Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

Yunus A. Cengel & Afshin J. Ghajar

McGraw-Hill, 2015

Chapter 6
FUNDAMENTALS OF CONVECTION

Mechanism and Types of Convection

6-1C A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is called an incompressible fluid. The flow of compressible fluid (such as air) is not necessarily compressible since the density of a compressible fluid may still remain constant during flow.

6-2C In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The convection caused by winds is natural convection for the earth, but it is forced convection for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

6-3C If the fluid is forced to flow over a surface, it is called external forced convection. If it is forced to flow in a tube, it is called internal forced convection. A heat transfer system can involve both internal and external convection simultaneously. Example: A pipe transporting a fluid in a windy area.

6-4C The convection heat transfer coefficient will usually be higher in forced convection since heat transfer coefficient depends on the fluid velocity, and forced convection involves higher fluid velocities.

6-5C The potato will normally cool faster by blowing warm air to it despite the smaller temperature difference in this case since the fluid motion caused by blowing enhances the heat transfer coefficient considerably.

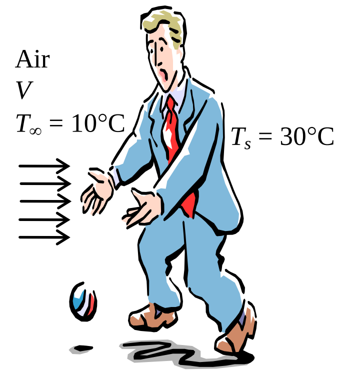
6-6C Nusselt number is the dimensionless convection heat transfer coefficient, and it represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. It is defined as $Nu = \frac{hL_c}{k}$ where L_c is the characteristic length of the surface and k is the thermal conductivity of the fluid.

6-7C Heat transfer through a fluid is conduction in the absence of bulk fluid motion, and convection in the presence of it. The rate of heat transfer is higher in convection because of fluid motion. The value of the convection heat transfer coefficient depends on the fluid motion as well as the fluid properties. Thermal conductivity is a fluid property, and its value does not depend on the flow.

6-8 The rate of heat loss from an average man walking in still air is to be determined at different walking velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different walking velocities are



(*a*) $h = 8.6V^{0.53} = 8.6(0.5\text{ m/s})^{0.53} = 5.956\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (5.956\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-10)°\text{C} = \mathbf{214\ W}$

(*b*) $h = 8.6V^{0.53} = 8.6(1.0\text{ m/s})^{0.53} = 8.60\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (8.60\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-10)°\text{C} = \mathbf{310\ W}$

(*c*) $h = 8.6V^{0.53} = 8.6(1.5\text{ m/s})^{0.53} = 10.66\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (10.66\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-10)°\text{C} = \mathbf{384\ W}$

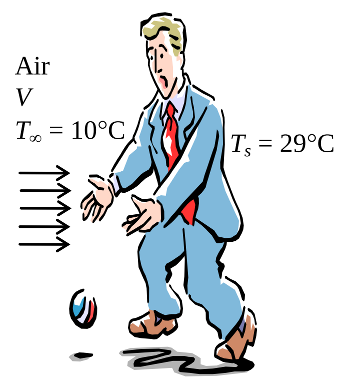
(*d*) $h = 8.6V^{0.53} = 8.6(2.0\text{ m/s})^{0.53} = 12.42\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (12.42\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-10)°\text{C} = \mathbf{447\ W}$

6-9 The rate of heat loss from an average man in windy air is to be determined at different wind velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different wind velocities are



(*a*) $h = 14.8V^{0.69} = 14.8(0.5\text{ m/s})^{0.69} = 9.174\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (9.174\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{296.3\ W}$

(*b*) $h = 14.8V^{0.69} = 14.8(1.0\text{ m/s})^{0.69} = 14.8\text{ W/m}^2.°\text{C}$

$\dot{Q} = hA_s(T_s - T_\infty) = (14.8\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{478.0\ W}$

(*c*) $h = 14.8V^{0.69} = 14.8(1.5\text{ m/s})^{0.69} = 19.58\text{ W/m}^2.°\text{C}$

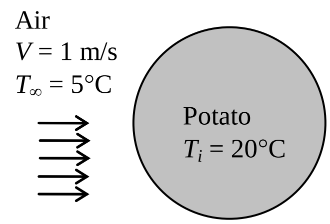
$\dot{Q} = hA_s(T_s - T_\infty) = (19.58\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{632.4\ W}$

6-10 Heat transfer coefficients at different air velocities are given during air cooling of potatoes. The initial rate of heat transfer from a potato and the temperature gradient at the potato surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Potato is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the potato at the film temperature of $T_f = (T_s + T_\infty)/2$ = (20°C + 5°C)/2 = 12.5°C is k_{fluid} = 0.02458 W/m·K (from Table A-15).

Analysis The initial rate of heat transfer from a potato is



$$A_s = \pi D^2 = \pi(0.08\,\text{m})^2 = 0.02011\,\text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (19.1\,\text{W/m}^2.°\text{C})(0.02011\,\text{m}^2)(20-5)°\text{C} = \mathbf{5.8\ W}$$

where the heat transfer coefficient is obtained from the table at 1 m/s velocity. The initial value of the temperature gradient at the potato surface is

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}}\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

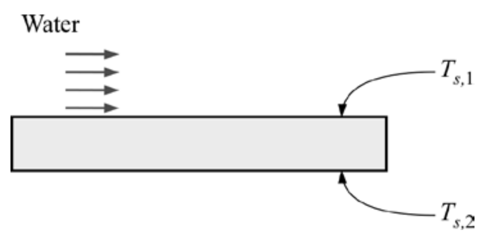
$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(19.1\,\text{W/m}^2.°\text{C})(20-5)°\text{C}}{0.02458\,\text{W/m}.°\text{C}} = \mathbf{-11{,}666°C/m}$$

6-11 The upper surface of a solid plate is being cooled by water. The water convection heat transfer coefficient and the water temperature gradient at the upper plate surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Heat conduction in solid is one-dimensional. **4** No-slip condition at the plate surface.

Properties The thermal conductivity of the solid plate is given as k = 237 W/m·K. The thermal conductivity of water at the film temperature of $T_f = (T_{s,1} + T_\infty)/2$ = (60°C + 20°C)/2 = 40°C is k_{fluid} = 0.631 W/m·K (from Table A-9).

Analysis Applying energy balance on the upper surface of the solid plate (x = 0), we have



$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad\rightarrow\quad k\frac{T_{s,2} - T_{s,1}}{L} = h(T_{s,1} - T_\infty)$$

The convection heat transfer coefficient for the water is

$$h = \frac{k}{L}\left(\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty}\right) = \frac{237\,\text{W/m}\cdot\text{K}}{0.50\,\text{m}}\left(\frac{120-60}{60-20}\right) = \mathbf{711\ W/m^2\cdot K}$$

The temperature gradient at the upper plate surface (x = 0) for the water is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{h}{k_{\text{fluid}}}(T_{s,1} - T_\infty) = -\frac{711\,\text{W/m}^2\cdot\text{K}}{0.631\,\text{W/m}\cdot\text{K}}(60-20)\,\text{K} = \mathbf{-45{,}071\,K/m}$$

Discussion The film temperature is used to evaluate the thermal conductivity of water (k_{fluid}). This is to account for the effect of temperature on the thermal conductivity.

6-12 Airflow over a plate surface has a given temperature profile. The heat flux on the plate surface and the convection heat transfer coefficient of the airflow are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** No-slip condition at the plate surface. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity and thermal diffusivity of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (220°C + 20°C)/2 = 120°C$ are $k_{\text{fluid}} = 0.03235$ W/m·K and $\alpha_{\text{fluid}} = 3.565 \times 10^{-5}$ m^2/s (from Table A-15).

Analysis For no-slip condition, heat flux from the solid surface to the fluid layer adjacent to the surface is

$$\dot{q} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} = -(0.03235\ \text{W/m}\cdot\text{K})(-4.488\times10^5\ °\text{C/m}) = \mathbf{1.452\times10^4\ W/m^2}$$

where the temperature gradient at the plate surface is

$$\begin{aligned}\left.\frac{\partial T}{\partial y}\right|_{y=0} &= (T_\infty - T_s)\left(\frac{V}{\alpha_{\text{fluid}}}\right)\exp\left(-\frac{V}{\alpha_{\text{fluid}}}y\right)\Bigg|_{y=0} \\ &= (T_\infty - T_s)\left(\frac{V}{\alpha_{\text{fluid}}}\right) = (20°\text{C} - 220°\text{C})\left(\frac{0.08\ \text{m/s}}{3.565\times10^{-5}\ \text{m}^2/\text{s}}\right) \\ &= -4.488\times10^5\ °\text{C/m}\end{aligned}$$

The convection heat transfer coefficient of the airflow is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-(0.03235\ \text{W/m}\cdot\text{K})(-4.488\times10^5\ °\text{C/m})}{(220°\text{C} - 20°\text{C})} = \mathbf{72.6\ W/m^2\cdot K}$$

Discussion The positive heat flux means that the plate is being cooled by the airflow that passes over the surface of the plate.

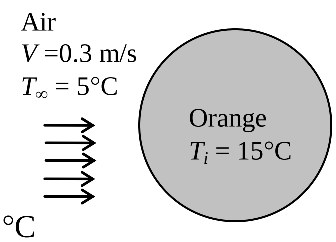
6-13 The expression for the heat transfer coefficient for air cooling of some fruits is given. The initial rate of heat transfer from an orange, the temperature gradient at the orange surface, and the value of the Nusselt number are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Orange is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Properties of water is used for orange.

Properties The thermal conductivity of the orange is given to be k = 0.50 W/m.°C. The thermal conductivity and the kinematic viscosity of air at the film temperature of $(T_s + T_\infty)/2 = (15+5)/2 = 10$°C are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}, \qquad \nu = 1.426\times10^{-5} \text{ m}^2/\text{s}$$

Analysis (*a*) The Reynolds number, the heat transfer coefficient, and the initial rate of heat transfer from an orange are



$$A_s = \pi D^2 = \pi(0.07 \text{ m})^2 = 0.01539 \text{ m}^2$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.3 \text{ m/s})(0.07 \text{ m})}{1.426\times10^{-5} \text{ m}^2/\text{s}} = 1473$$

$$h = \frac{5.05 k_{air} \text{ Re}^{1/3}}{D} = \frac{5.05(0.02439 \text{ W/m.°C})(1473)^{1/3}}{0.07 \text{ m}} = 20.0 \text{ W/m}^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (20.0 \text{ W/m}^2.\text{°C})(0.01539 \text{ m}^2)(15-5)\text{°C} = \mathbf{3.08\ W}$$

(*b*) The temperature gradient at the orange surface is determined from

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(20.0 \text{ W/m}^2.\text{°C})(15-5)\text{°C}}{0.50 \text{ W/m.°C}} = \mathbf{-400°C/m}$$

(*c*) The Nusselt number is

$$Nu = \frac{hD}{k} = \frac{(20.0 \text{ W/m}^2.\text{°C})(0.07 \text{ m})}{0.02439 \text{ W/m.°C}} = \mathbf{57.4}$$

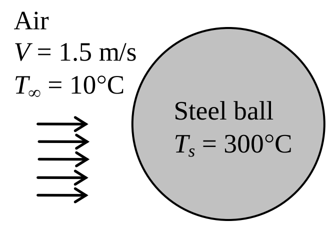
6-14 Heat transfer coefficient as a function of air velocity is given during air cooling of steel balls. The initial values of the heat flux and the temperature gradient in the steel ball at the surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity is constant. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the steel ball is given to be k = 15 W/m·K.

Analysis The initial value of the heat flux in the steel ball at the surface is

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = h(T_s - T_\infty) = (22.28\ \text{W/m}^2 \cdot \text{K})(300 - 10)\ \text{K} = \mathbf{6462\ W/m^2}$$



where

$$h = 17.9V^{0.54} = 17.9(1.5)^{0.54} = 22.28\ \text{W/m}^2 \cdot \text{K}$$

The initial value of the temperature gradient in the steel ball at the surface is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{\dot{q}_s}{k} = -\frac{6462\ \text{W/m}^2}{15\ \text{W/m} \cdot \text{K}} = -\ \mathbf{431\ K/m}$$

Discussion The higher the temperature gradient in the steel ball, the higher the value of the heat flux would be.

6-15 Heat transfer coefficient as a function of air velocity is given during air cooling of chromium steel balls. The effect of air velocity on the temperature gradient in the chromium steel ball at the surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity is constant. **3** Convection heat transfer coefficient is constant over the entire surface.

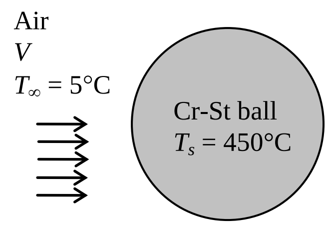
Properties The thermal conductivity of the steel ball is given to be k = 40 W/m·K.

Analysis The equation for the temperature gradient in the chromium steel ball at the surface is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

or

$$\left(\frac{\partial T}{\partial r}\right)_{r=R} = -\frac{h(T_s - T_\infty)}{k} \quad \text{where} \quad h = 18.05V^{0.56}$$



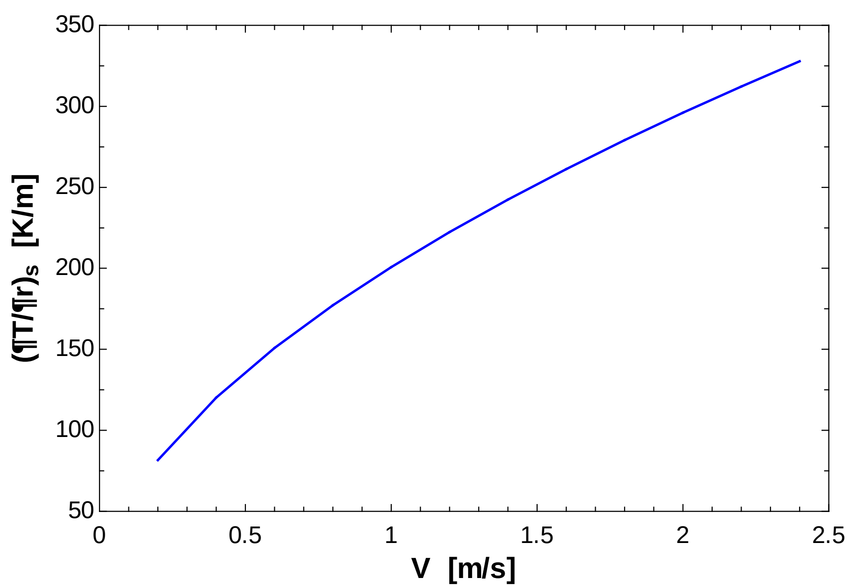
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_s=450 [C]
T_inf=5 [C]

"PROPERTIES"
k=40 [W/m-K]

"ANALYSIS"
h=18.05*V^0.56
dTdr_s=h*(T_s-T_inf)/k   "Temperature gradient at the surface"
```

V [m/s]	$(\partial T/\partial r)_{r=R}$ [K/m]
0.2	81.54
0.4	120.2
0.6	150.8
0.8	177.2
1.0	200.8
1.2	222.4
1.4	242.4
1.6	261.3
1.8	279.1
2.0	296.0
2.2	312.3
2.4	327.9



Discussion As the air velocity increases the surface temperature gradient in the chromium steel ball increases as well. Thus, the rate of heat removal from the chromium steel ball increases with increasing air velocity.

6-16 A metal plate surface is being cooled by convection. The ratio of the temperature gradient in the fluid to the temperature gradient in the plate at the surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivities of the plate and the fluid are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** No-slip condition at the plate surface.

Properties The thermal conductivities of the plate and the fluid are constant.

Analysis The temperature gradient in the fluid at the plate surface is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{\text{fluid},y=0}}{T_s - T_\infty} \quad \rightarrow \quad \left(\frac{\partial T}{\partial y}\right)_{\text{fluid},y=0} = -\frac{h}{k_{\text{fluid}}}(T_s - T_\infty)$$

The temperature gradient in the plate at the surface is

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k_{\text{plate}}\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = h(T_s - T_\infty)$$

or

$$\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = -\frac{h}{k_{\text{plate}}}(T_s - T_\infty)$$

Thus, the ratio of the temperature gradient in the fluid to the temperature gradient in the plate at the surface is

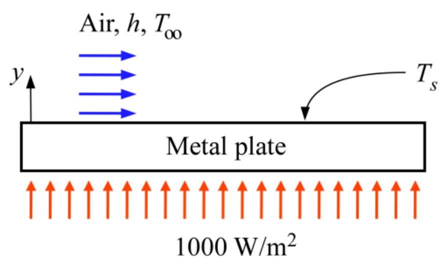
$$\frac{(\partial T/\partial y)_{\text{fluid},y=0}}{(\partial T/\partial y)_{\text{plate},y=0}} = \frac{-\dfrac{h}{k_{\text{fluid}}}(T_s - T_\infty)}{-\dfrac{h}{k_{\text{plate}}}(T_s - T_\infty)} = \frac{k_{\text{plate}}}{k_{\text{fluid}}}$$

Discussion The temperature gradient in the fluid is larger than the temperature gradient in the plate, because the thermal conductivity of solid is generally larger than the thermal conductivity of fluid ($k_{\text{plate}} > k_{\text{fluid}}$).

6-17 The upper surface of a metal plate is being cooled by air while the bottom surface is subjected to uniform heat flux. The temperature gradient in the air at the upper plate surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity of the fluid is constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Heat conduction in solid is one-dimensional. **5** No-slip condition at the plate surface.

Properties The thermal conductivity of air is given as k_{fluid} = 0.259 W/m·K



Analysis The heat flux through the plate is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad \dot{q}_s = h(T_s - T_\infty)$$

The temperature gradient in the air at the upper plate surface is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{h(T_s - T_\infty)}{k_{\text{fluid}}} = -\frac{\dot{q}_s}{k_{\text{fluid}}} = -\frac{1000\ \text{W/m}^2}{0.259\ \text{W/m}\cdot\text{K}} = \mathbf{-3861\ K/m}$$

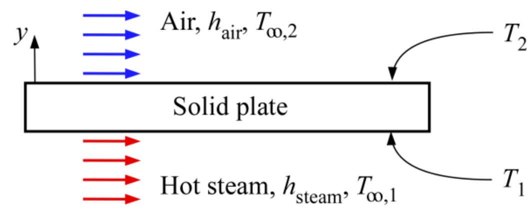
Discussion The temperature gradient in the air at the plate surface increases with decreasing k_{fluid}.

6-18 The top surface of a metal plate is being cooled by air while the bottom surface is subjected to a hot steam. The temperature gradient in the air and the temperature gradient in the plate at the top surface of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity of fluid and the plate are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Heat conduction in solid is one-dimensional. **5** No-slip condition at the plate surface.

Properties The thermal conductivity of air is given as k_{air} = 0.243 W/m·K and the thermal conductivity of the plate is given as k_{plate} = 237 W/m·K.

Analysis The heat transfer rate through the plate is



$$\dot{Q}_{conv,1} = \dot{Q}_{conv,2} \quad \rightarrow \quad h_{steam} A(T_{\infty 1} - T_1) = h_{air} A(T_2 - T_{\infty 2})$$

The temperature gradient in the air at the top surface of the plate is

$$h_{air} = \frac{-k_{air}(\partial T / \partial y)_{air,y=0}}{T_2 - T_{\infty,2}}$$

$$\left(\frac{\partial T}{\partial y}\right)_{air,y=0} = -\frac{h_{air}(T_2 - T_{\infty 2})}{k_{air}} = -\frac{h_{steam}(T_{\infty 1} - T_1)}{k_{air}}$$

Thus,

$$\left(\frac{\partial T}{\partial y}\right)_{air,y=0} = -\frac{(30\ \text{W/m}^2\cdot\text{K})(100-80)\ \text{K}}{0.243\ \text{W/m}\cdot\text{K}} = \mathbf{-2469\ K/m}$$

The temperature gradient in the plate at the top surface is

$$-k_{plate}\left(\frac{\partial T}{\partial y}\right)_{plate,y=0} = h_{air}(T_2 - T_{\infty 2}) = h_{steam}(T_{\infty 1} - T_1) \quad \rightarrow \quad \left(\frac{\partial T}{\partial y}\right)_{plate,y=0} = -\frac{h_{steam}(T_{\infty 1} - T_1)}{k_{plate}}$$

Thus,

$$\left(\frac{\partial T}{\partial y}\right)_{plate,y=0} = -\frac{(30\ \text{W/m}^2\cdot\text{K})(100-80)\ \text{K}}{237\ \text{W/m}\cdot\text{K}} = \mathbf{-2.53\ K/m}$$

Discussion The temperature gradient in the air is larger than the temperature gradient in the plate, because the thermal conductivity of the plate is larger than the thermal conductivity of air ($k_{plate} > k_{air}$).

6-19 Heat transfer coefficient as a function of air velocity is given during air cooling of a flat plate. The effect of air velocity on the air temperature gradient at the plate surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivities or air and plate are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** No-slip condition at the plate surface.

Properties The thermal conductivity of the plate is given as k_{plate} = 1.4 W/m·K and the thermal conductivity of air is given as k_{air} = 0.266 W/m·K.

Analysis The air temperature gradient at the plate surface is

$$h = \frac{-k_{\text{air}}(\partial T/\partial y)_{\text{air},y=0}}{T_s - T_\infty} \quad \rightarrow \quad \left(\frac{\partial T}{\partial y}\right)_{\text{air},y=0} = -\frac{h}{k_{\text{air}}}(T_s - T_\infty) \quad \text{where} \quad h = 27V^{0.85}$$

The temperature gradient in the plate at the surface is

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k_{\text{plate}}\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = h(T_s - T_\infty) \quad \text{or} \quad \left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = -\frac{h}{k_{\text{plate}}}(T_s - T_\infty)$$

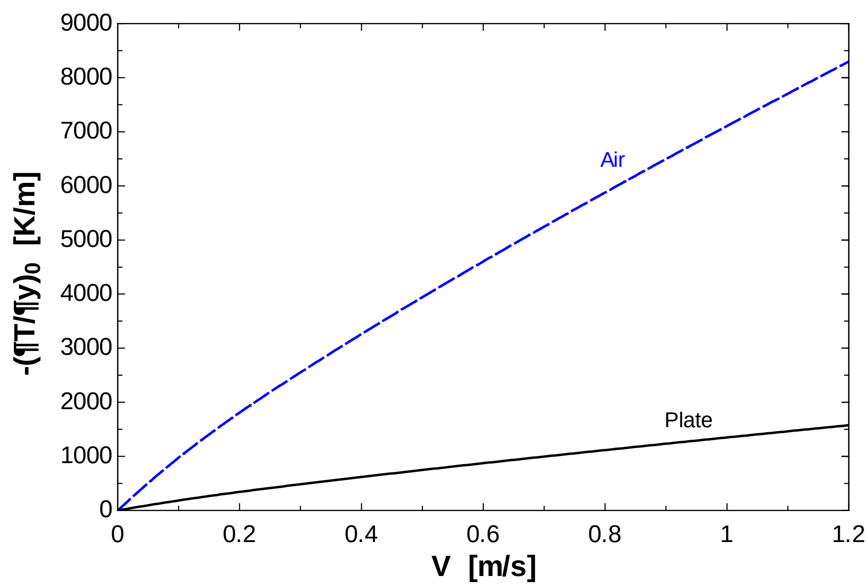
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_s=75 [C]
T_inf=5 [C]

"PROPERTIES"
k_plate=1.4 [W/m-K]
k_air=0.266 [W/m-K]

"ANALYSIS"
h=27*V^0.85
dTdy_air=h*(T_s-T_inf)/k_air   "Air temperature gradient at the surface"
dTdy_plate=h*(T_s-T_inf)/k_plate   "Plate temperature gradient at the surface"
```

