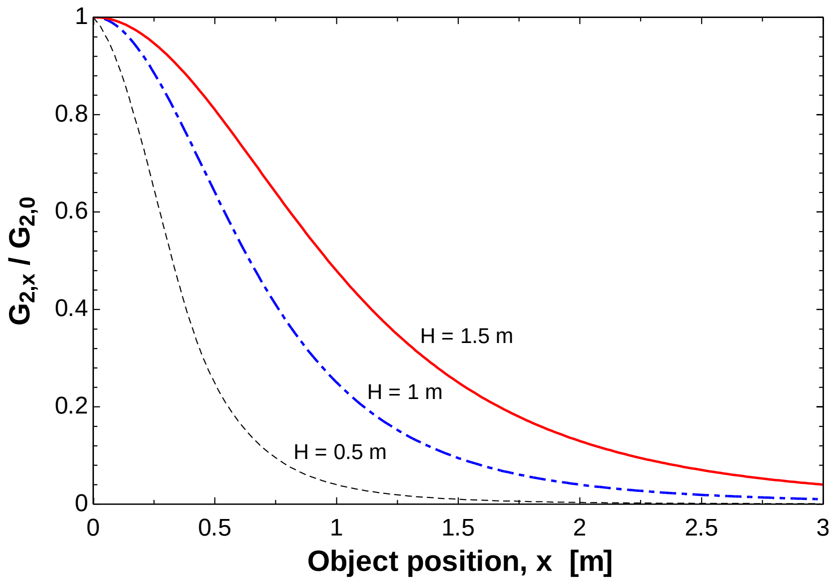
12-59 A radiometer is used to measure the position of an approaching hot object. The effect of the approaching object position on the irradiation measured by the radiometer is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
H_1=0.5 [m]
H_2=1.0 [m]
H_3=1.5 [m]

