Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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McGraw-Hill, 2015

Chapter 9
NATURAL CONVECTION

(Last updated August 1, 2015)

Physical Mechanisms of Natural Convection

9-1C Natural convection is the mode of heat transfer that occurs between a solid and a fluid which moves under the influence of natural means. Natural convection differs from forced convection in that fluid motion in natural convection is caused by natural effects such as buoyancy.

9-2C The convection heat transfer coefficient is usually higher in forced convection because of the higher fluid velocities involved.

9-3C The hot boiled egg in a spacecraft will cool faster when the spacecraft is on the ground since there is no gravity in space, and thus there will be no natural convection currents which is due to the buoyancy force.

9-4C The buoyancy force is proportional to the density of the medium, and thus is larger in sea water than it is in fresh water. Therefore, the hull of a ship will sink deeper in fresh water because of the smaller buoyancy force acting upwards.

9-5C The upward force exerted by a fluid on a body completely or partially immersed in it is called the buoyancy or "lifting" force. The buoyancy force is proportional to the density of the medium. Therefore, the buoyancy force is the largest in mercury, followed by in water, air, and the evacuated chamber. Note that in an evacuated chamber there will be no buoyancy force because of absence of any fluid in the medium.

9-6C There cannot be any natural convection heat transfer in a medium that experiences no change in volume with temperature.

9-7C The greater the volume expansion coefficient, the greater the change in density with temperature, the greater the buoyancy force, and thus the greater the natural convection currents.

9-8C The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid. The inertial forces in Reynolds number is replaced by the buoyancy forces in Grashof number.

9-9 The volume expansion coefficient is defined as $\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$. For an ideal gas, $P = \rho RT$ or $\rho = \frac{P}{RT}$, and thus

$$\beta = -\frac{1}{\rho}\left(\frac{\partial (P/RT)}{\partial T}\right)_P = \frac{-1}{\rho}\left(\frac{-P}{RT^2}\right) = \frac{1}{\rho T}\left(\frac{P}{RT}\right) = \frac{1}{\rho T}(\rho) = \frac{1}{T}$$

9-10 The volume expansion coefficient of saturated liquid water at 70°C is to be determined using its definition and the values tabulated in Table A-9.

Assumptions Density depends on temperature only and not pressure.

Properties The properties of sat. liq. water are listed in the following table:

T, °C	***ρ*, kg/m³**	***β*, K⁻¹**
65	980.4	
70	977.5	0.578×10^{-3}
75	974.7	

Analysis The volume expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

For density varying with temperature at constant pressure, we can approximate

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_1 - \rho_2}{T_1 - T_2} \quad \text{where} \quad T_1 = 65°\text{C}, \quad T_2 = 75°\text{C}, \quad \text{and} \quad \rho = 977.5\ \text{kg/m}^3$$

Hence, the volume expansion coefficient is calculated to be

$$\beta \approx -\frac{1}{977.5\ \text{kg/m}^3}\frac{(980.4 - 974.7)\ \text{kg/m}^3}{(65 - 75)\ \text{K}} = \mathbf{5.83 \times 10^{-4}\ K^{-1}}$$

Discussion The calculated volume expansion coefficient is about 1% higher than the value listed in Table A-9 (5.78×10^{-4} K^{-1}).

9-11 Using the given $\rho(T)$ correlation, the volume expansion coefficient of liquid water at 70°C is to be determined.

Assumptions Density depends on temperature only and not pressure.

Properties The volume expansion coefficient of liquid water at 70°C is 5.78×10^{-4} K^{-1} (Table A-9).

Analysis The volume expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{d\rho}{dT}\right) = -\frac{1}{\rho}(-0.0736 - 0.0071T)$$

Hence, at T = 70°C the volume expansion coefficient is

$$\beta = -\frac{(-0.0736 - 0.0071T)\ \text{kg/m}^3\cdot\text{K}}{(1000 - 0.0736T - 0.00355T^2)\ \text{kg/m}^3}$$

$$= -\frac{[-0.0736 - 0.0071(70)]\ \text{kg/m}^3\cdot\text{K}}{[1000 - 0.0736(70) - 0.00355(70)^2]\ \text{kg/m}^3}$$

$$= \mathbf{5.84 \times 10^{-4}\ K^{-1}}$$

Discussion The calculated volume expansion coefficient is about 1% higher than the value listed in Table A-9 (5.78×10^{-4} K^{-1}).

9-12 The Grashof numbers for a plate placed in various fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Air behaves as an ideal gas.

Properties The properties of air, liq. water, and engine oil are listed in the following table:

Fluid	T_f, **°C**	ρ, **kg/m³**	μ, **kg/m·s**	β, **K⁻¹**
Air (Table A-15)	90	0.9718	2.139×10^{-5}	2.755×10^{-3}
Liq. water (Table A-9)	90	965.3	0.315×10^{-3}	0.702×10^{-3}
Engine oil (Table A-13)	80	852.0	3.232×10^{-2}	0.700×10^{-3}

For air (ideal gas) $\beta = 1/T_f$

Analysis The Grashof number is given as

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)L_c^3}{(\mu/\rho)^2}$$

For air,

$$\mathrm{Gr}_{L,\,\mathrm{air}} = \frac{(9.81\,\mathrm{m/s^2})(2.755\times10^{-3}\ \mathrm{K^{-1}})(150-30)\mathrm{K}(0.1\,\mathrm{m})^3}{(2.139\times10^{-5}/0.9718)^2\ \mathrm{m^4/s^2}} = \mathbf{6.69\times10^6}$$

The Grashof number for liquid water and engine oil are calculated similar to the calculation done for air above

$$\mathrm{Gr}_{L,\,\mathrm{water}} = \mathbf{7.76\times10^9}$$

$$\mathrm{Gr}_{L,\,\mathrm{oil}} = \mathbf{6.68\times10^5}$$

Discussion Higher value of the Grashof number implies increase in buoyancy force over the viscous force, which means increase in natural convection flow.

9-13 The Grashof and Rayleigh numbers for a rod submerged in various fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Air behaves as an ideal gas. **4** The rod is orientated such that the characteristic length is its diameter.

Properties The properties of air, liq. water, and engine oil are listed in the following table:

Fluid	T_f, **°F**	ρ, **lbm/ft³**	μ, **lbm/ft·s**	**Pr**	β, **R⁻¹**
Liq. water (Table A-9E)	120	61.71	3.744×10^{-4}	3.63	0.246×10^{-3}
Liq. ammonia (Table A-11E)	120	35.26	7.444×10^{-5}	1.313	1.74×10^{-3}
Engine oil (Table A-13E)	125	54.24	7.617×10^{-2}	1607	0.389×10^{-3}
Air (Table A-15E)	120	0.06843	1.316×10^{-5}	0.723	1.72×10^{-3}

For air (ideal gas), $\beta = 1/T_f$.

Analysis The Grashof and Rayleigh numbers are given as

$$\mathrm{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)D^3}{(\mu/\rho)^2} \qquad \text{and} \qquad \mathrm{Ra}_D = \mathrm{Gr}_D\,\mathrm{Pr}$$

(*a*) For liquid water,

$$\mathrm{Gr}_{D,\,\mathrm{water}} = \frac{(32.2\ \mathrm{ft/s^2})(0.246\times10^{-3}\ \mathrm{R^{-1}})(200-40)\mathrm{R}(2/12\ \mathrm{ft})^3}{(3.744\times10^{-4}/61.71)^2\ \mathrm{ft^4/s^2}} = \mathbf{1.59\times10^8}$$

$$\mathrm{Ra}_{D,\,\mathrm{water}} = (1.59\times10^8)(3.63) = \mathbf{5.79\times10^8}$$

The Grashof and Rayleigh numbers for liquid ammonia, engine oil, and air are calculated similar to the calculation done for liquid water above

(*b*) $\mathrm{Gr}_{D,\,\mathrm{ammonia}} = \mathbf{9.31\times10^9}$ and $\mathrm{Ra}_{D,\,\mathrm{ammonia}} = \mathbf{1.22\times10^{10}}$

(*c*) $\mathrm{Gr}_{D,\,\mathrm{oil}} = \mathbf{4.41\times10^3}$ and $\mathrm{Ra}_{D,\,\mathrm{oil}} = \mathbf{7.09\times10^6}$

(*d*) $\mathrm{Gr}_{D,\,\mathrm{air}} = \mathbf{1.11\times10^6}$ and $\mathrm{Ra}_{D,\,\mathrm{air}} = \mathbf{8.02\times10^5}$

Discussion For the rod’s characteristic length to be its diameter, the rod has to be placed horizontally.

Natural Convection over Surfaces

9-14C Rayleigh number is the product of the Grashof and Prandtl numbers.

9-15C No, a hot surface will cool slower when facing down since the warmer air in this position cannot rise and escape easily.

9-16C The heat flux will be higher at the bottom of the plate since the thickness of the boundary layer which is a measure of thermal resistance is the lowest there.

9-17C A vertical cylinder can be treated as a vertical plate when $D \ge \dfrac{35L}{Gr^{1/4}}$.

9-18 A vertical plate separates the hot water from the cold water. The temperature of the plate surface on the cold water side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature of each surface is constant. **3** The plate thermal conductivity is constant. **4** Radiation heat transfer is negligible.

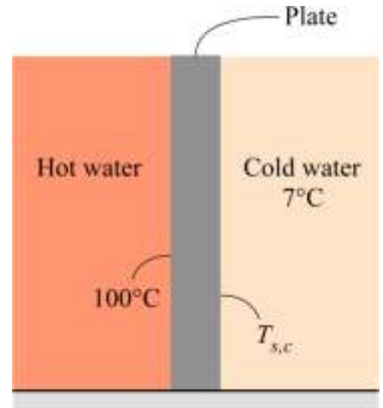
Properties Assuming the surface temperature on the cold water side is $T_{s,c}$ = (100 + 7)°C/2=53.5°C, thus $T_{f,c} = (T_{s,c} + T_{\infty,c})/2$ = (53.5 + 7)°C/2 = 30.25°C. Then, the properties of water at $T_{f,c}$ are k = 0.6033 W/m·K, ρ = 995.5 kg/m^3, μ = 0.0007935 kg/m·s, $\nu = \mu/\rho = 7.971 \times 10^{-7}$ m^2/s, Pr = 5.502, β = 0.0003072 K^{-1} (Table A-9).

The thermal conductivity of the plate is given as k_{plate} = 15 W/m·K.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,c} - T_{\infty,c})L_c^{\,3}}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.0003072\ \text{K}^{-1})(53.5-7)\ \text{K}\ (0.2\ \text{m})^3}{(7.971\times10^{-7}\ \text{m}^2/\text{s})^2}(5.502) = 9.708\times10^9$$

The Nusselt number relation for vertical plate is



$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(9.705\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{5.502}\right)^{9/16}\right]^{8/27}}\right\}^2 = 307.2$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.6033\ \text{W/m}\cdot\text{K}}{0.2\ \text{m}}(307.2) = 926.7\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the plate thickness and natural convection is

$$\frac{k_{\text{plate}}}{l}(T_{s,h} - T_{s,c}) = h(T_{s,c} - T_{\infty,c}) \quad \rightarrow \quad T_{s,c} = 43.5°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,c}$ converges to $T_{s,c}$ = **46°C**.

Discussion The results from the iterative solution are listed in the following table:

Iter	$T_{s,c}$ [°C]	Ra	Nu
1	53.5	9.708×10^9	307.2
2	43.5	5.915×10^9	264.6
3	47.2	7.194×10^9	280.7
4	45.8	6.694×10^9	274.7
5	46.3	6.870×10^9	276.8
6	**46.1**	6.799×10^9	276.0

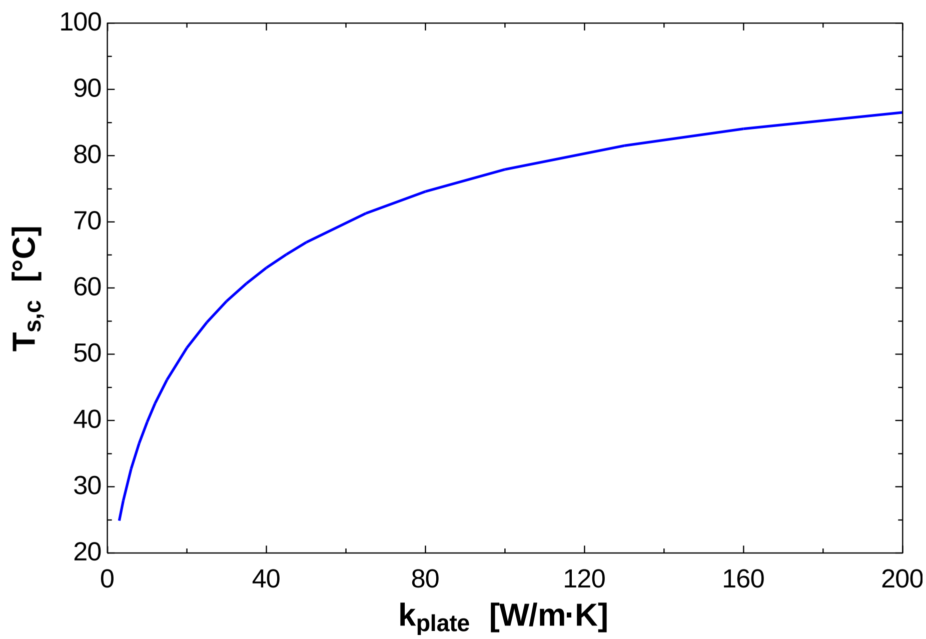
As $T_{s,c}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-19 Reconsider Prob. 9-18. A vertical plate separates the hot water from the cold water. The effect of k_{plate} on $T_{s,c}$ is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]    "Plate length"
thickness=0.025 [m]    "Plate thickness"
T_infinity_c=7 [C]    "Cold water T"
T_s_h=100 [C]    "Surface T, hot water side"

"PROPERTIES"
g=9.81 [m/s^2]    "gravitational acceleration"
Fluid$='water'
k=Conductivity(Fluid$, T=T_film_c, x=0)
Pr=Prandtl(Fluid$, T=T_film_c, x=0)
rho=Density(Fluid$, T=T_film_c, x=0)
mu=Viscosity(Fluid$, T=T_film_c, x=0)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film_c, x=0)
T_film_c=1/2*(T_s_c+T_infinity_c)

"ANALYSIS"
Ra=(g*beta*(T_s_c-T_infinity_c)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=k_plate/thickness*(T_s_h-T_s_c)    "Heat conduction through the plate"
q_dot=h*(T_s_c-T_infinity_c)    "Natural heat convection"
```

k_{plate} [W/m·K]	$T_{s,c}$ [°C]
3	24.89
4	27.88
6	32.71
8	36.57
10	39.80
12	42.59
15	46.16
20	50.97
25	54.82
30	58.01
35	60.72
40	63.05
45	65.09
50	66.90
65	71.27
80	74.57
100	77.90
130	81.49
160	84.06
200	86.53



Discussion As k_{plate} increases, the thermal resistance of the plate reduces, therefore $T_{s,c}$ increases with increasing value of k_{plate}.

9-20 A vertical plate separates the hot water from the cold air. The surface exposed to the cold air is subjected to radiation heat transfer also. The temperature of the plate surface exposed to the cold air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature on each surface is constant. **3** The plate thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the cold air temperature.

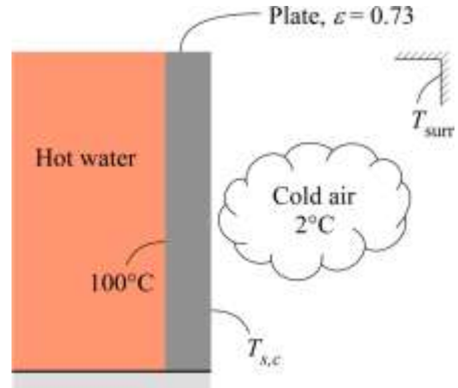
Properties Assuming the surface temperature on the cold air side is $T_{s,c}$ = (100 + 2)°C/2=51°C, thus $T_{f,c} = (T_{s,c} + T_{\infty,c})/2$ = (51 + 2)°C/2 = 26.5°C. Then, the properties of air at $T_{f,c}$ and 1 atm pressure are k = 0.02562 W/m·K, $\nu = 1.576 \times 10^{-5}$ m²/s, Pr = 0.7277 (Table A-15), and $\beta = 1/T_{f,c}$ = 1/ 299.5 K.

The thermal conductivity and the emissivity of the plate are given as k_{plate} = 1.5 W/m·K and ε_{plate} = 0.73, respectively.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,c} - T_{\infty,c})L_c^{\,3}}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(26.5+273\,\text{K})^{-1}(51-2)\,\text{K}\,(0.2\,\text{m})^3}{(1.576\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7277) = 3.762\times10^7$$

The Nusselt number relation for vertical plate is



$$\text{Nu} = \left\{0.825 + \frac{0.387\,\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.762\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7277}\right)^{9/16}\right]^{8/27}}\right\}^2$$
$$= 45.89$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02562\ \text{W/m}\cdot\text{K}}{0.2\,\text{m}}(45.89) = 5.879\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the plate thickness l, natural convection and radiation is

$$\frac{k_{\text{plate}}}{l}(T_{s,h} - T_{s,c}) = h(T_{s,c} - T_{\infty,c}) + \varepsilon_{\text{plate}}\sigma(T_{s,c}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,c} = 84.6°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,c}$ converges to $T_{s,c}$ = **83.7°C**.

Discussion The results from the iterative solution are listed in the following table:

Iter	$T_{s,c}$ [°C]	Ra	Nu	h [W/m²·K]
1	51	3.762×10^7	45.89	5.879
2	84.6	4.934×10^7	49.66	6.670
3	83.7	4.912×10^7	49.60	6.653
4	83.72	4.913×10^7	49.60	6.654
5	**83.71**	4.912×10^7	49.60	6.653

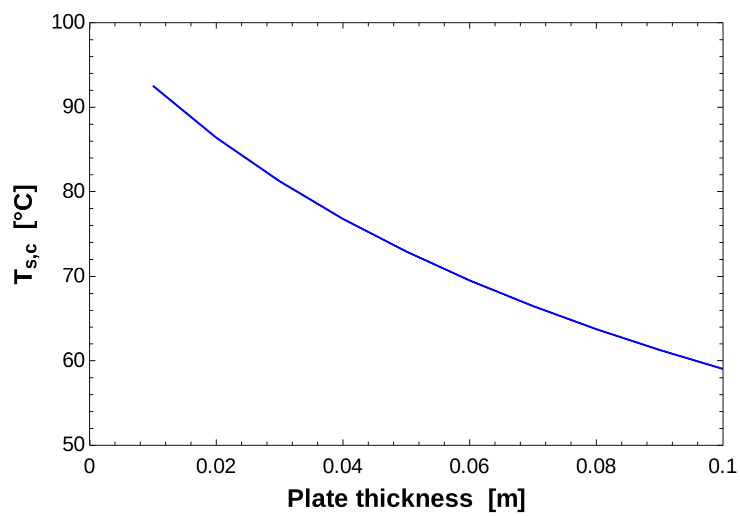
As $T_{s,c}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-21 Reconsider Prob. 9-20. A vertical plate separates the hot water from the cold air. The surface exposed to the cold air is subjected to radiation heat transfer also. The effect of the plate thickness on $T_{s,c}$ is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]
T_infinity_c=2 [C]
T_s_h=100 [C]

"PROPERTIES"
g=9.81 [m/s^2]     "gravitational acceleration"
Fluid$='air'
"Cold air"
k=Conductivity(Fluid$, T=T_film_c)
Pr=Prandtl(Fluid$, T=T_film_c)
rho=Density(Fluid$, T=T_film_c, P=101.3)
mu=Viscosity(Fluid$, T=T_film_c)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film_c)
T_film_c=1/2*(T_s_c+T_infinity_c)
"Plate"
k_plate=1.5 [W/m-K]
epsilon_plate=0.73

"ANALYSIS"
Ra=(g*beta*(T_s_c-T_infinity_c)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=k_plate/thickness*(T_s_h-T_s_c)
q_dot=h*(T_s_c-T_infinity_c)+sigma#*epsilon_plate*((T_s_c+273)^4-(T_infinity_c+273)^4)
```

Thickness [m]	$T_{s,c}$ [°C]
0.01	92.54
0.02	86.40
0.03	81.23
0.04	76.79
0.05	72.92
0.06	69.51
0.07	66.47
0.08	63.74
0.09	61.27
0.10	59.03



Discussion As the plate thickness increases, the thermal resistance of the plate increases, thus reducing the surface temperature on the cold air side.

9-22 A thin vertical plate is subjected to uniform heat flux on one side and exposed to cool air on the other side. The heat flux on the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the cool air temperature.

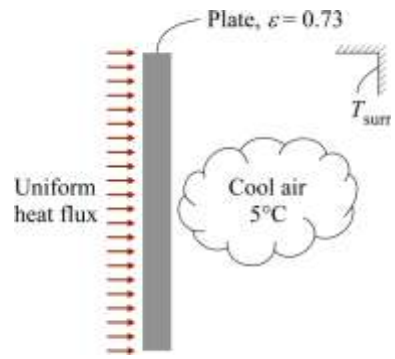
Properties The film temperature is determined with the plate midpoint temperature, $T_f = (T_{L/2} + T_\infty)/2 = (55 + 5)°C/2 = 30°C$. Then, the properties of air at $T_f = 30°C$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f = 1/303 \text{ K} = 0.0033 \text{ K}^{-1}$.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^{\,3}}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(55-5)\,\text{K}\,(0.5\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 5.699\times10^8$$

The Nusselt number relation for vertical plate is

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.699\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 103.7$$



$$h = \frac{k}{L}\text{Nu} = \frac{0.02588\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}}(103.7) = 5.368\,\text{W/m}^2\cdot\text{K}$$

Thus, the heat flux on the plate surface can be determined from the heat loss by natural convection and radiation on the other side of the plate:

$$\begin{aligned}\dot{q}_s &= h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{\text{surr}}^4) \\ &= (5.368\,\text{W/m}^2\cdot\text{K})(55-5)\,\text{K} + (5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(0.73)(328^4 - 278^4)\,\text{K}^4 \\ &= \mathbf{500\ W/m^2}\end{aligned}$$

Discussion Natural convection contributes to about 54% of the 500 W/m^2 heat flux. For constant surface heat flux, the plate midpoint temperature is used instead of the surface temperature in the evaluation of the fluid properties.

9-23 A thin vertical plate is subjected to uniform heat flux on one side and exposed to hydrogen gas on the other side. The plate midpoint temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The film temperature is 50°C (this assumption will be verified).

Properties The properties of H_2 gas at T_f = 50°C are k = 0.1881 W/m·K, ν = 1.240 × 10^{-4} m^2/s, Pr = 0.7191 (Table A-16), and $\beta = 1/T_f$ = 1/ 323 K = 0.003096 K^{-1} (Table A-16).

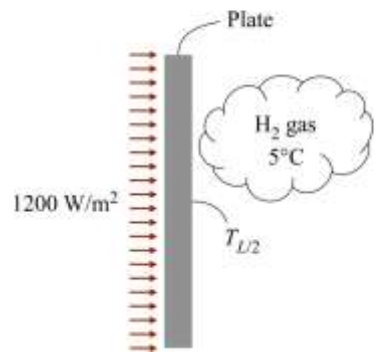
Analysis With the assumption that T_f = 50°C, the plate midpoint temperature is estimated as

$$T_{L/2} = 2T_f - T_\infty = 95°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50+273\,\text{K})^{-1}(95-5)\,\text{K}\,(0.5\,\text{m})^3}{(1.240\times10^{-4}\ \text{m}^2/\text{s})^2}(0.7191) = 1.598\times10^7$$

The Nusselt number relation for vertical plate is



$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.598\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7191}\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.74$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.1881\ \text{W/m}\cdot\text{K}}{0.5\,\text{m}}(35.74) = 13.45\ \text{W/m}^2\cdot\text{K}$$

Thus, the plate midpoint temperature from the heat flux as

$$\dot{q}_s = h(T_{L/2} - T_\infty) \quad \rightarrow \quad T_{L/2} = \frac{\dot{q}_s}{h} + T_\infty$$

$$T_{L/2} = \frac{1200\ \text{W/m}^2}{13.45\ \text{W/m}^2\cdot\text{K}} + 5°\text{C} = \mathbf{94.2°C}$$

Discussion The assumed film temperature T_f = 50°C is an appropriate assumption, since the determined $T_{L/2}$ = 94.2°C would give a film temperature of T_f = 49.6°C. Otherwise, $T_{L/2}$ would have to be solved iteratively.

For constant surface heat flux, the plate midpoint temperature is used instead of the surface temperature in the evaluation of the fluid properties.

9-24 A thin vertical plate is subjected to uniform heat flux on one side and exposed to air on the other side. The plate midpoint temperatures for (*a*) a highly polish copper surface and (*b*) a black oxidized copper surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature.

Properties We first assume the film temperature is T_f = 30°C. Then, the properties of air at T_f are k = 0.02588 W/m·K, ν = 1.608 × 10^{-5} m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f =$ 1/ 303 K = 0.0033 K^{-1}.

The emissivity of highly polished copper is ε = 0.02 and of black oxidized copper is ε = 0.78 (TableA-18).

Analysis With the assumption that T_f = 30°C, the plate midpoint temperature is estimated as

$$T_{L/2} = 2T_f - T_\infty = 55°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^{\,3}}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(55-5)\,\text{K}\,(0.5\,\text{m})^3}{(1.608\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7282) = 5.699\times10^8$$

The Nusselt number relation for vertical plate is

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.699\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 103.7$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02588\ \text{W/m}\cdot\text{K}}{0.5\,\text{m}}(103.7) = 5.368\ \text{W/m}^2\cdot\text{K}$$

(*a*) With the known surface heat flux of 1000 W/m^2 and ε = 0.02 (highly polished copper), the plate midpoint temperature can be determined as

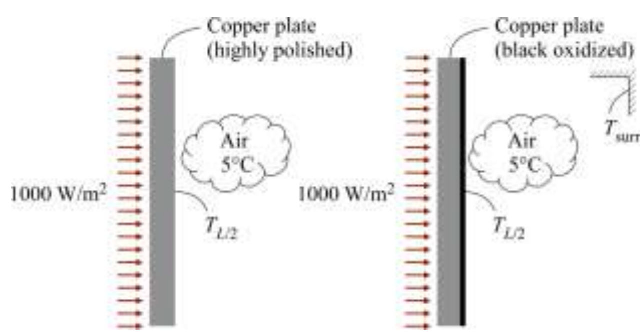
$$\dot{q}_s = h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{surr}^4)$$

$$1000\ \text{W/m}^2 = (5.368\ \text{W/m}^2\cdot\text{K})(T_{L/2} - 5)\,\text{K} + (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.02)[(T_{L/2}+273)^4 - (5+273)^4]\,\text{K}^4$$

Solving for the plate midpoint temperature yields $T_{L/2}$ = 183.4°C (first iteration)

The above solution is repeated iteratively until $T_{L/2}$ converges to $T_{L/2}$ = **147.2°C**. The results from the iterations are as follows:

Iter	$T_{L/2}$ [°C]	Ra	Nu	h [W/m²·K]
1	55	5.699 × 10^8	103.7	5.368
2	183.4	8.427 × 10^8	116.6	7.122
3	141.6	8.423 × 10^8	116.7	6.778
4	148.3	8.457 × 10^8	116.8	6.842
5	147.0	8.452 × 10^8	116.8	6.830
6	**147.2**	8.453 × 10^8	116.8	6.832



(*b*) With the known surface heat flux of 1000 W/m^2 and ε = 0.78 (black oxidized copper), the plate midpoint temperature can be determined as

$$\dot{q}_s = h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{surr}^4)$$

$$1000\ \text{W/m}^2 = (5.368\ \text{W/m}^2\cdot\text{K})(T_{L/2} - 5)\,\text{K} + (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.78)[(T_{L/2}+273)^4 - (5+273)^4]\,\text{K}^4$$

Solving for the plate midpoint temperature yields $T_{L/2}$ = 92.9°C (first iteration)

The above solution is repeated iteratively until $T_{L/2}$ converges to $T_{L/2}$ = **88.5°C**. The results from the iterations are as follows:

Iter	$T_{L/2}$ [°C]	Ra	Nu	h [W/m²·K]
1	55	5.699 × 10^8	103.7	5.368
2	92.9	7.564 × 10^8	113.0	6.165
3	88.1	7.401 × 10^8	112.3	6.085
4	88.6	7.419 × 10^8	112.3	6.093
5	**88.5**	7.415 × 10^8	112.3	6.092

Discussion For part (*a*), the highly polished copper surface has a low emissivity, which limits the heat loss on the surface by radiation. For part (*b*), the black oxidized copper surface has a higher emissivity, which increases the heat loss on the surface by radiation. Therefore the plate midpoint temperature for part (*a*) is higher than that of part (*b*).

Note that as $T_{L/2}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-25 Reconsider Prob. 9-24. A thin vertical copper plate is subjected to uniform heat flux on one side and exposed to air on the other side. The effect of the heat flux on the plate midpoint temperature for (*a*) a highly polished copper surface and (*b*) a black oxidized copper surface is to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.5 [m]
T_infinity=5 [C]
epsilon=0.02    "0.02 for (a) and 0.78 for (b)"

"PROPERTIES"
g=9.81 [m/s^2]    "gravitational acceleration"
Fluid$='air'
"Air"
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film)
T_film=1/2*(T_0.5L+T_infinity)

"ANALYSIS"
Ra=(g*beta*(T_0.5L-T_infinity)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=h*(T_0.5L-T_infinity)+sigma#*epsilon*((T_0.5L+273)^4-(T_infinity+273)^4)
```

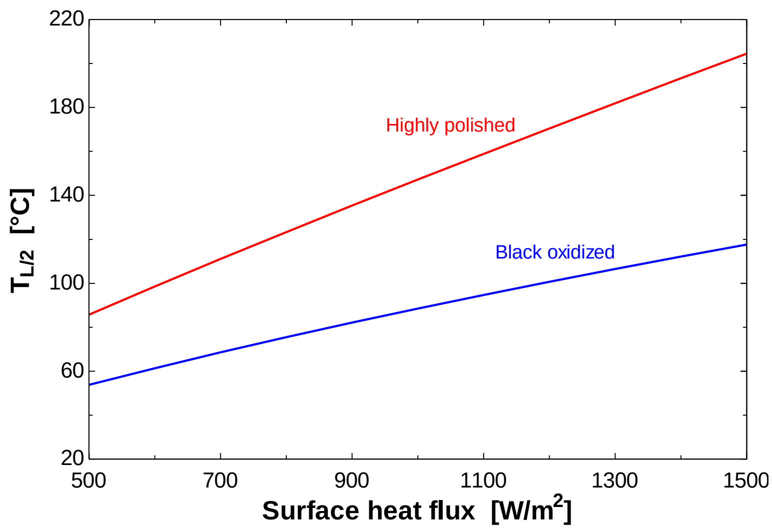
(*a*) Highly polished surface (ε = 0.02)

$\dot{q}_s$ [W/m^2]	$T_{s,c}$ [°C]	Ra
500	85.74	7.314E+08
600	98.57	7.735E+08
700	111.1	8.036E+08
800	123.3	8.243E+08
900	135.3	8.377E+08
1000	147.2	8.453E+08
1100	158.9	8.481E+08
1200	170.5	8.473E+08
1300	181.9	8.434E+08
1400	193.3	8.371E+08
1500	204.5	8.288E+08

(*b*) Black oxidized surface (ε = 0.78)

$\dot{q}_s$ [W/m^2]	$T_{s,c}$ [°C]	Ra
500	53.77	5.598E+08
600	61.34	6.106E+08
700	68.55	6.527E+08
800	75.46	6.877E+08
900	82.11	7.170E+08
1000	88.52	7.416E+08
1100	94.71	7.621E+08
1200	100.7	7.794E+08
1300	106.5	7.938E+08
1400	112.2	8.058E+08
1500	117.7	8.157E+08

Discussion The plate midpoint temperature for the highly polished copper surface is higher than that of the black oxidized copper surface. The highly polished copper surface has very low emissivity, thus the heat loss from the surface is mainly by natural convection. The black oxidized copper surface has significant heat loss by radiation in addition to natural convection, which causes the plate midpoint temperature to be lower.



9-26 A street sign surface is subjected to radiation, the surface temperature of the street sign is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** The street sign is treated as a vertical plate. **5** Air is an ideal gas.

Properties The properties of air (1 atm) at 30°C are given in Table A-15: k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, and Pr = 0.7282. Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number is

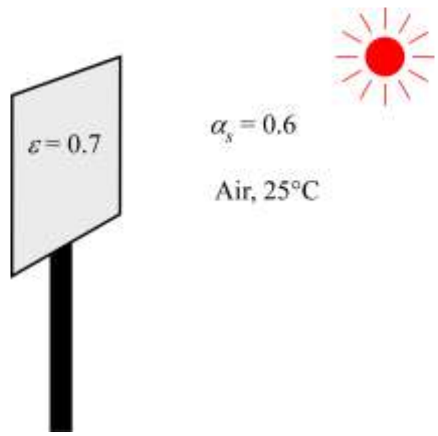
$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.0033\,\mathrm{K^{-1}})(T_s - 293)\mathrm{K}(0.2\,\mathrm{m})^3}{(1.608\times10^{-5}\,\mathrm{m^2/s})^2}(0.7282) \;\;\ldots\ldots\,(1)$$

Assuming the Rayleigh number is within $10^4 < \mathrm{Ra}_L < 10^9$, the Nusselt number for vertical plate is

$$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$$

or

$$h = \left(\frac{0.02588\ \mathrm{W/m\cdot K}}{0.2\ \mathrm{m}}\right)0.59\mathrm{Ra}_L^{1/4} \;\;\ldots\ldots\,(2)$$



From energy balance, we obtain

$$\alpha_s \dot{q}_{\mathrm{solar}} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\mathrm{surr}}^4] \;\;\ldots\ldots\,(3)$$

Equations (1), (2), and (3) can be solved simultaneously to get the surface temperature. Copy the following lines and paste on a blank EES screen to solve the above equation:

```
g=9.81
k=0.02588
L=0.2
Pr=0.7282
q_incindent=200
T_inf=25+273
T_surr=25+273
alpha=0.6
beta=1/(273+30)
epsilon=0.7
nu=1.608e-5
sigma=5.670e-8
Ra_L=g*beta*(T_s-T_inf)*L^3/nu^2*Pr
(h*L/(0.59*k))^4=Ra_L
alpha*q_incindent=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)
```

Solving by EES software, we get

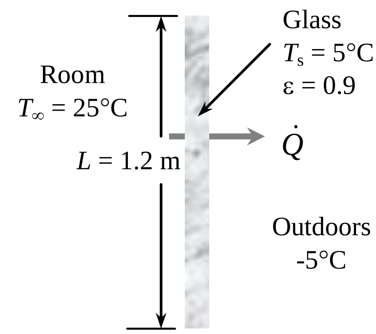
$$\mathrm{Ra}_L = 9.961\times10^6, \qquad h = 4.289\ \mathrm{W/m^2\cdot K}, \qquad \text{and} \qquad T_s = 311.7\ \mathrm{K}$$

Therefore, the surface temperature of the street sign is

$$T_s = \mathbf{38.7°C}$$

Discussion The assumption that the Rayleigh number is within $10^4 < \mathrm{Ra}_L < 10^9$ turned out to be appropriate. Note that absolute temperatures must be used in calculations involving the radiation heat transfer equation.

9-27 A glass window is considered. The convection heat transfer coefficient on the inner side of the window, the rate of total heat transfer through the window, and the combined natural convection and radiation heat transfer coefficient on the outer surface of the window are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (5+25)/2 = 15$°C are (Table A-15)

$$k = 0.02476 \text{ W/m.°C}$$
$$\nu = 1.470\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7323$$
$$\beta = \frac{1}{T_f} = \frac{1}{(15+273)\text{K}} = 0.003472 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window, $L_c = L = 1.2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003472 \text{ K}^{-1})(25-5 \text{ K})(1.2 \text{ m})^3}{(1.470\times10^{-5} \text{ m}^2/\text{s})^2}(0.7323) = 3.989\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.989\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7323}\right)^{9/16}\right]^{8/27}}\right\}^2 = 189.7$$

$$h = \frac{k}{L}Nu = \frac{0.02476 \text{ W/m.°C}}{1.2 \text{ m}}(189.7) = \mathbf{3.915 \ W/m^2.°C}$$
$$A_s = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA_s(T_\infty - T_s) = (3.915 \text{ W/m}^2.°\text{C})(2.4 \text{ m}^2)(25-5)°\text{C} = 187.9 \text{ W}$$
$$\dot{Q}_{\text{radiation}} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(25+273 \text{ K})^4 - (5+273 \text{ K})^4] = 234.3 \text{ W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 187.9 + 234.3 = \mathbf{422.2 \ W}$$

(*c*) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA_s}{t}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}}t}{kA_s} = 5°\text{C} - \frac{(422.2 \text{ W})(0.006 \text{ m})}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 3.65°\text{C}$$

Then the combined natural convection and radiation heat transfer coefficient on the outer window surface becomes

$$\dot{Q}_{\text{total}} = h_{\text{combined}}A_s(T_{s,o} - T_{\infty,o})$$

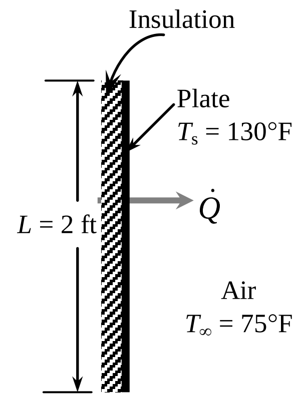
or $$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A_s(T_{s,o} - T_{\infty,o})} = \frac{422.2 \text{ W}}{(2.4 \text{ m}^2)[3.65-(-5)]°\text{C}} = \mathbf{20.35 \ W/m^2.°C}$$

Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta T_{\text{total}}} = \frac{5-3.65}{25-(-5)} = 0.045 \quad (\text{or } 4.5\%)$$

which is low. Thus it is reasonable to neglect the thermal resistance of the glass.

9-28E A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (130+75)/2 = 102.5$°F are (Table A-15)

$$k = 0.01535 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1823\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{T_f} = \frac{1}{(102.5+460)\text{R}} = 0.001778 \text{ R}^{-1}$$

Analysis (*a*) When the plate is vertical, the characteristic length is the height of the plate. $L_c = L = 2$ ft. Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001778\text{ R}^{-1})(130-75\text{ R})(2\text{ ft})^3}{(0.1823\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7256) = 5.503\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.503\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7256}\right)^{9/16}\right]^{8/27}}\right\}^2 = 102.6$$

$$h = \frac{k}{L}Nu = \frac{0.01535\text{ Btu/h.ft.°F}}{2\text{ ft}}(102.6) = 0.7869\text{ Btu/h.ft}^2\text{.°F}$$
$$A_s = L^2 = (2\text{ ft})^2 = 4\text{ ft}^2$$

and $$\dot{Q} = hA_s(T_s-T_\infty) = (0.7869\text{ Btu/h.ft}^2\text{.°F})(4\text{ ft}^2)(130-75)\text{°C} = \mathbf{173.1\ Btu/h}$$

(*b*) When the plate is horizontal with hot surface facing up, the characteristic length is determined from

$$L_s = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{2\text{ ft}}{4} = 0.5\text{ ft} \ .$$

Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001778\text{ R}^{-1})(130-75\text{ R})(0.5\text{ ft})^3}{(0.1823\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7256) = 8.598\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(8.598\times10^6)^{1/4} = 29.24$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535\text{ Btu/h.ft.°F}}{0.5\text{ ft}}(29.24) = 0.8975\text{ Btu/h.ft}^2\text{.°F}$$

and $$\dot{Q} = hA_s(T_s-T_\infty) = (0.8975\text{ Btu/h.ft}^2\text{.°F})(4\text{ ft}^2)(130-75)\text{°C} = \mathbf{197.4\ Btu/h}$$

(*c*) When the plate is horizontal with hot surface facing down, the characteristic length is again L_c = 0.5 ft and the Rayleigh number is $Ra = 8.598\times10^6$. Then,

$$Nu = 0.27Ra^{1/4} = 0.27(8.598\times10^6)^{1/4} = 14.62$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535\text{ Btu/h.ft.°F}}{0.5\text{ ft}}(14.62) = 0.4487\text{ Btu/h.ft}^2\text{.°F}$$

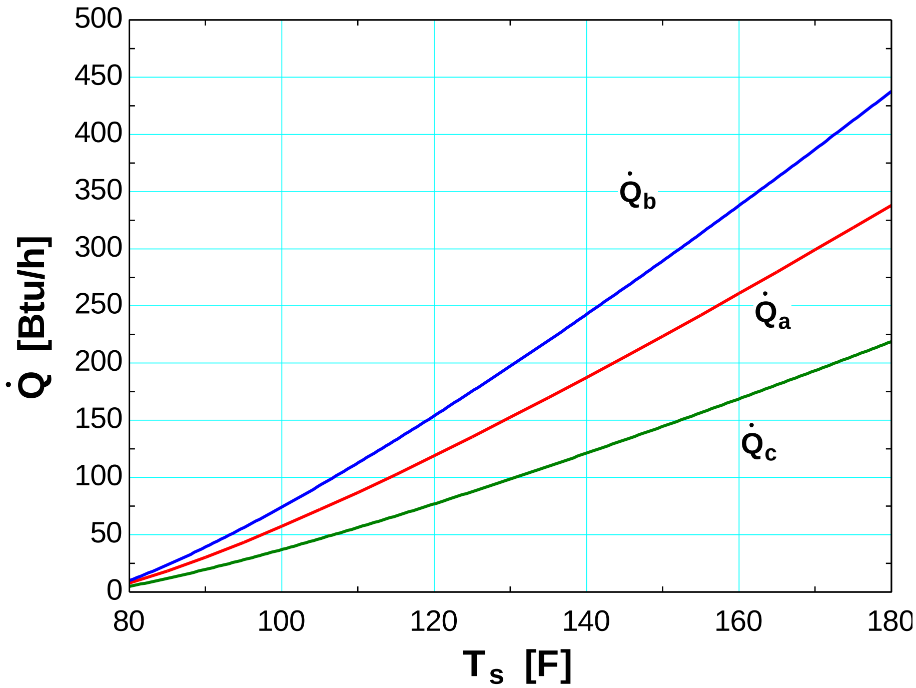
and $$\dot{Q} = hA_s(T_s-T_\infty) = (0.4487\text{ Btu/h.ft}^2\text{.°F})(4\text{ ft}^2)(130-75)\text{°C} = \mathbf{98.7\ Btu/h}$$

9-29E Prob. 9-28E is reconsidered. The rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

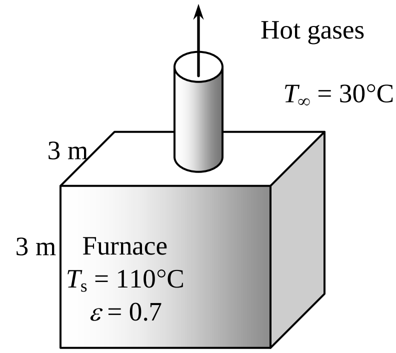
```
"GIVEN"
L=2 [ft]
T_infinity=75 [F]
T_s=130 [F]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_film+460)
T_film=1/2*(T_s+T_infinity)
g=32.2 [ft/s^2]

