Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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Chapter 4
TRANSIENT HEAT CONDUCTION

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Lumped System Analysis

4-1C Biot number represents the ratio of conduction resistance within the body to convection resistance at the surface of the body. The Biot number is more likely to be larger for poorly conducting solids since such bodies have larger resistances against heat conduction.

4-2C In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

4-3C The lumped system analysis is more likely to be applicable in air than in water since the convection heat transfer coefficient and thus the Biot number is much smaller in air.

4-4C The lumped system analysis is more likely to be applicable for a golden apple than for an actual apple since the thermal conductivity is much larger and thus the Biot number is much smaller for gold.

4-5C The lumped system analysis is more likely to be applicable to slender bodies than the well-rounded bodies since the characteristic length (ratio of volume to surface area) and thus the Biot number is much smaller for slender bodies.

4-6C The lumped system analysis is more likely to be applicable for the body cooled naturally since the Biot number is proportional to the convection heat transfer coefficient, which is proportional to the air velocity. Therefore, the Biot number is more likely to be less than 0.1 for the case of natural convection.

4-7C The lumped system analysis is more likely to be applicable for the body allowed to cool in the air since the Biot number is proportional to the convection heat transfer coefficient, which is larger in water than it is in air because of the larger thermal conductivity of water. Therefore, the Biot number is more likely to be less than 0.1 for the case of the solid cooled in the air

4-8C The temperature drop of the potato during the second minute will be less than 4°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

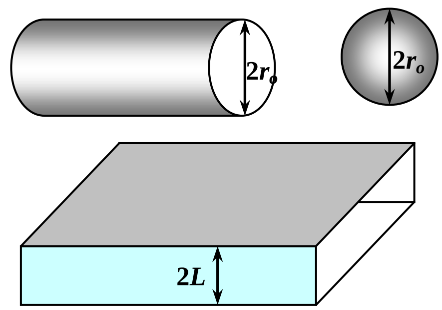
4-9C The temperature rise of the potato during the second minute will be less than 5°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

4-10C The heat transfer is proportional to the surface area. Two half pieces of the roast have a much larger surface area than the single piece and thus a higher rate of heat transfer. As a result, the two half pieces will cook much faster than the single large piece.

4-11C The cylinder will cool faster than the sphere since heat transfer rate is proportional to the surface area, and the sphere has the smallest area for a given volume.

4-12 Relations are to be obtained for the characteristic lengths of a large plane wall of thickness 2L, a very long cylinder of radius r_o and a sphere of radius r_o.

Analysis Relations for the characteristic lengths of a large plane wall of thickness 2L, a very long cylinder of radius r_o and a sphere of radius r_o are



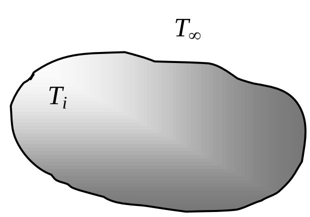
$$L_{c,wall} = \frac{V}{A_{\text{surface}}} = \frac{2LA}{2A} = L$$

$$L_{c,cylinder} = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 h}{2\pi r_o h} = \frac{r_o}{2}$$

$$L_{c,sphere} = \frac{V}{A_{\text{surface}}} = \frac{4\pi r_o^3/3}{4\pi r_o^2} = \frac{r_o}{3}$$

4-13 A relation for the time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ is to be obtained.

Analysis The relation for time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ can be determined as



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{\dfrac{T_i + T_\infty}{2} - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$\frac{T_i - T_\infty}{2(T_i - T_\infty)} = e^{-bt} \longrightarrow \frac{1}{2} = e^{-bt}$$

$$-bt = -\ln 2 \longrightarrow t = \mathbf{\frac{\ln 2}{b}} = \mathbf{\frac{0.693}{b}}$$

4-14 The time required to cool a brick from 1100°C to a temperature difference of 5°C from the ambient air temperature is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the brick are given as ρ = 1920 kg/m^3, c_p = 790 J/kg · K, and k = 0.90 W/m · K.

Analysis For a brick, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{(0.203 \times 0.102 \times 0.057)\ \text{m}^3}{\left[2(0.203 \times 0.102) + 2(0.102 \times 0.057) + 2(0.203 \times 0.057)\right]\text{m}^2} = 0.01549\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(5\ \text{W/m}^2 \cdot \text{K})(0.01549\ \text{m})}{0.90\ \text{W/m} \cdot \text{K}} = 0.0861 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the time required to cool a brick from 1100°C to a temperature difference of 5°C from the ambient air temperature is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{5\ \text{W/m}^2 \cdot \text{K}}{(1920\ \text{kg/m}^3)(790\ \text{J/kg} \cdot \text{K})(0.01549\ \text{m})} = 2.128 \times 10^{-4}\ \text{s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

or

$$t = -\frac{1}{b}\ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{2.128 \times 10^{-4}\ \text{s}^{-1}}\ln\left[\frac{5}{1100 - 30}\right] = 2.522 \times 10^4\ \text{s} = \mathbf{7\ hours}$$

Discussion In practice, it takes days to cool bricks coming out of kilns, since they are being burned and cooled in bulk.

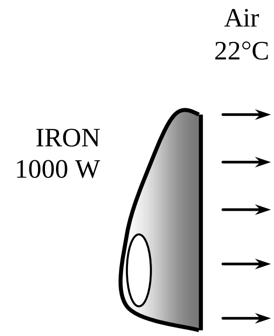
4-15 An iron whose base plate is made of an aluminum alloy is turned on. The time for the plate temperature to reach 140°C and whether it is realistic to assume the plate temperature to be uniform at all times are to be determined.

Assumptions **1** 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The density, specific heat, and thermal diffusivity of the aluminum alloy plate are given to be ρ = 2770 kg/m^3, c_p = 875 kJ/kg.°C, and $\alpha = 7.3\times10^{-5}$ m^2/s. The thermal conductivity of the plate can be determined from $k = \alpha\rho c_p$ = 177 W/m.°C (or it can be read from Table A-3).

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770\ \text{kg/m}^3)(0.005\ \text{m})(0.03\ \text{m}^2) = 0.4155\ \text{kg}$$



Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

$$\dot{Q}_{\text{in}} = 0.85\times1000\ \text{W} = 850\ \text{W}$$

The temperature of the plate, and thus the rate of heat transfer from the plate, changes during the process. Using the average plate temperature, the average rate of heat loss from the plate is determined from

$$\dot{Q}_{\text{loss}} = hA(T_{\text{plate,ave}} - T_\infty) = (12\ \text{W/m}^2.°\text{C})(0.03\ \text{m}^2)\left(\frac{140+22}{2} - 22\right)°\text{C} = 21.2\ \text{W}$$

Energy balance on the plate can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{plate}} \ \rightarrow\ \dot{Q}_{\text{in}}\Delta t - \dot{Q}_{\text{out}}\Delta t = \Delta E_{\text{plate}} = mc_p\Delta T_{\text{plate}}$$

Solving for Δt and substituting,

$$\Delta t = \frac{mc_p\Delta T_{\text{plate}}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}} = \frac{(0.4155\ \text{kg})(875\ \text{J/kg.°C})(140-22)°\text{C}}{(850-21.2)\ \text{J/s}} = \mathbf{51.8\ s}$$

which is the time required for the plate temperature to reach 140 °C. To determine whether it is realistic to assume the plate temperature to be uniform at all times, we need to calculate the Biot number,

$$L_c = \frac{V}{A_s} = \frac{LA}{A} = L = 0.005\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\ \text{W/m}^2.°\text{C})(0.005\ \text{m})}{(177.0\ \text{W/m.°C})} = 0.00034 < 0.1$$

It is realistic to assume uniform temperature for the plate since Bi < 0.1.

Discussion This problem can also be solved by obtaining the differential equation from an energy balance on the plate for a differential time interval, and solving the differential equation. It gives

$$T(t) = T_\infty + \frac{\dot{Q}_{\text{in}}}{hA}\left(1 - \exp(-\frac{hA}{mc_p}t)\right)$$

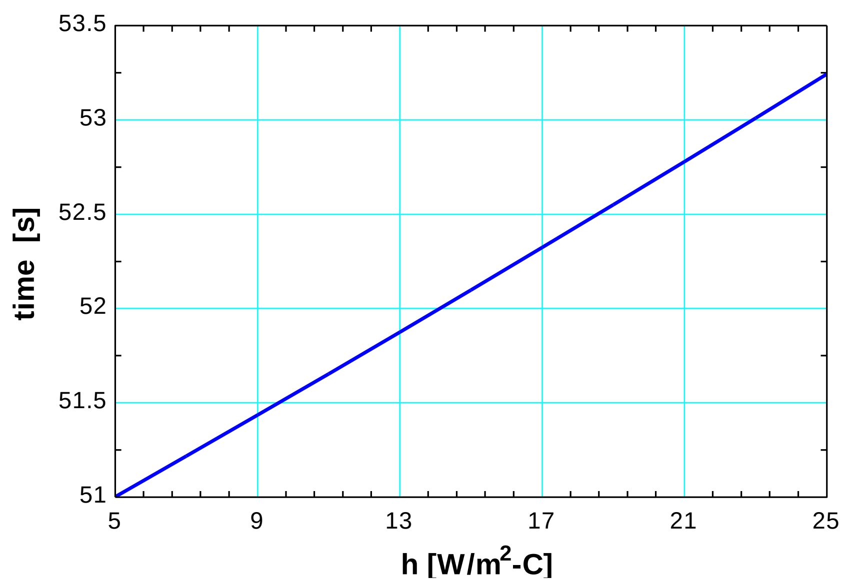
Substituting the known quantities and solving for t again gives 51.8 s.

4-16 Prob. 4-15 is reconsidered. The effects of the heat transfer coefficient and the final plate temperature on the time it will take for the plate to reach this temperature are to be investigated.

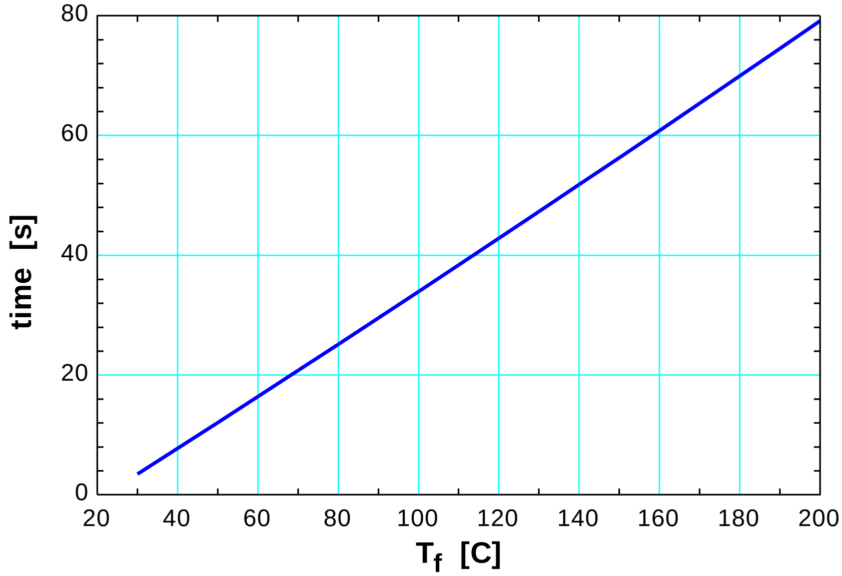
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
E_dot=1000 [W]
L=0.005 [m]
A=0.03 [m^2]
T_infinity=22 [C]
T_i=T_infinity
h=12 [W/m^2-C]
f_heat=0.85
T_f=140 [C]

"PROPERTIES"
rho=2770 [kg/m^3]
c_p=875 [J/kg-C]
alpha=7.3E-5 [m^2/s]

"ANALYSIS"
V=L*A
m=rho*V
Q_dot_in=f_heat*E_dot
Q_dot_out=h*A*(T_ave-T_infinity)
T_ave=1/2*(T_i+T_f)
(Q_dot_in-Q_dot_out)*time=m*c_p*(T_f-T_i) "energy balance on the plate"
```

h [W/m^2.C]	time [s]
5	51
7	51.22
9	51.43
11	51.65
13	51.88
15	52.1
17	52.32
19	52.55
21	52.78
23	53.01
25	53.24



T_f [C]	time [s]
30	3.428
40	7.728
50	12.05
60	16.39
70	20.74
80	25.12
90	29.51
100	33.92
110	38.35
120	42.8
130	47.28
140	51.76
150	56.27
160	60.8
170	65.35
180	69.92
190	74.51
200	79.12



4–17 Metal plates are heated in an oven. The temperature of the plates exiting the oven is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface of all the metal plates. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plates are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\ \text{mm}}{2} = 5\ \text{mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(200\ \text{W/m}^2\cdot\text{K})(5\times10^{-3}\ \text{m})}{(180\ \text{W/m}\cdot\text{K})} = 0.00556 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200\ \text{W/m}^2\cdot\text{K}}{(2800\ \text{kg/m}^3)(880\ \text{J/kg}\cdot\text{K})(5\times10^{-3}\ \text{m})} = 0.01623\ \text{s}^{-1}$$

Thus, the temperature of the plates as they exit the oven is (at t = 120 s)

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

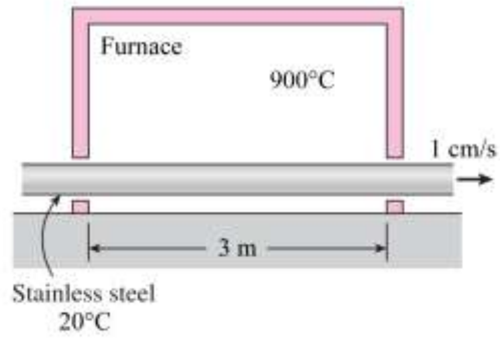
$$T(t) = 800°\text{C} + (20-800)(°\text{C})\exp[-(0.01623\ \text{s}^{-1})(120\ \text{s})] = \mathbf{689°C}$$

Discussion As the metal plates exit the oven, they have reached about 88% of the initial temperature difference.

4-18 Stainless steel strip is heat treated as it moves through a furnace. The temperature of the strip exiting the furnace is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.



Analysis The characteristic length and the Biot number of the stainless steel strip

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\,\text{mm}}{2} = 2.5\,\text{mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(80\ \text{W/m}^2\cdot\text{K})(2.5\times10^{-3}\ \text{m})}{(21\,\text{W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{80\ \text{W/m}^2\cdot\text{K}}{(8000\ \text{kg/m}^3)(570\ \text{J/kg}\cdot\text{K})(2.5\times10^{-3}\ \text{m})} = 0.007018\,\text{s}^{-1}$$

The time for the stainless steel strip being heated can be determined from the furnace length and the speed of the moving strip:

$$t = \frac{3\,\text{m}}{0.01\,\text{m/s}} = 300\,\text{s}$$

Thus, the temperature of the strip as it exits the oven is (at t = 300 s)

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

$$T(t) = 900°\text{C} + (20 - 900)(°\text{C})\exp[-(0.007018\,\text{s}^{-1})(300\,\text{s})] = \mathbf{793°C}$$

Discussion As the stainless steel strip exits the furnace, it has reached about 90% of the initial temperature difference.

4–19 Stainless steel plates are heat treated as they move through a furnace. The effect of the plate velocity on the plate temperature at the furnace exit is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{2\text{ cm}}{2} = 1\text{ cm} \quad \text{and} \quad \text{Bi} = \frac{hL_c}{k} = \frac{(150\text{ W/m}^2\cdot\text{K})(1\times10^{-2}\text{ m})}{(21\text{ W/m}\cdot\text{K})} = 0.07143 < 0.1$$

Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below.

```
"GIVEN"
h=150 [W/m^2-K]
L_c=2e-2/2 [m]
L=3 [m]   "length of furnace"
T_infinity=950 [C]
T_i=18 [C]

"PROPERTIES"
c_p=570 [J/kg-K]
k=21 [W/m-K]
rho=8000 [kg/m^3]

"ANALYSIS"
t=L/Velocity
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_plate-T_infinity)/(T_i-T_infinity)=exp(-b*t)
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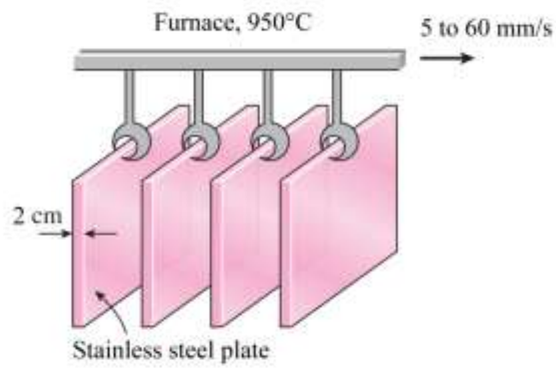
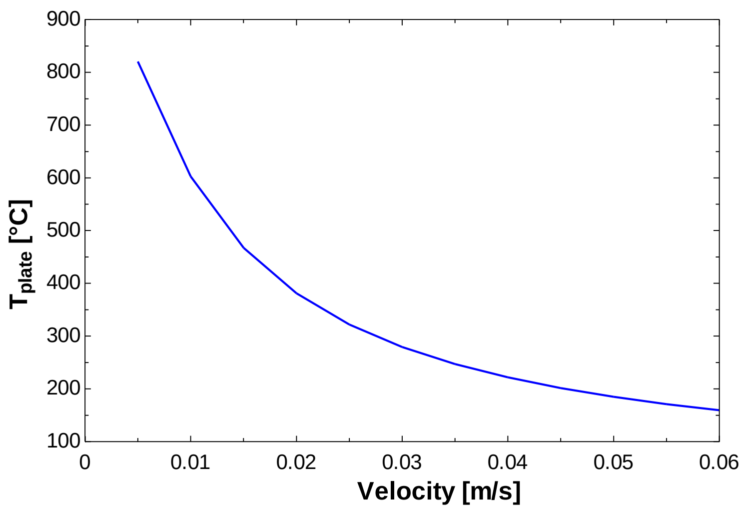


Velocity [m/s]	T_{plate} [°C]
0.005	820.5
0.010	602.6
0.015	467.3
0.020	381.0
0.025	322.0
0.030	279.3
0.035	247.0
0.040	221.8
0.045	201.5
0.050	184.9
0.055	171.1
0.060	159.3



Discussion As the plate velocity increases, the duration of the plates being heated in the furnace decreases. Thus, the plate temperature at the furnace exit decreases with increasing plate velocity.