"ANALYSIS"
G_ratio1=(H_1^2/(H_1^2+x^2))^2
G_ratio2=(H_2^2/(H_2^2+x^2))^2
G_ratio3=(H_3^2/(H_3^2+x^2))^2
```

	$G_{2,x}$ / $G_{2,0}$		
x [m]	H = 0.5 m	1 m	1.5 m
0	1	1	1
0.2	0.7432	0.9246	0.9654
0.4	0.3718	0.7432	0.8716
0.6	0.1680	0.5407	0.7432
0.8	0.0789	0.3718	0.6061
1.0	0.0400	0.2500	0.4793
1.2	0.02188	0.1680	0.3718
1.4	0.01280	0.1141	0.2856
1.6	0.007915	0.07890	0.2188
1.8	0.005131	0.05562	0.1680
2.0	0.003460	0.04000	0.12960
2.2	0.002412	0.02932	0.10070
2.4	0.001730	0.02188	0.07890
2.6	0.001272	0.01661	0.06236
2.8	0.0009550	0.01280	0.04973
3.0	0.0007305	0.01000	0.04000



Discussion When the placement of the radiometer is nearest to the *x*-axis (H = 0.5 m), $G_{2,x}$ / $G_{2,0}$ increases most rapidly as the object approaches to x = 0. Therefore, the radiometer should be placed appropriately based on its response to the approaching object.

12-60 A radiometer A_2 is placed normal to the direction of viewing from the plate A_1 at a distance L. The irradiation on the radiometer, if the distance L is doubled, is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the plate A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2}$$

The irradiation on the radiometer at L and $2L$ are

$$G_{2,L} = \frac{\sigma T_1^4 A_1 (\cos\theta_1)(\cos\theta_2)}{\pi L^2} \quad \text{and} \quad G_{2,2L} = \frac{\sigma T_1^4 A_1 (\cos\theta_1)(\cos\theta_2)}{\pi (2L)^2}$$

Thus,

$$\frac{G_{2,2L}}{G_{2,L}} = \left[\frac{\sigma T_1^4 A_1 (\cos\theta_1)(\cos\theta_2)}{\pi (2L)^2}\right]\left[\frac{\pi L^2}{\sigma T_1^4 A_1 (\cos\theta_1)(\cos\theta_2)}\right] = \mathbf{\frac{1}{4}} \quad \rightarrow \quad G_{2,2L} = \frac{1}{4} G_{2,L}$$

Discussion Doubling the distance L would quarter the irradiation on the radiometer.

12-61 A blackbody plate is subjected to uniform heat flux at the bottom and the top surface is exposed to ambient surrounding. A radiometer is placed above the plate and the irradiation detected by the radiometer is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis By performing energy balance on the plate, we can determine the surface temperature of the plate:

$$\dot{q} = \dot{q}_{\text{emit}} - \dot{q}_{\text{incident}} + \dot{q}_{\text{conv}}$$

$$\dot{q} = \sigma T_1^4 - \sigma T_{\text{surr}}^4 + h(T_1 - T_\infty)$$

$$1000\ \text{W/m}^2 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(T_1^4 - 278^4)\ \text{K}^4 + (5\ \text{W/m}^2\cdot\text{K})(T_1 - 278)\ \text{K}$$

$$\rightarrow \quad T_1 = 358\ \text{K}$$

The solid angle subtended by the radiometer when viewed from the plate is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted by the plate A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(358^4)\ \text{K}^4(5\times10^{-4}\ \text{m}^2)}{\pi(1\,\text{m})^2} = \mathbf{0.148\ W/m^2}$$

Discussion If the radiometer is placed at a distance farther away from the plate, it would detect a smaller value of irradiation G_2.

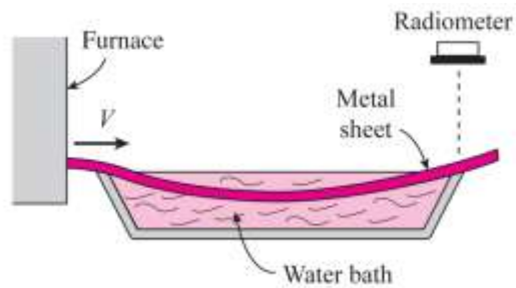
12-62 A radiometer is used to monitor the surface temperature of a metal sheet exiting a water bath. The irradiation detected by the radiometer when the surface temperature is unsafe to touch is to be determined.

Assumptions **1** The metal sheet surface emits diffusely as a blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Metal sheet temperature at the water bath exit is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The solid angle subtended by the radiometer when viewed from the metal sheet surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$



The intensity of the radiation emitted by the target surface of the metal sheet A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2}$$

The irradiation measured by the radiometer when the metal sheet temperature is at 45°C or higher is

$$G_2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(318\ \text{K})^4(1\times10^{-4}\ \text{m}^2)}{\pi(0.5\ \text{m})^2} = \mathbf{0.0738\ W/m^2}$$

Discussion When the radiometer detects an irradiation of 0.0738 W/m^2 or higher, an alarm should be triggered warning people that the metal sheet is unsafe to touch.

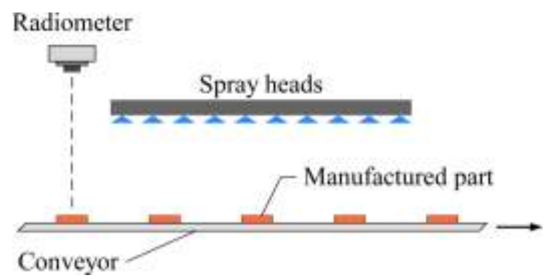
12-63 A radiometer is used to monitor temperatures of manufactured parts. The irradiation detected by the radiometer when the temperature of a part is unsafe to touch is to be determined.

Assumptions **1** The manufactured parts emit diffusely as blackbody. **2** Both part surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Part surface temperature is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The solid angle subtended by the radiometer when viewed from the manufactured parts is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$



The intensity of the radiation emitted by a manufactured part A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer measures the radiation emitted by a manufactured part when it is normal in the direction of viewing ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2}$$

The irradiation measured by the radiometer when a manufactured part temperature is at 45°C or higher is

$$G_2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(318\ \text{K})^4(10\times10^{-4}\ \text{m}^2)}{\pi(1\ \text{m})^2} = \mathbf{0.185\ W/m^2}$$

Discussion When the radiometer detects an irradiation of 0.185 W/m^2 or higher, the spray heads should release mist to cool the parts to prevent thermal burn hazards at the end of the conveyor.

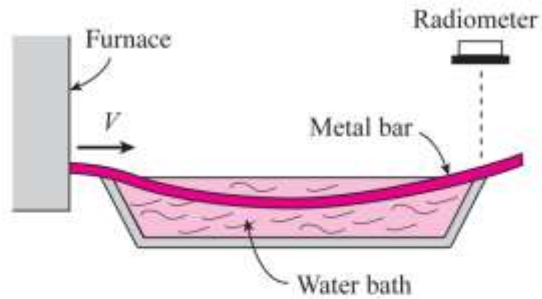
12-64 A radiometer is used to monitor temperature of a metal bar leaving a quenching process. The speed of the metal bar is to be determined from the irradiation detected by the radiometer.

Assumptions **1** The metal bar emits diffusely as blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** The metal bar temperature at the water bath exit is uniform.

Properties The specific heat and the density of metal bar are given as $c_{p,ss}$ = 450 J/kg·K and ρ_{ss} = 7900 kg/m^3, respectively. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The solid angle subtended by the radiometer when viewed from the metal bar surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$



The intensity of the radiation emitted by the target surface of the metal bar A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} \quad \rightarrow \quad T_1 = \left(\frac{G_2 \pi L^2}{\sigma A_1}\right)^{1/4}$$

$$T_1 = \left[\frac{(0.015\ \text{W/m}^2)\pi(1\ \text{m})^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1\times10^{-4}\ \text{m}^2)}\right]^{1/4} = 302\ \text{K} = 29°\text{C} = T_{\text{out}} < 45°\text{C}$$

The mass of the metal bar being conveyed enters and exits the water bath at a rate of

$$\dot{m} = \rho_{ss} V w t$$

The rate of heat removed from the metal bar is

$$\dot{Q}_{\text{removed}} = \dot{m} c_{p,ss}(T_{\text{in}} - T_{\text{out}}) = \rho_{ss} V w t c_{p,ss}(T_{\text{in}} - T_{\text{out}})$$

The speed of the bar is

$$V = \frac{\dot{Q}_{\text{removed}}}{\rho_{ss} w t c_{p,ss}(T_{\text{in}} - T_{\text{out}})} = \frac{500\times10^3\ \text{J/s}}{(7900\ \text{kg/m}^3)(0.030\ \text{m})(0.015\ \text{m})(450\ \text{J/kg}\cdot\text{K})(700-29)\ \text{K}} = \mathbf{0.466\ m/s}$$

Discussion At a speed of about 0.47 m/s, the metal bar can be cooled down to below 45°C for prevention from thermal burn hazards.

Radiation Properties

12-65C A body whose surface properties are independent of wavelength is said to be a graybody. The emissivity of a blackbody is one for all wavelengths, the emissivity of a graybody is between zero and one. A surface whose properties change with wavelength and direction is called a diffuse gray surface.

12-66C The emissivity ε is the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The fraction of radiation absorbed by the surface is called the absorptivity α,

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \text{and} \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G}$$

When the surface temperature is equal to the temperature of the source of radiation, the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature $\varepsilon_\lambda(T) = \alpha_\lambda(T)$.

12-67C The fraction of irradiation reflected by the surface is called reflectivity ρ and the fraction transmitted is called the transmissivity τ

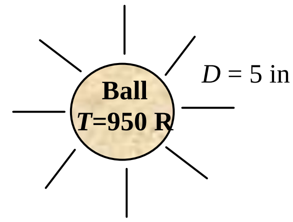
$$\rho = \frac{G_{ref}}{G} \quad \text{and} \quad \tau = \frac{G_{tr}}{G}$$

Surfaces are assumed to reflect in a perfectly spectral or diffuse manner for simplicity. In spectral (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam. In diffuse reflection, radiation is reflected equally in all directions.

12-68C The heating effect which is due to the non-gray characteristic of glass, clear plastic, or atmospheric gases is known as the greenhouse effect since this effect is utilized primarily in greenhouses. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth, acting like a heat trap. There is a concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

12-69C Glass has a transparent window in the wavelength range 0.3 to 3 μm and it is not transparent to the radiation which has wavelength range greater than 3 μm. Therefore, because the microwaves are in the range of 10^2 to10^5 μm, the harmful microwave radiation cannot escape from the glass door.

12-70E A spherical ball emits radiation at a certain rate. The average emissivity of the ball is to be determined at the given temperature.



Analysis The surface area of the ball is

$$A = \pi D^2 = \pi(5/12\text{ ft})^2 = 0.5454\text{ ft}^2$$

Then the average emissivity of the ball at this temperature is determined to be

$$E = \varepsilon A\sigma T^4 \longrightarrow \varepsilon = \frac{E}{A\sigma T^4} = \frac{550\text{ Btu/h}}{(0.5454\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)(950\text{ R})^4} = \mathbf{0.722}$$

12-71 A radiation sensor (A_2) is placed normal to the direction of viewing from another surface (A_1). An optical filter with specified spectral transmissivity is placed in front of the sensor. The irradiation that is measured by the sensor is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis For T = 1000 K, the blackbody radiation functions corresponding to $\lambda_1 T$ is determined from Table 12-2 to be

$$\lambda_1 T = (2\ \mu\text{m})(1000\text{ K}) = 2000\ \mu\text{m}\cdot\text{K} \qquad \rightarrow \qquad f_{\lambda_1} = 0.066728$$

Hence, the transmissivity of the optical filter is

$$\tau = \tau_1(f_{\lambda_1}) + \tau_2(1 - f_{\lambda_1}) = 0(0.066728) + 0.5(1 - 0.066728) = 0.4666$$

The rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1\cos\theta_1)\omega_{2-1}\,\tau$$

where

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2\cos\theta_2}{L^2} \qquad \text{and} \qquad I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

Since the radiation sensor is placed normal to the direction of viewing from A_1, we have $\theta_1 = \theta_2 = 0°$, hence

$$\dot{Q}_{1-2} = \frac{\sigma T_1^4 A_1 A_2}{L^2\pi}\tau$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation sensor A_2 is

$$G_{\text{tr}} = G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{\sigma T_1^4 A_1}{L^2\pi}\tau$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000\text{ K})^4(5\times10^{-4}\text{ m}^2)}{(0.5\text{ m})^2\pi}(0.4666)$$

$$= \mathbf{16.8\ W/m^2}$$

Discussion If the optical filter is removed, the irradiation measured by the radiation sensor would be

$$G = \frac{G_{\text{tr}}}{0.4666} = \frac{16.8\text{ W/m}^2}{0.4666} = 36\text{ W/m}^2$$

12-72 A radiometer is used to monitor the surface temperature of a tank surface to prevent thermal burn hazard. The tank surface temperature is to be determined from the irradiation measured by the radiometer.

Assumptions **1** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **2** Tank surface temperature and properties are uniform. **3** Kirchhoff's law is applicable. **4** The tank wall is opaque.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis From Kirchhoff's law, the emissivity of the plate is

$$\varepsilon = \alpha = 0.85$$

The tank wall is opaque and the reflectivity of the tank is

$$\rho = 1 - \alpha = 1 - 0.85 = 0.15$$

The solid angle subtended by the radiometer when viewed from the target surface of the tank A_1 is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted and reflected (radiosity) by the target surface A_1 is

$$I_1 = \frac{J}{\pi} = \frac{E + G_{\text{ref}}}{\pi} = \frac{\varepsilon\sigma T_1^4 + \rho G_1}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1 (A_1 \cos\theta_1)\omega_{2-1}}{A_2} = \frac{(\varepsilon\sigma T_1^4 + \rho G)\, A_1 (\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{(\varepsilon\sigma T_1^4 + \rho G)A_1}{\pi L^2} \quad \rightarrow \quad T_1 = \left[\left(\frac{G_2 \pi L^2}{A_1} - \rho G_1\right)\left(\frac{1}{\varepsilon\sigma}\right)\right]^{0.25}$$