V [m/s]	$-(\partial T/\partial y)_{0,\text{air}}$ [K/m]	$-(\partial T/\partial y)_{0,\text{plate}}$ [K/m]
0	0	0
0.1	1003	190.5
0.2	1809	343.7
0.3	2553	485.2
0.4	3261	619.6
0.5	3942	749.0
0.6	4603	874.5
0.7	5247	996.9
0.8	5878	1117
0.9	6497	1234
1.0	7105	1350
1.1	7705	1464
1.2	8296	1576



Discussion The air temperature gradient is larger than the temperature gradient in the plate, because the thermal conductivity of the plate is larger than the thermal conductivity of air ($k_{\text{plate}} > k_{\text{air}}$).

6-20 A metal plate is being cooled by air, the plate temperature gradient at the surface after 2 minutes of cooling is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plate are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\text{ mm}}{2} = 5\text{ mm} \qquad \text{(Note: the plate thickness is } 2L = 10\text{ mm)}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(30\text{ W/m}^2\cdot\text{K})(5\times10^{-3}\text{ m})}{(180\text{ W/m}\cdot\text{K})} = 0.0008333 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{30\text{ W/m}^2\cdot\text{K}}{(2800\text{ kg/m}^3)(880\text{ J/kg}\cdot\text{K})(5\times10^{-3}\text{ m})} = 0.002435\text{ s}^{-1}$$

The temperature of the plate after 2 minutes (t = 120 s) of cooling is

$$\frac{T_s(t) - T_\infty}{T_{s,i} - T_\infty} = e^{-bt} \quad \rightarrow \quad T_s(t) = T_\infty + (T_{s,i} - T_\infty)e^{-bt}$$

$$T_s(t) = 5°\text{C} + (300-5)(°\text{C})\exp[-(0.002435\text{ s}^{-1})(120\text{ s})] = 225.3°\text{C}$$

The plate temperature gradient at the surface at t = 120 s is

$$-k\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = h[T_s(t) - T_\infty] \quad \rightarrow \quad \left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{h[T_s(t) - T_\infty]}{k}$$

Thus,

$$\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{(30\text{ W/m}^2\cdot\text{K})(225.3-5)\text{ K}}{180\text{ W/m}\cdot\text{K}} = \mathbf{-36.7\text{ K/m}}$$

Discussion With respect to the y-direction on the plate, the negative temperature gradient at the surface indicates that heat is being removed from the plate.

6-21 Metal plates are being cooled by air. The effect of cooling time on the plates' temperature gradient at the surface is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plate are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plates are (note: plate thickness is 2L = 10 mm),

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\text{ mm}}{2} = 5\text{ mm}, \qquad \text{Bi} = \frac{hL_c}{k} = \frac{(30\text{ W/m}^2\cdot\text{K})(5\times10^{-3}\text{ m})}{(180\text{ W/m}\cdot\text{K})} = 0.0008333 < 0.1$$

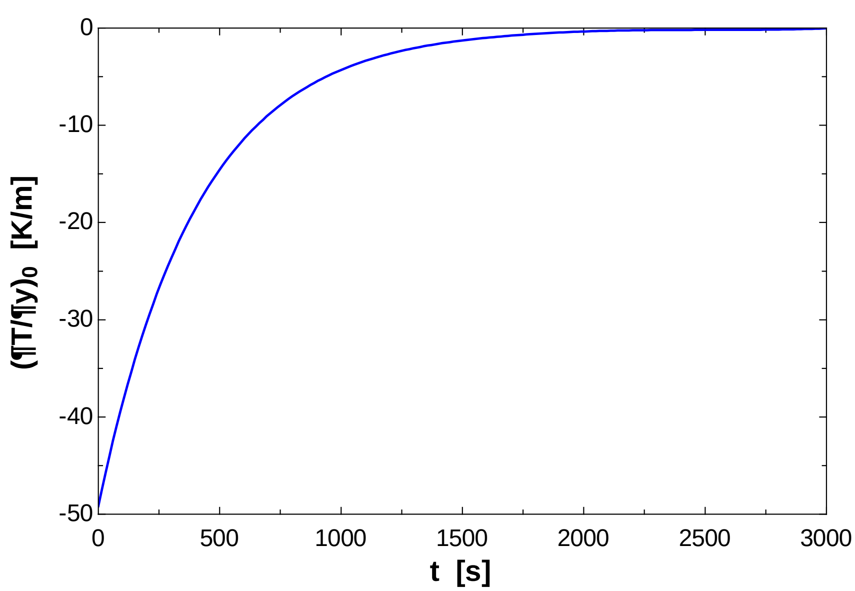
Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below:

```
"GIVEN"
h=30 [W/m^2-K]
L_c=10e-3/2 [m]
T_infinity=5 [C]
T_i=300 [C]

"PROPERTIES"
k=180 [W/m-K]
c_p=880 [J/kg-K]
rho=2800 [kg/m^3]

"ANALYSIS"
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_s-T_infinity)/(T_i-T_infinity)=exp(-b*t)
dTdy_s=-h*(T_s-T_infinity)/k   "Plate temperature gradient at the surface"
```

t [s]	$(\partial T/\partial y)_{0,\text{plate}}$ [K/m]
0	−49.17
100	−38.54
200	−30.21
300	−23.68
400	−18.56
500	−14.55
600	−11.41
700	−8.941
800	−7.009
900	−5.494
1000	−4.307
1200	−2.646
1600	−0.9991
2200	−0.2318
3000	−0.03304



Discussion As the plates cool to the air temperature, the temperature gradient at the surface approaches zero.

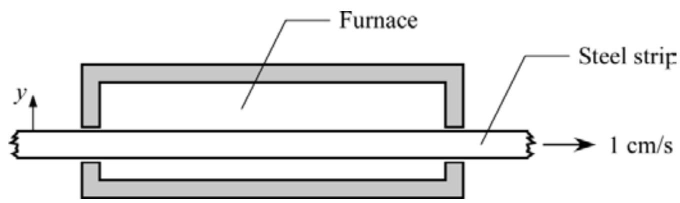
6-22 A stainless steel strip is heat treated as it moves through a furnace. The surface temperature gradient of the strip at mid-length of the furnace is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the stainless steel strip

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\,\text{mm}}{2} = 2.5\,\text{mm}$$



(Note: the strip thickness is 2L = 5 mm)

$$\text{Bi} = \frac{hL_c}{k} = \frac{(80\,\text{W/m}^2\cdot\text{K})(2.5\times10^{-3}\,\text{m})}{(21\,\text{W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{80\,\text{W/m}^2\cdot\text{K}}{(8000\,\text{kg/m}^3)(570\,\text{J/kg}\cdot\text{K})(2.5\times10^{-3}\,\text{m})} = 0.007018\,\text{s}^{-1}$$

The time of the stainless steel strip being heated can be determined from the furnace mid-length and the speed of the moving strip:

$$t = \frac{3\,\text{m}/2}{0.01\,\text{m/s}} = 150\,\text{s}$$

Thus, the temperature of the strip at mid-length of the furnace is (at t = 150 s)

$$\frac{T_s(t) - T_\infty}{T_{s,i} - T_\infty} = e^{-bt} \quad \rightarrow \quad T_s(t) = T_\infty + (T_{s,i} - T_\infty)e^{-bt}$$

$$T_s(t) = 900°\text{C} + (20 - 900)(°\text{C})\exp[-(0.007018\,\text{s}^{-1})(150\,\text{s})] = 592.9°\text{C}$$

The strip temperature gradient at the surface at t = 150 s is

$$-k\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = h[T_s(t) - T_\infty] \quad \rightarrow \quad \left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{h[T_s(t) - T_\infty]}{k}$$

Thus,

$$\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{(80\,\text{W/m}^2\cdot\text{K})(592.9 - 900)\,\text{K}}{21\,\text{W/m}\cdot\text{K}} = \mathbf{1170\,K/m}$$

Discussion With respect to the y-direction on the strip, the positive temperature gradient at the surface indicates that heat is being added to the strip.

6-23 A steel strip is heat treated as it moves through a furnace. The surface temperature gradient of the strip as a function of the furnace location is to be determined.

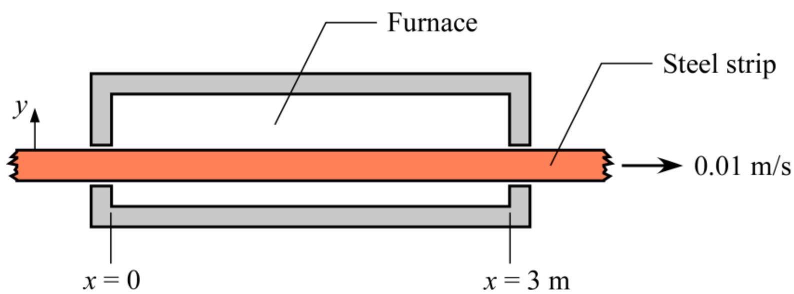
Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the steel strip are (note: plate thickness is 2L = 5 mm),

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\text{ mm}}{2} = 2.5\text{ mm}, \qquad \text{Bi} = \frac{hL_c}{k} = \frac{(80\text{ W/m}^2\cdot\text{K})(2.5\times10^{-3}\text{ m})}{(21\text{ W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below:

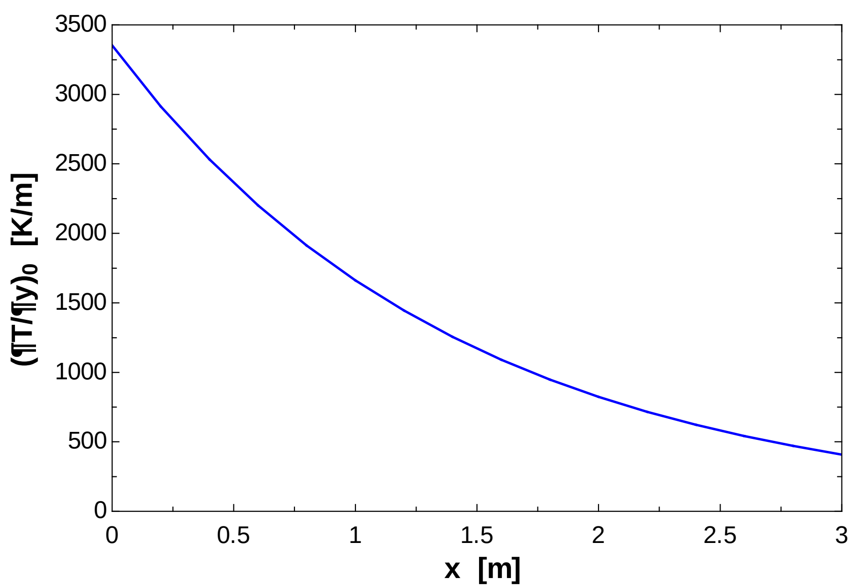



```
"GIVEN"
h=80 [W/m^2-K]
L_c=5e-3/2 [m]
T_infinity=900 [C]
T_i=20 [C]
V=0.01 [m/s]

"PROPERTIES"
k=21 [W/m-K]
c_p=570 [J/kg-K]
rho=8000 [kg/m^3]

"ANALYSIS"
t=x/V  "Cooling time"
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_s-T_infinity)/(T_i-T_infinity)=exp(-b*t)
dTdy_s=-h*(T_s-T_infinity)/k  "Surface temperature gradient of the strip"
```

x [m]	$(\partial T/\partial y)_{0,\text{strip}}$ [K/m]
0	3352
0.2	2913
0.4	2532
0.6	2200
0.8	1912
1.0	1662
1.2	1444
1.4	1255
1.6	1091
1.8	947.9
2.0	823.8
2.2	715.9
2.4	622.2
2.6	540.7
2.8	469.9
3.0	408.4



Discussion As the strip heats to the hot air in the furnace, the surface temperature gradient of the strip approaches zero. With respect to the y-direction on the strip, the positive temperature gradient at the surface indicates that heat is being added to the strip.

Boundary Layers and Flow Regimes

6-24C A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

6-25C The fluids whose shear stress is proportional to the velocity gradient are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oil are Newtonian fluids.

6-26C Viscosity is a measure of the "stickiness" or "resistance to deformation" of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

6-27C The ball reaches the bottom of the container first in water due to lower viscosity of water compared to oil.

6-28C (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature.

6-29C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

6-30C The Prandtl number $\Pr = \nu / \alpha$ is a measure of the relative magnitudes of the diffusivity of momentum (and thus the development of the velocity boundary layer) and the diffusivity of heat (and thus the development of the thermal boundary layer). The Pr is a fluid property, and thus its value is independent of the type of flow and flow geometry. The Pr changes with temperature, but not pressure.

6-31C A thermal boundary layer will not develop in flow over a surface if both the fluid and the surface are at the same temperature since there will be no heat transfer in that case.

6-32C Reynolds number is the ratio of the inertial forces to viscous forces, and it serves as a criterion for determining the flow regime. For flow over a plate of length L it is defined as Re = VL/ν where V is flow velocity and ν is the kinematic viscosity of the fluid.

6-33C A fluid motion is laminar when it involves smooth streamlines and highly ordered motion of molecules, and turbulent when it involves velocity fluctuations and highly disordered motion. The heat transfer coefficient is higher in turbulent flow.

6-34C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

6-35C In turbulent flow, it is the *turbulent eddies* due to enhanced mixing that cause the friction factor to be larger.

6-36C Turbulent viscosity μ_t is caused by turbulent eddies, and it accounts for momentum transport by turbulent eddies. It is expressed as $\tau_t = -\rho\overline{u'v'} = \mu_t \dfrac{\partial \overline{u}}{\partial y}$ where $\overline{u}$ is the mean value of velocity in the flow direction and u' and u' are the fluctuating components of velocity.

6-37C Turbulent thermal conductivity k_t is caused by turbulent eddies, and it accounts for thermal energy transport by turbulent eddies. It is expressed as $\dot{q}_t = \rho c_p \overline{v'T'} = -k_t \dfrac{\partial \overline{T}}{\partial y}$ where T' is the eddy temperature relative to the mean value, and $\dot{q}_t = \rho c_p v'T'$ the rate of thermal energy transport by turbulent eddies.

6-38 Using the given velocity profile, the wall shear stresses for air and liquid water, are to be determined.

Assumptions **1** The fluid is Newtonian. **2** Properties are constant.

Properties The dynamic viscosities for air and liquid water at 20°C are 1.825×10^{-5} kg/m·s (Table A-15) and 1.002×10^{-3} kg/m·s (Table A-9), respectively.

Analysis The shear stress at the wall surface is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = 100\mu\left[1 + 4y - 1.5y^2\right]_{y=0} = 100\mu$$

(*a*) For air

$$\tau_{s,\,\mathrm{air}} = 100\mu_{\mathrm{air}} = 100(1.825\times10^{-5})\ \mathrm{N/m^2} = \mathbf{1.825\times10^{-3}\ N/m^2}$$

(*b*) For water

$$\tau_{s,\,\mathrm{H2O}} = 100\mu_{\mathrm{H2O}} = 100(1.002\times10^{-3})\ \mathrm{N/m^2} = \mathbf{0.1002\ N/m^2}$$

Discussion For the same velocity profile, the wall shear stress ratio for liquid water and air is simply the ratio of the dynamic viscosity for both fluids:

$$\frac{\tau_{s,\,\mathrm{H2O}}}{\tau_{s,\,\mathrm{air}}} = \frac{\mu_{\mathrm{H2O}}\left.(\partial u/\partial y)\right|_{y=0}}{\mu_{\mathrm{air}}\left.(\partial u/\partial y)\right|_{y=0}} = \frac{\mu_{\mathrm{H2O}}}{\mu_{\mathrm{air}}} = 54.9$$

Hence the wall shear stress of liquid water flow over the surface is approximately fifty five times larger than that of air flow. Since liquid water is about fifty five times more viscous than air.

6-39 Using the given velocity and temperature profiles, the expressions for friction coefficient and convection heat transfer coefficient are to be determined.

Assumptions **1** The fluid is Newtonian. **2** Properties are constant. **3** No-slip condition at the plate surface.

Analysis The shear stress at the wall surface is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = C_1\mu\left[1 + 2y - 3y^2\right]_{y=0} = C_1\mu$$

The friction coefficient is

$$C_f = \frac{2\tau_s}{\rho V^2} = 2C_1\frac{\mu}{\rho V^2} = 2C_1\frac{\nu}{V^2}$$

The heat transfer convection coefficient is

$$h = \frac{-k_{\mathrm{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-k_{\mathrm{fluid}}\left[2C_2 e^{-2C_2 y}\right]_{y=0}}{T_s - T_\infty} = \frac{-2C_2 k_{\mathrm{fluid}}}{C_2 - 1 - T_\infty} \quad \text{where} \quad T_s = T(0) = C_2 - 1$$

Discussion Obtaining the expressions for friction and convection heat transfer coefficients is simple if the velocity and temperature profiles are known. However, determining the velocity and temperature profiles is generally not a simple matter in practice.

6-40 For air flowing over a flat plate, the wall shear stress and the air velocity gradient on the plate surface at mid-length of the plate are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are $\mu = 1.825 \times 10^{-5}$ kg/m·s, $\nu = 1.516 \times 10^{-5}$ m^2/s, and $\rho = 1.204$ kg/m^3 (Table A-15).

Analysis At mid-length of the plate ($x = 0.5$ m), the friction coefficient is

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} = 0.664\left[\frac{(7\text{ m/s})(0.5\text{ m})}{1.516\times10^{-5}\text{ m}^2\text{/s}}\right]^{-0.5} = 0.001382$$

The wall shear stress on the plate can be determined using

$$\tau_s = C_f\frac{\rho V^2}{2} = (0.001382)\frac{(1.204\text{ kg/m}^3)(7\text{ m/s})^2}{2} = \mathbf{0.04076\ N/m^2}$$

The velocity gradient at the plate surface is

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\tau_w}{\mu} = \frac{0.04076\text{ N/m}^2}{1.825\times10^{-5}\text{ kg/m}\cdot\text{s}} = \mathbf{2233.4\ s^{-1}}$$

Discussion For Newtonian fluids, such as air, the shear stress is proportional to the velocity gradient at the wall surface.