4–20 Stainless steel strip exiting an oven is allowed to cool within a distance of 5 m. The maximum speed of the strip, such that it is cooled to 45°C within 5 m, is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis We take the thickness of the strip as 2L = 6 mm, and the top (or bottom) area to be A and neglect the edge areas. Then, the total surface area is the sum of top and bottom areas, which is 2A. Then, the characteristic length and the Biot number of the stainless steel strip are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{6\,\text{mm}}{2} = 3\,\text{mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(120\ \text{W/m}^2\cdot\text{K})(3\times10^{-3}\ \text{m})}{(21\ \text{W/m}\cdot\text{K})} = 0.01714 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{120\ \text{W/m}^2\cdot\text{K}}{(8000\ \text{kg/m}^3)(570\ \text{J/kg}\cdot\text{K})(3\times10^{-3}\ \text{m})} = 0.008772\ \text{s}^{-1}$$

The time for the stainless steel strip to cool can be determined from the cooling distance of x = 5 m and the speed of the moving strip V:

$$\text{t} = \frac{\text{x}}{\text{V}}$$

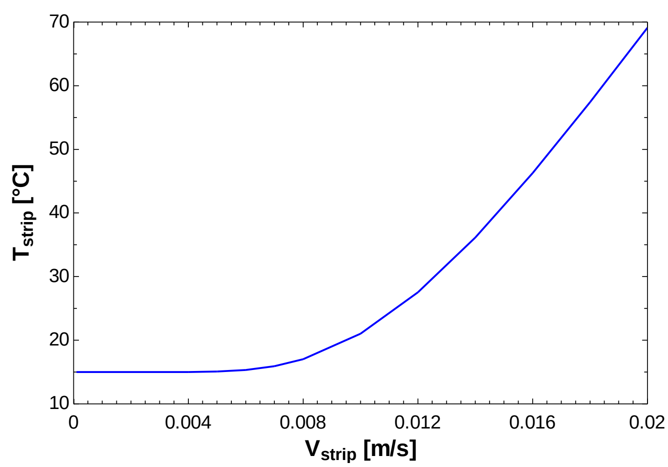
At the end of the cooling distance, the temperature of the strip should be 45°C, thus

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad \ln\left[\frac{\text{T(t)}-\text{T}_\infty}{\text{T}_\text{i}-\text{T}_\infty}\right] = -\text{bt} = -\text{b}\frac{\text{x}}{\text{V}}$$

$$V = -bx\left\{\ln\left[\frac{T(t)-T_\infty}{T_i-T_\infty}\right]\right\}^{-1} = -(0.008772\ \text{s}^{-1})(5\ \text{m})\left[\ln\left(\frac{45-15}{500-15}\right)\right]^{-1} = \mathbf{0.0157\ m/s}$$

Discussion To prevent thermal burn by cooling the strip from 500°C to 45°C within the distance of 5 m, the maximum speed of the moving strip should be 0.0157 m/s. The effect of the moving strip speed on the temperature at the end of the cooling distance is shown in the figure. As long as the strip is moving slower than 0.0157 m/s, the temperature of the strip at the end of the cooling distance would stay below 45°C.

In many industrial applications, air jets/blowers are used in the buffer zone for cooling of the heat treated metal parts. This may speed up the process without reducing the speed of stainless steel strip.



4–21 Metal plates exiting an oven are being cooled by air in a cooling chamber. The temperatures of the plates exiting the cooling chamber at different speed as a function of the air velocity are to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plates are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L_c=0.02/2 [m]
L_chamber=10 [m]
T_i=500 [C]
T_inf=10 [C]
V_plate1=0.02 [m/s]
V_plate2=0.05 [m/s]
V_plate3=0.08 [m/s]

"PROPERTIES"
c_p=880 [J/kg-K]
k=180 [W/m-K]
rho=2800 [kg/m^3]

"ANALYSIS"
t_1=L_chamber/V_plate1
t_2=L_chamber/V_plate2
t_3=L_chamber/V_plate3
h=33*V_air^0.8 [W/m^2-K]
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
((T_plate1-T_inf)/(T_i-T_inf))=exp(-b*t_1)
((T_plate2-T_inf)/(T_i-T_inf))=exp(-b*t_2)
((T_plate3-T_inf)/(T_i-T_inf))=exp(-b*t_3)
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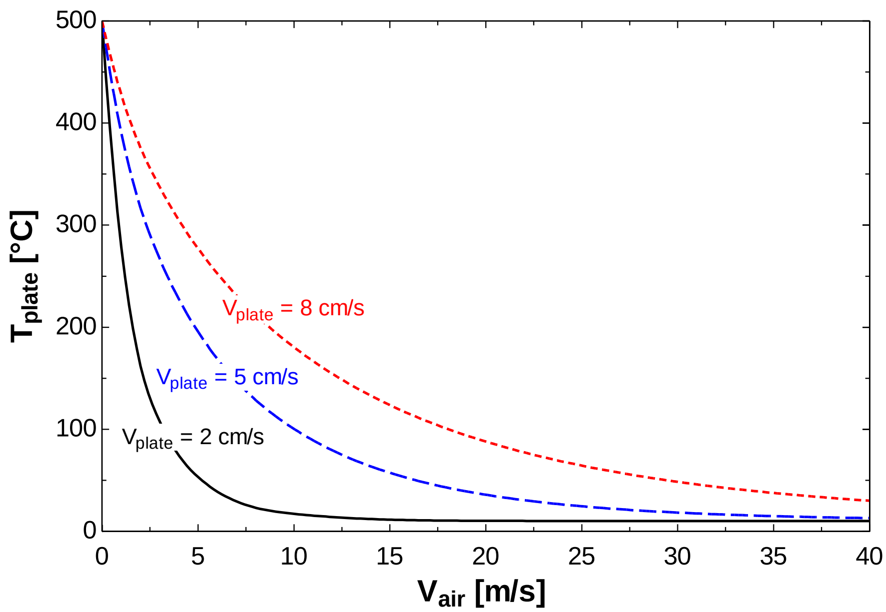


	T_{plate} [°C]		
V_{air} [m/s]	V_{plate} = 0.02 m/s	0.05 m/s	0.08 m/s
0	500	500	500
2	163	317	376
3	108	267	337
4	74.4	228	305
5	53.3	196	277
10	17.2	100	180
15	11.4	57.3	124
20	10.3	35.8	87.9
25	10.1	24.5	64.4
30	10.0	18.4	48.5
35	10.0	14.9	37.6
40	10.0	12.9	29.9

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied. In this problem, the highest Bi occurs at V_{air} = 40 m/s, which gives h =631 W/m^2·K.

$$\mathrm{Bi}=\frac{hL_c}{k}=\frac{(631\ \mathrm{W/m^2\cdot K})(0.01\,\mathrm{m})}{180\ \mathrm{W/m\cdot K}}=0.0351<0.1$$

Discussion The air velocities required to cool the plates to 50°C before exiting the cooling chamber are 5.2, 16.4, and 29.4 m/s for the plates moving at the speed of 0.02, 0.05, and 0.08 m/s, respectively.

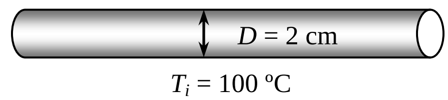
4-22 A long copper rod is cooled to a specified temperature. The cooling time is to be determined.

Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of copper are k = 401 W/m·ºC, ρ = 8933 kg/m^3, and c_p = 0.385 kJ/kg·ºC (Table A-3).

Analysis For cylinder, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.02\text{ m}}{4} = 0.005\text{ m}$$



$$Bi = \frac{hL_c}{k} = \frac{(200\text{ W/m}^2.°\text{C})(0.005\text{ m})}{(401\text{ W/m.°C})} = 0.0025 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200\text{ W/m}^2.°\text{C}}{(8933\text{ kg/m}^3)(385\text{ J/kg.°C})(0.005\text{ m})} = 0.01163\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-20}{100-20} = e^{-(0.01163\text{s}^{-1})t} \longrightarrow t = 238\text{ s} = \mathbf{4.0\,min}$$

4-23 The ambient temperature in the oven necessary to heat the steel rods from 20°C to 450°C within 10 minutes is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the steel rods are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, and k = 63.9 W/m · K.

Analysis For a cylindrical rod, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.025\text{ m}}{4} = 0.00625\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(20\text{ W/m}^2\cdot\text{K})(0.00625\text{ m})}{63.9\text{ W/m}\cdot\text{K}} = 0.00196 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the ambient temperature in the oven is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{20\text{ W/m}^2\cdot\text{K}}{(7832\text{ kg/m}^3)(434\text{ J/kg}\cdot\text{K})(0.00625\text{ m})} = 9.414\times10^{-4}\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt}$$

or

$$T_\infty = \frac{T_i e^{-bt} - T(t)}{e^{-bt} - 1} = \frac{(20\,°\text{C})e^{-(9.414\times10^{-4})(600)} - 450\,°\text{C}}{e^{-(9.414\times10^{-4})(600)} - 1} = \mathbf{1016\,°C}$$

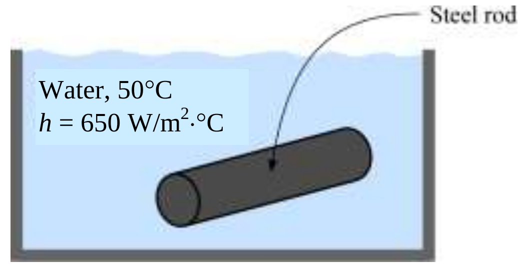
Discussion By increasing the ambient temperature in the oven, the time required to heat the steel rods to the desired temperature would be reduced.

4-24 Steel rods are quenched in a hardening process. The average temperature of rods when they are taken out of oven is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the steel rod are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, and k = 63.9 W/m·K.

Analysis (a) For a cylindrical rod, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.040\ \text{m}}{4} = 0.01\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(650\ \text{W/m}^2\cdot\text{K})(0.01\ \text{m})}{63.9\ \text{W/m}\cdot\text{K}} = 0.102 \approx 0.1$$

Since Biot number is close to 0.1, we can use the lumped system analysis. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{650\ \text{W/m}^2\cdot\text{K}}{(7832\ \text{kg/m}^3)(434\ \text{J/kg}\cdot\text{K})(0.01\ \text{m})} = 0.01912\ \text{s}^{-1}$$

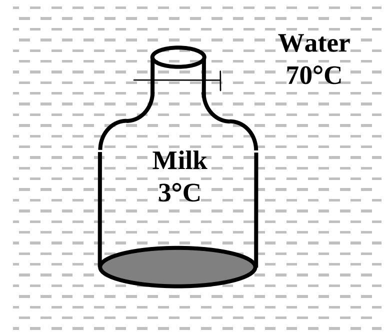
The average temperature of rods when they are taken out of the water bath is determined from

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-50}{850-50} = e^{-(0.01912\ \text{s}^{-1})(40\ \text{s})} \longrightarrow T(t) = \mathbf{422.3°C}$$

Discussion For the temperature of the water bath to remain constant, it is assumed that the heat capacity of the water is much larger than that of the steel rod.

4-25 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The thermal conductivity, density, and specific heat of the milk at 20°C are k = 0.598 W/m.°C, ρ = 998 kg/m^3, and c_p = 4.182 kJ/kg.°C (Table A-9).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03\,\text{m})^2(0.07\,\text{m})}{2\pi(0.03\,\text{m})(0.07\,\text{m}) + 2\pi(0.03\,\text{m})^2} = 0.01050\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\ \text{W/m}^2.^\circ\text{C})(0.0105\,\text{m})}{(0.598\ \text{W/m.}^\circ\text{C})} = 2.107 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{120\ \text{W/m}^2.^\circ\text{C}}{(998\ \text{kg/m}^3)(4182\ \text{J/kg.}^\circ\text{C})(0.0105\,\text{m})} = 0.002738\,\text{s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38-70}{3-70} = e^{-(0.002738\,\text{s}^{-1})t} \longrightarrow t = \mathbf{270\,s = 4.50\,min}$$

Therefore, it will take 4.5 minutes to warm the milk from 3 to 38°C.

4-26 A body is found while still warm. The time of death is to be estimated.

Assumptions **1** The body can be modeled as a 30-cm-diameter, 1.70-m-long cylinder. **2** The thermal properties of the body and the heat transfer coefficient are constant. **3** The radiation effects are negligible. **4** The person was healthy(!) when he or she died with a body temperature of 37°C.

Properties The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37 + 25)/2 = 31°C; k = 0.617 W/m · °C, ρ = 996 kg/m^3, and c_p = 4178 J/kg·°C (Table A–9).

Analysis The characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi (0.15\text{ m})^2 (1.7\text{ m})}{2\pi (0.15\text{ m})(1.7\text{ m}) + 2\pi (0.15\text{ m})(0.15\text{ m})^2} = 0.0689\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(8\text{ W/m}^2.°\text{C})(0.0689\text{ m})}{0.617\text{ W/m.°C}} = 0.89 > 0.1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a "rough" estimate of the time of death. Then,

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{8\text{ W/m}^2.°\text{C}}{(996\text{ kg/m}^3)(4178\text{ J/kg.°C})(0.0689\text{ m})} = 2.79 \times 10^{-5}\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25 - 20}{37 - 20} = \exp[(-2.79 \times 10^{-5}\text{ s}^{-1})t] \longrightarrow t = 43{,}860\text{ s} = \mathbf{12.2\,h}$$

Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is 5 AM.

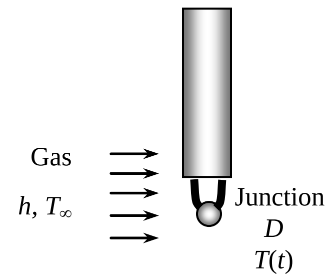
Discussion This example demonstrates how to obtain "ball park" values using a simple analysis. A similar analysis is used in practice by incorporating constants to account for deviation from lumped system analysis.

4-27 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

Assumptions **1** The junction is spherical in shape with a diameter of D = 0.0012 m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35 \text{ W/m.°C}$, $\rho = 8500 \text{ kg/m}^3$, and $c_p = 320 \text{ J/kg.°C}$.

Analysis The characteristic length of the junction and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012 \text{ m}}{6} = 0.0002 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(110 \text{ W/m}^2.\text{°C})(0.0002 \text{ m})}{35 \text{ W/m.°C}} = 0.000629 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{90 \text{ W/m}^2.\text{°C}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg.°C})(0.0002 \text{ m})} = 0.1654 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1654 \text{s}^{-1})t} \longrightarrow t = \mathbf{27.8 \text{ s}}$$

4–28 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT should be within 5 s, and the junction diameter is to be determined.

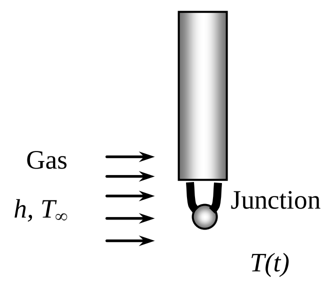
Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the junction surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be k = 35 W/m·K, ρ = 8500 kg/m^3, and c_p = 320 J/kg·K.

Analysis The characteristic length of the thermocouple junction is

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6}$$

The time period for the thermocouple to read 99% of the initial temperature difference is determined from



$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{6h}{\rho c_p D} = \frac{6(250\ \text{W/m}^2\cdot\text{K})}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})}\frac{1}{D}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = 0.01 \quad \rightarrow \quad 0.01 = \exp\left(-\frac{6h}{\rho c_p D}t\right)$$

$$0.01 = \exp\left[-\frac{6(250\ \text{W/m}^2\cdot\text{K})(5\ \text{s})}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})}\frac{1}{D}\right] = \exp\left(-\frac{0.002757\ \text{m}}{D}\right)$$

Solving for the junction diameter yields

$$D = 5.99\times 10^{-4}\ \text{m} = \mathbf{0.599\,mm}$$

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied:

$$\text{Bi} = \frac{\text{hL}_\text{c}}{\text{k}} = \frac{\text{hD}}{6\text{k}} = \frac{(250\ \text{W/m}^2\cdot\text{K})(0.000599\ \text{m})}{6(35\ \text{W/m}\cdot\text{K})} = 0.000713 < 0.1$$

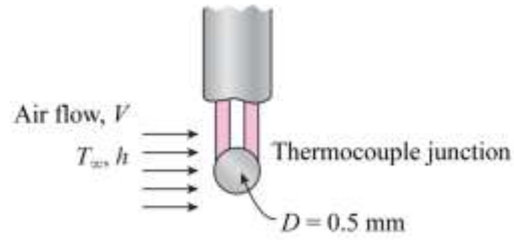
Thus, the lumped system analysis is applicable.

Discussion For the thermocouple to register 99% of the initial temperature difference within 5 s, the junction diameter should be less than 0.6 mm. The smaller the junction size, the faster the thermocouple would respond.

4–29 The temperature of an air flow is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT should be within 5 s, and the air flow velocity (given as a function of h) is to be determined.

Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be k = 35 W/m·K, ρ = 8500 kg/m^3, and c_p = 320 J/kg·K.



Analysis The characteristic length of the thermocouple junction is

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6}$$

The time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{6h}{\rho c_p D} = \frac{6h}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})(0.0005\ \text{m})}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} = 0.01 \quad \rightarrow \quad 0.01 = \exp\left(-\frac{6h}{\rho c_p D}t\right)$$

$$0.01 = \exp\left[-\frac{6h}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})(0.0005\ \text{m})}(5\ \text{s})\right]$$

Solving for the convection heat transfer coefficient yields

$$h = 208.77\ \text{W/m}^2\cdot\text{K}$$

The lowest air flow velocity that the thermocouple can be used to register 99% of the initial temperature difference in 5 s is determined from

$$h = 2.2\left(\frac{V}{D}\right)^{0.5} \quad \text{where} \quad D = 0.0005\ \text{m} \quad \text{and} \quad h = 208.77\ \text{W/m}^2\cdot\text{K}$$

Thus, $\quad V = \mathbf{4.50\ m/s}$

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied

$$\text{Bi} = \frac{hL_c}{k} = \frac{hD}{6k} = \frac{(208.77\ \text{W/m}^2\cdot\text{K})(0.0005\ \text{m})}{6(35\ \text{W/m}\cdot\text{K})} = 0.000497 < 0.1$$

Thus, the lumped system analysis is applicable.

Discussion The lower the convection heat transfer coefficient is, the longer the time it takes for the thermocouple to register 99% of the initial temperature difference.