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=L^2
Q_dot_a=h_a*A*(T_s-T_infinity)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=4*L
Ra_b=(g*beta*(T_s-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot_b=h_b*A*(T_s-T_infinity)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot_c=h_c*A*(T_s-T_infinity)
```

T_s [F]	$\dot{Q}_a$ [Btu/h]	$\dot{Q}_b$ [Btu/h]	$\dot{Q}_c$ [Btu/h]
80	7.714	9.985	4.993
85	18.32	23.72	11.86
90	30.38	39.32	19.66
95	43.47	56.26	28.13
100	57.37	74.26	37.13
105	71.97	93.15	46.58
110	87.15	112.8	56.4
115	102.8	133.1	66.56
120	119	154	77.02
125	135.6	175.5	87.75
130	152.5	197.4	98.72
135	169.9	219.9	109.9
140	187.5	242.7	121.3
145	205.4	265.9	132.9
150	223.7	289.5	144.7
155	242.1	313.4	156.7
160	260.9	337.7	168.8
165	279.9	362.2	181.1
170	299.1	387.1	193.5
175	318.5	412.2	206.1
180	338.1	437.6	218.8



9-30 It is proposed that the side surfaces of a cubic industrial furnace be insulated for $550 in order to reduce the heat loss by 90 percent. The thickness of the insulation and the payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (110+30)/2 = 70°C are (Table A-15)

$$k = 0.02881\ \text{W/m.°C}$$

$$\nu = 1.995\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7177$$

$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915\ \text{K}^{-1}$$

Analysis The characteristic length in this case is the height of the furnace, $L_c = L = 3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.002915\ \text{K}^{-1})(110-30\ \text{K})(3\ \text{m})^3}{(1.995\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7177) = 1.114\times10^{11}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.114\times10^{11})^{1/6}}{\left[1+\left(\frac{0.492}{0.7177}\right)^{9/16}\right]^{8/27}}\right\}^2 = 545.1$$

$$h = \frac{k}{L_c}Nu = \frac{0.02881\ \text{W/m.°C}}{3\ \text{m}}(545.1) = 5.235\ \text{W/m}^2.°\text{C}$$

$$A_s = 4(3\ \text{m})^2 = 36\ \text{m}^2$$

Then the heat loss by combined natural convection and radiation becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)\\ &= (5.235\ \text{W/m}^2.°\text{C})(36\ \text{m}^2)(110-30)°\text{C}\\ &\quad + (0.7)(36\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(110+273\ \text{K})^4 - (30+273\ \text{K})^4]\\ &= 33{,}779\ \text{W}\end{aligned}$$

Noting that insulation will reduce the heat losses by 90%, the rate of heat loss after insulation will be

$$\dot{Q}_{\text{saved}} = 0.9\dot{Q}_{\text{no insulation}} = 0.9\times33{,}779\ \text{W} = 30{,}401\ \text{W}$$

$$\dot{Q}_{\text{loss}} = (1-0.9)\dot{Q}_{\text{no insulation}} = 0.1\times33{,}779\ \text{W} = 3378\ \text{W}$$

The furnace operates continuously and thus 8760 h. Then the amount of energy and money the insulation will save becomes

$$\text{Energy saved} = \dot{Q}_{saved}\Delta t = \frac{30.401\ \text{kJ/s}}{0.78}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right)(8760\times3600\ \text{s/yr}) = 11{,}651\ \text{therms/yr}$$

$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (11{,}651\ \text{therms})(\$1.10/\text{therm}) = \$12{,}816$$

Therefore, the money saved by insulation will pay for the cost of $550 in

550/($12,816/yr)=0.04292 yr = **16 days**.

Insulation will lower the outer surface temperature, the Rayleigh and Nusselt numbers, and thus the convection heat transfer coefficient. For the evaluation of the heat transfer coefficient, we assume the surface temperature in this case to be 50°C. The properties of air at the film temperature of $(T_s+T_\infty)/2$ = (50+30)/2 = 40°C are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(3 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 4.239\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(4.239\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 400.5$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{3 \text{ m}}(400.5) = 3.554 \text{ W/m}^2\text{.°C}$$
$$A_s = 4\times(3 \text{ m})(3+2t_{insul}) \text{ m}$$

The total rate of heat loss from the outer surface of the insulated furnace by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$3378 \text{ W} = (3.554 \text{ W/m}^2\text{.°C})A(T_s - 30)\text{°C}$$
$$+ (0.7)A(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273 \text{ K})^4 - (30+273 \text{ K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

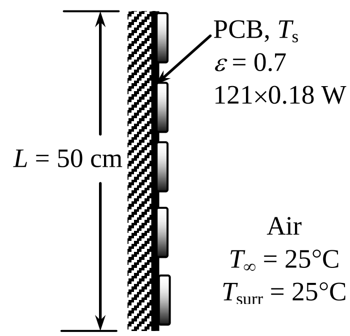
$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{(T_{\text{furnace}} - T_s)}{t_{ins}} \rightarrow 3378 \text{ W} = (0.038 \text{ W/m.°C})A_s\frac{(110 - T_s)\text{°C}}{t_{insul}}$$

Solving the two equations above by trial-and error (or better yet, an equation solver) gives

T_s = 41.2°C and t_{insul} = 0.0284 m = **2.84 cm**

9-31 A circuit board containing square chips is mounted on a vertical wall in a room. The surface temperature of the chips is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.



Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30°C$ are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$

$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7282$$

$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the board, $L_c = L = 0.5\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.0033\text{ K}^{-1})(35-25\text{ K})(0.5\text{ m})^3}{(1.608\times10^{-5}\text{ m}^2\text{/s})^2}(0.7282) = 1.140\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.140\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 63.72$$

$$h = \frac{k}{L}Nu = \frac{0.02588\text{ W/m.°C}}{0.5\text{ m}}(63.72) = 3.30\text{ W/m}^2\text{.°C}$$

$$A_s = (0.5\text{ m})^2 = 0.25\text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$

$$(121\times0.18)\text{ W} = (3.30\text{ W/m}^2\text{.°C})(0.25\text{ m}^2)(T_s - 25)\text{°C}$$
$$+(0.7)(0.25\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(T_s + 273\text{ K})^4 - (25+273\text{ K})^4]$$

Its solution is

T_s = **36.2°C**

which is sufficiently close to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations by reevaluating the properties and h at the new film temperature.

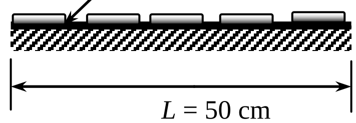
Discussion The assumed film temperature of T_f = 30°C is an appropriate assumption, since the determined T_s = 36.2°C would give a film temperature of T_f = 30.6°C. Otherwise, T_s would have to be solved iteratively.

9-32 A circuit board containing square chips is positioned horizontally in a room. The surface temperature of the chips is to be determined for two orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30°C$ are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033\,\text{K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. The characteristic length for both cases is determined from

$$L_c = \frac{A_s}{p} = \frac{(0.5\,\text{m})^2}{2[(0.5\,\text{m})+(0.5\,\text{m})]} = 0.125\,\text{m}.$$

Then, $$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(35-25\,\text{K})(0.125\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 1.781\times10^6$$

(a) Chips (hot surface) facing up:

$$Nu = 0.54Ra^{1/4} = 0.54(1.781\times10^6)^{1/4} = 19.73$$
$$h = \frac{k}{L_c}Nu = \frac{0.02588 \text{ W/m.°C}}{0.125\,\text{m}}(19.73) = 4.08 \text{ W/m}^2\text{.°C}$$
$$A_s = (0.5\,\text{m})^2 = 0.25\,\text{m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$(121\times0.18)\text{ W} = (4.08 \text{ W/m}^2\text{.°C})(0.25\,\text{m}^2)(T_s - 25)°\text{C}$$
$$+ (0.7)(0.25\,\text{m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273\,\text{K})^4 - (25+273\,\text{K})^4]$$

Its solution is T_s = **35.2°C**

which is sufficiently close to the assumed value. Therefore, there is no need to repeat calculations.

(b) Chips (hot surface) facing up:

$$Nu = 0.27Ra^{1/4} = 0.27(1.781\times10^6)^{1/4} = 9.863$$
$$h = \frac{k}{L_c}Nu = \frac{0.02588 \text{ W/m.°C}}{0.125\,\text{m}}(9.863) = 2.04 \text{ W/m}^2\text{.°C}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$(121\times0.18)\text{ W} = (2.04 \text{ W/m}^2\text{.°C})(0.25\,\text{m}^2)(T_s - 25)°\text{C}$$
$$+ (0.7)(0.25\,\text{m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273\,\text{K})^4 - (25+273\,\text{K})^4]$$

Its solution is T_s = **38.3°C**

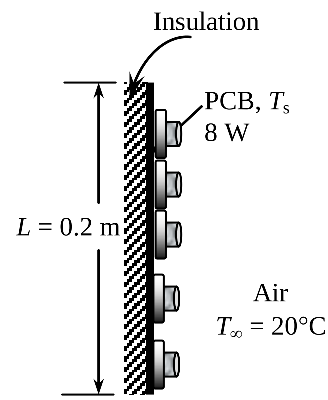
which is sufficiently close to the assumed value of 35°C in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

Discussion The assumed film temperature of T_f = 30°C is an appropriate assumption, since the determined T_s = 38.3°C would give a film temperature of T_f = 31.7°C. Otherwise, T_s would have to be solved iteratively.

9-33 A printed circuit board (PCB) is placed in a room. The average temperature of the hot surface of the board is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** The heat loss from the back surface of the board is negligible.

Properties We evaluate air properties at a film temperature of $(T_s+T_\infty)/2$ = 32.5°C and 1 atm based on the problem statement. Then, for an air temperature of T_∞ = 20°C, the corresponding surface temperature is T_s = 45°C. The properties of air at 1 atm and 32.5°C are (Table A-15)



$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.631\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32.5+273)\text{K}} = 0.003273\,\text{K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown

(*a*) **Vertical PCB** . We start the solution process by "guessing" the surface temperature to be 45°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the PCB, $L_c = L = 0.2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003273\,\text{K}^{-1})(45-20\,\text{K})(0.2\,\text{m})^3}{(1.631\times10^{-5}\text{ m}^2/\text{s})^2}(0.7275) = 1.756\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.756\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7275}\right)^{9/16}\right]^{8/27}}\right\}^2 = 36.78$$

$$h = \frac{k}{L}Nu = \frac{0.02607 \text{ W/m.°C}}{0.2\,\text{m}}(36.78) = 4.794 \text{ W/m}^2.\text{°C}$$
$$A_s = (0.15\,\text{m})(0.2\,\text{m}) = 0.03\,\text{m}^2$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$8\text{ W} = (4.794\text{ W/m}^2.\text{°C})(0.03\,\text{m}^2)(T_s - 20)\text{°C} + (0.8)(0.03\,\text{m}^2)(5.67\times10^{-8})\left[(T_s+273)^4 - (20+273\,\text{K})^4\right]$$

Its solution is

$$T_s = \mathbf{46.6°C}$$

which is sufficiently close to the assumed value of 45°C for the evaluation of the properties and *h*.

(*b*) **Horizontal, hot surface facing up** Again we assume the surface temperature to be 45 °C and use the properties evaluated above. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.20\,\text{m})(0.15\,\text{m})}{2(0.2\,\text{m}+0.15\,\text{m})} = 0.0429\,\text{m}.$$

Then

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003273\,\text{K}^{-1})(45-20\,\text{K})(0.0429\,\text{m})^3}{(1.631\times10^{-5}\text{ m}^2/\text{s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.728\times10^5)^{1/4} = 11.01$$

$$h = \frac{k}{L_c}Nu = \frac{0.02607 \text{ W/m.°C}}{0.0429\,\text{m}}(11.01) = 6.696 \text{ W/m}^2.\text{°C}$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8\text{ W} = (6.696\text{ W/m}^2.°\text{C})(0.03\text{ m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\text{ K})^4]$$

Its solution is

$$T_s = \mathbf{42.6°C}$$

which is sufficiently close to the assumed value of 45°C in the evaluation of the properties and h.

(c) **Horizontal, hot surface facing down** Again we assume the surface temperature to be 45°C and use the properties evaluated above. The characteristic length in this case is, from part (b), L_c = 0.0429 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003273\text{ K}^{-1})(45 - 20\text{ K})(0.0429\text{ m})^3}{(1.631\times10^{-5}\text{ m}^2\text{/s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.27(1.728\times10^5)^{1/4} = 5.505$$

$$h = \frac{k}{L_c}Nu = \frac{0.02607\text{ W/m.°C}}{0.0429\text{ m}}(5.505) = 3.3.345\text{ W/m}^2.°\text{C}$$

Considering both natural convection and radiation heat loses

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8\ \text{ W} = (3.345\text{ W/m}^2.°\text{C})(0.03\text{ m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\text{ K})^4]$$

Its solution is

$$T_s = \mathbf{50.7°C}$$

which is sufficiently close to the assumed value. Therefore, there is no need to repeat the calculations.

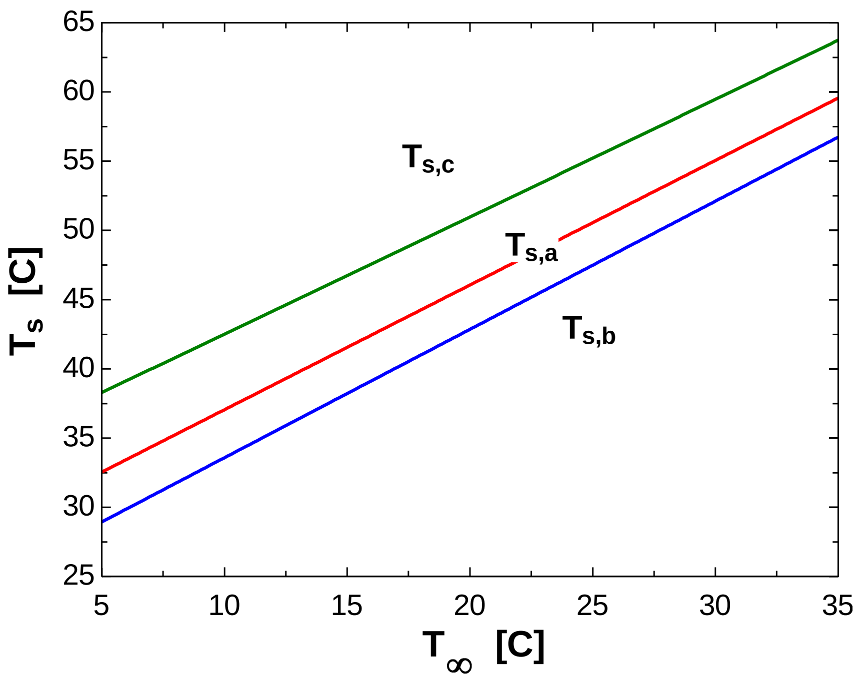
Discussion The assumed film temperature of T_f = 32.5°C is an appropriate assumption, since the determined T_s = 46.6°C, T_s = 42.6°C, and T_s = 50.7°C in parts *a*, *b*, and *c*, respectively would give film temperatures of T_f = 33.3°C, T_f = 31.3°C, and T_f = 35.4°C, respectively. Otherwise, T_s would have to be solved iteratively.

9-34 Prob. 9-33 is reconsidered. The effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]
w=0.15 [m]
T_infinity=20 [C]
Q_dot=8 [W]
epsilon=0.8
T_surr=T_infinity

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s_a+T_infinity)
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
g=9.807 [m/s^2] "gravitational acceleration"

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s_a-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=w*L
Q_dot=h_a*A*(T_s_a-T_infinity)+epsilon*A*sigma*((T_s_a+273)^4-(T_surr+273)^4)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=2*(w+L)
Ra_b=(g*beta*(T_s_b-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot=h_b*A*(T_s_b-T_infinity)+epsilon*A*sigma*((T_s_b+273)^4-(T_surr+273)^4)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot=h_c*A*(T_s_c-T_infinity)+epsilon*A*sigma*((T_s_c+273)^4-(T_surr+273)^4)
```

T_∞ [F]	$T_{s,a}$ [C]	$T_{s,b}$ [C]	$T_{s,c}$ [C]
5	32.54	28.93	38.29
7	34.34	30.79	39.97
9	36.14	32.65	41.66
11	37.95	34.51	43.35
13	39.75	36.36	45.04
15	41.55	38.22	46.73
17	43.35	40.07	48.42
19	45.15	41.92	50.12
21	46.95	43.78	51.81
23	48.75	45.63	53.51
25	50.55	47.48	55.21
27	52.35	49.33	56.91
29	54.16	51.19	58.62
31	55.96	53.04	60.32
33	57.76	54.89	62.03
35	59.56	56.74	63.74



9-35 A vertical plate with length L is placed in a quiescent air, and the expressions, having the form $\mathrm{Nu} = CRa_L^n$, for the average heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties.

Properties The properties of air at T_f = 20°C are k = 0.02514 W/m·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f = 0.003413$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.003413\,\mathrm{K^{-1}})\Delta T L^3}{(1.516\times10^{-5}\ \mathrm{m^2/s})^2}(0.7309) = 1.065\times10^8\,\Delta T L^3$$

For $10^4 < \mathrm{Ra}_L < 10^9$, we have

$$\mathrm{Nu} = \frac{hL}{k} = 0.59\mathrm{Ra}_L^{1/4} \qquad \rightarrow \qquad h = 0.59\frac{k}{L}\mathrm{Ra}_L^{1/4}$$

Substituting the Ra_L yields

$$h = 0.59\left(\frac{0.02514}{L}\right)(1.065\times10^8\,\Delta T L^3)^{1/4} = 1.51(\Delta T / L)^{1/4} \qquad 10^4 < \mathrm{Ra}_L < 10^9$$

For $10^{10} < \mathrm{Ra}_L < 10^{13}$, we have

$$\mathrm{Nu} = \frac{hL}{k} = 0.1\mathrm{Ra}_L^{1/3} \qquad \rightarrow \qquad h = 0.1\frac{k}{L}\mathrm{Ra}_L^{1/3}$$

Substituting the Ra_L yields

$$h = 0.1\left(\frac{0.02514}{L}\right)(1.065\times10^8\,\Delta T L^3)^{1/3} = 1.19\Delta T^{1/3} \qquad 10^{10} < \mathrm{Ra}_L < 10^{13}$$

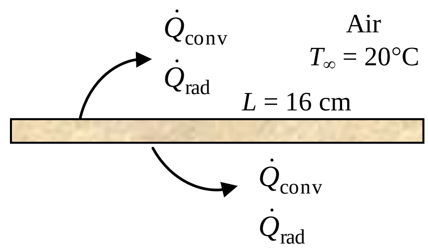
Discussion The average heat transfer coefficient for laminar conditions ($10^4 < \mathrm{Ra}_L < 10^9$) is dependent on ΔT and L. In turbulent conditions ($10^{10} < \mathrm{Ra}_L < 10^{13}$), the average heat transfer coefficient is not influenced by L.

9-36 Heat is generated in a horizontal plate while heat is lost from it by convection and radiation. The temperature of the plate when steady operating conditions are reached is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties We assume the surface temperature to be 50°C based on the problem statement. Then the properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (50+20)/2 = 35°C are (Table A-15)

$$k = 0.02625 \text{ W/m}\cdot\text{K}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7268$$
$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)\text{K}} = 0.003247 \text{ K}^{-1}$$



Analysis The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.16\text{ m})(0.20\text{ m})}{2[(0.16\text{ m})+(0.20\text{ m})]} = 0.04444\text{ m}$$

The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003247\text{ K}^{-1})(50-20\text{ K})(0.04444\text{ m})^3}{(1.655\times10^{-5}\text{ m}^2\text{/s})^2}(0.7268) = (306{,}195)(0.7268) = 222{,}543$$

The Nusselt number relation for the top surface of the plate is

$$\text{Nu} = 0.54\text{Ra}_L^{0.25} = 0.54(222{,}543)^{0.25} = 11.73$$

Then

$$h = \frac{k}{L_c}Nu = \frac{0.02625\text{ W/m}\cdot\text{K}}{0.04444\text{ m}}(11.73) = 6.929\text{ W/m}^2\cdot\text{K}$$

and

$$\dot{Q}_{top} = hA(T_s - T_\infty) = (6.929\text{ W/m}^2.°\text{C})(0.16\times0.20\text{ m}^2)(T_s - 20)°\text{C} = 0.2217(T_s - 20)$$

The Nusselt number relation for the bottom surface of the plate is

$$\text{Nu} = 0.27\text{Ra}_L^{0.25} = 0.27(222{,}543)^{0.25} = 5.864$$

Then

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02625\text{ W/m}\cdot\text{K}}{0.04444\text{ m}}(5.864) = 3.464\text{ W/m}^2\cdot\text{K}$$

$$\dot{Q}_{bottom} = hA(T_s - T_\infty) = (3.464\text{ W/m}^2.°\text{C})(0.16\times0.20\text{ m}^2)(T_s - 20)°\text{C} = 0.1109(T_s - 20)$$

Considering that radiation heat loss to surroundings occur both from top and bottom surfaces, it may be expressed as

$$\begin{aligned}\dot{Q}_{rad} &= 2\varepsilon A\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (2)(0.9)(0.16\times0.20\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(T_s+273\text{ K})^4 - (17+273\text{ K})^4\right] \\ &= 3.266\times10^{-9}\left[(T_s+273\text{ K})^4 - (17+273\text{ K})^4\right]\end{aligned}$$

When the heat lost from the plate equals to the heat generated, the steady operating conditions are reached. The surface temperature in this case can be determined by trial-error or using EES to be

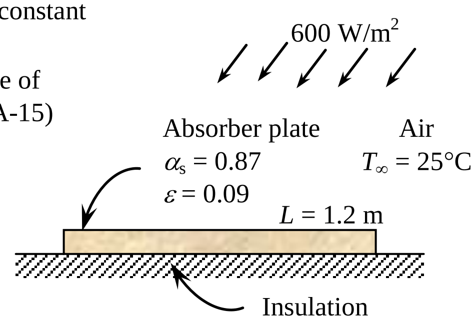
$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{rad}$$
$$20\text{ W} = 0.2217(T_s - 20) + 0.1109(T_s - 20) + 3.266\times10^{-9}\left[(T_s+273\text{ K})^4 - (17+273\text{ K})^4\right]$$
$$T_s \approx \mathbf{47°C}$$

This is close to the assumed surface temperature of 50°C for property evaluation. Therefore, there is no need to repeat the calculations.

9-37 Absorber plates whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (115+25)/2 = 70°C$ based on the problem statement are (Table A-15)



$$k = 0.02881 \text{ W/m.°C}$$
$$\nu = 1.995\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 115°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.002915\text{ K}^{-1})(115-25\text{ K})(0.24\text{ m})^3}{(1.995\times10^{-5}\text{ m}^2\text{/s})^2}(0.7177) = 6.414\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(6.414\times10^7)^{1/3} = 60.04$$

$$h = \frac{k}{L_c}Nu = \frac{0.02881\text{ W/m.°C}}{0.24\text{ m}}(60.04) = 7.208\text{ W/m}^2\text{.°C}$$

$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.87)(600\text{ W/m}^2)(0.96\text{ m}^2) = 501.1\text{ W}$$

$$\dot{Q} = hA_s(T_s-T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{sky}^{\,4})$$

$$501.1\text{ W} = (7.208\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s-25)\text{°C}$$
$$+(0.09)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $\quad T_s = 89.7°C$

which is not very close to the assumed value of 115°C. We repeat the calculations at a new anticipated surface temperature of 95°C. The properties are to be evaluated at the film temperature of (95+25)/2=60°C.

$$k = 0.02808\text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5}\text{ m}^2\text{/s}$$
$$\Pr = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003\text{ K}^{-1}$$

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003003\text{ K}^{-1})(95-25\text{ K})(0.24\text{ m})^3}{(1.896\times10^{-5}\text{ m}^2\text{/s})^2}(0.7202) = 5.711\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(5.711\times10^7)^{1/3} = 57.77$$

$$h = \frac{k}{L_c}Nu = \frac{0.02808\text{ W/m.°C}}{0.24\text{ m}}(57.77) = 6.759\text{ W/m}^2\text{.°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma(T_s{}^4 - T_{sky}{}^4)$$

$$501.1\text{ W} = (6.759\text{ W/m}^2.°\text{C})(0.96\text{ m}^2)(T_s - 25)°\text{C}$$
$$+ (0.09)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (10 + 273\text{ K})^4]$$

$$T_s = \mathbf{93.5°C}$$

This is close to the assumed surface temperature of 95°C. Therefore, there is no need to repeat the calculations.

If the absorber plate is made of ordinary aluminum which has a solar absorptivity of 0.28 and an emissivity of 0.07, the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A_s = (0.28)(600\text{ W/m}^2)(0.96\text{ m}^2) = 161.3\text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience,

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma(T_s{}^4 - T_{sky}{}^4)$$

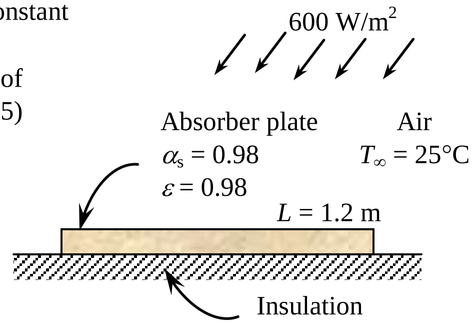
$$161.3\text{ W} = (6.629\text{ W/m}^2.°\text{C})(0.96\text{ m}^2)(T_s - 25)°\text{C} + (0.07)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (10 + 273\text{ K})^4]$$

Its solution is T_s = 47.8°C

Repeating the calculations at the new anticipated surface temperature of 55°C and the film temperature of (55+25)/2=40°C, we obtain

h = 5.312 W/m^2.°C and T_s = **53.0°C**

9-38 An absorber plate whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (65+25)/2 = 45°C based on the problem statement are (Table A-15)

$$k = 0.02699 \text{ W/m.°C}$$
$$\nu = 1.750\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7241$$
$$\beta = \frac{1}{T_f} = \frac{1}{(45+273)\text{K}} = 0.003145 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 65°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.00312\text{ K}^{-1})(65-25\text{ K})(0.24\text{ m})^3}{(1.750\times10^{-5}\text{ m}^2/\text{s})^2}(0.7241) = 4.033\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(4.033\times10^7)^{1/3} = 51.44$$

$$h = \frac{k}{L_c}Nu = \frac{0.02699\text{ W/m.°C}}{0.24\text{ m}}(51.44) = 5.785\text{ W/m}^2\text{.°C}$$

$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.98)(600\text{ W/m}^2)(0.96\text{ m}^2) = 564.5\text{ W}$$

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$564.5\text{ W} = (5.785\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C}$$
$$+ (0.98)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $\quad T_s = \mathbf{64.2°C}$

which is very close to the assumed value. Therefore there is no need to repeat calculations.

For a white painted absorber plate, the solar absorptivity is 0.26 and the emissivity is 0.90. Then the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A_s = (0.26)(600\text{ W/m}^2)(0.96\text{ m}^2) = 149.8\text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience (actually, we should calculate the new h using data at a lower temperature, and iterating if necessary for better accuracy),

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$149.8\text{ W} = (5.785\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C}$$
$$+ (0.90)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $\quad T_s = 32.1\text{°C}$

Repeating the calculations at the new anticipated surface temperature of 35°C and the film temperature of (35+25)/2=30°C, we obtain

h = 3.764 W/m^2.°C and T_s = **33.6°C**

9-39 The required electrical power to maintain a specified surface temperature of a grill is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant.

Properties The properties of air at $T_f = (T_s + T_\infty)/2$ = 90°C are k = 0.03024 W/m·K, $\nu = 2.201 \times 10^{-5}$ m^2/s, Pr = 0.7132 (from Table A-15). Also, $\beta = 1/T_f = 2.755 \times 10^{-3}$ K^{-1}.

Analysis Treating the grill as a horizontal circular plate, the characteristic length is

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = 0.0625\ \text{m}$$

The Rayleigh number ($L_c = D/4$) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.002755\ \text{K}^{-1})(150-30)\ \text{K}(0.0625\ \text{m})^3}{(2.201\times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7132)$$
$$= 1.166\times 10^6$$

Since the grill has a hot upper surface, we use

$$\text{Nu} = 0.54\text{Ra}_L^{1/4} = 0.54(1.166\times 10^6)^{1/4} = 17.74$$

$$h = \text{Nu}\frac{k}{L_c} = (17.74)\frac{0.03024\ \text{W/m}\cdot\text{K}}{0.0625\ \text{m}} = 8.583\ \text{W/m}^2\cdot\text{K}$$

The total rate of heat transfer on the grill surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$
$$= \frac{\pi}{4}(0.25\ \text{m})^2[(8.583\ \text{W/m}^2\cdot\text{K})(150-30)\ \text{K} + 0.8(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(423^4 - 303^4)\ \text{K}^4]$$
$$= \mathbf{103\ W}$$

Discussion To maintain a surface temperature of 150°C, the grill needs at least 103 W of electrical power.

9-40 A can of engine oil placed vertically in the trunk of a car and the heat transfer from the ends of the can are negligible, determine the heat transfer rate from the can surface.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2$ = 30°C are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m²/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\mathrm{Ra}_L = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.0033\,\mathrm{K^{-1}})(43-17)\mathrm{K}(0.15\,\mathrm{m})^3}{(1.608\times10^{-5}\,\mathrm{m^2/s})^2}(0.7282)$$
$$= 8.00\times10^6$$

Then

$$\frac{35L}{Gr_L^{1/4}} = \frac{35(0.15\,\mathrm{m})}{(1.099\times10^7)^{1/4}} = 0.0912\,\mathrm{m} < D$$

Since $D \ge 35L/Gr_L^{1/4}$ is satisfied, we can treat this vertical cylinder as a vertical plate, and the Nusselt may be calculated with

$$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4} = 0.59(8.00\times10^6)^{1/4} = 31.38$$

Then, the heat transfer coefficient is

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02588\ \mathrm{W/m\cdot K}}{0.15\,\mathrm{m}}(31.38) = 5.414\ \mathrm{W/m^2\cdot K}$$

Hence, the rate of heat transfer is

$$\dot{Q} = hA_s(T_\infty - T_s) = h\pi DL(T_\infty - T_s)$$
$$= (5.414\ \mathrm{W/m^2\cdot K})\pi(0.1\,\mathrm{m})(0.15\,\mathrm{m})(43-17)\,\mathrm{K}$$
$$= \mathbf{6.63\ W}$$

Discussion For vertical cylinder, the characteristic length is its length.

9-41 Flue gases are released to atmosphere using a cylindrical stack. The rates of heat transfer from the stack with and without wind cases are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

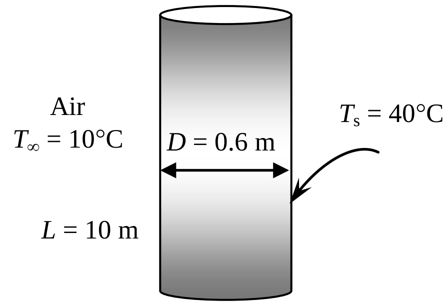
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+10)/2 = 25°C$ are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$

$$\nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7296$$

$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356 \text{ K}^{-1}$$



Analysis (*a*) When there is no wind heat transfer is by natural convection. The characteristic length in this case is the height of the stack, $L_c = L = 10$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003356 \text{ K}^{-1})(40-10 \text{ K})(10 \text{ m})^3}{(1.562\times10^{-5} \text{ m}^2/\text{s})^2}(0.7296) = 2.953\times10^{12}$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(10)}{(2.953\times10^{12}/0.7296)^{1/4}} = 0.246 < 0.6 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$ The

Nusselt number is determined from

$$Nu = 0.1Ra^{1/3} = 0.1(2.953\times10^{12})^{1/3} = 1435 \qquad \text{(from Table 9-1)}$$

Then

$$h = \frac{k}{L_c}Nu = \frac{0.02551 \text{ W/m.°C}}{10 \text{ m}}(1435) = 3.660 \text{ W/m}^2.°\text{C}$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (3.660 \text{ W/m}^2.°\text{C})(\pi\times0.6\times10 \text{ m}^2)(40-10)°\text{C} = \mathbf{2070 \text{ W}}$$

(*b*) When the stack is exposed to 20 km/h winds, the heat transfer will be by forced convection. We have flow of air over a cylinder and the heat transfer rate is determined as follows:

$$\text{Re} = \frac{VD}{\nu} = \frac{(20\times1000/3600 \text{ m/s})(0.6 \text{ m})}{1.562\times10^{-5} \text{ m}^2/\text{s}} = 213{,}400$$

$$\text{Nu} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(213{,}400)^{0.805}(0.7296)^{1/3} = 473.9 \qquad \text{(from Table 7-1)}$$

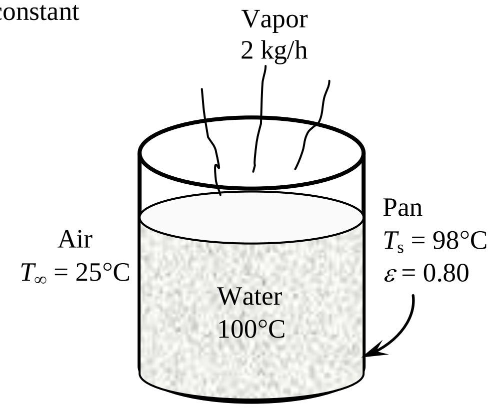
$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m.°C}}{0.6 \text{ m}}(473.9) = 20.15 \text{ W/m}^2.°\text{C}$$

$$\dot{Q} = hA(T_s - T_\infty) = (20.15 \text{ W/m}^2.°\text{C})(\pi\times0.6\times10 \text{ m}^2)(40-10)°\text{C} = \mathbf{11{,}390 \text{ W}}$$

Discussion There is more than five-fold increase in heat transfer due to winds.

9-42 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5$°C are (Table A-15)

$$k = 0.02819 \text{ W/m.°C}$$
$$\nu = 1.910\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6 / 0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2 \text{ W}}$$

(*b*) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.80)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{47.3 \text{ W}}$$

(*c*) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (1.5/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 0.9404 \text{ kW} = 940 \text{ W}$$

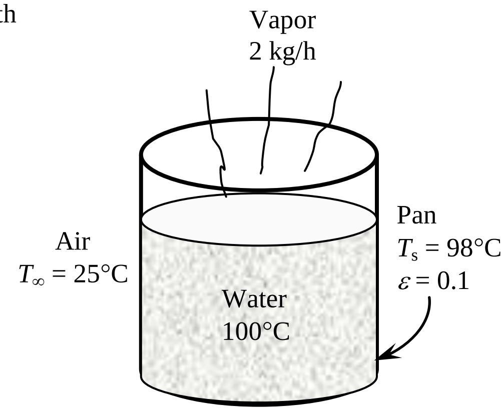
Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{46.2+47.3}{940} = 0.099 = \mathbf{9.9\%}$$

9-43 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5$°C are (Table A-15)



$$k = 0.02819 \text{ W/m.°C}$$
$$\nu = 1.910\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6 / 0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2 \text{ W}}$$

(b) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.10)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{5.9 \text{ W}}$$

(c) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (1.5/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 0.9404 \text{ kW} = 940 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

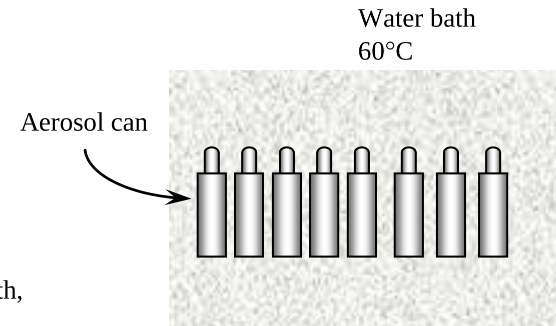
$$f = \frac{46.2+5.9}{940} = 0.055 = \mathbf{5.5\%}$$

9-44 Some cans move slowly in a hot water container made of sheet metal. The rate of heat loss from the four side surfaces of the container and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (60+20)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m}\cdot\text{K}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the bath, $L_c = L = 0.5 \text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(60-20 \text{ K})(0.5 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2/\text{s})^2}(0.7255) = 3.925\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.925\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 92.50$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{0.5 \text{ m}}(92.50) = 4.925 \text{ W/m}^2\cdot\text{K}$$

$$A_s = 2[(0.5 \text{ m})(1 \text{ m}) + (0.5 \text{ m})(3.5 \text{ m})] = 4.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.925 \text{ W/m}^2\cdot\text{K})(4.5 \text{ m}^2)(60-20)°\text{C} = 886.5 \text{ W}$$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4) \\ &= (0.7)(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(60+273 \text{ K})^4 - (20+273 \text{ K})^4\right] \\ &= 879.9 \text{ W}\end{aligned}$$

Then the total rate of heat loss becomes

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural \\ convection}} + \dot{Q}_{rad} = 886.5 + 879.9 = \mathbf{1766\ W}$$

The amount and cost of the heat loss during one year is

$$Q_{total} = \dot{Q}_{total}\Delta t = (1.766 \text{ kW})(8760 \text{ h}) = 15{,}470 \text{ kWh}$$
$$\text{Cost} = (15{,}470 \text{ kWh})(\$0.085/\text{kWh}) = \mathbf{\$1315}$$

9-45 Some cans move slowly in a hot water container made of sheet metal. It is proposed to insulate the side and bottom surfaces of the container for \$350. The simple payback period of the insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature, which is unknown.