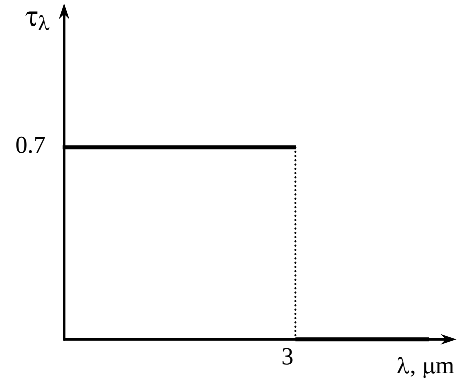
$$\begin{aligned} T_1 &= \left\{\left[\frac{(0.085\ \text{W/m}^2)\pi(0.5\ \text{m})^2}{1\times10^{-4}\ \text{m}^2} - (0.15)(390\ \text{W/m}^2)\right]\left[\frac{1}{(0.85)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right]\right\}^{0.25} \\ &= 335\ \text{K} \\ &= \mathbf{62°C} > 45°\text{C} \end{aligned}$$

Discussion The irradiation measured by the radiometer indicates that the tank surface temperature is higher than 45°C. Therefore there is a risk of thermal burn hazard.

12-73 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

Assumptions The window glass behaves as a black body.

Analysis The fraction of radiation at wavelengths smaller than 3 μm is



$$\lambda T = (3\ \mu\text{m})(1200\text{ K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$

The fraction of radiation coming from the furnace and transmitted through the window is

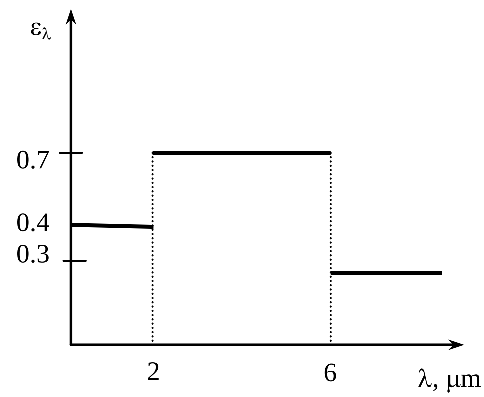
$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2(1 - f_\lambda)\\ &= (0.7)(0.403607) + (0)(1 - 0.403607)\\ &= \mathbf{0.2825}\end{aligned}$$

Then the rate of radiation coming from the furnace and transmitted through the window becomes

$$G_{tr} = \tau A\sigma T^4 = 0.2825(0.40 \times 0.40\text{ m}^2)(5.67 \times 10^{-8}\text{ W/m}^2.\text{K}^4)(1200\text{ K})^4 = \mathbf{5315\ W}$$

12-74 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from



$$\begin{aligned}\varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4}\\ &= \varepsilon_1 f_{0\text{-}\lambda_1} + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2} + \varepsilon_3 f_{\lambda_2\text{-}\infty}\\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})\end{aligned}$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (2\ \mu\text{m})(1000\text{ K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\text{ K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.737818$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } \mathrm{f}_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and,

$$\varepsilon = (0.4)(0.066728) + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = \mathbf{0.5751}$$

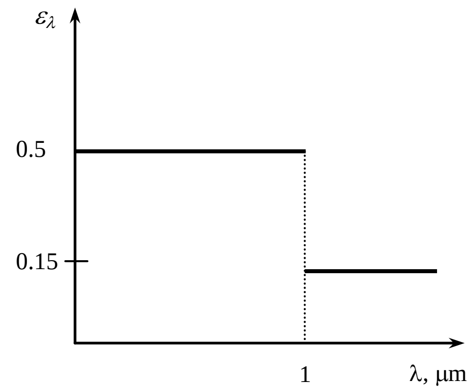
Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = (0.5751)(5.67 \times 10^{-8}\text{ W/m}^2.\text{K}^4)(1000\text{ K})^4 = 32{,}610\text{ W/m}^2 = \mathbf{32.6\ kW/m^2}$$

12-75 The variation of emissivity of a tungsten filament with wavelength is given. The average emissivity, absorptivity, and reflectivity of the filament are to be determined for two temperatures.

Analysis (*a*) T = 2000 K

$$\lambda_1 T = (1\ \mu\text{m})(2000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.5)(0.066728) + (0.15)(1 - 0.066728) \\ &= \mathbf{0.173}\end{aligned}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.173} \quad \text{(at 2000 K)}$$

and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.173 = \mathbf{0.827}$$

(*b*) T = 3000 K

$$\lambda_1 T = (1\ \mu\text{m})(3000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

Then

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) = (0.5)(0.273232) + (0.15)(1 - 0.273232) = \mathbf{0.246}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.246} \quad \text{(at 3000 K)}$$

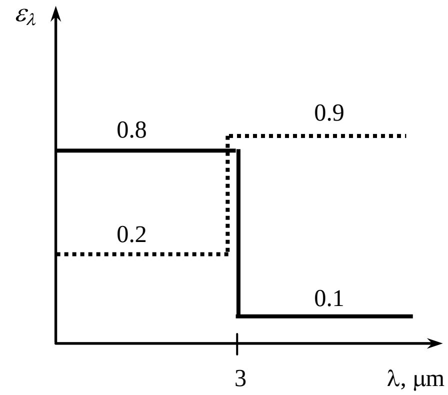
and

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.246 = \mathbf{0.754}$$

12-76 The variations of emissivity of two surfaces are given. The average emissivity, absorptivity, and reflectivity of each surface are to be determined at the given temperature.

Analysis For the first surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.890029) + (0.9)(1 - 0.890029) \\ &= \mathbf{0.28}\end{aligned}$$

The absorptivity and reflectivity are determined from Kirchhoff's law

$$\varepsilon = \alpha = \mathbf{0.28} \quad \text{(at 3000 K)}$$

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.28 = \mathbf{0.72}$$

For the second surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.8)(0.890029) + (0.1)(1 - 0.890029) \\ &= \mathbf{0.72}\end{aligned}$$

Then,

$$\varepsilon = \alpha = \mathbf{0.72} \quad \text{(at 3000 K)}$$

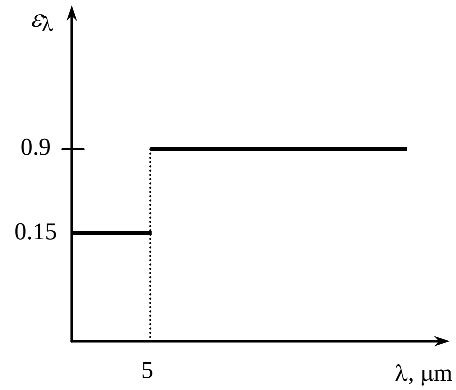
$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha = 1 - 0.72 = \mathbf{0.28}$$

Discussion The second surface is more suitable to serve as a solar absorber since its absorptivity for short wavelength radiation (typical of radiation emitted by a high-temperature source such as the sun) is high, and its emissivity for long wavelength radiation (typical of emitted radiation from the absorber plate) is low.

12-77 The variation of emissivity of a surface with wavelength is given. The average emissivity and absorptivity of the surface are to be determined for two temperatures.

Analysis (*a*) <u>For *T* = 5800 K:</u>

$$\lambda_1 T = (5\ \mu\text{m})(5800\ \text{K}) = 29{,}000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.994715$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.15)(0.994715) + (0.9)(1 - 0.994715) \\ &= \mathbf{0.154}\end{aligned}$$

(*b*) <u>For *T* = 300 K:</u>

$$\lambda_1 T = (5\mu\text{m})(300\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

and

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.15)(0.013754) + (0.9)(1 - 0.013754) \\ &= \mathbf{0.89}\end{aligned}$$

The absorptivities of this surface for radiation coming from sources at 5800 K and 300 K are, from Kirchhoff's law,

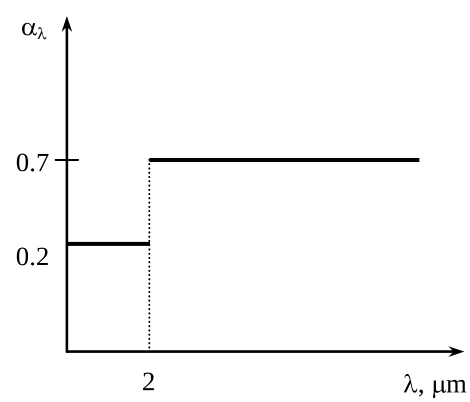
$$\alpha = \varepsilon = \mathbf{0.154} \quad \text{(at 5800 K)}$$

$$\alpha = \varepsilon = \mathbf{0.89} \quad \text{(at 300 K)}$$

12-78 The variation of absorptivity of a surface with wavelength is given. The average absorptivity, reflectivity, and emissivity of the surface are to be determined at given temperatures.

Analysis <u>For *T* = 2500 K:</u>

$$\lambda_1 T = (2\ \mu\text{m})(2500\ \text{K}) = 5000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.633747$$



The average absorptivity of this surface is

$$\begin{aligned}\alpha(T) &= \alpha_1 f_{\lambda_1} + \alpha_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.633747) + (0.7)(1 - 0.633747) \\ &= \mathbf{0.383}\end{aligned}$$

Then the reflectivity of this surface becomes

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.383 = \mathbf{0.617}$$

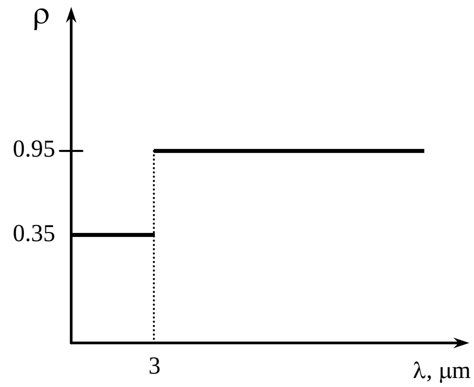
Using Kirchhoff's law, $\alpha = \varepsilon$, the average emissivity of this surface at T = 3000 K is determined to be

$$\lambda T = (2\mu\text{m})(3000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda} = 0.737818$$

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.737818) + (0.7)(1 - 0.737818) \\ &= \mathbf{0.331}\end{aligned}$$

12-79 The variation of reflectivity of a surface with wavelength is given. The average reflectivity, emissivity, and absorptivity of the surface are to be determined for two source temperatures.

Analysis The average reflectivity of this surface for solar radiation (T = 5800 K) is determined to be



$$\lambda T = (3\,\mu\text{m})(5800\text{ K}) = 17400\,\mu\text{mK} \to f_{\lambda} = 0.978746$$

$$\begin{aligned}\rho(T) &= \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T)\\ &= \rho_1 f_{\lambda_1} + \rho_2(1 - f_{\lambda_1})\\ &= (0.35)(0.978746) + (0.95)(1 - 0.978746)\\ &= \mathbf{0.362}\end{aligned}$$

Noting that this is an opaque surface, $\tau = 0$

At T = 5800 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at T = 300 K,

$$\lambda T = (3\,\mu\text{m})(300\text{ K}) = 900\,\mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1 - 0.0001685) = \mathbf{0.95}$$

At T = 300 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

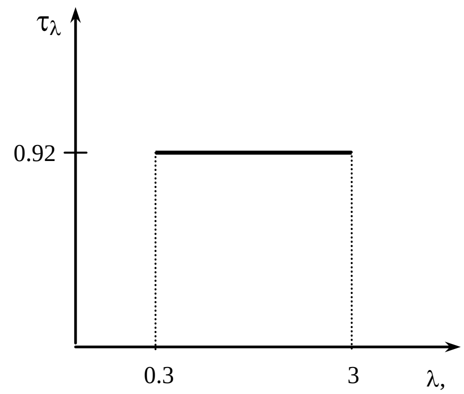
The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

which makes it suitable as a solar collector. ($\alpha_s = 1$ and $\varepsilon_{\text{room}} = 0$ for an ideal solar collector)

12-80 The variation of transmissivity of a glass is given. The average transmissivity of the pane at two temperatures and the amount of solar radiation transmitted through the pane are to be determined.

Analysis <u>For *T*=5800 K:</u>



$$\lambda_1 T_1 = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T_1 = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$

The average transmissivity of this surface is

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.978746 - 0.033454) = \mathbf{0.870}$$

<u>For *T*=300 K:</u>

$$\lambda_1 T_2 = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.0001685 - 0.0) = \mathbf{0.00016 \approx 0}$$

The amount of solar radiation transmitted through this glass is

$$G_{\text{tr}} = \tau G_{\text{incident}} = 0.870(650\ \text{W/m}^2) = \mathbf{566\ W/m^2}$$

12-81 An opaque horizontal plate that is well insulated on the edges and the lower surface has a constant temperature of 500 K and α = 0.51, (*a*) the total hemispherical emissivity and (*b*) the radiosity of the plate surface are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Properties The total hemispherical absorptivity of the plate is given to be 0.51.

Analysis (*a*) The average emissivity of the surface can be determined from

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\infty} E_{b\lambda} d\lambda}{E_b} = \varepsilon_1 f_{0-\lambda_1} + \varepsilon_2 f_{\lambda_1-\infty} = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1})$$

For T = 500 K, the blackbody radiation functions corresponding to $\lambda_1 T$ is determined from Table 12-2 to be

$$\lambda_1 T = (4\ \mu\text{m})(500\ \text{K}) = 2000\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.066728$$

Hence, the total hemispherical emissivity is

$$\varepsilon(T) = 0.4(0.066728) + 0.8(1 - 0.066728) = \mathbf{0.773}$$

(*b*) The radiosity of the plate surface can be determined from the following expression:

$$J = E + G_{\text{ref}} = \varepsilon E_b + \rho G$$

Since the plate is opaque (τ = 0), the reflectivity is then, $\rho = 1 - \alpha$. Hence,

$$\begin{aligned} J &= \varepsilon\sigma T^4 + (1-\alpha)G \\ &= (0.773)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(500\ \text{K})^4 + (1-0.51)(5600\ \text{W/m}^2) \\ &= \mathbf{5480\ W/m^2} \end{aligned}$$

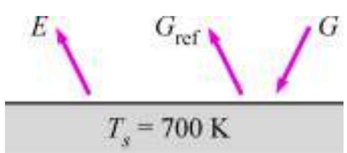
Discussion Both emissive power (E) and reflected irradiation (G_{ref}) contributed equal amount to the radiosity, with $E = G_{\text{ref}}$ = 2740 W/m^2.

12-82 An opaque horizontal plate that is well insulated on the edges and the lower surface experiences irradiation, the total emissivity and absorptivity of the plate are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Analysis The total emissivity of the plate can be determined using

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{5000\ \text{W/m}^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(700\ \text{K})^4} = \mathbf{0.367}$$



The total absorptivity of the plate is determined using

$$\alpha + \rho + \tau = 1 \quad \rightarrow \quad \alpha = 1 - \rho \quad \text{(for opaque surface, } \tau = 0\text{)}$$

The reflectivity of the plate is

$$\rho = \frac{G_{\text{ref}}}{G} = \frac{500}{3000} = 0.167$$

Hence, the total absorptivity of the plate is

$$\alpha = 1 - 0.167 = \mathbf{0.833}$$

Discussion The plate has a total absorptivity that is about 5 times the reflectivity.

12-83 Irradiation is on a semi-transparent medium. The medium's absorptivity, reflectivity, transmissivity, and emissivity are to be determined.

Assumptions **1** Properties are constant. **2** Kirchhoff's law is applicable.

Analysis The irradiation on the medium is

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}}$$

The irradiation absorbed by the medium is

$$G_{\text{abs}} = G - G_{\text{ref}} - G_{\text{tr}} = (520 - 160 - 130)\ \text{W/m}^2 = 230\ \text{W/m}^2$$

The absorptivity is

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{230}{520} = \mathbf{0.442}$$

The reflectivity is

$$\rho = \frac{G_{\text{ref}}}{G} = \frac{160}{520} = \mathbf{0.308}$$

The transmissivity is

$$\tau = \frac{G_{\text{tr}}}{G} = \frac{130}{520} = \mathbf{0.25}$$

From Kirchhoff's law, the emissivity of the medium is

$$\varepsilon = \alpha = \mathbf{0.442}$$

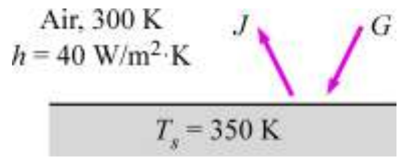
Discussion Having determined α and ρ, the transmissivity can also be determined using $\tau = 1 - \alpha - \rho = 0.25$.

12-84 An opaque horizontal plate that is well insulated on the edges and the lower surface is uniformly irradiated from above, (*a*) the irradiation on the plate, (*b*) the total reflectivity of the plate, (*c*) the emissive power of the plate, and (*d*) the total emissivity of the plate are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Properties The total absorptivity of the plate is given to be 0.40.

Analysis (*a*) Applying energy balance on the surface,