4-30 Coal particles suspended in hot air flow. The time it takes for the particles to reach 2/3 of the initial temperature difference is to be determined.

Assumptions **1** The thermal properties of coal particles are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of coal are k = 0.26 W/m·K, ρ = 1350 kg/m^3, and c_p = 1260 J/kg·K (from Table A-8).

Analysis The characteristic length and the Biot number of the coal particle are

$$L_c = \frac{V}{A_s} = \frac{0.5\,\text{mm}^3}{3.1\,\text{mm}^2} = 0.1613\,\text{mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(100\ \text{W/m}^2\cdot\text{K})(0.1613\times10^{-3}\ \text{m})}{(0.26\ \text{W/m}\cdot\text{K})} = 0.062 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the time period for a coal particle to heat up to 2/3 of the initial temperature difference is determined from

$$b = \frac{hA_s}{\rho c_p V} = \frac{(100\ \text{W/m}^2\cdot\text{K})(3.1\times10^{-6}\ \text{m}^2)}{(1350\ \text{kg/m}^3)(1260\ \text{J/kg}\cdot\text{K})(0.5\times10^{-9}\ \text{m}^3)} = 0.3645\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = \frac{1}{3} \quad\rightarrow\quad \exp[-(0.3645\,\text{s}^{-1})\,t] = \frac{1}{3}$$

Thus,

$$t = \mathbf{3.01\,s}$$

Discussion If the coal particles are treated as spheres, then the diameter of the coal particles would be about $D = 6L_c$ = 0.97 mm and the time required to heat the coal particles to two thirds of the initial temperature difference is 2.88s.

4–31 Coal particles suspended in hot air are flowing through a heated tube at V = 2 m/s. The temperature of the particles exiting the heated tube is to be determined.

Assumptions **1** The thermal properties of coal particles are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of coal are k = 0.26 W/m·K, ρ = 1350 kg/m^3, and c_p = 1260 J/kg·K (from Table A-8).

Analysis The characteristic length and the Biot number of the coal particle are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{300\ \mu\text{m}}{6} = 50\ \mu\text{m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(250\ \text{W/m}^2\cdot\text{K})(50\times10^{-6}\ \text{m})}{(0.26\ \text{W/m}\cdot\text{K})} = 0.0481 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{250\ \text{W/m}^2\cdot\text{K}}{(1350\ \text{kg/m}^3)(1260\ \text{J/kg}\cdot\text{K})(50\times10^{-6}\ \text{m})} = 2.939\ \text{s}^{-1}$$

The time of the coal particles being heated in the tube can be determined from the tube length and the particle velocity:

$$t = \frac{3\ \text{m}}{2\ \text{m/s}} = 1.5\ \text{s}$$

Thus, the temperature of the coal particles exiting the heated tube is (at t = 1.5 s)

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

$$T(t) = 900°\text{C} + (20-900)(°\text{C})\exp[-(2.939\ \text{s}^{-1})(1.5\ \text{s})] = \mathbf{889°C}$$

Discussion As the coal particles exit the heated tube, they have reached about 99% of the initial temperature difference.

4–32 Alumina particles are injected into a plasma jet. The time it would take for the particles to reach their melting point is to be determined.

Assumptions **1** The thermal properties of alumina are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of alumina are given as k = 30 W/m·K, ρ = 3970 kg/m^3, and c_p = 800 J/kg·K.

Analysis The characteristic length and the Biot number of the alumina particle are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{60\ \mu\text{m}}{6} = 10\ \mu\text{m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(10{,}000\ \text{W/m}^2\cdot\text{K})(10\times10^{-6}\ \text{m})}{(30\ \text{W/m}\cdot\text{K})} = 0.00333 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{10{,}000\ \text{W/m}^2\cdot\text{K}}{(3970\ \text{kg/m}^3)(800\ \text{J/kg}\cdot\text{K})(10\times10^{-6}\ \text{m})} = 314.86\ \text{s}^{-1}$$

Thus, the time for the alumina particles to reach their melting point is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad t = -\frac{1}{b}\ln\left[\frac{T(t)-T_\infty}{T_i - T_\infty}\right]$$

$$t = -\frac{1}{314.86\ \text{s}^{-1}}\ln\left(\frac{2300-15{,}000}{20-15{,}000}\right) = \mathbf{5.24\times10^{-4}\ s}$$

Discussion It takes about half a millisecond for the particles to be heated to their melting point.

4-33 The satellite shell temperature after 5 minutes of reentry is to be determined

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer is uniform over the outer surface of the shell. **4** Heat transfer is limited to the shell only. **5** Heat transfer by radiation is negligible.

Properties The properties of stainless steel are given as ρ = 8238 kg/m^3, c_p = 468 J/kg · K, and k = 13.4 W/m · K.

Analysis For a spherical shell, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi/6\left[D^3-(D-2L)^3\right]}{\pi D^2} = \frac{4^3-[4-2(0.01)]^3}{6(4)^2}\text{ m} = 0.00995\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(130\text{ W/m}^2\cdot\text{K})(0.00995\text{ m})}{13.4\text{ W/m}\cdot\text{K}} = 0.0965 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the shell temperature after 5 minutes of reentry is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{130\text{ W/m}^2\cdot\text{K}}{(8283\text{ kg/m}^3)(468\text{ J/kg}\cdot\text{K})(0.00995\text{ m})} = 0.003389\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt}$$

or

$$T(t) = (T_i-T_\infty)e^{-bt}+T_\infty$$

$$T(5\text{ min}) = (10\,^\circ\text{C}-1250\,^\circ\text{C})e^{-(0.003389)(300)}+1250\,^\circ\text{C} = \mathbf{801\,^\circ C}$$

Discussion The analysis to this problem has been simplified by assuming the shell temperature to be uniform during the reentry.

4-34 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

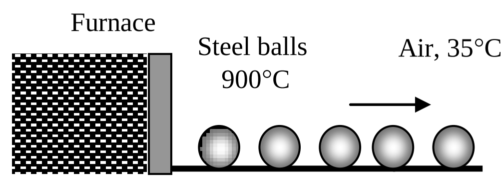
Assumptions **1** The balls are spherical in shape with a radius of r_o = 4 mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be k = 54 W/m.°C, ρ = 7833 kg/m^3, and c_p = 0.465 kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008\text{ m}}{6} = 0.0013\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75\text{ W/m}^2.°\text{C})(0.0013\text{ m})}{(54\text{ W/m.°C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{75\text{ W/m}^2.°\text{C}}{(7833\text{ kg/m}^3)(465\text{ J/kg.°C})(0.0013\text{ m})} = 0.01584\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-35}{900-35} = e^{-(0.01584\text{s}^{-1})t} \longrightarrow t = \mathbf{163\text{ s} = 2.7\text{ min}}$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (7833\text{ kg/m}^3)\frac{\pi(0.008\text{ m})^3}{6} = 0.0021\text{ kg}$$

$$Q = mc_p[T_f - T_i] = (0.0021\text{ kg})(465\text{ J/kg.°C})(900-100)°\text{C} = 781\text{ J} = 0.781\text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

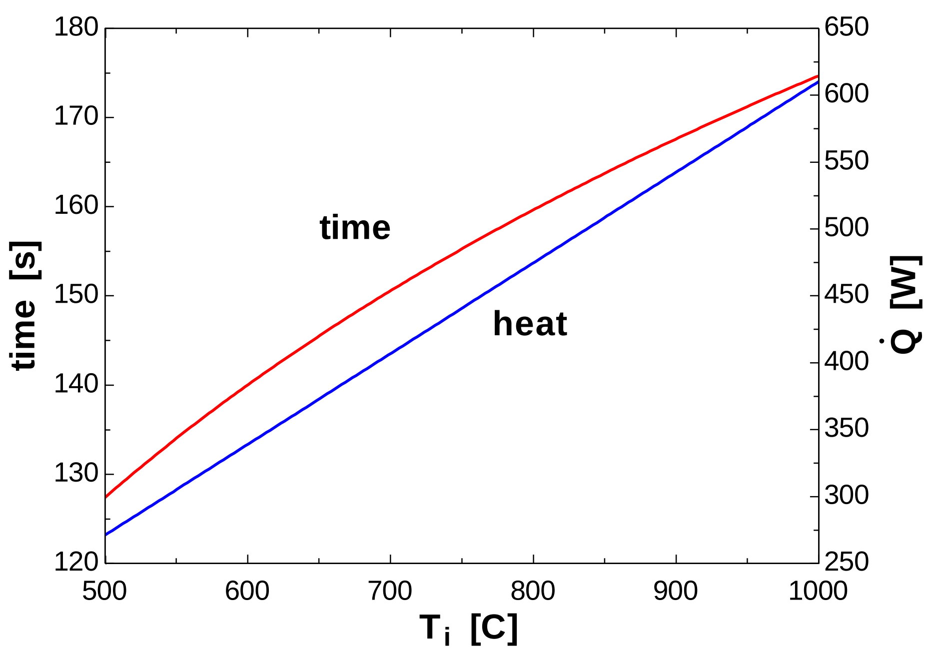
$$\dot{Q} = \dot{n}_{\text{ball}}Q = (2500\text{ balls/h})\times(0.781\text{ kJ/ball}) = 1{,}953\text{ kJ/h} = \mathbf{543\text{ W}}$$

4-35 Prob. 4-34 is reconsidered. The effect of the initial temperature of the balls on the annealing time and the total rate of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.008 [m]
T_i=900 [C]
T_f=100 [C]
T_infinity=35 [C]
h=75 [W/m^2-C]
n_dot_ball=2500 [1/h]

"PROPERTIES"
rho=7833 [kg/m^3]
k=54 [W/m-C]
c_p=465 [J/kg-C]
alpha=1.474E-6 [m^2/s]