6-41 For air flowing over a flat plate, the effect of air velocity on the wall shear stress at x = 0.5 m and 1 m is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are $\nu = 1.516 \times 10^{-5}$ m^2/s and ρ = 1.204 kg/m^3 (Table A-15).

Analysis The friction coefficient and the wall shear stress can be determined with

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} \quad \text{and} \quad \tau_s = C_f \frac{\rho V^2}{2}$$

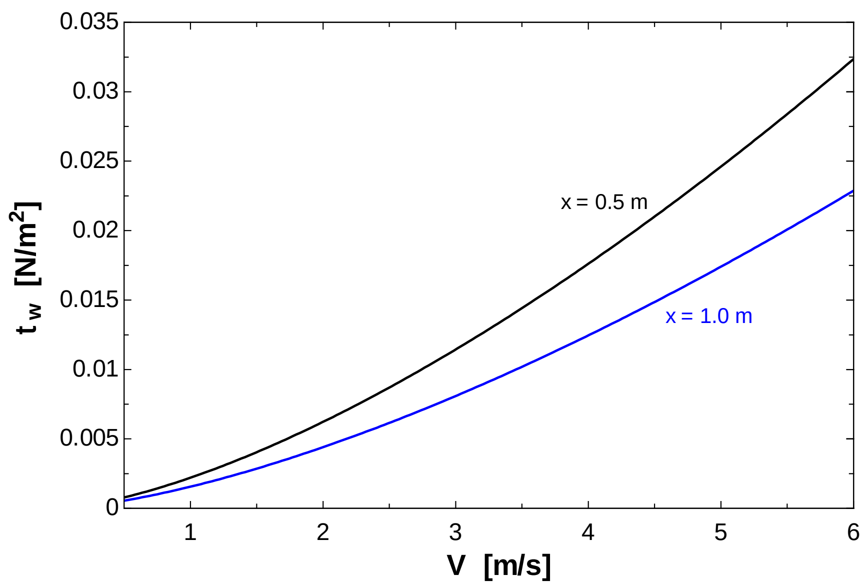
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
x_1=0.5 [m]
x_2=1.0 [m]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]

"ANALYSIS"
C_f1=0.664*(V*x_1/nu)^(-0.5)
C_f2=0.664*(V*x_2/nu)^(-0.5)
tau_w1=C_f1*rho*V^2/2   "Wall shear stress at x = 0.5 m"
tau_w2=C_f2*rho*V^2/2   "Wall shear stress at x = 1.0 m"
```

V [m/s]	τ_w (x = 0.5 m) [N/m^2]	τ_w (x = 1 m) [N/m^2]
0.5	0.0007782	0.0005503
1.0	0.002201	0.001556
1.5	0.004044	0.002859
2.0	0.006225	0.004402
2.5	0.008700	0.006152
3.0	0.01144	0.008087
3.5	0.01441	0.01019
4.0	0.01761	0.01245
4.5	0.02101	0.01486
5.0	0.02461	0.01740
5.5	0.02839	0.02008
6.0	0.03235	0.02287



Discussion As the air velocity increases, the wall shear stress also increases.

6-42 For air flowing over a flat plate at 5 m/s, the effect of plate location on the wall shear stress is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are $\nu = 1.516 \times 10^{-5}$ m^2/s and $\rho = 1.204$ kg/m^3 (Table A-15).

Analysis The friction coefficient and the wall shear stress can be determined with

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} \qquad \text{and} \qquad \tau_s = C_f \frac{\rho V^2}{2}$$

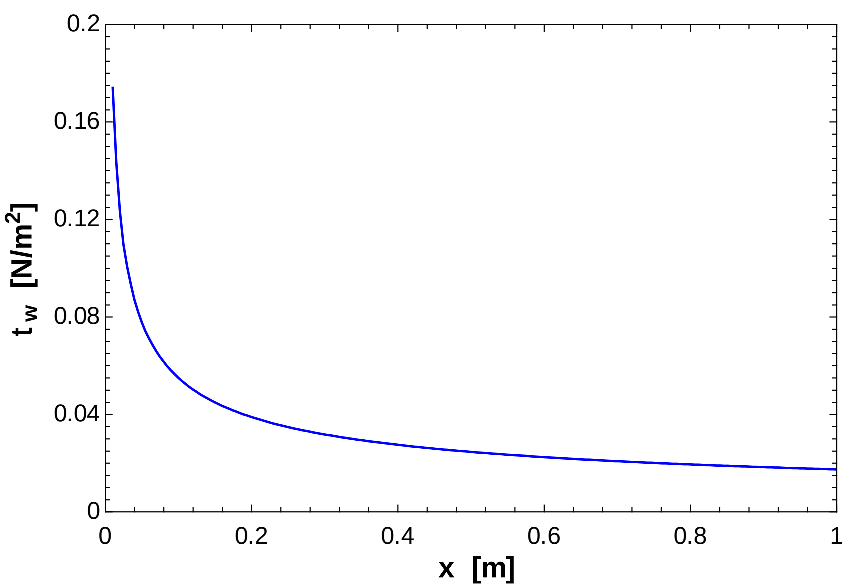
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=5 [m/s]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]

"ANALYSIS"
C_f=0.664*(V*x/nu)^(-0.5)
tau_w=C_f*rho*V^2/2
```

x [m]	τ_w [N/m^2]
0.01	0.1740
0.02	0.1230
0.03	0.1005
0.04	0.0870
0.06	0.07104
0.08	0.06152
0.10	0.05503
0.15	0.04493
0.20	0.03891
0.30	0.03177
0.40	0.02751
0.50	0.02461
0.60	0.02246
0.80	0.01945
1.00	0.01740



Discussion The wall shear stress decreases with increasing x. This is because as the air flows along the plate, the velocity gradient decreases with increasing velocity boundary layer thickness.

6-43 A flat plate is positioned inside a wind tunnel. The minimum length of the plate necessary for the Reynolds number to reach 1×10^5 is to be determined. The type of flow regime at 0.2 m from the leading edge is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosity for air at 20°C is $\nu = 1.516 \times 10^{-5}$ m²/s (Table A-15).

Analysis The Reynolds number is given as

$$\mathrm{Re} = \frac{VL_c}{\nu}$$

For the Reynolds number to reach 2×10^7, we need the minimum length of

$$L_c = \frac{\nu\,\mathrm{Re}}{V} = \frac{(1.516\times10^{-5}\ \mathrm{m^2/s})(2\times10^7)}{60\ \mathrm{m/s}} = \mathbf{5.053\ m}$$

At L_c = 0.2 m, the flow regime is

$$\mathrm{Re} = \frac{VL_c}{\nu} = \frac{(60\ \mathrm{m/s})(0.2\ \mathrm{m})}{1.516\times10^{-5}\ \mathrm{m^2/s}} = 7.92\times10^5 > 5\times10^5 \quad \rightarrow \quad \textbf{Flow is turbulent}$$

Discussion The distance from the leading edge necessary for the flow to reach turbulent regime is

$$x_{\mathrm{cr}} = \frac{\nu\,\mathrm{Re_{cr}}}{V} = \frac{(1.516\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{60\ \mathrm{m/s}} = 0.1263\ \mathrm{m}$$

6-44 For air flowing over a flat plate, the plate length to achieve a Reynolds number of 1×10^8 at the end of the plate and the distance from the leading edge of the plate at which transition would occur are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The kinematic viscosity of air at 25°C is $\nu = 1.562 \times 10^{-5}$ m²/s (Table A-15).

Analysis (a) The Reynolds number is given as

$$\mathrm{Re} = \frac{VL_c}{\nu}$$

For the Reynolds number to reach 1×10^8, we need a plate length of

$$L_c = \frac{\nu\,\mathrm{Re}}{V} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(1\times10^8)}{40\ \mathrm{m/s}} = \mathbf{39.1\ m}$$

(b) The distance from the leading edge for the transition to take place with a critical Reynolds number of 5×10^5 is

$$x_{\mathrm{cr}} = \frac{\nu\,\mathrm{Re_{cr}}}{V} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{40\ \mathrm{m/s}} = \mathbf{0.195\ \ m}$$

Discussion The expressions for the Reynolds number and the critical Reynolds number when combined would provide the following expression.

$$x_{cr} = \frac{\mathrm{Re}_{cr}}{\mathrm{Re}}\ L_c = \frac{5\times10^5}{1\times10^8}\ (39.1\,\mathrm{m}) = \mathbf{0.195\ \ m}$$

The above expression the knowledge of Re and $\mathrm{Re_{cr}}$ can be used to quickly establish the location of the transition along the plate.

6-45 Fluid is flowing over a flat plate. The distance from the leading edge at which the transition from laminar to turbulent flow occurs for different fluids is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosities for the different fluids are listed in the following table:

Fluid	*Table*	*Kinematic viscosity*, m^2/s
Air (1 atm, 20°C)	A-15	1.516×10^{-5}
Liq. water (20°C)	A-9	1.004×10^{-6}
Methanol (20°C)	A-13	7.429×10^{-7}
Engine oil (20°C)	A-13	9.429×10^{-4}
Mercury (25°C)	A-14	1.133×10^{-7}

Analysis The distance from the leading edge at which the transition from laminar to turbulent flow occurs is calculated using

$$x_{\mathrm{cr}} = \frac{\nu\,\mathrm{Re}_{\mathrm{cr}}}{V}$$

Substituting the appropriate kinematic viscosities, we have

Fluid	x_c, m
Air (1 atm, 20°C)	1.52
Liq. water (20°C)	0.100
Methanol (20°C)	0.0743
Engine oil (20°C)	94.3
Mercury (25°C)	0.0113

Discussion The distance required by the flow to reach turbulent regime increases with increasing value of kinematic viscosity. Engine oil has the highest kinematic viscosity and requires the longest length and mercury with the smallest value of kinematic viscosity requires the shortest distance.

6-46E Fluid is flowing over a flat plate. The distance from the leading edge at which the transition from laminar to turbulent flow occurs for different fluids is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosities for the different fluids at 50°F are listed in the following table:

Fluid	*Table*	*Kinematic viscosity*, ft^2/s
Air (1 atm)	A-15E	1.535×10^{-4}
Liq. water	A-9E	1.407×10^{-5}
Isobutane	A-13E	3.368×10^{-6}
Engine oil	A-13E	2.169×10^{-2}
Mercury	A-14E	1.289×10^{-6}

Analysis The distance from the leading edge at which the transition from laminar to turbulent flow occurs is calculated using

$$x_{\text{cr}} = \frac{\nu \, \text{Re}_{\text{cr}}}{V}$$

Substituting the appropriate kinematic viscosities, we have

Fluid	x_c, ft
Air (1 atm)	76.8
Liq. water	7.04
Isobutane	1.68
Engine oil	10845
Mercury	0.645

Discussion The distance required by the flow to reach turbulent regime increases with increasing value of kinematic viscosity. Mercury due to its low kinematic viscosity value, can achieve turbulent flow at a relatively short distance from the leading edge.

Convection Equations and Similarity Solutions

6-47C For steady, laminar, two-dimensional, incompressible flow with constant properties and a Prandtl number of unity and a given geometry, yes, it is correct to say that both the average friction and heat transfer coefficients depend on the Reynolds number only since $C_f = f_4(\mathrm{Re}_L)$ and $\mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr})$ from non-dimensionalized momentum and energy equations.

6-48C The continuity equation for steady two-dimensional flow is expressed as $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. When multiplied by density, the first and the second terms represent net mass fluxes in the *x* and *y* directions, respectively.

6-49C *Steady* simply means no change with time at a specified location (and thus $\partial u / \partial t = 0$), but the value of a quantity may change from one location to another (and thus $\partial u / \partial x$ and $\partial u / \partial y$ may be different from zero). Even in steady flow and thus constant mass flow rate, a fluid may accelerate. In the case of a water nozzle, for example, the velocity of water remains constant at a specified point, but it changes from inlet to the exit (water accelerates along the nozzle).

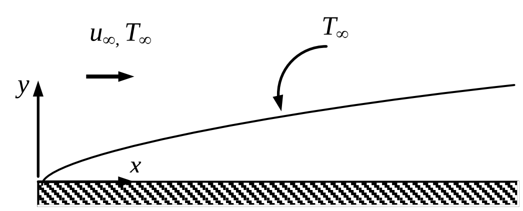
6-50C In a boundary layer during steady two-dimensional flow, the velocity component in the flow direction is much larger than that in the normal direction, and thus $u >> v$, and $\partial v / \partial x$ and $\partial v / \partial y$ are negligible. Also, *u* varies greatly with *y* in the normal direction from zero at the wall surface to nearly the free-stream value across the relatively thin boundary layer, while the variation of *u* with *x* along the flow is typically small. Therefore, $\partial u / \partial y >> \partial u / \partial x$. Similarly, if the fluid and the wall are at different temperatures and the fluid is heated or cooled during flow, heat conduction will occur primarily in the direction normal to the surface, and thus $\partial T / \partial y >> \partial T / \partial x$. That is, the velocity and temperature gradients normal to the surface are much greater than those along the surface. These simplifications are known as the **boundary layer approximations**.

6-51C For flows with low velocity and for fluids with low viscosity the viscous dissipation term in the energy equation is likely to be negligible.

6-52C For steady two-dimensional flow over an isothermal flat plate in the *x*-direction, the boundary conditions for the velocity components *u* and *v*, and the temperature *T* at the plate surface and at the edge of the boundary layer are expressed as follows:

At $y = 0$: $u(x, 0) = 0$, $v(x, 0) = 0$, $T(x, 0) = T_s$

As $y \to \infty$: $u(x, \infty) = u_\infty$, $T(x, \infty) = T_\infty$



6-53C An independent variable that makes it possible to transforming a set of partial differential equations into a single ordinary differential equation is called a **similarity variable**. A similarity solution is likely to exist for a set of partial differential equations if there is a function that remains unchanged (such as the non-dimensional velocity profile on a flat plate).

6-54C During steady, laminar, two-dimensional flow over an isothermal plate, the thickness of the velocity boundary layer (*a*) increases with distance from the leading edge, (*b*) decreases with free-stream velocity, and (*c*) and increases with kinematic viscosity

6-55C During steady, laminar, two-dimensional flow over an isothermal plate, the wall shear stress decreases with distance from the leading edge

6-56C A major advantage of nondimensionalizing the convection equations is the significant reduction in the number of parameters [the original problem involves 6 parameters ($L, V, T_\infty, T_s, \nu, \alpha$), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr)]. Nondimensionalization also results in similarity parameters (such as Reynolds and Prandtl numbers) that enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters.

6-57C A curved surface can be treated as a flat surface if there is no flow separation and the curvature effects are negligible.

6-58 A shaft rotating in a bearing is considered. The power required to rotate the shaft is to be determined for different fluids in the gap.

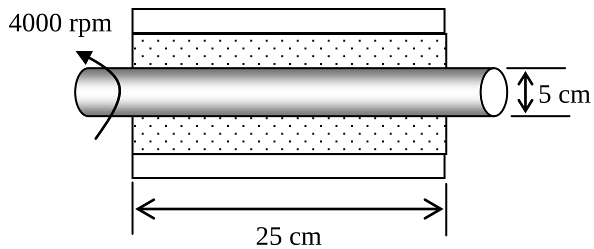
Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of air, water, and oil at 40°C are (Tables A-15, A-9, A-13)

Air: $\mu = 1.918\times10^{-5}$ N·s/m^2

Water: $\mu = 0.653\times10^{-3}$ N·s/m^2

Oil: $\mu = 0.2177$ N·s/m^2



Analysis A shaft rotating in a bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Therefore, we solve this problem considering such a flow with the plates separated by a L=0.5 mm thick fluid film similar to the problem given in Example 6-2. By simplifying and solving the continuity, momentum, and energy equations it is found in Example 6-2 that

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{2kL}(1-0) = -A\frac{\mu V^2}{2L} = -A\frac{\mu V^2}{2L}$$

First, the velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.05\text{ m})(4000\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10.47\text{ m/s}$$

$$A = \pi DL_{\text{bearing}} = \pi(0.05\text{ m})(0.25\text{ m}) = 0.03927\text{ m}^2$$

(*a*) Air:

$$\dot{W}_{\text{mech}} = -A\frac{\mu V^2}{2L} = -(0.03927\text{ m}^2)\frac{(1.918\times10^{-5}\text{ N}\cdot\text{s/m}^2)(10.47\text{ m/s})^2}{2(0.0005\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-0.083\ W}$$

(*b*) Water:

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.03927\text{ m}^2)\frac{(0.653\times10^{-3}\text{ N}\cdot\text{s/m}^2)(10.47\text{ m/s})^2}{2(0.0005\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-2.81\ W}$$

(*c*) Oil:

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.03927\text{ m}^2)\frac{(0.2177\text{ N}\cdot\text{s/m}^2)(10.47\text{ m/s})^2}{2(0.0005\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-937\ W}$$

6-59 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

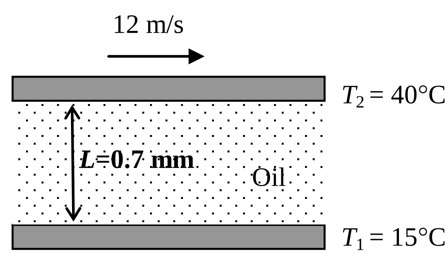
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in *z* direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

$k = 0.145$ W/m·K and $\mu = 0.605$ kg/m·s = 0.605 N·s/m^2

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation (Eq. 6-28) reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y)=\frac{T_2-T_1}{L}y+T_1+\frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=L\left(k\frac{T_2-T_1}{\mu V^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this y, whose numeric value is

$$y = L\left(k\frac{T_2 - T_1}{\mu V^2} + \frac{1}{2}\right) = (0.0007\text{ m})\left[(0.145\text{ W/m.°C})\frac{(40-15)\text{°C}}{(0.605\text{ N.s/m}^2)(12\text{ m/s})^2} + \frac{1}{2}\right]$$
$$= 0.0003791\text{ m} = \mathbf{0.379\ mm}$$

Then

$$T_{\max} = T(0.0003791) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$
$$= \frac{(40-15)\text{°C}}{0.0007\text{ m}}(0.0003791\text{ m}) + 15\text{°C} + \frac{(0.605\text{ N}\cdot\text{s/m}^2)(12\text{ m/s})^2}{2(0.145\text{ W/m}\cdot\text{°C})}\left(\frac{0.0003791\text{ m}}{0.0007\text{ m}} - \frac{(0.0003791\text{ m})^2}{(0.0007\text{ m})^2}\right)$$
$$= \mathbf{103°C}$$

(c) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$
$$= -(0.145\text{ W/m.°C})\frac{(40-15)\text{°C}}{0.0007\text{ m}} - \frac{(0.605\text{ N}\cdot\text{s/m}^2)(12\text{ m/s})^2}{2(0.0007\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-6.74\times10^4\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$
$$= -(0.145\text{ W/m.°C})\frac{(40-15)\text{°C}}{0.0007\text{ m}} + \frac{(0.605\text{ N}\cdot\text{s/m}^2)(12\text{ m/s})^2}{2(0.0007\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{5.71\times10^4\ W/m^2}$$

Discussion A temperature rise of about 75°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 65°C to improve accuracy.

6-60 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

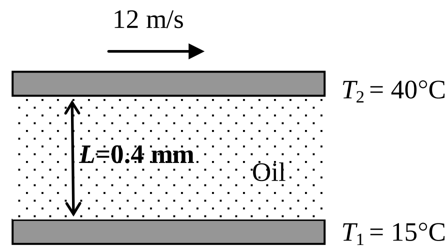
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in *z* direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

$k = 0.145$ W/m·K and $\mu = 0.605$ kg/m·s = 0.605 N·s/m^2

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation (Eq. 6-21) reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{\mathrm{V}}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3 y+C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y)=\frac{T_2-T_1}{L}y+T_1+\frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=L\left(k\frac{T_2-T_1}{\mu V^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this y, whose numeric value is

$$y = L\left(k\frac{T_2 - T_1}{\mu V^2} + \frac{1}{2}\right) = (0.0004\ \text{m})\left[(0.145\ \text{W/m.°C})\frac{(40-15)\text{°C}}{(0.605\ \text{N.s/m}^2)(12\ \text{m/s})^2} + \frac{1}{2}\right]$$

$$= 0.0002166\ \text{m} = \mathbf{0.217\ mm}$$

Then

$$T_{\max} = T(0.0002166) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$= \frac{(40-15)\text{°C}}{0.0004\ \text{m}}(0.0002166\ \text{m}) + 15\text{°C} + \frac{(0.605\ \text{N}\cdot\text{s/m}^2)(12\ \text{m/s})^2}{2(0.145\ \text{W/m}\cdot\text{°C})}\left(\frac{0.0002166\ \text{m}}{0.0004\ \text{m}} - \frac{(0.0002166\ \text{m})^2}{(0.0004\ \text{m})^2}\right)$$

$$= \mathbf{103°C}$$

(c) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$

$$= -(0.145\ \text{W/m.°C})\frac{(40-15)\text{°C}}{0.0004\ \text{m}} - \frac{(0.605\ \text{N}\cdot\text{s/m}^2)(12\ \text{m/s})^2}{2(0.0004\ \text{m})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{-1.18\times10^5\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$

$$= -(0.145\ \text{W/m.°C})\frac{(40-15)\text{°C}}{0.0004\ \text{m}} + \frac{(0.605\ \text{N}\cdot\text{s/m}^2)(12\ \text{m/s})^2}{2(0.0004\ \text{m})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{9.98\times10^4\ W/m^2}$$

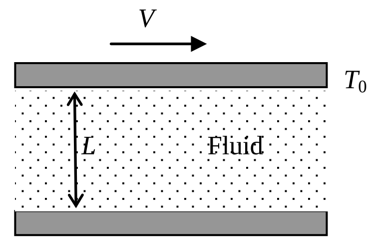
Discussion A temperature rise of about 35°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 45°C to improve accuracy.