$$\begin{aligned} G &= J + \dot{q}_{\text{conv}} = J + h(T_s - T_\infty) \\ &= 4000\ \text{W/m}^2 + (40\ \text{W/m}^2\cdot\text{K})(350-300)\ \text{K} \\ &= \mathbf{6000\ W/m^2} \end{aligned}$$

(*b*) The total reflectivity of the plate is determined using

$$\alpha + \rho + \tau = 1 \quad \rightarrow \quad \rho = 1 - \alpha - \tau \qquad \text{(for opaque surface, } \tau = 0\text{)}$$

$$\rho = 1 - 0.40 - 0 = \mathbf{0.60}$$

(*c*) The emissive power of the plate is

$$J = E + G_{\text{ref}} = E + \rho G \quad \rightarrow \quad E = J - \rho G$$

$$E = 4000\ \text{W/m}^2 - (0.60)(6000\ \text{W/m}^2) = \mathbf{400\ W/m^2}$$

(*d*) The total emissivity of the plate is

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{400\ \text{W/m}^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(350\ \text{K})^4} = \mathbf{0.470}$$

Discussion The emissivity may also be determined by applying energy balance on the plate:

$$E = J - \rho G \qquad \text{or} \qquad \varepsilon\sigma T_s^4 = J - \rho G \quad \rightarrow \quad \varepsilon = \frac{J - \rho G}{\sigma T_s^4}$$

12-85 An opaque plate is being heated uniformly at the bottom and the top surface is exposed to natural convection and irradiated uniformly. The radiosity of the plate is to be determined.

Assumptions **1** The plate has a uniform temperature. **2** Heat loss from plate's side surface is negligible. **3** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$. **4** Kirchhoff's law is applicable.

Properties The emissivity of the plate is given as $\varepsilon = 0.67$.

Analysis By performing energy balance on the plate surface gives

$$G + \dot{q}_{\text{elec}} = J + \dot{q}_{\text{conv}} = J + h(T_s - T_\infty) \qquad (1)$$

The reflectivity of the plate can be determined from

$$\alpha + \rho + \tau = 1$$

From Kirchhoff's law ($\varepsilon = \alpha$) and for an opaque medium ($\tau = 0$) we have

$$\begin{aligned} \rho &= 1 - \alpha - \tau \\ &= 1 - \varepsilon \\ &= 1 - 0.67 \\ &= 0.33 \end{aligned}$$

The radiosity of the plate is given as

$$J = E + G_{\text{ref}} = E + \rho G \quad \rightarrow \quad G = \frac{J - E}{\rho} \qquad (2)$$

Substituting Eq. (2) into (1) gives

$$\frac{J - E}{\rho} + \dot{q}_{\text{elec}} = J + h(T_s - T_\infty) \quad \rightarrow \quad J = \frac{\rho h(T_s - T_\infty) - \rho \dot{q}_{\text{elec}} + E}{1 - \rho}$$

Thus,

$$\begin{aligned} J &= \frac{\rho h(T_s - T_\infty) - \rho \dot{q}_{\text{elec}} + \varepsilon \sigma T_s^4}{1 - \rho} \\ &= \frac{(0.33)(7\ \text{W/m}^2 \cdot \text{K})(80 - 7)\ \text{K} - (0.33)(1000\ \text{W/m}^2) + (0.67)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(353\ \text{K})^4}{1 - 0.33} \\ &= \mathbf{639.6\ W/m^2} \end{aligned}$$

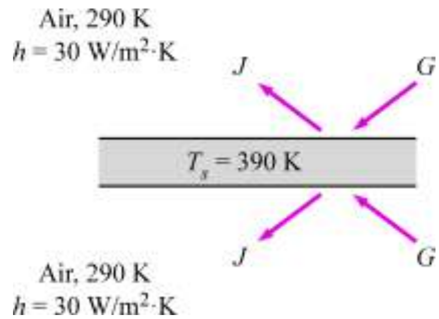
Discussion Radiosity contributed to about 56% of the total heat loss from the plate surface.

12-86 A horizontal non-opaque plate is experiencing uniform irradiation on the both upper and lower surfaces. The irradiation and emissivity of the plate are to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate has a uniform temperature. **3** The convection heat transfer coefficient is uniform.

Properties The absorptivity of the plate is given to be 0.527.

Analysis Applying energy balance on the plate, we have



$$2G = 2J + 2\dot{q}_{\text{conv}} \qquad \rightarrow \qquad G = J + h(T_s - T_\infty)$$

$$G = 4000 \text{ W/m}^2 + (30 \text{ W/m}^2 \cdot \text{K})(390 - 290) \text{ K} = \mathbf{7000 \text{ W/m}^2}$$

Applying the definition of radiosity, we have

$$J = E + G_{\text{ref}} + G_{\text{tr}} = E + \rho G + \tau G = E + (\rho + \tau)G$$

Also, we have

$$\alpha + \rho + \tau = 1 \qquad \text{or} \qquad \rho + \tau = 1 - \alpha$$

Hence,

$$J = E + (1 - \alpha)G \qquad \text{or} \qquad E = J - (1 - \alpha)G$$

Then, the emissivity of the plate is

$$\begin{aligned} \varepsilon &= \frac{E}{E_b} \\ &= \frac{J - (1-\alpha)G}{\sigma T_s^4} \\ &= \frac{[4000 - (1 - 0.527)7000] \text{ W/m}^2}{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(390 \text{ K})^4} = \mathbf{0.525} \end{aligned}$$

Discussion Since $\alpha \approx \varepsilon \approx 0.53$, the plate can be considered as a gray surface.

12-87 Irradiation is on a semi-transparent plate. A radiometer is placed above the plate and the irradiation detected by the radiometer is to be determined.

Assumptions **1** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **2** Plate surface temperature and properties are uniform. **3** Heat loss from plate's side surface is negligible. **4** Kirchhoff's law is applicable.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The irradiation on the plate is

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} \quad \rightarrow \quad G_{\text{abs}} = G - G_{\text{ref}} - G_{\text{tr}}$$

The irradiation absorbed by the medium is

$$G_{\text{abs}} = G - 0.3G - 0.5G = 500(1 - 0.3 - 0.5)\ \text{W/m}^2 = 100\ \text{W/m}^2$$

The absorptivity and reflectivity of the plate are

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{100}{500} = 0.2 \quad \text{and} \quad \rho = 0.3\ \text{(given)}$$

From Kirchhoff's law, the emissivity of the plate is

$$\varepsilon = \alpha = 0.2$$

The solid angle subtended by the radiometer when viewed from the plate is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted and reflected (radiosity) by the plate A_1 is

$$I_1 = \frac{J}{\pi} = \frac{E + G_{\text{ref}}}{\pi} = \frac{\varepsilon\sigma T_1^4 + \rho G}{\pi}$$
$$= \frac{(0.2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(350^4)\ \text{K}^4 + (0.3)(500\ \text{W/m}^2)}{\pi} = 101.91\ \text{W/m}^2$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{I_1 A_1(\cos\theta_1)(\cos\theta_2)}{L^2}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{I_1 A_1}{L^2} = \frac{(101.91\ \text{W/m}^2)(2\times10^{-4}\ \text{m}^2)}{(0.5\ \text{m})^2} = \mathbf{0.0815\ \ W/m^2}$$

Discussion The emissive power of the plate E contributed about 53% of the radiosity of the plate.

Atmospheric and Solar Radiation

12-88C The reason for different seasons is the tilt of the earth which causes the solar radiation to travel through a longer path in the atmosphere in winter, and a shorter path in summer. Therefore, the solar radiation is attenuated much more strongly in winter.

12-89C Because of different wavelengths of solar radiation and radiation originating from surrounding bodies, the surfaces usually have quite different absorptivities. Solar radiation is concentrated in the short wavelength region and the surfaces in the infrared region.

12-90C There is heat loss from both sides of the bridge (top and bottom surfaces of the bridge) which reduces temperature of the bridge surface to very low values. The relatively warm earth under a highway supply heat to the surface continuously, making the water on it less likely to freeze.

12-91C The amount of solar radiation incident on earth will decrease by a factor of

$$\text{Reduction factor} = \frac{\sigma T_{sun}{}^4}{\sigma T_{sun,new}{}^4} = \frac{5762^4}{2000^4} = 68.9$$

(or to 1.5% of what it was). Also, the fraction of radiation in the visible range would be much smaller.

12-92C The solar constant represents the rate at which solar energy is incident on a surface normal to sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun. It value is $G_s = 1353 \text{ W/m}^2$. The solar constant is used to estimate the effective surface temperature of the sun from the requirement that

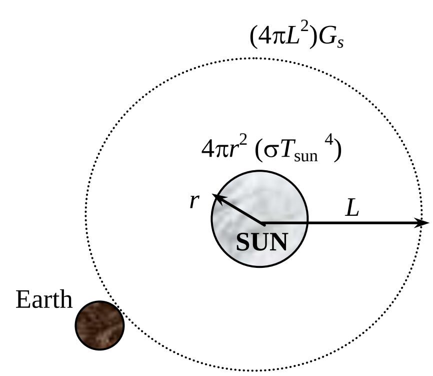
$$(4\pi L^2)G_{s1} = (4\pi r^2)\sigma T_{sun}{}^4$$

where L is the mean distance between the sun and the earth and r is the radius of the sun. If the distance between the earth and the sun doubled, the value of G_s drops to one-fourth since

$$4\pi(2L)^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = 4\pi L^2 G_{s1} \longrightarrow \mathbf{G_{s2} = \frac{G_{s1}}{4}}$$



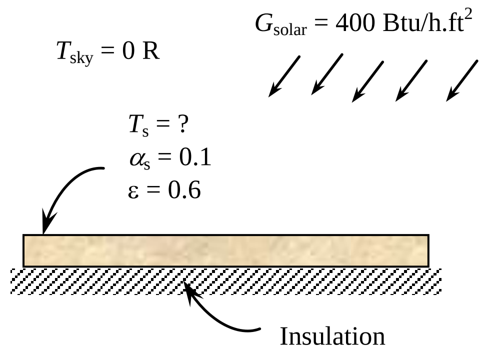
12-93C Air molecules scatter blue light much more than they do red light. This molecular scattering in all directions is what gives the sky its bluish color. At sunset, the light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, letting the red dominate.

12-94C The gas molecules and the suspended particles in the atmosphere emit radiation as well as absorbing it. Although this emission is far from resembling the distribution of radiation from a blackbody, it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy. This fictitious temperature is called the effective sky temperature T_{sky}.

12-95E A surface is exposed to solar and sky radiation. The equilibrium temperature of the surface is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to α_s = 0.10 and ε = 0.6.

Analysis The equilibrium temperature of the surface in this case is



$$\dot{q}_{net,rad} = \alpha_s G_{solar} - \varepsilon\sigma(T_s{}^4 - T_{sky}{}^4) = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s{}^4 - T_{sky}{}^4)$$

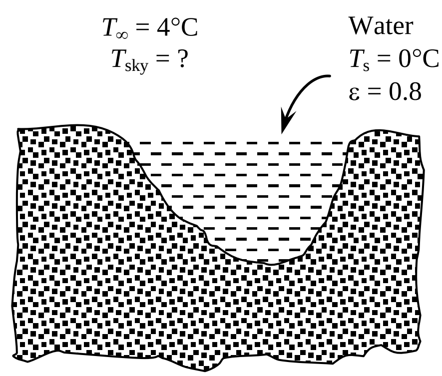
$$0.10(400\ \text{Btu/h.ft}^2) = 0.6(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)\left[T_s{}^4 - (0\ \text{R})^4\right]$$

$$T_s = \mathbf{444R}$$

12-96 Water is observed to have frozen one night while the air temperature is above freezing temperature. The effective sky temperature is to be determined.

Properties The emissivity of water is ε = 0.95 (Table A-18).

Analysis Assuming the water temperature to be 0°C, the value of the effective sky temperature is determined from an energy balance on water to be



$$h(T_{\text{air}} - T_{\text{surface}}) = \varepsilon\sigma(T_s^4 - T_{\text{sky}}^4)$$

and

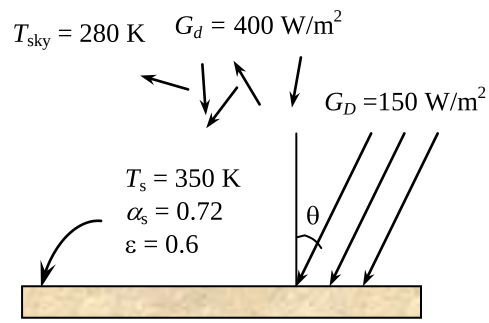
$$(18\ \text{W/m}^2\cdot{}^\circ\text{C})(4^\circ\text{C} - 0^\circ\text{C}) = 0.95(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(273\ \text{K})^4 - T_{sky}{}^4\right]$$

$$\longrightarrow T_{\text{sky}} = \mathbf{254.8K}$$

Therefore, the effective sky temperature must have been below 255 K.

12-97 A surface is exposed to solar and sky radiation. The net rate of radiation heat transfer is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.72$ and $\varepsilon = 0.6$.



Analysis The total solar energy incident on the surface is

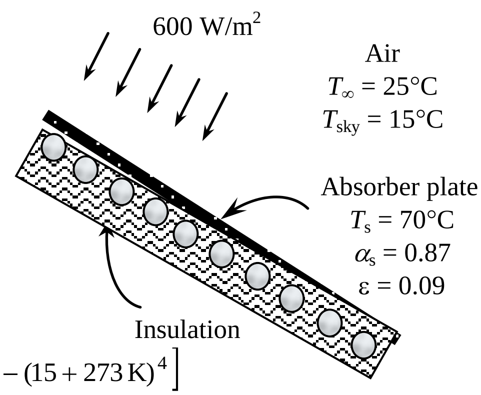
$$\begin{aligned} G_{solar} &= G_D \cos\theta + G_d \\ &= (350 \text{ W/m}^2)\cos 30° + (400 \text{ W/m}^2) \\ &= 703.1 \text{ W/m}^2 \end{aligned}$$

Then the net rate of radiation heat transfer in this case becomes

$$\begin{aligned} \dot{q}_{net,rad} &= \alpha_s G_{solar} - \varepsilon\sigma(T_s^{\,4} - T_{sky}^{\,4}) \\ &= 0.72(703.1 \text{ W/m}^2) - 0.6(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(350\text{ K})^4 - (280\text{ K})^4\right] \\ &= \mathbf{205\ W/m^2} \quad \text{(to the surface)} \end{aligned}$$

12-98 The absorber plate of a solar collector is exposed to solar and sky radiation. The net rate of solar energy absorbed by the absorber plate is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.87$ and $\varepsilon = 0.09$.



Analysis The net rate of solar energy delivered by the absorber plate to the water circulating behind it can be determined from an energy balance to be

$$\dot{q}_{net} = \dot{q}_{gain} - \dot{q}_{loss}$$

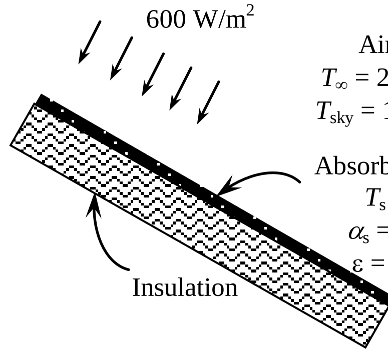
$$\dot{q}_{net} = \alpha_s G_{solar} - \left[\varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})\right]$$

Then,

$$\begin{aligned} \dot{q}_{net} &= 0.87(600 \text{ W/m}^2) - 0.09(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(70+273\text{ K})^4 - (15+273\text{ K})^4\right] \\ &\quad - (10 \text{ W/m}^2\cdot\text{K})(70°\text{C} - 25°\text{C}) \\ &= \mathbf{27.9\ W/m^2} \end{aligned}$$

Therefore, heat is gained by the plate and transferred to water at a rate of 27.9 W per m^2 surface area.

12-99 The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.



Properties The solar absorptivity and emissivity of the surface are given to α_s = 0.87 and ε = 0.09.

Analysis The backside of the absorbing plate is insulated (instead of being attached to water tubes), and thus

$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})$$

$$(0.87)(600\text{ W/m}^2) = (0.09)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\text{ K})^4\right] + (10\text{ W/m}^2\cdot\text{K})(T_s - 298\text{ K})$$

$$T_s = \mathbf{346 K}$$

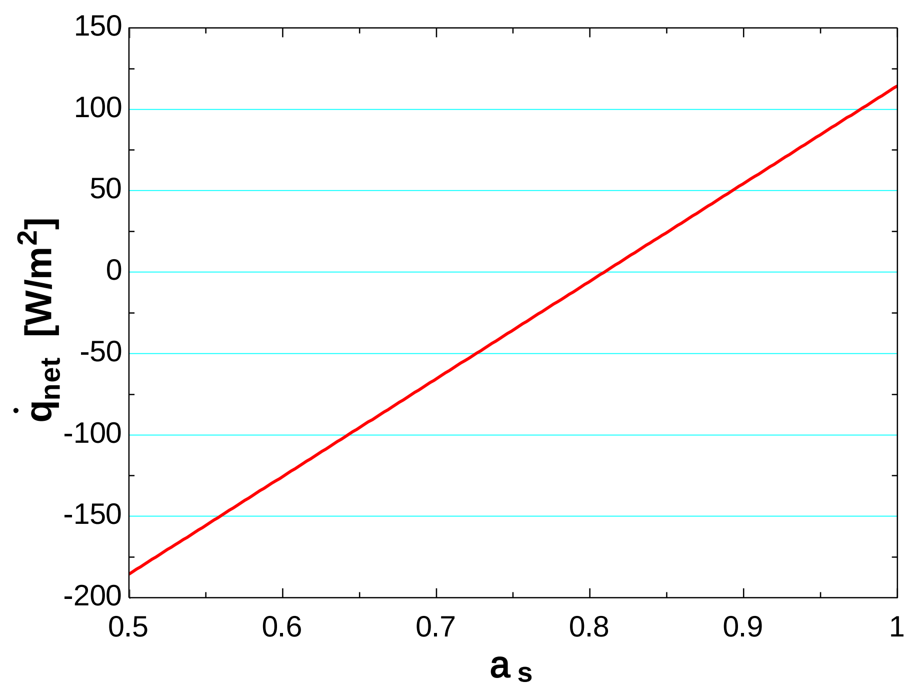
12-100 Prob. 12-98 is reconsidered. The the net rate of solar energy transferred to water as a function of the absorptivity of the absorber plate is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
alpha_s=0.87
epsilon=0.09
G_solar=600 [W/m^2]
T_air=(25+273) [K]
T_sky=(15+273) [K]
T_s=(70+273) [K]
h=10 [W/m^2-C]
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_net=q_dot_gain-q_dot_loss "energy balance"
q_dot_gain=alpha_s*G_solar
q_dot_loss=epsilon*sigma*(T_s^4-T_sky^4)+h*(T_s-T_air)
```