"ANALYSIS"
A=pi*D^2
V=pi*D^3/6
L_c=V/A
Bi=(h*L_c)/k "if Bi < 0.1, the lumped sytem analysis is applicable"
b=(h*A)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*time)
m=rho*V
Q=m*c_p*(T_i-T_f)
Q_dot=n_dot_ball*Q*Convert(J/h, W)
```

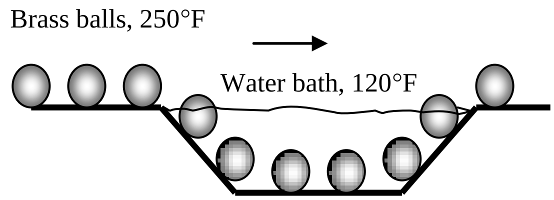
T_i [C]	time [s]	$\dot{Q}$ [W]
500	127.4	271.2
550	134	305.1
600	140	339
650	145.5	372.9
700	150.6	406.9
750	155.3	440.8
800	159.6	474.7
850	163.7	508.6
900	167.6	542.5
950	171.2	576.4
1000	174.7	610.3



4-36E A number of brass balls are to be quenched in a water bath at a specified rate. The temperature of the balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

Assumptions **1** The balls are spherical in shape with a radius of r_o = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the brass balls are given to be k = 64.1 Btu/h.ft.°F, ρ = 532 lbm/ft^3, and c_p = 0.092 Btu/lbm.°F.



Analysis (*a*) The characteristic length and the Biot number for the brass balls are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12\text{ ft}}{6} = 0.02778\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42\text{ Btu/h.ft}^2.°\text{F})(0.02778\text{ ft})}{(64.1\text{ Btu/h.ft.°F})} = 0.01820 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{42\text{ Btu/h.ft}^2.°\text{F}}{(532\text{ lbm/ft}^3)(0.092\text{ Btu/lbm.°F})(0.02778\text{ ft})} = 30.9\text{ h}^{-1} = 0.00858\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.00858\text{ s}^{-1})(120\text{s})} \longrightarrow T(t) = \mathbf{166\ °F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (532\text{ lbm/ft}^3)\frac{\pi(2/12\text{ ft})^3}{6} = 1.290\text{ lbm}$$

$$Q = mc_p[T_i - T(t)] = (1.29\text{ lbm})(0.092\text{ Btu/lbm.°F})(250-166)°\text{F} = 9.97\text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}}Q_{\text{ball}} = (120\text{ balls/min})\times(9.97\text{ Btu}) = \mathbf{1196\ Btu/min}$$

Therefore, heat must be removed from the water at a rate of 1196 Btu/min in order to keep its temperature constant at 120°F.

4-37 The heating times of a sphere, a cube, and a rectangular prism with similar dimensions are to be determined.

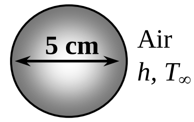
Assumptions **1** The thermal properties of the geometries are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of silver are given to be k = 429 W/m·ºC, ρ = 10,500 kg/m^3, and c_p = 0.235 kJ/kg·ºC.

Analysis For sphere, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$

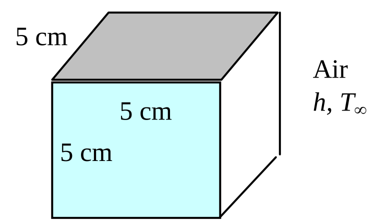


Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the sphere temperature to reach to 25ºC is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\ min}$$

Cube:



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L^3}{6L^2} = \frac{L}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$

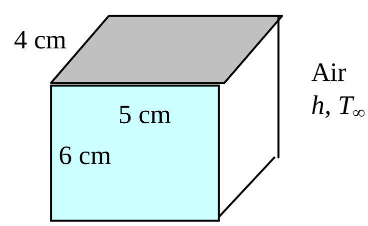
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\ min}$$

Rectangular prism:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(0.04\text{ m})(0.05\text{ m})(0.06\text{ m})}{2(0.04\text{ m})(0.05\text{ m}) + 2(0.04\text{ m})(0.06\text{ m}) + 2(0.05\text{ m})(0.06\text{ m})} = 0.008108\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008108\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$



$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$= \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008108\text{ m})} = 0.0005998\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005998\text{ s}^{-1})t} \longrightarrow t = 2363\text{ s} = \mathbf{39.4\ min}$$

The heating times are same for the sphere and cube while it is smaller in rectangular prism.

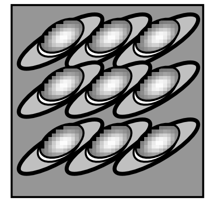
4-38 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The specific heat of the device is given to be c_p = 850 J/kg.°C. The specific heat of the aluminum sink is 903 J/kg.°C (Table A-3), but can be taken to be 850 J/kg.°C for simplicity in analysis.

Analysis **(*a*) Approximate solution**

Electronic device, 20 W



This problem can be solved approximately by using an average temperature for the device when evaluating the heat loss. An energy balance on the device can be expressed as

$$E_{\text{in}} - E_{\text{out}} + E_{\text{generation}} = \Delta E_{\text{device}} \longrightarrow -\dot{Q}_{\text{out}}\Delta t + \dot{E}_{\text{generation}}\Delta t = mc_p \Delta T_{\text{device}}$$

or,
$$\dot{E}_{\text{generation}}\Delta t - hA_s\left(\frac{T+T_\infty}{2} - T_\infty\right)\Delta t = mc_p(T - T_\infty)$$

Substituting the given values,

$$(20\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0004\text{ m}^2)\left(\frac{T-25}{2}\right)^{\text{o}}\text{C}(5\times 60\text{ s}) = (0.02\text{ kg})(850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **363.6°C**

If the device were attached to an aluminum heat sink, the temperature of the device would be

$$(20\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0084\text{ m}^2)\left(\frac{T-25}{2}\right)°\text{C}(5\times 60\text{ s})$$
$$= (0.20 + 0.02)\text{kg} \times (850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **54.7°C**

Note that the temperature of the electronic device drops considerably as a result of attaching it to a heat sink.

(*b*) Exact solution

This problem can be solved exactly by obtaining the differential equation from an energy balance on the device for a differential time interval *dt*. We will get

$$\frac{d(T - T_\infty)}{dt} + \frac{hA_s}{mc_p}(T - T_\infty) = \frac{\dot{E}_{\text{generation}}}{mc_p}$$

It can be solved to give

$$T(t) = T_\infty + \frac{\dot{E}_{\text{generation}}}{hA_s}\left(1 - \exp(-\frac{hA_s}{mc_p}t)\right)$$

Substituting the known quantities and solving for t gives **363.4°C** for the first case and **54.6°C** for the second case, which are practically identical to the results obtained from the approximate analysis.

Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

4-39C Yes. Although rapid boiling will not change the boiling temperature, it will increase the heat transfer coefficient because of the higher level of agitation of bubbles. As a result, the cooking time will be shortened.

4-40C A cylinder whose diameter is small relative to its length can be treated as an infinitely long cylinder. When the diameter and length of the cylinder are comparable, it is not proper to treat the cylinder as being infinitely long. It is also not proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder since heat transfer at those locations can be two-dimensional.

4-41C The Fourier number is a measure of heat conducted through a body relative to the heat stored. Thus a large value of Fourier number indicates faster propagation of heat through body. Since Fourier number is proportional to time, doubling the time will also double the Fourier number.

4-42C The solution for determination of the one-dimensional transient temperature distribution involves many variables that make the graphical representation of the results impractical. In order to reduce the number of parameters, some variables are grouped into dimensionless quantities.

4-43C Yes. A plane wall whose one side is insulated is equivalent to a plane wall that is twice as thick and is exposed to convection from both sides. The midplane in the latter case will behave like an insulated surface because of thermal symmetry.

4-44C This case can be handled by setting the heat transfer coefficient *h* to infinity ∞ since the temperature of the surrounding medium in this case becomes equivalent to the surface temperature.

4-45C When the Biot number is less than 0.1, the temperature of the sphere will be nearly uniform at all times. Therefore, it is more convenient to use the lumped system analysis in this case.

4-46C The maximum possible amount of heat transfer will occur when the temperature of the body reaches the temperature of the medium, and can be determined from $Q_{\max} = mc_p(T_\infty - T_i)$.

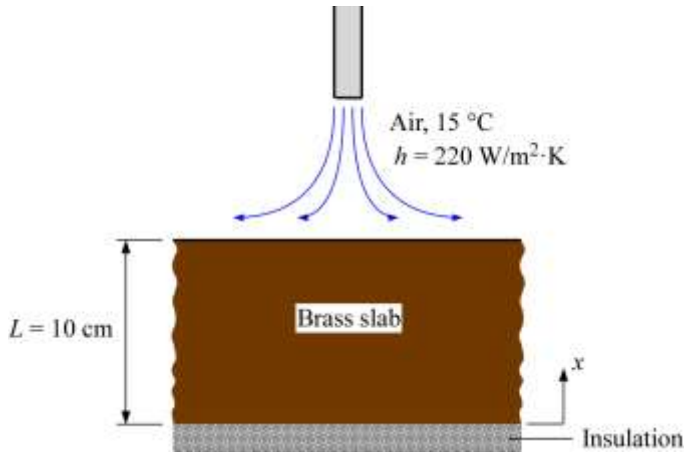
4-47 The temperature at the center plane of a brass plate after 3 minutes of cooling by impinging air jet is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the brass plate are given as ρ = 8530 kg/m^3, c_p = 380 J/kg · K, k = 110 W/m · K, and α = 33.9 × 10^{-6} m^2/s.

Analysis This geometry can be considered to be a large plane wall with a thickness of 2L = 20 cm subjected to convection at both sides. The surface with insulation becomes the center surface of the wall. Then the Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(220\ \text{W/m}^2\cdot\text{K})(0.10\ \text{m})}{110\ \text{W/m}\cdot\text{K}} = 0.2$$



From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times 10^{-6}\ \text{m}^2/\text{s})(3\times 60\ \text{s})}{(0.10\ \text{m})^2} = 0.6102 > 0.2$$

The temperature at the center plane of the plate (x/L = 0.5) after 3 minutes of cooling is

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 x / L)$$

$$T(x,t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 x / L) + T_\infty$$

$$\begin{aligned} T(0.05\ \text{m}, 180\ \text{s}) &= (650\ ^\circ\text{C} - 15\ ^\circ\text{C})(1.0311)e^{-(0.4328)^2(0.6102)}\cos\left[(0.4328)(0.5)\right] + 15\ ^\circ\text{C} \\ &= \mathbf{585\ ^\circ C} \end{aligned}$$

Discussion The insulated bottom surface of the brass plate is treated as a thermally symmetric boundary.

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The center temperature is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hL} = \frac{110\ \text{W/m}\cdot{}^\circ\text{C}}{(220\ \text{W/m}^2\cdot{}^\circ\text{C})(0.10\ \text{m})} = 5 \\ \tau &= \frac{\alpha t}{L^2} = \frac{(33.9\times 10^{-6}\ \text{m}^2/\text{s})(3\times 60\ \text{s})}{(0.10\ \text{m})^2} = 0.610 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 1 \ \text{(Fig. 4-17}a\text{)}$$

Therefore,

$$T_0 = T_\infty + 1(T_i - T_\infty) = 15 + 1(650 - 15) = 650^\circ\text{C}$$

The temperature at the center plane of the plate (x/L = 0.5) is

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hL} = 0.610 \\ \frac{x}{L} &= 0.5 \end{aligned}\right\} \frac{T - T_\infty}{T_0 - T_\infty} = 0.86 \ \text{(Fig. 4-17}b\text{)}$$

$$T = T_\infty + 0.86(T_0 - T_\infty) = 15 + 0.86(650 - 15) = 561^\circ\text{C}$$

The difference between the two results is due to the reading error of the charts.

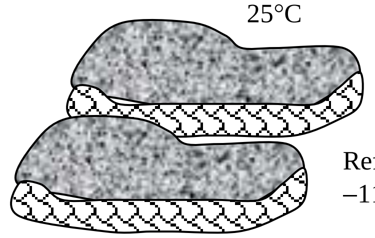
4-48 Steaks are cooled by passing them through a refrigeration room. The time of cooling is to be determined.

Assumptions **1** Heat conduction in the steaks is one-dimensional since the steaks are large relative to their thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steaks are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of steaks are given to be k = 0.45 W/m.°C and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hL}{k} = \frac{(9 \text{ W/m}^2.°\text{C})(0.01\text{ m})}{(0.45 \text{ W/m.°C})} = 0.200$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\frac{T(L,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L / L)$$

$$\frac{2-(-11)}{25-(-11)} = (1.0311)e^{-(0.4328)^2 \tau} \cos(0.4328) \longrightarrow \tau = 5.085 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the length of time for the steaks to be kept in the refrigerator is determined to be

$$t = \frac{\tau L^2}{\alpha} = \frac{(5.085)(0.01\text{ m})^2}{0.91\times10^{-7}\text{ m}^2/\text{s}} = 5590\text{ s} = \mathbf{93.1\,min}$$

Alternative solution: This problem can also be solved using transient temperature charts as follows:

First, the center temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{1}{0.2} = 5 \\ \dfrac{x}{L} = 1 \end{array}\right\} \frac{T(L,t) - T_\infty}{T_0 - T_\infty} = 0.9 \qquad \text{(Fig. 4-17}b\text{)}$$

Therefore,

$$T_0 = T_\infty + \frac{T(L,t) - T_\infty}{0.9} = -11 + \frac{2-(-11)}{0.9} = 3.44°\text{C}$$

The cooling time is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 5 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{3.44-(-11)}{25-(-11)} = 0.401 \end{array}\right\} \tau = \frac{\alpha t}{L^2} = 5 \qquad \text{(Fig. 4-17}a\text{)}$$

Then,

$$t = \frac{\tau L^2}{\alpha} = \frac{(5)(0.01\text{ m})^2}{0.91\times10^{-7}\text{ m}^2/\text{s}} = 5495\text{ s} = 91.6\text{ min}$$

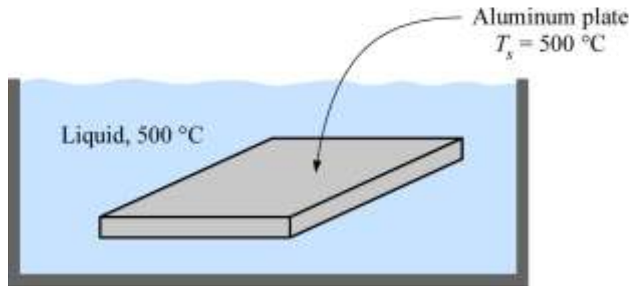
The difference between the two results is due to the reading error of the charts.

4-49 The temperature at the center plane of an aluminum plate with $T_s \approx T_\infty$, after 15 seconds of heating, is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The properties of the aluminum plate are given as ρ = 2702 kg/m^3, c_p = 903 J/kg · K, k = 237 W/m · K, and α = 97.1 × 10^{-6} m^2/s.

Analysis For $T_s \approx T_\infty$, it implies that $h \to \infty$. Thus, the Biot number is



$$Bi = \frac{hL}{k} \to \infty$$

From Table 4-2 with $Bi \to \infty$, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(97.1\times10^{-6}\ \text{m}^2/\text{s})(15\,\text{s})}{(0.05\,\text{m})^2} = 0.5826$$

The temperature at the center plane after 15 seconds of heating is

$$\theta_{0,\,\text{wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$T_0 = (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} + T_\infty$$

$$T_0 = (25\,°\text{C} - 500\,°\text{C})(1.2732) e^{-(1.5708)^2(0.5826)} + 500\,°\text{C} = \mathbf{356\,°C}$$

Discussion Since τ > 0.2, the one-term approximate solution is applicable for this problem.

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The center temperature is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{1}{\infty} = 0 \\ \tau &= \frac{\alpha t}{L^2} = \frac{(97.1\times10^{-6}\ \text{m}^2/\text{s})(15\,\text{s})}{(0.05\,\text{m})^2} = 0.5826 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.29 \qquad \text{(Fig. 4-17}a\text{)}$$

Therefore,

$$T_0 = T_\infty + 0.29(T_i - T_\infty) = 500 + (0.29)(25 - 500) = 362°\text{C}$$

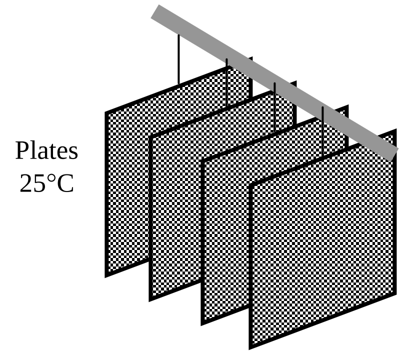
The difference between the two results is due to the reading error of the charts.

4-50 Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the plate are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of brass at room temperature are given to be k = 110 W/m·°C, $\alpha = 33.9\times10^{-6}$ m^2/s

Analysis The Biot number for this process is



$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.015\ \text{m})}{(110\ \text{W/m.°C})} = 0.0109$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.1035 \quad \text{and} \quad A_1 = 1.0018$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times 60\ \text{s/min})}{(0.015\ \text{m})^2} = 90.4 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the plates becomes

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0018)e^{-(0.1035)^2(90.4)}\cos(0.1035) = 0.378$$

$$\frac{T(L,t)-700}{25-700} = 0.378 \longrightarrow T(L,t) = \mathbf{445\ °C}$$

Discussion This problem can be solved easily using the lumped system analysis since Bi < 0.1, and thus the lumped system analysis is applicable. It gives

$$\alpha = \frac{k}{\rho c_p} \rightarrow \rho c_p = \frac{k}{\alpha} = \frac{110\ \text{W/m}\cdot°\text{C}}{33.9\times10^{-6}\ \text{m}^2/s} = 3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot°\text{C}$$

$$b = \frac{hA}{\rho V c_p} = \frac{hA}{\rho(LA)c_p} = \frac{h}{\rho L c_p} = \frac{h}{L(k/\alpha)} = \frac{80\ \text{W/m}^2\cdot°\text{C}}{(0.015\ \text{m})(3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot°\text{C})} = 0.001644\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt} = 700°\text{C} + (25 - 700°\text{C})e^{-(0.001644\text{s}^{-1})(600\text{s})} = \mathbf{448\ °C}$$

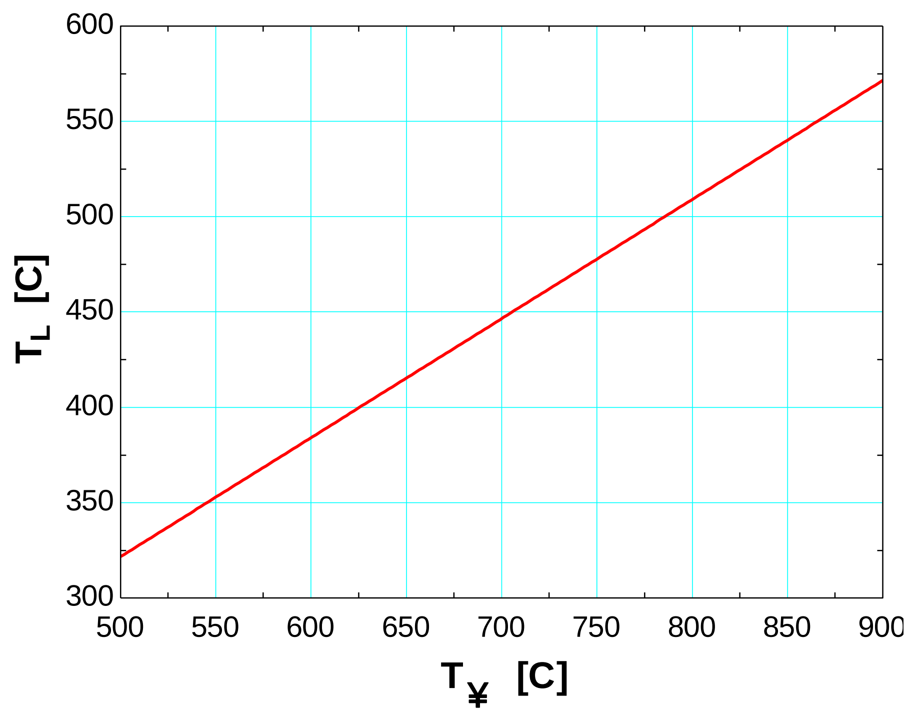
which is almost identical to the result obtained above.

4-51 Prob. 4-50 is reconsidered. The effects of the temperature of the oven and the heating time on the final surface temperature of the plates are to be investigated.

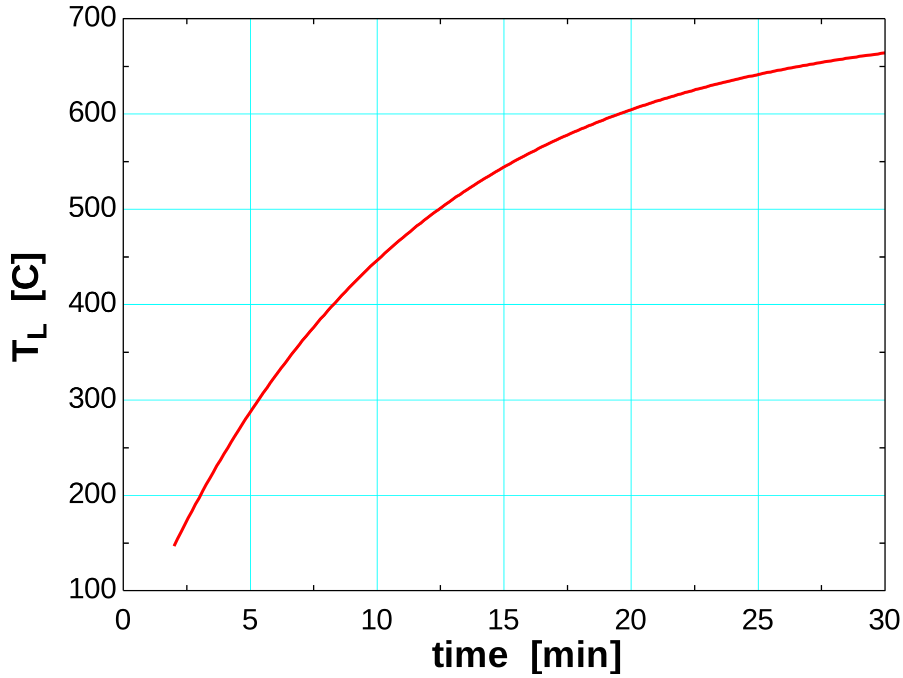
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=(0.03/2) [m]
T_i=25 [C]
T_infinity=700 [C]
time=10 [min]
h=80 [W/m^2-C]

"PROPERTIES"
k=110 [W/m-C]
alpha=33.9E-6 [m^2/s]

"ANALYSIS"
Bi=(h*L)/k
"From Table 4-2, corresponding to this Bi number, we read"
lambda_1=0.1039
A_1=1.0018
tau=(alpha*time*Convert(min, s))/L^2
(T_L-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Cos(lambda_1*L/L)
```

T_∞ [C]	T_L [C]
500	321.6
525	337.2
550	352.9
575	368.5
600	384.1
625	399.7
650	415.3
675	430.9
700	446.5
725	462.1
750	477.8
775	493.4
800	509
825	524.6
850	540.2
875	555.8
900	571.4



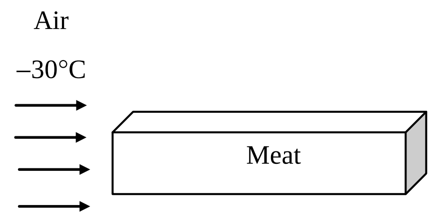
time [min]	T_L [C]
2	146.7
4	244.8
6	325.5
8	391.9
10	446.5
12	491.5
14	528.5
16	558.9
18	583.9
20	604.5
22	621.4
24	635.4
26	646.8
28	656.2
30	664



4-52 The center temperature of meat slabs is to be lowered to -18°C during cooling. The cooling time and the surface temperature of the slabs at the end of the cooling process are to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 11.5 cm. **2** Heat conduction in the meat slabs is one-dimensional because of the symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual cooling time will be much longer than the value determined.

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m²/s. These properties will be used for both fresh and frozen meat.



Analysis First we find the Biot number:

$$\text{Bi} = \frac{hr_o}{k} = \frac{(20\text{ W/m}^2.°\text{C})(0.115\text{ m})}{0.47\text{ W/m°C}} = 4.89$$

From Table 4-2 we read, for a plane wall, λ_1 = 1.308 and A_1=1.239.
Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{-18-(-30)}{7-(-30)} = 1.239 e^{-(1.308)^2 \tau} \rightarrow \tau = 0.783$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{L^2} \rightarrow t = \frac{\tau L^2}{\alpha} = \frac{(0.783)(0.115\text{ m})^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 79{,}650\text{ s} = \mathbf{22.1\,h}$$

The lowest temperature during cooling will occur on the surface (x/L = 1), and is determined to be

$$\frac{T(x) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x / L) \rightarrow \frac{T(L) - T_\infty}{T_i - T_\infty} = \theta_0 \cos(\lambda_1 L / L) = \frac{T_o - T_\infty}{T_i - T_\infty}\cos(\lambda_1)$$

Substituting,

$$\frac{T(L)-(-30)}{7-(-30)} = \left(\frac{-18-(-30)}{7-(-30)}\right)\cos(\lambda_1) = 0.3243\times0.2598 = 0.08425 \longrightarrow T(L) = -\mathbf{26.9°C}$$

which is close the temperature of the refrigerated air.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hL} = \dfrac{0.47\text{W/m.ºC}}{(20\text{ W/m².ºC})(0.115\text{ m})} = 0.204 \\ \dfrac{T_o - T_\infty}{T_i - T_\infty} = \dfrac{-18-(-30)}{7-(-30)} = 0.324 \end{array}\right\} \tau = \frac{\alpha t}{L^2} = 0.75 \quad \text{(Fig. 4-17a)}$$

Therefore, $t = \dfrac{\tau r_o^2}{\alpha} = \dfrac{(0.75)(0.115\text{ m})^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 76{,}300\text{s} \cong 21.2\text{ h}$

The surface temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hL} = 0.204 \\ \dfrac{x}{L} = 1 \end{array}\right\} \frac{T(x) - T_\infty}{T_o - T_\infty} = 0.22 \quad \text{(Fig. 4-17b)}$$

which gives $T_{surface} = T_\infty + 0.22(T_o - T_\infty) = -30 + 0.22[-18-(-30)] = -27.4°\text{C}$

The slight difference between the two results is due to the reading error of the charts.

4-53 The time that a stainless steel plate should be heated in the furnace to at least 600°C is to be determined using (a) Table 4-2 and (b) the Heisler chart (Figure 4-17).

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of stainless steel are given as $\rho = 8238$ kg/m^3, $c_p = 468$ J/kg · K, $k = 13.4$ W/m · K, and $\alpha = 3.48 \times 10^{-6}$ m^2/s.