6-61 The flow of fluid between two large parallel plates is considered. The relations for the maximum temperature of fluid, the location where it occurs, and heat flux at the upper plate are to be obtained.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Analysis We take the *x-axis* to be the flow direction, and y to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \quad\longrightarrow\quad \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $$0=k\frac{\partial^2T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad\longrightarrow\quad k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by k and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\quad -k\left.\frac{dT}{dy}\right|_{y=0}=0\longrightarrow C_3=0$

B.C. 2: $y=L$ $\quad T(L)=T_0\longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y = 0 \longrightarrow y = 0$$

Therefore, maximum temperature will occur at the lower plate surface, and it s value is

$$T_{\max} = T(0) = T_0 + \frac{\mu V^2}{2k}$$

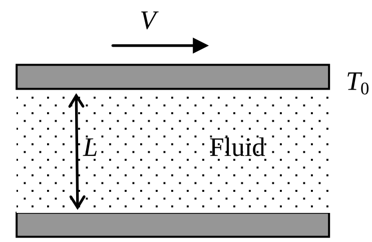
.The heat flux at the upper plate is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\,\frac{\mu V^2}{kL^2} L = \frac{\mu V^2}{L}$$

6-62 The flow of fluid between two large parallel plates is considered. Using the results of Problem 6-45, a relation for the volumetric heat generation rate is to be obtained using the conduction problem, and the result is to be verified.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Analysis The energy equation in Prob. 6-55 was determined to be



$$k\frac{d^2T}{dy^2} = -\mu\left(\frac{V}{L}\right)^2 \qquad (1)$$

The steady one-dimensional heat conduction equation with constant heat generation is

$$\frac{d^2T}{dy^2} + \frac{\dot{e}_{\text{gen}}}{k} = 0 \qquad (2)$$

Comparing the two equation above, the volumetric heat generation rate is determined to be

$$\dot{e}_{\text{gen}} = \mu\left(\frac{V}{L}\right)^2$$

Integrating Eq. (2) twice gives

$$\frac{dT}{dy} = -\frac{\dot{e}_{\text{gen}}}{k}y + C_3$$

$$T(y) = -\frac{\dot{e}_{\text{gen}}}{2k}y^2 + C_3 y + C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\quad -k\left.\frac{dT}{dy}\right|_{y=0} = 0 \longrightarrow C_3 = 0$

B.C. 2: $y=L$ $\quad T(L) = T_0 \longrightarrow C_4 = T_0 + \frac{\dot{e}_{\text{gen}}}{2k}L^2$

Substituting, the temperature distribution becomes

$$T(y) = T_0 + \frac{\dot{e}_{\text{gen}}L^2}{2k}\left(1 - \frac{y^2}{L^2}\right)$$

Maximum temperature occurs at $y = 0$, and it value is

$$T_{\max} = T(0) = T_0 + \frac{\dot{e}_{\text{gen}}L^2}{2k}$$

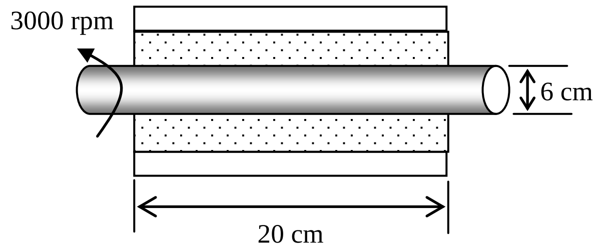
which is equivalent to the result $T_{\max} = T(0) = T_0 + \frac{\mu V^2}{2k}$ obtained in Prob. 6-55.

6-63 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

k = 0.17 W/m·K and μ = 0.05 N·s/m^2



Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x*-axis to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking x = 0 at the surface of the bearing, the boundary conditions are $u(0)$ = 0 and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with viscous dissipation reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by k and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy}=\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting dT/dy = 0 and solving for y,

$$\frac{dT}{dy}=\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=\frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane in the oil. The velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.06\ \text{m})(3000\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 9.425\ \text{m/s}$$

$$A = \pi D L_{bearing} = \pi(0.06\ \text{m})(0.20\ \text{m}) = 0.0377\ \text{m}^2$$

The maximum temperature is

$$T_{\max} = T(L/2) = T_0 + \frac{\mu V^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right)$$

$$= T_0 + \frac{\mu V^2}{8k} = 50°\text{C} + \frac{(0.05\ \text{N}\cdot\text{s/m}^2)(9.425\ \text{m/s})^2}{8(0.17\ \text{W/m}\cdot°\text{C})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{53.3°C}$$

(*b*) The rates of heat transfer are

$$\dot{Q}_0 = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{2kL}(1-0) = -A\frac{\mu V^2}{2L}$$

$$= -(0.0377\ \text{m}^2)\frac{(0.05\ \text{N}\cdot\text{s/m}^2)(9.425\ \text{m/s})^2}{2(0.0002\ \text{m})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{-419\ W}$$

$$\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=L} = -kA\frac{\mu V^2}{2kL}(1-2) = A\frac{\mu V^2}{2L} = -\dot{Q}_0 = \mathbf{419\ W}$$

(*c*) Therefore, rates of heat transfer at the two plates are equal in magnitude but opposite in sign. The mechanical power wasted is equal to the rate of heat transfer.

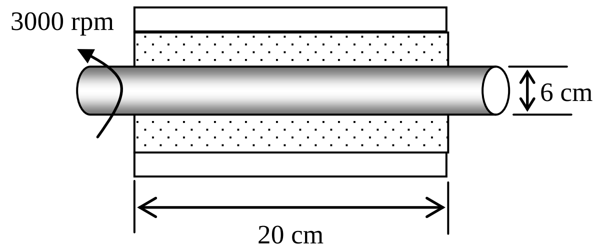
$$\dot{W}_{\text{mech}} = \dot{Q} = 2\times 419 = \mathbf{838\ W}$$

6-64 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

k = 0.17 W/m·K and μ = 0.05 N·s/m^2



Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x*-axis to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking x = 0 at the surface of the bearing, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $\quad T(0)=T_1 \longrightarrow C_4=T_1$

B.C. 2: y=L $\quad -k\left.\frac{dT}{dy}\right|_{y=L}=0 \longrightarrow C_3=\frac{\mu V^2}{kL}$

Substituting the constants give the temperature distribution to be

$$T(y) = T_1 + \frac{\mu V^2}{kL}\left(y - \frac{y^2}{2L}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{\mu V^2}{kL}\left(1 - \frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{\mu V^2}{kL}\left(1 - \frac{y}{L}\right) = 0 \longrightarrow y = L$$

This result is also known from the second boundary condition. Therefore, maximum temperature will occur at the shaft surface, for $y = L$. The velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.06\text{ m})(3000\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 9.425\text{ m/s}$$

$$A = \pi DL_{bearing} = \pi(0.06\text{ m})(0.20\text{ m}) = 0.0377\text{ m}^2$$

The maximum temperature is

$$T_{\max} = T(L) = T_1 + \frac{\mu V^2}{kL}\left(L - \frac{L^2}{2L}\right) = T_1 + \frac{\mu V^2}{k}\left(1 - \frac{1}{2}\right) = T_1 + \frac{\mu V^2}{2k}$$

$$= 50°\text{C} + \frac{(0.05\text{ N}\cdot\text{s/m}^2)(9.425\text{ m/s})^2}{2(0.17\text{ W/m}\cdot°\text{C})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{63.1°C}$$

(*b*) The rate of heat transfer to the bearing is

$$\dot{Q}_0 = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{kL}(1-0) = -A\frac{\mu V^2}{L}$$

$$= -(0.0377\text{ m}^2)\frac{(0.05\text{ N}\cdot\text{s/m}^2)(9.425\text{ m/s})^2}{0.0002\text{ m}}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-837\ W}$$

(*c*) The rate of heat transfer to the shaft is zero. The mechanical power wasted is equal to the rate of heat transfer,

$$\dot{W}_{\text{mech}} = \dot{Q} = \mathbf{837\ W}$$

6-65 Prob. 6-64 is reconsidered. The effect of shaft velocity on the mechanical power wasted by viscous dissipation is to be investigated.

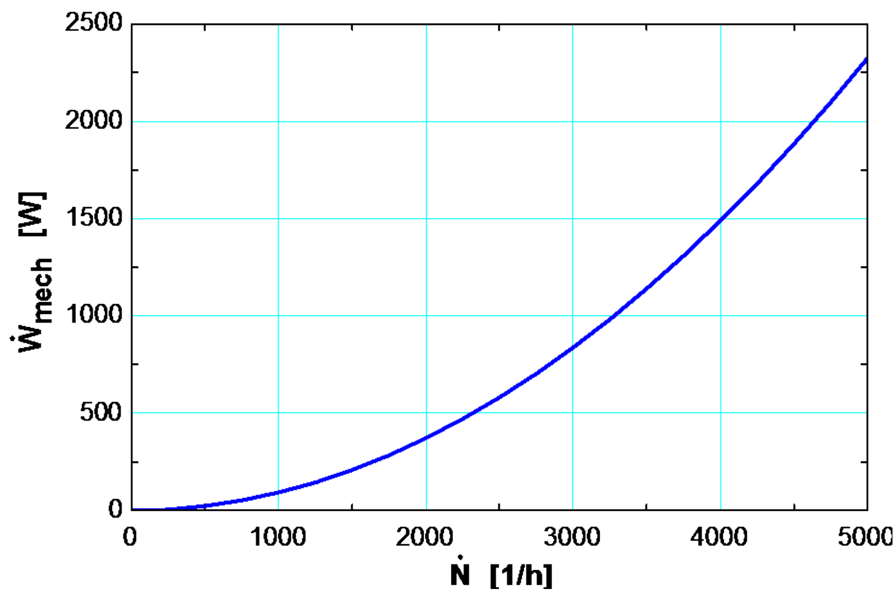
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.06 [m]
N_dot=3000 [1/h]
L_bearing=0.20 [m]
L=0.0002 [m]
T_0=50 [C]

"PROPERTIES"
k=0.17 [W/m-K]
mu=0.05 [N-s/m^2]