α_s	$\dot{q}_{net}$ [W/m^2]
0.5	-185.5
0.525	-170.5
0.55	-155.5
0.575	-140.5
0.6	-125.5
0.625	-110.5
0.65	-95.52
0.675	-80.52
0.7	-65.52
0.725	-50.52
0.75	-35.52
0.775	-20.52
0.8	-5.525
0.825	9.475
0.85	24.48
0.875	39.48
0.9	54.48
0.925	69.48
0.95	84.48
0.975	99.48
1	114.5



Special Topic: Solar Heat Gain through Windows

12-101C (*a*) The spectral distribution of solar radiation beyond the earth's atmosphere resembles the energy emitted by a black body at 5982°C, with about 39 percent in the visible region (0.4 to 0.7 μm), and the 52 percent in the near infrared region (0.7 to 3.5 μm). (*b*) At a solar altitude of 41.8°, the total energy of direct solar radiation incident at sea level on a clear day consists of about 3 percent ultraviolet, 38 percent visible, and 59 percent infrared radiation.

12-102C A window that transmits visible part of the spectrum while absorbing the infrared portion is ideally suited for minimizing the air-conditioning load since such windows provide maximum daylighting and minimum solar heat gain. The ordinary window glass approximates this behavior remarkably well.

12-103C A low-e coating on the inner surface of a window glass reduces both the (*a*) heat loss in winter and (*b*) heat gain in summer. This is because the radiation heat transfer to or from the window is proportional to the emissivity of the inner surface of the window. In winter, the window is colder and thus radiation heat loss from the room to the window is low. In summer, the window is hotter and the radiation transfer from the window to the room is low.

12-104C A device that blocks solar radiation and thus reduces the solar heat gain is called a shading device. External shading devices are more effective in reducing the solar heat gain since they intercept sun's rays before they reach the glazing. The solar heat gain through a window can be reduced by as much as 80 percent by exterior shading. *Light colored* shading devices maximize the back reflection and thus minimize the solar gain. *Dark colored* shades, on the other hand, minimize the back refection and thus maximize the solar heat gain.

12-105C The SC (shading coefficient) of a device represents the solar heat gain relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87. The shading coefficient of a 3-mm thick *clear glass* is SC = 1.0 whereas SC = 0.88 for 3-mm thick *heat absorbing glass*.

12-106C The **solar heat gain coefficient (**SHGC) is defined as the fraction of incident solar radiation that enters through the glazing. The solar heat gain of a glazing relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87, is called the **shading coefficient**. They are related to each other by

$$\mathrm{SC} = \frac{\text{Solar heat gain of product}}{\text{Solar heat gain of reference glazing}} = \frac{\mathrm{SHGC}}{\mathrm{SHGC}_{\mathrm{ref}}} = \frac{\mathrm{SHGC}}{0.87} = 1.15 \times \mathrm{SHGC}$$

For single pane clear glass window, SHGC = 0.87 and SC = 1.0.

12-107 A building at 40º N latitude has double pane heat absorbing type windows that are equipped with light colored venetian blinds. The total solar heat gains of the building through the south windows at solar noon in April for the cases of with and without the blinds are to be determined.

Assumptions The calculations are performed for an "average" day in April, and may vary from location to location.

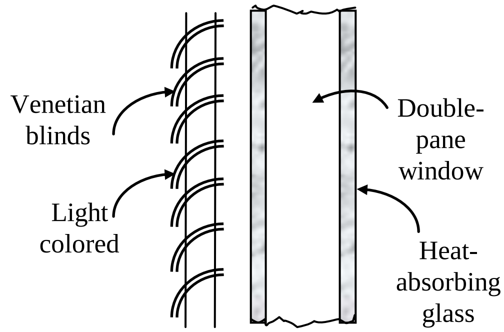
Properties The shading coefficient of a double pane heat absorbing type windows is SC = 0.58 (Table 12-5). It is given to be SC = 0.30 in the case of blinds. The solar radiation incident at a South-facing surface at 12:00 noon in April is 559 W/m^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows without the blinds is determined from Eq.12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.58 = 0.5046$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} \\ &= 0.5046(130\ \text{m}^2)(559\ \text{W/m}^2) \\ &= \mathbf{36{,}670\ \ W}\end{aligned}$$



In the case of windows equipped with venetian blinds, the SHGC and the rate of solar heat gain become

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.30 = 0.261$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} \\ &= 0.261(130\ \text{m}^2)(559\ \text{W/m}^2) \\ &= \mathbf{18{,}970\ \ W}\end{aligned}$$

Discussion Note that light colored venetian blinds significantly reduce the solar heat, and thus air-conditioning load in summers.

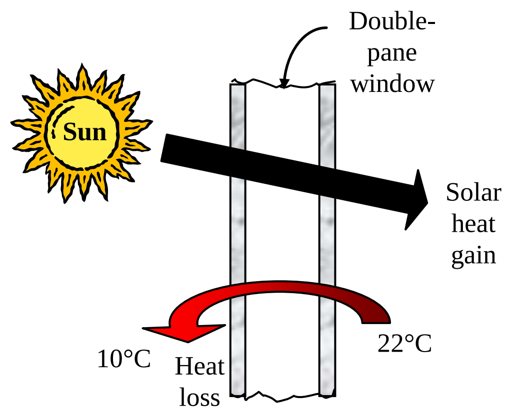
12-108 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through an east window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 12-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m^2.°C. (Table 9-6). The total solar radiation incident at an East-facing surface in January during a typical day is 1863 Wh/m^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.88 = 0.7656$$



Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC \times A_{\text{glazing}} \times q_{\text{solar, daily total}} \\ &= 0.7656(1\text{ m}^2)(1863\text{ Wh/m}^2) \\ &= \mathbf{1426\ Wh = 1.426\ kWh} \end{aligned}$$

The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}} A_{\text{window}}(T_i - T_{0,\text{ave}})(1\text{ day}) \\ &= (4.55\text{ W/m}^2\cdot{}^\circ\text{C})(1\text{ m}^2)(22-10)^\circ\text{C}(24\text{ h}) \\ &= \mathbf{1310\ Wh = 1.31\ kWh} \end{aligned}$$

Therefore, the house is loosing **less** heat than it is gaining through the East windows during a typical day in January.

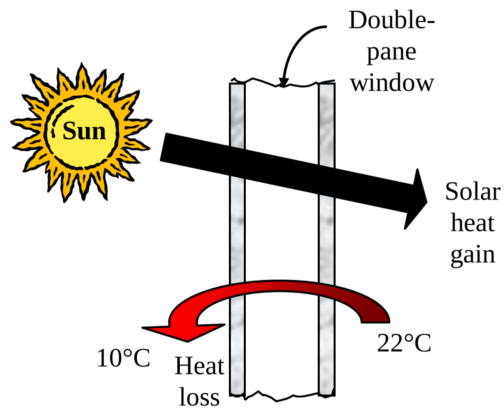
12-109 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through a South window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 12-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m^2.°C (Table 9-6). The total solar radiation incident at a South-facing surface in January during a typical day is 5897 Wh/m^2 (Table 12-5).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.88 = 0.7656$$



Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC\times A_{\text{glazing}}\times q_{\text{solar, daily total}} \\ &= 0.7656(1\text{ m}^2)(5897\text{ Wh/m}^2) \\ &= \mathbf{4515\ Wh = 4.515\ kWh} \end{aligned}$$

The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}}A_{\text{window}}(T_i - T_{0,\text{ave}})(1\text{ day}) \\ &= (4.55\text{ W/m}^2\cdot{}^\circ\text{C})(1\text{ m}^2)(22-10)^\circ\text{C}(24\text{ h}) \\ &= \mathbf{1310\ Wh = 1.31\,kWh} \end{aligned}$$

Therefore, the house is **loosing** much less heat than it is gaining through the South windows during a typical day in January.

12-110 A building located near 40º N latitude has equal window areas on all four sides. The side of the building with the highest solar heat gain in summer is to be determined.

Assumptions The shading coefficients of windows on all sides of the building are identical.

Analysis The reflective films should be installed on the side that receives the most incident solar radiation in summer since the window areas and the shading coefficients on all four sides are identical. The incident solar radiation at different windows in July are given to be (Table 12-5)

Month	Time	The daily total solar radiation incident on the surface, Wh/m^2			
		North	East	South	West
July	Daily total	1621	4313	2552	4313

Therefore, the reflective film should be installed on the **East** or **West** windows (instead of the South windows) in order to minimize the solar heat gain and thus the cooling load of the building.

12-111E A house has 1/8-in thick single pane windows with aluminum frames on a West wall. The rate of net heat gain (or loss) through the window at 3 PM during a typical day in January is to be determined.

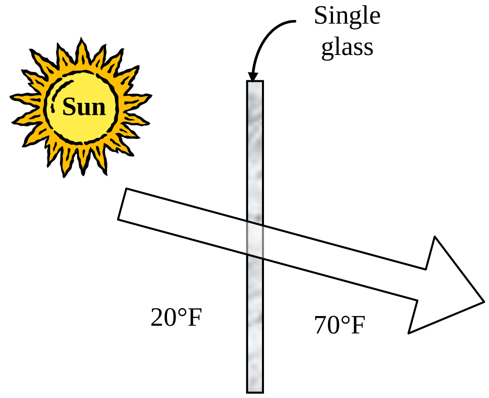
Assumptions **1** The calculations are performed for an "average" day in January. **2** The frame area relative to glazing area is small so that the glazing area can be taken to be the same as the window area.

Properties The shading coefficient of a 1/8-in thick single pane window is SC = 1.0 (Table 12-5). The overall heat transfer coefficient for 1/8-in thick single pane windows with aluminum frames is 6.63 W/m^2.°C = 1.17 Btu/h.ft^2.°F (Table 9-6). The total solar radiation incident at a West-facing surface at 3 PM in January during a typical day is 557 W/m^2 = 177 Btu/h.ft^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times1.0 = 0.87$$

The window area is: $A_{\text{window}} = (9\text{ ft})(15\text{ ft}) = 135\text{ ft}^2$



Then the rate of solar heat gain through the window at 3 PM becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain,3 PM}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar,3 PM}}\\ &= 0.87(135\text{ ft}^2)(177\text{ Btu/h.ft}^2)\\ &= 20{,}789\text{ Btu/h}\end{aligned}$$

The rate of heat loss through the window at 3 PM is

$$\begin{aligned}\dot{Q}_{\text{loss,window}} &= U_{\text{window}}A_{\text{window}}(T_i - T_0)\\ &= (1.17\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(135\text{ ft}^2)(70-20)°\text{F}\\ &= 7898\text{ Btu/h}\end{aligned}$$

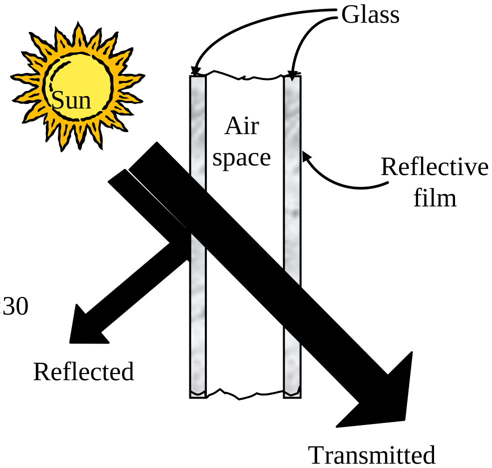
The house will be gaining heat at 3 PM since the solar heat gain is larger than the heat loss. The rate of net heat gain through the window is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{solar gain,3 PM}} - \dot{Q}_{\text{loss,window}} = 20{,}789 - 7898 = \mathbf{12{,}890\ Btu/h}$$

Discussion The actual heat gain will be less because of the area occupied by the window frame.