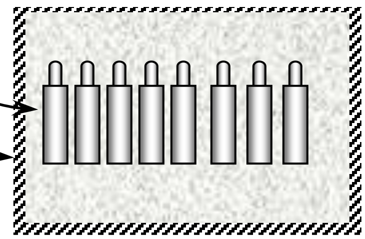
We evaluate air properties at a film temperature of $(T_s+T_\infty)/2$ = 23°C and 1 atm based on the problem statement. Then, for an air temperature of T_∞ = 20°C, the corresponding surface temperature is T_s = 26°C. The properties of air at 1 atm and 23°C are (Table A-15)

$$k = 0.02536 \text{ W/m.°C}$$

$$\nu = 1.543\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7301$$

$$\beta = \frac{1}{T_f} = \frac{1}{(23+273)\text{K}} = 0.00338 \text{ K}^{-1}$$



Analysis We start the solution process by "guessing" the outer surface temperature to be 26°C. We will check the accuracy of this guess later and repeat the calculations if necessary with a better guess based on the results obtained. The characteristic length in this case is the height of the tank, $L_c = L = 0.5\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00338 \text{ K}^{-1})(26-20 \text{ K})(0.5 \text{ m})^3}{(1.543\times10^{-5} \text{ m}^2\text{/s})^2}(0.7301) = 7.622\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.622\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7301}\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.53$$

$$h = \frac{k}{L}Nu = \frac{0.02536 \text{ W/m.°C}}{0.5 \text{ m}}(56.53) = 2.868 \text{ W/m}^2\text{.°C}$$

$$A_s = 2[(0.5 \text{ m})(1.10 \text{ m}) + (0.5 \text{ m})(3.60 \text{ m})] = 4.7 \text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated tank by convection and radiation becomes

$$\begin{aligned}\dot{Q} &= \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4) \\ &= (2.868 \text{ W/m}^2\text{.°C})(4.7 \text{ m}^2)(26-20)\text{°C} + (0.1)(4.7 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(26+273 \text{ K})^4 - (20+273 \text{ K})^4] \\ &= 97.5 \text{ W}\end{aligned}$$

In steady operation, the heat lost by the side surfaces of the tank must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. The second conditions requires the surface temperature to be

$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{T_{\text{tank}} - T_s}{L} \quad\rightarrow\quad 97.5 \text{ W} = (0.035 \text{ W/m.°C})(4.7 \text{ m}^2)\frac{(60-T_s)\text{°C}}{0.05 \text{ m}}$$

It gives T_s = 30.4°C, which is sufficiently close to the assumed temperature, 26°C. Therefore, there is no need to repeat the calculations. The total amount of heat loss and its cost during one year are

$$Q_{total} = \dot{Q}_{total}\Delta t = (97.5 \text{ W})(8760 \text{ h}) = 853.7 \text{ kWh}$$

$$\text{Cost} = (853.7 \text{ kWh})(\$0.085/\text{kWh}) = \$72.6$$

Then money saved during a one-year period due to insulation becomes

$$\text{Money saved} = \text{Cost}_{\substack{without\\insulation}} - \text{Cost}_{\substack{with\\insulation}} = \$1315 - \$72.6 = \$1242$$

where \$1116 is obtained from the solution of Problem 9-29. The insulation will pay for itself in

$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$350}{\$1242/yr} = \mathbf{0.282\ yr = 103\ days}$$

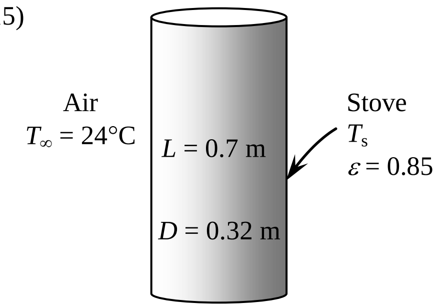
Discussion We would definitely recommend the installation of insulation in this case.

9-46 A room is to be heated by a cylindrical coal-burning stove. The surface temperature of the stove and the amount of coal burned during a 14-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the stove is constant. **5** The heat transfer from the bottom surface is negligible. **6** The heat transfer coefficient at the top surface is the same as that on the side surface.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (130+24)/2 = 77°C based on the problem statement are (Table A-15)

$$k = 0.02931 \text{ W/m.°C}$$
$$\nu = 2.066\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7161$$
$$\beta = \frac{1}{T_f} = \frac{1}{(77+273)\text{K}} = 0.002857 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the cylindir, $L_c = L = 0.70$ m. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(0.002857 \text{ K}^{-1})(130-24 \text{ K})(0.70 \text{ m})^3}{(2.066\times10^{-5} \text{ m}^2\text{/s})^2} = 2.387\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.32 \text{ m}) \geq \frac{35L}{\text{Gr}^{1/4}} = \frac{35(0.70 \text{ m})}{(1.203\times10^{10})^{1/4}} = 0.0740 \text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for side surfaces.

$$\text{Ra} = \text{GrPr} = (2.387\times10^9)(0.7161) = 1.709\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.709\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7161}\right)^{9/16}\right]^{8/27}}\right\}^2 = 145.2$$

$$h = \frac{k}{L}Nu = \frac{0.02931 \text{ W/m.°C}}{0.70 \text{ m}}(145.2) = 6.080 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + \pi D^2/4 = \pi(0.32 \text{ m})(0.70 \text{ m}) + \pi(0.32 \text{ m})^2/4 = 0.7841 \text{ m}^2$$

Then the surface temperature of the stove is determined from

$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$
$$1500 \text{ W} = (6.080 \text{ W/m}^2\text{.°C})(0.7841 \text{ m}^2)(T_s - 297)$$
$$+ (0.85)(0.7841 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)(T_s^4 - 287^4)$$
$$T_s = 419.6 \text{ K} = 146.6\text{°C}$$

This surface temperature is sufficiently close to the value assumed for the evaluation of properties and h. Therefore, there is no need to repeat the calculations. The amount of coal used is determined from

$$Q = \dot{Q}\Delta t = (1.5 \text{ kJ/s})(14 \text{ h/day}\times3600 \text{ s/h}) = 75{,}600 \text{ kJ}$$
$$m_{\text{coal}} = \frac{Q/\eta}{HV} = \frac{(75{,}600 \text{ kJ})/0.65}{30{,}000 \text{ kJ/kg}} = \mathbf{3.88 kg}$$

9-47 A cylinder with specified length and diameter, the orientation of the cylinder that would achieve higher heat transfer rate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 30°C$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis For vertical orientation, the Rayleigh number ($L_c = L$) is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.0033\,\mathrm{K^{-1}})(43-17)\mathrm{K}(0.15\,\mathrm{m})^3}{(1.608\times10^{-5}\,\mathrm{m^2/s})^2}(0.7282)$$

$$= 8.00\times10^6$$

Then

$$\frac{35L}{Gr_L^{1/4}} = \frac{35(0.15\,\mathrm{m})}{(1.099\times10^7)^{1/4}} = 0.0912\,\mathrm{m} < D$$

Since $D \geq 35L/Gr_L^{1/4}$ is satisfied, we can treat this vertical cylinder as a vertical plate, and the Nusselt may be calculated with

$$\mathrm{Nu}_{\mathrm{vert}} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(8.00\times10^6)^{1/6}}{[1+(0.492/0.7282)^{9/16}]^{8/27}}\right\}^2 = 29.39$$

For horizontal orientation, the Rayleigh number ($L_c = D$) is

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(0.0033\,\mathrm{K^{-1}})(43-17)\mathrm{K}(0.1\,\mathrm{m})^3}{(1.608\times10^{-5}\,\mathrm{m^2/s})^2}(0.7282)$$

$$= 2.37\times10^6$$

The Nusselt number for horizontal cylinder is

$$\mathrm{Nu}_{\mathrm{horiz}} = \left\{0.6 + \frac{0.387\mathrm{Ra}_D^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.37\times10^6)^{1/6}}{[1+(0.559/0.7282)^{9/16}]^{8/27}}\right\}^2 = 18.64$$

Hence, the ratio of heat transfer rate is

$$\frac{\dot{Q}_{\mathrm{vert}}}{\dot{Q}_{\mathrm{horiz}}} = \frac{h_{\mathrm{vert}}(k/L)A_s\Delta T}{h_{\mathrm{horiz}}(k/L)A_s\Delta T} = \frac{\mathrm{Nu}_{\mathrm{vert}}(k/L)A_s\Delta T}{\mathrm{Nu}_{\mathrm{horiz}}(k/L)A_s\Delta T} = \frac{\mathrm{Nu}_{\mathrm{vert}}}{\mathrm{Nu}_{\mathrm{horiz}}} = \frac{29.39}{18.64} = \mathbf{1.58}$$

Discussion For the same ΔT, the rate of heat transfer for the vertical orientation is 58% larger than that for the horizontal orientation.

9-48 A soda can placed horizontally in a refrigerator compartment and the heat transfer from the ends of the can are negligible, determine the heat transfer rate from the can surface.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 20°C$ are $k = 0.02514$ W/m·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f = 0.003413$ K^{-1}.

Analysis The Rayleigh number ($L_c = D$) is

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003413\,\text{K}^{-1})(36-4)\text{K}(0.06\,\text{m})^3}{(1.516\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7309)$$
$$= 7.36\times10^5$$

The Nusselt number for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(7.36\times10^5)^{1/6}}{[1+(0.559/0.7309)^{9/16}]^{8/27}}\right\}^2 = 13.39$$

Then, the heat transfer coefficient is

$$h = \frac{k}{D}\text{Nu} = \frac{0.02514\,\text{W/m}\cdot\text{K}}{0.06\,\text{m}}(13.39) = 5.61\,\text{W/m}^2\cdot\text{K}$$

Hence, the rate of heat transfer is

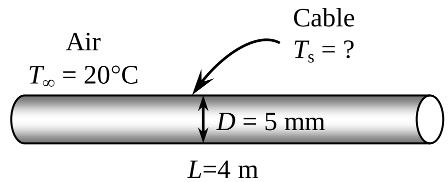
$$\dot{Q} = hA_s(T_s - T_\infty) = h\pi DL(T_s - T_\infty)$$
$$= (5.61\,\text{W/m}^2\cdot\text{K})\pi(0.06\,\text{m})(0.15\,\text{m})(36-4)\,\text{K}$$
$$= \mathbf{5.08\,W}$$

Discussion For horizontal cylinder, the characteristic length is its diameter.

9-49 Heat generated by the electrical resistance of a bare cable is dissipated to the surrounding air. The surface temperature of the cable is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the surface of the cable is constant.

Properties We evaluate air properties at a film temperature of $(T_s+T_\infty)/2$ = 60°C and 1 atm based on the problem statement. Then, for an air temperature of T_∞ = 20°C, the corresponding surface temperature is T_s = 100°C. The properties of air at 1 atm and 60°C are (Table A-15)



$$k = 0.02808\ \text{W/m.°C}$$
$$\nu = 1.896\times10^{-5}\ \text{m}^2/\text{s}$$
$$\Pr = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003\ \text{K}^{-1}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.005\ \text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81\ \text{m/s}^2)(0.003003\ \text{K}^{-1})(100-20\ \text{K})(0.005\ \text{m})^3}{(1.896\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7202) = 590.2$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(590.2)^{1/6}}{\left[1+(0.559/0.7202)^{9/16}\right]^{8/27}}\right\}^2 = 2.346$$

$$h = \frac{k}{D}Nu = \frac{0.02808\ \text{W/m.°C}}{0.005\ \text{m}}(2.346) = 13.17\ \text{W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.005\ \text{m})(4\ \text{m}) = 0.06283\ \text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60\ \text{V})(1.5\ \text{A}) = (13.17\ \text{W/m}^2.\text{°C})(0.06283\ \text{m}^2)(T_s - 20)\text{°C}$$
$$T_s = 128.8\text{°C}$$

which is not close to the assumed value of 100°C. Repeating calculations for an assumed surface temperature of 120°C, [T_f = $(T_s+T_\infty)/2$ = (120+20)/2 = 70°C]

$$k = 0.02881\ \text{W/m.°C}$$
$$\nu = 1.995\times10^{-5}\ \text{m}^2/\text{s}$$
$$\Pr = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915\ \text{K}^{-1}$$

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81\ \text{m/s}^2)(0.002915\ \text{K}^{-1})(120-20\ \text{K})(0.005\ \text{m})^3}{(1.995\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7177) = 644.6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(644.6)^{1/6}}{\left[1+(0.559/0.7177)^{9/16}\right]^{8/27}}\right\}^2 = 2.387$$

$$h = \frac{k}{D}Nu = \frac{0.02881\ \text{W/m.°C}}{0.005\ \text{m}}(2.387) = 13.76\ \text{W/m}^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60\ \text{V})(1.5A) = (13.76\ \text{W/m}^2.\text{°C})(0.06283\ \text{m}^2)(T_s - 20)\text{°C}$$
$$T_s = \mathbf{124.1°C}$$

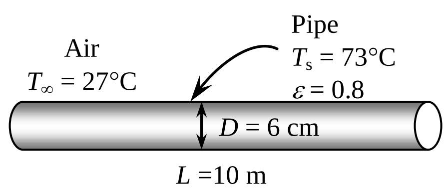
which is sufficiently close to the assumed value of 120°C.

9-50 A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the pipe is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (73+27)/2 = 50°C$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.06$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(73-27 \text{ K})(0.06 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2/\text{s})^2}(0.7228) = 6.747\times10^5$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(6.747\times10^5)^{1/6}}{\left[1+\left(0.559/0.7228\right)^{9/16}\right]^{8/27}}\right\}^2 = 13.05$$

$$h = \frac{k}{D}Nu = \frac{0.02735 \text{ W/m.°C}}{0.06 \text{ m}}(13.05) = 5.949 \text{ W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.06 \text{ m})(10 \text{ m}) = 1.885 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.949 \text{ W/m}^2.\text{°C})(1.885 \text{ m}^2)(73-27)\text{°C} = \mathbf{516 \ W}$$

(*b*) The radiation heat loss from the pipe is

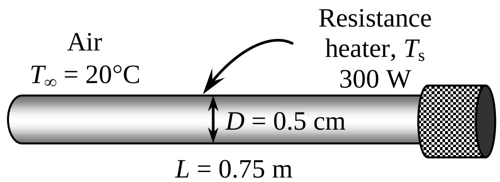
$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4) \\ &= (0.8)(1.885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(73+273 \text{ K})^4 - (27+273 \text{ K})^4\right] \\ &= \mathbf{533 \ W}\end{aligned}$$

9-51 A cylindrical resistance heater is placed horizontally in a fluid. The outer surface temperature of the resistance wire is to be determined for two different fluids.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer by radiation is ignored. **5** Properties are evaluated at 500°C for air and 40°C for water.

Properties The properties of air at 1 atm and 500°C are (Table A-15)

$$k = 0.05572 \text{ W/m.°C}$$
$$\nu = 7.804 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.6986,$$
$$\beta = \frac{1}{T_f} = \frac{1}{(500+273)\text{K}} = 0.001294 \text{ K}^{-1}$$



The properties of water at 40°C are (Table A-9)

$$k = 0.631 \text{ W/m.°C}$$
$$\nu = \mu/\rho = 0.6582 \times 10^{-6} \text{ m}^2/\text{s}$$
$$\text{Pr} = 4.32$$
$$\beta = 0.000377 \text{ K}^{-1}$$

Analysis (*a*) The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 1200°C for the calculation of *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.001294 \text{ K}^{-1})(1200-20)\text{°C}(0.005 \text{ m})^3}{(7.804\times10^{-5} \text{ m}^2/\text{s})^2}(0.6986) = 214.7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(214.7)^{1/6}}{\left[1+(0.559/0.6986)^{9/16}\right]^{8/27}}\right\}^2 = 1.919$$

$$h = \frac{k}{D}Nu = \frac{0.05572 \text{ W/m.°C}}{0.005 \text{ m}}(1.919) = 21.38 \text{ W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.005 \text{ m})(0.75 \text{ m}) = 0.01178 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow 300 \text{ W} = (21.38 \text{ W/m}^2.\text{°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \rightarrow T_s = \mathbf{1211°C}$$

which is close to the assumed value of 1200°C used in the evaluation of *h*. Therefore, there is no need to repeat calculations.

(*b*) For the case of water, we "guess" the surface temperature to be 40°C. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.000377 \text{ K}^{-1})(40-20 \text{ K})(0.005 \text{ m})^3}{(0.6582\times10^{-6} \text{ m}^2/\text{s})^2}(4.32) = 92{,}197$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(92{,}197)^{1/6}}{\left[1+(0.559/4.32)^{9/16}\right]^{8/27}}\right\}^2 = 8.986$$

$$h = \frac{k}{D}Nu = \frac{0.631 \text{ W/m.°C}}{0.005 \text{ m}}(8.986) = 1134 \text{ W/m}^2.\text{°C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow 300 \text{ W} = (1134 \text{ W/m}^2.\text{°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \longrightarrow T_s = \mathbf{42.5°C}$$

which is sufficiently close to the assumed value of 40°C in the evaluation of the properties and *h*. Therefore, there is no need to repeat calculations.

9-52 A thick fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The power rating of the electric resistance heater and the cost of electricity during a 10-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

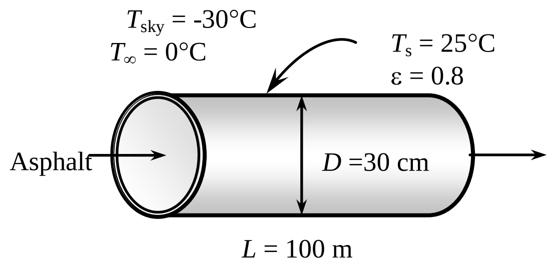
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (25+0)/2 = 12.5°C are (Table A-15)

$$k = 0.02458 \text{ W/m.°C}$$

$$\nu = 1.448\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7330$$

$$\beta = \frac{1}{T_f} = \frac{1}{(12.5+273)\text{K}} = 0.003503 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.3\,\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\,\text{m/s}^2)(0.003503\,\text{K}^{-1})(25-0\,\text{K})(0.3\,\text{m})^3}{(1.448\times10^{-5}\,\text{m}^2\text{/s})^2}(0.7330) = 8.106\times10^7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(8.106\times10^7)^{1/6}}{\left[1+\left(0.559/0.7330\right)^{9/16}\right]^{8/27}}\right\}^2 = 53.29$$

$$h = \frac{k}{L_c}Nu = \frac{0.02458\text{ W/m.°C}}{0.3\,\text{m}}(53.29) = 4.366 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.3\,\text{m})(100\,\text{m}) = 94.25\,\text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.366\text{ W/m}^2\text{.°C})(94.25\,\text{m}^2)(25-0)\text{°C} = 10{,}287\text{ W}$$

The radiation heat loss from the cylinder is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$= (0.8)(94.25\,\text{m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(25+273\,\text{K})^4 - (-30+273\,\text{K})^4] = 18{,}808\text{ W}$$

Then,

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural\\convection}} + \dot{Q}_{radiation} = 10{,}287 + 18{,}808 = 29{,}094\text{ W} = \mathbf{29.1kW}$$

The total amount and cost of heat loss during a 10 hour period is

$$Q = \dot{Q}\Delta t = (29.1\,\text{kW})(10\,\text{h}) = 291\,\text{kWh}$$

$$\text{Cost} = (291\,\text{kWh})(\$0.09/\text{kWh}) = \mathbf{\$26.2}$$

9-53 A fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The thickness of the insulation needed to reduce the losses by 85% and the money saved during 10-h are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

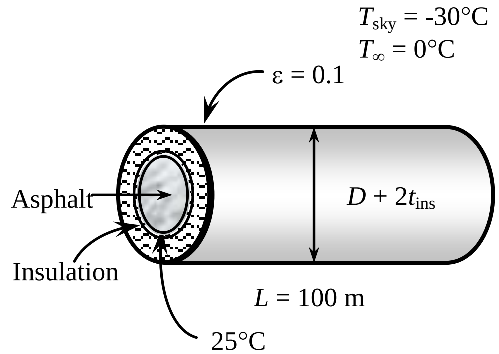
Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature, and possible below it because of the very low sky temperature for radiation heat loss. Based on the problem statemenet, we use the properties of air at 1 atm and 5°C (the anticipated film temperature) (Table A-15),

$$k = 0.02401 \text{ W/m.°C}$$

$$\nu = 1.382\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7350$$

$$\beta = \frac{1}{T_f} = \frac{1}{(5+273)\text{K}} = 0.003597 \text{ K}^{-1}$$



Analysis The rate of heat loss in the previous problem was obtained to be 29,094 W. Noting that insulation will cut down the heat losses by 85%, the rate of heat loss will be

$$\dot{Q} = (1-0.85)\dot{Q}_{\text{no insulation}} = 0.15\times 29{,}094 \text{ W} = 4364 \text{ W}$$

The amount of energy and money insulation will save during a 10-h period is simply determined from

$$Q_{saved,total} = \dot{Q}_{saved}\Delta t = (0.85\times 29.094 \text{ kW})(10 \text{ h}) = 247.3 \text{ kWh}$$

$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (247.3 \text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$22.3}$$

The characteristic length in this case is the outer diameter of the insulated pipe, $L_c = D + 2t_{insul} = 0.3 + 2t_{insul}$ where t_{insul} is the thickness of insulation in m. Then the problem can be formulated for T_s and t_{insul} as follows:

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003597 \text{ K}^{-1})[(T_s - 273)\text{K}](0.3+2t_{insul})^3}{(1.382\times10^{-5} \text{ m}^2\text{/s})^2}(0.7350)$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/0.7350\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02401 \text{ W/m.°C}}{L_c}Nu$$

$$A_s = \pi D_0 L = \pi(0.3+2t_{insul})(100 \text{ m})$$

The total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$4364 = hA_s(T_s - 273) + (0.1)A_s(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[T_s^4 - (-30+273 \text{ K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = \frac{2\pi kL(T_{\text{tank}} - T_s)}{\ln(D_o/D)} \rightarrow 4364 \text{ W} = \frac{2\pi(0.035 \text{ W/m.°C})(100 \text{ m})(298 - T_s)\text{K}}{\ln[(0.3+2t_{insul})/0.3]}$$

The solution of all of the equations above simultaneously using an equation solver gives

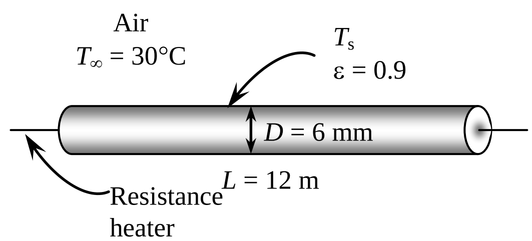
T_s = 281.5 K = 8.5°C and t_{insul} = **0.013 m = 1.3 cm**.

Note that the film temperature is (8.5+0)/2 = 4.25°C which is very close to the assumed value of 5°C. Therefore, there is no need to repeat the calculations using properties at this new film temperature.

9-54 An insulated electric wire is exposed to calm air. The temperature at the interface of the wire and the plastic insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ based on the problem statement are (Table A-15)



$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 50°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated wire $L_c = D = 0.006$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(0.006 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 339.3$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(339.3)^{1/6}}{\left[1+\left(0.559/0.7255\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.101$$

$$h = \frac{k}{D}Nu = \frac{0.02662 \text{ W/m.°C}}{0.006 \text{ m}}(2.101) = 9.327 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.006 \text{ m})(12 \text{ m}) = 0.2262 \text{ m}^2$$

The rate of heat generation, and thus the rate of heat transfer is

$$\dot{Q} = \mathbf{V}I = (7 \text{ V})(10 \text{ A}) = 70 \text{ W}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$70 \text{ W} = (9.327 \text{ W/m}^2\text{.°C})(0.226 \text{ m}^2)(T_s - 30)\text{°C}$$
$$+ (0.9)(0.2262 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (30+273 \text{ K})^4]$$

Its solution is

$$T_s = 49.9\text{°C}$$

which is very close to the assumed value of 50°C. Then the temperature at the interface of the wire and the plastic cover in steady operation becomes

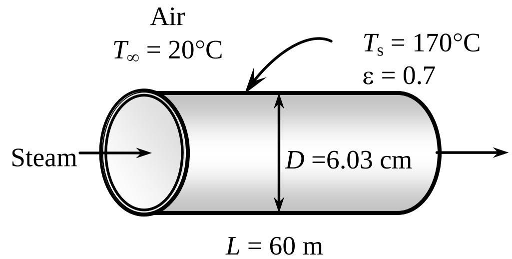
$$\dot{Q} = \frac{2\pi kL}{\ln(D_2/D_1)}(T_i - T_s) \longrightarrow T_i = T_s + \frac{\dot{Q}\ln(D_2/D_1)}{2\pi kL} = 49.9\text{°C} + \frac{(70 \text{ W})\ln(6/3)}{2\pi(0.20 \text{ W/m.°C})(12 \text{ m})} = \mathbf{53.1°C}$$

Discussion The assumed film temperature of T_f = 40°C is an appropriate assumption, since the determined T_s = 49.9°C would give a film temperature of T_f = 39.95°C. Otherwise, T_s would have to be solved iteratively.

9-55 A steam pipe extended from one end of a plant to the other with no insulation on it. The rate of heat loss from the steam pipe and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (170+20)/2 = 95°C$ are (Table A-15)



$$k = 0.0306 \text{ W/m.°C}$$
$$\nu = 2.254\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7122$$
$$\beta = \frac{1}{T_f} = \frac{1}{(95+273)\text{K}} = 0.002717 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.0603\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.002717\text{ K}^{-1})(170-20\text{ K})(0.0603\text{ m})^3}{(2.254\times10^{-5}\text{ m}^2\text{/s})^2}(0.7122) = 1.229\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.229\times10^6)^{1/6}}{\left[1+\left(0.559/0.7122\right)^{9/16}\right]^{8/27}}\right\}^2 = 15.41$$

$$h = \frac{k}{D}Nu = \frac{0.0306\text{ W/m.°C}}{0.0603\text{ m}}(15.41) = 7.820\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.0603\text{ m})(60\text{ m}) = 11.37\text{ m}^2$$

Then the total rate of heat transfer by natural convection and radiation becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (7.820\text{ W/m}^2\text{.°C})(11.37\text{ m}^2)(170-20)°\text{C} \\ &\quad + (0.7)(11.37\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(170+273\text{ K})^4 - (20+273\text{ K})^4] \\ &= 27{,}391\text{ W} = \mathbf{27.4\,kW}\end{aligned}$$

The total amount of gas consumption and its cost during a one-year period is

$$Q_{gas} = \frac{\dot{Q}\Delta t}{\eta} = \frac{27.391\text{ kJ/s}}{0.78}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right)(8760\text{ h/yr}\times3600\text{ s/h}) = 10{,}497\text{ therms/yr}$$

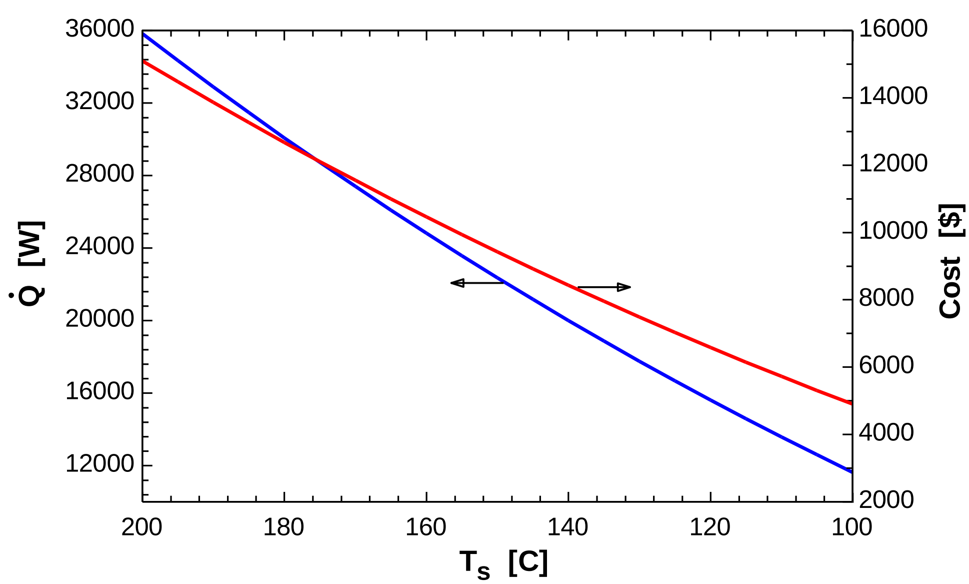
$$\text{Cost} = (10{,}497\text{ therms/yr})(\$1.10/\text{therm}) = \mathbf{\$11{,}547/yr}$$

9-56 Prob. 9-55 is reconsidered. The effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=60 [m]
D=0.0603 [m]
T_s=170 [C]
T_infinity=20 [C]
epsilon=0.7
T_surr=T_infinity
eta_furnace=0.78
UnitCost=1.10 [$/therm]
time=24*365 [h]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
g=9.807 [m/s^2] "gravitational acceleration"

"ANALYSIS"
delta=D
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
Nusselt=(0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27))^2
h=k/delta*Nusselt
A=pi*D*L
Q_dot=h*A*(T_s-T_infinity)+epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
Q_gas=(Q_dot*time)/eta_furnace*Convert(h, s)*Convert(J, kJ)*Convert(kJ, therm)
Cost=Q_gas*UnitCost
```

T_s[C]	$\dot{Q}$ [W]	Cost [$]
100	11635	4904
105	12593	5308
110	13577	5723
115	14585	6148
120	15618	6584
125	16677	7030
130	17760	7487
135	18870	7954
140	20006	8433
145	21168	8923
150	22357	9424
155	23574	9937
160	24817	10461
165	26089	10997
170	27390	11546
175	28719	12106
180	30078	12679
185	31466	13264
190	32885	13862
195	34335	14473
200	35816	15098

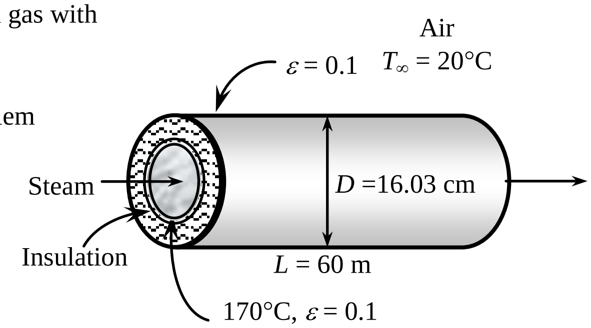


9-57 A steam pipe extended from one end of a plant to the other. It is proposed to insulate the steam pipe for \$750. The simple payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+20)/2 = 27.5$°C based on the problem statement are (Table A-15)

$$k = 0.0257 \text{ W/m.°C}$$
$$\nu = 1.584\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7289$$
$$\beta = \frac{1}{T_f} = \frac{1}{(27.5+273)\text{K}} = 0.003328 \text{ K}^{-1}$$



Analysis Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the outer surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated pipe, $L_c = D = 0.1603\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003328\text{ K}^{-1})(35-20\text{ K})(0.1603\text{ m})^3}{(1.584\times10^{-5}\text{ m}^2\text{/s})^2}(0.7289) = 5.856\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(5.856\times10^6)^{1/6}}{\left[1+\left(0.559/0.7289\right)^{9/16}\right]^{8/27}}\right\}^2 = 24.23$$

$$h = \frac{k}{D}Nu = \frac{0.0257\text{ W/m.°C}}{0.1603\text{ m}}(24.23) = 3.884\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.1603\text{ m})(60\text{ m}) = 30.22\text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\begin{aligned}\dot{Q} &= \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)\\ &= (3.884\text{ W/m}^2\text{.°C})(30.22\text{ m}^2)(35-20)\text{°C}\\ &\quad + (0.1)(30.22\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(35+273\text{ K})^4 - (20+273\text{ K})^4]\\ &= 2040\text{ W}\end{aligned}$$

In steady operation, the heat lost from the exposed surface of the insulation by convection and radiation must be equal to the heat conducted through the insulation. This requirement gives the surface temperature to be

$$\dot{Q} = \dot{Q}_{\text{insulation}} = \frac{T_{s.i} - T_s}{R_{ins}} = \frac{T_{s.i} - T_s}{\dfrac{\ln(D_2/D_1)}{2\pi kL}} \rightarrow 2040\text{ W} = \frac{(170 - T_s)\text{°C}}{\dfrac{\ln(16.03/6.03)}{2\pi(0.038\text{ W/m.°C})(60\text{ m})}}$$

It gives 30.8°C for the surface temperature, which is somewhat different than the assumed value of 35°C. Repeating the calculations with other surface temperatures gives

$$T_s = 34.3\text{°C} \quad \text{and } \dot{Q} = 1988\text{ W}$$

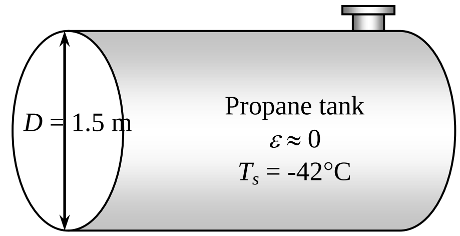
Heat loss and its cost without insulation were determined in the Prob. 9-42 to be 27.391 kW and \$11,547. Then the reduction in the heat losses becomes

$$\dot{Q}_{\text{saved}} = 27.391 - 1.988 = 25.403\text{ kW} \quad \text{or} \quad 25.403/27.391 = 0.927 \quad (92.7\%)$$

Therefore, the money saved by insulation will be 0.927 × (\$11,547/yr) = **\$10,700/yr** which will pay for the cost of \$750 in \$750/(\$10,700/yr)=0.070 year = **26 days**.

9-58 A cylindrical propane tank is exposed to calm ambient air. The propane is slowly vaporized due to a crack developed at the top of the tank. The time it will take for the tank to empty is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation heat transfer is negligible.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (-42+25)/2 = -8.5°C are (Table A-15)

$$k = 0.02299 \text{ W/m.°C}$$
$$\nu = 1.265\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7383$$
$$\beta = \frac{1}{T_f} = \frac{1}{(-8.5+273)\text{K}} = 0.003781\,\text{K}^{-1}$$

Analysis The tank gains heat through its cylindrical surface as well as its circular end surfaces. For convenience, we take the heat transfer coefficient at the end surfaces of the tank to be the same as that of its side surface. (The alternative is to treat the end surfaces as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the end surfaces is much smaller and it is circular in shape rather than being rectangular). The characteristic length in this case is the outer diameter of the tank, $L_c = D = 1.5\text{ m}$. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003781\text{ K}^{-1})[(25-(-42)\text{ K}](1.5\text{ m})^3}{(1.265\times10^{-5}\text{ m}^2\text{/s})^2}(0.7383) = 3.869\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.869\times10^{10})^{1/6}}{\left[1+\left(0.559/0.7383\right)^{9/16}\right]^{8/27}}\right\}^2 = 374.1$$

$$h = \frac{k}{D}Nu = \frac{0.02299\text{ W/m.°C}}{1.5\text{ m}}(374.1) = 5.733\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + 2\pi D^2/4 = \pi(1.5\text{ m})(4\text{ m}) + 2\pi(1.5\text{ m})^2/4 = 22.38\text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (5.733\text{ W/m}^2\text{.°C})(22.38\text{ m}^2)[(25-(-42)]\text{°C} = 8598\text{ W}$$

The total mass and the rate of evaporation of propane are

$$m = \rho \boldsymbol{V} = \rho\frac{\pi D^2}{4}L = (581\text{ kg/m}^3)\frac{\pi(1.5\text{ m})^2}{4}(4\text{ m}) = 4107\text{ kg}$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{8.598\text{ kJ/s}}{425\text{ kJ/kg}} = 0.02023\text{ kg/s}$$

and it will take

$$\Delta t = \frac{m}{\dot{m}} = \frac{4107\text{ kg}}{0.02023\text{ kg/s}} = 202{,}996\text{ s} = \mathbf{56.4\,hours}$$

for the propane tank to empty.

9-59 Hot water flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is exposed to cool air. The outer surface temperature of the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The pipe thermal conductivity is constant. **4** Radiation heat transfer is negligible. **5** Local atmospheric pressure is 1 atm. **6** The film temperature is 40°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 40°C are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, Pr = 0.7255 (Table A-15), and $\beta = 1/T_f = 1/313$ K.

Analysis With the assumption that T_f = 40°C, the pipe outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 68°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(40+273\,\text{K})^{-1}(68-12)\,\text{K}\,(0.045\,\text{m})^3}{(1.702\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7255) = 400{,}561$$

The Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(400{,}561)^{1/6}}{\left[1+\left(\frac{0.559}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.31$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02662\ \text{W/m}\cdot\text{K}}{0.045\,\text{m}}(11.31) = 6.6905\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the pipe wall and natural convection on the outer surface is

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}}$$

$$\frac{2\pi L k_{\text{cyl}}}{\ln(D_o/D_i)}(T_{s,i} - T_{s,o}) = hA_s(T_{s,o} - T_\infty) \quad \rightarrow \quad T_{s,o} = \mathbf{67.3°C}$$

where $A_s = \pi D_o L$

Discussion The assumed film temperature of T_f = 40°C is an appropriate assumption, since the determined $T_{s,o}$ = 67.3°C would give a film temperature of T_f = 39.7°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-60 Hot engine flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is covered with a layer of insulation. The outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Thermal conductivities of pipe and insulation are constant. **4** Contact resistance is negligible. **5** Radiation heat transfer is negligible. **6** Local atmospheric pressure is 1 atm.

Properties We first assume the film temperature is T_f = 50°C. Then, the properties of air at T_f = 50°C are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, Pr = 0.7228 (Table A-15), and $\beta = 1/T_f$ = 1/323 K .

The thermal conductivities of the pipe and the insulation are k_{pipe} = 15 W.m·K and k_{ins} = 0.15 W.m·K, respectively.

Analysis With the assumption that T_f = 50°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 90°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50+273\,\text{K})^{-1}(90-10)\,\text{K}\,(0.07\,\text{m})^3}{(1.798\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7228) = 1.863\times10^6$$

The Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.8633\times10^6)^{1/6}}{\left[1+\left(\frac{0.559}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 17.38$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02735\,\text{W/m}\cdot\text{K}}{0.07\,\text{m}}(17.38) = 6.791\,\text{W/m}^2\cdot\text{K}$$

From Chapter 3, the total thermal resistance for the pipe wall and the insulation is

$$R_{\text{total}} = \frac{\ln(D_{\text{interface}}/D_i)}{2\pi k_{\text{pipe}}L} + \frac{\ln(D_o/D_{\text{interface}})}{2\pi k_{\text{ins}}L}$$

$$= \frac{\ln(0.06/0.05)}{2\pi(15\,\text{W/m}\cdot\text{K})(2\,\text{m})} + \frac{\ln(0.07/0.06)}{2\pi(0.15\,\text{W/m}\cdot\text{K})(2\,\text{m})} = 0.08275\,\text{K/W}$$

Thus, the rate of heat transfer balance for conduction through the cylindrical layers and natural convection on the outer surface is

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}}$$

$$\frac{(T_{s,i} - T_{s,o})}{R_{\text{total}}} = h(\pi D_o L)(T_{s,o} - T_\infty) \quad \to \quad T_{s,o} = 74.15°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,o}$ converges to $T_{s,o}$ = **74.8°C**.

Discussion The results from the iterations are as follows:

Iter	$T_{s,o}$ [°C]	Ra	Nu	h [W/m^2·K]
1	90	1.863×10^6	17.38	6.791
2	74.15	1.672×10^6	16.86	6.447
3	74.80	1.681×10^6	16.88	6.462
4	**74.77**	1.681×10^6	16.88	6.462

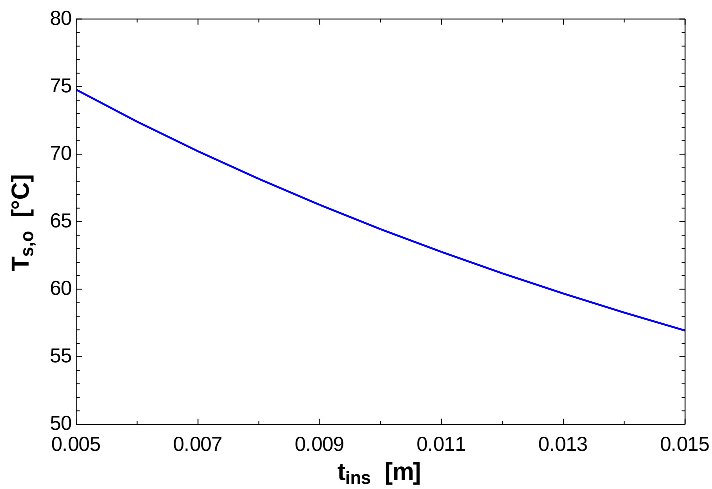
As $T_{s,o}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-61 Reconsider Prob. 9-60. Hot engine flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is covered with a layer of insulation. The effect of the insulation layer thickness on the outer surface temperature is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=2 [m]     "Length exposed to natural convection"
D_i=0.05 [m]     "Inner pipe diameter"
t_pipe=5e-3 [m]     "Pipe wall thickness"
T_infinity=10 [C]     "Cool air temperature"
T_s_i=90 [C]     "Inner surface temperature"

"PROPERTIES"
g=9.81 [m/s^2]     "Gravitational acceleration"
"Air"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film)
T_film=1/2*(T_s_o+T_infinity)
"Pipe and insulation"
k_ins=0.15 [W/m-K]     "Insulation layer thermal conductivity"
k_pipe=15 [W/m-K]     "Pipe thermal conductivity"

"ANALYSIS"
"Natural convection"
Gr=(g*beta*(T_s_o-T_infinity)*D_o^3)/nu^2
Ra=(g*beta*(T_s_o-T_infinity)*D_o^3)/nu^2*Pr
Nusselt=(0.6+0.387*Ra^(1/6)/((1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D_o*Nusselt
"Heat conduction through cylindrical layers"
D_interface=D_i+t_pipe*2
D_o=D_interface+t_ins*2
R_pipe=ln(D_interface/D_i)/(2*pi#*L*k_pipe)     "Pipe wall resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*L*k_ins)     "Insulation layer resistance"
R_total=R_pipe+R_ins
"Heat balance"
Q_dot=(T_s_i-T_s_o)/R_total     "Heat conduction through cylindrical layers"
Q_dot=h*A_s*(T_s_o-T_infinity)     "Heat loss by natural convection"
A_s=pi#*D_o*L     "Surface area exposed to natural convection"
```

t_{ins} [m]	$T_{s,o}$ [°C]
0.005	74.77
0.006	72.41
0.007	70.21
0.008	68.16
0.009	66.24
0.010	64.45
0.011	62.76
0.012	61.18
0.013	59.68
0.014	58.27
0.015	56.94



Discussion As the insulation layer thickness increases, the heat loss through the pipe is reduced. Therefore the outer surface temperature decreases with increasing thickness of insulation.