"ANALYSIS"
Vel=pi*D*N_dot*Convert(1/min, 1/s)
A=pi*D*L_bearing
T_max=T_0+(mu*Vel^2)/(8*k)
Q_dot=A*(mu*Vel^2)/(2*L)
W_dot_mech=2*Q_dot
```

$\dot{N}$ [rpm]	$\dot{W}_{mech}$ [W]
0	0
250	5.814
500	23.25
750	52.32
1000	93.02
1250	145.3
1500	209.3
1750	284.9
2000	372.1
2250	470.9
2500	581.4
2750	703.5
3000	837.2
3250	982.5
3500	1139
3750	1308
4000	1488
4250	1680
4500	1884
4750	2099
5000	2325

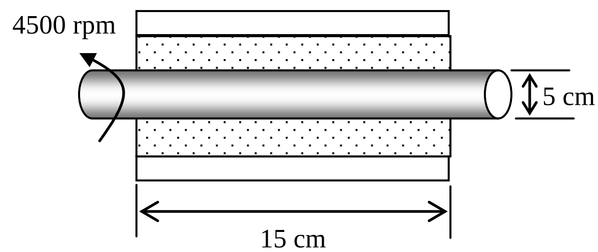


6-66 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be k = 0.14 W/m·K and μ = 0.03 N·s/m^2. The thermal conductivity of bearing is given to be k = 70 W/m·K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to



Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u=u(y)$$

Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

where

$$V=\pi D\dot{N}=\pi(0.05\text{ m})(4500\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)=11.78\text{ m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by k and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $$-k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$$

B.C. 2: y=L $$T(L)=T_0 \longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y) = T_0 + \frac{\mu V^2}{2k}\left(1 - \frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating *T*(*y*) with respect to *y*,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y$$

.The heat flux at the upper surface is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\frac{\mu V^2}{kL^2}L = \frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q} = A_s \dot{q}_L = (\pi D W)\frac{\mu V^2}{L} = \pi(0.05\,\text{m})(0.15\,\text{m})\frac{(0.03\,\text{N}\cdot\text{s/m}^2)(11.78\,\text{m/s})^2}{0.0006\,\text{m}} = \mathbf{163.5\ W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q} = k\frac{2\pi W(T_0 - T_s)}{\ln(D_0/D)} \quad \to (70\,\text{W/m}\cdot{}^\circ\text{C})\frac{2\pi(0.15\,\text{m})(T_0 - 40^\circ\text{C})}{\ln(8/5)} = 163.5\,\text{W}$$

which gives the surface temperature of the shaft to be

$$T_o = \mathbf{41.2^\circ C}$$

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

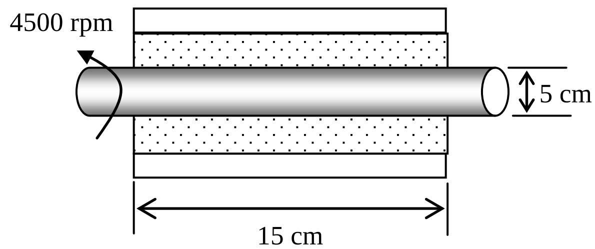
$$\dot{W}_{lost} = \dot{Q} = \mathbf{163.5\ W}$$

6-67 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be $k = 0.14$ W/m·K and $\mu = 0.03$ N·s/m^2. The thermal conductivity of bearing is given to be $k = 70$ W/m·K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x*-*axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to



Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

where

$$V=\pi D\dot{N}=\pi(0.05\text{ m})(4500\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)=11.78\text{ m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $$-k\left.\frac{dT}{dy}\right|_{y=0}=0\longrightarrow C_3=0$$

B.C. 2: $y=L$ $$T(L)=T_0\longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y) = T_0 + \frac{\mu V^2}{2k}\left(1 - \frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y$$

.The heat flux at the upper surface is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\frac{\mu V^2}{kL^2} L = \frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q} = A_s \dot{q}_L = (\pi D W)\frac{\mu V^2}{L} = \pi(0.05\,\text{m})(0.15\,\text{m})\frac{(0.03\,\text{N}\cdot\text{s/m}^2)(11.78\,\text{m/s})^2}{0.001\,\text{m}} = \mathbf{98.1\,W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q} = k\frac{2\pi W(T_0 - T_s)}{\ln(D_0/D)} \quad \to \; (70\,\text{W/m}\cdot{}^\circ\text{C})\frac{2\pi(0.15\,\text{m})(T_0 \text{ - } 40^\circ\text{C})}{\ln(8/5)} = 98.1\,\text{W}$$

which gives the surface temperature of the shaft to be

T_o = **40.7°C**

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

$$\dot{W}_{lost} = \dot{Q} = \mathbf{98.1\,W}$$

6-68E Glycerin is flowing over a flat plate. The velocity and thermal boundary layer thicknesses are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The properties of glycerin at 50°F are ν = 0.03594 ft^2/s and Pr = 34561 (Table A-13E).

Analysis The Reynolds number at x = 0.5 ft is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(6\text{ ft/s})(0.5\text{ ft})}{0.03594\text{ ft}^2/\text{s}} = 83.47 < 5\times10^5 \quad \text{(flow is laminar at } x = 0.5\text{ ft)}$$

The velocity boundary thickness for laminar flow over a flat plate is

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} = \frac{4.91(0.5\text{ ft})}{\sqrt{83.47}} = \mathbf{0.2687\ ft}$$

The thermal boundary thickness over a flat plate is

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{0.2687\text{ ft}}{34561^{1/3}} = \mathbf{0.008249\ ft}$$

Discussion The ratio of the velocity boundary thickness to the thermal boundary layer thickness can be expressed as

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3} = 32.57$$

Since glycerin has a large Prandtl number, this implies that the velocity boundary thickness is larger than the thermal boundary layer thickness, and this case by an order of about 33 times.

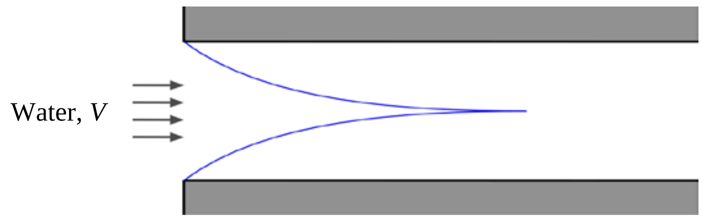
6-69 Water is flowing between two parallel flat plates. The distances from the entrance at which the velocity and thermal boundary layers meet are to be determined.

Assumptions **1** Isothermal condition exists between the flat plates and fluid flow. **2** Properties are constant.

Properties The properties of water at 20°C are ρ = 998.0 kg/m^3, μ = 1.002 × 10^{-3} kg/m·s and Pr = 7.01 (Table A-9).

Analysis The kinematic viscosity for water at 20°C is

$$\nu = \frac{\mu}{\rho} = \frac{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}{998.0\ \text{kg/m}^3} = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$



Both the velocity and thermal boundary layers meet at the centerline between the two parallel plates when

$$\delta = \delta_t = \frac{1\,\text{cm}}{2} = 0.005\ \text{m}$$

Assuming the flow is laminar, the velocity and thermal boundary layer thicknesses are

$$\delta = \frac{4.91}{\sqrt{V/(\nu x)}} \quad \text{and} \quad \delta_t = \frac{4.91}{\text{Pr}^{1/3}\sqrt{V/(\nu x_t)}}$$

The distance from the entrance at which the velocity boundary layers meet is

$$x = \frac{\delta^2 V}{(4.91)^2 \nu} = \frac{(0.005\ \text{m})^2(0.5\ \text{m/s})}{(4.91)^2(1.004\times10^{-6}\ \text{m}^2/\text{s})} = \mathbf{0.516\ m}$$

The distance from the entrance at which the thermal boundary layers meet is

$$x_t = \frac{\delta_t^2\,\text{Pr}^{2/3} V}{(4.91)^2 \nu} = \frac{(0.005\ \text{m})^2(7.01)^{2/3}(0.5\ \text{m/s})}{(4.91)^2(1.004\times10^{-6}\ \text{m}^2/\text{s})} = \mathbf{1.89\ m}$$

Discussion The analysis for this problem assumed that the flow is laminar. To check whether the flow is indeed laminar, the Reynolds number at x = 0.516 m is calculated:

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(0.5\ \text{m/s})(0.516\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 2.57\times10^5 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.516\text{ m)}$$

Therefore, the laminar flow assumption for this analysis is valid.

6-70E The δ / δ_t ratios for different fluids in laminar boundary layer flow over a flat plate are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The Prandtl numbers for the different fluids at 50°F are listed in the following table:

Fluid	*Table*	Pr
Air (1 atm)	A-15E	0.7336
Liq. water	A-9E	9.44
Isobutane	A-13E	4.114
Engine oil	A-13E	22963
Mercury	A-14E	0.02737

Analysis The velocity and thermal boundary layers for laminar flow can be related using

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \quad \rightarrow \quad \frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

Hence the δ / δ_t ratio for each fluid is

Fluid	δ / δ_t
Air (1 atm)	0.902
Liq. water	2.11
Isobutane	1.60
Engine oil	28.4
Mercury	0.301

Discussion For Pr > 1, the velocity boundary layer is thicker than the thermal boundary layer ($\delta > \delta_t$). For Pr < 1, the velocity boundary layer is thinner than the thermal boundary layer ($\delta < \delta_t$).

6-71 For laminar boundary layer flow over a flat plate with air (100ºC and 1 atm), δ_t is 15% larger than δ. Determine the ratio of δ/δ_t for engine oil (unused) at the same conditions.

Assumptions **1** Laminar flow.

Properties For air at 100ºC and 1 atm, Pr = 0.7111 (Table A-15). For engine oil (unused) at 100ºC, Pr = 279.1 (Table A-13)

Analysis The Prandtl number strongly influences growth of the velocity δ, and thermal δ_t, boundary layers. For laminar flow, the approximate relationship is given by

$$\delta/\delta_t \approx \mathrm{Pr}^n$$

Where n is a positive coefficient. Substituting the values for air

$$\delta/1.15\delta = (0.7111)^n \rightarrow n = 0.4099$$

For engine oil (unused) it follows that

$$\delta/\delta_t = (279.1)^{0.4099} = \mathbf{10.06}$$

Discussion Large value of Prandtl number for engine oil causes $\delta >> \delta_t$.

6-72 The hydrodynamic boundary layer and the thermal boundary layer both as a function of *x* are to be plotted for the flow of air over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-15)

ρ = 1.127 kg/m^3

c_p = 1007 J/kg·°C

μ = 0.00001918 kg/m·s

ν = 1.702×10^{-5} m^2/s

k = 0.02662 W/m·°C

Pr = 0.7255

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 15°C

T_s = 65°C

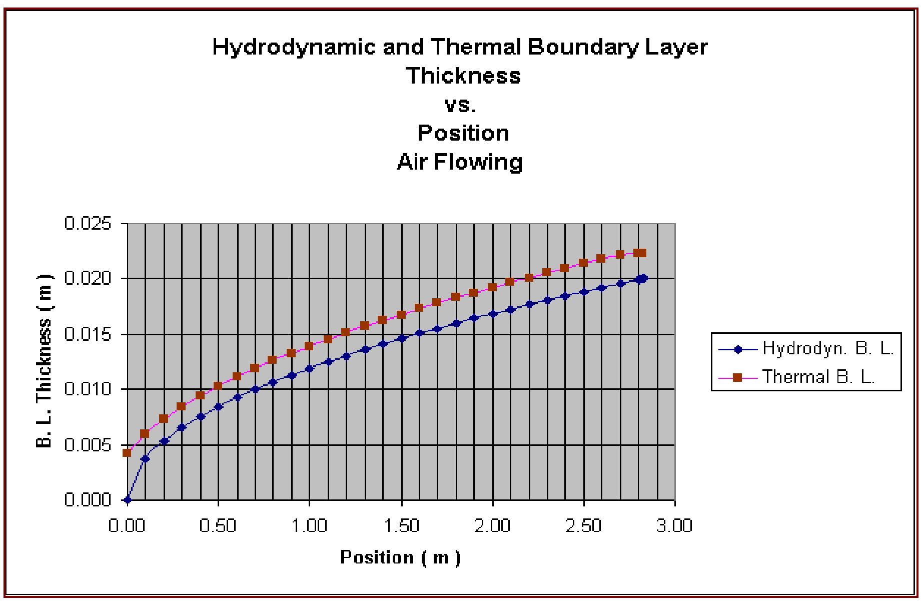
Analysis

The critical length: $\text{Re} = \dfrac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \dfrac{\text{Re}\,\nu}{V} = \dfrac{(500{,}000)(1.702\times10^{-5}\ \text{m}^2/\text{s})}{3\ \text{m/s}} = 2.84\ \text{m}$

Hydrodynamic boundary layer thickness: $\delta = \dfrac{4.91x}{\sqrt{\text{Re}_x}}$

Thermal boundary layer thickness: $\delta_t = \dfrac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$

x (m)	Re_x	δ	δ_t
0.00	0	0	0
0.10	17628	0.0038	0.0042
0.20	35255	0.0053	0.0059
0.30	52883	0.0065	0.0073
0.40	70511	0.0075	0.0084
0.50	88139	0.0084	0.0094
0.60	105766	0.0092	0.0103
0.70	123394	0.0100	0.0111
0.80	141022	0.0107	0.0119
0.90	158650	0.0113	0.0126
1.00	176277	0.0119	0.0133
1.10	193905	0.0125	0.0139
1.20	211533	0.0130	0.0145
1.30	229161	0.0136	0.0151
1.40	246788	0.0141	0.0157
1.50	264416	0.0146	0.0162
1.60	282044	0.0151	0.0168
1.70	299672	0.0155	0.0173
1.80	317299	0.0160	0.0178
1.90	334927	0.0164	0.0183
2.00	352555	0.0168	0.0187
2.10	370182	0.0173	0.0192
2.20	387810	0.0177	0.0197
2.30	405438	0.0181	0.0201
2.40	423066	0.0184	0.0205
2.50	440693	0.0188	0.0210
2.60	458321	0.0192	0.0214
2.70	475949	0.0196	0.0218
2.80	493577	0.0199	0.0222
2.81	495339	0.0200	0.0222
2.82	497102	0.0200	0.0223
2.83	498865	0.0200	0.0223



6-73 The hydrodynamic boundary layer and the thermal boundary layer both as a function of x are to be plotted for the flow of liquid water over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-9)

ρ = 992.1 kg/m^3

c_p = 4179 J/kg·°C

μ = 0.000653 kg/m·s

k = 0.631 W/m·°C

Pr = 4.32

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 15°C

T_s = 65°C

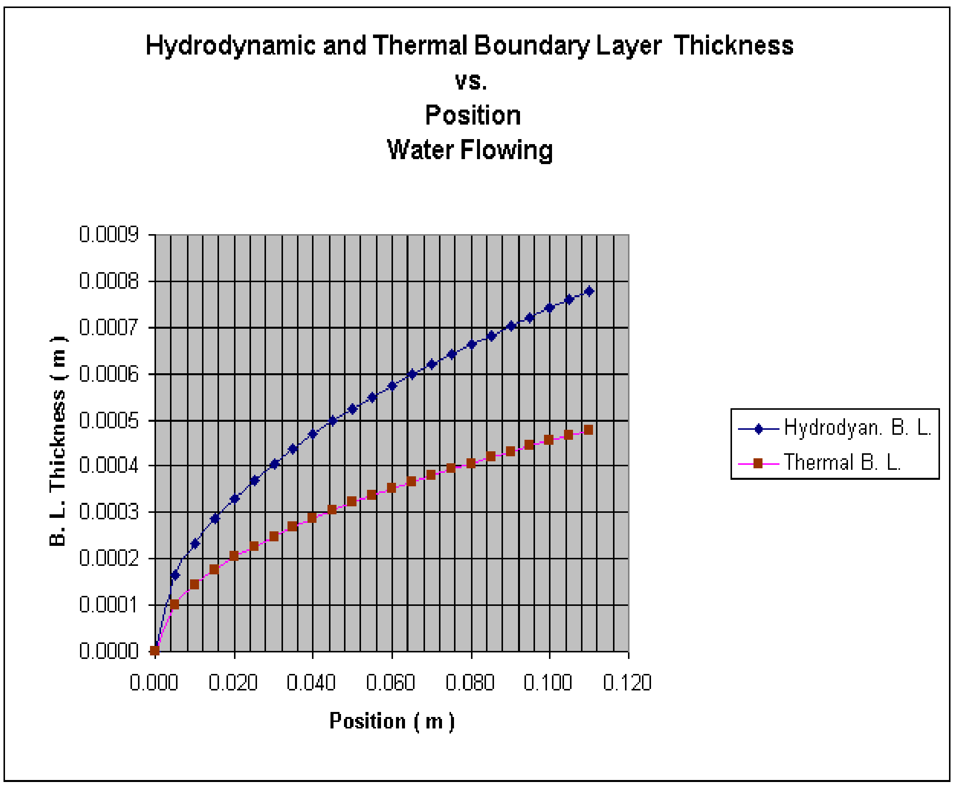
Analysis

The critical length: $$\text{Re} = \frac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \frac{\text{Re}\,\nu}{V} = \frac{\text{Re}\,\mu}{V\rho} = \frac{(500{,}000)(0.000653\,\text{kg/m}\cdot\text{s})}{(3\,\text{m/s})(992.1\,\text{kg/m}^3)} = 0.11\,\text{m}$$

Hydrodynamic boundary layer thickness: $$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}}$$

Thermal boundary layer thickness: $$\delta_t = \frac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$$

x (m)	Re_x	δ	δ_t
0.000	0.000	0	0
0.005	22789	0.0002	0.0001
0.010	45579	0.0002	0.0001
0.015	68368	0.0003	0.0002
0.020	91158	0.0003	0.0002
0.025	113947	0.0004	0.0002
0.030	136737	0.0004	0.0002
0.035	159526	0.0004	0.0003
0.040	182315	0.0005	0.0003
0.045	205105	0.0005	0.0003
0.050	227894	0.0005	0.0003
0.055	250684	0.0005	0.0003
0.060	273473	0.0006	0.0004
0.065	296263	0.0006	0.0004
0.070	319052	0.0006	0.0004
0.075	341842	0.0006	0.0004
0.080	364631	0.0007	0.0004
0.085	387420	0.0007	0.0004
0.090	410210	0.0007	0.0004
0.095	432999	0.0007	0.0004
0.100	455789	0.0007	0.0005
0.105	478578	0.0008	0.0005
0.110	501368	0.0008	0.0005



6-74 For saturated liquid water flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of saturated liquid water at 5°C are μ = 1.519 × 10^{-3} kg/m·s, ρ = 999.9 kg/m^3, and Pr = 11.2 (Table A-9).

Analysis The Reynolds number at x = 0.5m is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{(999.9\ kg/m^3)(1\ m/s)(0.5\ \mathrm{ft})}{1.519\times10^{-3}\ kg/m\cdot s} = 3.29\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \quad \text{where} \quad \mathrm{Re}_x = \frac{\rho V x}{\mu}$$

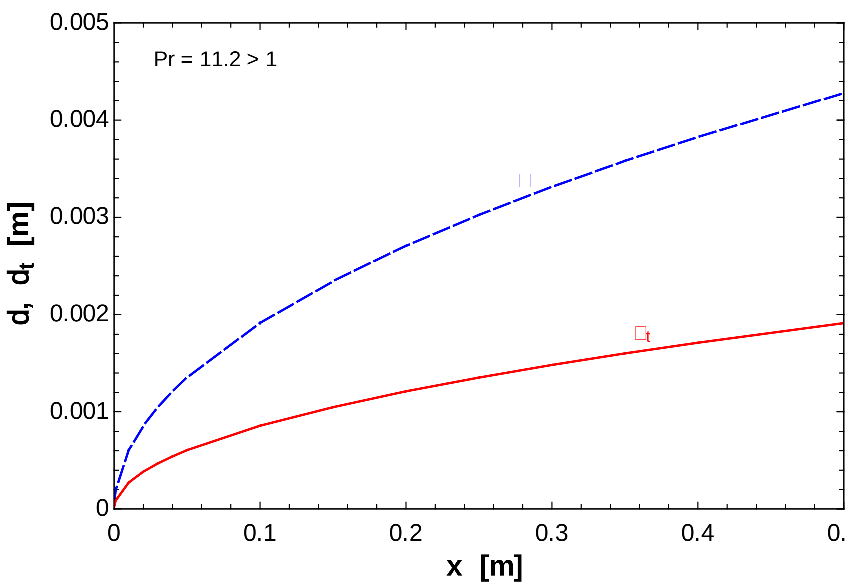
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=1 [m/s]

"PROPERTIES"
mu=1.519e-3 [kg/m-s]
rho=999.9 [kg/m^3]
Pr=11.2

"ANALYSIS"
Re_x=rho*V*x/mu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.00006052	0.00002705
0.001	0.0001914	0.00008553
0.01	0.0006052	0.0002705
0.02	0.0008558	0.0003825
0.03	0.001048	0.0004685
0.04	0.001210	0.0005410
0.05	0.001353	0.0006048
0.10	0.001914	0.0008553
0.15	0.002344	0.001048
0.20	0.002706	0.001210
0.25	0.003026	0.001352
0.30	0.003315	0.001482
0.35	0.003580	0.001600
0.40	0.003827	0.001711
0.50	0.004279	0.001913



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as δ/δ_t = Pr$^{1/3}$. Since Pr > 1 for saturated liquid water, the velocity boundary layer develops quicker than the thermal boundary layer along the plate.

6-75 For mercury flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of mercury at 0°C are $\nu = 1.241 \times 10^{-7}$ m^2/s, and Pr = 0.0289 (Table A-14).

Analysis The Reynolds number at x = 0.5 m is

$$\mathrm{Re}_x = \frac{\mathrm{V}x}{\nu} = \frac{(0.1\ \mathrm{m/s})(0.5\ \mathrm{m})}{1.241\times10^{-7}\ \mathrm{m}^2/\mathrm{s}} = 4.03\times10^5 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \quad \text{where} \quad \mathrm{Re}_x = \frac{Vx}{\nu}$$

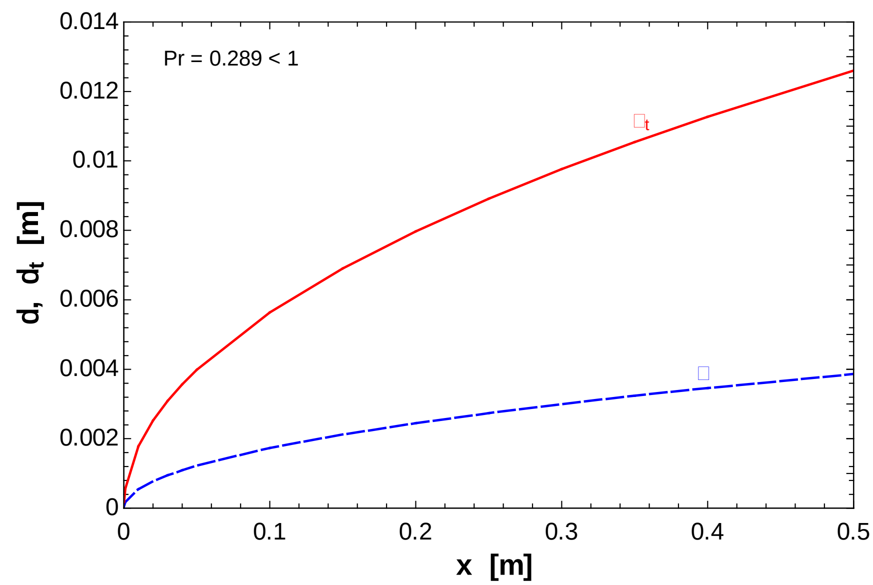
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=0.1 [m/s]

"PROPERTIES"
nu=1.241e-7 [m^2/s]
Pr=0.0289

"ANALYSIS"
Re_x=V*x/nu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.0000547	0.0001782
0.001	0.0001730	0.0005636
0.01	0.0005470	0.001782
0.02	0.0007735	0.002521
0.03	0.0009474	0.003087
0.04	0.001094	0.003565
0.05	0.001223	0.003986
0.10	0.001730	0.005636
0.15	0.002118	0.006903
0.20	0.002446	0.007971
0.25	0.002735	0.008912
0.30	0.002996	0.009763
0.35	0.003236	0.01054
0.40	0.003459	0.01127
0.50	0.003868	0.01260



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as δ/δ_t = Pr$^{1/3}$. Since Pr < 1 for mercury, the velocity boundary layer develops slower than the thermal boundary layer along the plate.

6-76 For water vapor flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of water vapor at 0°C and 1 atm are $\nu = 1.114 \times 10^{-5}$ m^2/s, and Pr = 1.0033 (Table A-16).

Analysis The Reynolds number at x = 0.5 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(10\ \mathrm{m/s})(0.5\ \mathrm{m})}{1.114\times10^{-5}\ \mathrm{m^2/s}} = 4.49\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \quad \text{where} \quad \mathrm{Re}_x = \frac{Vx}{\nu}$$

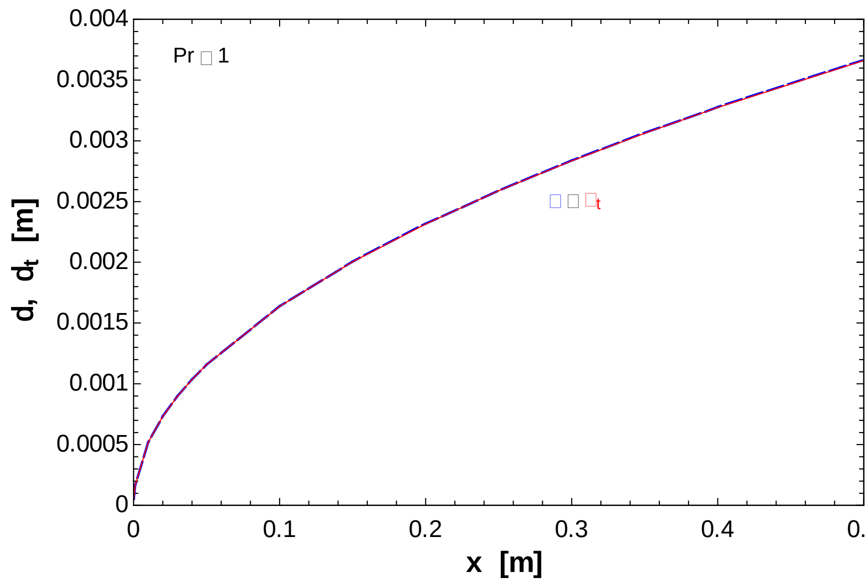
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=10 [m/s]

"PROPERTIES"
nu=1.114e-5 [m^2/s]
Pr=1.0033

"ANALYSIS"
Re_x=V*x/nu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.00005182	0.00005177
0.001	0.0001639	0.0001637
0.01	0.0005182	0.0005177
0.02	0.0007329	0.0007321
0.03	0.0008976	0.0008966
0.04	0.001036	0.001035
0.05	0.001159	0.001158
0.10	0.001639	0.001637
0.15	0.002007	0.002005
0.20	0.002318	0.002315
0.25	0.002591	0.002588
0.30	0.002838	0.002835
0.35	0.003066	0.003063
0.40	0.003278	0.003274
0.50	0.003664	0.003660



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as δ/δ_t = Pr$^{1/3}$. Since Pr ≈ 1 for water vapor, the velocity and thermal boundary layer thicknesses are approximately the same along the plate.

6-77 A laminar ideal gas flows over a flat plate. Using the given $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$ expression, the formulation for local convection heat transfer coefficient, $h_x = C[V/(xT)]^m$, is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Gas behaves as ideal gas.

Analysis Using the definitions for Nusselt, Prandtl, and Reynolds numbers, we have

$$\mathrm{Nu}_x = \frac{h_x x}{k}, \quad \mathrm{Pr} = \frac{c_p \mu}{k}, \quad \text{and} \quad \mathrm{Re}_x = \frac{\rho V x}{\mu}$$

Hence

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad \frac{h_x x}{k} = 0.332\left(\frac{\rho V x}{\mu}\right)^{1/2}\left(\frac{c_p \mu}{k}\right)^{1/3}$$

For ideal gas, the density is $\rho = P/(RT)$, thus

$$\frac{h_x x}{k} = 0.332\left(\frac{P V x}{R T \mu}\right)^{1/2}\left(\frac{c_p \mu}{k}\right)^{1/3}$$

Simplifying we get

$$h_x = 0.332\frac{c_p^{1/3} k^{2/3}}{\mu^{1/6}}\left(\frac{P}{R}\right)^{1/2}\left(\frac{V}{xT}\right)^{1/2}$$

Or $\quad h_x = C[V/(xT)]^m$

where

$$C = 0.332\frac{c_p^{1/3} k^{2/3}}{\mu^{1/6}}\left(\frac{P}{R}\right)^{1/2} \quad \text{and} \quad m = \frac{1}{2}$$

Discussion The temperature in the $h_x = C[V/(xT)]^m$ expression is an absolute temperature, since it was derived from the ideal gas law.

6-78 For air flowing over a flat plate, the convection heat transfer coefficients and the Nusselt numbers at x = 0.5 m and 0.75 m are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Edge effects are negligible. **4** Flow is laminar.

Properties The properties of air at 40°C are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, and Pr = 0.7255 (Table A-15).

Analysis The Reynolds number at x = 0.75 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(3\,\mathrm{m/s})(0.75\,\mathrm{m})}{1.702\times10^{-5}\ m^2/s} = 1.32\times10^5 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.75\text{m)}$$

For laminar flow over a flat plate, the local convection heat transfer coefficients can be determined using

$$h_x = 0.332\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu\,x}\right)^{0.5}$$

At x = 0.5 m

$$h_x = 0.332(0.7255)^{1/3}(0.02662\ \mathrm{W/m\cdot K})\left[\frac{3\,\mathrm{m/s}}{(1.702\times10^{-5}\ \mathrm{m^2/s})(0.5\,\mathrm{m})}\right]^{0.5} = \mathbf{4.715\ \ W/m^2\cdot K}$$

At x = 0.75 m

$$h_x = 0.332(0.7255)^{1/3}(0.02662\ \mathrm{W/m\cdot K})\left[\frac{3\,\mathrm{m/s}}{(1.702\times10^{-5}\ \mathrm{m^2/s})(0.75\,\mathrm{m})}\right]^{0.5} = \mathbf{3.850\ W/m^2\cdot K}$$

The Nusselt numbers are, at x = 0.5 m,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{(4.715\ \mathrm{W/m^2\cdot K})(0.5\,\mathrm{m})}{0.02662\ \mathrm{W/m\cdot K}} = \mathbf{88.6}$$

At x = 0.75 m

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{(3.850\ \mathrm{W/m^2\cdot K})(0.75\,\mathrm{m})}{0.02662\ \mathrm{W/m\cdot K}} = \mathbf{108.5}$$

Discussion The average convection heat transfer coefficient of the plate can be determined by integrating h_x over the plate length $0 \le x \le 1$ m.

6-79 For air flowing over a flat plate, the effect of the location along the plate (x) on the heat transfer coefficient and the surface temperature gradient of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at 40°C and 1 atm are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, and Pr = 0.7255 (Table A-15). The thermal conductivity of the plate is k_{plate} = 15 W/m·K.

Analysis The Reynolds number at x = 0.50 m is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(3\,\text{m/s})(0.50\,\text{m})}{1.702\times10^{-5}\,\text{m}^2/\text{s}} = 8.813\times10^4 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.50\text{m)}$$

For laminar flow over a flat plate, the local convection heat transfer coefficients can be determined using

$$h_x = 0.332\,\text{Pr}^{1/3}\,k\left(\frac{V}{\nu\,x}\right)^{0.5}$$

The surface temperature gradient of the plate can be determined using

$$\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = -\frac{h_x}{k_{\text{plate}}}(T_s - T_\infty)$$

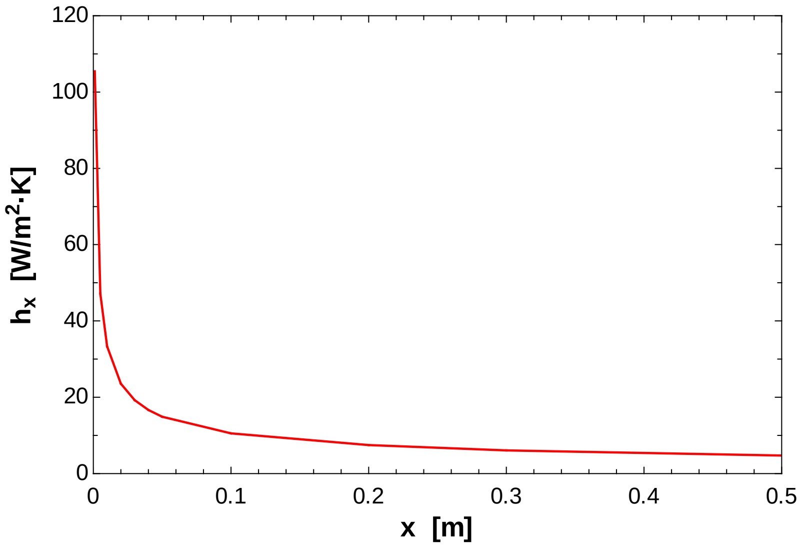
The problem is solved using EES, and the solution is given below:

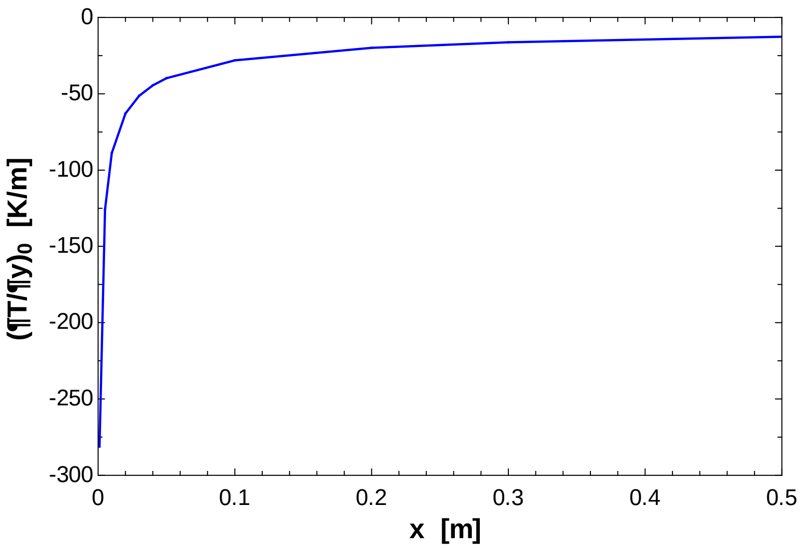
```
"GIVEN"
V=3 [m/s]
T_infinity=20 [C]
T_s=60 [C]