12-112 The net annual cost savings due to installing reflective coating on the West windows of a building and the simple payback period are to be determined.

Assumptions **1** The calculations given below are for an average year. **2** The unit costs of electricity and natural gas remain constant.



Analysis Using the daily averages for each month and noting the number of days of each month, the total solar heat flux incident on the glazing during summer and winter months are determined to be

$$Q_{\text{solar, summer}} = 4.24\times30 + 4.16\times31 + 3.93\times31 + 3.48\times30$$
$$= 482 \text{ kWh/year}$$

$$Q_{\text{solar, winter}} = 2.94\times31 + 2.33\times30 + 2.07\times31 + 2.35\times31 + 3.03\times28 + 3.62\times31 + 4.00\times30$$
$$= 615 \text{ kWh/year}$$

Then the decrease in the annual cooling load and the increase in the annual heating load due to reflective film become

$$\text{Cooling load decrease} = Q_{\text{solar, summer}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (482 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.35)$$
$$= 12{,}031 \text{ kWh/year}$$

$$\text{Heating load increase} = Q_{\text{solar, winter}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (615 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.35)$$
$$= 15{,}350 \text{ kWh/year} = 523.7 \text{ therms/year}$$

since 1 therm = 29.31 kWh. The corresponding decrease in cooling costs and increase in heating costs are

Decrease in cooling costs = (Cooling load decrease)(Unit cost of electricity)/COP

= (12,031 kWh/year)($0.09/kWh)/3.2 = $338.4/year

Increase in heating costs = (Heating load increase)(Unit cost of fuel)/Efficiency

= (523.7 therms/year)($0.45/therm)/0.90 = $261.9/year

Then the net annual cost savings due to reflective films become

Cost Savings = Decrease in cooling costs - Increase in heating costs = $338.4 – 261.9 = **$76.5/year**

The implementation cost of installing films is

$$\text{Implementation Cost} = (\$20/\text{m}^2)(60 \text{ m}^2) = \$1200$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$1200}{\$76.5/\text{year}} = \mathbf{15.7 \text{ years}}$$

Discussion The reflective films will pay for themselves in this case in about 16 years, which is unacceptable to most manufacturers since they are not usually interested in any energy conservation measure which does not pay for itself within 3 years.

12-113 A house located at 40ºN latitude has ordinary double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a double pane window with 6-mm thick glasses is SC = 0.82 (Table 12-5). The incident radiation at different windows at different times are given as (Table 12-4)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.12-57 to be

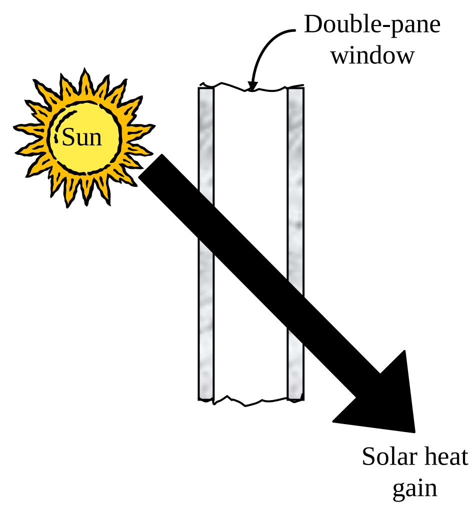
$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.82 = 0.7134$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}}$$
$$= 0.7134\times A_{\text{glazing}}\times\dot{q}_{\text{solar,incident}}$$

Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:



$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{334\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(4\text{ m}^2)\times(138\text{ W/m}^2) = \mathbf{394\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{334\ W}$$

East wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{3001\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{488\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{1084\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(8\text{ m}^2)\times(395\text{ W/m}^2) = \mathbf{2254\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{1084\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.7134\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{488\ W}$$
$$\dot{Q}_{\text{solar gain,12:00}} = 0.7134\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain,15:00}} = 0.7134\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{3001\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain,North}} = 0.7134\times(4\text{ m}^2)\times(446\text{ Wh/m}^2\cdot\text{day}) = 1273\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain,East}} = 0.7134\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 7974\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain,South}} = 0.7134\times(8\text{ m}^2)\times(5897\text{ Wh/m}^2\cdot\text{day}) = 33{,}655\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain,West}} = 0.7134\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 7974\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 1273+7974+33{,}655+7974 = 58{,}876\text{ Wh/day} \cong \mathbf{58.9\ kWh/day}$$

12-114 A house located at 40º N latitude has gray-tinted double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a gray-tinted double pane window with 6-mm thick glasses is SC = 0.58 (Table 12-5). The incident radiation at different windows at different times are given as (Table 12-4)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.58 = 0.5046$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}} = 0.5046 \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}}$$

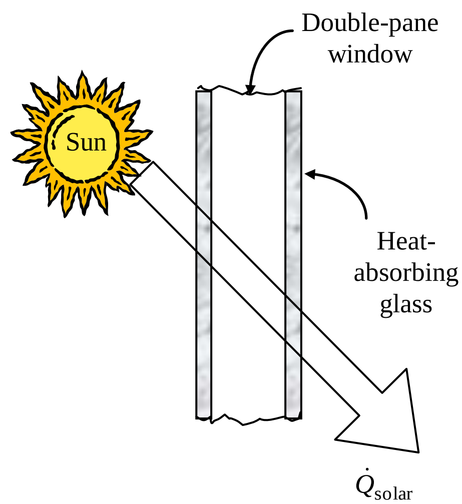
Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046 \times (4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\ W}$$

$$\dot{Q}_{\text{solar gain,1200}} = 0.5046 \times (4\text{ m}^2)\times(138\text{ W/m}^2) = \mathbf{279\ W}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046 \times (4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\ W}$$



East wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046 \times (6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\ W}$$

$$\dot{Q}_{\text{solar gain,1200}} = 0.5046 \times (6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{451\ W}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046 \times (6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046 \times (8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{767\ W}$$

$$\dot{Q}_{\text{solar gain,1200}} = 0.5046 \times (8\text{ m}^2)\times(395\text{ W/m}^2) = \mathbf{1595\ W}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046 \times (8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{767\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain,9:00}} = 0.5046 \times (6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\ W}$$

$$\dot{Q}_{\text{solar gain,1200}} = 0.5046 \times (6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{451\ W}$$

$$\dot{Q}_{\text{solar gain,15:00}} = 0.5046 \times (6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain,North}} = 0.5046 \times (4\text{ m}^2)\times(446\text{ Wh/m}^2\cdot\text{day}) = 900\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,East}} = 0.5046 \times (6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,South}} = 0.5046 \times (8\text{ m}^2)\times(5897\text{ Wh/m}^2\cdot\text{day}) = 23{,}805\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,West}} = 0.5046 \times (6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 900 + 5640 + 23{,}805 + 5640 = 35{,}985\text{ Wh/day} = \mathbf{35.895\ kWh/day}$$

Review Problems

12-115 A hole is drilled in a spherical cavity. The maximum rate of radiation energy streaming through the hole is to be determined.

Analysis The maximum rate of radiation energy streaming through the hole is the blackbody radiation, and it can be determined from

$$E = A\sigma T^4 = \pi(0.0025\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(600\,\text{K})^4 = \mathbf{0.144\,W}$$

The result would not change for a different diameter of the cavity.

12-116 The wavelengths at maximum emission of radiation for both daylight and incandescent light are to be determined.

Assumptions **1** The sun and the incandescent light filament behave as black bodies.

Analysis The wavelength at maximum emission of radiation can be determined using the Wien's displacement law:

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

For daylight,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{T} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{5800\,\text{K}} = \mathbf{0.50\,\mu m} \qquad \text{(daylight)}$$

For incandescent liht,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{T} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2800\,\text{K}} = \mathbf{1.04\,\mu m} \qquad \text{(incandescent light)}$$

Discussion For daylight, the peak of emissive power is at 0.50 μm, which is within the visible spectrum. On the other hand, the peak of emissive power for incandescent light (1.04 μm) is outside the visible spectrum.

12-117 The fraction of the incident solar radiation that is absorbed by the human skin is to be determined.

Assumptions **1** The sun behaves as a blackbody.

Analysis For solar radiation (T = 5800 K), the blackbody radiation functions corresponding to λ_1 = 0.517 μm to λ_2 = 1.552 μm are determined from Table 12-2 to be

$$\lambda_1 T = (0.517\ \mu\text{m})(5800\ \text{K}) = 3000\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (1.552\ \mu\text{m})(5800\ \text{K}) = 9000\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_2} = 0.890029$$

The fraction of radiation emitted for $0 \le \lambda < \lambda_1$ is

$$f_{0-\lambda_1} = 0.273232$$

The fraction of radiation emitted for $\lambda_1 \le \lambda < \lambda_2$ is

$$f_{\lambda_1-\lambda_2} = 0.890029 - 0.273232 = 0.616797$$

The fraction of radiation emitted for $\lambda_2 \le \lambda < \infty$ is

$$f_{\lambda_2-\infty} = 1 - 0.890029 = 0.109971$$

Thus, the fraction of the incident solar radiation that is absorbed by the human skin is

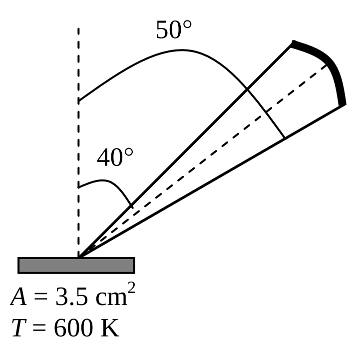
$$\begin{aligned} 1.0(f_{0-\lambda_1}) + 0.5(f_{\lambda_1-\lambda_2}) + 1.0(f_{\lambda_2-\infty}) &= 1.0(0.273232) + 0.5(0.616797) + 1.0(0.109971) \\ &= \mathbf{0.6916} \end{aligned}$$

Discussion The calculation shows that human skin absorbs about 69% of the incident solar radiation.

12-118 A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.

Assumptions Surface A emits diffusely as a blackbody.

Analysis The rate of radiation emission from a surface per unit surface area in the direction (θ,ϕ) is given as



$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

The total rate of radiation emission through the band between 40° and 50° can be expressed as

$$\begin{aligned} E &= \int_{\phi=0}^{2\pi}\int_{\theta=40}^{50} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b(0.1736\pi) \\ &= \frac{\sigma T^4}{\pi}(0.1736\pi) = 0.1736\sigma T^4 \end{aligned}$$

since the blackbody radiation intensity is constant (I_b = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 50 - \sin^2 40) = 0.1736\pi$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 3.5 cm^2 in the specified band becomes

$$\dot{Q}_e = EdA = 0.1736\sigma T^4 dA = 0.1736 \times (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(600\ \text{K})^4(3.5\times10^{-4}\ \text{m}^2) = \mathbf{0.446\ W}$$

12-119 The intensity of radiation emitting from a surface is given. The emissive power from the surface into the surrounding hemisphere and the rate of radiation emission from the surface are to be determined.

Assumptions **1** The intensity is a function of both the azimuth angle ϕ and the zenith angle θ.

Analysis The emissive power from surface A can be determined by integration as

$$
\begin{aligned}
E &= \int_{\text{hemisphere}} dE = \int_0^{2\pi}\int_0^{\pi/2} I_e(\theta,\ \phi)\cos\theta\sin\theta\, d\theta\, d\phi \\
&= \int_0^{2\pi}\int_0^{\pi/2} 100\phi\cos^2\theta\ \sin\theta\, d\theta\, d\phi \\
&= \int_0^{2\pi} 100\phi\left[-\frac{(\cos\theta)^3}{3}\right]_0^{\pi/2} d\phi \\
&= \int_0^{2\pi}\frac{100\phi}{3}\, d\phi \\
&= \left[\frac{100\phi^2}{6}\right]_0^{2\pi} \\
&= \frac{200\pi^2}{3} = \mathbf{658\ W/m^2}
\end{aligned}
$$

Then, the rate of radiation emission from the surface is

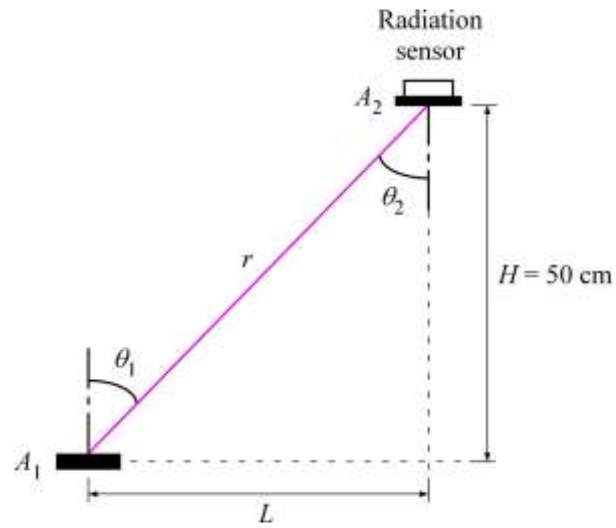
$$\dot{Q}_e = AE = (3\times10^{-4}\ \text{m}^2)(658\ \text{W/m}^2) = \mathbf{0.197\ W}$$

Discussion The rate of radiation emission from the black surface, which is a diffuse emitter, is simply $\dot{Q}_e = A\sigma T^4$.