9-62 A hot fluid flowing as a fully-developed laminar flow inside a horizontal pipe with constant surface temperature. The pipe outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Radiation heat transfer is negligible. **4** Local atmospheric pressure is 1 atm. **5** The film temperature is 50°C (this assumption will be verified).

Properties We first assume the film temperature is T_f = 50°C. Then, the properties of air at T_f = 50°C and 1 atm are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, Pr = 0.7228 (Table A-15), and $\beta = 1/T_f$ = 1/323 K .

Analysis With the assumption that T_f = 50°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 90°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50+273\,\text{K})^{-1}(90-10)\,\text{K}\,(0.045\,\text{m})^3}{(1.798\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7228) = 495{,}031$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(495{,}031)^{1/6}}{\left[1+\left(\frac{0.559}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.98$$

$$h_o = \frac{k}{D_o}\text{Nu} = \frac{0.02735\ \text{W/m}\cdot\text{K}}{0.045\,\text{m}}(11.98) = 7.2812\ \text{W/m}^2\cdot\text{K}$$

At the inner surface, the forced convection heat transfer coefficient for laminar fully-developed flow with constant surface temperature is (see Chapter 8)

$$h_i = 3.66\frac{k_{\text{fluid}}}{D_i} = 3.66\frac{0.72\ \text{W/m}\cdot\text{K}}{0.035\,\text{m}} = 75.29\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for internal forced convection at the pipe inner surface and the natural convection at the outer surface is

$$\dot{Q}_{\text{conv},i} = \dot{Q}_{\text{conv},o}$$

$$h_i(\pi D_i L)\Delta T_{\text{avg}} = h_o(\pi D_o L)(T_{s,o} - T_\infty) \quad \rightarrow \quad T_{s,o} = \frac{h_i D_i}{h_o D_o}\Delta T_{\text{avg}} + T_\infty$$

$$T_{s,o} = \frac{(75.29\ \text{W/m}^2\cdot\text{K})(0.035\,\text{m})}{(7.2812\ \text{W/m}^2\cdot\text{K})(0.045\,\text{m})}(10°\text{C}) + 10°\text{C} = \mathbf{90.4°C}$$

Discussion The assumed film temperature T_f = 50°C is an appropriate assumption, since the determined $T_{s,o}$ = 90.4°C would give a film temperature of T_f = 50.2°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-63 A hot liquid flowing inside a horizontal pipe with a known mass flow rate and temperature difference of the pipe inlet and outlet. The pipe outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Local atmospheric pressure is 1 atm. **4** The film temperature is 35°C (this assumption will be verified). **5** The T_{surr} is the same as the air temperature.

Properties We first assume the film temperature is T_f = 35°C. Then, the properties of air at T_f = 35°C are k = 0.02625 W/m·K, ν = 1.655 × 10^{-5} m^2/s, Pr = 0.7268 (Table A-15), and $\beta = 1/T_f = 1/308$ K.

The emissivity of the black oxidized copper pipe outer surface is ε = 0.78 (Table A-18)

Analysis With the assumption that T_f = 35°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 60°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(35+273\,\text{K})^{-1}(60-10)\,\text{K}\,(0.055\,\text{m})^3}{(1.655\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7268) = 703{,}065$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(703{,}065)^{1/6}}{\left[1+\left(\frac{0.559}{0.7268}\right)^{9/16}\right]^{8/27}}\right\}^2 = 13.21$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02625\ \text{W/m}\cdot\text{K}}{0.055\,\text{m}}(13.21) = 6.3048\ \text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q} = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4)$$

$$\dot{m}c_p\Delta T = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4) \quad\rightarrow\quad T_{s,o} = \mathbf{60.3°C}$$

where $\dot{m} = 0.05\,\text{kg/s}$, $c_p = 1000\ \text{J/kg}\cdot\text{K}$, $\Delta T = 10°\text{C}$

Discussion The assumed film temperature T_f = 35°C is an appropriate assumption, since the determined $T_{s,o}$ = 60.3°C would give a film temperature of T_f = 35.2°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-64 A hot liquid flowing inside a horizontal pipe with a known mass flow rate and temperature difference of the pipe inlet and outlet. The pipe outer surface temperature is to be determined whether or not it is safe from thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Local atmospheric pressure is 1 atm. **4** The film temperature is 35°C (this assumption will be verified). **5** The T_{surr} is the same as the air temperature.

Properties The properties of air at T_f = 25°C and 1 atm pressure are k = 0.02551 W/m·K, $\nu = 1.562 \times 10^{-5}$ m^2/s, Pr = 0.7296 (Table A-15), and $\beta = 1/T_f$ = 1/298 K . The emissivity of the black painted surface is given as ε = 0.88.

Analysis With the assumption that T_f = 25°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(25+273\ \text{K})^{-1}(40-10)\ \text{K}\ (0.055\ \text{m})^3}{(1.562\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7296) = 491{,}343$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(491{,}343)^{1/6}}{\left[1+\left(\frac{0.559}{0.7296}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.97$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02551\ \text{W/m}\cdot\text{K}}{0.055\ \text{m}}(11.97) = 5.5519\ \text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q} = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4)$$

$$\dot{m}c_p\Delta T = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{39.9°C}$$

where

$$\dot{m} = 0.028\ \text{kg/s}, \quad c_p = 1000\ \text{J/kg}\cdot\text{K}, \quad \Delta T = 10°\text{C}$$

Discussion The black painted surface is sufficient to keep the outer surface temperature below 45°C to prevent thermal burn hazards.

The assumed film temperature T_f = 25°C is appropriate for evaluating the properties of air, since the determined $T_{s,o}$ = 39.9°C would give a film temperature of T_f = 24.95°C.

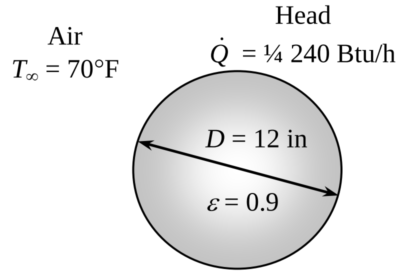
The emissivity of black paint is also listed in Table A-18 as ε = 0.88.

9-65E The average surface temperature of a human head is to be determined when it is not covered.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The head can be approximated as a 12-in.-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 90°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (90+70)/2 = 80°F are (Table A-15E)

$$k = 0.01481 \text{ Btu/h.ft.°F}$$
$$\nu = 1.697\times10^{-4} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7290$$
$$\beta = \frac{1}{T_f} = \frac{1}{(80+460)\text{R}} = 0.001852 \text{ R}^{-1}$$



Analysis The characteristic length for a spherical object is $L_c = D$ = 12/12 = 1 ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001852 \text{ R}^{-1})(90-70 \text{ R})(1 \text{ ft})^3}{(1.697\times10^{-4} \text{ ft}^2\text{/s})^2}(0.7290) = 3.019\times10^7$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(3.019\times10^7)^{1/4}}{\left[1+\left(\frac{0.469}{0.7290}\right)^{9/16}\right]^{4/9}} = 35.79$$

$$h = \frac{k}{D}Nu = \frac{0.01481 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(35.79) = 0.5300 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_s = \pi D^2 = \pi(1 \text{ ft})^2 = 3.142 \text{ ft}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$(240/4 \text{ Btu/h}) = (0.5300 \text{ Btu/h.ft}^2\text{.°F})(3.142 \text{ ft}^2)(T_s - 70)\text{°F}$$
$$+(0.9)(3.142 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[(T_s + 460 \text{ R})^4 - (70+460 \text{ R})^4]$$

Its solution is

T_s = **82.9°F**

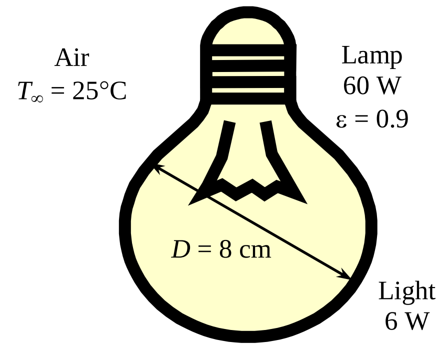
which is sufficiently close to the assumed value in the evaluation of the properties and *h*. Therefore, there is no need to repeat calculations.

9-66 The equilibrium temperature of a light glass bulb in a room is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The light bulb is approximated as an 8-cm-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 170°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (170+25)/2 = 97.5$°C are (Table A-15)

$$k = 0.03077 \text{ W/m.°C}$$
$$\nu = 2.279\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7116$$
$$\beta = \frac{1}{T_f} = \frac{1}{(97.5+273)\text{K}} = 0.002699 \text{ K}^{-1}$$



Analysis The characteristic length in this case is $L_c = D = 0.08$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)(0.002699 \text{ K}^{-1})(170-25 \text{ K})(0.08 \text{ m})^3}{(2.279\times10^{-5} \text{ m}^2\text{/s})^2}(0.7116) = 2.694\times10^6$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.694\times10^6)^{1/4}}{\left[1+(0.469/0.7116)^{9/16}\right]^{4/9}} = 20.42$$

Then

$$h = \frac{k}{D}Nu = \frac{0.03077 \text{ W/m.°C}}{0.08 \text{ m}}(20.42) = 7.854 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi D^2 = \pi(0.08 \text{ m})^2 = 0.02011 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$
$$(0.90\times60) \text{ W} = (7.854 \text{ W/m}^2\text{.°C})(0.02011 \text{ m}^2)(T_s - 25)\text{°C}$$
$$+ (0.9)(0.02011 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (25+273 \text{ K})^4]$$

Its solution is

T_s = **169°C**

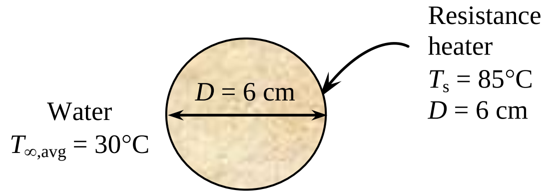
which is very close to the value assumed in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

9-67 Water in a tank is to be heated by a spherical heater. The heating time is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature of the outer surface of the sphere is constant.

Properties Using the average temperature for water (15+45)/2=30°C as the fluid temperature, the properties of water at the film temperature of $(T_s+T_\infty)/2$ = (85+30)/2 = 57.5°C are (Table A-9)

$$k = 0.6515 \text{ W/m.°C}$$
$$\nu = 0.493\times10^{-6} \text{ m}^2\text{/s}$$
$$\Pr = 3.12$$
$$\beta = 0.501\times10^{-3} \text{ K}^{-1}$$



Also, the properties of water at 30°C are (Table A-15)

$$\rho = 996 \text{ kg/m}^3 \text{ and } c_p = 4178 \text{ J/kg.°C}$$

Analysis The characteristic length in this case is $L_c = D$ = 0.06 m. Then,

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.501\times10^{-3} \text{ K}^{-1})(85-30 \text{ K})(0.06 \text{ m})^3}{(0.493\times10^{-6} \text{ m}^2\text{/s})^2}(3.12) = 7.495\times10^8$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(0.469/\Pr\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(7.495\times10^8)^{1/4}}{\left[1+\left(0.469/3.12\right)^{9/16}\right]^{4/9}} = 87.44$$

$$h = \frac{k}{D}Nu = \frac{0.6515 \text{ W/m.°C}}{0.06 \text{ m}}(87.44) = 949.5 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2 = \pi(0.06 \text{ m})^2 = 0.01131 \text{ m}^2$$

The rate of heat transfer by convection is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (949.5 \text{ W/m}^2\text{.°C})(0.01131 \text{ m}^2)(85-30) = 590.6 \text{ W}$$

The mass of water in the container is

$$m = \rho \boldsymbol{V} = (996 \text{ kg/m}^3)(0.040 \text{ m}^3) = 39.84 \text{ kg}$$

The amount of heat transfer to the water is

$$Q = mc_p(T_2 - T_1) = (39.84 \text{ kg})(4178 \text{ J/kg.°C})(45-15)\text{°C} = 4.994\times10^6 \text{ J}$$

Then the time the heater should be on becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{4.994\times10^6 \text{ J}}{590.6 \text{ J/s}} = 8456 \text{ s} = \mathbf{2.35 \text{ hours}}$$

9-68 Chemical reaction in a spherical tank is releasing heat and the surface temperature is known. The heat rate from the reaction is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is constant. **3** Thermal resistance on the tank wall is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = (50 + 20)/2 = 35°C$ are $k = 0.02625$ W/m·K, $\nu = 1.655 \times 10^{-5}$ m^2/s, Pr = 0.7268 (Table A-15), and $\beta = 1/T_f = 1/308$ K.

The emissivity of the tank is given as $\varepsilon = 0.35$.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(35+273\,\text{K})^{-1}(50-20)\,\text{K}\,(2\,\text{m})^3}{(1.655\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7268) = 2.0284\times10^{10}$$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\dfrac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.0284\times10^{10})^{1/4}}{\left[1+\left(\dfrac{0.469}{0.7268}\right)^{9/16}\right]^{4/9}} = 173.96$$

$$h = \frac{k}{D}\text{Nu} = \frac{0.02625\ \text{W/m}\cdot\text{K}}{2\,\text{m}}(173.96) = 2.2832\ \text{W/m}^2\cdot\text{K}$$

Thus, the heat rate from the reaction is

$$\dot{Q} = h(\pi D^2)(T_s - T_\infty) + \varepsilon\sigma(\pi D^2)(T_s^4 - T_{surr}^4)$$

$$\dot{Q} = \pi(2\,\text{m})^2[(2.2832\ \text{W/m}^2\cdot\text{K})(50-20)\,\text{K} + (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.35)(323^4 - 293^4)\,\text{K}^4]$$

$$= \mathbf{1737\ W}$$

Discussion At the tank surface, natural convection contributed to about 50% of the 1737 W heat loss. From Table A-18, the emissivity value of 0.35 is a suitable value for lightly oxidized stainless steel surface.

9-69 A spherical tank is filled with a hot liquid and the inner surface temperature is known. The outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The tank wall thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature. **6** The film temperature is 40°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 40°C are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, Pr = 0.7255 (Table A-15), and $\beta = 1/T_f$ = 1/313 K.

The thermal conductivity and the emissivity of the tank are given as k_{sph} = 0.15 W/m·K and ε = 0.35, respectively.

Analysis With the assumption that T_f = 40°C, the tank outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 60°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(40+273\,\text{K})^{-1}(60-20)\,\text{K}\,(3.06\,\text{m})^3}{(1.702\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7255) = 8.9964\times10^{10}$$

where

$$D_o = D_i + 2t = 3.06\,\text{m}$$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\dfrac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(8.9964\times10^{10})^{1/4}}{\left[1+\left(\dfrac{0.469}{0.7255}\right)^{9/16}\right]^{4/9}} = 251.5$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02662\ \text{W/m}\cdot\text{K}}{3.06\,\text{m}}(251.5) = 2.1879\ \text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{2\pi k_{\text{sph}}D_oD_i}{D_o - D_i}(T_{s,i} - T_{s,o}) = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{61.1°C}$$

Discussion The assumed film temperature T_f = 40°C is an appropriate assumption, since the determined $T_{s,o}$ = 61.1°C would give a film temperature of T_f = 40.6°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-70 A metal spherical tank is filled with chemicals in exothermic reaction and the heat generation is known. The type of surface to be used so that the outer surface temperature is safe from thermal burn is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is constant. **3** Local atmospheric pressure is 1 atm. **4** The T_{surr} is the same as the air temperature.

Properties The properties of air at the assumed T_f = 30°C and 1 atm pressure are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f = 1/303$ K .

The emissivities for the unpainted and painted outer surface are $\varepsilon_{\text{unpaint}}$ = 0.20 and $\varepsilon_{\text{paint}}$ = 0.88, respectively.

Analysis With the assumption that T_f = 30°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 45°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(45-15)\,\text{K}\,(3.02\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 7.5344\times10^{10}$$

where $\quad D_o = D_i + 2t_{\text{tank}} = 3.02\,\text{m}$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(7.5344\times10^{10})^{1/4}}{\left[1+\left(\frac{0.469}{0.7282}\right)^{9/16}\right]^{4/9}} = 240.78$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02588\,\text{W/m}\cdot\text{K}}{3.02\,\text{m}}(240.78) = 2.0634\,\text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{reaction}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\dot{e}_{\text{gen}}(\pi D_i^3/6) = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4)$$

where $\quad \dot{e}_{\text{gen}} = 450\,\text{W/m}^3$

For an unpainted outer surface ($\varepsilon = \varepsilon_{\text{unpaint}}$ = 0.20), we have $\quad\rightarrow\quad T_{s,o} = 77.5°\text{C}$ (above 45°C)

For a painted outer surface ($\varepsilon = \varepsilon_{\text{paint}}$ = 0.88), we have $\quad\rightarrow\quad T_{s,o} = 44.2°\text{C}$ (below 45°C)

Discussion To prevent thermal burn hazards, the outer surface should be painted with black paint. The black paint increases the radiation heat transfer from the outer surface, thus reducing the surface temperature.

Note that 30°C is an appropriate temperature to evaluate the air properties for the painted surface, since $T_{s,o}$ = 44.2°C would give a film temperature of T_f = 29.6°C.

For the unpainted surface, a more appropriate temperature for evaluating the air properties would be 45°C. By performing an iterative solution, the outer surface temperature for the unpainted surface would converge to about 73°C.

9-71 A metal spherical tank is filled with hot liquid and the inner surface temperature is known. The tank is covered with a layer of insulation and outer surface temperature is to be determined whether it is safe for thermal burn prevention.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The tank wall thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature. **6** The film temperature is 30°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 30°C and 1 atm pressure are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f$ = 1/303 K. The thermal conductivity of the tank is given as k_{tank} = 15 W/m·K.

The thermal conductivity and the emissivity of the insulation are given as k_{ins} = 0.15 W/m·K and ε = 0.35, respectively.

Analysis With the assumption that T_f = 30°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 44°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(44-16)\,\text{K}\,(3.16\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 8.0561\times10^{10}$$

where $\quad D_o = D_i + 2(t_{\text{tank}} + t_{\text{ins}}) = 3.16\,\text{m}$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\dfrac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(8.0561\times10^{10})^{1/4}}{\left[1+\left(\dfrac{0.469}{0.7282}\right)^{9/16}\right]^{4/9}} = 244.8$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02588\ \text{W/m}\cdot\text{K}}{3.16\,\text{m}}(244.8) = 2.0049\ \text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{(T_{s,i} - T_{s,o})}{R_{\text{total}}} = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{43.5°C}$$

where the total thermal resistance for the spherical layers is

$$R_{\text{total}} = \frac{D_{\text{interface}} - D_i}{2\pi k_{\text{tank}} D_{\text{interface}} D_i} + \frac{D_o - D_{\text{interface}}}{2\pi k_{\text{ins}} D_o D_{\text{interface}}} = 0.01559\ \text{K/W}$$

Discussion The insulation layer is able to keep the outer surface temperature 1.5°C below the safe temperature of 45°C. For additional safety precaution, the outer surface temperature can be further reduced by increasing insulation thickness or paint the outer surface to achieve higher emissivity.

The assumed film temperature T_f = 30°C is an appropriate assumption, since the determined $T_{s,o}$ = 43.5°C would give a film temperature of T_f = 29.8°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

Natural Convection from Finned Surfaces and PCBs

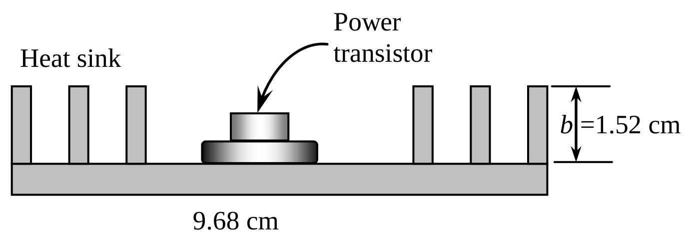
9-72C Finned surfaces are frequently used in practice to enhance heat transfer by providing a larger heat transfer surface area. Finned surfaces are referred to as heat sinks in the electronics industry since they provide a medium to which the waste heat generated in the electronic components can be transferred effectively.

9-73C A heat sink with closely packed fins will have greater surface area for heat transfer, but smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages.

9-74C Removing some of the fins on the heat sink will decrease heat transfer surface area, but will increase heat transfer coefficient. The decrease on heat transfer surface area more than offsets the increase in heat transfer coefficient, and thus heat transfer rate will decrease. In the second case, the decrease on heat transfer coefficient more than offsets the increase in heat transfer surface area, and thus heat transfer rate will again decrease.

9-75 An aluminum heat sink of rectangular profile oriented vertically is used to cool a power transistor. The average natural convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation heat transfer from the sink is negligible. **4** The entire sink is at the base temperature.



Analysis The total surface area of the heat sink is

$$A_{fins} = 2nLb = (2)(6)(0.0762\text{ m})(0.0152\text{ m}) + (2)(0.0048\text{ m})(0.0762\text{ m}) = 0.01463\text{ m}^2$$

$$A_{unfinned} = (4)(0.0145\text{ m})(0.0762\text{ m}) + (0.0317\text{ m})(0.0762\text{ m}) = 0.006835\text{ m}^2$$

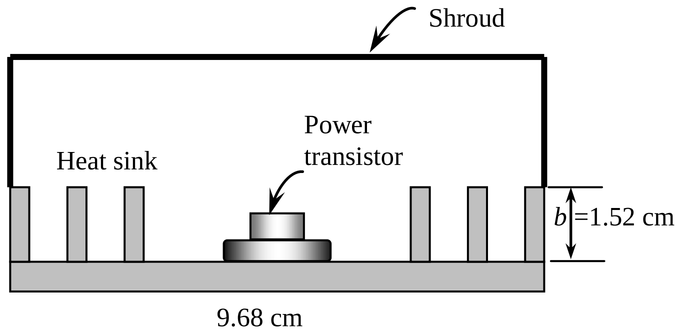
$$A_{total} = A_{fins} + A_{unfinned} = 0.01463 + 0.006835 = 0.021465\text{ m}^2$$

Then the average natural convection heat transfer coefficient becomes

$$\dot{Q} = hA_{total}(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_{total}(T_s - T_\infty)} = \frac{15\text{ W}}{(0.021465\text{ m}^2)(120-22)°\text{C}} = \mathbf{7.13\ W/m^2 \cdot °C}$$

9-76 Aluminum heat sinks of rectangular profile oriented vertically are used to cool a power transistor. A shroud is placed very close to the tips of fins. The average natural convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. 3 Radiation heat transfer from the sink is negligible. **4** The entire sink is at the base temperature.



Analysis The total surface area of the shrouded heat sink is

$$A_{fins} = 2nLb = (2)(6)(0.0762\text{ m})(0.0152\text{ m}) = 0.013898\text{ m}^2$$

$$A_{unfinned} = (4)(0.0145\text{ m})(0.0762\text{ m}) + (0.0317\text{ m})(0.0762\text{ m}) = 0.006835\text{ m}^2$$

$$A_{shroud} = (2)(0.0968\text{ m})(0.0762\text{ m}) = 0.014752\text{ m}^2$$

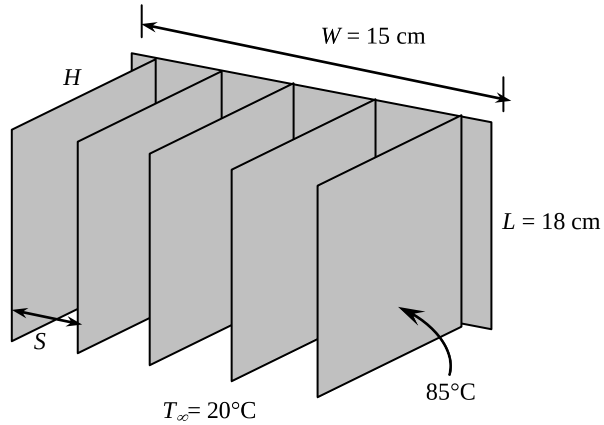
$$A_{total} = A_{fins} + A_{unfinned} + A_{shroud} = 0.013898 + 0.006835 + 0.014752 = 0.035486\text{ m}^2$$

Then the average natural convection heat transfer coefficient becomes

$$\dot{Q} = hA_{total}(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_{total}(T_s - T_\infty)} = \frac{15\text{ W}}{(0.035486\text{ m}^2)(108-22)^\circ\text{C}} = \mathbf{4.92\ W/m^2.^\circ C}$$

9-77 A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer from the heat sink are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (85+20)/2 = 52.5°C are (Table A-15)

$$k = 0.02753 \text{ W/m.°C}$$

$$\nu = 1.823\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7222$$

$$\beta = \frac{1}{T_f} = \frac{1}{(52.5+273)\text{K}} = 0.003072 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the surface $L_c = L$ = 0.18 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003072 \text{ K}^{-1})(85-20 \text{ K})(0.18 \text{ m})^3}{(1.823\times10^{-5} \text{ m}^2/\text{s})^2}(0.7222) = 2.483\times10^7$$

The optimum fin spacing is

$$S = 2.714\frac{L}{Ra^{1/4}} = 2.714\frac{0.18 \text{ m}}{(2.483\times10^7)^{1/4}} = 0.006921 \text{ m} = \mathbf{6.921 \text{ mm}}$$

The heat transfer coefficient for this optimum fin spacing case is

$$h = 1.307\frac{k}{S} = 1.307\frac{0.02753 \text{ W/m.°C}}{0.006921 \text{ m}} = 5.199 \text{ W/m}^2.\text{°C}$$

The number of fins and the total heat transfer surface area is

$$n = \frac{w}{S+t} \cong \frac{w}{s} = \frac{0.15}{0.006921} \cong 22 \text{ fins}$$

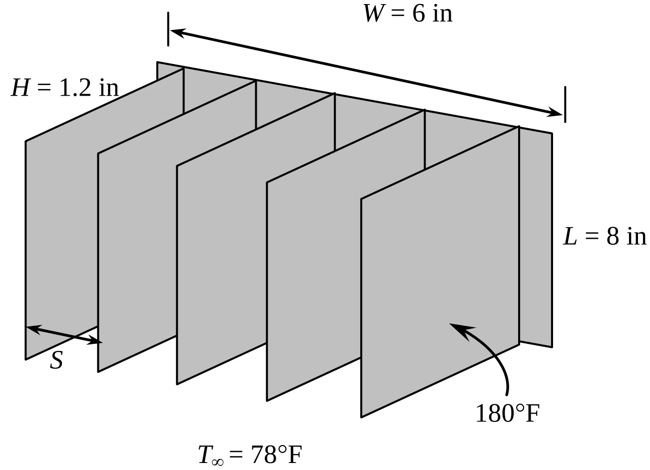
$$A_s = 2nLH = 2\times22\times(0.18 \text{ m})(0.04 \text{ m}) = 0.3168 \text{ m}^2$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.199 \text{ W/m}^2.\text{°C})(0.3168 \text{ m}^2)(85-20)\text{°C} = \mathbf{107.1 \text{ W}}$$

9-78E A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer from the heat sink are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm. **4** The thickness t of the fins is very small relative to the fin spacing S so that Eqs. 9-32 and 9-33 for optimum fin spacing are applicable.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (180+78)/2=129°F$ are (Table A-15E)

$$k = 0.01597 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1975\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7217$$
$$\beta = \frac{1}{T_f} = \frac{1}{(129+460)\text{ R}} = 0.001698 \text{ R}^{-1}$$

Analysis The characteristic length in this case is the fin height, $L_c = L = 8$ in. Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001698 \text{ R}^{-1})(180-78 \text{ R})(8/12 \text{ ft})^3}{(0.1975\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7217) = 3.058\times10^7$$

The optimum fin spacing is

$$S = 2.714\frac{L}{Ra^{1/4}} = 2.714\frac{8/12 \text{ ft}}{(3.058\times10^7)^{1/4}} = 0.02433 \text{ ft } = \mathbf{0.292 in}$$

The heat transfer coefficient for this optimum spacing case is

$$h = 1.307\frac{k}{S} = 1.307\frac{0.01597 \text{ Btu/h.ft.°F}}{0.02433 \text{ ft}} = 0.8578 \text{ Btu/h.ft}^2\text{.°F}$$

The number of fins and the total heat transfer surface area is

$$n = \frac{w}{S+t} = \frac{6}{0.2916+0.08} = 16 \text{ fins}$$

$$\begin{aligned}A_s &= 2nLH + ntL + 2ntH \\ &= 2\times16\times(8/12 \text{ ft})(1.2/12 \text{ ft}) + 16\times(0.08/12 \text{ ft})(8/12 \text{ ft}) + 2\times16\times(0.08/12 \text{ ft})(1.2/12 \text{ ft}) \\ &= 2.226 \text{ ft}^2\end{aligned}$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.8578 \text{ Btu/h.ft}^2\text{.°F})(2.226 \text{ ft}^2)(180-78)\text{°F} = \mathbf{194.7 \text{ Btu/h}}$$

Discussion If the fin thickness is disregarded, the number of fins and the rate of heat transfer become

$$n = \frac{w}{s+t} \cong \frac{w}{s} = \frac{6}{0.2916} \cong 21 \text{ fins}$$

$$A_s = 2nLH = 2\times21\times(8/12 \text{ ft})(1.2/12 \text{ ft}) = 2.8 \text{ ft}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.8578 \text{ Btu/h.ft}^2\text{.°F})(2.8 \text{ ft}^2)(180-78)\text{°F} = \mathbf{245 \text{ Btu/h}}$$

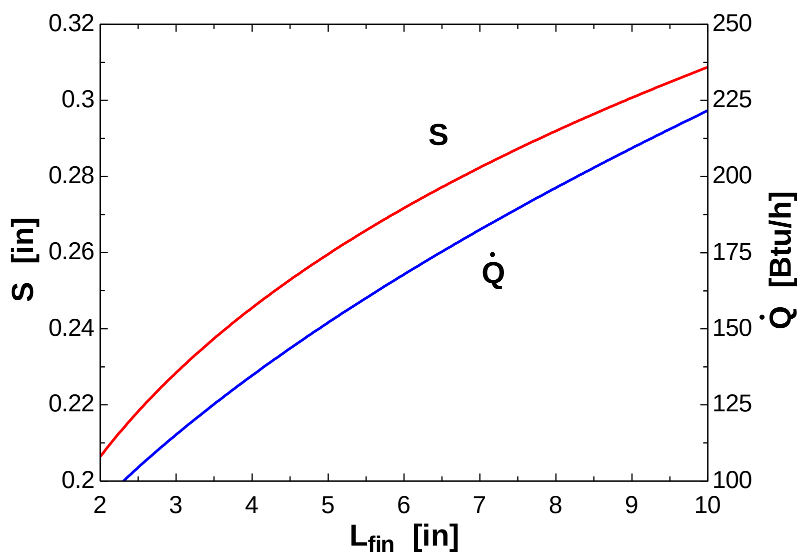
Therefore, the fin tip area is significant in this case.

9-79E Prob. 9-78E is reconsidered. The effect of the length of the fins in the vertical direction on the optimum fin spacing and the rate of heat transfer by natural convection is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
w_s=(6/12) [ft]
H_s=(8/12) [ft]
T_infinity=78 [F]
t_fin=(0.08/12) [ft]
L_fin=8 [in]
H_fin=(1.2/12) [ft]
T_s=180 [F]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_film+460)
T_film=1/2*(T_s+T_infinity)
g=32.2 [ft/s^2] "gravitational acceleration"