"PROPERTIES"
"For Air"
nu=1.702e-5 [m^2/s]
k=0.02662 [W/m-K]
Pr=0.7255
"For Plate"
k_plate=15 [W/m-K]

"ANALYSIS"
h_x=0.332*Pr^(1/3)*k*(V/(nu*x))^0.5
dTdy_0=-h_x/k_plate*(T_s-T_infinity)   "Surface temperature gradient of the plate"
```

x [m]	h_x [W/m^2·K]	$(\partial T/\partial r)_{0,\text{plate}}$ [K/m]
0.001	105.4	−281.2
0.005	47.15	−125.7
0.01	33.34	−88.91
0.02	23.58	−62.87
0.03	19.25	−51.33
0.04	16.67	−44.45
0.05	14.91	−39.76
0.1	10.54	−28.12
0.2	7.455	−19.88
0.3	6.087	−16.23
0.5	4.715	−12.57





Discussion As the magnitude of the surface temperature gradient decreases along *x*, so does the value of the convection heat transfer coefficient.

6-80 The ratio of the average convection heat transfer coefficient (h) to the local convection heat transfer coefficient (h_x) is to be determined from a given relationship for h_x.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis From the given relationship for $h_x = Cx^{-0.5}$

At $x = L$, the local convection heat transfer coefficient is $h_{x=L} = CL^{-0.5}$. The average convection heat transfer coefficient over the entire plate length is

$$h = \frac{1}{L}\int_0^L h_x dx = \frac{C}{L}\int_0^L x^{-0.5} dx = \frac{2C}{L}L^{0.5} = 2CL^{-0.5}$$

Taking the ratio of h to h_x at $x = L$, we get

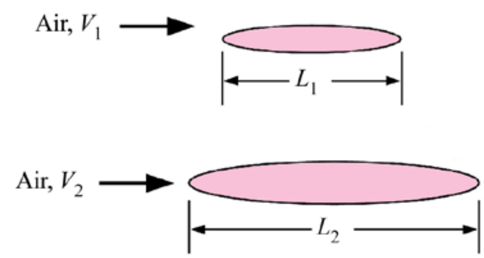
$$\frac{h}{h_{x=L}} = \frac{2CL^{-0.5}}{CL^{-0.5}} = \mathbf{2}$$

Discussion For laminar flow and constant properties, it should be noted that ratio of the average Nusselt number over the entire plate length to the local Nusselt number at the end of the plate is also $\text{Nu}/\text{Nu}_{x=L} = 2$.

6-81E Two airfoils with different characteristic lengths are placed in airflow of different free stream velocities at 1 atm and 60°F. The heat flux from the second airfoil is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Both airfoils are geometrically similar.

Analysis The relation for Nusselt, Prandtl, and Reynolds numbers is given as



$$\text{Nu} = g(\text{Re}, \text{Pr})$$

where $\text{Nu} = \dfrac{hL}{k}$, $\text{Pr} = \dfrac{c_p \mu}{k}$, and $\text{Re} = \dfrac{VL}{\nu}$

Then

Airfoil 1: $\text{Re}_1 = \dfrac{VL_1}{\nu} = \dfrac{(150\text{ ft/s})(0.2\text{ ft})}{\nu} = \dfrac{30\text{ ft}^2/\text{s}}{\nu}$ and $\text{Pr}_1 = \dfrac{c_p \mu}{k}$

Airfoil 2: $\text{Re}_2 = \dfrac{VL_2}{\nu} = \dfrac{(75\text{ ft/s})(0.4\text{ ft})}{\nu} = \dfrac{30\text{ ft}^2/\text{s}}{\nu}$ and $\text{Pr}_2 = \dfrac{c_p \mu}{k}$

Since the fluid properties are constant, we have $\text{Re}_1 = \text{Re}_2$ and $\text{Pr}_1 = \text{Pr}_2$, which implies

$$\text{Nu}_1 = g(\text{Re}_1, \text{Pr}_1) = \text{Nu}_2 = g(\text{Re}_2, \text{Pr}_2) \quad \rightarrow \quad \text{Nu}_1 = \text{Nu}_2$$

Hence

$$\frac{h_1 L_1}{k} = \frac{h_2 L_2}{k} \quad \rightarrow \quad h_2 = h_1 \frac{L_1}{L_2} = h_1 \frac{0.2}{0.4} = 0.5h_1$$

Therefore the average convection heat transfer coefficient for airfoil 2 is

$$h_2 = 0.5h_1 = 0.5(21\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}) = 10.5\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

The heat flux from the airfoil 2 is

$$\dot{q}_2 = h_2(T_s - T_\infty) = (10.5\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(180-60)^\circ\text{F} = \mathbf{1260\,Btu/h\cdot ft^2}$$

Discussion The relation for the Nusselt numbers $\text{Nu}_1 = \text{Nu}_2$ is valid due to $\text{Re}_1 = \text{Re}_2$ and $\text{Pr}_1 = \text{Pr}_2$.

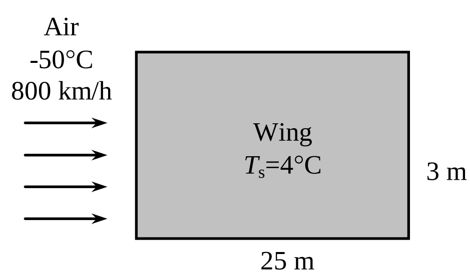
Momentum and Heat Transfer Analogies

6-82C Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2}=\mathrm{Nu}_x$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is limited to flow of fluids with a Prandtl number of near unity (such as gases), and negligible pressure gradient in the flow direction (such as flow over a flat plate).

6-83C Modified Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2}=\mathrm{Nu}_x\,\mathrm{Pr}^{-1/3}$ or $\dfrac{C_{f,x}}{2}=\dfrac{h_x}{\rho c_p V}\mathrm{Pr}^{2/3}\equiv j_H$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is valid for a Prandtl number range of 0.6 < Pr < 60. This relation is developed using relations for laminar flow over a flat plate, but it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

6-84 An airplane cruising is considered. The average heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at -50°C and 1 atm are (Table A-15)

$$c_p = 0.999\ \text{kJ/kg}\cdot\text{K} \qquad \mathrm{Pr} = 0.7440$$

The density of air at -50°C and 26.5 kPa is

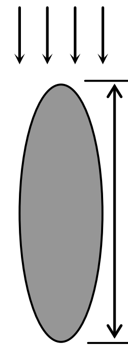
$$\rho=\frac{P}{RT}=\frac{26.5\,\text{kPa}}{(0.287\,\text{kJ/kg.K})(\text{-}50+273)\text{K}}=0.4141\,\text{kg/m}^3$$

Analysis The average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h=\frac{C_f}{2}\frac{\rho V c_p}{\mathrm{Pr}^{2/3}}=\frac{0.0016}{2}\frac{(0.4141\,\text{kg/m}^3)(800/3.6\,\text{m/s})(999\,\text{J/kg}\cdot{}^\circ\text{C})}{(0.7440)^{2/3}}=\mathbf{89.6\ W/m^2\cdot C}$$

6-85 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

ρ = 1.184 kg/m^3, c_p =1.007 kJ/kg·K, Pr = 0.7296

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mc_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\ \text{J/kg}\cdot°\text{C})(160-150)°\text{C}}{(2\times 60\ \text{s})} = 2083\ \text{W}$$

Then the average heat transfer coefficient is

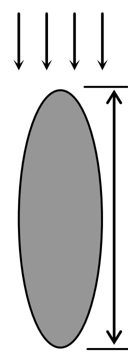
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\ \text{W}}{(12\ \text{m}^2)(155-25)°\text{C}} = 1.335\ \text{W/m}^2\cdot°\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(1.335\ \text{W/m}^2\cdot°\text{C})(0.7296)^{2/3}}{(1.184\ \text{kg/m}^3)(5\ \text{m/s})(1007\ \text{J/kg}\cdot°\text{C})} = \mathbf{0.000363}$$

6-86 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

ρ = 1.184 kg/m^3, c_p =1.007 kJ/kg·K, Pr = 0.7296

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mc_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\ \text{J/kg}\cdot°\text{C})(160-150)°\text{C}}{(2\times 60\ \text{s})} = 2083\ \text{W}$$

Then the average heat transfer coefficient is

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\ \text{W}}{(12\ \text{m}^2)(155-25)°\text{C}} = 1.335\ \text{W/m}^2\cdot°\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

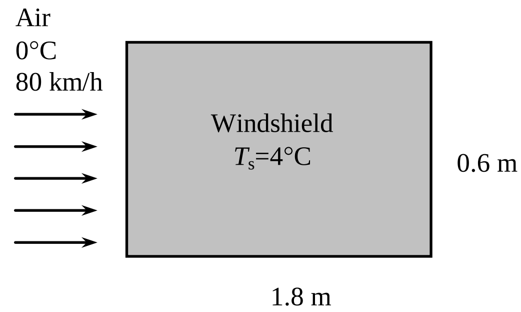
$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(1.335\ \text{W/m}^2\cdot°\text{C})(0.7296)^{2/3}}{(1.184\ \text{kg/m}^3)(10\ \text{m/s})(1007\ \text{J/kg}\cdot°\text{C})} = \mathbf{0.000181}$$

6-87 The windshield of a car is subjected to parallel winds. The drag force the wind exerts on the windshield is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 0°C and 1 atm are (Table A-15)

$\rho = 1.292$ kg/m^3, c_p =1.006 kJ/kg·K, Pr = 0.7362



Analysis The average heat transfer coefficient is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)}$$

$$= \frac{50 \text{ W}}{(0.6 \times 1.8 \text{ m}^2)(4-0)°\text{C}} = 11.57 \text{ W/m}^2 \cdot °\text{C}$$

The average friction coefficient is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(11.57 \text{ W/m}^2 \cdot °\text{C})(0.7362)^{2/3}}{(1.292 \text{ kg/m}^3)(80/3.6 \text{ m/s})(1006 \text{ J/kg} \cdot °\text{C})} = 0.0006534$$

The drag force is determined from

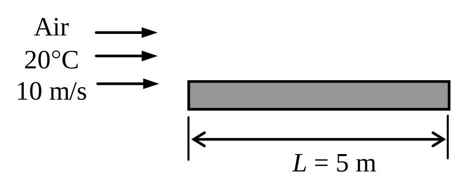
$$F_f = C_f A_s \frac{\rho V^2}{2} = (0.0006534)(0.6 \times 1.8 \text{ m}^2)\frac{(1.292 \text{ kg/m}^3)(80/3.6 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg.m/s}^2}\right) = \mathbf{0.225 \text{ N}}$$

6-88 A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection heat transfer coefficient and the rate of heat transfer on the upper surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 20°C and 1 atm are (Table A-15)

$\rho = 1.204$ kg/m^3, c_p =1.007 kJ/kg-K, Pr = 0.7309



Analysis The flow is along the 5-m side of the plate, and thus the characteristic length is L = 5 m. The surface area of the upper surface is

$$A_s = WL = (5 \text{ m})(5 \text{ m}) = 25 \text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \longrightarrow C_f = \frac{F_f}{\rho A_s V^2/2} = \frac{2.4 \text{ N}}{(1.204 \text{ kg/m}^3)(25 \text{ m}^2)(10 \text{ m/s})^2/2}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 0.001595$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\text{Pr}^{2/3}} = \frac{0.001595}{2}\frac{(1.204 \text{ kg/m}^3)(10 \text{ m/s})(1007 \text{ J/kg} \cdot °\text{C})}{(0.7309)^{2/3}} = \mathbf{11.91 \text{ W/m}^2 \cdot \text{C}}$$

Them the rate of heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (11.91 \text{ W/m}^2 \cdot °\text{C})(25 \text{ m}^2)(80-20)°\text{C} = \mathbf{17{,}900 \text{ W}}$$

6-89 Air is flowing in parallel to a stationary thin flat plate over the top and bottom surfaces. The rate of heat transfer from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The edge effects are negligible.

Properties The properties of air (1 atm) at the film temperature of $T_f = (T_s + T_\infty)/2 = 20°C$ are given in Table A-15: $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, and Pr = 0.7309.

Analysis The flow is over the top and bottom surfaces of the plate, hence the total surface area is

$$A_s = 2(1\,\text{m})(1\,\text{m}) = 2\,\text{m}^2$$

For flat plate, the friction force can be determined using

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad \rightarrow \quad C_f = \frac{2F_f}{\rho A_s V^2}$$

Using the Chilton-Colburn analogy, the convection heat transfer coefficient is determined to be:

$$\frac{C_f}{2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad \rightarrow \quad h = \frac{C_f}{2}\rho c_p V\,\text{Pr}^{-2/3}$$

$$h = \frac{F_f}{A_s V}c_p\,\text{Pr}^{-2/3} = \frac{(0.1\,\text{N})}{(2\,\text{m}^2)(2\,\text{m/s})}(1007\,\text{J/kg}\cdot\text{K})(0.7309)^{-2/3} = 31.03\,\text{W/m}^2\cdot\text{K}$$

The rate of heat transfer from the flat plate is

$$\dot{Q} = hA_s(T_s - T_\infty) = (31.03\,\text{W/m}^2\cdot\text{K})(2\,\text{m}^2)(35-5)\,\text{K} = \mathbf{1862\,W}$$

Discussion The friction force asserted on the flat plate is due to the shear stress on the plate surfaces.

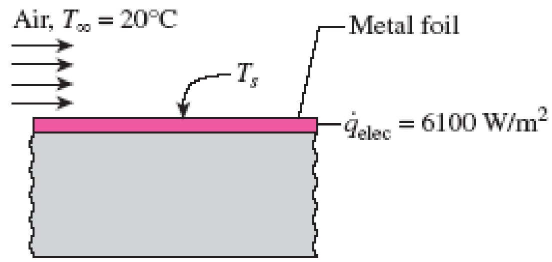
6-90 Air is flowing in parallel to a stationary thin flat plate over the top surface. The rate of heat transfer from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at the 100°C are given in Table A-15: $\rho = 0.9458$ kg/m^3, $c_p = 1009$ J/kg·K, $\nu = 2.306 \times 10^{-5}$ m^2/s, and Pr = 0.7111.

Analysis The flow is over the top surface of the metal foil, hence the surface area is

$$A_s = (0.2\text{ m})(0.5\text{ m}) = 0.1\text{ m}^2$$



For flat plate, the friction force can be determined using

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad \rightarrow \quad C_f = \frac{2F_f}{\rho A_s V^2}$$

Using the Chilton-Colburn analogy, the convection heat transfer coefficient is determined to be:

$$\frac{C_f}{2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad \rightarrow \quad h = \frac{C_f}{2}\rho c_p V\,\text{Pr}^{-2/3}$$

$$h = \frac{F_f}{A_s V}c_p\,\text{Pr}^{-2/3} = \frac{(0.3\text{ N})}{(0.1\text{ m}^2)(100\text{ m/s})}(1009\text{ J/kg}\cdot\text{K})(0.7111)^{-2/3} = 38\text{ W/m}^2\cdot\text{K}$$

The surface temperature of the metal foil is

$$\dot{q} = h(T_s - T_\infty) \quad \rightarrow \quad T_s = \frac{\dot{q}}{h} + T_\infty = \frac{6100\text{ W/m}^2}{38\text{ W/m}^2\cdot\text{K}} + 20°\text{C} = \mathbf{181°C}$$

Discussion The temperature, at 100°C, used for evaluating the fluid properties turned out to be appropriate, since the film temperature is

$$T_f = \frac{T_s + T_\infty}{2} = 101°\text{C} \approx 100°\text{C}$$

6-91 Air at 1 atm is flowing over a flat plate. The friction coefficient and wall shear stress at a location 2 m from the leading edge are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at 20°C are given in Table A-15: $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, and Pr = 0.7309.

Analysis At the location $x = 2$ m from the leading edge, the Reynolds number is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(70\text{ m/s})(2\text{ m})}{1.516\times10^{-5}\text{ m}^2/\text{s}} = 9.235\times10^6$$

Applying the modified Reynolds analogy,

$$\frac{C_{f,x}}{2} = \text{St}_x\,\text{Pr}^{2/3} = \frac{\text{Nu}_x}{\text{Re}_x\,\text{Pr}}\text{Pr}^{2/3}$$

Substituting the given correlation for Nusselt number, we get

$$\frac{C_{f,x}}{2} = \frac{0.03\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}}{\text{Re}_x\,\text{Pr}}\text{Pr}^{2/3} = 0.03\,\text{Re}_x^{-0.2} \qquad \text{or} \qquad C_{f,x} = 0.06\,\text{Re}_x^{-0.2}$$

The friction coefficient at $x = 2$ m is

$$C_{f,x} = 0.06\,\text{Re}_x^{-0.2} = 0.06(9.235\times10^6)^{-0.2} = \mathbf{0.002427}$$

The wall shear stress is

$$\tau_s = C_{f,x}\frac{\rho V^2}{2} = (0.002427)\frac{(1.204\text{ kg/m}^3)(70\text{ m/s})^2}{2} = \mathbf{7.16\ N/m^2}$$

Discussion At $x = 2$ m from the leading edge, the flow is turbulent. Since the Reynolds number at that location is greater than 5×10^5.

6-92 Metal plates are subject to parallel air flow cooling. The average convection heat transfer coefficient for the plates are to be determined from a given average friction coefficient over each plate.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at 20°C and 1 atm are $\nu = 1.516 \times 10^{-5}$ m^2/s, $\rho = 1.204$ kg/m^3, $c_p = 1007$J/kg·K, and Pr = 0.7309 (Table A-15).

Analysis The Reynolds number for the plate is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(5\text{ m/s})(1\text{ m})}{1.516\times10^{-5}\text{ m}^2/\text{s}} = 3.298\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 1\text{m)}$$

The average friction coefficient over the plate is

$$C_f = 1.33\,\text{Re}_L^{-0.5} = 1.33(3.298\times10^5)^{-0.5} = 0.002316$$

The average convection heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\text{Pr}^{2/3}} = \frac{0.002316}{2}\frac{(1.204\text{ kg/m}^3)(5\text{ m/s})(1007\text{ J/kg}\cdot\text{K})}{(0.7309)^{2/3}} = \mathbf{8.65\ \ W/m^2\cdot K}$$

Discussion The given $C_f = 1.33(\text{Re}_L)^{-0.5}$ equation is valid up to $V = 7.58$ m/s to satisfy the $\text{Re}_L < 5 \times 10^5$ condition.

6-93 A flat plate is subjected to air flow parallel to its surface. The effect of air velocity on the average convection heat transfer coefficient for the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at 20°C and 1 atm are $\nu = 1.516 \times 10^{-5}$ m^2/s, $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, and Pr = 0.7309 (Table A-15).

Analysis The air velocity for $\text{Re}_L < 5 \times 10^5$ is

$$\text{Re}_L = \frac{VL}{\nu} \quad \rightarrow \quad V = \frac{\nu\,\text{Re}_L}{L} = \frac{(1.516\times10^{-5}\ \text{m}^2/\text{s})(5\times10^5)}{1\ \text{m}} = 7.58\ \text{m/s} \quad \rightarrow \quad V < 7.58\ \text{m/s}$$

For $V < 7.58$ m/s, we have $\text{Re}_L < 5 \times 10^5$; and for $V > 7.58$ m/s, we have $\text{Re}_L > 5 \times 10^5$.

Thus,

$$C_f = 1.33\,\text{Re}_L^{-1/2} \quad \text{for} \quad V < 7.58\ \text{m/s} \quad \text{(laminar flow)}$$

$$C_f = 0.074\,\text{Re}_L^{-1/5} \quad \text{for} \quad 7.58 \le V \le 20\ \text{m/s} \quad \text{(turbulent flow)}$$

The average convection heat transfer coefficient can be determined from the modified Reynolds analogy

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\text{Pr}^{2/3}}$$

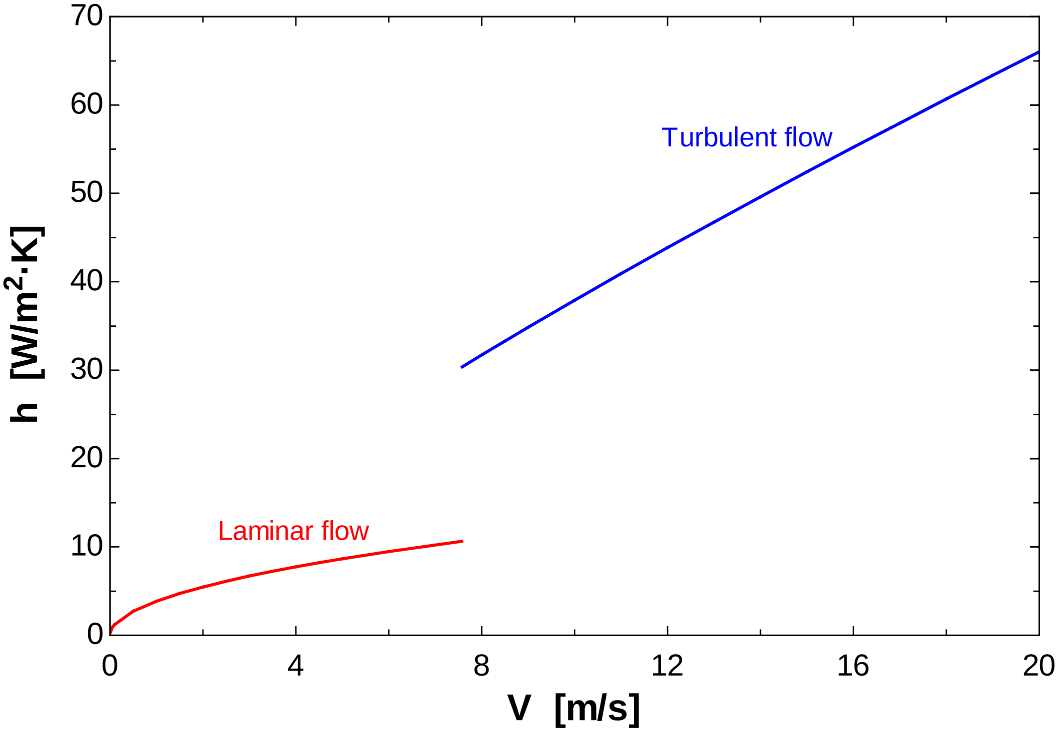
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
L=1 [m]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]
c_p=1007 [J/kg-K]
Pr=0.7309

"ANALYSIS"
C_f_lam=1.33*(V_lam*L/nu)^(-1/2)
C_f_turb=0.074*(V_turb*L/nu)^(-1/5)
h_lam=(C_f_lam/2)*(rho*V_lam*c_p)/(Pr^(2/3))
h_turb=(C_f_turb/2)*(rho*V_turb*c_p)/(Pr^(2/3))
```