12-120 A radiation sensor is measuring radiation rate emitted by another surface (A_1). The distance at which the sensor is measuring two-thirds of the radiation rate corresponding to the position of A_1 directly under the sensor is to be determined.

Assumptions **1** The surface A_1 emits diffusely as a blackbody. **2** Both surface A_1 and sensor can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{A_2 H}{r^2 r} = \frac{A_2 H}{(H^2 + L^2)^{3/2}} \qquad (1)$$

Note that

$$r = (H^2 + L^2)^{1/2}$$

and

$$\cos\theta_1 = \cos\theta_2 = \frac{H}{r} = \frac{H}{(H^2 + L^2)^{1/2}}$$

Then, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} = I_1 A_1 \frac{H}{(H^2 + L^2)^{1/2}}\omega_{2-1} \qquad (2)$$

Substituting Eq. (1) into (2) yields

$$\dot{Q}_{1-2} = I_1 A_1 A_2 \frac{H^2}{(H^2 + L^2)^2}$$

Also, when the surface A_1 is positioned directly under the sensor at $L = 0$, we have

$$\dot{Q}_{1-2,\,0} = I_1 A_1 A_2 \frac{H^2}{H^4} = I_1 A_1 A_2 \frac{1}{H^2}$$

Hence, the distance L at which the sensor is measuring two-thirds of the radiation rate emitted from surface A_1 corresponding to the position directly under the sensor at $L = 0$ can be determined as

$$\frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \frac{I_1 A_1 A_2 \dfrac{H^2}{(H^2 + L^2)^2}}{I_1 A_1 A_2 \dfrac{1}{H^2}} = \frac{2}{3} \qquad \rightarrow \qquad \frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \left(\frac{H^2}{H^2 + L^2}\right)^2 = \frac{2}{3}$$

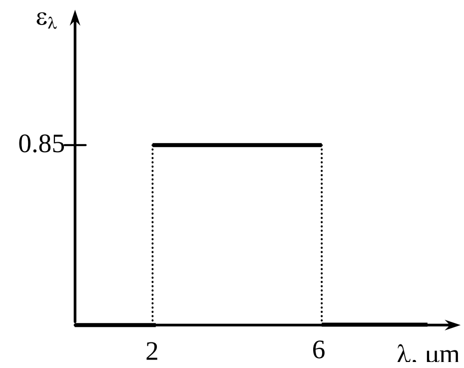
$$L = 0.4741H = 0.4741(0.5\,\text{m}) = \mathbf{0.237\,m}$$

Discussion In this orientation of the radiation sensor and surface A_1, the $(\dot{Q}_{1-2} / \dot{Q}_{1-2,\,0})$ ratio is simply expressed as

$$\frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \left(\frac{H^2}{H^2 + L^2}\right)^2 = \left(\frac{H}{r}\right)^4$$

12-121 The variation of emissivity of an opaque surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from



$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (2\ \mu\text{m})(1500\text{ K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (6\ \mu\text{m})(1500\text{ K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.890029$$

and

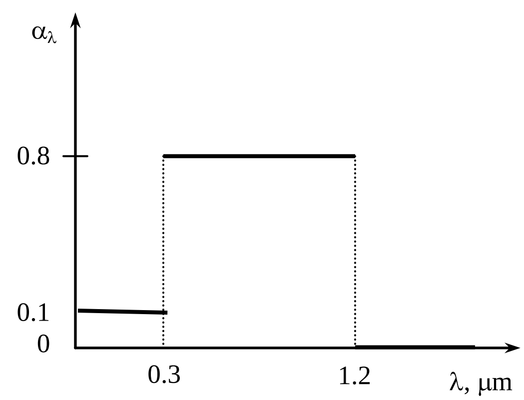
$$\varepsilon = (0.0)(0.273232) + (0.85)(0.890029 - 0.273232) + (0.0)(1 - 0.890029) = \mathbf{0.5243}$$

Then the emissive flux of the surface becomes

$$E = \varepsilon\sigma T^4 = (0.5243)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(1500\text{ K})^4 = \mathbf{150{,}500 W/m^2}$$

12-122 The variation of absorptivity of a surface with wavelength is given. The average absorptivity of the surface is to be determined for two source temperatures.

Analysis (*a*) <u>T = 1000 K</u>. The average absorptivity of the surface can be determined from



$$\alpha(T) = \alpha_1 f_{0\text{-}\lambda_1} + \alpha_2 f_{\lambda_1\text{-}\lambda_2} + \alpha_3 f_{\lambda_2\text{-}\infty}$$
$$= \alpha_1 f_{\lambda_1} + \alpha_2 (f_{\lambda_2} - f_{\lambda_1}) + \alpha_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (0.3\ \mu\text{m})(1000\text{ K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T = (1.2\ \mu\text{m})(1000\text{ K}) = 1200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.002134$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and

$$\alpha = (0.1)0.0 + (0.8)(0.002134 - 0.0) + (0.0)(1 - 0.002134) = \mathbf{0.00171}$$

(*a*) <u>T = 3000 K</u>.

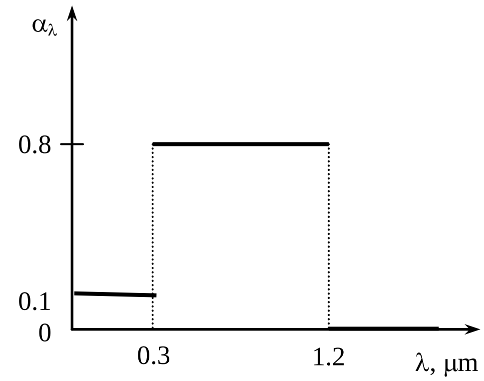
$$\lambda_1 T = (0.3\ \mu\text{m})(3000\text{ K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.000169$$

$$\lambda_2 T = (1.2\ \mu\text{m})(3000\text{ K}) = 3600\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.403607$$

$$\alpha = (0.1)0.000169 + (0.8)(0.403607 - 0.000169) + (0.0)(1 - 0.403607) = \mathbf{0.323}$$

12-123 The variation of absorptivity of a surface with wavelength is given. The surface receives solar radiation at a specified rate. The solar absorptivity of the surface and the rate of absorption of solar radiation are to be determined.

Analysis For solar radiation, T = 5800 K. The solar absorptivity of the surface is



$$\lambda_1 T = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \rightarrow f_{\lambda_1} = 0.033454$$
$$\lambda_2 T = (1.2\ \mu\text{m})(5800\ \text{K}) = 6960\ \mu\text{mK} \rightarrow f_{\lambda_2} = 0.805713$$

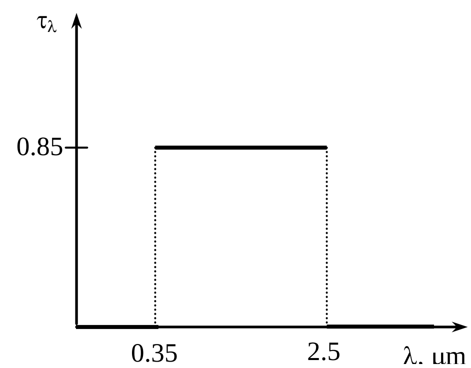
$$\alpha = (0.1)0.033454 + (0.8)(0.805713 - 0.033454) + (0.0)(1 - 0.805713)$$
$$= \mathbf{0.6212}$$

The rate of absorption of solar radiation is determined from

$$E_{\text{absorbed}} = \alpha I = 0.6212(470\ \text{W/m}^2) = \mathbf{292\ W/m^2}$$

12-124 The variation of transmissivity of glass with wavelength is given. The transmissivity of the glass for solar radiation and for light are to be determined.

Analysis For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from



$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2(f_{\lambda_2} - f_{\lambda_1}) + \tau_3(1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (0.35\ \mu\text{m})(5800\ \text{K}) = 2030\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.071852$$
$$\lambda_2 T = (2.5\ \mu\text{m})(5800\ \text{K}) = 14{,}500\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.966440$$

and

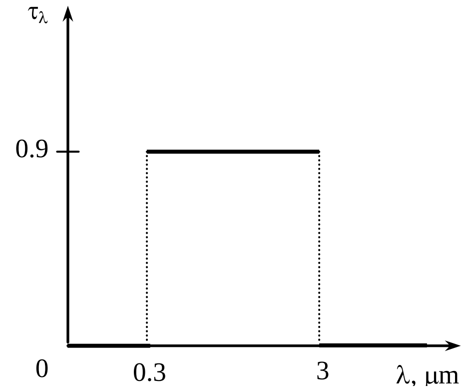
$$\tau = (0.0)(0.071852) + (0.85)(0.966440 - 0.071852) + (0.0)(1 - 0.966440) = \mathbf{0.760}$$

For light, we take T = 300 K. Repeating the calculations at this temperature we obtain

$$\lambda_1 T = (0.35\ \mu\text{m})(300\ \text{K}) = 105\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.00$$
$$\lambda_2 T = (2.5\ \mu\text{m})(300\ \text{K}) = 750\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000012$$

$$\tau = (0.0)(0.00) + (0.85)(0.000012 - 0.00) + (0.0)(1 - 0.000012) = \mathbf{0.00001}$$

12-125 The spectral transmissivity of a glass cover used in a solar collector is given. Solar radiation is incident on the collector. The solar flux incident on the absorber plate, the transmissivity of the glass cover for radiation emitted by the absorber plate, and the rate of heat transfer to the cooling water are to be determined.



Analysis (*a*) For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$

and

$$\tau = (0.0)(0.033454) + (0.9)(0.978746 - 0.033454) + (0.0)(1 - 0.978746) = 0.851$$

Since the absorber plate is black, all of the radiation transmitted through the glass cover will be absorbed by the absorber plate and therefore, the solar flux incident on the absorber plate is same as the radiation absorbed by the absorber plate:

$$E_{\text{abs.plate}} = \tau I = 0.851(950\ \text{W/m}^2) = \mathbf{808.5\ W/m^2}$$

(*b*) For radiation emitted by the absorber plate, we take T = 300 K, and calculate the transmissivity as follows:

$$\lambda_1 T = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000169$$

$$\tau = (0.0)(0.0) + (0.9)(0.000169 - 0.0) + (0.0)(1 - 0.000169) = \mathbf{0.00015}$$

(*c*) The rate of heat transfer to the cooling water is the difference between the radiation absorbed by the absorber plate and the radiation emitted by the absorber plate, and it is determined from

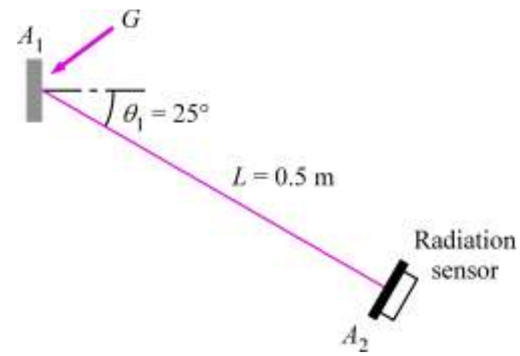
$$\dot{Q}_{\text{water}} = (\tau_{\text{solar}} - \tau_{\text{room}}) I = (0.851 - 0.00015)(950\ \text{W/m}^2) = \mathbf{808.3\ W/m^2}$$

12-126 In a configuration involving a small opaque surface A_1 and a radiation sensor, the rate at which radiation emitted from A_1 that is intercepted by the sensor is to be determined.