Analysis The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(215\ \text{W/m}^2\cdot\text{K})(0.025\ \text{m})}{13.4\ \text{W/m}\cdot\text{K}} = 0.4$$

(a) From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

For plane wall with the temperature at the center plane being 600°C, we have

$$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \quad \rightarrow \quad \frac{600-1000}{230-1000} = 1.0580 e^{-(0.5932)^2 \tau} \quad \rightarrow \quad \tau = 2.021$$

Hence, the time that the plate should be heated in the furnace is

$$\tau = \frac{\alpha t}{L^2} = 2.021 > 0.2 \quad \rightarrow \quad t = \frac{2.021 L^2}{\alpha} = \frac{2.021(0.025\ \text{m})^2}{3.48\times 10^{-6}\ \text{m}^2/\text{s}} = \mathbf{363\ s}$$

(b) From Figure 4-16(a) with

$$\frac{1}{Bi} = \frac{1}{0.4} = 2.5 \quad \text{and} \quad \theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{600-1000}{230-1000} \approx 0.52$$

the corresponding Fourier number is $\tau \approx 2.1$. Hence, the time that the plate should be heated in the furnace is

$$\tau = \frac{\alpha t}{L^2} = 2.1 > 0.2 \quad \rightarrow \quad t = \frac{2.1 L^2}{\alpha} = \frac{2.1(0.025\ \text{m})^2}{3.48\times 10^{-6}\ \text{m}^2/\text{s}} = \mathbf{377\ s}$$

Discussion The results for parts (a) and (b) are in comparable agreement, with the result from part (b) approximately 4% larger than the result from part (a).

4-54 The heat transfer from the Pyroceram plate during the cooling process of 286 seconds is to be determined using (a) Table 4-2 and (b) Figure 4-17.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the Pyroceram plate are given as ρ = 2600 kg/m^3, c_p = 808 J/kg · K, k = 3.98 W/m · K, and α = 1.89 × 10^{-6} m^2/s.

Analysis The maximum amount heat transfer from the Pyroceram plate is

$$Q_{\max} = mc_p(T_i - T_\infty) = (10\text{ kg})(808\text{ J/kg}\cdot\text{K})(500-25)\text{ K} = 3.838\times10^6\text{ J}$$

The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(13.3\text{ W/m}^2\cdot\text{K})(0.003\text{ m})}{3.98\text{ W/m}\cdot\text{K}} = 0.01$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.89\times10^{-6}\text{ m}^2\text{/s})(286\text{ s})}{(0.003\text{ m})^2} = 60.06$$

(a) From Table 4-2 with Bi = 0.01, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.0998 \quad \text{and} \quad A_1 = 1.0017$$

For plane wall, we have

$$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = 1.0017e^{-(0.0998)^2 60.06} = 0.5507$$

The heat transfer from the Pyroceram plate during the cooling process of 286 seconds is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{ wall}}\frac{\sin\lambda_1}{\lambda_1} = 1-(0.5507)\frac{\sin(0.0998)}{0.0998} = 0.4502$$

$$Q = 0.4502Q_{\max} = \mathbf{1.73\times10^6\ J}$$

(b) From Figure 4-17c with

$$Bi = 0.01 \quad \text{and} \quad Bi^2\tau = (0.01)^2(60.06) = 0.006$$

we have $Q/Q_{\max} \approx 0.45$

The heat transfer from the Pyroceram plate during the cooling process of 286 seconds is

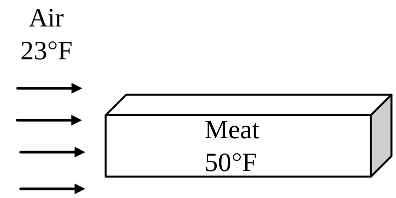
$$Q = 0.45Q_{\max} = \mathbf{1.73\times10^6\ J}$$

Discussion The method for part (b) involved fewer calculations than the method for part (a). However, results obtained using the method in part (a) are generally more accurate than that of part (b).

4-55E The center temperature of meat slabs is to be lowered to 36°F during 12-h of cooling. The average heat transfer coefficient during this cooling process is to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 3-in. **2** Heat conduction in the meat slabs is one-dimensional because of symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.26 Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft²/s.



Analysis The average heat transfer coefficient during this cooling process is determined from the transient temperature charts for a flat plate as follows:

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{L^2} = \frac{(1.4\times10^{-6}\ \text{ft}^2/\text{s})(12\times3600\ \text{s})}{(3/12\ \text{ft})^2} = 0.968 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{36-23}{50-23} = 0.481 \end{aligned}\right\} \frac{1}{Bi} = 0.7 \quad \text{(Fig. 4-17a)}$$

Therefore,

$$h = \frac{kBi}{L} = \frac{(0.26\text{Btu/h.ft.}^\circ\text{F})(1/0.7)}{(3/12)\ \text{ft}} = \mathbf{1.5\ Btu/h.ft^2.^\circ F}$$

Discussion We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the analytical one-term solution relation for an infinite plane wall, but it would require a trial and error approach since the Bi number is not known.

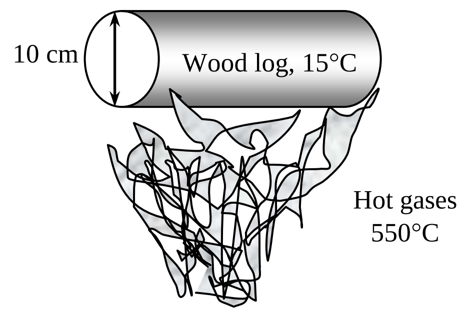
4-56 A long cylindrical wood log is exposed to hot gases in a fireplace. The time for the ignition of the wood is to be determined.

Assumptions **1** Heat conduction in the wood is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the wood are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of wood are given to be k = 0.17 W/m.°C, $\alpha = 1.28\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(13.6\ \text{W/m}^2.\text{°C})(0.05\ \text{m})}{(0.17\ \text{W/m.°C})} = 4.00$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.9081 \quad \text{and} \quad A_1 = 1.4698$$

Once the constant J_0 is determined from Table 4-3 corresponding to the constant λ_1=1.9081, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r_o / r_o)$$

$$\frac{420-550}{15-550} = (1.4698)e^{-(1.9081)^2 \tau}(0.2771) \longrightarrow \tau = 0.1419$$

which is not above the value of 0.2. We use one-term approximate solution (or the transient temperature charts) knowing that the result may be somewhat in error. Then the length of time before the log ignites is

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1419)(0.05\ \text{m})^2}{1.28\times10^{-7}\ \text{m}^2\text{/s}} = 2771\ \text{s} = \mathbf{46.2\ min}$$

Alternative solution: This problem can also be solved using transient temperature charts as follows:

First, the center temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{k}{hr_o} = \dfrac{0.17\ \text{W/m}\cdot\text{°C}}{(13.6\ \text{W/m}^2\cdot\text{°C})(0.05\ \text{m})} = 0.25 \\ \dfrac{x}{r_o} = 1 \end{array}\right\} \frac{T(r_o,t)-T_\infty}{T_0 - T_\infty} \approx 0.25 \qquad \text{(Fig. 4-18}b\text{)}$$

Therefore, $$T_0 = T_\infty + \frac{T(r_o,t)-T_\infty}{0.22} = 550 + \frac{420-550}{0.25} = 30\text{°C}$$

The time for the ignition is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 0.25 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{30-550}{15-550} = 0.972 \end{array}\right\} \tau = \frac{\alpha t}{L^2} \approx 0.15 \qquad \text{(Fig. 4-18}a\text{)}$$

Note that it is very difficult to get an accurate reading for τ in Fig. 4-18*a*.

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.15)(0.05\ \text{m})^2}{1.28\times10^{-7}\ \text{m}^2\text{/s}} = 2930\ \text{s} = 48.8\ \text{min}$$

The difference between the two results is due to the reading error of the charts. For the values of this problem, using transient temperature charts is not recommended as it is very difficult to get accurate readings.

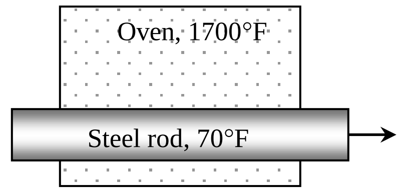
4-57E Long cylindrical steel rods are heat-treated in an oven. Their centerline temperature when they leave the oven is to be determined.

Assumptions **1** Heat conduction in the rods is one-dimensional since the rods are long and they have thermal symmetry about the center line. **2** The thermal properties of the rod are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of AISI stainless steel rods are given to be k = 7.74 Btu/h.ft.°F, α = 0.135 ft^2/h.

Analysis The time the steel rods stays in the oven can be determined from

$$t = \frac{\text{length}}{\text{velocity}} = \frac{21\text{ ft}}{7\text{ ft/min}} = 3\text{ min} = 180\text{ s}$$



The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(20\text{ Btu/h.ft}^2.°\text{F})(2/12\text{ ft})}{(7.74\text{ Btu/h.ft.°F})} = 0.4307$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.8790 \quad \text{and} \quad A_1 = 1.0996$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.135\text{ ft}^2/\text{h})(3/60\text{ h})}{(2/12\text{ ft})^2} = 0.243$$

Then the temperature at the center of the rods becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0996)e^{-(0.8790)^2(0.243)} = 0.911$$

$$\frac{T_0 - 1700}{70 - 1700} = 0.911 \longrightarrow T_o = \mathbf{215°F}$$

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The center temperature is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{7.74\text{ Btu/h.ft.°F}}{(20\text{ Btu/h.ft}^2.°\text{F})(2/12\text{ ft})} = 2.32 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(0.135\text{ ft}^2/\text{h})(3/60\text{ h})}{(2/12\text{ ft})^2} = 0.243 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.9 \qquad \text{(Fig. 4-18}a\text{)}$$

Therefore,

$$T_0 = T_\infty + 0.9(T_i - T_\infty) = 1700 + (0.9)(70 - 1700) = 233°\text{F}$$

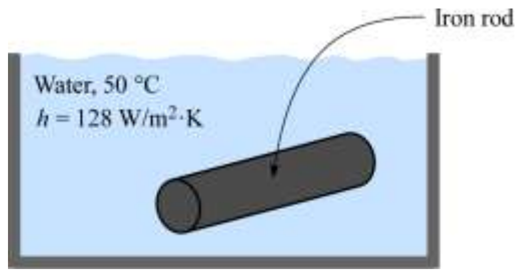
The difference between the two results is due to the reading error of the charts.

4-58 The time required for a long iron rod surface temperature to cool to 200°C in a water bath is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of iron rod are given as ρ = 7870 kg/m^3, c_p = 447 J/kg · K, k = 80.2 W/m · K, and α = 23.1 × 10^{-6} m^2/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(128\ \text{W/m}^2 \cdot \text{K})(0.0125\ \text{m})}{80.2\ \text{W/m} \cdot \text{K}} = 0.02$$

From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.1995 \quad \text{and} \quad A_1 = 1.0050$$

For the temperature at the rod surface ($r = r_o$) to be 200°C, we have

$$\theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_o)$$

From Table 4-3, we have $J_0(0.1995) \approx 0.9900$. Hence

$$\frac{200-50}{700-50} = (1.0050)e^{-(0.1995)^2 \tau}(0.9900) \quad \rightarrow \quad \tau = 36.72$$

The time required for the iron rod surface to cool to 200°C is

$$\tau = \frac{\alpha t}{r_o^2} = 36.72 \quad \rightarrow \quad t = \frac{36.72 r_o^2}{\alpha} = \frac{36.72(0.0125\ \text{m})^2}{23.1 \times 10^{-6}\ \text{m}^2/\text{s}} = \mathbf{248\ s}$$

Discussion Since $\tau > 0.2$, the one-term approximate solution is applicable for this problem.

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The center temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{k}{hr_o} = \dfrac{80.2\ \text{Btu/h.ft.°F}}{(128\ \text{Btu/h.ft}^2\text{.°F})(0.0125\ \text{ft})} = 50 \\ \dfrac{x}{r_o} = 1 \end{array}\right\} \frac{T(r_o,t) - T_\infty}{T_0 - T_\infty} = 0.97 \quad \text{(Fig. 4-18}b\text{)}$$

Therefore, $$T_0 = T_\infty + \frac{T(r_o,t) - T_\infty}{0.97} = 50 + \frac{200-50}{0.97} = 204.6°\text{C}$$

The time period is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 50 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{204.6-50}{700-50} = 0.238 \end{array}\right\} \tau = \frac{\alpha t}{r_o^2} = 37 \qquad \text{(Fig. 4-18}a\text{)}$$

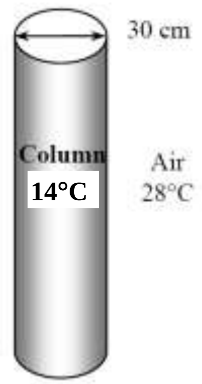
Then, $$t = \frac{37 r_o^2}{\alpha} = \frac{37(0.0125\ \text{m})^2}{23.1 \times 10^{-6}\ \text{m}^2/\text{s}} = 250\ \text{s}$$

The difference between the two results is due to the reading error of the charts.

4–59 A cold cylindrical concrete column is exposed to warm ambient air during the day. The time it will take for the surface temperature to rise to a specified value, the amounts of heat transfer for specified values of center and surface temperatures are to be determined using the approximate analytical solutions.

Assumptions **1** Heat conduction in the column is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the column are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions is applicable (this assumption will be verified).

Properties The properties of concrete are given to be $k = 0.79$ W/m.K, $\alpha = 5.94\times10^{-7}$ m^2/s, $\rho = 1600$ kg/m^3 and $c_p = 0.84$ kJ/kg.K



Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(14 \text{ W/m}^2\cdot\text{K})(0.15\text{ m})}{(0.79 \text{ W/m}\cdot\text{K})} = 2.658$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.7240 \quad \text{and} \quad A_1 = 1.3915$$

Once the constant $J_0 = 0.3841$ is determined from Table 4-3 corresponding to the constant λ_1, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o) \longrightarrow \frac{27-28}{14-28} = (1.3915)e^{-(1.7240)^2\tau}(0.3841) \longrightarrow \tau = 0.6771$$

which is greater than 0.2. Therefore, the one-term approximate solution can be used. Then the time it will take for the column surface temperature to rise to 27°C becomes

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.6771)(0.15\text{ m})^2}{5.94\times10^{-7}\text{ m}^2\text{/s}} = 25{,}650\text{ s} = \mathbf{7.1\,hours}$$

(*b*) The heat transfer to the column will stop when the center temperature of column reaches to the ambient temperature, which is 28°C. That is, we are asked to determine the maximum heat transfer between the ambient air and the column.

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2 L = (1600\text{ kg/m}^3)[\pi(0.15\text{ m})^2(4\text{ m})] = 452.4\text{ kg}$$
$$Q_{\max} = mc_p[T_\infty - T_i] = (452.4\text{ kg})(0.84\text{ kJ/kg}.K)(28-14)°\text{C} = \mathbf{5320\,kJ}$$

(*c*) To determine the amount of heat transfer until the surface temperature reaches to 27°C, we first determine

$$\frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.3915)e^{-(1.7240)^2(0.6771)} = 0.1860$$

Once the constant $J_1 = 0.5787$ is determined from Table 4-3 corresponding to the constant λ_1, the amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\times0.1860\times\frac{0.5787}{1.7240} = 0.875$$
$$Q = 0875 Q_{\max}$$
$$Q = 0.875(5320\text{ kJ}) = \mathbf{4660\,kJ}$$

Alternative solution is on the next page.

Alternative solution: This problem can also be solved using transient temperature and heat transfer charts as follows:

(*a*) The center temperature is determined from

$$\left.\begin{array}{l}\dfrac{1}{\text{Bi}}=\dfrac{k}{hr_o}=\dfrac{0.79\ \text{W/m}\cdot{}^\circ\text{C}}{(14\ \text{W/m}^2\cdot{}^\circ\text{C})(0.15\,\text{m})}=0.376\\[2ex] \dfrac{x}{L}=1\end{array}\right\}\dfrac{T_s-T_\infty}{T_0-T_\infty}=0.27 \qquad \text{(Fig. 4-18}b\text{)}$$

Therefore,

$$T_0=T_\infty+\frac{T_s-T_\infty}{0.9}=28+\frac{27-28}{0.27}=24.3^\circ\text{C}$$

The time period is determined from

$$\left.\begin{array}{l}\dfrac{1}{\text{Bi}}=0.376\\[1ex] \dfrac{T_0-T_\infty}{T_i-T_\infty}=\dfrac{24.3-28}{14-28}=0.265\end{array}\right\}\tau=\frac{\alpha t}{r_o^2}=0.6 \qquad \text{(Fig. 4-18}a\text{)}$$

Then,

$$t=\frac{\tau r_o^2}{\alpha}=\frac{(0.6)(0.15\,\text{m})^2}{5.94\times10^{-7}\ \text{m}^2/\text{s}}=22{,}727\ \text{s}=6.3\,\text{hours}$$

(*c*) The amount of heat transfer is determined from

$$\left.\begin{array}{l}\text{Bi}^2\tau=(2.658)^2(0.6)=4.24\\ \text{Bi}=2.658\end{array}\right\}\frac{Q}{Q_{\max}}=0.83 \qquad \text{(Fig. 4-18}c\text{)}$$

Therefore,

$$Q=0.83Q_{\max}=0.83(5320\,\text{kJ})=4416\,\text{kJ}$$

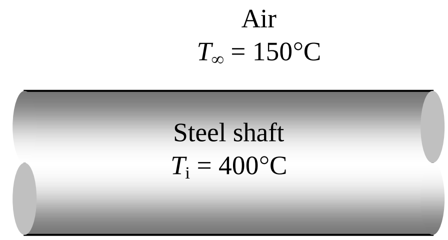
The difference between the results is due to the reading error of the charts.