V [m/s]	h [W/m^2·K]	V [m/s]	h [W/m^2·K]
0.01	0.3869	7.58	30.37
0.05	0.8651	8.0	31.71
0.1	1.223	8.5	33.29
0.5	2.736	9.0	34.85
1.0	3.869	10	37.91
1.5	4.738	11	40.91
2.0	5.471	12	43.86
2.5	6.117	13	46.76
3.0	6.701	14	49.62
3.5	7.238	15	52.44
4.0	7.738	16	55.21
4.5	8.207	17	57.96
5.0	8.651	18	60.67
6.0	9.477	19	63.35
7.57	10.64	20	66.01

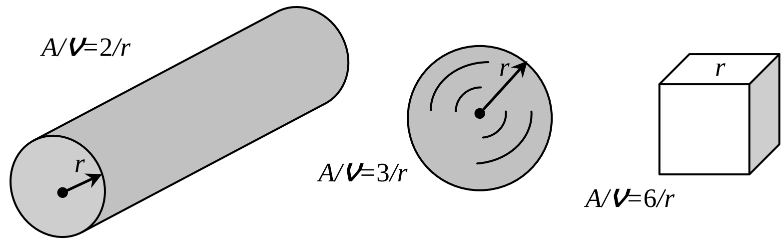


Discussion The convection heat transfer coefficient in turbulent flow is significantly higher than that of laminar flow.

Special Topic: Microscale Heat Transfer

6-94 It is to be shown that the rate of heat transfer is inversely proportional to the size of an object.

Analysis Consider a cylinder of radius *r* and length *l*. The surface area of this cylinder is $A = 2\pi r(l + r)$ and its volume is $A = \pi r^2 l$. Therefore, the area per unit volume is $\frac{2(l+r)}{rl}$ which, for a long tube *l* <<*r*, becomes $\frac{A}{V} = \frac{2}{r}$. Similarly, it can be shown that the surface area to volume ratio is $\frac{3}{r}$ for a sphere of radius *r*, and $\frac{6}{r}$ for a cube of side *r*.



Note that as *r* becomes smaller, the surface to volume ratio increases. Specifically, this means that while the surface area is about the same order of that of the volume of macroscale (meter, centimeter scale) objects, but the surface becomes million or more times the volume as the size of the object goes to micrometer scale or below. Since, convective heat transfer is proportional to $A(T - T_\infty)$, heat flow increases as *A* increases.

6-95 For specified wall and fluid temperatures, the heat flux at the wall of a microchannel is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties for both cases are given.

Analysis**:** (*a*) The gas and wall temperatures are T_g =100°C = 373 K, T_w = 50°C = 323 K. Then,

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\left(\frac{\lambda}{\Pr}\right)\left(\frac{\partial T}{\partial y}\right)_w = \left(\frac{2-1}{1}\right)\left(\frac{2\times 1.667}{2.667}\right)(0.5)\left(\frac{\partial T}{\partial y}\right)_w$$

$$\left(\frac{\partial T}{\partial y}\right)_w = \frac{T_g - T_w}{0.625} = \frac{373-323}{0.625} = 80\ \text{K/m}$$

Therefore, the wall heat flux is

$$-k\left(\frac{\partial T}{\partial y}\right)_w = (0.15\ \text{W/m}\cdot\text{K})(80\ \text{K/m}) = \mathbf{12\ W/m^2}$$

(*b*) Repeating the same calculations for a different set of properties,

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\left(\frac{\lambda}{\Pr}\right)\left(\frac{\partial T}{\partial y}\right)_w = \left(\frac{2-0.8}{0.8}\right)\left(\frac{2\times 2}{2+1}\right)(5)\left(\frac{\partial T}{\partial y}\right)_w$$

$$\left(\frac{\partial T}{\partial y}\right)_w = \frac{T_g - T_w}{10} = \frac{373-323}{10} = 5\ \text{K/m}$$

$$-k\left(\frac{\partial T}{\partial y}\right)_w = (0.1\ \text{W/m}\cdot\text{K})(5\ \text{K/m}) = \mathbf{0.5\ W/m^2}$$

6-96 For a specified temperature gradient, the Nusselt numbers associated with ambient air and nitrogen gas are to be determined.

Assumptions Steady operating conditions exist.

***Analysis*:** At the outer surface of the microchannel (assuming it to be infinitesimally thin), the heat transferred through the channel fluid (gas-wall interface) outward should balance the heat convected outside, and

$$h(T_a - T_w) = -k\left(\frac{\partial T}{\partial y}\right)_w$$

Therefore, the Nusselt number for cooling is

$$\text{Nu} = \frac{h}{k}L = \frac{-(\partial T/dy)_w}{(T_a - T_w)}L$$

The channel is 1.2 μm thick, i.e., $L = 1.2 \times 10^{-6}$ m.

(*a*) For an ambient air temperature of 30°C,

$$\frac{h}{k} = \frac{-(\partial T/dy)_w}{(T_a - T_w)} = \frac{80\text{ K/m}}{(50-30)\text{K}} = 4.0\text{ m}^{-1}$$

Thus,

$$\text{Nu} = hL/k = (4.0\text{ m}^{-1})(1.2\times10^{-6}\text{ m}) = \mathbf{4.80 \times 10^{-6}}$$

(*b*) For a nitrogen gas temperature of -100°C,

$$\frac{h}{k} = \frac{-(\partial T/dy)_w}{(T_a - T_w)} = \frac{80\text{ K/m}}{[50-(-100)]\text{K}} = 0.533\text{ m}^{-1}$$

Thus,

$$\text{Nu} = hL/k = (0.533\text{ m}^{-1})(1.2\times10^{-6}\text{ m}) = \mathbf{6.40 \times 10^{-7}}$$

Review problems

6-97E Prantl number is to be determined for a given set of properties.

Assumptions None.

Properties The given properties are: c_p = 0.5 Btu/lbm·R, k = 2 Btu/h·ft·R, μ = 0.3 lbm/ft·s.

Analysis The Prandtl number is

$$\text{Pr} = \frac{c_p\,\mu}{k} = \frac{(0.5\,\text{Btu/lbm}\cdot\text{R})(0.3\,\text{lbm/ft}\cdot\text{s})(3600\,\text{s/h})}{2\,\text{Btu/h}\cdot\text{ft}\cdot\text{R}} = \mathbf{270}$$

Discussion Typically Prandtl number of liquids and gases is listed along with other properties in the property tables in the Appendix. However, if it is not listed, it can be obtained from the other properties. An alternative to the above equation is Pr = ν/α where $\nu = \mu/\rho$ and $\alpha = k/\rho c_p$.

6-98 Determine whether the flow is laminar or turbulent over a flat plate for different fluids at a given temperature.

Assumptions **1** Transition from laminar to turbulent flow over the flat plate occurs at a Reynolds number of 5×10^5

Properties The properties of the fluids are evaluated at 50°C. For Air: $\nu = 1.798 \times 10^{-5}$ m²/s (Table A-15). For CO_2: $\nu = 9.714 \times 10^{-6}$ m²/s (Table A-16). For Water: $\rho = 988.1$ kg/m³, $\mu = 0.547 \times 10^{-3}$ kg/m·s $\rightarrow$ $\nu = \mu/\rho = 5.536 \times 10^{-7}$ m²/s (Table A-9). For Engine oil (unused): $\nu = 1.671 \times 10^{-4}$ m²/s (Table A-13).

Analysis The Reynolds number for the plate is

$$\text{Re}_L = \frac{VL}{\nu}$$

For Air: $$\text{Re}_L = \frac{VL}{\nu} = \frac{(5\text{ m/s})(0.15\text{ m})}{1.798\times10^{-5}\text{ m}^2\text{/s}} = \mathbf{4.17\times10^4} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

For CO_2: $$\text{Re}_L = \frac{VL}{\nu} = \frac{(5\text{ m/s})(0.15\text{ m})}{9.714\times10^{-6}\text{ m}^2\text{/s}} = \mathbf{7.72\times10^4} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

For Water: $$\text{Re}_L = \frac{VL}{\nu} = \frac{(5\text{ m/s})(0.15\text{ m})}{5.536\times10^{-7}\text{ m}^2\text{/s}} = \mathbf{1.35\times10^6} < 5\times10^5 \quad (\textbf{Turbulent Flow})$$

For Engine oil (unused): $$\text{Re}_L = \frac{VL}{\nu} = \frac{(5\text{ m/s})(0.15\text{ m})}{1.671\times10^{-4}\text{ m}^2\text{/s}} = \mathbf{4.49\times10^3} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

Discussion The value of the kinematic viscosity dictates whether the flow will be laminar or turbulent for the four fluids. The smaller the value, the higher the Reynolds number.

6-99E Fluid is flowing over a flat plate. The characteristic length (L_c) at which the Reynolds number is 1×10^5 is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Analysis Using the definition for Reynolds number, we have

$$\text{Re} = \frac{VL_c}{\nu} \quad \text{and} \quad \text{Re}_{\text{cr}} = \frac{Vx_{\text{cr}}}{\nu}$$

Taking the ratio yields

$$\frac{\text{Re}}{\text{Re}_{\text{cr}}} = \frac{VL_c/\nu}{Vx_{\text{cr}}/\nu} \quad \rightarrow \quad \frac{\text{Re}}{\text{Re}_{\text{cr}}} = \frac{L_c}{x_{\text{cr}}}$$

The characteristic length (L_c) at which Re = 1×10^5 is

$$L_c = x_{\text{cr}}\frac{\text{Re}}{\text{Re}_{\text{cr}}} = (7\text{ ft})\frac{1\times10^5}{5\times10^5} = \mathbf{1.4\,ft}$$

Discussion In some flow conditions, the value of Re_{cr} may change substantially, depending on the free stream turbulence level.

6-100 The Coutte flow of a fluid between two parallel plates is considered. The temperature distribution is to be sketched and determined, and the maximum temperature of the fluid, as well as the temperature of the fluid at the contact surfaces with the lower and upper plates are to be determined.

Assumptions Steady operating conditions exist.

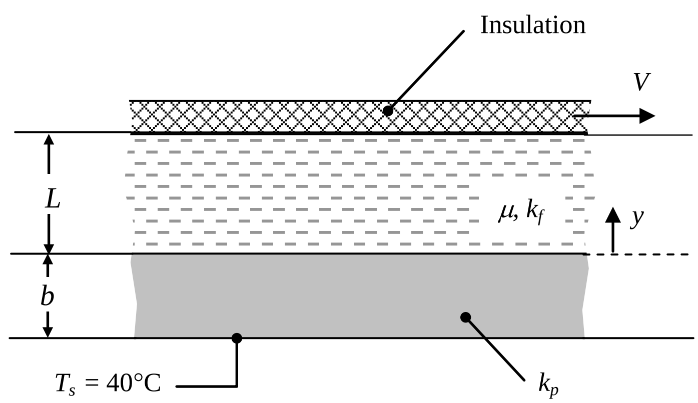
Properties The viscosity and thermal conductivity of the fluid are given to be $\mu = 0.8$ N·s/m^2 and $k_f = 0.145$ W/m·K. The thermal conductivity of lower plate is given to be $k_p = 1.5$ W/m·K.

***Analysis*:** (*a*) The general solution of the relevant differential equation is obtained as follows:

$$u = \frac{y}{L}V \longrightarrow \frac{du}{dy} = \frac{V}{L}$$

$$\frac{d^2T}{dy^2} = \frac{-\mu}{k_f}\frac{V^2}{L^2} \longrightarrow \frac{dT}{dy} = \frac{-\mu}{k_f}\frac{V^2}{L^2}y + C_1$$

$$T = \frac{-\mu}{2k_f}\frac{V^2}{L^2}y^2 + C_1 y + C_2$$



Applying the boundary conditions:

$y = 0$

$$\dot{q}_f = \dot{q}_p \longrightarrow -k_f \left.\frac{dT}{dy}\right|_0 = \frac{T(0) - T_s}{b/k_p}$$

$$k_f C_1 = \frac{k_p}{b}(C_2 - T_s) \qquad (1)$$

$y = L$, adiabatic $\quad \left.\frac{dT}{dy}\right|_L = 0 \longrightarrow C_1 = \frac{\mu}{k_f}\frac{V^2}{L}$

From Eq. (1), $\quad C_2 = b\frac{k_f}{k_p}C_1 + T_s = b\frac{\mu}{k_p}\frac{V^2}{L} + T_s$

Substituting the coefficients, the temperature distribution becomes

$$T(y) = \frac{-\mu}{2k_f}\frac{V^2}{L^2}y^2 + \frac{\mu}{k_f}\frac{V^2}{L}y + \frac{\mu}{k_p}\frac{V^2}{L}b + T_s$$

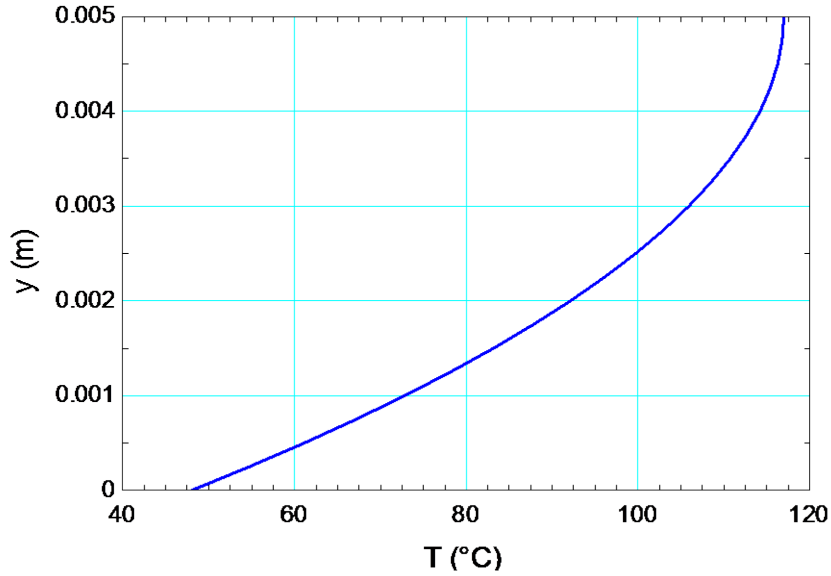
(b) Then the temperatures at the contact surfaces are determined to be

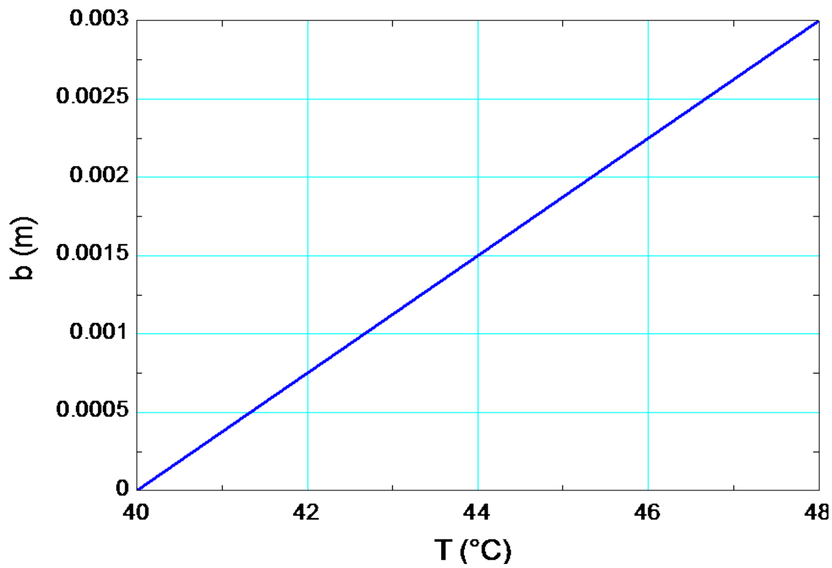
$$T(0) = 0 + 0 + \frac{0.8}{1.5}\frac{5^2}{0.005}0.003 + 40 = \mathbf{48.0°C}$$

$$T(y) = \frac{-0.8}{2(0.145)}\frac{5^2}{0.005^2}0.005^2 + \frac{0.8}{0.145}\frac{5^2}{0.005}0.005 + \frac{0.8}{1.5}\frac{5^2}{0.005}0.003 + 40 = \mathbf{117°C}$$

The maximum temperature is $T_{max} = T(L) = 117°C$ because of the adiabatic condition at $y = L$.