"ANALYSIS"
L_fin_ft=L_fin*Convert(in, ft)
delta=L_fin_ft
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
S_ft=2.714*L_fin_ft/Ra^0.25
S=S_ft*Convert(ft, in)
h=1.307*k/S_ft
n_fin=w_s/(S_ft+t_fin)
A=2*n_fin*L_fin_ft*H_fin+n_fin*t_fin*L_fin_ft+2*n_fin*t_fin*H_fin
Q_dot=h*A*(T_s-T_infinity)
```

L_{fin} [in]	S [in]	$\dot{Q}$ [Btu/h]
2	0.2065	92.73
2.5	0.2183	104.5
3	0.2285	115.3
3.5	0.2375	125.3
4	0.2455	134.7
4.5	0.2529	143.6
5	0.2596	152
5.5	0.2659	160.1
6	0.2717	167.9
6.5	0.2772	175.4
7	0.2824	182.6
7.5	0.2873	189.6
8	0.292	196.3
8.5	0.2964	202.9
9	0.3007	209.3
9.5	0.3048	215.6
10	0.3087	221.7



Natural Convection inside Enclosures

9-80C We would recommend putting the hot fluid into the upper compartment of the container. In this case no convection currents will develop in the enclosure since the lighter (hot) fluid will always be on top of the heavier (cold) fluid.

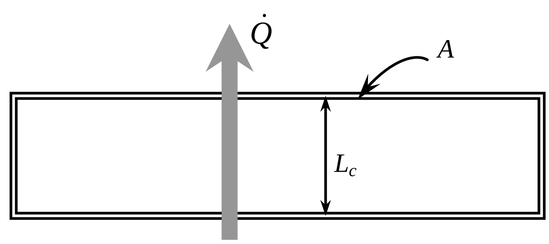
9-81C We would disagree with this recommendation since the air space introduces some thermal resistance to heat transfer. The thermal resistance of air space will be zero only when the convection coefficient approaches infinity, which is never the case. However, when the air space is eliminated, so is its thermal resistance.

9-82C Yes, dividing the air space into two compartments will retard air motion in the air space, and thus slow down heat transfer by natural convection. The vinyl sheet will also act as a radiation shield and reduce heat transfer by radiation.

9-83C The effective thermal conductivity of an enclosure represents the enhancement on heat transfer as result of convection currents relative to conduction. The ratio of the effective thermal conductivity to the ordinary thermal conductivity yields Nusselt number $Nu = k_{eff} / k$.

9-84 Conduction thermal resistance of a medium is expressed as $R = L/(kA)$. Thermal resistance of a rectangular enclosure can be expressed by replacing L with characteristic length of enclosure L_c, and thermal conductivity k with effective thermal conductivity k_{eff} to give

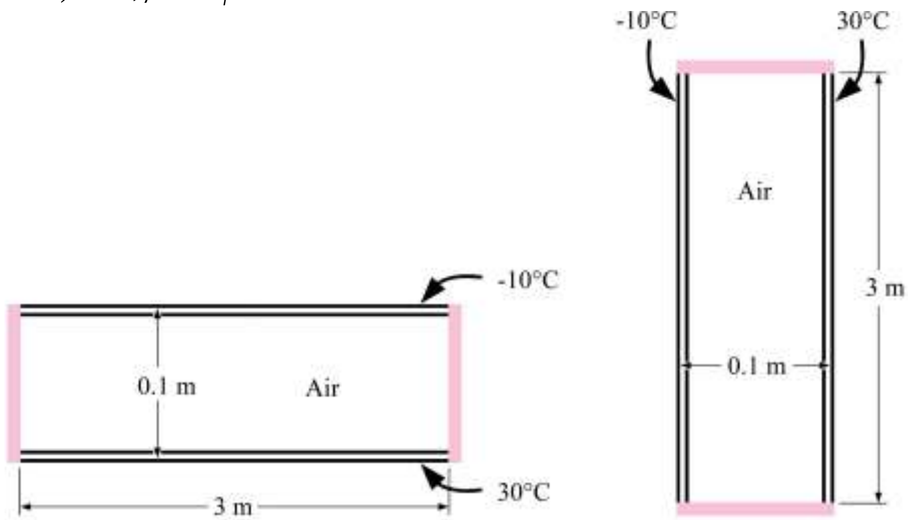
$$R = L_c/(k_{eff}A) = L_c/(kNuA)$$



9-85 A rectangular enclosure consists of two surfaces separated by an air gap, and the ratio of the heat transfer rate for the horizontal orientation (with hotter surface at the bottom) to that of vertical orientation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 10°\text{C}$ are $k = 0.02439$ W/m·K, $\nu = 1.426 \times 10^{-5}$ m^2/s, Pr = 0.7336 (from Table A-15). Also, $\beta = 1/T_f = 3.534 \times 10^{-3}$ K^{-1}.



Analysis The characteristic length for both cases is $L_c = 0.1$ m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003534\,\text{K}^{-1})(30+10)\,\text{K}(0.1\,\text{m})^3}{(1.426\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7336)$$

$$= 5.003\times10^6$$

The aspect ratio for this rectangular enclosure is H/L = 30 m. The Nusselt numbers for horizontal and vertical orientations are

$$\text{Nu}_{\text{horiz}} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^+ = 10.94$$

$$\text{Nu}_{\text{vert}} = 0.42Ra^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 7.134$$

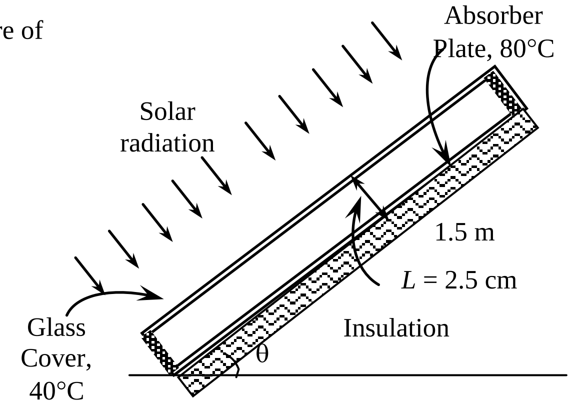
Hence, the ratio of heat transfer rate is

$$\frac{\dot{Q}_{\text{horiz}}}{\dot{Q}_{\text{vert}}} = \frac{h_{\text{horiz}}(k/L)A_s\Delta T}{h_{\text{vert}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{horiz}}(k/L)A_s\Delta T}{\text{Nu}_{\text{vert}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{horiz}}}{\text{Nu}_{\text{vert}}} = \frac{10.94}{7.134} = \mathbf{1.53}$$

Discussion For the same ΔT, the rate of heat transfer for the horizontal orientation is 53% larger than that for the vertical orientation.

9-86 The absorber plate and the glass cover of a flat-plate solar collector are maintained at specified temperatures. The rate of heat loss from the absorber plate by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **4** The air pressure in the enclusure is 1 atm.



Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+40)/2 = 60°C$ are (Table A-15)

$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003\ \text{K}^{-1}$$

Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.025$ m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003003\text{ K}^{-1})(80-40\text{ K})(0.025\text{ m})^3}{(1.896\times10^{-5}\text{ m}^2\text{/s})^2}(0.7202) = 3.689\times10^4$$

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{3.689\times10^4}\right]^+ + \left[\frac{(3.689\times10^4)^{1/3}}{18}-1\right]^+ = 3.223$$

Then

$$A_s = H\times W = (1.5\text{ m})(3\text{ m}) = 4.5\text{ m}^2$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02808\text{ W/m.°C})(3.223)(4.5\text{ m}^2)\frac{(80-40)°\text{C}}{0.025\text{ m}} = \mathbf{652\ W}$$

For $\theta = 30°$, we obtain

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1-\frac{1708(\sin1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right]+\left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{(3.689\times10^4)\cos(30)}\right]^+\left[1-\frac{1708[\sin(1.8\times30)]^{1.6}}{(3.689\times10^4)\cos(30)}\right]+\left[\frac{[(3.689\times10^4)\cos(30)]^{1/3}}{18}-1\right]^+$$
$$= 3.074$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02808\text{ W/m.°C})(3.074)(4.5\text{ m}^2)\frac{(80-40)°\text{C}}{0.025\text{ m}} = \mathbf{621 W}$$

For $\theta = 90°$, we have vertical rectangular enclosure. The Nusselt number for this geometry and orientation can be determined from ($Ra = 3.689\times10^4$ - same as that for horizontal case)

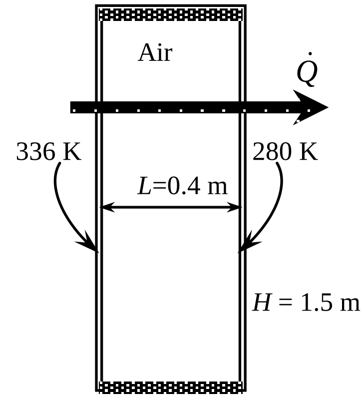
$$Nu = 0.42Ra^{1/4}\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(3.689\times10^4)^{1/4}(0.7202)^{0.012}\left(\frac{1.5\text{ m}}{0.025\text{ m}}\right)^{-0.3} = 1.69$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02808\text{ W/m.°C})(1.69)(4.5\text{ m}^2)\frac{(80-40)°\text{C}}{0.025\text{ m}} = \mathbf{344\ W}$$

Discussion Caution is advised for the vertical case since the condition $H/L < 40$ is not satisfied.

9-87 Two glasses of a double pane window are maintained at specified temperatures. The fraction of heat transferred through the enclosure by radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm.



Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2$ = (280+336)/2 = 308 K = 35°C are (Table A-15E)

$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7268$$
$$\beta = \frac{1}{T_f} = \frac{1}{308 \text{ K}} = 0.003247 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the two glasses, $L_c = L$ = 0.4 m. Then,

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(336-280 \text{ K})(0.4 \text{ m})^3}{(1.655\times10^{-5} \text{ m}^2\text{/s})^2}(0.7268) = 3.029\times10^8$$

The aspect ratio of the geometry is H/L = 1.5/0.4 = 3.75. For this value of H/L the Nusselt number can be determined from

$$Nu = 0.22\left(\frac{\text{Pr}}{0.2+\text{Pr}}Ra\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.7268}{0.2+0.7268}(3.029\times10^8)\right)^{0.28}\left(\frac{1.5}{0.4}\right)^{-1/4} = 35.00$$

Then,

$$A_s = H\times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\dot{Q}_{\text{conv}} = kNuA_s\frac{T_1 - T_2}{L} = (0.02625 \text{ W/m.°C})(35.00)(4.5 \text{ ft}^2)\frac{(336-280)\text{K}}{0.4 \text{ m}} = 578.9 \text{ W}$$

The effective emissivity is

$$\frac{1}{\varepsilon_{\text{eff}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{0.15} + \frac{1}{0.90} - 1 = 6.778 \longrightarrow \varepsilon_{\text{eff}} = 0.1475$$

The rate of heat transfer by radiation is

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon_{\text{eff}}A_s\sigma(T_1^{\,4} - T_2^{\,4}) \\ &= (0.1475)(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(336 \text{ K})^4 - (280 \text{ K})^4] = 248.4 \text{ W}\end{aligned}$$

Then the fraction of heat transferred through the enclosure by radiation becomes

$$f_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}} = \frac{248.4}{578.9+248.4} = \mathbf{0.30}$$

9-88 A double pane window with an air gap is considered. The rate of heat transfer through the window by natural convection the temperature of the outer glass layer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm. **4** Radiation heat transfer is neglected.

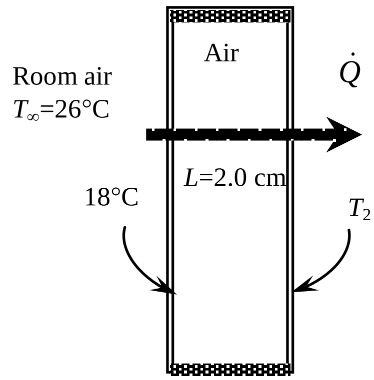
Properties For natural convection between the inner surface of the window and the room air, the properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (18+26)/2 = 22°C are (Table A-15)

$$k = 0.02529 \text{ W/m.°C}$$

$$\nu = 1.534\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7304$$

$$\beta = \frac{1}{T_f} = \frac{1}{(22+273)\text{K}} = 0.00339 \text{ K}^{-1}$$



For natural convection between the two glass sheets separated by an air gap, the properties of air at 1 atm and the anticipated average temperature of $(T_1+T_2)/2$ = (18+0)/2 = 9°C are (Table A-15)

$$k = 0.02431 \text{ W/m.°C},\quad \nu = 1.417\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7339,\quad \beta = \frac{1}{T_f} = \frac{1}{(9+273)\text{K}} = 0.003546 \text{ K}^{-1}$$

Analysis We first calculate the natural convection heat transfer between the room air and the inner surface of the window.

$$L_c = H = 1.5 \text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00339 \text{ K}^{-1})(26-18)\text{K}(1.5 \text{ m})^3}{(1.534\times10^{-5} \text{ m}^2\text{/s})^2}(0.7304) = 2.787\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.787\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7304}\right)^{9/16}\right]^{8/27}}\right\}^2 = 169.5$$

$$h = \frac{k}{H}Nu = \frac{0.02529 \text{ W/m.°C}}{1.5 \text{ m}}(169.5) = 2.858 \text{ W/m}^2\text{.°C}$$

$$A_s = H\times W = (1.5 \text{ m})(2.8 \text{ m}) = 4.2 \text{ m}^2$$

$$\dot{Q}_{\text{conv}} = hA_s(T_\infty - T_s) = (2.858 \text{ W/m}^2\text{.°C})(4.2 \text{ m}^2)(26-18)\text{°C} = \mathbf{96.0 \text{ W}}$$

Next, we consider the natural convection between the two glass sheets separated by an air gap.

$$L_c = L = 2.0 \text{ cm}$$

$$\text{Ra} = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003546 \text{ K}^{-1})(18-0)\text{K}(0.020 \text{ m})^3}{(1.417\times10^{-5} \text{ m}^2\text{/s})^2}(0.7339) = 18{,}309$$

$$Nu = 0.42Ra^{1/4}\Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(18{,}309)^{1/4}(0.7339)^{0.012}\left(\frac{1.5 \text{ m}}{0.020 \text{ m}}\right)^{-0.3} = 1.333$$

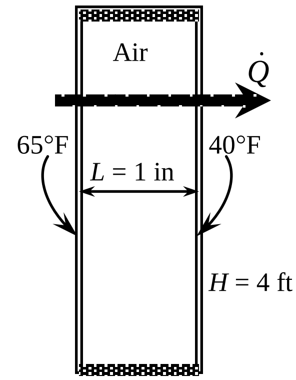
Under steady operation, the rate of heat transfer between the room air and the inner surface of the window is equal to the heat transfer through the air gap. Setting these two equal to each other we obtain the temperature of the outer glass sheet

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} \longrightarrow 96.0 \text{ W} = (0.02431 \text{ W/m.°C})(1.333)(4.2 \text{ m}^2)\frac{(18-T_2)\text{°C}}{0.020 \text{ m}} \longrightarrow T_2 = \mathbf{3.9°C}$$

which is sufficiently close to the assumed temperature 0°C. Therefore, there is no need to repeat the calculations.

Discussion The aspect ratio for this geometry is H/L = (1.5 m)/(0.020 m) = 75. There is no Nusselt number relation given in the text covering this value of aspect ratio. One of the closest aspect ratio ranges is given by Eq. 9-54 as 10 < H/L < 40. We used Eq. 9-54 knowing that there will be some error in the calculation of Nusselt number.

9-89E Two glasses of a double pane window are maintained at specified temperatures. The rate of heat transfer through the window by natural convection and radiation, and the R-value of insulation are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (65+40)/2 = 52.5$°F are (Table A-15E)

$$k = 0.01415 \text{ Btu/h.ft.°F}$$

$$\nu = 0.1548\times10^{-3} \text{ ft}^2\text{/s}$$

$$\text{Pr} = 0.7332$$

$$\beta = \frac{1}{T_f} = \frac{1}{(52.5+460)\text{ R}} = 0.001951 \text{ R}^{-1}$$

Analysis (*a*) The characteristic length in this case is the distance between the two glasses, $L_c = L = 1$ in. Then,

$$Ra_L = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001951 \text{ R}^{-1})(65-40 \text{ R})(1/12 \text{ ft})^3}{(0.1548\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7332) = 27{,}809$$

The aspect ratio of the geometry is $H/L = 4\times12/1 = 48$ (which is somewhat over 40, but still close enough for an approximate analysis). For these values of H/L and Ra_L, the Nusselt number can be determined from

$$Nu = 0.42Ra^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(27{,}809)^{1/4}(0.7332)^{0.012}\left(\frac{4 \text{ ft}}{1/12 \text{ ft}}\right)^{-0.3} = 1.692$$

Then,

$$A_s = H\times W = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.01415 \text{ Btu/h.ft.°F})(1.692)(24 \text{ ft}^2)\frac{(65-40)\text{°F}}{(1/12)\text{ft}} = \mathbf{172.4 \text{ Btu/h}}$$

(*b*) The rate of heat transfer by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_1^4 - T_2^4)$$
$$= (0.82)(24 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2.\text{R}^4)[(65+460 \text{ R})^4 - (40+460 \text{ R})^4] = \mathbf{454.3 \text{ Btu/h}}$$

Then the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{convection} + \dot{Q}_{rad} = 172.4 + 454.3 = 626.7 \text{ Btu/h}$$

Then the effective thermal conductivity of the air, which also accounts for the radiation effect and the *R*-value become

$$\dot{Q}_{total} = k_{eff}A_s\frac{T_1-T_2}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{A_s(T_1-T_2)} = \frac{(626.7 \text{ Btu/h})(1/12 \text{ ft})}{(24 \text{ ft}^2)(65-40)\text{°F}} = 0.08704 \text{ Btu/h.ft.°F}$$

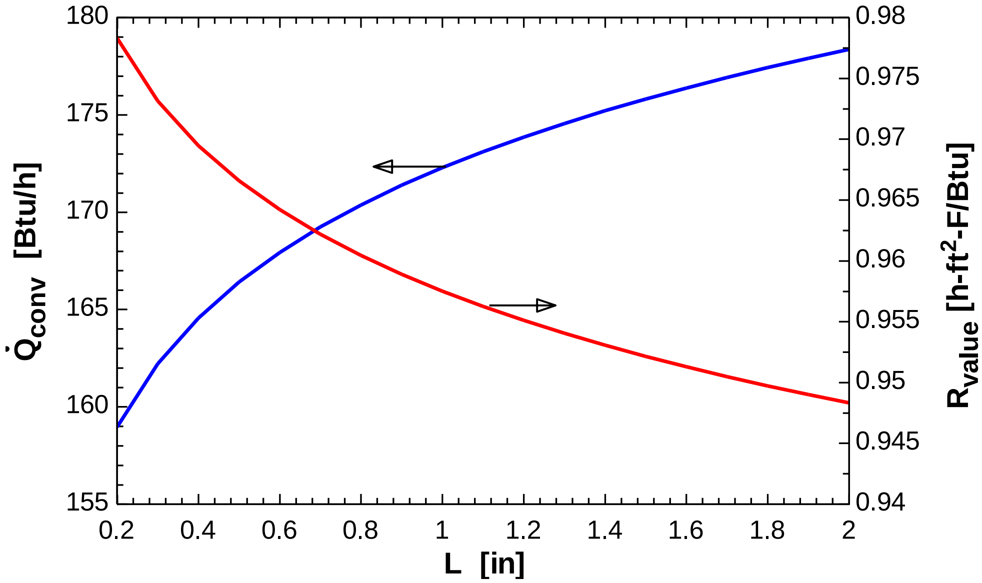
$$R_{value} = \frac{L}{k_{eff}} = \frac{(1/12 \text{ ft})}{0.08704 \text{ Btu/h.ft.°F}} = 0.957 \text{ h.ft}^2\text{.°F/Btu} = \mathbf{R-0.96}$$

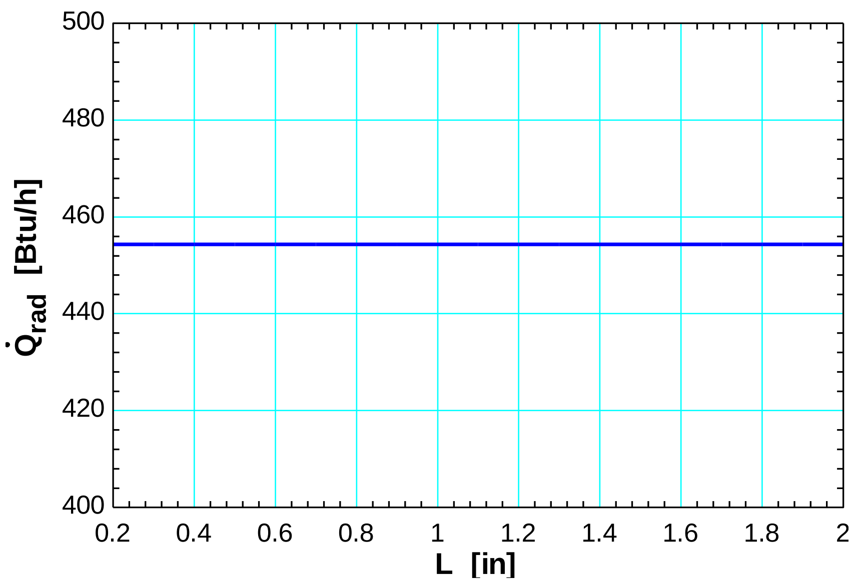
9-90E Prob. 9-89E is reconsidered. The effect of the air gap thickness on the rates of heat transfer by natural convection and radiation, and the *R*-value of insulation is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
H=4 [ft]
W=6 [ft]
L=1 [in]
T_1=65 [F]
T_2=40 [F]
epsilon_eff=0.82

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=14.7)
mu=Viscosity(Fluid$, T=T_ave)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_ave+460)
T_ave=1/2*(T_1+T_2)
g=32.2 [ft/s^2]
sigma=0.1714E-8 [Btu/h-ft^2-R^4]

"ANALYSIS"
L_ft=L*Convert(in, ft)
Ra=(g*beta*(T_1-T_2)*L_ft^3)/nu^2*Pr
Ratio=H/L_ft
Nusselt=0.42*Ra^0.25*Pr^0.012*(H/L_ft)^(-0.3)
A=H*W
Q_dot_conv=k*Nusselt*A*(T_1-T_2)/L_ft
Q_dot_rad=epsilon_eff*A*sigma*((T_1+460)^4-(T_2+460)^4)
Q_dot_total=Q_dot_conv+Q_dot_rad
Q_dot_total=k_eff*A*(T_1-T_2)/L_ft
R_value=L_ft/k_eff
```

L [in]	$\dot{Q}_{conv}$ [Btu/h]	$\dot{Q}_{rad}$ [Btu/h]	R-value [h.ft^2.F/Btu]
0.2	159	454.3	0.9783
0.3	162.2	454.3	0.9731
0.4	164.6	454.3	0.9695
0.5	166.4	454.3	0.9666
0.6	167.9	454.3	0.9642
0.7	169.2	454.3	0.9622
0.8	170.4	454.3	0.9604
0.9	171.4	454.3	0.9589
1	172.3	454.3	0.9575
1.1	173.1	454.3	0.9563
1.2	173.9	454.3	0.9551
1.3	174.6	454.3	0.9541
1.4	175.2	454.3	0.9531
1.5	175.8	454.3	0.9522
1.6	176.4	454.3	0.9513
1.7	176.9	454.3	0.9505
1.8	177.4	454.3	0.9497
1.9	177.9	454.3	0.949
2	178.4	454.3	0.9483


180
175
170
165
160
155
0.98
0.975
0.97
0.965
0.96
0.955
0.95
0.945
0.94
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
L [in]
Q̇conv [Btu/h]
Rvalue [h-ft2-F/Btu]


500
480
460
440
420
400
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
L [in]
Q̇rad [Btu/h]

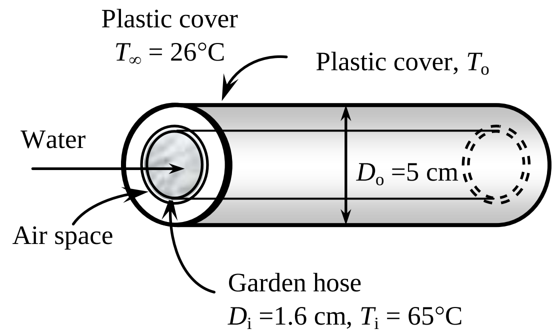
9-91 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose per meter of its length by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **3** The air pressure in the enclosure is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated average temperature of $(T_i+T_o)/2$ = (65+35)/2 = 50°C are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}, \quad \nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7228, \quad \beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis We assume the plastic tube temperature to be 35°C. We will check this assumption later, and repeat calculations, if necessary. The characteristic length in this case is

$$L_c = \frac{D_o - D_i}{2} = \frac{5-1.6}{2} = 1.7 \text{ cm}$$

Then,

$$Ra = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(65-35 \text{ K})(0.017 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2/\text{s})^2}(0.7228) = 10{,}000$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(0.05/0.016)]^4}{(0.017 \text{ m})^3[(0.016 \text{ m})^{-3/5} + (0.05 \text{ m})^{-3/5}]^5} = 0.1821$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02735 \text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}[(0.1821)(10{,}000)]^{1/4} = 0.05670 \text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.05670 \text{ W/m.°C})}{\ln(0.05/0.016)}(65 - T_0) \quad \text{(Eq. 1)}$$

Now we will calculate heat transfer from plastic tube to the ambient air by natural convection. Note that we should find a result close to the value we have already calculated since in steady operation they must be equal to each other. Also note that we neglect radiation heat transfer. We will use the same assumption for the plastic tube temperature (i.e., 35°C). The properties of air at 1 atm and the film temperature of $T_{avg} = (T_s + T_\infty)/2 = (35+26)/2 = 30.5°\text{C}$ are

$$k = 0.02592 \text{ W/m.°C}, \quad \nu = 1.613\times10^{-5} \text{ m}^2/\text{s},$$

$$\text{Pr} = 0.7281, \text{ and } \beta = 1/T_f = 1/(30.5+273)\text{K} = 0.003295 \text{ K}^{-1}$$

The characteristic length in this case is the outer diameter of the solar collector $L_c = D_o$ = 0.05 m Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003295 \text{ K}^{-1})(35-26 \text{ K})(0.05 \text{ m})^3}{(1.613\times10^{-5} \text{ m}^2/\text{s})^2}(0.7281) = 1.018\times10^5$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.018\times10^5)^{1/6}}{\left[1+(0.559/0.7281)^{9/16}\right]^{8/27}}\right\}^2 = 7.838$$

$$A_o = \pi D_0 L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2 \qquad h_o = \frac{k}{D_o}Nu = \frac{0.02592 \text{ W/m.°C}}{0.05 \text{ m}}(7.838) = 4.063 \text{ W/m}^2.\text{°C}$$

$$\dot{Q} = hA_o(T_o - T_\infty) = (4.063 \text{ W/m}^2.\text{°C})(0.1571 \text{ m}^2)(T_o - 26)\text{°C} \quad \text{(Eq. 2)}$$

Solving Eq. 1 and Eq. 2 simultaneously, we find

$$T_o = 38.8\text{°C}, \quad \dot{Q} = 8.18 \text{ W}$$

Repeating the calculations at the new average temperature for enclosure analysis and at the new film temperature for convection at the outer surface analysis using the new calculated temperature 38.8°C, we find

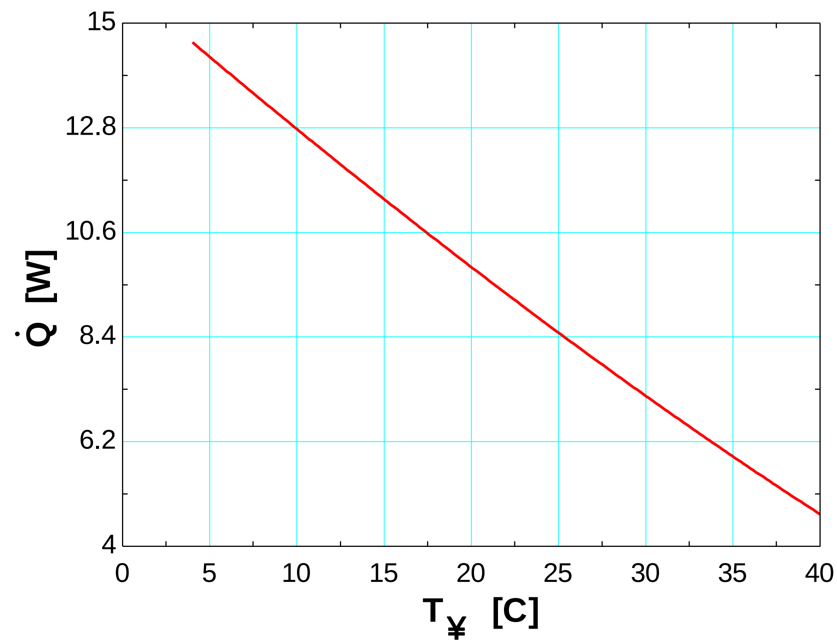
$$T_o = 39.0\text{°C}, \quad \dot{Q} = \mathbf{8.22 \text{ W}}$$

9-92 Prob. 9-91 is reconsidered. The rate of heat loss from the water by natural convection as a function of the ambient air temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_1=0.016 [m]
D_2=0.05 [m]
T_1=65 "[C]"
T_infinity=26 [C]
Length=1 [m] "unit length of the tube is considered"

"PROPERTIES for enclosure"
Fluid$='air'
k_1=Conductivity(Fluid$, T=T_ave)
Pr_1=Prandtl(Fluid$, T=T_ave)
rho_1=Density(Fluid$, T=T_ave, P=101.3)
mu_1=Viscosity(Fluid$, T=T_ave)
nu_1=mu_1/rho_1
beta_1=1/(T_ave+273)
T_ave=1/2*(T_1+T_2)
g=9.807 [m/s^2]

"ANALYSIS for enclosure"
L=(D_2-D_1)/2
Ra_1=(g*beta_1*(T_1-T_2)*L^3)/nu_1^2*Pr_1
F_cyl=(ln(D_2/D_1))^4/(L^3*(D_1^(-3/5)+D_2^(-3/5))^5)
k_eff=0.386*k_1*(Pr_1/(0.861+Pr_1))^0.25*(F_cyl*Ra_1)^0.25
Q_dot=(2*pi*k_eff)/ln(D_2/D_1)*(T_1-T_2)

"PROPERTIES for convection on the outer surface"
k_2=Conductivity(Fluid$, T=T_film)
Pr_2=Prandtl(Fluid$, T=T_film)
rho_2=Density(Fluid$, T=T_film, P=101.3)
mu_2=Viscosity(Fluid$, T=T_film)
nu_2=mu_2/rho_2
beta_2=1/(T_film+273)
T_film=1/2*(T_2+T_infinity)

"ANALYSIS for convection on the outer surface"
delta=D_2
Ra_2=(g*beta_2*(T_2-T_infinity)*delta^3)/nu_2^2*Pr_2
Nusselt=(0.6+(0.387*Ra_2^(1/6))/(1+(0.559/Pr_2)^(9/16))^(8/27))^2
h=k_2/delta*Nusselt
A=pi*D_2*Length
Q_dot=h*A*(T_2-T_infinity)
```

T_∞ [W]	$\dot{Q}$ [W]
4	14.6
6	13.98
8	13.37
10	12.77
12	12.18
14	11.59
16	11.01
18	10.44
20	9.871
22	9.314
24	8.764
26	8.222
28	7.688
30	7.163
32	6.647
34	6.139
36	5.641
38	5.153
40	4.675



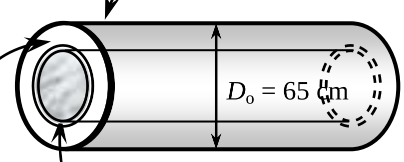
9-93 The space between the two concentric cylinders is filled with water or air. The rate of heat transfer from the outer cylinder to the inner cylinder by natural convection is to be determined for both cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm. **4** Heat transfer by radiation is negligible.

Properties The properties of water air at the average temperature of $(T_i+T_o)/2$ = (54+106)/2 =80°C are (Table A-9)

$$k = 0.670\ \text{W/m.°C},\quad \nu = 3.653\times10^{-7}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 2.22,\qquad \beta = 0.653\times10^{-3}\ \text{K}^{-1}$$



The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2$ = (54+106)/2 = 80°C are (Table A-15)

$$k = 0.02953\ \text{W/m.°C},\quad \nu = 2.097\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7154,\qquad \beta = \frac{1}{T_f} = \frac{1}{(80+273)\text{K}} = 0.002833\,\text{K}^{-1}$$

Analysis (*a*) The fluid is water:

$$L_c = \frac{D_o - D_i}{2} = \frac{65-55}{2} = 5\,\text{cm}.$$

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.653\times10^{-3}\ \text{K}^{-1})(106-54)\text{K}(0.05\,\text{m})^3}{(3.653\times10^{-7}\ \text{m}^2/\text{s})^2}(2.22) = 6.927\times10^8$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.65\,\text{m}}{0.55\,\text{m}}\right]^4}{(0.05\,\text{m})^3\left[(0.55\,\text{m})^{-7/5} + (0.65\,\text{m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.670\ \text{W/m.°C})\left(\frac{2.22}{0.861+2.22}\right)^{1/4}\left[(0.04136)(6.927\times10^8)\right]^{1/4} = 17.43\ \text{W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_0 - T_i) = \frac{2\pi(17.43\ \text{W/m.°C})}{\ln\left(\dfrac{0.65\,\text{m}}{0.55\,\text{m}}\right)}(106-54) = 34{,}090\ \text{W} = \mathbf{34.1\,kW}$$

(*b*) The fluid is air:

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.002833\,\text{K}^{-1})(106-54)\text{K}(0.05\,\text{m})^3}{(2.097\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7154) = 2.939\times10^5$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.65\,\text{m}}{0.55\,\text{m}}\right]^4}{(0.05\,\text{m})^3\left[(0.55\,\text{m})^{-7/5} + (0.65\,\text{m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02953\ \text{W/m.°C})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}\left[(0.04136)(2.939\times10^5)\right]^{1/4} = 0.09824\ \text{W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_o - T_i) = \frac{2\pi(0.09824\ \text{W/m.°C})}{\ln\left(\dfrac{0.65\,\text{m}}{0.55\,\text{m}}\right)}(106-54) = \mathbf{192\ W}$$

9-94 A hot fluid flowing inside a horizontal tube with a known mass flow rate and temperature difference between the tube inlet and outlet. The tube is enclosed in a concentric cylindrical thin cover. The concentric outer cover temperature is to be determined whether it is safe from thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Air is an ideal gas with constant properties. **4** Heat loss by radiation is negligible. **5** The air pressure in the enclosure is 1 atm.

Properties The properties of air at the assumed T_{avg} = 80°C and 1 atm pressure are k = 0.02953 W/m·K, $\nu = 2.097 \times 10^{-5}$ m^2/s, Pr = 0.7154 (Table A-15), and $\beta = 1/T_{avg}$ = 1/353 K.

Analysis With the assumption that T_{avg} = 80°C, the outer surface temperature is estimated as

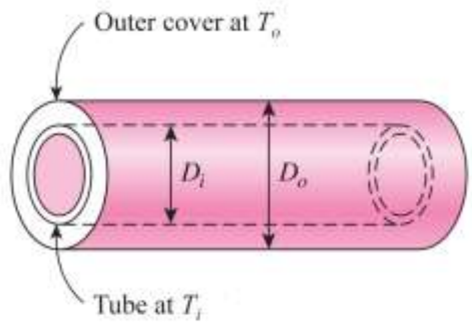
$$T_o = 2T_{avg} - T_i = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(80+273\text{ K})^{-1}(120-40)\text{ K }(0.0125\text{ m})^3}{(2.097\times10^{-5}\text{ m}^2/\text{s})^2}(0.7154) = 7064$$

where

$$L_c = (D_o - D_i)/2 = 0.0125\text{ m}$$



The effective thermal conductivity is

$$F_{cyl} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

$$= \frac{\ln(0.05/0.025)^4}{(0.0125\text{ m})^3[(0.025\text{ m})^{-3/5} + (0.05\text{ m})^{-3/5}]^5} = 0.1466$$

$$k_{eff} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{cyl}\text{Ra})^{1/4}$$

$$= 0.386(0.02953\text{ W/m}\cdot\text{K})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}[(0.1466)(7064)]^{1/4} = 0.05307\text{ W/m}\cdot\text{K}$$

Thus, the temperature of the outer cylinder can be determined using

$$\dot{Q} = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o)$$

$$\dot{m}c_p\Delta T = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) \quad \rightarrow \quad T_o = \mathbf{41°C}$$

where

$$\dot{m} = 0.005\text{ kg/s},\quad c_p = 950\text{ J/kg}\cdot\text{K},\quad \Delta T = 8°\text{C},\quad T_i = 120°\text{C}$$

Discussion The air gap between the concentric cylinders is sufficient to keep the outer cover temperature below 45°C to prevent thermal burn hazards.

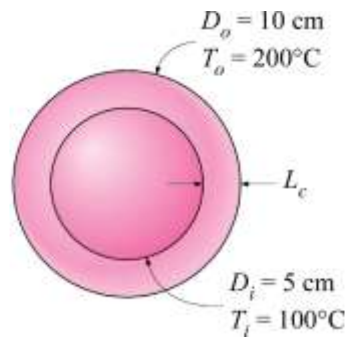
The assumed average temperature T_{avg} = 80°C is appropriate for evaluating the air properties, since the determined T_o = 41°C would give an average temperature of T_{avg} = 80.5°C.

9-95 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Nitrogen is an ideal gas with constant properties. **3** Radiation heat transfer is negligible

Properties The properties of nitrogen at $T_f = (T_s + T_\infty)/2 = 150°C$ are $k = 0.03416$ W/m·K, $\nu = 2.851 \times 10^{-5}$ m^2/s, Pr = 0.7025 (from Table A-16). Also, $\beta = 1/T_f = 0.002364$ K^{-1}.

Analysis The characteristic length in this case is determined from



$$L_c = \frac{D_o - D_i}{2} = \frac{10-5}{2}\text{ cm} = 2.5\text{ cm}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\text{ m/s}^2)(2.364\times10^{-3}\text{ K}^{-1})(200-100)\text{K}(0.025\text{ m})^3}{(2.851\times10^{-5}\text{ m}^2\text{/s})^2}(0.7025)$$
$$= 3.132\times10^4$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} = \frac{(0.025\text{ m})}{[(0.1\text{ m})(0.05\text{ m})]^4[(0.05\text{ m})^{-7/5} + (0.1\text{ m})^{-7/5}]^5}$$
$$= 0.006268$$

$$k_{\text{eff}} = 0.74\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}k$$
$$= 0.74\left(\frac{0.7025}{0.861+0.7025}\right)^{1/4}[(0.006268)(3.132\times10^4)]^{1/4}(0.03416\text{ W/m}\cdot\text{K})$$
$$= 0.07747\text{ W/m}\cdot\text{K}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\frac{\pi D_iD_o}{L_c}(T_o - T_i) = (0.07747\text{ W/m}\cdot\text{K})\frac{\pi(0.1\text{ m})(0.05\text{ m})}{(0.025\text{ m})}(200-100)\text{ K} = \mathbf{4.87\text{ W}}$$

Discussion Note that if $k_{\text{eff}} < k$, then we use $k = k_{\text{eff}}$.

9-96 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclusure is 1 atm.

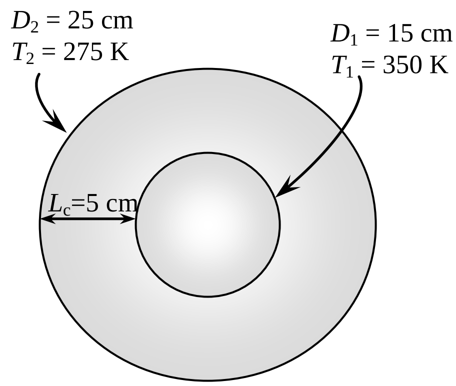
Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)

$$k = 0.02658 \text{ W/m.°C}$$

$$\nu = 1.697\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7256$$

$$\beta = \frac{1}{T_f} = \frac{1}{312.5\text{ K}} = 0.003200\text{ K}^{-1}$$



Analysis The characteristic length in this case is determined from

$$L_c = \frac{D_2 - D_1}{2} = \frac{25-15}{2} = 5\,\text{cm.}$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003200\text{ K}^{-1})(350-275\text{ K})(0.05\text{ m})^3}{(1.697\times10^{-5}\text{ m}^2\text{/s})^2}(0.7256) = 7.415\times10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5}+D_o^{-7/5})^5} = \frac{0.05\text{ m}}{[(0.15\text{ m})(0.25\text{ m})]^4\left[(0.15\text{ m})^{-7/5}+(0.25\text{ m})^{-7/5}\right]^5} = 0.005900$$

$$\begin{aligned} k_{\text{eff}} &= 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}Ra)^{1/4} \\ &= 0.74(0.02658\text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}\left[(0.00590)(7.415\times10^5)\right]^{1/4} \\ &= 0.1315\text{ W/m.°C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

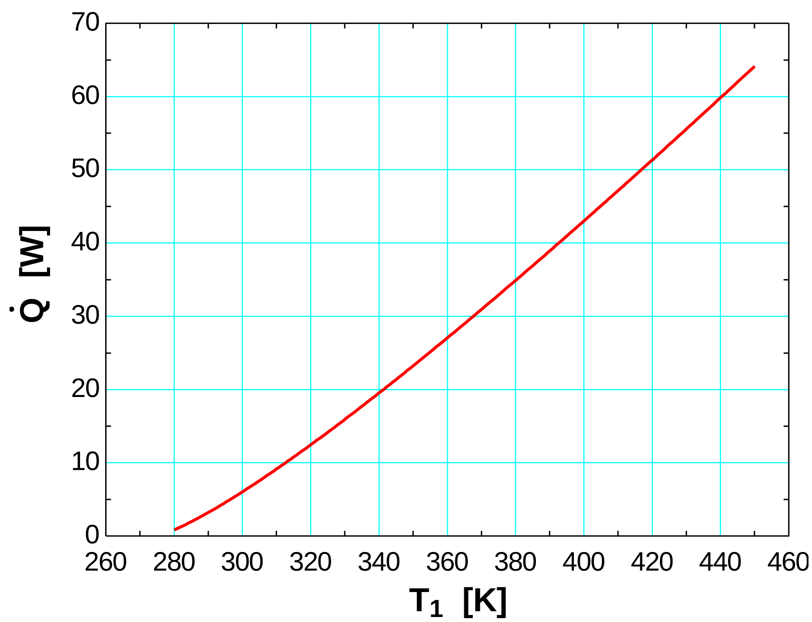
$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) = (0.1315\text{ W/m.°C})\pi\left[\frac{(0.15\text{ m})(0.25\text{ m})}{(0.05\text{ m})}\right](350-275)\text{K} = \mathbf{23.3\,W}$$

9-97 Prob. 9-96 is reconsidered. The rate of natural convection heat transfer as a function of the hot surface temperature of the sphere is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

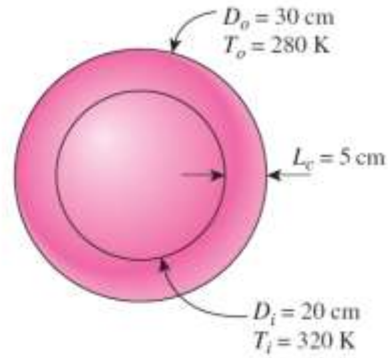
```
"GIVEN"
D_1=0.15 [m]
D_2=0.25 [m]
T_1=350 [K]
T_2=275 [K]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
beta=1/T_ave
T_ave=1/2*(T_1+T_2)
g=9.807 [m/s^2]