Assumptions **1** Surface A_1 is an opaque, diffuse emitter and reflector. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} = \frac{A_2}{L^2} = \frac{1\times10^{-4}\ \text{m}^2}{(0.5\ \text{m})^2} = 4\times10^{-4}\ \text{sr} \qquad (\text{for } \theta_2 = 0°)$$



The radiosity of surface A_1 is expressed as

$$\begin{aligned} J &= E + G_{\text{ref}} \\ &= E + \rho G \\ &= \varepsilon E_b + (1-\alpha)G \end{aligned}$$

where

$$\alpha + \rho + \tau = 1 \qquad \rightarrow \qquad \rho = 1-\alpha \qquad (\text{for opaque surface, } \tau = 0)$$

Hence, the radiosity can be calculated as

$$\begin{aligned} J &= \varepsilon\sigma T_1^4 + (1-\alpha)G \\ &= (0.7)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(400\ \text{K})^4 + (0.5)(2000\ \text{W/m}^2) \\ &= 2016\ \text{W/m}^2 \end{aligned}$$

Since surface A_1 is a diffuse emitter and reflector, the sum of the emitted and reflected intensities is

$$I_{e+r} = \frac{J}{\pi} = \frac{2016\ \text{W/m}^2}{\pi} = 641.7\ \text{W/m}^2\cdot\text{sr}$$

Therefore, the rate at which radiation emitted from A_1 that is intercepted by the sensor is

$$\begin{aligned} \dot{Q}_{1-2} &= I_{e+r}(A_1 \cos\theta_1)\omega_{2-1} \\ &= (641.7\ \text{W/m}^2\cdot\text{sr})(3\times10^{-4}\ \text{m}^2)\cos 25°(4\times10^{-4}\ \text{sr}) \\ &= \mathbf{6.98\times10^{-5}\ W} \end{aligned}$$

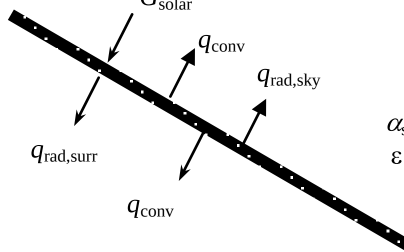
Discussion From the radiation rate intercepted by the sensor, the irradiation on the sensor can be calculated to be

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{6.98\times10^{-5}\ \text{W}}{1\times10^{-4}\ \text{m}^2} = 0.698\ \text{W/m}^2$$

12-127 Solar radiation is incident on front surface of a plate. The equilibrium temperature of the plate is to be determined.

Assumptions The plate temperature is uniform.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.63$ and $\varepsilon = 0.93$.



Analysis The solar radiation is

$$G_{\text{solar}} = G_{\text{direct}} \cos\alpha + G_{\text{diffuse}}$$
$$= (300 \text{ W/m}^2)\cos(30°) + 250 \text{ W/m}^2 = 509.8 \text{ W/m}^2$$

The front surface is exposed to solar and sky radiation and convection while the back surface is exposed to convection and radiation with the surrounding surfaces. An energy balance can be written as

$$\dot{q}_{\text{in}} = \dot{q}_{\text{out}}$$
$$\alpha_s G_{\text{solar}} + \varepsilon\sigma T_{\text{sky}}^4 + \varepsilon\sigma T_{\text{surr}}^4 = 2\varepsilon\sigma T_s^4 + 2h(T_s - T_{\text{air}})$$

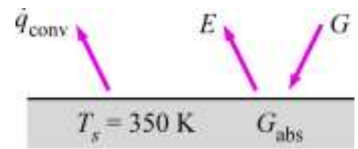
Substituting,

$$(0.63)(509.8) + (0.93)(5.67\times10^{-8})(-33+273)^4 + (0.93)(5.67\times10^{-8})(5+273)^4$$
$$= 2(0.93)(5.67\times10^{-8})T_s^4 + 2(20 \text{ W/m}^2\cdot\text{K})(T_s - 278) \longrightarrow T_s = 281.7\text{ K} = \mathbf{8.7°C}$$

12-128 A horizontal opaque flat plate is well insulated on the edges and the lower surface is experiencing irradiation and heat loss by convection. The absorptivity, reflectivity, and emissivity of the plate are to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate has a uniform temperature. **3** The plate is well insulated on the edges and the lower surface.

Analysis The irradiation on the plate is



$$G = \frac{5000\ \text{W}}{5\ \text{m}^2} = 1000\ \text{W/m}^2$$

The irradiation absorbed by the plate is

$$G_{\text{abs}} = \frac{4000\ \text{W}}{5\ \text{m}^2} = 800\ \text{W/m}^2$$

The convection heat flux is

$$\dot{q}_{\text{conv}} = \frac{500\ \text{W}}{5\ \text{m}^2} = 100\ \text{W/m}^2$$

Applying energy balance on the surface, the emissive power is

$$E = G_{\text{abs}} - \dot{q}_{\text{conv}} = 800\ \text{W/m}^2 - 100\ \text{W/m}^2 = 700\ \text{W/m}^2$$

Hence, the absorptivity of the plate is

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{800\ \text{W/m}^2}{1000\ \text{W/m}^2} = \mathbf{0.80}$$

Then, the reflectivity of the plate is determined using

$$\alpha + \rho + \tau = 1 \qquad \rightarrow \qquad \rho = 1 - \alpha \qquad \text{(for opaque surface, } \tau = 0\text{)}$$

$$\rho = 1 - 0.80 = \mathbf{0.20}$$

Finally, the emissivity of the plate is

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{700\ \text{W/m}^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(350\ \text{K})^4} = \mathbf{0.823}$$

Discussion Since the plate is opaque, that means it is reflecting $G_{ref} = G - G_{abs} = 200$ W/m^2 of the irradiation.

Fundamentals of Engineering (FE) Exam Problems

12-129 Consider a surface at -5ºC in an environment at 25ºC. The maximum rate of heat that can be emitted from this surface by radiation is

(a) 0 W/m^2 (b) 155 W/m^2 (c) 293 W/m^2 (d) 354 W/m^2 (e) 567 W/m^2

Answer (c) 293 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=-5 [C]
T_infinity=25 [C]
sigma=5.67E-8 [W/m^2-K^4]
E_b=sigma*(T+273)^4
"Some Wrong Solutions with Common Mistakes"
W1_E_b=sigma*T^4 "Using C unit for temperature"
W2_E_b=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
```

12-130 Consider a surface at 500 K. The spectral blackbody emissive power at a wavelength of 50 μm is

(a) 1.54 W/m^2·μm (b) 26.3 W/m^2·μm (c) 108.4 W/m^2·μm (d) 2750 W/m^2·μm (e) 8392 W/m^2·μm

Answer (a) 1.54 W/m^2·μm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=500 [K]
lambda=50 [micrometer]
C1=3.742E8 [W-micrometer^4/m^2]
C2=1.439E4 [micrometer-K]
E_b_lambda=C1/(lambda^5*(exp(C2/(lambda*T))-1))
```

12-131 The wavelength at which the blackbody emissive power reaches its maximum value at 300 K is

(a) 5.1 μm (b) 9.7 μm (c) 15.5 μm (d) 38.0 μm (e) 73.1 μm

Answer (b) 9.7 μm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=300 [K]
lambda*T=2897.8 [micrometer-K] "Wien's displacement law"
```

12-132 A surface absorbs 10% of radiation at wavelengths less than 3 μm and 50% of radiation at wavelengths greater than 3 μm. The average absorptivity of this surface for radiation emitted by a source at 3000 K is

(a) 0.14 (b) 0.22 (c) 0.30 (d) 0.38 (e) 0.42

Answer (a) 0.14

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Abs1=0.1
Abs2=0.5
T=3000
Wave= 3
LT=Wave*T
F1=0.890029 "The radiation fraction corresponding to lamda-T = 9000, from Table 12-2"
Abs =F1*Abs1+(1-F1)*Abs2
```

12-133 A surface at 300°C has an emissivity of 0.7 in the wavelength range of 0-4.4 μm and 0.3 over the rest of the wavelength range. At a temperature of 300°C, 19 percent of the blackbody emissive power is in wavelength range up to 4.4 μm. The total emissivity of this surface is

(a) 0.300 (b) 0.376 (c) 0.624 (d) 0.70 (e) 0.50

Answer (b) 0.376

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
f=0.19
e1=0.7
e2=0.3
e=f*e1+(1-f)*e2
```

12-134 Consider a 4-cm-diameter and 6-cm-long cylindrical rod at 1000 K. If the emissivity of the rod surface is 0.75, the total amount of radiation emitted by all surfaces of the rod in 20 min is

(a) 43 kJ (b) 385 kJ (c) 434 kJ (d) 513 kJ (e) 684 kJ

Answer (d) 513 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
L=0.06 [m]
T=1000 [K]
epsilon=0.75
time=20*60 [s]
sigma=5.67E-8 [W/m^2-K^4]
A_s=2*pi*D^2/4+pi*D*L
q_dot_emission=epsilon*sigma*T^4
Q_emission=Q_dot_emission*A_s*time

"Some Wrong Solutions with Common Mistakes"
W1_Q_emission=q_dot_emission "Using rate of emission as the answer"
W2_A_s=pi*D*L "Ignoring bottom and top surfaces of the rod"
W2_Q_emission=q_dot_emission*W2_A_s*time
W3_q_dot_emission=sigma*T^4 "Assuming the surface to be a blackbody"
W3_Q_emission=W3_q_dot_emission*A_s*time
```

12-135 Solar radiation is incident on a semi-transparent body at a rate of 500 W/m^2. If 150 W/m^2 of this incident radiation is reflected back and 225 W/m^2 is transmitted across the body, the absorptivity of the body is

(a) 0 (b) 0.25 (c) 0.30 (d) 0.45 (e) 1

Answer (b) 0.25

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G=500 [W/m^2]
G_ref=150 [W/m^2]
G_tr=225 [W/m^2]
G_abs=G-G_ref-G_tr
alpha=G_abs/G

"Some Wrong Solutions with Common Mistakes"
W1_alpha=G_ref/G "Definition for reflectivity"
W2_alpha=G_tr/G "Definition for transmissivity"
```

12-136 Solar radiation is incident on an opaque surface at a rate of 400 W/m^2. The emissivity of the surface is 0.65 and the absorptivity to solar radiation is 0.85. The convection coefficient between the surface and the environment at 25ºC is 6 W/m^2·ºC. If the surface is exposed to atmosphere with an effective sky temperature of 250 K, the equilibrium temperature of the surface is

(a) 281 K (b) 298 K (c) 303 K (d) 317 K (e) 339 K

Answer (d) 317 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G_solar=400 [W/m^2]
epsilon=0.65
alpha_s=0.85
h=6 [W/m^2-C]
T_infinity=25[C]+273 [K]
T_sky=250 [K]
sigma=5.67E-8 [W/m^2-K^4]
E_in=E_out
E_in=alpha_s*G_solar+epsilon*sigma*T_sky^4
E_out=epsilon*sigma*T_s^4+h*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
W1_E_in=W1_E_out "Ignoring atmospheric radiation"
W1_E_in=alpha_s*G_solar
W1_E_out=epsilon*sigma*W1_T_s^4+h*(W1_T_s-T_infinity)
W2_E_in=W2_E_out  "Ignoring convection heat transfer"
W2_E_in=alpha_s*G_solar+epsilon*sigma*T_sky^4
W2_E_out=epsilon*sigma*W2_T_s^4
```

12-137 A surface is exposed to solar radiation. The direct and diffuse components of solar radiation are 350 and 250 W/m^2, and the direct radiation makes a 35º angle with the normal of the surface. The solar absorptivity and the emissivity of the surface are 0.24 and 0.41, respectively. If the surface is observed to be at 315 K and the effective sky temperature is 256 K, the net rate of radiation heat transfer to the surface is

(a) -129 W/m^2 (b) -44 W/m^2 (c) 0 W/m^2 (d) 129 W/m^2 (e) 537 W/m^2

Answer (c) 0 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G_direct=350 [W/m^2]
G_diffuse=250 [W/m^2]
theta=35 [degrees]
alpha_s=0.24
epsilon=0.41
T_s=315 [K]
T_sky=256 [K]
sigma=5.67E-8 [W/m^2-K^4]
G_solar=G_direct*cos(theta)+G_diffuse
q_dot_net=alpha_s*G_solar+epsilon*sigma*(T_sky^4-T_s^4)

"Some Wrong Solutions with Common Mistakes"
W1_q_dot_net=G_solar "Using solar radiation as the answer"
W2_q_dot_net=alpha_s*G_solar "Using absorbed solar radiation as the answer"
W3_q_dot_net=epsilon*sigma*(T_sky^4-T_s^4) "Ignoring solar radiation"
```

12-138 ….. 12-139 Design and Essay Problems