(*c*) The sketch of temperature distribution is given in the following figures. They are obtained using the temperature distribution obtained in part (a). The top figure shows the distribution in the fluid and the bottom one in the plate. We observe from this figure that there are different slopes at the interface ($y = 0$) because of different conductivities ($k_p > k_f$). The slope is zero at the upper plate ($y = L$) because of adiabatic condition.


0.005
0.004
0.003
0.002
0.001
0
y (m)
40
60
80
100
120
T (°C)


0.003
0.0025
0.002
0.0015
0.001
0.0005
0
b (m)
40
42
44
46
48
T (°C)

6-101 The hydrodynamic boundary layer and the thermal boundary layer both as a function of *x* are to be plotted for the flow of engine oil over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-13)

ρ = 876 kg/m^3

c_p = 1964 J/kg·°C

μ = 0.2177 kg/m·s

k = 0.1444 W/m·°C

Pr = 2962

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 15°C

T_s = 65°C

Analysis

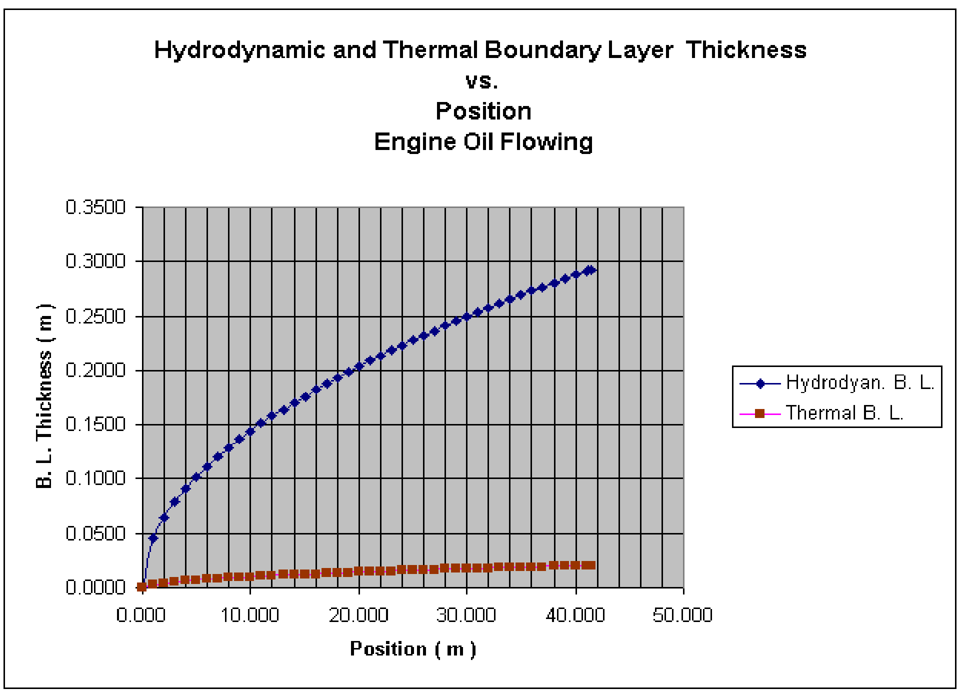
The critical length: $$\text{Re} = \frac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \frac{\text{Re}\,\nu}{V} = \frac{\text{Re}\,\mu}{V\rho} = \frac{(500{,}000)(0.2177\ \text{kg/m}\cdot\text{s})}{(3\ \text{m/s})(876\ \text{kg/m}^3)} = 41.42\ \text{m}$$

Hydrodynamic boundary layer thickness: $$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}}$$

Thermal boundary layer thickness: $$\delta_t = \frac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$$

x (m)	**Re$_x$**	**δ**	**δ$_t$**
0.000	0.000	0	0
1.000	12072	0.0455	0.0032
2.000	24143	0.0644	0.0045
3.000	36215	0.0788	0.0055
4.000	48287	0.0910	0.0063
5.000	60358	0.1018	0.0071
6.000	72430	0.1115	0.0078
7.000	84502	0.1204	0.0084
8.000	96573	0.1287	0.0090
9.000	108645	0.1365	0.0095
10.000	120717	0.1439	0.0100
11.000	132788	0.1509	0.0105

12.000	144860	0.1576	0.0110
13.000	156932	0.1641	0.0114
14.000	169003	0.1703	0.0119
15.000	181075	0.1763	0.0123
16.000	193147	0.1820	0.0127
17.000	205218	0.1876	0.0131
18.000	217290	0.1931	0.0134
19.000	229362	0.1984	0.0138
20.000	241433	0.2035	0.0142
21.000	253505	0.2085	0.0145
22.000	265576	0.2135	0.0149
23.000	277648	0.2182	0.0152
24.000	289720	0.2229	0.0155
25.000	301791	0.2275	0.0158
26.000	313863	0.2320	0.0162
27.000	325935	0.2365	0.0165
28.000	338006	0.2408	0.0168
29.000	350078	0.2451	0.0171
30.000	362150	0.2493	0.0174
31.000	374221	0.2534	0.0176
32.000	386293	0.2574	0.0179
33.000	398365	0.2614	0.0182
34.000	410436	0.2654	0.0185
35.000	422508	0.2692	0.0187
36.000	434580	0.2730	0.0190
37.000	446651	0.2768	0.0193
38.000	458723	0.2805	0.0195
39.000	470795	0.2842	0.0198
40.000	482866	0.2878	0.0200
41.000	494938	0.2914	0.0203
41.210	497473	0.2921	0.0203
41.420	500008	0.2929	0.0204



6-102 Object 1 and object 2 with same shape and geometry, but different characteristic lengths, are placed in airflow of different free stream velocities at 1 atm and 20°C. The average convection heat transfer coefficient for object 2 is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis The relation for Nusselt, Prandtl, and Reynolds numbers is given as

$$\mathrm{Nu} = g(\mathrm{Re}, \mathrm{Pr}) \quad \text{where} \quad \mathrm{Nu} = \frac{hL}{k}, \quad \mathrm{Pr} = \frac{c_p \mu}{k}, \quad \text{and} \quad \mathrm{Re} = \frac{VL}{\nu}$$

Then

$$\textit{Object 1}: \quad \mathrm{Re}_1 = \frac{VL_1}{\nu} = \frac{(50\ \mathrm{m/s})(0.5\ \mathrm{m})}{\nu} = \frac{25\ \mathrm{m^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_1 = \frac{c_p \mu}{k}$$

$$\textit{Object 2}: \quad \mathrm{Re}_2 = \frac{VL_2}{\nu} = \frac{(5\ \mathrm{m/s})(5\ \mathrm{m})}{\nu} = \frac{25\ \mathrm{m^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_2 = \frac{c_p \mu}{k}$$

Since the fluid properties are constant, we have $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$, which implies

$$\mathrm{Nu}_1 = g(\mathrm{Re}_1, \mathrm{Pr}_1) = \mathrm{Nu}_2 = g(\mathrm{Re}_2, \mathrm{Pr}_2) \quad \rightarrow \quad \mathrm{Nu}_1 = \mathrm{Nu}_2$$

Hence

$$\frac{h_1 L_1}{k} = \frac{h_2 L_2}{k} \quad \rightarrow \quad h_2 = h_1 \frac{L_1}{L_2} = h_1 \frac{0.5}{5} = 0.1 h_1$$

The average convection heat transfer coefficient for object 1 is

$$\dot{q}_1 = h_1 (T_s - T_\infty) \quad \rightarrow \quad h_1 = \frac{\dot{q}_1}{T_s - T_\infty} = \frac{12000\ \mathrm{W/m^2}}{(120 - 20)\ \mathrm{K}} = 120\ \mathrm{W/m^2 \cdot K}$$

Therefore the average convection heat transfer coefficient for object 2 is

$$h_2 = 0.1 h_1 = 0.1(120\ \mathrm{W/m^2 \cdot K}) = \mathbf{12\ W/m^2 \cdot K}$$

Discussion The conditions where $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$ allowed one to easily relate the Nusselt numbers as $\mathrm{Nu}_1 = \mathrm{Nu}_2$. If $\mathrm{Re}_1 \neq \mathrm{Re}_2$, then the specific expression for $g(\mathrm{Re}, \mathrm{Pr})$ is needed to relate Nu_1 and Nu_2.

6-103 A rectangular bar is placed in a free stream flow. Using the given expression for Nusselt number, the heat transfer coefficients, for different characteristic lengths and free stream velocities, are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis From the given expression for Nusselt number

$$\mathrm{Nu} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n \quad \rightarrow \quad \frac{hL}{k} = C\left(\frac{VL}{\nu}\right)^m \left(\frac{\nu}{\alpha}\right)^n$$

From the given information, we have

Case 1: $h_1 = 100\ \mathrm{W/m^2 \cdot K}$ when $V_1 = 25\ \mathrm{m/s}$ and $L_1 = 0.5\ \mathrm{m}$

Case 2: $h_2 = 50\ \mathrm{W/m^2 \cdot K}$ when $V_2 = 5\ \mathrm{m/s}$ and $L_2 = 0.5\ \mathrm{m}$

Hence

$$\frac{\mathrm{Nu}_1}{\mathrm{Nu}_2} = \frac{C\,\mathrm{Re}_1^m\,\mathrm{Pr}_1^n}{C\,\mathrm{Re}_2^m\,\mathrm{Pr}_2^n} = \frac{\mathrm{Re}_1^m}{\mathrm{Re}_2^m} \quad \rightarrow \quad \frac{h_1 L_1}{h_2 L_2} = \left(\frac{V_1 L_1}{V_2 L_2}\right)^m$$

where $\mathrm{Pr}_1 = \mathrm{Pr}_2$ is due to constant properties. Then

$$\frac{h_1 L_1}{h_2 L_2} = \left(\frac{V_1 L_1}{V_2 L_2}\right)^m \quad \rightarrow \quad \frac{(100\ \mathrm{W/m^2 \cdot K})(0.5\ \mathrm{m})}{(50\ \mathrm{W/m^2 \cdot K})(0.5\ \mathrm{m})} = \left[\frac{(25\ \mathrm{m/s})(0.5\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^m \quad \rightarrow \quad 2 = 5^m$$

Solving for the constant m yields $m = 0.4307$.

(*a*) For $L = 1$ m and $V = 5$ m/s, the convection heat transfer coefficient is

$$h = h_2 \frac{L_2}{L}\left(\frac{VL}{V_2 L_2}\right)^{0.4307} = (50\ \mathrm{W/m^2 \cdot K})\frac{(0.5\ \mathrm{m})}{(1\ \mathrm{m})}\left[\frac{(5\ \mathrm{m/s})(1\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^{0.4307} = \mathbf{33.7\ W/m^2 \cdot K}$$

(*b*) For $L = 2$ m and $V = 50$ m/s, the convection heat transfer coefficient is

$$h = h_2 \frac{L_2}{L}\left(\frac{VL}{V_2 L_2}\right)^{0.4307} = (50\ \mathrm{W/m^2 \cdot K})\frac{(0.5\ \mathrm{m})}{(2\ \mathrm{m})}\left[\frac{(50\ \mathrm{m/s})(2\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^{0.4307} = \mathbf{61.2\ W/m^2 \cdot K}$$

Discussion The Nusselt number relation, $\mathrm{Nu} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$, is in general a reasonably accurate representation for convection heat transfer coefficient. However, more complex relations for Nusselt number are used for better accuracy.

6-104 Electrical heaters are embedded inside the wing to prevent formation of ice. The heat flux necessary to keep the wing surface above 0°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at −10°C are given in Table A-15: $\nu = 1.252 \times 10^{-5}$ m^2/s, k = 0.02288 W/m·K, and Pr = 0.7387.

Analysis With a characteristic length of 2.5 m, the Reynolds number is

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{(200\ \mathrm{m/s})(2.5\ \mathrm{m})}{1.252\times10^{-5}\ \mathrm{m^2/s}} = 3.994\times10^{7}$$

Applying the modified Reynolds analogy,

$$\frac{C_f\ \mathrm{Re}}{2} = \mathrm{Nu\,Pr}^{-1/3} \quad \rightarrow \quad \mathrm{Nu} = \frac{C_f}{2}\mathrm{Re\,Pr}^{1/3} \quad \text{or} \quad h = \frac{C_f}{2}\frac{k}{L}\mathrm{Re\,Pr}^{1/3}$$

$$h = \frac{0.001}{2}\frac{(0.02288\ \mathrm{W/m\cdot K})}{(2.5\ \mathrm{m})}(3.994\times10^{7})(0.7387)^{1/3} = 165.2\ \mathrm{W/m^2\cdot K}$$

The heat flux necessary to keep the wing surface above 0°C is

$$\dot{q} \geq h(T_s - T_\infty) = (165.2\ \mathrm{W/m^2\cdot K})[0-(-20)]\,\mathrm{K} = 3304\ \mathrm{W/m^2} \quad \rightarrow \quad \dot{q} \geq \mathbf{3304\ W/m^2}$$

Discussion The modified Reynolds analogy is applicable approximately for turbulent flow over a surface, even when pressure gradient is present.

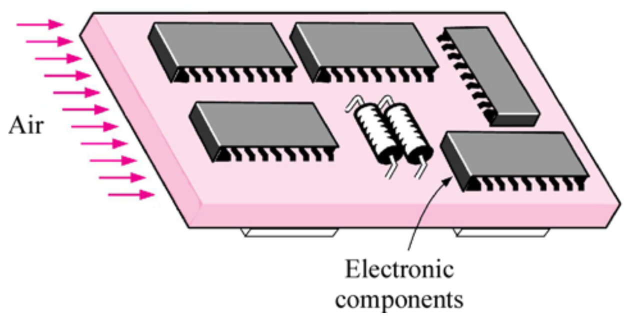
6-105 Forced convection of air is used for cooling the surface of a circuit board. The temperature difference between the circuit board surface temperature and the airstream temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at 40°C are given in Table A-15: $\rho = 1.127$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, and Pr = 0.7255.

Analysis Applying the modified Reynolds analogy

$$\frac{C_f}{2} = \text{St}\,\text{Pr}^{2/3}$$



with $$\text{St} = \frac{h}{\rho c_p V}$$

and $$\tau_s = C_f \frac{\rho V^2}{2}$$

Substituting yields

$$\frac{\tau_s}{\rho V^2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad \rightarrow \quad h = \frac{\tau_s}{V} c_p\, \text{Pr}^{-2/3}$$

The convection heat transfer coefficient is

$$h = \frac{\tau_s}{V} c_p\, \text{Pr}^{-2/3} = \frac{(0.075\ \text{N/m}^2)}{(3\ \text{m/s})}(1007\ \text{J/kg}\cdot\text{K})(0.7255)^{-2/3} = 31.18\ \text{W/m}^2\cdot\text{K}$$

Therefore the temperature difference between the circuit board surface temperature and the airstream temperature is

$$T_s - T_\infty = \frac{\dot{q}}{h} = \frac{1000\ \text{W/m}^2}{31.18\ \text{W/m}^2\cdot\text{K}} = \mathbf{32.1°C}$$

Discussion To reduce the temperature difference between the circuit board surface temperature and the airstream temperature, the value of convection heat transfer coefficient needs to be increased. This can be achieved by increasing the airstream velocity.

Fundamentals of Engineering (FE) Exam Problems

6-106 The transition from laminar flow to turbulent flow in a forced convection situation is determined by which one of the following dimensionless numbers?

(a) Grasshof (b) Nusselt (c) Reynolds (d) Stanton (e) Mach

Answer (c) Reynolds

6-107 The ______ number is a significant dimensionless parameter for forced convection and the _____ number is a significant dimensionless parameter for natural convection.

(a) Reynolds, Grashof (b) Reynolds, Mach (c) Reynolds, Eckert

(d) Reynolds, Schmidt (e) Grashof, Sherwood

Answer (a) Reynolds, Grashof

6-108 In any forced or natural convection situation, the velocity of the flowing fluid is zero where the fluid wets any stationary surface. The magnitude of heat flux where the fluid wets a stationary surface is given by

(a) $k(T_{\text{fluid}} - T_{\text{wall}})$ (b) $k\left.\dfrac{dT}{dy}\right|_{\text{wall}}$ (c) $k\left.\dfrac{d^2T}{dy^2}\right|_{\text{wall}}$ (d) $h\left.\dfrac{dT}{dy}\right|_{\text{wall}}$ (e) None of them

Answer (b) $k\left.\dfrac{dT}{dy}\right|_{\text{wall}}$

6-109 The coefficient of friction C_f for a fluid flowing across a surface in terms of the surface shear stress, τ_s, is given by

(a) $2\rho V^2/\tau_s$ (b) $2\tau_s/\rho V^2$ (c) $2\tau_s/\rho V^2\Delta T$ (d) $4\tau_s/\rho V^2$ (e) None of them

Answer (b) $2\tau_s/\rho V^2$

6-110 Most correlations for the convection heat transfer coefficient use the dimensionless Nusselt number, which is defined as

(a) h / k (b) k / h (c) hL_c / k (d) kL_c / h (e) $k/\rho c_p$

Answer (c) hL_c / k

6-111 For the same initial conditions, one can expect the laminar thermal and momentum boundary layers on a flat plate to have the same thickness when the Prandtl number of the flowing fluid is

(a) Close to zero (b) Small (c) Approximately one

(d) Large (e) Very large

Answer (c) Approximately one

6-112 One can expect the heat transfer coefficient for turbulent flow to be ____ for laminar flow

(a) less than (b) same as (c) greater than

Answer (c) greater than

6-113 An electrical water (k = 0.61 W/m·K) heater uses natural convection to transfer heat from a 1-cm diameter by 0.65-m long, 110 V electrical resistance heater to the water. During operation, the surface temperature of this heater is 120°C while the temperature of the water is 35°C, and the Nusselt number (based on the diameter) is 5. Considering only the side surface of the heater (and thus $A = \pi DL$), the current passing through the electrical heating element is

(a) 2.2 A (b) 2.7 A (c) 3.6 A (d) 4.8 A (e) 5.6 A

Answer (d) 4.8 A

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.61 [W/m-K]
d=0.01 [m]
L=0.65 [m]
Nus=5
DT=85 [K]
DV=110 [Volt]
h=Nus*k/d
Q=h*pi*d*L*DT
I=Q/DV
```

6-114 In turbulent flow, one can estimate the Nusselt number using the analogy between heat and momentum transfer (Colburn analogy). This analogy relates the Nusselt number to the coefficient of friction, C_f, as

(a) $\text{Nu} = 0.5\, C_f \text{Re}\, \text{Pr}^{1/3}$ (b) $\text{Nu} = 0.5\, C_f \text{Re}\, \text{Pr}^{2/3}$ (c) $\text{Nu} = C_f \text{Re}\, \text{Pr}^{1/3}$

(d) $\text{Nu} = C_f \text{Re}\, \text{Pr}^{2/3}$ (e) $\text{Nu} = C_f \text{Re}^{1/2}\, \text{Pr}^{1/3}$

Answer (a) $\text{Nu} = 0.5\, C_f \text{Re}\, \text{Pr}^{1/3}$

6-115, 6-116 Design and Essay Problems