"ANALYSIS"
L=(D_2-D_1)/2
Ra=(g*beta*(T_1-T_2)*L^3)/nu^2*Pr
F_sph=L/((D_1*D_2)^4*(D_1^(-7/5)+D_2^(-7/5))^5)
k_eff=0.74*k*(Pr/(0.861+Pr))^0.25*(F_sph*Ra)^0.25
Q_dot=k_eff*pi*(D_1*D_2)/L*(T_1-T_2)
```

T_1 [K]	$\dot{Q}$ [W]
250	-
260	-
270	-
280	0.8153
290	3.202
300	6.032
310	9.139
320	12.45
330	15.92
340	19.52
350	23.23
360	27.04
370	30.93
380	34.89
390	38.92
400	43.01
410	47.15
420	51.33
430	55.56
440	59.83
450	64.13



9-98 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.



Properties The properties of air at the average temperature of $T_{avg} = (T_i + T_o)/2 = (320 + 280)/2 = 300$ K = 27°C and 1 atm pressure are: $k = 0.02566$ W/m·K, Pr = 0.7290, $\nu = 1.58 \times 10^{-5}$ m^2/s (Table A-15), and $\beta = 1/T_{avg} = 1/300$ K.

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05\text{ m}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(300\text{ K})^{-1}(320-280)\text{ K}(0.05\text{ m})^3}{(1.58\times 10^{-5}\text{ m}^2/\text{s})^2}(0.729)$$
$$= 4.775 \times 10^5$$

The effective thermal conductivity is

$$F_{sph} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05\text{ m}}{[(0.2\text{ m})(0.3\text{ m})]^4[(0.2\text{ m})^{-7/5} + (0.3\text{ m})^{-7/5}]^5} = 0.0005229$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{sph}\text{Ra})^{1/4}$$
$$= 0.74(0.02566\text{ W/m}\cdot\text{K})\left(\frac{0.729}{0.861 + 0.729}\right)^{1/4}[(0.0005229)(4.775\times 10^5)]^{1/4}$$
$$= 0.1105\text{ W/m}\cdot\text{K}$$

Then, the rate of heat transfer between spheres becomes

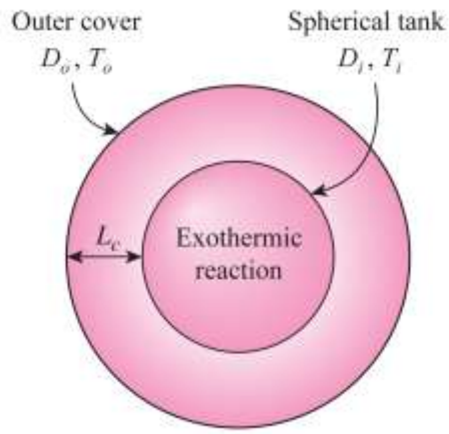
$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)$$

$$= (0.1105)\pi\left(\frac{0.2\text{ m}\times 0.3\text{ m}}{0.05\text{ m}}\right)(320-280)\text{ K} = \mathbf{16.7\text{ W}}$$

Discussion Note that the air in the spherical enclosure acts like a stationary fluid whose thermal conductivity is k_{eff}/k = 0.1105/0.02566 = 4.3 times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually significant, and should be considered in a complete analysis.

9-99 A metal spherical tank is filled with a solution undergoing an exothermic reaction and the heat generation is known. The tank in enclosed by a concentric outer cover to prevent thermal burn hazards. The temperature of the outer cover is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Air is an ideal gas with constant properties. **4** Heat loss by radiation is negligible. **5** The air pressure in the enclosure is 1 atm.



Properties The properties of air at the assumed T_{avg} = 80°C and 1 atm pressure are k = 0.02953 W/m·K, $\nu = 2.097 \times 10^{-5}$ m²/s, Pr = 0.7154 (Table A-15), and $\beta = 1/T_f$ = 1/ 353 K.

Analysis With the assumption that T_{avg} = 80°C, the outer surface temperature is estimated as

$$T_o = 2T_{\text{avg}} - T_i = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81\,\text{m/s}^2)(80+273\,\text{K})^{-1}(120-40)\,\text{K}\,(0.05\,\text{m})^3}{(2.097\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7154) = 452{,}112$$

where

$$L_c = (D_o - D_i)/2 = 0.05\,\text{m}$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05\,\text{m}}{[(3.0\,\text{m})(3.1\,\text{m})]^4[(3.0\,\text{m})^{-7/5} + (3.1\,\text{m})^{-7/5}]^5} = 0.0005117$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}\text{Ra})^{1/4}$$

$$= 0.74(0.02953\ \text{W/m}\cdot\text{K})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}[(0.0005117)(452{,}112)]^{1/4} = 0.06995\ \text{W/m}\cdot\text{K}$$

Thus, the temperature of the outer cover can be determined using

$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o)$$

$$\dot{e}_{\text{gen}}(\pi D_i^3/6) = k_{\text{eff}}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) \quad \rightarrow \quad T_o = \mathbf{39.4°C}$$

where

$$\dot{e}_{\text{gen}} = 233\ \text{W/m}^3$$

Discussion The air gap between the concentric spheres is sufficient to keep the outer cover temperature below 45°C to prevent thermal burn hazards.

The assumed average temperature T_{avg} = 80°C is appropriate for evaluating the air properties, since the determined T_o = 39.4°C would give an average temperature of T_{avg} = 79.7°C.

Combined Natural and Forced Convection

9-100C In combined natural and forced convection, the natural convection is negligible when $Gr/\mathrm{Re}^2 < 0.1$. Otherwise it is not.

9-101C In assisting or transverse flows, natural convection enhances forced convection heat transfer while in opposing flow it hurts forced convection.

9-102C When neither natural nor forced convection is negligible, it is not correct to calculate each separately and to add them to determine the total convection heat transfer. Instead, the correlation

$$\mathrm{Nu}_{\text{combined}} = \left(\mathrm{Nu}_{\text{forced}}^{n} + \mathrm{Nu}_{\text{natural}}^{n}\right)^{1/n}$$

based on the experimental studies should be used.

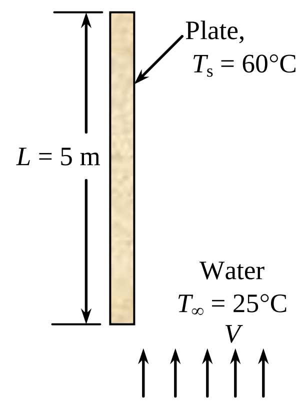
9-103 A vertical plate in water is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The properties of water at the film temperature of $(T_s+T_\infty)/2 = (60+25)/2 = 42.5$°C are (Table A-9)

$$\nu = \mu/\rho = 0.630\times10^{-6}\ \mathrm{m^2/s}$$

$$\beta = 0.396\times10^{-3}\ \mathrm{K^{-1}}$$



Analysis The characteristic length is the height of the plate $L_c = L = 5$ m. The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \mathrm{m/s^2})(0.396\times10^{-3}\ \mathrm{K^{-1}})(60-25\ \mathrm{K})(5\ \mathrm{m})^3}{(0.630\times10^{-6}\ \mathrm{m^2/s})^2} = 4.28\times10^{13}$$

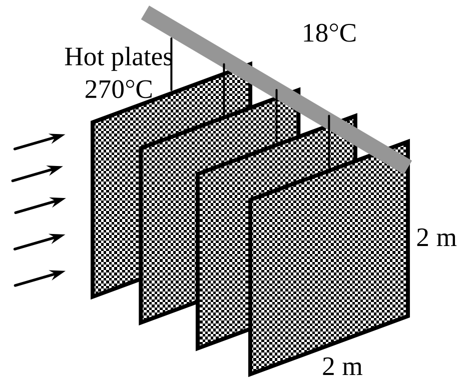
$$\mathrm{Re} = \frac{V_\infty L}{\nu} = \frac{V(5\ \mathrm{m})}{0.630\times10^{-6}\ \mathrm{m^2/s}} = 7.94\times10^{6}V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\mathrm{Re}^2} = 0.1 \longrightarrow \frac{4.28\times10^{13}}{(7.94\times10^6 V)^2} = 0.1 \longrightarrow V = \mathbf{2.61\ m/s}$$

9-104 Thin square plates coming out of the oven in a production facility are cooled by blowing ambient air horizontally parallel to their surfaces. The air velocity above which the natural convection effects on heat transfer are negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (270+18)/2 = 144$°C are (Table A-15)

$$\nu = 2.791\times10^{-5}\ \text{m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(144+273)\text{K}} = 0.002398\ \text{K}^{-1}$$

Analysis The characteristic length is the height of the plate $L_c = L = 3$ m. The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\,\text{m/s}^2)(0.002398\,\text{K}^{-1})(270-18\,\text{K})(2\,\text{m})^3}{(2.791\times10^{-5}\ \text{m}^2/\text{s})^2} = 6.088\times10^{10}$$

$$\text{Re} = \frac{VL}{\nu} = \frac{V(2\,\text{m})}{2.791\times10^{-5}\ \text{m}^2/\text{s}} = 7.166\times10^4 V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\text{Re}^2} = 0.1 \longrightarrow \frac{6.088\times10^{10}}{(7.166\times10^4 V)^2} = 0.1 \longrightarrow V = \mathbf{10.9\,m/s}$$

9-105 The significance of natural convection to the heat transfer process on a vertical rod with water flowing across its outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The rod is orientated such that the characteristic length is its length.

Properties The properties of water at $T_f = (T_s + T_\infty)/2 = 80$°C are $\rho = 971.8$ kg/m^3, $\mu = 0.355\times10^{-3}$ kg/m·s, and $\beta = 0.653\times10^{-3}$ K^{-1} (Table A-9).

Analysis The Reynolds number for the cross flow is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(971.8\,\text{kg/m}^3)(0.5\,\text{m/s})(0.150\,\text{m})}{0.355\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 2.05\times10^5$$

For vertical cylinder, the Grashof number with $L_c = L$ is

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)L_c^3}{(\mu/\rho)^2}$$

$$= \frac{(9.81\,\text{m/s}^2)(0.653\times10^{-3}\ \text{K}^{-1})(120-40)\ \text{K}(1\,\text{m})^3}{(0.355\times10^{-3}/971.8)^2\ \text{m}^4/\text{s}^2} = 3.84\times10^{12}$$

Hence,

$$\frac{\text{Gr}_L}{\text{Re}^2} = \frac{3.84\times10^{12}}{(2.05\times10^5)^2} = \mathbf{91.4}$$

Since $\text{Gr}_L/\text{Re}^2 >> 1$, therefore natural convection effects dominate forced convection effects. Natural convection effects are important to the heat transfer process.

Discussion If $\text{Gr}_L/\text{Re}^2 << 1$, then forced convection effects dominate natural convection effects.

9-106 The significance of natural convection to the heat transfer process on a horizontal rod with water flowing across its outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The rod is orientated such that the characteristic length is its diameter.

Properties The properties of water at $T_f = (T_s + T_\infty)/2 = 80°C$ are $\rho = 971.8\ \text{kg/m}^3$, $\mu = 0.355 \times 10^{-3}$ kg/m·s, and $\beta = 0.653 \times 10^{-3}\ \text{K}^{-1}$ (Table A-9).

Analysis The Reynolds number for the cross flow is

$$\text{R}e = \frac{\rho VD}{\mu} = \frac{(971.8\ \text{kg/m}^3)(0.2\ \text{m/s})(0.150\ \text{m})}{0.355\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 8.21\times10^4$$

For horizontal cylinder, the Grashof number with $L_c = D$ is

$$\text{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)D^3}{(\mu/\rho)^2}$$

$$= \frac{(9.81\ \text{m/s}^2)(0.653\times10^{-3}\ \text{K}^{-1})(120-40)\ \text{K}(0.15\ \text{m})^3}{(0.355\times10^{-3}/971.8)^2\ \text{m}^4/\text{s}^2} = 1.296\times10^{10}$$

Hence,

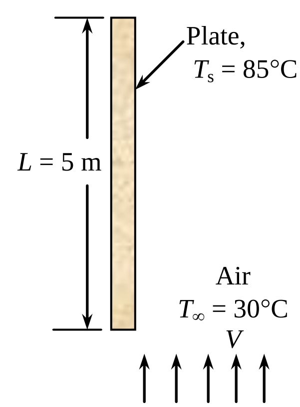
$$\frac{\text{Gr}_D}{\text{Re}^2} = \frac{6.48\times10^8}{(8.21\times10^4)^2} = \mathbf{1.92}$$

Since $\text{Gr}_D/\text{Re}^2 \approx 1$, therefore natural convection and forced convection effects are both important to the heat transfer process.

Discussion When both natural convection and forced convection effects are important, the process is called combined natural and forced convection or mixed convection.

9-107 A vertical plate in air is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (85+30)/2 = 57.5°C$ are (Table A-15)

$$\nu = 1.872\times10^{-5}\ \text{m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(57.5+273)\text{K}} = 0.003026\ \text{K}^{-1}$$

Analysis The characteristic length is the height of the plate, $L_c = L = 5$ m.
The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \text{m/s}^2)(0.003026\ \text{K}^{-1})(85-30\ \text{K})(5\ \text{m})^3}{(1.872\times10^{-5}\ \text{m}^2/\text{s})^2} = 5.824\times10^{11}$$

$$\text{Re} = \frac{VL}{\nu} = \frac{V_\infty(5\ \text{m})}{1.872\times10^{-5}\ \text{m}^2/\text{s}} = 2.671\times10^5 V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

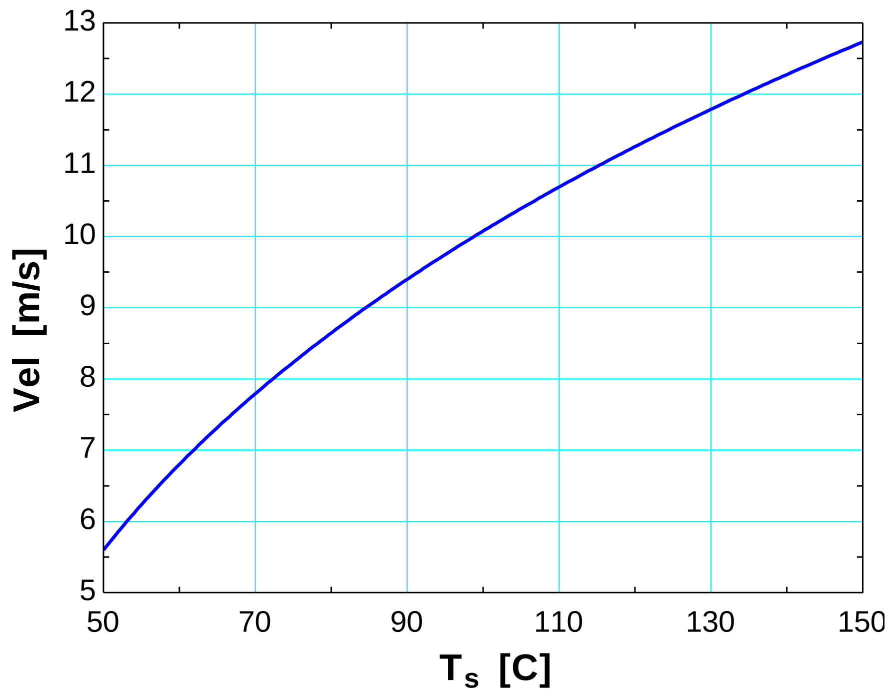
$$\frac{Gr}{\text{Re}^2} = 0.1 \longrightarrow \frac{5.824\times10^{11}}{(2.671\times10^5 V)^2} = 0.1 \longrightarrow V = \mathbf{9.04\ m/s}$$

9-108 Prob. 9-106 is reconsidered. The forced motion velocity above which natural convection heat transfer is negligible as a function of the plate temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=5 [m]
T_s=85 [C]
T_infinity=30 [C]

"PROPERTIES"
Fluid$='air'
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
g=9.807 [m/s^2]

"ANALYSIS"
Gr=(g*beta*(T_s-T_infinity)*L^3)/nu^2
Re=(Vel*L)/nu
Gr/Re^2=0.1
```

T_s [C]	Vel [m/s]
50	5.598
55	6.233
60	6.801
65	7.318
70	7.793
75	8.233
80	8.646
85	9.033
90	9.4
95	9.747
100	10.08
105	10.39
110	10.69
115	10.98
120	11.26
125	11.53
130	11.79
135	12.03
140	12.27
145	12.51
150	12.73

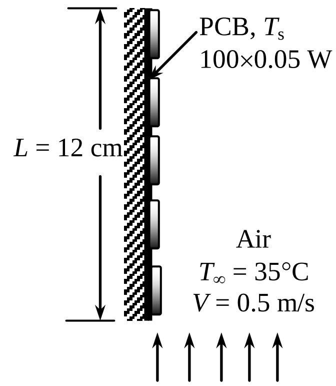


9-109 A circuit board is cooled by a fan that blows air upwards. The average temperature on the surface of the circuit board is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (60+35)/2 = 47.5°C are (Table A-15)

$$k = 0.02717 \text{ W/m.°C}$$
$$\nu = 1.774\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7235$$
$$\beta = \frac{1}{T_f} = \frac{1}{(47.5+273)\text{K}} = 0.00312 \text{ K}^{-1}$$



Analysis We assume the surface temperature to be 60°C. We will check this assumption later on and repeat calculations with a better assumption, if necessary. The characteristic length in this case is the length of the board in the flow (vertical) direction, $L_c = L$ = 0.12 m. Then the Reynolds number becomes

$$\text{Re} = \frac{VL}{\nu} = \frac{(0.5\text{ m/s})(0.12\text{ m})}{1.774\times10^{-5}\text{ m}^2\text{/s}} = 3383$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar and the forced convection Nusselt number and h are determined from

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}^{0.5}\,\text{Pr}^{1/3} = 0.664(3383)^{0.5}(0.7235)^{1/3} = 34.67$$

$$h = \frac{k}{L}Nu = \frac{0.02717 \text{ W/m.°C}}{0.12\text{ m}}(34.67) = 7.85 \text{ W/m}^2\text{.°C}$$

$$A_s = L\times W = (0.12\text{ m})(0.2\text{ m}) = 0.024\text{ m}^2$$

Then, $$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 35\text{°C} + \frac{(100)(0.05\text{ W})}{(7.85\text{ W/m}^2\text{.°C})(0.024\text{ m}^2)} = \mathbf{61.5°C}$$

which is sufficiently close to the assumed value in the evaluation of properties. Therefore, there is no need to repeat calculations.

(b) The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.00312\text{ K}^{-1})(60-35\text{ K})(0.12\text{ m})^3}{(1.774\times10^{-5}\text{ m}^2\text{/s})^2}(0.7235) = 3.041\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.041\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7235}\right)^{9/16}\right]^{8/27}}\right\}^2 = 22.42$$

This is an assisting flow and the combined Nusselt number is determined from

$$Nu_{combined} = (Nu_{forced}{}^n + Nu_{natural}{}^n)^{1/n} = (34.67^3 + 22.42^3)^{1/3} = 37.55$$

Then, $$h = \frac{k}{L}Nu_{combined} = \frac{0.02717\text{ W/m.°C}}{0.12\text{ m}}(37.55) = 8.502\text{ W/m}^2\text{.°C}$$

and $$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 35\text{°C} + \frac{(100)(0.05\text{ W})}{(8.502\text{ W/m}^2\text{.°C})(0.024\text{ m}^2)} = \mathbf{59.5°C}$$

which is sufficiently close to the assumed surface temperature value of 60°C used in the evaluation of properties. Therefore, there is no need to repeat calculations. The natural convection lowers the surface temperature in this case by about 2°C.

9-110 Water is flowing across a horizontal cylinder, the Nusselt number for water flowing (*a*) upward and (*b*) downward is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The cylinder is orientated such that the characteristic length is its diameter. **4** Heat transfer by radiation is negligible.

Properties The properties of water at $T_f = (T_s + T_\infty)/2 = 80°\text{C}$ are $\rho = 971.8\ \text{kg/m}^3$, $\mu = 0.355 \times 10^{-3}\ \text{kg/m·s}$, Pr = 2.22 and $\beta = 0.653 \times 10^{-3}\ \text{K}^{-1}$ (Table A-9).

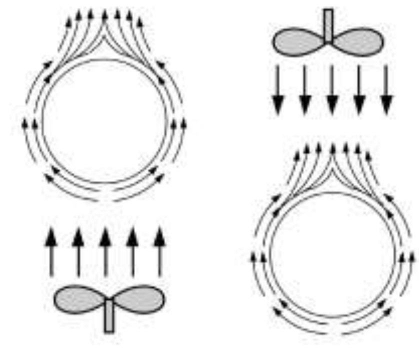
Analysis To check if combined natural and force convection exists, we find

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(971.8\ \text{kg/m}^3)(0.2\ \text{m/s})(0.150\ \text{m})}{0.355\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 8.21\times10^4$$

$$\text{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)D^3}{(\mu/\rho)^2}$$

$$= \frac{(9.81\ \text{m/s}^2)(0.653\times10^{-3}\ \text{K}^{-1})(120-40)\ \text{K}(0.15\ \text{m})^3}{(0.355\times10^{-3}/971.8)^2\ \text{m}^4/\text{s}^2}$$

$$= 1.296\times10^{10}$$



(*a*) (*b*)

Hence,

$$\frac{\text{Gr}_D}{\text{Re}^2} = \frac{1.296\times10^{10}}{(8.21\times10^4)^2} = 1.92$$

Since $\text{Gr}_D/\text{Re}^2 \approx 1$, therefore combined natural and force convection exists. The Nusselt number for natural convection of a horizontal cylinder is

$$\text{Nu}_{\text{natural}} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.6 + \frac{0.387(1.296\times10^{10}\times2.22)^{1/6}}{[1+(0.559/2.22)^{9/16}]^{8/27}}\right\}^2 = 390$$

Then, the Nusselt number for forced convection across a cylinder is calculated using the correlation listed in Table 7.1,

$$\text{Nu}_{\text{forced}} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(8.21\times10^4)^{0.805}(2.22)^{1/3} = 318$$

(*a*) For water flowing upward (assisting flow) the combined Nusselt number is (use *n* = 4 for horizontal surfaces)

$$\text{Nu}_{\text{combined}} = (\text{Nu}_{\text{forced}}^4 + \text{Nu}_{\text{natural}}^4)^{1/3} = (318^4 + 390^4)^{1/3} = \mathbf{427}$$

(*b*) For water flowing downward (opposing flow) the combined Nusselt number is (use *n* = 4 for horizontal surfaces)

$$\text{Nu}_{\text{combined}} = (\text{Nu}_{\text{natural}}^4 - \text{Nu}_{\text{forced}}^4)^{1/3} = (390^4 - 318^4)^{1/3} = \mathbf{337}$$

Discussion The Nusselt number for the assisting flow is about 21% higher than that for the opposing flow.

Special Topic: Heat Transfer through Windows

9-111C Windows are considered in three regions when analyzing heat transfer through them because the structure and properties of the frame are quite different than those of the glazing. As a result, heat transfer through the frame and the edge section of the glazing adjacent to the frame is two-dimensional. Even in the absence of solar radiation and air infiltration, heat transfer through the windows is more complicated than it appears to be. Therefore, it is customary to consider the windows in three regions when analyzing heat transfer through them: (1) the *center-of-glass*, (2) the *edge-of-glass*, and (3) the *frame* regions. When the heat transfer coefficient for all three regions are known, the overall U-value of the window is determined from

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}}$$

where A_{window} is the window area, and A_{center}, A_{edge}, and A_{frame} are the areas of the center, edge, and frame sections of the window, respectively, and U_{center}, U_{edge}, and U_{frame} are the heat transfer coefficients for the center, edge, and frame sections of the window.

9-112C Of the three similar double pane windows with air gab widths of 5, 10, and 20 mm, the U-factor and thus the rate of heat transfer through the window will be a minimum for the window with 10-mm air gab, as can be seen from Fig. 9-37.

9-113C In an ordinary double pane window, about half of the heat transfer is by radiation. A practical way of reducing the radiation component of heat transfer is to reduce the emissivity of glass surfaces by coating them with low-emissivity (or "low-e") material.

9-114C When a thin polyester film is used to divide the 20-mm wide air of a double pane window space into two 10-mm wide layers, both (*a*) convection and (*b*) radiation heat transfer through the window will be reduced.

9-115C When a double pane window whose air space is flashed and filled with argon gas, (*a*) convection heat transfer will be reduced but (*b*) radiation heat transfer through the window will remain the same.

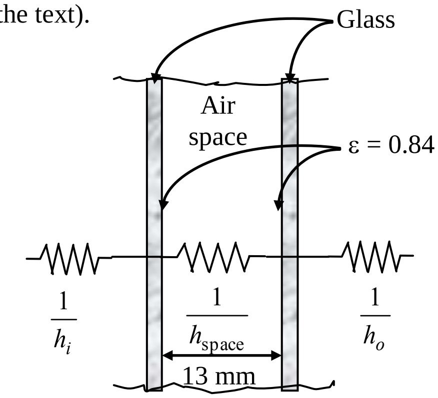
9-116C The heat transfer rate through the glazing of a double pane window is higher at the edge section than it is at the center section because of the two-dimensional effects due to heat transfer through the frame.

9-117C The *U*-factors of windows with aluminum frames will be highest because of the higher conductivity of aluminum. The *U*-factors of wood and vinyl frames are comparable in magnitude.

9-118 The U-factor for the center-of-glass section of a double pane window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** The thermal resistance of glass sheets is negligible.

Properties The emissivity of clear glass is given to be 0.84. The values of h_i and h_o for winter design conditions are h_i = 8.29 W/m^2.°C and h_o = 34.0 W/m^2.°C (from the text).



Analysis Disregarding the thermal resistance of glass sheets, which are small, the U-factor for the center region of a double pane window is determined from

$$\frac{1}{U_{\text{center}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

where h_i, h_{space}, and h_o are the heat transfer coefficients at the inner surface of window, the air space between the glass layers, and the outer surface of the window, respectively. The effective emissivity of the air space of the double pane window is

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} = 0.72$$

For this value of emissivity and an average air space temperature of 10°C with a temperature difference across the air space to be 15°C, we read h_{space} = 5.7 W/m^2.°C from Table 9-3 for 13-mm thick air space. Therefore,

$$\frac{1}{U_{\text{center}}} = \frac{1}{8.29} + \frac{1}{5.7} + \frac{1}{34.0} \longrightarrow U_{\text{center}} = \mathbf{3.07\ W/m^2 \cdot °C}$$

Discussion The overall U-factor of the window will be higher because of the edge effects of the frame.

9-119 The overall U-factor of a window is given to be U = 2.76 W/m^2.°C for 12 km/h winds outside. The new U-factor when the wind velocity outside is doubled is to be determined.

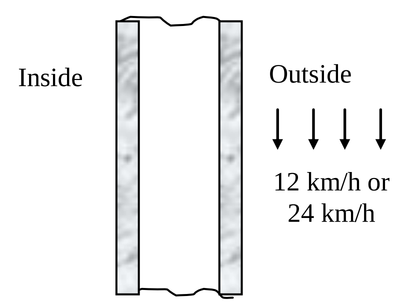
Assumptions Thermal properties of the windows and the heat transfer coefficients are constant.

Properties The heat transfer coefficients at the outer surface of the window are h_o = 22.7 W/m^2.°C for 12 km/h winds, and h_o = 34.0 W/m^2.°C for 24 km/h winds (from the text).

Analysis The corresponding convection resistances for the outer surfaces of the window are

$$R_{\text{o,12km/h}} = \frac{1}{h_{\text{o,12km/h}}} = \frac{1}{22.7\ \text{W/m}^2.\text{°C}} = 0.044\ \text{m}^2.\text{°C/W}$$

$$R_{\text{o,24km/h}} = \frac{1}{h_{\text{o,24km/h}}} = \frac{1}{34.0\ \text{W/m}^2.\text{°C}} = 0.029\ \text{m}^2.\text{°C/W}$$



Also, the R-value of the window at 12 km/h winds is

$$R_{\text{window,12km/h}} = \frac{1}{U_{\text{window,12km/h}}} = \frac{1}{2.76\ \text{W/m}^2.\text{°C}} = 0.362\ \text{m}^2.\text{°C/W}$$

Noting that all thermal resistances are in series, the thermal resistance of the window for 24 km/h winds is determined by replacing the convection resistance for 12 km/h winds by the one for 24 km/h:

$$R_{\text{window,24km/h}} = R_{\text{window,12km/h}} - R_{\text{o,12km/h}} + R_{\text{o,24km/h}} = 0.362 - 0.044 + 0.029 = 0.347\ \text{m}^2.\text{°C/W}$$

Then the U-factor for the case of 24 km/h winds becomes

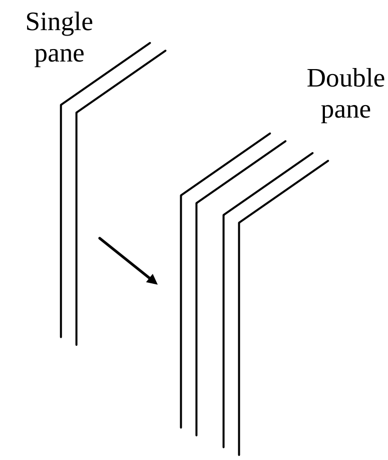
$$U_{\text{window,24km/h}} = \frac{1}{R_{\text{window,24km/h}}} = \frac{1}{0.347\ \text{m}^2.\text{°C/W}} = \mathbf{2.88\ W/m^2.°C}$$

Discussion Note that doubling of the wind velocity increases the U-factor only slightly (about 4%) from 2.76 to 2.88 W/m^2.°C.

9-120 The existing wood framed single pane windows of an older house in Wichita are to be replaced by double-door type vinyl framed double pane windows with an air space of 6.4 mm. The amount of money the new windows will save the home owner per month is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are 5.57 W/m^2.°C for the old single pane windows, and 3.20 W/m^2.°C. for the new double pane windows (Table 9-6).



Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Noting that the heaters will turn on only when the outdoor temperature drops below 18°C, the rates of heat transfer due to electric heating for the old and new windows are determined to be

$$\dot{Q}_{\text{window,old}} = (5.57\text{ W/m}^2.°\text{C})(17\text{ m}^2)(18-7.1)°\text{C} = 1032\text{ W}$$

$$\dot{Q}_{\text{window,new}} = (3.20\text{ W/m}^2.°\text{C})(17\text{ m}^2)(18-7.1)°\text{C} = 593\text{ W}$$

$$\dot{Q}_{\text{saved}} = \dot{Q}_{\text{window,old}} - \dot{Q}_{\text{window,new}} = 1032 - 593 = 439\text{ W}$$

Then the electrical energy and cost savings per month becomes

$$\text{Energy savings} = \dot{Q}_{\text{saved}} \Delta t = (0.439\text{ kW})(30 \times 24\text{ h/month}) = 316\text{ kWh/month}$$

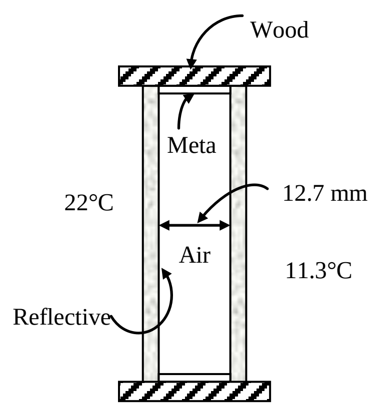
$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy}) = (316\text{ kWh/month})(\$0.085/\text{kWh}) = \mathbf{\$26.9/month}$$

Discussion We would obtain the same result if we used the actual indoor temperature (probably 22°C) for T_i instead of the balance point temperature of 18°C.

9-121 The windows of a house in Atlanta are of double door type with wood frames and metal spacers. The average rate of heat loss through the windows in winter is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factor of the window is given in Table 9-6 to be 2.13 W/m^2.°C.



Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window,avg}} = U_{\text{overall}} A_{\text{window}} (T_i - T_{o,avg})$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

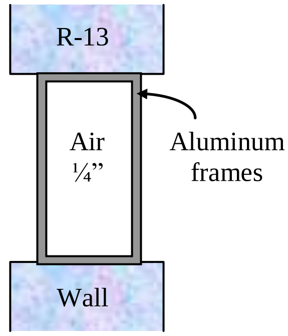
$$\dot{Q}_{\text{window,avg}} = (2.13\text{ W/m}^2.°\text{C})(14\text{ m}^2)(22-11.3)°\text{C} = \mathbf{319\ W}$$

Discussion This is the "average" rate of heat transfer through the window in winter in the absence of any infiltration.

9-122E The *R*-value of the common double door windows that are double pane with 1/4-in of air space and have aluminum frames is to be compared to the *R*-value of *R*-13 wall. It is also to be determined if more heat is transferred through the windows or the walls.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factor of the window is given in Table 9-6 to be $4.55 \times 0.176 = 0.801$ Btu/h.ft^2.°F.



Analysis The *R*-value of the windows is simply the inverse of its *U*-factor, and is determined to be

$$R_{\text{window}} = \frac{1}{U} = \frac{1}{0.801\,\text{Btu/h}\cdot\text{ft}^2.\text{°F}} = 1.25\,\text{h}\cdot\text{ft}^2.\text{°F/Btu}$$

which is less than 13. Therefore, the *R*-value of a double pane window is **much less** than the *R*-value of an *R*-13 wall.

Now consider a 1-ft^2 section of a wall. The solid wall and the window areas of this section are $A_{\text{wall}} = 0.8$ ft^2 and $A_{\text{window}} = 0.2$ ft^2. Then the rates of heat transfer through the two sections are determined to be

$$\dot{Q}_{\text{wall}} = U_{\text{wall}} A_{\text{wall}} (T_i - T_o) = A_{\text{wall}} \frac{T_i - T_o}{R-\text{value, wall}} = (0.8\,\text{ft}^2)\frac{\Delta T(\text{°F})}{13\,\text{h.ft}^2.\text{°F/Btu}} = 0.0615\Delta T \text{ Btu/h}$$

$$\dot{Q}_{\text{window}} = U_{\text{window}} A_{\text{window}} (T_i - T_o) = A_{\text{window}} \frac{T_i - T_o}{R-\text{value}} = (0.2\,\text{ft}^2)\frac{\Delta T(\text{°F})}{1.25\,\text{h.ft}^2.\text{°F/Btu}} = 0.160\Delta T \text{ Btu/h}$$

Therefore, the rate of heat transfer through a double pane window is **much more** than the rate of heat transfer through an *R*-13 wall.

Discussion The ratio of heat transfer through the wall and through the window is

$$\frac{\dot{Q}_{\text{window}}}{\dot{Q}_{\text{wall}}} = \frac{0.160\,\text{Btu/h}}{0.0615\,\text{Btu/h}} = 2.60$$

Therefore, 2.6 times more heat is lost through the windows than through the walls although the windows occupy only 20% of the wall area.

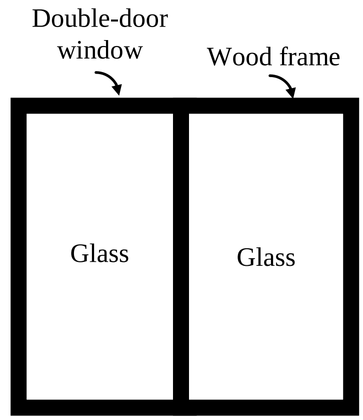
9-123 The rate of heat loss through a double-door wood framed window and the inner surface temperature are to be determined for the cases of single pane, double pane, and low-e triple pane windows.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are given in Table 9-6.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$



where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area which is determined to be

$$A_{window} = \text{Height} \times \text{Width} = (1.2\text{ m})(1.8\text{ m}) = 2.16\text{ m}^2$$

The U-factors for the three cases can be determined directly from Table 9-6 to be 5.57, 2.86, and 1.46 W/m^2.°C, respectively. Also, the inner surface temperature of the window glass can be determined from Newton's law,

$$\dot{Q}_{\text{window}} = h_i A_{\text{window}} (T_i - T_{\text{glass}}) \longrightarrow T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}}$$

where h_i is the heat transfer coefficient on the inner surface of the window which is determined from Table 9-5 to be h_i = 8.3 W/m^2.°C. Then the rate of heat loss and the interior glass temperature for each case are determined as follows:

(*a*) Single glazing:

$$\dot{Q}_{\text{window}} = (5.57\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)\left[20-(-8)°\text{C}\right] = \mathbf{337\text{ W}}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20°\text{C} - \frac{337\text{ W}}{(8.29\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)} = \mathbf{1.2°C}$$

(*b*) Double glazing (13 mm air space):

$$\dot{Q}_{\text{window}} = (2.86\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)\left[20-(-8)°\text{C}\right] = \mathbf{173\text{ W}}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20°\text{C} - \frac{173\text{ W}}{(8.29\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)} = \mathbf{10.3°C}$$

(*c*) Triple glazing (13 mm air space, low-e coated):

$$\dot{Q}_{\text{window}} = (1.46\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)\left[20-(-8)°\text{C}\right] = \mathbf{88.3\text{ W}}$$

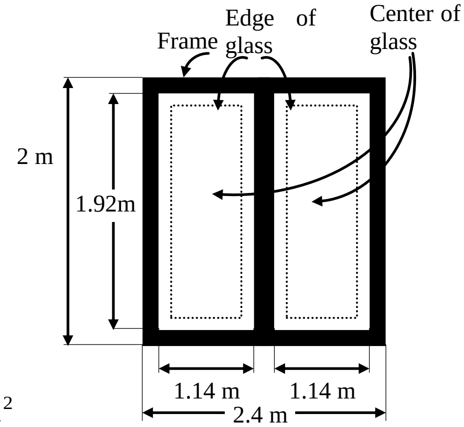
$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{window}}{h_i A_{window}} = 20 - \frac{88.3\text{ W}}{(8.3\text{ W/m}^2.°\text{C})(2.16\text{ m}^2)} = \mathbf{15.1°C}$$

Discussion Note that heat loss through the window will be reduced by 49 percent in the case of double glazing and by 74 percent in the case of triple glazing relative to the single glazing case. Also, in the case of single glazing, the low inner glass surface temperature will cause considerable discomfort in the occupants because of the excessive heat loss from the body by radiation. It is raised from 1.2°C to 10.3°C in the case of double glazing and to 15.1°C in the case of triple glazing.

9-124 The overall U-factor for a double-door type window is to be determined, and the result is to be compared to the value listed in Table 9-6.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional.

Properties The U-factors for the various sections of windows are given in Table 9-6.



Analysis The areas of the window, the glazing, and the frame are

$$A_{\text{window}} = \text{Height} \times \text{Width} = (2\text{ m})(2.4\text{ m}) = 4.80\text{ m}^2$$

$$A_{\text{glazing}} = 2 \times \text{Height} \times \text{Width} = 2(1.92\text{ m})(1.14\text{ m}) = 4.38\text{ m}^2$$

$$A_{\text{frame}} = A_{\text{window}} - A_{\text{glazing}} = 4.80 - 4.38 = 0.42\text{ m}^2$$

The edge-of-glass region consists of a 6.5-cm wide band around the perimeter of the glazings, and the areas of the center and edge sections of the glazing are determined to be

$$A_{\text{center}} = 2(\text{Height} \times \text{Width}) = 2(1.92 - 0.13\text{ m})(1.14 - 0.13\text{ m}) = 3.62\text{ m}^2$$

$$A_{\text{edge}} = A_{\text{glazing}} - A_{\text{center}} = 4.38 - 3.62 = 0.76\text{ m}^2$$

The U-factor for the frame section is determined from Table 9-4 to be U_{frame} = 2.8 W/m^2.°C. The U-factor for the center and edge sections are determined from Table 9-6 to be U_{center} = 2.78 W/m^2.°C and U_{edge} =3.40 W/m^2.°C. Then the overall U-factor of the entire window becomes

$$\begin{aligned} U_{\text{window}} &= (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}} \\ &= (2.78 \times 3.62 + 3.40 \times 0.76 + 2.8 \times 0.42) / 4.80 \\ &= \mathbf{2.88\ W/m^2 \cdot {}^\circ C} \end{aligned}$$

Discussion The overall U-factor listed in Table 9-6 for the specified type of window is 2.86 W/m^2.°C, which is sufficiently close to the value obtained above.

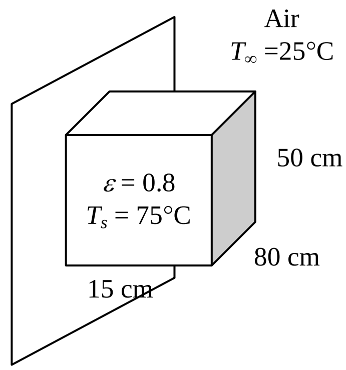
Review Problems

9-125 An electric resistance space heater filled with oil is placed against a wall. The power rating of the heater and the time it will take for the heater to reach steady operation when it is first turned on are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the back, bottom, and top surfaces are disregarded. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (75+25)/2 = 50$°C are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis Heat transfer from the top and bottom surfaces are said to be negligible, and thus the heat transfer area in this case consists of the three exposed side surfaces. The characteristic length is the height of the box, $L_c = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(75-25 \text{ K})(0.5 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2\text{/s})^2}(0.7228) = 4.244\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(4.244\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 94.68$$

$$h = \frac{k}{L}Nu = \frac{0.02735 \text{ W/m.°C}}{0.5 \text{ m}}(94.68) = 5.179 \text{ W/m}^2\text{.°C}$$

$$A_s = (0.5 \text{ m})(0.8 \text{ m}) + 2(0.15 \text{ m})(0.5 \text{ m}) = 0.55 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.179 \text{ W/m}^2\text{.°C})(0.55 \text{ m}^2)(75-25)\text{°C} = 142.4 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.8)(0.55 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(75+273 \text{ K})^4 - (25+273 \text{ K})^4] = 169.1 \text{ W}$$

Then the total rate of heat transfer, thus the power rating of the heater becomes

$$\dot{Q}_{total} = 142.4 + 169.1 = \mathbf{311.5 \text{ W}}$$

The specific heat of the oil at the average temperature of the oil is 2006 J/kg.°C (Table A-13). Then the amount of heat transfer needed to raise the temperature of the oil to the steady operating temperature and the time it takes become

$$Q = mc_p(T_2 - T_1) = (45 \text{ kg})(2006 \text{ J/kg.°C})(75-25)\text{°C} = 4.514\times10^6 \text{ J}$$

$$Q = \dot{Q}\Delta t \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{4.514\times10^6 \text{ kJ}}{311.5 \text{ J/s}} = 14{,}490 \text{ s} = \mathbf{4.03 \text{ h}}$$

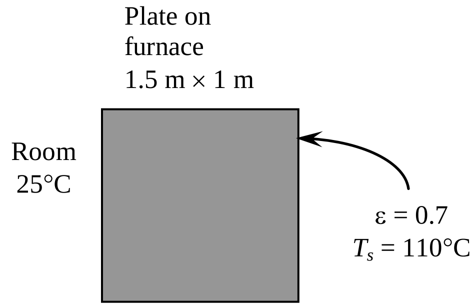
which is not practical. Therefore, the surface temperature of the heater must be allowed to be higher than 75°C.