4-60 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, c_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be



$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.°\text{C})(0.175\ \text{m})}{(14.9\ \text{W/m.°C})} = 0.705$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times 60\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{400-150} = 0.9607 \longrightarrow T_0 = \mathbf{390.2°C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(400-150)°\text{C} = 90{,}642\ \text{kJ}$$

Once the constant J_1 = 0.4679 is determined from Table 4-3 corresponding to the constant λ_1 =1.0904, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390.2-150}{400-150}\right)\frac{0.4679}{1.0904} = 0.1754$$

$$Q = 0.1754(90{,}642\ \text{kJ}) = \mathbf{15{,}900\ kJ}$$

Alternative solution: This problem can also be solved using transient temperature and heat transfer charts as follows:

The center temperature is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{14.9\ \text{W/m}\cdot°\text{C}}{(60\ \text{W/m}^2\cdot°\text{C})(0.175\ \text{m})} = 1.42 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times 60\ \text{s})}{(0.175\ \text{m})^2} = 0.155 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.95 \qquad \text{(Fig. 4-18}a\text{)}$$

Therefore,

$$T_0 = T_\infty + 0.95(T_i - T_\infty) = 150 + (0.95)(400-150) = 388°\text{C}$$

The amount of heat transfer is determined from

$$\left.\begin{array}{l}\mathrm{Bi}^2\tau = (1/1.42)^2(0.155) = 0.077 \\ \mathrm{Bi} = 1/1.42 = 0.704\end{array}\right\} \frac{Q}{Q_{\max}} = 0.18 \qquad \text{(Fig. 4-18}c\text{)}$$

Therefore,

$$Q = 0.18Q_{\max}\ 0.18(90{,}642\ \mathrm{kJ}) = 16{,}300\ \mathrm{kJ}$$

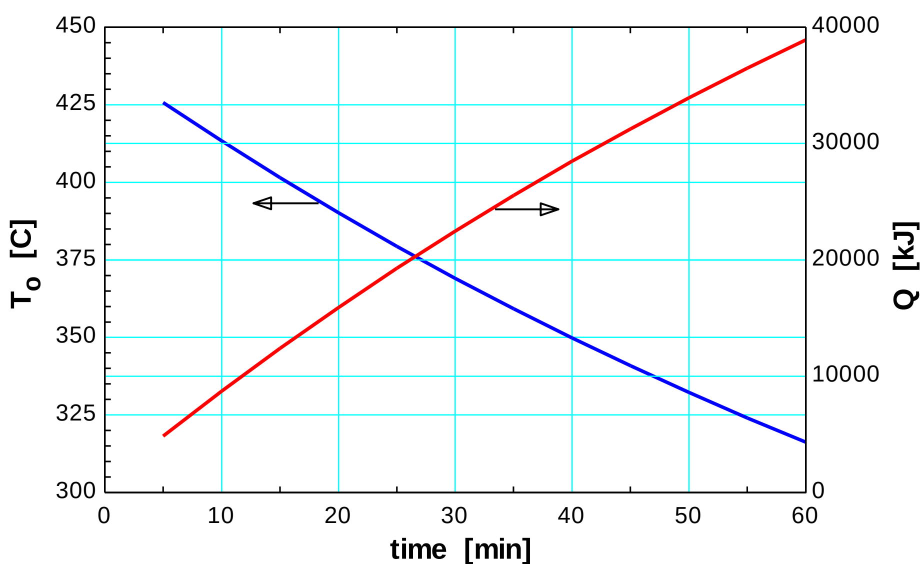
The difference between the results is due to the reading error of the charts.

4-61 Prob. 4-60 is reconsidered. The effect of the cooling time on the final center temperature of the shaft and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_o=(0.35/2) [m]
T_i=400 [C]
T_infinity=150 [C]
h=60 [W/m^2-C]
time=20 [min]

 "PROPERTIES"
k=14.9 [W/m-C]
rho=7900 [kg/m^3]
c_p=477 [J/kg-C]
alpha=3.95E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=1.0904
A_1=1.1548
J_1=0.4679 "From Table 4-3, corresponding to lambda_1"
tau=(alpha*time*Convert(min, s))/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
L=1 "[m], 1 m length of the cylinder is considered"
V=pi*r_o^2*L
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)*Convert(J, kJ)
Q/Q_max=1-2*(T_o-T_infinity)/(T_i-T_infinity)*J_1/lambda_1
```

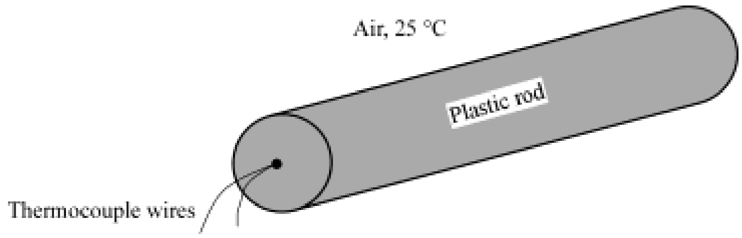
time [min]	T_o [C]	Q [kJ]
5	425.7	4849
10	413.3	8706
15	401.5	12390
20	390.2	15908
25	379.4	19268
30	369.1	22477
35	359.2	25542
40	349.8	28469
45	340.8	31265
50	332.2	33934
55	324	36484
60	316.2	38919



4-62 The convection heat transfer coefficient for a plastic rod being cooled is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the plastic rod are given as ρ = 1190 kg/m^3, c_p = 1465 J/kg · K, and k = 0.19 W/m · K.



Analysis The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = \frac{(0.19\ \text{W/m}\cdot\text{K})(1388\ \text{s})}{(1190\ \text{kg/m}^3)(1465\ \text{J/kg}\cdot\text{K})(0.01\ \text{m})^2} = 1.513$$

After 1388 s of cooling, the temperature at the center of the rod is 30°C. So, we have

$$\theta_{0,\text{cyl}} = A_1 e^{-\lambda_1^2 \tau} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{30-25}{70-25} = 0.111$$

To determine the convection heat transfer coefficient, we need to find the corresponding Biot number by trial-and-error:

Trial 1: Let *Bi* = 0.8 and from Table 4-2 we have

$$\lambda_1 = 1.1490 \quad \text{and} \quad A_1 = 1.1724$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.1724)e^{-(1.1490)^2(1.513)} = 0.159 > 0.111 \quad \text{(does no match)}$$

Trial 2: Let *Bi* = 2.0 and from Table 4-2 we have

$$\lambda_1 = 1.5995 \quad \text{and} \quad A_1 = 1.3384$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.3384)e^{-(1.5995)^2(1.513)} = 0.0279 < 0.111 \quad \text{(does no match)}$$

Trial 3: Let *Bi* = 1.0 and from Table 4-2 we have

$$\lambda_1 = 1.2558 \quad \text{and} \quad A_1 = 1.2071$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.2071)e^{-(1.2558)^2(1.513)} = 0.111 = 0.111 \quad \text{(match)}$$

Therefore the Biot number for this process is

$$Bi = \frac{hr_o}{k} = 1.0 \quad \rightarrow \quad h = \frac{k}{r_o} = \frac{0.19\ \text{W/m}\cdot\text{K}}{0.01\ \text{m}} = \mathbf{19\ W/m^2 \cdot K}$$

Discussion Speeding up the cooling process can be achieved by increasing the convection heat transfer coefficient.

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The Biot number is determined from

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = \frac{(0.19\ \text{W/m}\cdot\text{K})(1388\ \text{s})}{(1190\ \text{kg/m}^3)(1465\ \text{J/kg}\cdot\text{K})(0.01\ \text{m})^2} = 1.51 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{30-25}{70-25} = 0.111 \end{aligned}\right\} \frac{1}{\text{Bi}} = 1 \qquad \text{(Fig. 4-18}a\text{)}$$

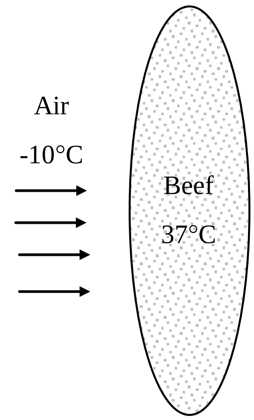
Therefore, $$h = \frac{k\text{Bi}}{r_o} = \frac{(0.19\ \text{W/m}\cdot\text{K})(1)}{0.01\text{m}} = 19\ \text{W/m}^2\cdot\text{K}$$

4-63 The center temperature of a beef carcass is to be lowered to 4°C during cooling. The cooling time and if any part of the carcass will suffer freezing injury during this cooling process are to be determined.

Assumptions **1** The beef carcass can be approximated as a cylinder with insulated top and base surfaces having a radius of r_o = 12 cm and a height of H = 1.4 m. **2** Heat conduction in the carcass is one-dimensional in the radial direction because of the symmetry about the centerline. **3** The thermal properties of the carcass are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of carcass are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{(22\ \mathrm{W/m^2.°C})(0.12\ \mathrm{m})}{0.47\ \mathrm{W/m°C}} = 5.62$$

From Table 4-2 we read, for a cylinder, λ_1 = 2.027 and A_1 = 1.517. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{4-(-10)}{37-(-10)} = 1.517 e^{-(2.027)^2 \tau} \rightarrow \tau = 0.396$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.396)(0.12\ \mathrm{m})^2}{0.13\times10^{-6}\ \mathrm{m^2/s}} = 43{,}865\ \mathrm{s} = \mathbf{12.2\,h}$$

The lowest temperature during cooling will occur on the surface (r/r_o = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o) \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 J_0(\lambda_1 r/r_o) = \frac{T_o - T_\infty}{T_i - T_\infty} J_0(\lambda_1 r_o/r_o)$$

Substituting,
$$\frac{T(r_0)-(-10)}{37-(-10)} = \left(\frac{4-(-10)}{37-(-10)}\right) J_0(\lambda_1) = 0.2979\times0.2084 = 0.0621 \longrightarrow T(r_o) = -7.1°\mathrm{C}$$

which is below the freezing temperature of -1.7 °C. Therefore, the outer part of the beef carcass will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

The cooling time is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.47\mathrm{W/m.°C}}{(22\ \mathrm{W/m^2.°C})(0.12\ \mathrm{m})} = 0.178 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{4-(-10)}{37-(-10)} = 0.298 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.4 \qquad \text{(Fig. 4-18a)}$$

Therefore,
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.4)(0.12\ \mathrm{m})^2}{0.13\times10^{-6}\ \mathrm{m^2/s}} = 44{,}308\mathrm{s} \cong 12.3\mathrm{h}$$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.178 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r)-T_\infty}{T_0 - T_\infty} = 0.17 \qquad \text{(Fig. 4-18b)}$$

which gives $T_{surface} = T_\infty + 0.17(T_0 - T_\infty) = -10 + 0.17[4-(-10)] = -7.6°\mathrm{C}$

The difference between the two results is due to the reading error of the charts.

4-64 The temperature at the center of a Pyroceram rod after 3 minutes of cooling is to be determined using (a) Table 4-2 and (b) the Heisler chart (Figure 4-18).

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of Pyroceram rod are given as ρ = 2600 kg/m^3, c_p = 808 J/kg · K, k = 3.98 W/m · K, and α = 1.89 × 10^{-6} m^2/s

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2\cdot\text{K})(0.005\ \text{m})}{3.98\ \text{W/m}\cdot\text{K}} = 0.10$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.89\times10^{-6}\ \text{m}^2\text{/s})(3\times60\ \text{s})}{(0.005\ \text{m})^2} = 13.61$$

From Table 4-2 with Bi = 0.10, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.4417 \quad \text{and} \quad A_1 = 1.0246$$

The temperature at the center of the rod after 3 minutes is

$$\theta_{0,\,\text{cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}$$

$$T_0 = (T_i - T_\infty)A_1 e^{-\lambda_1^2\tau} + T_\infty = (1000\,°\text{C} - 25\,°\text{C})(1.0246)e^{-(0.4417)^2(13.61)} + 25\,°\text{C} = \mathbf{95.2\,°C}$$

(b) From Figure 4-17a with

$$1/Bi = 1/0.10 = 10 \quad \text{and} \quad \tau = 13.61$$

we get $\theta_0 \approx 0.075$. Hence, the temperature at the center of the rod after 3 minutes is

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.075 \quad \to \quad T_0 = 0.075(1000\,°\text{C} - 25\,°\text{C}) + 25\,°\text{C} = \mathbf{98.1\,°C}$$

Discussion The results for part (a) and (b) are in comparable agreement. The result from part (b) is approximately 3% larger than the result from part (a).

4-65 The amount of heat transfer to a steel rod being drawn through an oven is to be determined using (a) Table 4.2 and (b) Figure 4-18.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the steel rod are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, k = 63.9 W/m · K, and α = 18.8 × 10^{-6} m^2/s.

Analysis The maximum amount of heat transfer to a steel rod is

$$\begin{aligned} Q_{\max} &= \rho V c_p (T_\infty - T_i) = \rho \pi L r_o^2 c_p (T_\infty - T_i) \\ &= (7832\ \text{kg/m}^3)\pi(2\ \text{m})(0.03\ \text{m})^2(434\ \text{J/kg}\cdot\text{K})(800-30)\ \text{K} = 1.48\times10^7\ \text{J} \end{aligned}$$

The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(128\ \text{W/m}^2\cdot\text{K})(0.030\ \text{m})}{63.9\ \text{W/m}\cdot\text{K}} = 0.06$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(18.8\times10^{-6}\ \text{m}^2/\text{s})(133\ \text{s})}{(0.03\ \text{m})^2} = 2.778$$

(a) From Table 4-2 with Bi = 0.06, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.3438 \quad \text{and} \quad A_1 = 1.0148$$

For cylindrical rod, we have

$$\theta_{0,\,\text{cyl}} = A_1 e^{-\lambda_1^2 \tau} = (1.0148)e^{-(0.3438)^2 2.778} = 0.7308$$

The heat transfer to a steel rod after 133 s is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}} \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7308)\frac{0.1694}{0.3438} = 0.2798$$

where from Table 4-3, $J_1(0.3438)$ = 0.1694. Thus

$$Q = 0.2798 Q_{\max} = \mathbf{4.14\times10^6\ J}$$

(b) From Figure 4-18c with

$$Bi = 0.06 \quad \text{and} \quad Bi^2\tau = (0.06)^2(2.778) = 0.01$$

we have $Q/Q_{\max} \approx 0.3$

The heat transfer to a steel rod after 133 s is

$$Q = 0.3 Q_{\max} = \mathbf{4.44\times10^6\ J}$$

Discussion The value of the Bessel function $J_1(\lambda_1)$ for part (a) can also be calculated using the EES with the following line:

```
J_1=Bessel_J1(0.3438)
```

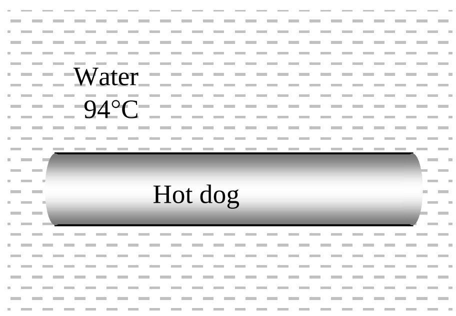
4–66 A hot dog is dropped into boiling water, and temperature measurements are taken at certain time intervals. The thermal diffusivity and thermal conductivity of the hot dog and the convection heat transfer coefficient are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the centerline. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the transient temperature charts (or the one-term approximate solutions) are applicable (this assumption will be verified).

Properties The properties of hot dog available are given to be ρ = 980 kg/m^3 and c_p = 3900 J/kg.K.

Analysis (*a*) From Fig. 4-18b we have

$$\left.\begin{array}{r} \dfrac{T-T_\infty}{T_0-T_\infty}=\dfrac{88-94}{59-94}=0.17 \\ \dfrac{r}{r_o}=\dfrac{r_o}{r_o}=1 \end{array}\right\} \dfrac{1}{Bi}=\dfrac{k}{hr_o}=0.15$$



The Fourier number is determined from Fig. 4-18a to be

$$\left.\begin{array}{r} \dfrac{1}{Bi}=\dfrac{k}{hr_o}=0.15 \\ \dfrac{T_0-T_\infty}{T_i-T_\infty}=\dfrac{59-94}{20-94}=0.47 \end{array}\right\} \tau=\dfrac{\alpha t}{r_o^{\,2}}=0.20$$

Note: With τ = 0.2, it is not a bad assumption to use the one-term approximate solutions (or the transient temperature charts).

The thermal diffusivity of the hot dog is determined to be

$$\frac{\alpha t}{r_o^{\,2}}=0.20 \longrightarrow \alpha=\frac{0.2r_o^{\,2}}{t}=\frac{(0.2)(0.011\,\text{m})^2}{120\,\text{s}}=\mathbf{2.02\times10^{-7}\ m^2/s}$$

(*b*) The thermal conductivity of the hot dog is determined from

$$k=\alpha\rho C_p=(\mathbf{2.02\times10^{-7}}\ \text{m}^2/\text{s})(980\ \text{kg/m}^3)(3900\ \text{J/kg.}K)=\mathbf{0.772\,W/m.K}$$

(*c*) From part (*a*) we have $\dfrac{1}{Bi}=\dfrac{k}{hr_o}=0.15$. Then,

$$\frac{k}{h}=0.15r_o=(0.15)(0.011\,\text{m})=0.00165\,\text{m}$$

Therefore, the heat transfer coefficient is

$$\frac{k}{h}=0.00165 \longrightarrow h=\frac{0.772\ \text{W/m}\cdot\text{K}}{0.00165\,\text{m}}=\mathbf{468\ W/m^2.K}$$

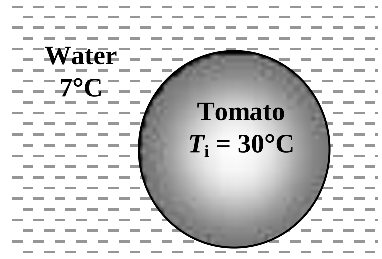
Discussion For known values of food and meat properties, the transient charts (or one-term approximation solutions) can give estimate of their cooking time. We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the analytical one-term solutions for a cylinder, but it would require a trial and error approach since the Bi number is not known.

4-67 Tomatoes are placed into cold water to cool them. The heat transfer coefficient and the amount of heat transfer are to be determined.

Assumptions **1** The tomatoes are spherical in shape. **2** Heat conduction in the tomatoes is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the tomatoes are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the tomatoes are given to be k = 0.59 W/m.°C, $\alpha = 0.141\times10^{-6}$ m^2/s, ρ = 999 kg/m^3 and c_p = 3.99 kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.141\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}{(0.04\ \text{m})^2} = 0.635$$

which is greater than 0.2. Therefore one-term solution is applicable. The ratio of the dimensionless temperatures at the surface and center of the tomatoes are

$$\frac{\theta_{\text{s,sph}}}{\theta_{0,\text{sph}}} = \frac{\dfrac{T_s - T_\infty}{T_i - T_\infty}}{\dfrac{T_0 - T_\infty}{T_i - T_\infty}} = \frac{T_s - T_\infty}{T_0 - T_\infty} = \frac{A_1 e^{-\lambda_1^2\tau}\ \dfrac{\sin(\lambda_1)}{\lambda_1}}{A_1 e^{-\lambda_1^2\tau}} = \frac{\sin(\lambda_1)}{\lambda_1}$$

Substituting,

$$\frac{7.1-7}{10-7} = \frac{\sin(\lambda_1)}{\lambda_1} \longrightarrow \lambda_1 = 3.0401$$

From Table 4-2, the corresponding Biot number and the heat transfer coefficient are

$$\text{Bi} = 31.1$$

$$\text{Bi} = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.59\ \text{W/m.°C})(31.1)}{(0.04\ \text{m})} = \mathbf{459\ W/m^2.°C}$$

The maximum amount of heat transfer is

$$m = 8\rho \boldsymbol{V} = 8\rho\pi D^3/6 = 8(999\ \text{kg/m}^3)[\pi(0.08\ \text{m})^3/6] = 2.143\ \text{kg}$$

$$Q_{\max} = mc_p[T_i - T_\infty] = (2.143\ \text{kg})(3.99\ \text{kJ/kg.°C})(30-7)\text{°C} = 196.6\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 3\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} = 1 - 3\left(\frac{10-7}{30-7}\right)\frac{\sin(3.0401) - (3.0401)\cos(3.0401)}{(3.0401)^3} = 0.9565$$

$$Q = 0.9565 Q_{\max}$$

$$Q = 0.9565(196.6\ \text{kJ}) = \mathbf{188\ kJ}$$

Alternative solution: This problem can also be solved using transient temperature and heat transfer charts as follows:

The Biot number is determined from

$$\left.\begin{array}{l} \dfrac{r}{r_o} = 1 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{7.1-7}{10-7} = 0.033 \end{array}\right\} \frac{1}{\text{Bi}} = 0.03 \qquad \text{(Fig. 4-19}b\text{)}$$

From the definition of Biot number,

$$h = \frac{k\text{Bi}}{r_o} = \frac{(0.59\ \text{W/m}\cdot\text{K})(1/0.03)}{0.04\ \text{m}} = 492\ \text{W/m}^2\cdot\text{K}$$

The amount of heat transfer is

$$\left.\begin{array}{l}\mathrm{Bi}^2\tau = (1/0.03)^2(0.635) = 706 \\ \mathrm{Bi} = 1/0.03 = 33.3\end{array}\right\} \frac{Q}{Q_{\max}} = 0.97 \qquad \text{(Fig. 4-19}c\text{)}$$

Therefore,

$$Q = 0.97Q_{\max}\ 0.97(196.6\ \mathrm{kJ}) = 191\ \mathrm{kJ}$$

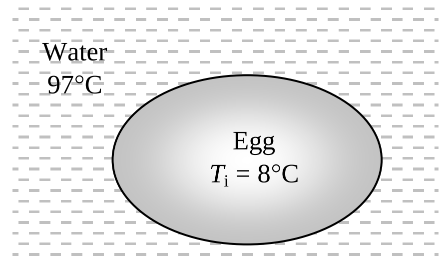
The difference between the results is due to the reading error of the charts.

4-68 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_o = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be k = 0.6 W/m.°C and $\alpha = 0.14\times10^{-6}$ m²/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(1400\text{ W/m}^2.°\text{C})(0.0275\text{ m})}{(0.6\text{ W/m.°C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 3.0877 \quad \text{and} \quad A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-97}{8-97} = (1.9969)e^{-(3.0877)^2 \tau} \longrightarrow \tau = 0.1977 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1977)(0.0275\text{ m})^2}{0.14\times10^{-6}\text{ m}^2/\text{s}} = 1068\text{ s} = \mathbf{17.8\,min}$$

Alternative solution: This problem can also be solved using the transient chart Fig. 4-19*a*,

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{1}{64.2} = 0.0156 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{70-97}{8-97} = 0.303 \end{array}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.18$$

Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.18)(0.0275\text{ m})^2}{0.14\times10^{-6}\text{ m}^2/\text{s}} = 972\text{ s} = 16.2\text{ min}$$

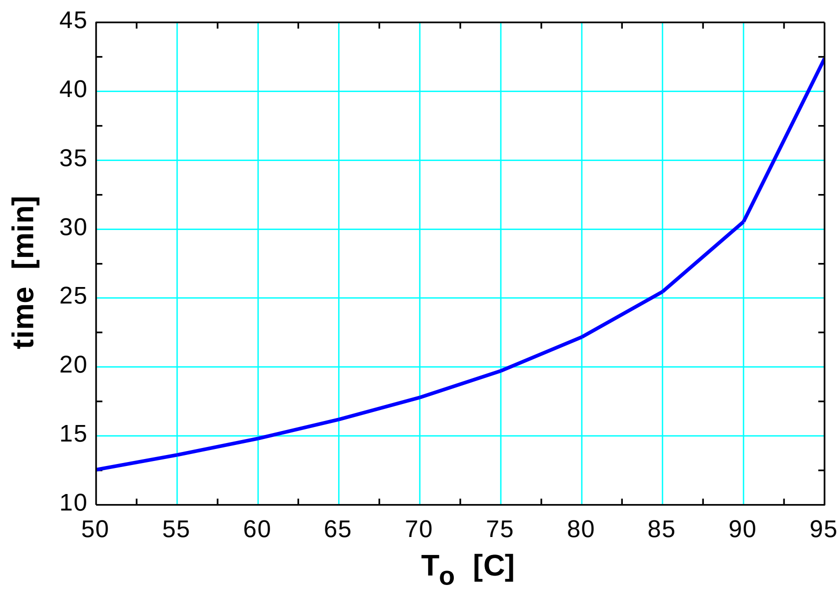
The difference is due to the reading error of the chart.

4-69 Prob. 4-68 is reconsidered. The effect of the final center temperature of the egg on the time it will take for the center to reach this temperature is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.055 [m]
T_i=8 [C]
T_o=70 [C]
T_infinity=97 [C]
h=1400 [W/m^2-C]

"PROPERTIES"
k=0.6 [W/m-C]
alpha=0.14E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
r_o=D/2
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=3.0863
A_1=1.9969
 (T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
time=(tau*r_o^2)/alpha*Convert(s, min)
```

T_o [C]	time [min]
50	12.56
55	13.62
60	14.82
65	16.19
70	17.79
75	19.73
80	22.16
85	25.45
90	30.54
95	42.37

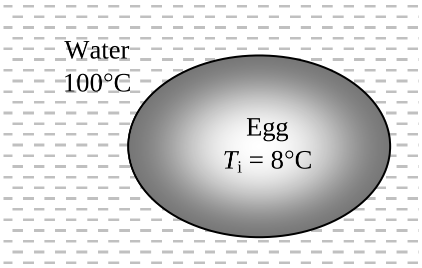


4-70 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho c_p = 0.146\times10^{-6}$ m^2/s (Table A-9).

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(800\ \text{W/m}^2.°\text{C})(0.0275\ \text{m})}{(0.607\ \text{W/m.°C})} = 36.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 3.0533 \quad \text{and} \quad A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-100}{8-100} = (1.9925)e^{-(3.0533)^2\tau} \longrightarrow \tau = 0.1633$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1633)(0.0275\ \text{m})^2}{0.146\times10^{-6}\ \text{m}^2/\text{s}} = 846\ \text{s} = \mathbf{14.1\ min}$$

Alternative solution: This problem can also be solved using the transient chart Fig. 4-19*a*,

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{1}{36.2} = 0.0276 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{60-100}{8-100} = 0.435 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.15$$

Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.15)(0.0275\ \text{m})^2}{0.14\times10^{-6}\ \text{m}^2/\text{s}} = 810\ \text{s} = 13.5\ \text{min}$$

The difference is due to the reading error of the chart.

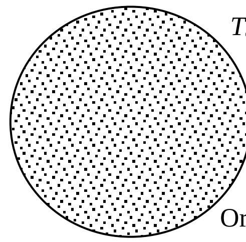
4-71 An orange is exposed to very cold ambient air. It is to be determined whether the orange will freeze in 4 h in subfreezing temperatures.

Assumptions **1** The orange is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the orange is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the orange are constant, and are those of water. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the orange are approximated by those of water at the average temperature of about 5°C, k = 0.571 W/m.°C and $\alpha = k/\rho c_p = 0.571/(999.9\times 4205) = 0.136\times 10^{-6}\ \text{m}^2/\text{s}$ (Table A-9).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(15\ \text{W/m}^2.\text{°C})(0.04\ \text{m})}{(0.571\ \text{W/m.°C})} = 1.051 \approx 1.0$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.136\times 10^{-6}\ \text{m}^2/\text{s})(4\ \text{h}\times 3600\ \text{s/h})}{(0.04\ \text{m})^2} = 1.224 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the oranges becomes

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.2732)e^{-(1.5708)^2(1.224)}\frac{\sin(1.5708\ \text{rad})}{1.5708} = 0.0396$$

$$\frac{T(r_o,t)-(-6)}{15-(-6)} = 0.0396 \longrightarrow T(r_o,t) = \mathbf{-5.2°C}$$

which is less than 0°C. Therefore, the oranges will freeze.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{0.571\ \text{W/m.°C}}{(15\ \text{W/m}^2\cdot\text{°C})(0.04\ \text{m})} = 0.952 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(0.136\times 10^{-6}\ \text{m}^2/\text{s})(4\ \text{h}\times 3600\ \text{s/h})}{(0.04\ \text{m})^2} = 1.224 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.07 \qquad \text{(Fig. 4-19}a\text{)}$$

Therefore, $\quad T_0 = T_\infty + 0.07(T_i - T_\infty) = (-6) + (0.07)[15-(-6)] = -4.53\text{°C}$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.952 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r)-T_\infty}{T_o - T_\infty} = 0.62 \qquad \text{(Fig. 4-19}b\text{)}$$

which gives $\quad T_{surface} = T_\infty + 0.62(T_o - T_\infty) = (-6) + 0.62[(-4.53)-(-6)] = -5.1\text{°C}$

The slight difference between the two results is due to the reading error of the charts.

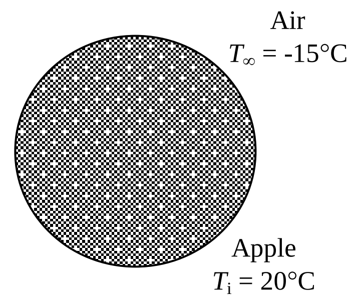
4-72 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418$ W/m.°C, $\rho = 840$ kg/m^3, $c_p = 3.81$ kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8\ \text{W/m}^2.°\text{C})(0.045\ \text{m})}{(0.418\ \text{W/m.°C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3\times10^{-7}\ \text{m}^2\text{/s})(1\ \text{h}\times 3600\ \text{s/h})}{(0.045\ \text{m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2390)e^{-(1.476)^2(0.231)} = 0.749$$

$$\frac{T_0 - (-15)}{20-(-15)} = 0.749 \longrightarrow T_0 = \mathbf{11.2°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)}\frac{\sin(1.476\ \text{rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t) - (-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7°C}$$

The maximum possible heat transfer is

$$m = \rho \boldsymbol{V} = \rho \frac{4}{3}\pi r_o^3 = (840\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^3\right] = 0.3206\ \text{kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (0.3206\ \text{kg})(3.81\ \text{kJ/kg.°C})[20-(-15)]°\text{C} = 42.8\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\frac{\sin(1.476\ \text{rad}) - (1.476)\cos(1.476\ \text{rad})}{(1.476)^3} = 0.402$$

$$Q = 0.402 Q_{\max} = (0.402)(42.8\ \text{kJ}) = \mathbf{17.2\ kJ}$$

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{0.418\ \text{W/m.°C}}{(8\ \text{W/m}^2\cdot°\text{C})(0.045\ \text{m})} = 1.16 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(1.3\times10^{-7}\ \text{m}^2\text{/s})(1\ \text{h}\times3600\ \text{s/h})}{(0.045\ \text{m})^2} = 0.231 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.75 \qquad \text{(Fig. 4-19}a\text{)}$$

Therefore, $T_0 = T_\infty + 0.75(T_i - T_\infty) = (-15) + (0.75)[20 - (-15)] = 11.3°\text{C}$

The surface temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hr_o} = 1.16 \\ \dfrac{r}{r_o} = 1 \end{array}\right\} \dfrac{T(r) - T_\infty}{T_o - T_\infty} = 0.63 \qquad \text{(Fig. 4-19}b\text{)}$$

which gives

$$T_{surface} = T_\infty + 0.63(T_0 - T_\infty) = (-15) + 0.63[11.3 - (-15)] = 1.6°\text{C}$$

The amount of heat transfer is determined from

$$\left.\begin{array}{l} \text{Bi}^2\tau = (0.861)^2(0.231) = 0.171 \\ \text{Bi} = 0.861 \end{array}\right\} \dfrac{Q}{Q_{\max}} = 0.42 \qquad \text{(Fig. 4-19}c\text{)}$$

Therefore,

$$Q = 0.42Q_{\max}\ 0.42(42.8\ \text{kJ}) = 18.0\ \text{kJ}$$

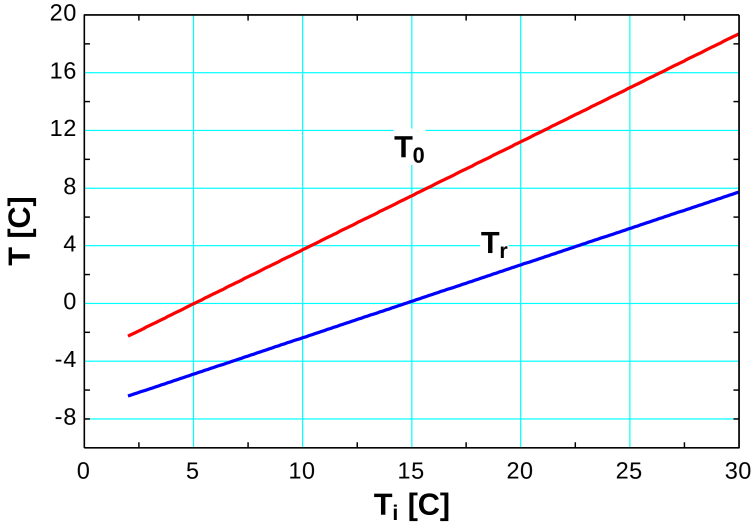
The difference between the results is due to the reading error of the charts.

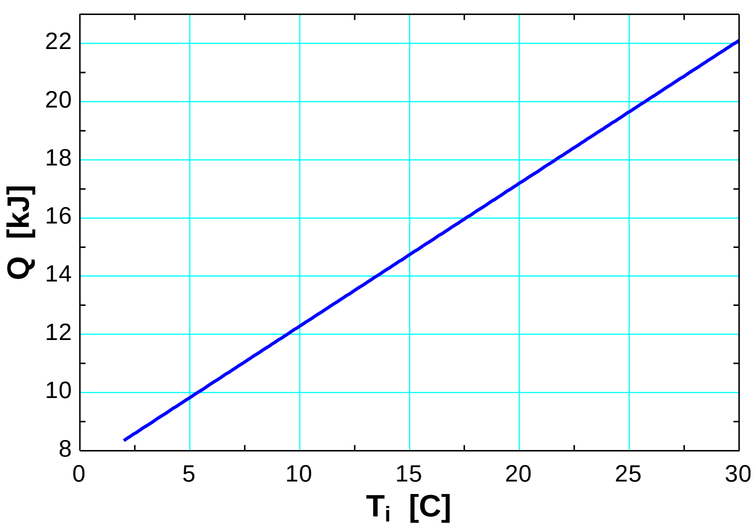
4-73 Prob. 4-72 is reconsidered. The effect of the initial temperature of the apples on the final center and surface temperatures and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity=-15 [C]
T_i=20 [C]
h=8 [W/m^2-C]
r_o=(0.09/2) [m]
time=1*3600 [s]