9-126 A hot part of the vertical front section of a natural gas furnace in a plant is considered. The rate of heat loss from this section and the annual cost of this heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from other surfaces of the tank is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+25)/2 = 67.5°C$ are (Table A-15)

$$k = 0.02863 \text{ W/m.°C}$$
$$\nu = 1.97\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7183$$
$$\beta = \frac{1}{T_f} = \frac{1}{(67.5+273)\text{K}} = 0.002937 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of that section of furnace, $L_c = L = 1.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002937 \text{ K}^{-1})(110-25 \text{ K})(1.5 \text{ m})^3}{(1.97\times10^{-5} \text{ m}^2\text{/s})^2}(0.7183) = 1.530\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.530\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7183}\right)^{9/16}\right]^{8/27}}\right\}^2 = 289.1$$

$$h = \frac{k}{L}Nu = \frac{0.02863 \text{ W/m.°C}}{1.5 \text{ m}}(289.1) = 5.518 \text{ W/m}^2\text{.°C}$$
$$A_s = (1.5 \text{ m})(1 \text{ m}) = 1.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.518 \text{ W/m}^2\text{.°C})(1.5 \text{ m}^2)(110-25)°\text{C} = 703.5 \text{ W}$$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)\\ &= (0.7)(1.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(110+273 \text{ K})^4 - (25+273 \text{ K})^4]\\ &= 811.6 \text{ W}\end{aligned}$$

$$\dot{Q}_{total} = 703.5 + 811.6 = \mathbf{1515 \text{ W}}$$

The amount and cost of natural gas used to overcome this heat loss per year is

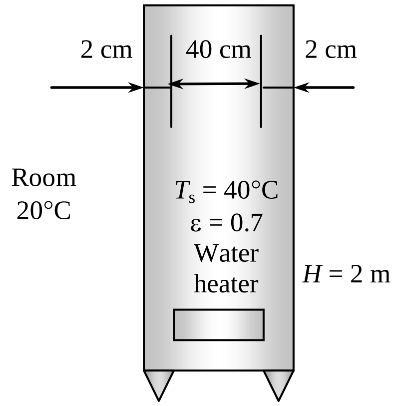
$$Q_{gas} = \dot{Q}_{gas}\Delta t = \frac{\dot{Q}_{total}}{0.79}\Delta t = \frac{(1.515 \text{ kJ/s})}{0.79}(310 \text{ days/yr}\times10 \text{ hr/day}\times3600 \text{ s/hr}) = 2.140\times10^7 \text{ kJ}$$

$$\text{Cost} = (2.140\times10^7 / 105{,}500 \text{ therm})(\$1.20\text{/therm}) = \mathbf{\$243}$$

9-127 An electric hot water heater is located in a small room. A hot water tank insulation kit is available for \$60. The payback period of this insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the top and bottom surfaces of the tank is disregarded. **5** The thermal resistance of the metal sheet is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (40+20)/2 = 30°C are (Table A-15)



$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033\,\text{K}^{-1}$$

Analysis The characteristic length in this case is the height of the heater, $L_c = L$ = 2 m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(40-20\,\text{K})(2\,\text{m})^3}{(1.608\times10^{-5}\text{ m}^2/\text{s})^2}(0.7282) = 1.459\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.459\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 285.4$$

$$h = \frac{k}{L}Nu = \frac{0.02588\text{ W/m.°C}}{2\,\text{m}}(285.4) = 3.693\text{ W/m}^2.\text{°C}$$
$$A_s = \pi DL = \pi(0.44\,\text{m})(2\,\text{m}) = 2.765\,\text{m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (3.693\text{ W/m}^2.\text{°C})(2.765\,\text{m}^2)(40-20)\text{°C} = 204.2\text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.7)(2.765\,\text{m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(40+273\,\text{K})^4 - (20+273\,\text{K})^4] = 244.5\text{ W}$$

and

$$\dot{Q}_{total} = 204.2 + 244.5 = 448.7\text{ W}$$

The reduction in heat loss after adding insulation is

$$\dot{Q} = (0.80)(448.7) = 359.0\text{ W}$$

The amount of heat and money saved per hour is

$$Q_{saved} = \dot{Q}_{saved}\Delta t = (0.3590\,\text{kW})(1\,\text{h}) = 0.3590\,\text{kWh}$$
$$\text{Money saved} = (0.3590\,\text{kWh})(\$0.08/\text{kWh}) = \$0.02872$$

Then it will take

$$\Delta t = \frac{\$60}{\$0.02872} = 2089\,\text{h} = \mathbf{87.0\,days}$$

for the additional insulation to pay for itself from the energy it saves.

9-128 A plate inclined at 30° with the top surface of the plate well insulated. The rate of heat loss from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2$ = 30°C are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\begin{aligned}\mathrm{Ra}_L &= \frac{g\cos\theta\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} \\ &= \frac{(9.81\ \mathrm{m/s^2})(\cos 30°)(0.0033\ \mathrm{K^{-1}})(60-0)\mathrm{K}(0.5\ \mathrm{m})^3}{(1.608\times10^{-5}\ \mathrm{m^2/s})^2}(0.7282) \\ &= 5.922\times10^8\end{aligned}$$

The Nusselt number is calculated using the correlation for vertical plate,

$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.922\times10^8)^{1/6}}{[1+(0.492/0.7282)^{9/16}]^{8/27}}\right\}^2 = 104.9$$

The heat transfer coefficient is

$$h = \mathrm{Nu}\frac{k}{L} = (104.9)\frac{0.02588\ \mathrm{W/m\cdot K}}{0.5\ \mathrm{m}} = 5.43\ \mathrm{W/m^2\cdot K}$$

Hence, the rate of heat loss is

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.43\ \mathrm{W/m^2\cdot K})(0.5\ \mathrm{m})(0.5\ \mathrm{m})(60-0)\ \mathrm{K} = \mathbf{81.5\ W}$$

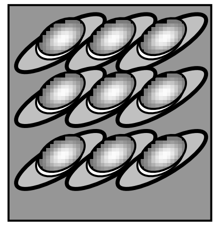
Discussion The inclined plate with well insulated top surface can be treated as an inclined plate with hot surface down.

9-129 A group of 25 transistors are cooled by attaching them to a square aluminum plate and mounting the plate on the wall of a room. The required size of the plate to limit the surface temperature to 50°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The Rayleigh number can be determined in terms of the characteristic length (length of the plate) to be

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(L)^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 1.571\times10^9 L^3$$

The Nusselt number relation is

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.571\times10^9 L^3)^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2$$

The heat transfer coefficient is

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{L}Nu$$
$$A_s = L^2$$

Noting that both the surface and surrounding temperatures are known, the rate of convection and radiation heat transfer are expressed as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = \frac{0.02662 \text{ W/m.°C}}{L}NuL^2(50-30)\text{°C}$$
$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_s{}^4 - T_{sky}{}^4) = (0.9)L^2(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(50+273)^4-(30+273)^4]\text{K}^4 = 125.3L^2$$

The rate of total heat transfer is expressed as

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$
$$25\times(1.5 \text{ W}) = \frac{0.02662 \text{ W/m.°C}}{L}NuL^2(50-30)\text{°C} + 125.3L^2$$

Substituting Nusselt number expression above into this equation and solving for L, the length of the plate is determined to be

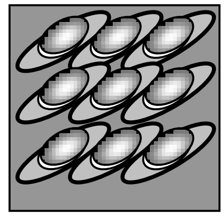
L = **0.426 m**

9-130 A group of 25 transistors are cooled by attaching them to a square aluminum plate and positioning the plate horizontally in a room. The required size of the plate to limit the surface temperature to 50°C is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The characteristic length and the Rayleigh number for the horizontal case are determined to be

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4}$$

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(L/4)^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 2.454\times10^7 L^3$$

Noting that both the surface and surrounding temperatures are known, the rate of radiation heat transfer is determined to be

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma(T_s{}^4 - T_{sky}{}^4) = (0.9)L^2(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(50+273)^4 - (30+273)^4]\text{K}^4 = 125.3L^2$$

(*a*) **Hot surface facing up:** We assume Ra < 10^7 and thus L <0.74 m so that we can determine the Nu number from Eq. 9-22. Then the Nusselt number and the convection heat transfer coefficient become

$$Nu = 0.54Ra^{1/4} = 0.54(2.454\times10^7 L^3)^{1/4} = 38.0L^{3/4}$$

Then,

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{L/4}(38.0L^{3/4}) = 4.047L^{-1/4} \text{ W/m}^2.\text{°C}$$
$$A_s = L^2$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (4.047L^{-1/4})L^2(50-30) = 80.94L^{7/4} \quad \text{W}$$

Then,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \longrightarrow 25\times(1.5 \text{ W}) = 80.94L^{7/4} + 125.3L^2 \text{ W}$$

Solving for L, the length of the plate is determined to be

L = **0.407 m**

Note that L < 0.75 m, and therefore the assumption of Ra < 10^7 is verified. That is,

(*b*) **Hot surface facing down:** The Nusselt number in this case is determined from

$$Nu = 0.27Ra^{1/4} = 0.27(2.454\times10^7 L^3)^{1/4} = 19.0L^{3/4}$$

Then, $$h = \frac{k}{L_c}Nu = \frac{0.02662 \text{ W/m.°C}}{L/4}(19.0L^{3/4}) = 2.023L^{-1/4}$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (2.023L^{-1/4})L^2(50-30) = 40.47L^{7/4} \quad \text{W}$$

Then, $$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \longrightarrow 25\times(1.5 \text{ W}) = 40.47L^{7/4} + 125.3L^2 \text{ W}$$

Solving for L, the length of the plate is determined to be

L = **0.464 m**

9-131 An $L \times L$ horizontal plate is placed in a quiescent air with the hot surface facing up, and the expressions, having the form $\text{Nu} = CRa_L^n$, for the average heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties.

Properties The properties of air at T_f = 20°C are k = 0.02514 W/m·K, ν = 1.516 × 10^{-5} m^2/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f$ = 0.003413 K^{-1}.

Analysis For horizontal plate, the characteristic length is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4}$$

The Rayleigh number ($L_c = L/4$) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003413\,\text{K}^{-1})\Delta T(L/4)^3}{(1.516\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7309)$$

$$= 1.664\times10^6\,\Delta TL^3$$

For $10^4 < \text{Ra}_L < 10^7$, we have

$$\text{Nu} = \frac{hL_c}{k} = \frac{hL}{4k} = 0.54\text{Ra}_L^{1/4} \qquad \rightarrow \qquad h = 2.16\frac{k}{L}\text{Ra}_L^{1/4}$$

Substituting the Ra_L yields

$$h = 2.16\left(\frac{0.02514}{L}\right)(1.664\times10^6\,\Delta TL^3)^{1/4} = 1.95(\Delta T/L)^{1/4} \qquad 10^4 < \text{Ra}_L < 10^7$$

For $10^7 < \text{Ra}_L < 10^{11}$, we have

$$\text{Nu} = \frac{hL_c}{k} = \frac{hL}{4k} = 0.15\text{Ra}_L^{1/3} \qquad \rightarrow \qquad h = 0.6\frac{k}{L}\text{Ra}_L^{1/3}$$

Substituting the Ra_L yields

$$h = 0.6\left(\frac{0.02514}{L}\right)(1.664\times10^6\,\Delta TL^3)^{1/3} = 1.79\Delta T^{1/3} \qquad 10^7 < \text{Ra}_L < 10^{11}$$

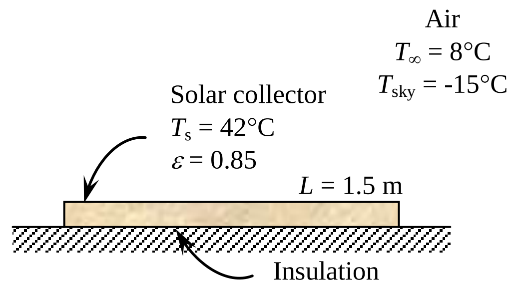
Discussion The average heat transfer coefficient for $10^4 < \text{Ra}_L < 10^7$ is dependent on ΔT and L. For $10^{10} < \text{Ra}_L < 10^{13}$, the average heat transfer coefficient is not influenced by L.

9-132 A flat-plate solar collector placed horizontally on the flat roof of a house is exposed to the calm ambient air. The rate of heat loss from the collector by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (42+8)/2 = 25°C$ are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7296$$
$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356 \text{ K}^{-1}$$



Analysis The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1.5\text{ m})(4.5\text{ m})}{2(1.5\text{ m} + 4.5\text{ m})} = 0.5625\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003356\text{ K}^{-1})(42-8\text{ K})(0.5625\text{ m})^3}{(1.562\times10^{-5}\text{ m}^2\text{/s})^2}(0.7296) = 5.957\times10^8$$

$$Nu = 0.15Ra^{1/3} = 0.15(5.957\times10^8)^{1/3} = 126.2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02551\text{ W/m.°C}}{0.5625\text{ m}}(126.2) = 5.724\text{ W/m}^2\text{.°C}$$

$$A_s = (1.5\text{ m})(4.5\text{ m}) = 6.75\text{ m}^2$$

and

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (5.724\text{ W/m}^2\text{.°C})(6.75\text{ m}^2)(42-8)\text{°C} = \mathbf{1314\text{ W}}$$

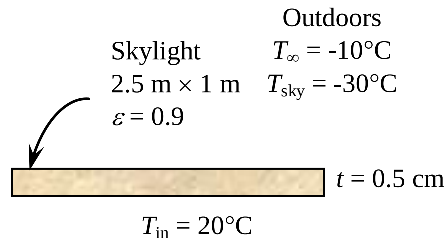
Heat transfer rate by radiation is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma({T_{surr}}^4 - {T_s}^4)\\ &= (0.85)(6.75\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(42+273\text{ K})^4 - (-15+273\text{ K})^4]\\ &= \mathbf{1762\text{ W}}\end{aligned}$$

9-133 A horizontal skylight made of a single layer of glass on the roof of a house is considered. The rate of heat loss through the skylight is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (-4-10)/2 = –7°C are (Table A-15)



$$k = 0.02311\ \text{W/m.°C}$$
$$\nu = 1.278\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.738$$
$$\beta = \frac{1}{T_f} = \frac{1}{(-7+273)\text{K}} = 0.003759\ \text{K}^{-1}$$

Analysis We assume radiation heat transfer inside the house to be negligible. We start the calculations by "guessing" the glass temperature to be -4°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1\,\text{m})(2.5\,\text{m})}{2(1\,\text{m}+2.5\,\text{m})} = 0.357\ \text{m}\,.$$

Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003759\,\text{K}^{-1})[-4-(-10)\,\text{K}](0.357\,\text{m})^3}{(1.278\times10^{-5}\ \text{m}^2/\text{s})^2}(0.738) = 4.553\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(4.553\times10^7)^{1/3} = 53.56$$

$$h_o = \frac{k}{L_c}Nu = \frac{0.02311\ \text{W/m.°C}}{0.357\ \text{m}}(53.56) = 3.467\ \text{W/m}^2.\text{°C}$$

$$A_s = (1\,\text{m})(2.5\,\text{m}) = 2.5\,\text{m}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_s+T_{sky})(T_s{}^2+T_{sky}{}^2) \\ &= 0.9(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(-4+273)+(-30+273)][(-4+273)^2+(-30+273)^2]\text{K}^3 \\ &= 3.433\ \text{W/m}^2.\text{K} \end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 3.467+3.433 = 6.90\ \text{W/m}^2$$

Again we take the glass temperature to be -4°C for the evaluation of the properties and h for the inner surface of the skylight. The properties of air at 1 atm and the film temperature of T_f = (-4+20)/2 = 8°C are (Table A-15)

$$k = 0.02424\ \text{W/m.°C}$$
$$\nu = 1.408\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7342$$
$$\beta = \frac{1}{T_f} = \frac{1}{(8+273)\text{K}} = 0.003559\ \text{K}^{-1}$$

The characteristic length in this case is also 0.357 m. Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003559\,\text{K}^{-1})[20-(-4)\,\text{K}](0.357\,\text{m})^3}{(1.408\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7342) = 1.412\times10^8$$

$$Nu = 0.27Ra^{1/4} = 0.27(1.412\times10^8)^{1/4} = 29.43$$

$$h_i = \frac{k}{L_c}Nu = \frac{0.02424\ \text{W/m.°C}}{0.357\ \text{m}}(29.43) = 1.998\ \text{W/m}^2.\text{°C}$$

Using the thermal resistance network, the rate of heat loss through the skylight is determined to be

$$\dot{Q}_{skylight} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond,glas} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + \dfrac{t_{\text{glass}}}{k_{glass}} + \dfrac{1}{h}} = \frac{(2.5\,\text{m}^2)[20 - (-10)]°\text{C}}{\dfrac{1}{1.998\ \text{W/m}^2.°\text{C}} + \dfrac{0.005\,\text{m}}{0.78\ \text{W/m}.°\text{C}} + \dfrac{1}{6.90\ \text{W/m}^2.°\text{C}}} = 115\,\text{W}$$

Using the same heat transfer coefficients for simplicity, the rate of heat loss through the roof in the case of R-5.34 construction is determined to be

$$\dot{Q}_{roof} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + R_{glass} + \dfrac{1}{h}} = \frac{(2.5\,\text{m}^2)[20 - (-10)]°\text{C}}{\dfrac{1}{1.998\ \text{W/m}^2.°\text{C}} + 5.34\,\text{m}^2.°\text{C/W} + \dfrac{1}{6.90\ \text{W/m}^2.°\text{C}}} = 12.5\,\text{W}$$

Therefore, a house loses 115/12.5 ≅ 9 times more heat through the skylights than it does through an insulated wall of the same size.

Using Newton's law of cooling, the glass temperature corresponding to a heat transfer rate of 115 W is calculated to be –3.3°C, which is sufficiently close to the assumed value of -4°C. Therefore, there is no need to repeat the calculations.

Discussion The assumed film temperature of T_f = -7°C is an appropriate assumption, since the determined T_s = -3.3°C would give a film temperature of T_f = -6.65°C. Otherwise, T_s would have to be solved iteratively.

9-134 An electronic box is cooled internally by a fan blowing air into the enclosure. The fraction of the heat lost from the outer surfaces of the electronic box is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base surface is disregarded. **4** The pressure of air inside the enclosure is 1 atm.

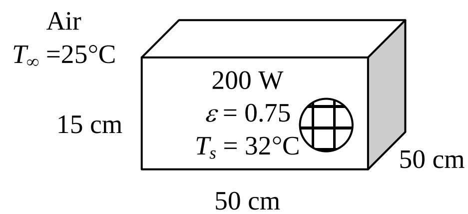
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (32+15)/2 = 28.5°C$ are (Table A-15)

$$k = 0.02577 \text{ W/m.°C}$$

$$\nu = 1.594\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7286$$

$$\beta = \frac{1}{T_f} = \frac{1}{(28.5+273)\text{K}} = 0.003317 \text{ K}^{-1}$$



Analysis <u>Heat loss from the horizontal top surface:</u>

The characteristic length in this case is

$$L_c = \frac{A}{P} = \frac{(0.5\text{ m})^2}{2[(0.5\text{ m})+(0.5\text{ m})]} = 0.125\text{ m}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003317\text{ K}^{-1})(32-25\text{ K})(0.125\text{ m})^3}{(1.594\times10^{-5}\text{ m}^2\text{/s})^2}(0.7286) = 1.275\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.275\times10^6)^{1/4} = 18.15$$

$$h = \frac{k}{L_c}Nu = \frac{0.02577\text{ W/m.°C}}{0.125\text{ m}}(18.15) = 3.741\text{ W/m}^2\text{.°C}$$

$$A_{top} = (0.5\text{ m})^2 = 0.25\text{ m}^2$$

and $\dot{Q}_{top} = hA_{top}(T_s - T_\infty) = (3.741\text{ W/m}^2\text{.°C})(0.25\text{ m}^2)(32-25)\text{°C} = 6.55\text{ W}$

<u>Heat loss from vertical side surfaces:</u>

The characteristic length in this case is the height of the box $L_c = L$ =0.15 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003317\text{ K}^{-1})(32-25\text{ K})(0.15\text{ m})^3}{(1.594\times10^{-5}\text{ m}^2\text{/s})^2}(0.7286) = 2.204\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.204\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7286}\right)^{9/16}\right]^{8/27}}\right\}^2 = 20.55$$

$$h = \frac{k}{L}Nu = \frac{0.02577\text{ W/m.°C}}{0.15\text{ m}}(20.55) = 3.530\text{ W/m}^2\text{.°C}$$

$$A_{side} = 4(0.15\text{ m})(0.5\text{ m}) = 0.3\text{ m}^2$$

and $\dot{Q}_{side} = hA_{side}(T_s - T_\infty) = (3.530\text{ W/m}^2\text{.°C})(0.3\text{ m}^2)(32-25)\text{°C} = 7.41\text{ W}$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma({T_s}^4 - {T_{surr}}^4)\\ &= (0.75)(0.25+0.3\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(32+273\text{ K})^4 - (25+273\text{ K})^4] = 17.95\text{ W}\end{aligned}$$

Then the fraction of the heat loss from the outer surfaces of the box is determined to be

$$f = \frac{(6.55+7.41+17.95)\text{W}}{200\text{ W}} = 0.160 = \mathbf{16.0\%}$$

9-135E The components of an electronic device located in a horizontal duct of rectangular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

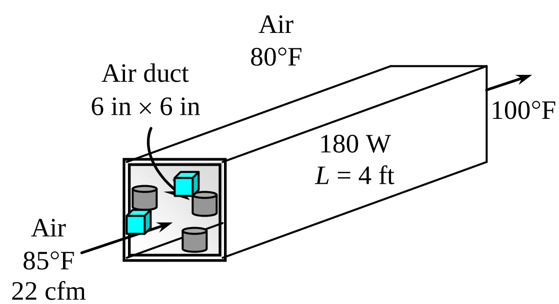
Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (120+80)/2 = 100°F$ are (Table A-15E)

$$k = 0.01529 \text{ Btu/h.ft.°F}$$

$$\nu = 0.1809\times10^{-3} \text{ ft}^2\text{/s}$$

$$\text{Pr} = 0.726$$

$$\beta = 1/T_f = 1/(100+460)\text{R} = 0.001786 \text{ R}^{-1}$$



Analysis (*a*) Using air density at the inlet temperature of 85°F and the specific heat at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{V} = (0.07284 \text{ lbm/ft}^3)(22 \text{ ft}^3\text{/min}) = 1.602 \text{ lbm/min}$$

$$\dot{Q}_{forced} = \dot{m}c_p(T_{out} - T_{in}) = (1.602\times60 \text{ lbm/h})(0.2404 \text{ Btu/lbm.°F})(100-85)\text{°F} = 346.6 \text{ Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (180\times3.412) - 346.6 = \mathbf{268 \text{ Btu/h}}$$

(*b*) We start the calculations by "guessing" the surface temperature to be 120°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary.

Horizontal top surface: The characteristic length is $L_c = \dfrac{A_s}{P} = \dfrac{(4 \text{ ft})(6/12 \text{ ft})}{2(4 \text{ ft} + 6/12 \text{ ft})} = 0.2222 \text{ ft}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001786 \text{ R}^{-1})(120-80 \text{ R})(0.2222 \text{ ft})^3}{(0.1809\times10^{-3} \text{ ft}^2\text{/s})^2}(0.726) = 5.599\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(5.599\times10^5)^{1/4} = 14.77$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01529 \text{ Btu/h.ft.°F}}{0.2222 \text{ ft}}(14.77) = 1.016 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_{top} = (4 \text{ ft})(6/12 \text{ ft}) = 2 \text{ ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(5.599\times10^5)^{1/4} = 7.386$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01529 \text{ Btu/h.ft.°F}}{0.2222 \text{ ft}}(7.386) = 0.5082 \text{ Btu/h.ft}^2\text{.°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L = 6$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001786 \text{ R}^{-1})(120-80 \text{ R})(0.5 \text{ ft})^3}{(0.1809\times10^{-3} \text{ ft}^2\text{/s})^2}(0.726) = 6.379\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(6.379\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.726}\right)^{9/16}\right]^{8/27}}\right\}^2 = 27.57$$

$$h_{side} = \frac{k}{L} Nu = \frac{0.01529 \text{ Btu/h.ft.°F}}{0.5 \text{ ft}}(27.57) = 0.843 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_{side} = 2(4 \text{ ft})(0.5 \text{ ft}) = 4 \text{ ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$268 \text{ Btu/h} = [(1.016 \times 2 + 0.5082 \times 2 + 0.843 \times 4) \text{ Btu/h.°F}](T_s - 80)\text{°F}$$

T_s = **121.7°F**

which is sufficiently close to the assumed value of 120°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

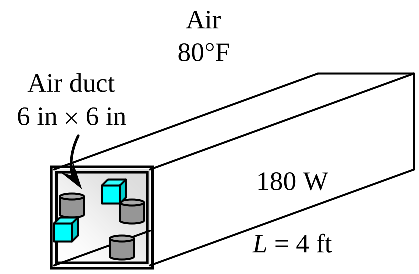
Discussion The assumed film temperature of T_f = 100°C is an appropriate assumption, since the determined T_s = 121.7°C would give a film temperature of T_f = 100.1°C. Otherwise, T_s would have to be solved iteratively.

9-136E The components of an electronic system located in a horizontal duct of rectangular cross section is cooled by natural convection. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (160+80)/2 = 120°F are (Table A-15E)

$$k = 0.01576 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1923\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.723$$
$$\beta = 1/T_f = 1/(120+460 \text{ R}) = 0.001724 \text{ R}^{-1}$$



Analysis (a) Noting that radiation heat transfer is negligible and no heat is removed by forced convection because of the failure of the fan, the entire 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} = \mathbf{180\ W}$$

(b) We start the calculations by "guessing" the surface temperature to be 160°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary.

Horizontal top surface: The characteristic length is $L_c = \dfrac{A_s}{p} = \dfrac{(4\text{ ft})(6/12\text{ ft})}{2(4\text{ ft}+6/12\text{ ft})} = 0.2222\text{ ft}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001724\text{ R}^{-1})(160-80\text{ R})(0.2222\text{ ft})^3}{(0.1923\times10^{-3}\text{ ft}^2\text{/s})^2}(0.723) = 9.534\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(9.534\times10^5)^{1/4} = 16.87$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01576\text{ Btu/h.ft.°F}}{0.2222\text{ ft}}(16.87) = 1.197\text{ Btu/h.ft}^2\text{.°F}$$

$$A_{top} = (4\text{ ft})(6/12\text{ ft}) = 2\text{ ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(9.534\times10^5)^{1/4} = 8.437$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01576\text{ Btu/h.ft.°F}}{0.2222\text{ ft}}(8.437) = 0.5983\text{ Btu/h.ft}^2\text{.°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L$ = 6 in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001724\text{ R}^{-1})(160-80\text{ R})(0.5\text{ ft})^3}{(0.1923\times10^{-3}\text{ ft}^2\text{/s})^2}(0.723) = 1.086\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.086\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.723}\right)^{9/16}\right]^{8/27}}\right\}^2 = 32.03$$

$$h_{side} = \frac{k}{L}Nu = \frac{0.01576\text{ Btu/h.ft.°F}}{0.5\text{ ft}}(32.03) = 1.009\text{ Btu/h.ft}^2\text{.°F}$$

$$A_{side} = 2(4\text{ ft})(0.5\text{ ft}) = 4\text{ ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$180\text{ W}\left(\frac{3.41214\text{ Btu/h}}{1\text{ W}}\right)=[(1.197\times 2+0.5983\times 2+1.009\times 4)\text{ Btu/h.°F}](T_s-80)\text{°F}$$

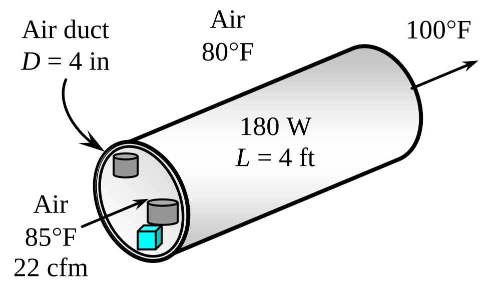
T_s = **161°F**

which is sufficiently close to the assumed value of 160°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

Discussion The assumed film temperature of T_f = 120°F is an appropriate assumption, since the determined T_s = 161°F would give a film temperature of T_f = 120.5°F. Otherwise, T_s would have to be solved iteratively.

9-137E The components of an electronic system located in a horizontal duct of circular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.



Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (150+80)/2 = 115°\text{F}$ are (Table A-15E)

$$k = 0.01564\ \text{Btu/h.ft.°F}$$

$$\nu = 0.1895\times10^{-3}\ \text{ft}^2/\text{s}$$

$$\text{Pr} = 0.7238$$

$$\beta = \frac{1}{T_f} = \frac{1}{(115+460)\ \text{R}} = 0.001739\ \text{R}^{-1}$$

Analysis (*a*) Using air density at the inlet temperature of 85°F and the specific heat at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{\boldsymbol{V}} = (0.07284\ \text{lbm/ft}^3)(22\ \text{ft}^3/\text{min}) = 1.602\ \text{lbm/min}$$

$$\dot{Q}_{forced} = \dot{m}c_p(T_{out}-T_{in}) = (1.602\times60\ \text{lbm/h})(0.2404\ \text{Btu/lbm.°F})(100-85)\text{°F} = 346.6\ \text{Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (180\times3.412) - 346.6 = \mathbf{268\ Btu/h}$$

(b) We start the calculations by "guessing" the surface temperature to be 150°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the duct, $L_c = D = 4$ in. Then,

$$Ra = \frac{g\beta(T_1-T_2)D^3}{\nu^2}\text{Pr} = \frac{(32.2\ \text{ft/s}^2)(0.001739\ \text{R}^{-1})(150-80\ \text{R})(4/12\ \text{ft})^3}{(0.1895\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.7238) = 2.926\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.926\times10^6)^{1/6}}{\left[1+(0.559/0.7238)^{9/16}\right]^{8/27}}\right\}^2 = 19.79$$

$$h = \frac{k}{D}Nu = \frac{0.01564\ \text{Btu/h.ft.°F}}{4/12\ \text{ft}}(19.79) = 0.9285\ \text{Btu/h.ft}^2.\text{°F}$$

$$A_s = \pi DL = \pi(4/12\ \text{ft})(4\ \text{ft}) = 4.19\ \text{ft}^2$$

Then the surface temperature is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) \ \rightarrow\ T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 80\text{°F} + \frac{268\ \text{Btu/h}}{(0.9285\ \text{Btu/h.ft}^2.\text{°F})(4.19\ \text{ft}^2)} = \mathbf{149°F}$$

which is very close to the assumed value of 150°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

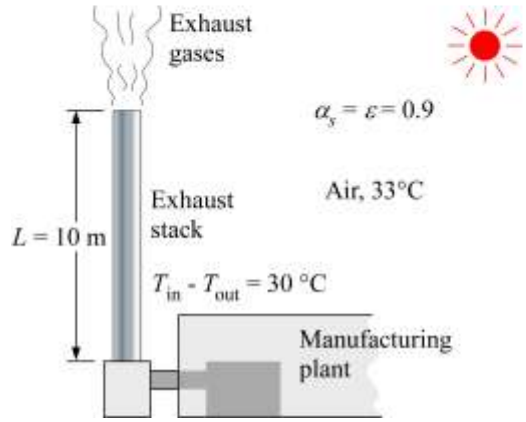
Discussion The assumed film temperature of T_f = 115°F is an appropriate assumption, since the determined T_s = 149°F would give a film temperature of T_f = 114.5°F. Otherwise, T_s would have to be solved iteratively.

9-138 A 10-m tall exhaust stack discharging exhaust gases at a rate of 0.125 kg/s is subjected to solar radiation and natural convection at the outer surface. The outer surface temperature of the exhaust stack is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** Air is an ideal gas.

Properties The properties of air at 60°C are k = 0.02808 W/m·K, $\nu = 1.896 \times 10^{-5}$ m^2/s, Pr = 0.7202 (from Table A-15). Also, $\beta = 1/T_f = 0.003003$ K^{-1}.

Analysis Assume that the exhaust stack can be treated as a vertical plate, the Rayleigh number is



$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$
$$= \frac{(9.81\,\mathrm{m/s^2})(0.003003\,\mathrm{K^{-1}})(T_s - 306)\mathrm{K}(10\,\mathrm{m})^3}{(1.896\times10^{-5}\,\mathrm{m^2/s})^2}(0.7202) \qquad (1)$$

Assuming the Rayleigh number is within $10^{10} < \mathrm{Ra}_L < 10^{13}$, the Nusselt number for vertical plate is

$$\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3} \qquad \text{or} \qquad h = \left(\frac{0.02808\ \mathrm{W/m\cdot K}}{10\,\mathrm{m}}\right)0.1\mathrm{Ra}_L^{1/3} \qquad (2)$$

The outer surface area of the exhaust stack is

$$A_s = \pi DL = \pi(1\,\mathrm{m})(10\,\mathrm{m}) = 31.42\,\mathrm{m^2}$$

The rate of heat loss from the exhaust gases in the exhaust stack can be determined from

$$\dot{Q}_{\mathrm{loss}} = \dot{m}c_p(T_{\mathrm{in}} - T_{\mathrm{out}}) = (0.125\,\mathrm{kg/s})(1600\,\mathrm{J/kg\cdot °C})(30)\,°\mathrm{C} = 6000\,\mathrm{W}$$

The heat loss on the outer surface of the exhaust stack by radiation and convection can be expressed as

$$\frac{\dot{Q}_{\mathrm{loss}}}{A_s} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\mathrm{surr}}^4] - \alpha_s\dot{q}_{\mathrm{solar}} \qquad (3)$$

Equations (1), (2), and (3) can be solved simultaneously to get the surface temperature. Copy the following lines and paste on a blank EES screen to solve the above equation:

```
A_s=31.42
D=1
g=9.81
k=0.02808
L=10
Pr=0.7202
q_incindent=500
Q_loss=6000
T_inf=33+273
T_surr=33+273
alpha=0.9
beta=1/(273+60)
epsilon=0.9
nu=1.896e-5
sigma=5.670e-8
Ra_L=g*beta*(T_s-T_inf)*L^3/nu^2*Pr
(h*L/(0.1*k))^3=Ra_L
Q_loss/A_s=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)-alpha*q_incindent
```

Solving by EES software, we get

$$\mathrm{Ra}_L = 3.219\times10^{12}, \qquad h = 4.146\ \mathrm{W/m^2\cdot K}, \qquad \text{and} \qquad T_s = 360.5\,\mathrm{K}$$

Therefore, the exhaust stack outer surface temperature is

$$T_s = \mathbf{87.5°C}$$

Now, we need to check if the assumption that the exhaust stack can be treated as a vertical plate is valid:

$$\frac{35L}{Gr_L^{1/4}} = \frac{35L}{(\mathrm{Ra}_L/\mathrm{Pr})^{1/4}} = \frac{35(10\,\mathrm{m})}{(3.219\times10^{12}/0.7202)^{1/4}} = 0.2407 < D$$

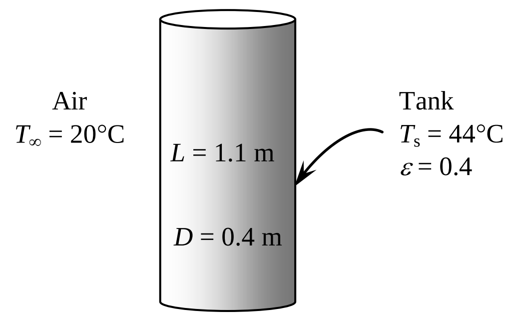
Since $D \ge 35L/Gr_L^{1/4}$ is satisfied, we can treat the exhaust stack as a vertical plate.

Discussion The assumption that the Rayleigh number is within $10^{10} < \mathrm{Ra}_L < 10^{13}$ turned out to be appropriate. Also, the assumed film temperature of 60 °C is valid, since $(T_s + T_\infty)/2$ = 60.3°C.

9–139 A vertically oriented cylindrical hot water tank is located in a bathroom. The rate of heat loss from the tank by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the tank is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (44+20)/2 = 32$°C are (Table A-15)



$$k = 0.02603 \text{ W/m}\cdot K$$
$$\nu = 1.627\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7276$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32+273)\text{K}} = 0.003279 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the cylinder, $L_c = L = 1.1$ m. Then,

$$\text{Gr} = \frac{g\beta(T_s-T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(1.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2\text{/s})^2} = 3.883\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.4 \text{ m}) \ge \frac{35L}{\text{Gr}^{1/4}} = \frac{35(1.1 \text{ m})}{(3.883\times10^9)^{1/4}} = 0.1542 \text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for the side surfaces. For the top and bottom surfaces we use the relevant Nusselt number relations. First, for the side surfaces,

$$\text{Ra} = \text{GrPr} = (3.883\times10^9)(0.7276) = 2.825\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.825\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7276}\right)^{9/16}\right]^{8/27}}\right\}^2 = 170.2$$

$$h = \frac{k}{L}Nu = \frac{0.02603 \text{ W/m}\cdot K}{1.1 \text{ m}}(170.2) = 4.027 \text{ W/m}^2\cdot K$$

$$A_s = \pi DL = \pi(0.4 \text{ m})(1.1 \text{ m}) = 1.382 \text{ m}^2$$

$$\dot{Q}_{\text{side}} = hA_s(T_s - T_\infty) = (4.027 \text{ W/m}^2\cdot K)(1.382 \text{ m}^2)(44-20)°\text{C} = 133.6 \text{ W}$$

For the top surface,

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4 \text{ m}}{4} = 0.1 \text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(0.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2\text{/s})^2}(0.7276) = 2.123\times10^6$$

$$\text{Nu} = 0.54\text{Ra}^{1/4} = 0.54(2.123\times10^6)^{1/4} = 20.61$$

$$h = \frac{k}{L_c}Nu = \frac{0.02603 \text{ W/m}\cdot K}{0.1 \text{ m}}(20.61) = 5.365 \text{ W/m}^2\cdot K$$

$$A_s = \pi D^2/4 = \pi(0.4 \text{ m})^2/4 = 0.1257 \text{ m}^2$$

$$\dot{Q}_{\text{top}} = hA_s(T_s - T_\infty) = (5.365 \text{ W/m}^2\cdot K)(0.1257 \text{ m}^2)(44-20)°\text{C} = 16.2 \text{ W}$$

For the bottom surface,

$$\text{Nu} = 0.27\text{Ra}^{1/4} = 0.27(2.123\times10^6)^{1/4} = 10.31$$

$$h = \frac{k}{L_c} Nu = \frac{0.02603 \text{ W/m} \cdot K}{0.1 \text{ m}}(10.31) = 2.683 \text{ W/m}^2 \cdot K$$

$$\dot{Q}_{\text{bottom}} = hA_s(T_s - T_\infty) = (2.683 \text{ W/m}^2 \cdot K)(0.1257 \text{ m}^2)(44 - 20)°\text{C} = 8.1 \text{ W}$$

The total heat loss by natural convection is

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{side}} + \dot{Q}_{\text{top}} + \dot{Q}_{\text{bottom}} = 133.6 + 16.2 + 8.1 = \mathbf{157.9 \ W}$$

The radiation heat loss from the tank is

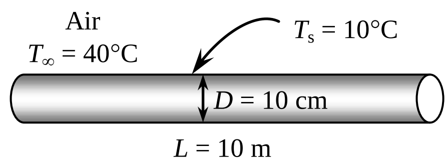
$$\begin{aligned}
\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4) \\
&= (0.4)(1.382 + 0.1257 + 0.1257 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)\left[(44 + 273 \text{ K})^4 - (20 + 273 \text{ K})^4\right] \\
&= \mathbf{101.1 \ W}
\end{aligned}$$

9-140 A cold cylinder is placed horizontally in hot air. The rates of heat transfer from the stack with and without wind cases are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+10)/2 = 25°C$ are (Table A-15)

$$k = 0.02551\ \text{W/m.°C}$$
$$\nu = 1.562\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7296$$
$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356\ \text{K}^{-1}$$



Analysis (*a*) When the stack is exposed to 10 m/s winds, the heat transfer will be by forced convection. We have flow of air over a cylinder and the heat transfer rate is determined as follows:

$$\text{Re} = \frac{VD}{\nu} = \frac{(10\ \text{m/s})(0.10\ \text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 64{,}020$$

$$\text{Nu} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(64{,}020)^{0.805}(0.7296)^{1/3} = 179.8 \qquad \text{(from Table 7-1)}$$

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551\ \text{W/m.°C}}{0.10\ \text{m}}(179.8) = 45.87\ \text{W/m}^2.°\text{C}$$

$$\dot{Q}_{\text{forced conv}} = hA(T_\infty - T_s) = (45.87\ \text{W/m}^2.°\text{C})(\pi\times0.10\times10\ \text{m}^2)(40-10)°\text{C} = \mathbf{4323\ W}$$

(b) Without wind the heat transfer will be by natural convection. The characteristic length in this case is the outer diameter of the cylinder, $L_c = D = 0.10$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003356\ \text{K}^{-1})(40-10\ \text{K})(0.10\ \text{m})^3}{(1.562\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7296) = 2.953\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.953\times10^6)^{1/6}}{\left[1+\left(0.559/0.7296\right)^{9/16}\right]^{8/27}}\right\}^2 = 19.86$$

$$h = \frac{k}{D}Nu = \frac{0.02551\ \text{W/m.°C}}{0.10\ \text{m}}(19.86) = 5.066\ \text{W/m}^2.°\text{C}$$

$$\dot{Q}_{\text{nat.conv}} = hA(T_\infty - T_s) = (5.066\ \text{W/m}^2.°\text{C})(\pi\times0.10\times10\ \text{m}^2)(40-10)°\text{C} = \mathbf{477\ W}$$

9-141E A hot water pipe passes through a basement. The temperature drop of water in the basement due to heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

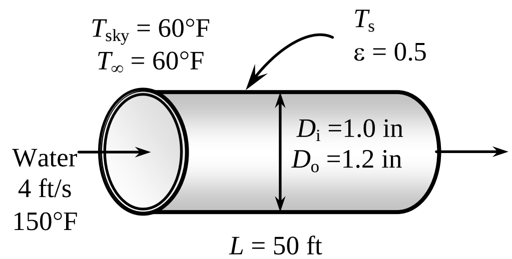
Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (150+60)/2 =$ 105°F are (Table A-15E)

$$k = 0.01541\ \text{Btu/h.ft.°F}$$

$$\nu = 0.1838\times10^{-3}\ \text{ft}^2\text{/s}$$

$$\text{Pr} = 0.7253$$

$$\beta = \frac{1}{T_f} = \frac{1}{(105+460)\text{R}} = 0.00177\ \text{R}^{-1}$$



Analysis We expect the pipe temperature to be very close to the water temperature, and start the calculations by "guessing" the average outer surface temperature of the pipe to be 150°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the pipe, $L_c = D_o$ = 1.2 in. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\nu^2}\text{Pr} = \frac{(32.2\ \text{ft/s}^2)(0.00177\ \text{R}^{-1})(150-60\ \text{R})(1.2/12\ \text{ft})^3}{(0.1838\times10^{-3}\ \text{ft}^2\text{/s})^2}(0.7253) = 1.101\times10^5$$

The natural convection Nusselt number can be determined from

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.101\times10^5)^{1/6}}{\left[1+\left(0.559/0.7253\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.998$$

$$h_o = \frac{k}{D_o}Nu = \frac{0.01541\ \text{Btu/h.ft.°F}}{(1.2/12)\ \text{ft}}(7.998) = 1.232\ \text{Btu/h.ft}^2\text{.°F}$$

$$A_i = \pi D_i L = \pi(1/12\ \text{ft})(50\ \text{ft}) = 13.09\ \text{ft}^2$$

$$A_o = \pi D_o L = \pi(1.2/12\ \text{ft})(50\ \text{ft}) = 15.708\ \text{ft}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_s + T_{surr})(T_s^2 + T_{surr}^2) \\ &= (0.5)(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\text{.R}^4)[(150+460)+(60+460)][(150+460)^2+(60+460)^2]R^3 \\ &= 0.6222\ \text{Btu/h.ft}^2\text{.R} \end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 1.232 + 0.6222 = 1.854\ \text{Btu/h.ft}^2\text{.R}$$

and
$$\dot{Q} = \frac{T_{water} - T_\infty}{\dfrac{1}{h_i A_i} + \dfrac{\ln(D_o/D_i)}{4\pi kL} + \dfrac{1}{h_o A_o}} = \frac{150-60}{\dfrac{1}{(30)(13.09)} + \dfrac{\ln(1.2/1)}{4\pi(30)(50)} + \dfrac{1}{(1.854)(15.708)}} = 2440\ \text{Btu/h}$$

The mass flow rate of water

$$\dot{m} = \rho A_c V = (61.2\ \text{lbm/ft}^3)\left[\pi(1/12\ \text{ft})^2/4\right](4\ \text{ft/s}) = 1.335\ \text{lbm/s} = 4807\ \text{lbm/h}$$

Then the temperature drop of water as it flows through the pipe becomes

$$\dot{Q} = \dot{m}c_p\Delta T \ \rightarrow\ \Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{2440\ \text{Btu/h}}{(4807\ \text{lbm/h})(1.0\ \text{Btu/lbm.°F})} = \mathbf{0.51°F}$$

Discussion The outer surface temperature of the pipe can be determined from

$$\dot{Q} = h_o A_o(T_{s,o} - T_\infty) \longrightarrow 2440\ \text{Btu/h} = (1.854\ \text{Btu/h.ft}^2\text{.R})(15.708\ \text{ft}^2)(T_{s,o} - 60) \longrightarrow T_{s,o} = 143.8°\text{F}$$

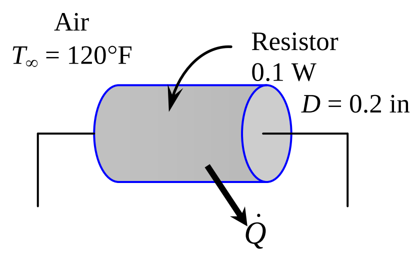
which is sufficiently close to the assumed value of 150°C. Therefore, there is no need to repeat the calculations.

Discussion The assumed film temperature of T_f = 105°F is an appropriate assumption, since the determined T_s = 143.8°F would give a film temperature of T_f = 101.9°F. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-142E A small cylindrical resistor mounted on the lower part of a vertical circuit board. The approximate surface temperature of the resistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** Heat transfer through the connecting wires is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (220+120)/2 =$ 170°F are (Table A-15E)



$$k = 0.01692 \text{ Btu/h.ft.°F}$$
$$\nu = 0.222\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7161$$
$$\beta = \frac{1}{T_f} = \frac{1}{(170+460)\text{R}} = 0.001587 \text{ R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 220°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the diameter of resistor, $L_c = D = 0.2$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001587 \text{ R}^{-1})(220-120 \text{ R})(0.2/12 \text{ ft})^3}{(0.222\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7161) = 343.8$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(343.8)^{1/6}}{\left[1+\left(0.559/0.7161\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.105$$

$$h = \frac{k}{D}Nu = \frac{0.01692 \text{ Btu/h.ft.°F}}{0.2/12 \text{ ft}}(2.105) = 2.138 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL + 2\pi D^2/4 = \pi(0.2/12 \text{ ft})(0.3/12 \text{ ft}) + 2\pi(0.2/12 \text{ ft})^2/4 = 0.00175 \text{ ft}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 120\text{°F} + \frac{(0.1\times3.412) \text{ Btu/h}}{(2.138 \text{ Btu/h.ft}^2\text{.°F})(0.00175 \text{ ft}^2)} = \mathbf{211°F}$$

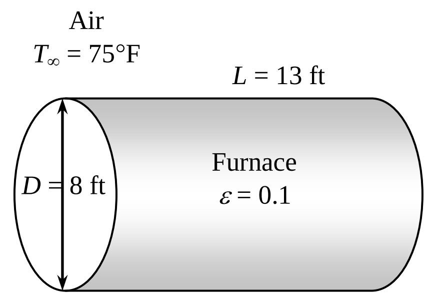
which is sufficiently close to the assumed temperature for the evaluation of properties. Therefore, there is no need to repeat calculations.

Discussion The assumed film temperature of T_f = 170°F is an appropriate assumption, since the determined T_s = 211°F would give a film temperature of T_f = 165.5°F. Otherwise, T_s would have to be solved iteratively.

9–143E An industrial furnace that resembles a horizontal cylindrical enclosure whose end surfaces are well insulated. The highest allowable surface temperature of the furnace and the annual cost of this loss to the plant are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (140+75)/2=107.5°F are (Table A-15E)



$$k = 0.01546 \text{ Btu/h.ft.°F}$$

$$\nu = 0.1852\times10^{-3} \text{ ft}^2\text{/s}$$

$$\text{Pr} = 0.7249$$

$$\beta = \frac{1}{T_f} = \frac{1}{(107.5+460)\text{R}} = 0.001762 \text{ R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 140°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the furnace, $L_c = D = 8$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001762 \text{ R}^{-1})(140-75 \text{ R})(8 \text{ ft})^3}{(0.1852\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7249) = 3.991\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.991\times10^{10})^{1/6}}{\left[1+\left(0.559/0.7249\right)^{9/16}\right]^{8/27}}\right\}^2 = 376.8$$

$$h = \frac{k}{D}Nu = \frac{0.01546 \text{ Btu/h.ft.°F}}{8 \text{ ft}}(376.8) = 0.7287 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL = \pi(8 \text{ ft})(13 \text{ ft}) = 326.7 \text{ ft}^2$$

The total rate of heat generated in the furnace is

$$\dot{Q}_{gen} = (0.82)(48 \text{ therms/h})(100{,}000 \text{ Btu/therm}) = 3.936\times10^6 \text{ Btu/h}$$

Noting that 1% of the heat generated can be dissipated by natural convection and radiation,

$$\dot{Q} = (0.01)(3.936\times10^6 \text{ Btu/h}) = 39{,}360 \text{ Btu/h}$$

The total rate of heat loss from the furnace by natural convection and radiation can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$39{,}360 \text{ Btu/h} = (0.7287 \text{ Btu/h.ft}^2\text{.°F})(326.7 \text{ ft}^2)[T_s - (75+460 \text{ R})]$$
$$+ (0.85)(326.7 \text{ m}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[T_s{}^4 - (75+460 \text{ R})^4]$$

Its solution is

$$T_s = 601.8 \text{ R} = \mathbf{141.8°F}$$

which is very close to the assumed value. Therefore, there is no need to repeat calculations. The total amount of heat loss and its cost during a-2800 hour period is

$$Q_{total} = \dot{Q}_{total}\Delta t = (39{,}360 \text{ Btu/h})(2800 \text{ h}) = 1.102\times10^8 \text{ Btu}$$

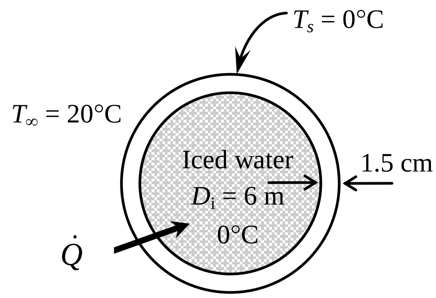
$$\text{Cost} = (1.102\times10^8 / 100{,}000 \text{ therm})(\$1.15/\text{therm}) = \mathbf{\$1267}$$

9-144 A spherical tank made of stainless steel is used to store iced water. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Thermal resistance of the tank is negligible. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (0+20)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is $L_c = D_o = 6.03$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(20-0 \text{ K})(6.03 \text{ m})^3}{(1.426\times10^{-5} \text{ m}^2\text{/s})^2}(0.7336) = 5.485\times10^{11}$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(0.469/\text{Pr}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(5.485\times10^{11})^{1/4}}{\left[1+\left(0.469/0.7336\right)^{9/16}\right]^{4/9}} = 394.5$$

$$h = \frac{k}{D_o}Nu = \frac{0.02439 \text{ W/m.°C}}{6.03 \text{ m}}(394.5) = 1.596 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D_o^2 = \pi(6.03 \text{ m})^2 = 114.2 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (1.596 \text{ W/m}^2\text{.°C})(114.2 \text{ m}^2)(20-0)\text{°C} = 3646 \text{ W}$$

Heat transfer by radiation and the total rate of heat transfer are

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (1)(114.2 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(20+273 \text{ K})^4 - (0+273 \text{ K})^4] = 11{,}759 \text{ W}$$
$$\dot{Q}_{total} = 3646 + 11{,}759 = 15{,}404 \text{ W} \cong \mathbf{15.4 kW}$$

(*b*) The total amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (15.4 \text{ kJ/s})(24 \text{ h/day}\times3600 \text{ s/h}) = 1.331\times10^6 \text{ kJ/day}$$

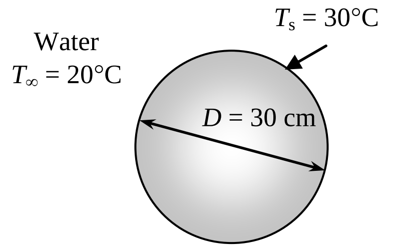
Then the amount of ice that melts during this period becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1.331\times10^6 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{3988 \text{ kg}}$$

9-145 A spherical vessel is completely submerged in a large water-filled tank. The rates of heat transfer from the vessel by natural convection, conduction, and forced convection are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature is constant.

Properties The properties of water at the film temperature of $(T_s+T_\infty)/2 = (30+20)/2 = 25°C$ are (Table A-9)



$$\rho = 997\ \text{kg/m}^3 \qquad k = 0.607\ \text{W/m.°C}$$
$$\mu = 0.891\times10^{-3}\ \text{kg/m}\cdot\text{s} \qquad \nu = \mu/\rho = 8.937\times10^{-7}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 6.14 \qquad \beta = 0.247\times10^{-3}\ \text{K}^{-1}$$

Analysis (*a*) Heat transfer in this case will be by natural convection.
The characteristic length in this case is $L_c = D = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.247\times10^{-3}\ \text{K}^{-1})(30-20\ \text{K})(0.3\ \text{m})^3}{(8.937\times10^{-7}\ \text{m}^2/\text{s})^2}(6.14) = 5.029\times10^9$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589(5.029\times10^9)^{1/4}}{\left[1+(0.469/6.14)^{9/16}\right]^{4/9}} = 144.8$$

Then

$$h = \frac{k}{D}Nu = \frac{0.607\ \text{W/m.°C}}{0.3\ \text{m}}(144.8) = 293.0\ \text{W/m}^2.°\text{C}$$
$$A_s = \pi D^2 = \pi(0.3\ \text{m})^2 = 0.2827\ \text{m}^2$$

The rate of heat transfer is

$$\dot{Q}_{\text{nat.conv}} = hA(T_s - T_\infty) = (293.0\ \text{W/m}^2.°\text{C})(0.2827\ \text{m}^2)(30-20)°\text{C} = \mathbf{828\ W}$$

(*b*) When buoyancy force is neglected, there will be no convection currents (since β = 0) and the heat transfer will be by conduction. Then Rayleigh number becomes zero (Ra = 0). The Nusselt number in this case is

Nu = 2

Then

$$h = \frac{k}{D}Nu = \frac{0.607\ \text{W/m.°C}}{0.3\ \text{m}}(2) = 4.047\ \text{W/m}^2.°\text{C}$$
$$\dot{Q}_{\text{cond}} = hA(T_s - T_\infty) = (4.047\ \text{W/m}^2.°\text{C})(0.2827\ \text{m}^2)(30-20)°\text{C} = \mathbf{11.4\ W}$$

(*c*) In this case, the heat transfer from the vessel is by forced convection. The properties of water at the free stream temperature of 20°C are (Table A-9)

$$\rho = 998\ \text{kg/m}^3 \qquad k = 0.598\ \text{W/m.°C}$$
$$\mu_\infty = 1.002\times10^{-3}\ \text{kg/m.s} \qquad \nu = \mu_\infty/\rho = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$
$$\mu_{s,@30°\text{C}} = 0.798\times10^{-3}\ \text{kg/m.s} \qquad \text{Pr} = 7.01$$

The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.2\ \text{m/s})(0.3\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 59{,}760$$

The Nusselt number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(59{,}760)^{0.5} + 0.06(59{,}760)^{2/3}\right](7.01)^{0.4}\left(\frac{1.002\times10^{-3}}{0.798\times10^{-3}}\right)^{1/4} = 439.1$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.598\ \text{W/m.°C}}{0.3\ \text{m}}(439.1) = 875.3\ \text{W/m}^2.°\text{C}$$

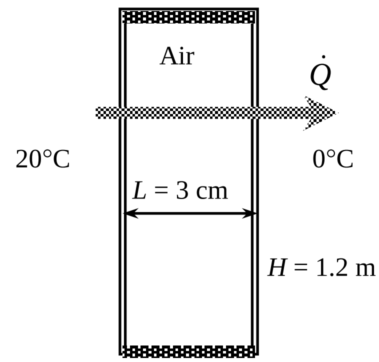
The rate of heat transfer is

$$\dot{Q}_{\text{forced conv}} = hA(T_s - T_\infty) = (875.3\ \text{W/m}^2.°\text{C})(0.2827\ \text{m}^2)(30-20)°\text{C} = \mathbf{2474\ W}$$

9-146 A double-pane window consisting of two layers of glass separated by an air space is considered. The rate of heat transfer through the window and the temperature of its inner surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **4** The pressure of air inside the enclosure is 1 atm.

Properties We expect the average temperature of the air gap to be roughly the average of the indoor and outdoor temperatures. Based on the problem statement, the properties of air at 1 atm and the average temperature of $(T_{\infty 1}+T_{\infty 2})/2$ = (20 +0)/2 = 10°C are (Table A-15)



$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{ K}^{-1}$$

Analysis We "guess" the temperature difference across the air gap to be 15°C = 15 K for use in the Ra relation. The characteristic length in this case is the air gap thickness, $L_c = L$ = 0.03 m. Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(15 \text{ K})(0.03 \text{ m})^3}{(1.426\times10^{-5} \text{ m}^2/\text{s})^2}(0.7336) = 5.065\times10^4$$

The aspect ratio of this geometry is H/L = (1.2 m)/(0.03 m) = 40. We can use Eq. 9-54 for the calculation of Nusselt number and an aspect rato of 40 satisfies the limitation of Eq. 9-54: 10 < H/L < 40. Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = 0.42Ra^{1/4}\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(5.065\times10^4)^{1/4}(0.7336)^{0.012}\left(\frac{1.2 \text{ m}}{0.03 \text{ m}}\right)^{-0.3} = 2.076$$

$$h_{air} = \frac{k}{L}Nu = \frac{0.02439 \text{ W/m.°C}}{0.03 \text{ m}}(2.076) = 1.688 \text{ W/m}^2.\text{°C}$$

Then the rate of heat transfer through this double pane window is determined to be

$$A_s = H\times W = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty,i}-T_{\infty,o}}{R_{conv,i}+R_{cond,glasses}+R_{conv,air}+R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{1}{h_i A_s}+\frac{2t_{\text{glass}}}{k_{glass}A_s}+\frac{1}{h_{air}A_s}+\frac{1}{h_o A_s}}$$

$$= \frac{20-0}{\frac{1}{(10)(2.4)}+\frac{2(0.003)}{(0.78)(2.4)}+\frac{1}{(1.688)(2.4)}+\frac{1}{(25)(2.4)}} = \mathbf{64.9\ W}$$

Check: The temperature drop across the air gap is determined from

$$\dot{Q} = hA_s\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{hA_s} = \frac{64.9 \text{ W}}{(1.688 \text{ W/m}^2.\text{°C})(2.4 \text{ m}^2)} = 16.0\text{°C}$$

which is very close to the assumed value of 15°C used in the evaluation of the Ra number.

9-147 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. Water is heated in the tube, and the annular space between the copper and glass tube is filled with air. The rate of heat loss from the collector by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **3** The pressure of air in the enclosure is 1 atm.

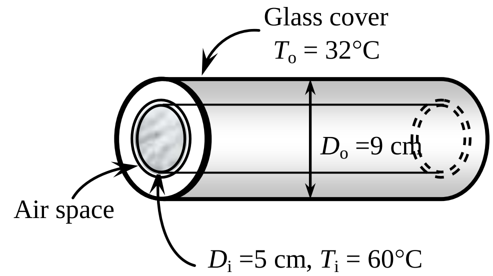
Properties The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2 = (60+32)/2 = 46$°C are (Table A-15)

$$k = 0.02706 \text{ W/m.°C}$$

$$\nu = 1.760\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7238$$

$$\beta = \frac{1}{T_f} = \frac{1}{(46+273)\text{K}} = 0.003135 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the distance between the two cylinders

$$L_c = \frac{D_o - D_i}{2} = \frac{(9-5)\text{ cm}}{2} = 2\text{ cm}$$

and

$$Ra = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003135\text{ K}^{-1})(60-32\text{ K})(0.02\text{ m})^3}{(1.760\times10^{-5}\text{ m}^2\text{/s})^2}(0.7238) = 16{,}100$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.09\text{ m}}{0.05\text{ m}}\right]^4}{(0.02\text{ m})^3\left[(0.05\text{ m})^{-7/5} + (0.09\text{ m})^{-7/5}\right]^5} = 0.1303$$

$$k_{\text{eff}} = 0.386k\left(\frac{\Pr}{0.861+\Pr}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02706\text{ W/m.°C})\left(\frac{0.7238}{0.861+0.7238}\right)^{1/4}\left[(0.1303)(16{,}100)\right]^{1/4} = 0.05811\text{ W/m.°C}$$

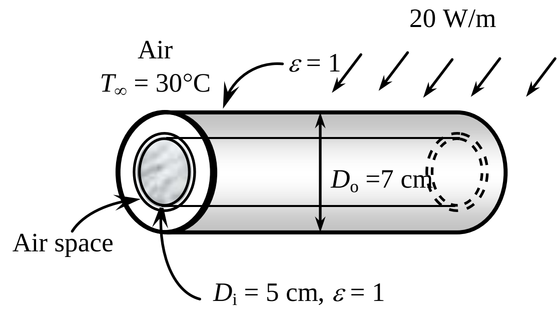
Then the heat loss from the collector per meter length of the tube becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_i - T_o) = \frac{2\pi(0.05811\text{ W/m.°C})}{\ln\left(\dfrac{0.09\text{ m}}{0.05\text{ m}}\right)}(60-32)\text{°C} = \mathbf{17.4\ W}$$

9-148 A solar collector consists of a horizontal tube enclosed in a concentric thin glass tube is considered. The pump circulating the water fails. The temperature of the aluminum tube when equilibrium is established is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties We assume a surface temperature of 33°C for glass cover based on the problem statement. Then the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (33+30)/2 = 31.5°C are (Table A-15)



$$k = 0.02599 \text{ W/m.°C}$$
$$\nu = 1.622\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7278$$
$$\beta = \frac{1}{T_f} = \frac{1}{(31.5+273)\text{K}} = 0.003284 \text{ K}^{-1}$$

Analysis This problem involves heat transfer from the aluminum tube to the glass cover, and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfers will be equal to the rate of heat gain. That is,

$$\dot{Q}_{tube-glass} = \dot{Q}_{glass-ambient} = \dot{Q}_{solar gain} = 20 \text{ W (per meter length)}$$

Now we assume the surface temperature of the glass cover to be 33°C. We will check this assumption later on, and repeat calculations with a better assumption, if necessary.

The characteristic length for the outer diameter of the glass cover $L_c = D_o$ =0.07 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003284 \text{ K}^{-1})(33-30 \text{ K})(0.07 \text{ m})^3}{(1.622\times10^{-5} \text{ m}^2\text{/s})^2}(0.7278) = 91{,}700$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(91{,}700)^{1/6}}{\left[1+\left(0.559/0.7278\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.626$$

$$A_s = \pi D_o L = \pi(0.07 \text{ m})(1 \text{ m}) = 0.2199 \text{ m}^2$$

$$h = \frac{k}{D_o}Nu = \frac{0.02599 \text{ W/m.°C}}{0.07 \text{ m}}(7.626) = 2.832 \text{ W/m}^2\text{.°C}$$

and,

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ 4}) = (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$
$$+ (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

Its solution is

$$T_{glass} = 33.34\text{°C}$$

which is sufficiently close to the assumed value of 33°C. Therefore, there is no need to repeat the calculations.

Now we will calculate heat transfer through the air layer between aluminum tube and glass cover. We will assume the aluminum tube temperature to be 45°C based on the problem statement and evaluate properties at the average temperature of $(T_i+T_o)/2$ = (45+33.34)/2 = 39.17°C are (Table A-15)

$$k = 0.02656 \text{ W/m.°C}$$
$$\nu = 1.694 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7257$$
$$\beta = \frac{1}{T_f} = \frac{1}{(39.17+273)\text{K}} = 0.003203 \text{ K}^{-1}$$

The characteristic length in this case is the distance between the two cylinders,

$$L_c = (D_o - D_i)/2 = (7-5)/2 \text{ cm} = 1 \text{ cm}$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003203 \text{ K}^{-1})(45-33.34 \text{ K})(0.01 \text{ m})^3}{(1.694\times10^{-5} \text{ m}^2\text{/s})^2}(0.7257) = 926.5$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.07 \text{ m}}{0.05 \text{ m}}\right]^4}{(0.01 \text{ m})^3\left[(0.05 \text{ m})^{-3/5} + (0.07 \text{ m})^{-3/5}\right]^5} = 0.08085$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02656 \text{ W/m.°C})\left(\frac{0.7257}{0.861+0.7257}\right)^{1/4}\left[(0.08085)(926.5)\right]^{1/4} = 0.02480 \text{ W/m.°C}$$

The heat transfer expression is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_1 - T_2) = \frac{2\pi(0.02480 \text{ W/m.°C})}{\ln\left(\dfrac{0.07 \text{ m}}{0.05 \text{ m}}\right)}(T_{tube} - 33.34)\text{°C}$$

The radiation heat loss is

$$A_s = \pi D_i L = \pi(0.05 \text{ m})^2(1 \text{ m}) = 0.1571 \text{ m}^2$$
$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$= (1)(0.1571 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(T_{tube} + 273 \text{ K})^4 - (33.34 + 273 \text{ K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = \frac{2\pi(0.02480 \text{ W/m.°C})}{\ln\left(\dfrac{0.07 \text{ m}}{0.05 \text{ m}}\right)}(T_{tube} - 33.34)\text{°C}$$
$$+ (1)(0.1571 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(T_{tube} + 273 \text{ K})^4 - (33.34 + 273 \text{ K})^4\right]$$

Its solution is

$$T_{tube} = \mathbf{46.3°C},$$

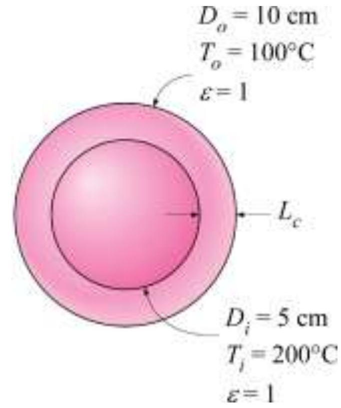
which is sufficiently close to the assumed value of 45°C. Therefore, there is no need to repeat the calculations.

9-149 Two surfaces of a spherical enclosure are maintained at specified temperatures. Both inner and outer surfaces are black, and the rate of heat transfer on the inner surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Nitrogen is an ideal gas with constant properties. **3** The inner and outer surfaces are black.

Properties The properties of nitrogen at $T_f = (T_s + T_\infty)/2 = 150°C$ are $k = 0.03416$ W/m·K, $\nu = 2.851 \times 10^{-5}$ m^2/s, Pr = 0.7025 (from Table A-16). Also, $\beta = 1/T_f = 0.002364$ K^{-1}.

Analysis The characteristic length in this case is determined from



$$L_c = \frac{D_o - D_i}{2} = \frac{10-5}{2}\text{ cm} = 2.5\text{ cm}$$

The Rayleigh number is

$$\begin{aligned}\text{Ra}_L &= \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}\\ &= \frac{(9.81\text{ m/s}^2)(2.364\times10^{-3}\text{ K}^{-1})(200-100)\text{K}(0.025\text{ m})^3}{(2.851\times10^{-5}\text{ m}^2\text{/s})^2}(0.7025)\\ &= 3.132\times10^4\end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned}F_{\text{sph}} &= \frac{L_c}{(D_iD_o)^4(D_i^{-7/5}+D_o^{-7/5})^5} = \frac{(0.025\text{ m})}{[(0.1\text{ m})(0.05\text{ m})]^4[(0.05\text{ m})^{-7/5}+(0.1\text{ m})^{-7/5}]^5}\\ &= 0.006268\end{aligned}$$

$$\begin{aligned}k_{\text{eff}} &= 0.74\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}k\\ &= 0.74\left(\frac{0.7025}{0.861+0.7025}\right)^{1/4}[(0.006268)(3.132\times10^4)]^{1/4}(0.03416\text{ W/m}\cdot\text{K})\\ &= 0.07747\text{ W/m}\cdot\text{K}\end{aligned}$$

Then the rate of heat transfer by natural convection is

$$\dot{Q}_{\text{conv}} = k_{\text{eff}}\frac{\pi D_iD_o}{L_c}(T_i - T_o) = (0.07747\text{ W/m}\cdot\text{K})\frac{\pi(0.1\text{ m})(0.05\text{ m})}{(0.025\text{ m})}(200-100)\text{ K} = 4.87\text{ W}$$

The rate of heat transfer by radiation on the inner surface is

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_{s,i}(T_i^4 - T_o^4) = \dot{Q}_{\text{rad}} = \sigma\pi D_i^2(T_i - T_o)\\ &= (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\pi(0.05\text{ m})^2(473^4 - 373^4)\text{ K}^4 = 13.67\text{ W}\end{aligned}$$

Hence, the total rate of heat transfer on the inner surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 4.87\text{ W} + 13.67\text{ W} = \mathbf{18.5\text{ W}}$$

Discussion In this problem, radiation heat transfer plays an important role, since the heat transfer rate by radiation is almost 3 times of that by natural convection.

Fundamentals of Engineering (FE) Exam Problems

9–150 Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times. Disregarding any radiation effect, will the egg cool faster or slower when the spacecraft is in space instead of on the ground?

(a) faster (b) no differencce (c) slower (d) insufficient information

Answer (c) slower [there is no gravity and thus no natural convections currents in space]

9–151 A hot object suspended by a string is to be cooled by natural convection in fluids whose volume changes differently with temperature at constant pressure. In which fluid will the rate of cooling be lowest?

With increasing temperature, a fluid whose volume

(a) increases a lot (b) increases slightly (c) does not change (d) decreases slightly (e) decreases a lot

Answer (c) A fluid whose volume **does not change** [since there will be no natural convection currents in this case]

9-152 A spherical block of dry ice at –79°C is exposed to atmospheric air at 30°C. The general direction in which the air moves in this situation is

(a) horizontal (b) up (c) down

(d) recirculation around the sphere (e) no motion

Answer (c) down

9-153 The primary driving force for natural convection is

(a) shear stress forces (b) buoyancy forces (c) pressure forces

(d) surface tension forces (e) None of them

Answer (b) buoyancy forces

9-154 Consider a horizontal 0.7-m-wide and 0.85-m-long plate in a room at 30ºC. Top side of the plate is insulated while the bottom side is maintained at 0ºC. The rate of heat transfer from the room air to the plate by natural convection is

(a) 36.8 W (b) 43.7 W (c) 128.5 W (d) 92.7 W (e) 69.7 W

(For air, use k = 0.02476 W/m·°C, Pr = 0.7323, $\nu = 1.470\times10^{-5}$ m^2/s)

Answer (b) 43.7 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Width=0.7 [m]
Height=0.85 [m]
T_infinity=30 [C]
T_s=0 [C]

"The properties of air at (0+30)/2 = 15 C are (Table A-15)"
k=0.02476 [W/m-C]
nu=1.470E-5 [m^2/s]
Pr=0.7323
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]

L=(Width*Height)/(2*(Width+Height))
Ra=(g*beta*(T_infinity-T_s)*L^3)/nu^2*Pr
Nus=0.27*Ra^0.25
h=k/L*Nus
A_s=Width*Height
Q_dot=h*A_s*(T_infinity-T_s)
```

9-155 A 4-m-long section of a 5-cm-diameter horizontal pipe in which a refrigerant flows passes through a room at 20ºC. The pipe is not well insulated and the outer surface temperature of the pipe is observed to be -10ºC. The emissivity of the pipe surface is 0.85 and the surrounding surfaces are at 15ºC. The fraction of heat transferred to the pipe by radiation is

(a) 0.24 (b) 0.30 (c) 0.37 (d) 0.48 (e) 0.58

(For air, use k = 0.02401 W/m·°C, Pr = 0.735, $\nu = 1.382\times10^{-5}$ m^2/s)

Answer (c) 0.37

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=4 [m]
D=0.05 [m]
T_infinity=20 [C]
T_s=-10 [C]
T_surr=15 [C]
epsilon=0.85

"The properties of air at (20-10)/2 = 5 C are (Table A-15)"
k=0.02401 [W/m-C]
nu=1.382E-5 [m^2/s]
Pr=0.7350
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]
sigma=5.67E-8 [W/m^2-K^4]

Ra=(g*beta*(T_infinity-T_s)*D^3)/nu^2*Pr
Nus=((0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D*Nus
A_s=pi*D*L
Q_dot_conv=h*A_s*(T_infinity-T_s)
Q_dot_rad=epsilon*A_s*sigma*((T_surr+273)^4-(T_s+273)^4)
f_rad=Q_dot_rad/(Q_dot_conv+Q_dot_rad)

"Some Wrong Solutions with Common Mistakes"
W_f_rad=Q_dot_rad/Q_dot_conv "Finding the ratio of radiation to convection"
```

9-156 Consider a 0.3-m-diameter, 1.8-m-long horizontal cylinder in a room at 20ºC. If the outer surface temperature of the cylinder is 40ºC, the natural convection heat transfer coefficient is

(a) 3.0 W/m^2·ºC (b) 3.5 W/m^2·ºC (c) 3.9 W/m^2·ºC (d) 4.6 W/m^2·ºC (e) 5.7 W/m^2·ºC

(For air, use k = 0.02588 W/m·°C, Pr = 0.7282, ν = 1.608×10^{-5} m^2/s)

Answer (c) 3.9 W/m^2·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.3 [m]
L=1.8 [m]
T_infinity=20 [C]
T_s=40 [C]

"The properties of air at (20+40)/2 = 30 C are (Table A-15)"
k=0.02588 [W/m-C]
nu=1.608E-5 [m^2/s]
Pr=0.7282
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]

Ra=(g*beta*(T_s-T_infinity)*D^3)/nu^2*Pr
Nus=((0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D*Nus
```

9-157 A 4-m-diameter spherical tank contains iced water at 0ºC. The tank is thin-shelled and thus its outer surface temperature may be assumed to be same as the temperature of the iced water inside. Now the tank is placed in a large lake at 20ºC. The rate at which the ice melts is

(a) 0.42 kg/s (b) 0.58 kg/s (c) 0.70 kg/s (d) 0.83 kg/s (e) 0.98 kg/s

(For lake water, use k = 0.580 W/m·°C, Pr = 9.45, ν = 0.1307×10^{-5} m^2/s, β = 0.138×10^{-3} K^{-1})

Answer (a) 0.42 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=4 [m]
T_infinity=20 [C]
T_s=0 [C]

"The properties of water at (20+0)/2 = 10 C are (Table A-9)"
k=0.580 [W/m-C]
nu=0.1307E-5 [m^2/s]
Pr=9.45
beta=0.138E-3 [1/K]
g=9.81 [m/s^2]

Ra=(g*beta*(T_infinity-T_s)*D^3)/nu^2*Pr
Nus=2+(0.589*Ra^(1/4))/(1+(0.469/Pr)^(9/16))^(4/9)
h=k/D*Nus
A_s=pi*D^2
Q_dot=h*A_s*(T_infinity-T_s)
h_if=333700 [J/kg]
m_dot_melt=Q_dot/h_if
```

9-158 A vertical double-pane window consists of two sheets of glass separated by a 1.5-cm air gap at atmospheric pressure. The glass surface temperatures across the air gap are measured to be 278 K and 288 K. If it is estimated that the heat transfer by convection through the enclosure is 1.5 times that by pure conduction and that the rate of heat transfer by radiation through the enclosure is about the same magnitude as the convection, the effective emissivity of the two glass surfaces is

(a) 0.47 (b) 0.53 (c) 0.61 (d) 0.65 (e) 0.72

Answer (a) 0.47

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.015 [m]
T1=288 [C]
T2=278 [C]
Nus=1.5

"The properties of air at (278+288)/2 = 283 K =10 C are (Table A-15)"
k=0.02439 [W/m-K]
sigma=5.67E-8 [W/m^2-K^4]

q_dot_conv=k*Nus*(T1-T2)/L
q_dot_rad=q_dot_conv
q_dot_rad=epsilon_eff*sigma*(T1^4-T2^4)
```

9-159 A horizontal 1.5-m-wide, 4.5-m-long double-pane window consists of two sheets of glass separated by a 3.5-cm gap filled with water. If the glass surface temperatures at the bottom and the top are measured to be 60ºC and 40ºC, respectively, the rate of heat transfer through the window is

(a) 27.6 kW (b) 39.4 kW (c) 59.6 kW (d) 66.4 kW (e) 75.5 kW

(For water, use k = 0.644 W/m·°C, Pr = 3.55, $\nu = 0.554\times10^{-6}$ m^2/s, $\beta = 0.451\times10^{-3}$ K^{-1}. Also, the applicable correlation is $\mathrm{Nu} = 0.069\mathrm{Ra}^{1/3}\,\mathrm{Pr}^{0.074}$).

Answer (d) 66.4 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Width=1.5 [m]
Length=4.5 [m]
L=0.035 [m]
T1=60 [C]
T2=40 [C]

"The properties of water at (60+40)/2 = 50 C are (Table A-9)"
k=0.644 [W/m-C]
nu=0.554E-6 [m^2/s]
Pr=3.55
beta=0.451E-3 [1/K]
g=9.81 [m/s^2]

Ra=(g*beta*(T1-T2)*L^3)/nu^2*Pr
Nus=0.069*Ra^(1/3)*Pr^(0.074)
A_s=Width*Length
Q_dot=k*Nus*A_s*(T1-T2)/L

"Some Wrong Solutions with Common Mistakes"
W1_Nus=0.068*Ra^(1/3) "Relation for air gap, Eq. 9-45"
W1_Q_dot=k*W1_Nus*A_s*(T1-T2)/L
W2_Nus=0.195*Ra^(1/4) "Relation for air gap, Eq. 9-44"
W2_Q_dot=k*W2_Nus*A_s*(T1-T2)/L
```

9-160 A vertical 0.9-m-high and 1.8-m-wide double-pane window consists of two sheets of glass separated by a 2.2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 20ºC and 30ºC, the rate of heat transfer through the window is

(a) 19.8 W (b) 26.1 W (c) 30.5 W (d) 34.7 W (e) 55.0 W

(For air, use k = 0.02551 W/m·°C, Pr = 0.7296, $\nu = 1.562\times10^{-5}$ m^2/s. Also, the applicable correlation is $\mathrm{Nu} = 0.42\mathrm{Ra}^{1/4}\,\mathrm{Pr}^{0.012}(H/L)^{-0.3}$)

Answer (b) 26.1 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
H=0.9 [m]
W=1.8 [m]
L=0.022 [m]
T1=30 [C]
T2=20 [C]

"The properties of air at (20+30)/2 = 25 C are (Table A-15)"
k=0.02551 [W/m-C]
nu=1.562E-5 [m^2/s]
Pr=0.7296
beta=1/T_ave
T_ave=(T1+T2)/2+273
g=9.81 [m/s^2]

Ra=(g*beta*(T1-T2)*L^3)/nu^2*Pr
Nus=0.42*Ra^(1/4)*Pr^0.012*(H/L)^(-0.3)
A_s=H*W
Q_dot=k*Nus*A_s*(T1-T2)/L
```

9-161 Two concentric cylinders of diameters D_i = 30 cm and D_o = 40 cm and L = 5 m are separated by air at 1 atm pressure. Heat is generated within the inner cylinder uniformly at a rate of 1100 W/m^3 and the inner surface temperature of the outer cylinder is 300 K. The steady-state outer surface temperature of the inner cylinder is

(a) 402 K (b) 415 K (c) 429 K (d) 442 K (e) 456 K

(For air, use k = 0.03095 W/m·°C, Pr = 0.7111, $\nu = 2.306\times10^{-5}$ m^2/s.)

Answer (c) 429 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D_i=0.30 [m]
D_o=0.40 [m]
L=5 [m]
g_dot=1100 [W/m^3]
T_o=300 [K]

"The properties of air at 100 C are (Table A-15)"
k=0.03095 [W/m-K]
nu=2.306E-5 [m^2/s]
Pr=0.7111
beta=1/T_ave
T_ave=100 [C]+273 [K]
g=9.81 [m/s^2]

L_c=(D_o-D_i)/2
Ra=(g*beta*(T_i-T_o)*L_c^3)/nu^2*Pr
F_cyl=(ln(D_o/D_i))^4/(L_c^3*(D_i^(-3/5)+D_o^(-3/5))^5)
k_eff=k*0.386*(Pr/(0.861+Pr))^0.25*(F_cyl*Ra)^0.25

Vol=pi*D_i^2/4*L
Q_dot=g_dot*Vol
Q_dot=(2*pi*k_eff)/ln(D_o/D_i)*(T_i-T_o)
```

9-162 ….. 9-165 Design and Essay Problems