"PROPERTIES"
k=0.418 [W/m-C]
rho=840 [kg/m^3]
c_p=3.81 [kJ/kg-C]
alpha=1.3E-7 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=1.476
A_1=1.2390
tau=(alpha*time)/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
(T_r-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Sin(lambda_1*r_o/r_o)/(lambda_1*r_o/r_o)
V=4/3*pi*r_o^3
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q/Q_max=1-3*(T_o-T_infinity)/(T_i-T_infinity)*(Sin(lambda_1)-lambda_1*Cos(lambda_1))/lambda_1^3
```

T_i [C]	T_o [C]	T_r [C]	Q [kJ]
2	-2.269	-6.414	8.35
4	-0.7715	-5.403	9.333
6	0.7263	-4.393	10.31
8	2.224	-3.383	11.3
10	3.722	-2.373	12.28
12	5.22	-1.363	13.26
14	6.717	-0.3525	14.24
16	8.215	0.6577	15.23
18	9.713	1.668	16.21
20	11.21	2.678	17.19
22	12.71	3.688	18.17
24	14.21	4.698	19.16
26	15.7	5.709	20.14
28	17.2	6.719	21.12
30	18.7	7.729	22.1


T [C]
20
16
12
8
4
0
-4
-8
0
5
10
15
20
25
30
T_i [C]
T_0
T_r

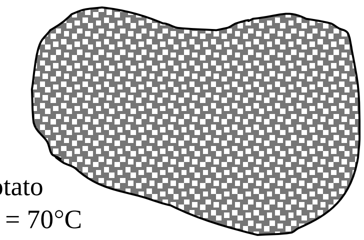

Q [kJ]
22
20
18
16
14
12
10
8
0
5
10
15
20
25
30
T_i [C]

4-74 A hot baked potato is taken out of the oven and wrapped so that no heat is lost from it. The time the potato is baked in the oven and the final equilibrium temperature of the potato after it is wrapped are to be determined.

Assumptions **1** The potato is spherical in shape with a diameter of 9 cm. **2** Heat conduction in the potato is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the potato are given to be $k = 0.6$ W/m.°C, $\rho = 1100$ kg/m^3, $c_p = 3.9$ kJ/kg.°C, and $\alpha = 1.4\times10^{-7}$ m^2/s.

Oven
$T_\infty = 170°C$



Potato
$T_0 = 70°C$

Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.045\ \text{m})}{(0.6\ \text{W/m.°C})} = 3$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 2.2889 \quad \text{and} \quad A_1 = 1.6227$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-170}{25-170} = 0.69 = (1.6227)e^{-(2.2889)^2 \tau} \longrightarrow \tau = 0.163$$

which is not greater than 0.2 but it is close. We may use one-term approximation knowing that the result may be somewhat in error. Then the baking time of the potatoes is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.163)(0.045\ \text{m})^2}{1.4\times10^{-7}\ \text{m}^2/\text{s}} = 2358\ \text{s} = \mathbf{39.3\ min}$$

(*b*) The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho \frac{4}{3}\pi r_o^3 = (1100\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^3\right] = 0.420\ \text{kg}$$

$$Q_{\max} = mc_p(T_\infty - T_i) = (0.420\ \text{kg})(3.900\ \text{kJ/kg.°C})(170-25)°\text{C} = 237\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.69)\frac{\sin(2.2889) - (2.2889)\cos(2.2889)}{(2.2889)^3} = 0.610$$

$$Q = 0.610 Q_{\max} = (0.610)(237\ \text{kJ}) = 145\ \text{kJ}$$

The final equilibrium temperature of the potato after it is wrapped is

$$Q = mc_p(T_{eqv} - T_i) \longrightarrow T_{eqv} = T_i + \frac{Q}{mc_p} = 25°\text{C} + \frac{145\ \text{kJ}}{(0.420\ \text{kg})(3.9\ \text{kJ/kg.°C})} = \mathbf{114°C}$$

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{0.6\ \text{W/m.}^\circ\text{C}}{(40\ \text{W/m}^2.^\circ\text{C})(0.045\ \text{m})} = 0.333 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{70-170}{25-170} = 0.690 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.15 \qquad \text{Fig. (4-19}a\text{)}$$

Therefore,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.15)(0.045)^2}{1.4\times10^{-7}\ \text{m}^2/s} = 2170\ \text{s} = 36.2\ \text{min}$$

The amount of heat transfer is determined from

$$\left.\begin{array}{l}\mathrm{Bi}^2\tau = (3)^2(0.15) = 1.35 \\ \mathrm{Bi} = 3\end{array}\right\}\ \frac{Q}{Q_{\max}} = 0.6 \qquad\qquad \text{(Fig. 4-19}c\text{)}$$

Then,

$$Q = 0.6Q_{\max}\ 0.6(237\ \mathrm{kJ}) = 142\ \mathrm{kJ}$$

The difference between the results is due to the reading error of the charts.

4-75 Chickens are to be chilled by holding them in agitated brine for 2.75 h. The center and surface temperatures of the chickens are to be determined, and if any part of the chickens will freeze during this cooling process is to be assessed.

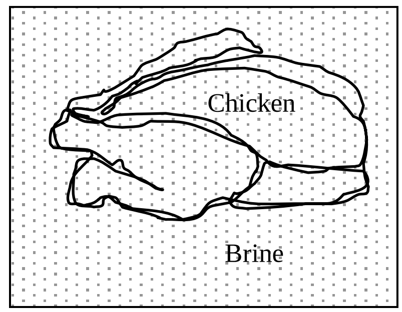
Assumptions **1** The chickens are spherical in shape. **2** Heat conduction in the chickens is one-dimensional in the radial direction because of symmetry about the midpoint. **3** The thermal properties of the chickens are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual the temperatures will be much higher than the values determined since a considerable part of the cooling process will occur during phase change (freezing of chicken).

Properties The thermal conductivity, thermal diffusivity, and density of chickens are given to be $k = 0.45$ W/m·°C, $\alpha = 0.13\times10^{-6}$ m^2/s, and $\rho = 950$ kg/ m^3. These properties will be used for both fresh and frozen chicken.

Analysis We first find the volume and equivalent radius of the chickens:

$$V = m/\rho = 1700\text{g}/(0.95\text{g/cm}^3) = 1789\text{cm}^3$$

$$r_o = \left(\frac{3}{4\pi}V\right)^{1/3} = \left(\frac{3}{4\pi}1789\text{ cm}^3\right)^{1/3} = 7.53\text{ cm} = 0.0753\text{ m}$$



Then the Biot and Fourier numbers become

$$\text{Bi} = \frac{hr_o}{k} = \frac{(440\text{ W/m}^2.°\text{C})(0.0753\text{ m})}{0.45\text{ W/m°C}} = 73.6$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.13\times10^{-6}\text{ m}^2/\text{s})(2.75\times3600\text{ s})}{(0.0753\text{ m})^2} = 0.2270$$

Note that $\tau = 0.2270 > 0.2$, and thus the one-term solution is applicable. From Table 4-2 we read, for a sphere, $\lambda_1 = 3.094$ and $A_1 = 1.998$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{T_0 - (-7)}{15-(-7)} = 1.998e^{-(3.094)^2(0.2270)} = 0.2274 \longrightarrow T_0 = \mathbf{-2.0°C}$$

The lowest temperature during cooling will occur on the surface ($r/r_o = 1$), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting,

$$\frac{T(r_o)-(-7)}{15-(-7)} = 0.2274\frac{\sin(3.094\text{ rad})}{3.094} \rightarrow T(r_o) = \mathbf{-6.9°C}$$

Most parts of chicken will freeze during this process since the freezing point of chicken is -2.8°C.

Discussion We could also solve this problem using transient temperature charts, but the data in this case falls at a point on the chart which is very difficult to read:

$$\left.\begin{aligned}\tau &= \frac{\alpha t}{r_o^2} = \frac{(0.13\times10^{-6}\text{ m}^2/\text{s})(2.75\times3600\text{ s})}{(0.0753\text{ m})^2} = 0.227\\ \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.45\text{W/m.°C}}{(440\text{W/m}^2.°\text{C})(0.0753\text{m})} = 0.0136\end{aligned}\right\}\frac{T_o - T_\infty}{T_i - T_\infty} = 0.15\ldots0.30 \ ?? \quad \text{Fig. (4-19}a\text{)}$$

4-76 The time it takes for the surface of a falling hailstone to reach melting point is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of ice at 253 K are ρ = 922 kg/m^3, c_p = 1945 J/kg · K, and k = 2.03 W/m · K (from Table A-8).

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(163\ \text{W/m}^2\cdot\text{K})(0.010\ \text{m})}{2.03\ \text{W/m}\cdot\text{K}} = 0.80$$

From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 1.4320 \quad \text{and} \quad A_1 = 1.2236$$

For a sphere, we have

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

For the hailstone surface ($r = r_o$) to reach melting point (0°C), the Fourier number is

$$\frac{0-15}{-20-15} = (1.2236)e^{-(1.4320)^2\tau}\frac{\sin(1.4320)}{1.4320} \quad \rightarrow \quad \tau = 0.3318$$

The time required for hailstone surface to reach melting point is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = 0.3318$$

$$t = \frac{0.3318 r_o^2 \rho c_p}{k} = \frac{0.3318(0.01\ \text{m})^2(922\ \text{kg/m}^3)(1945\ \text{J/kg}\cdot\text{K})}{2.03\ \text{W/m}\cdot\text{K}} = \mathbf{29.3\,s}$$

Discussion Depending on the altitude in which the hailstone is formed, its surface may not even reach melting point before hitting the ground.

Alternative solution: This problem can also be solved using transient temperature charts as follows:

The center temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{k}{hr_o} = \dfrac{2.03\ \text{W/m}\cdot{}^\circ\text{C}}{(163\ \text{W/m}^2\cdot{}^\circ\text{C})(0.010\ \text{m})} = 1.25 \\ \dfrac{r}{r_o} = 1 \end{array}\right\} \frac{T - T_\infty}{T_0 - T_\infty} = 0.63 \qquad \text{(Fig. 4-19}b\text{)}$$

Therefore, $$T_0 = T_\infty + \frac{T - T_\infty}{0.63} = 15 + \frac{0-15}{0.63} = -8.8^\circ\text{C}$$

The time period is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 1.25 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{-8.8-15}{-20-15} = 0.680 \end{array}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.3 \qquad \text{(Fig. 4-19}a\text{)}$$

Then,

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = 0.3 \ \rightarrow\ t = \frac{0.3 r_o^2 \rho c_p}{k} = \frac{0.3(0.01\ \text{m})^2(922\ \text{kg/m}^3)(1945\ \text{J/kg}\cdot\text{K})}{2.03\ \text{W/m}\cdot\text{K}} = 26.5\,\text{s}$$

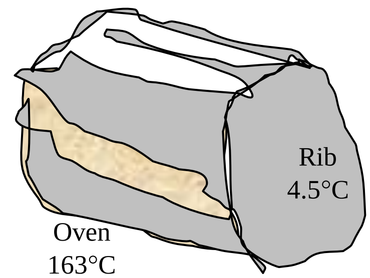
The difference between the two results is due to the reading error of the charts.

4-77 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (*a*) The radius of the roast is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(2\times3600 + 45\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when $Bi = 30$, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{ m})} = \mathbf{156.9\ W/m^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5\,°C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.65)\frac{\sin(3.0372)-(3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{\max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898)e^{-(3.0372)^2 \tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1336)(0.08603\,\text{m})^2}{(0.91\times10^{-7}\ \text{m}^2/\text{s})} = 10{,}866\,\text{s} = 181\,\text{min} \cong \mathbf{3\,hr}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

Alternative solution: This problem can also be solved using transient temperature and heat transfer charts as follows:

(*a*) The Biot number is determined from

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\ \text{m}^2/\text{s})(2\times3600 + 45\times60)\text{s}}{(0.08603\,\text{m})^2} = 0.1217 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{60-163}{4.5-163} = 0.65 \end{aligned}\right\} \frac{1}{\text{Bi}} = ??? \quad \text{(Fig. 4-19}a\text{)}$$

It is very difficult to get a reasonably accurate reading at these values in Fig. 4-18a. In order to continue with the chart solution, we copy the Bi number from the analytical solution as Bi = 30. Then, the heat transfer coefficient is

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\ \text{W/m.°C})(30)}{(0.08603\,\text{m})} = 156.9\ \text{W/m}^2.°\text{C}$$

(*b*) The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{1}{30} = 0.033 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_o - T_\infty} = 0.025 \qquad \text{Fig. (4-19}b\text{)}$$

which gives $T_{surface} = T_\infty + 0.025(T_0 - T_\infty) = 163 + 0.025(60-163) = 160.4°\text{C}$

(*c*) The amount of heat transfer is determined from

$$\left.\begin{aligned} \text{Bi}^2\tau &= (30)^2(0.1217) = 109.5 \\ \text{Bi} &= 30 \end{aligned}\right\} \frac{Q}{Q_{\max}} = 0.77 \qquad \text{(Fig. 4-19}c\text{)}$$

$$Q = 0.77Q_{\max}\ 0.77(2080\,\text{kJ}) = 1602\,\text{kJ}$$

(*d*) The cooking time for medium-done rib is determined from

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{1}{30} = 0.033 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{71-163}{4.5-163} = 0.58 \end{aligned}\right\} \tau = \frac{\alpha \text{t}}{r_o^2} \approx 0.1 \quad \text{Fig. (4-19}a\text{)}$$

Agan, it is very difficult to get a good reading at these values. Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1)(0.08603\,\text{m})^2}{(0.91\times10^{-7}\ \text{m}^2/\text{s})} = 8133 = 2.26\,\text{h}$$

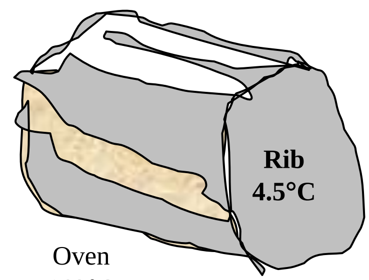
The difference between the results is due to the reading error of the charts.

4-78 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is well-done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis (*a*) The radius of the rib is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.00267\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.00267\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2\text{/s})(4\times3600+15\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1881$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution formulation can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{77-163}{4.5-163} = 0.543 = A_1 e^{-\lambda_1^2(0.1881)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when $Bi = 4.3$, which corresponds to $\lambda_1 = 2.4900$ and $A_1 = 1.7402$. Then the heat transfer coefficient can be determined from.

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(4.3)}{(0.08603\text{ m})} = \mathbf{22.5\ W/m^2.°C}$$

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.7402)e^{-(2.49)^2(0.1881)}\frac{\sin(2.49)}{2.49}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.132 \longrightarrow T(r_o,t) = \mathbf{142.1°C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1-3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.543)\frac{\sin(2.49)-(2.49)\cos(2.49)}{(2.49)^3} = 0.727$$

$$Q = 0.727Q_{\max} = (0.727)(2080\text{ kJ}) = \mathbf{1512\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.7402)e^{-(2.49)^2\tau} \longrightarrow \tau = 0.177$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.177)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2\text{/s})} = 14{,}400\text{ s} = 240\text{ min} = \mathbf{4\ hr}$$

This result is close to the listed value of 4 hours and 15 minutes. The difference between the two results is probably due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

Alternative solution: This problem can also be solved using transient temperature and heat transfer charts as follows:

(*a*) The Biot number is determined from

$$\left.\begin{array}{l} \tau = \dfrac{\alpha t}{r_o^2} = \dfrac{(0.91\times10^{-7}\ \text{m}^2/\text{s})(4\times3600+15\times60)\text{s}}{(0.08603\,\text{m})^2} = 0.1881 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{77-163}{4.5-163} = 0.543 \end{array}\right\} \dfrac{1}{\text{Bi}} \approx 0.2 \qquad \text{(Fig. 4-19}a\text{)}$$

It is very difficult to get a reasonably accurate reading at these values in Fig. 4-19a. Then,

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\ \text{W/m.°C})(1/0.2)}{(0.08603\,\text{m})} = 26.2\ \text{W/m}^2.\text{°C}$$

(*b*) The surface temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 0.2 \\ \dfrac{r}{r_o} = 1 \end{array}\right\} \dfrac{T(r) - T_\infty}{T_o - T_\infty} = 0.15 \qquad \text{Fig. (4-19}b\text{)}$$

which gives $T_{surface} = T_\infty + 0.15(T_0 - T_\infty) = 163 + 0.15(77-163) = 150.1\text{°C}$

(*c*) The amount of heat transfer is determined from

$$\left.\begin{array}{l} \text{Bi}^2\tau = (5)^2(0.1881) = 4.70 \\ \text{Bi} = 1/0.2 = 5 \end{array}\right\} \dfrac{Q}{Q_{\max}} = 0.73 \qquad \text{(Fig. 4-19}c\text{)}$$

$$Q = 0.77Q_{\max}\ 0.73(2080\ \text{kJ}) = 1518\ \text{kJ}$$

(*d*) The cooking time for medium-done rib is determined from

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = 0.2 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{71-163}{4.5-163} = 0.58 \end{array}\right\} \tau = \dfrac{\alpha \text{t}}{r_o^2} \approx 0.16 \qquad \text{Fig. (4-19}a\text{)}$$

Agan, it is very difficult to get a good reading at these values. Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.16)(0.08603\,\text{m})^2}{(0.91\times10^{-7}\ \text{m}^2/\text{s})} = 13{,}013 = 3.61\,\text{h}$$

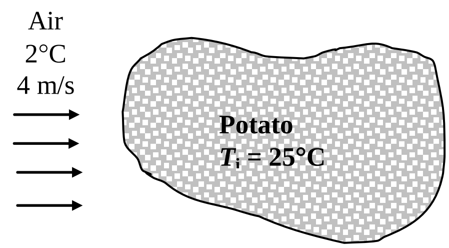
The difference between the results is due to the reading error of the charts.

4-79 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions **1** The potatoes are spherical in shape with a radius of r_0 = 3 cm. **2** Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be k = 0.50 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\mathrm{Bi} = \frac{hr_o}{k} = \frac{(19\ \mathrm{W/m^2.°C})(0.03\ \mathrm{m})}{0.5\ \mathrm{W/m°C}} = 1.14$$

From Table 4-2 we read, for a sphere, λ_1 = 1.635 and A_1 = 1.302. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{6-2}{25-2} = 1.302 e^{-(1.635)^2\tau} \rightarrow \tau = 0.7531$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.7531)(0.03\ \mathrm{m})^2}{0.13\times10^{-6}\ \mathrm{m^2/s}} = 5213\ \mathrm{s} = \mathbf{1.45\ h}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting, $$\frac{T(r_o)-2}{25-2} = \left(\frac{6-2}{25-2}\right)\frac{\sin(1.635\ \mathrm{rad})}{1.635} \longrightarrow T(r_o) = 4.44°\mathrm{C}$$

which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, **no part** of the potatoes will experience chilling injury during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{array}{l} \dfrac{1}{\mathrm{Bi}} = \dfrac{k}{hr_o} = \dfrac{0.50\ \mathrm{W/m.^oC}}{(19\ \mathrm{W/m^2.^oC})(0.03\ \mathrm{m})} = 0.877 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{6-2}{25-2} = 0.174 \end{array}\right\} \tau = \frac{\alpha \mathrm{t}}{r_o^2} = 0.75 \qquad \text{Fig. (4-19}a\text{)}$$

Therefore, $$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.75)(0.03)^2}{0.13\times10^{-6}\ \mathrm{m^2}/s} = 5192\ \mathrm{s} = \mathbf{1.44h}$$

The surface temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hr_o} = 0.877 \\ \dfrac{r}{r_o} = 1 \end{array}\right\} \frac{T(r)-T_\infty}{T_o - T_\infty} = 0.6 \qquad \text{Fig. (4-19}b\text{)}$$

which gives $T_{surface} = T_\infty + 0.6(T_o - T_\infty) = 2 + 0.6(6-2) = 4.4°\mathrm{C}$

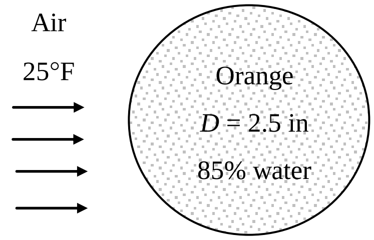
The slight difference between the two results is due to the reading error of the charts.

4-80E The center temperature of oranges is to be lowered to 40°F during cooling. The cooling time and if any part of the oranges will freeze during this cooling process are to be determined.

Assumptions **1** The oranges are spherical in shape with a radius of r_o =1.25 in = 0.1042 ft. **2** Heat conduction in the orange is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the orange are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of oranges are given to be k = 0.26 Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(4.6\,\text{Btu/h.ft}^2.°\text{F})(1.25/12\,\text{ft})}{0.26\,\text{Btu/h.ft.°F}} = 1.843$$

From Table 4-2 we read, for a sphere, λ_1 = 1.9569 and A_1 = 1.447. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \;\rightarrow\; \frac{40-25}{78-25} = 1.447 e^{-(1.9569)^2 \tau} \;\rightarrow\; \tau = 0.426$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \;\rightarrow\; t = \frac{\tau r_o^2}{\alpha} = \frac{(0.426)(1.25/12\,\text{ft})^2}{1.4\times10^{-6}\,\text{ft}^2/\text{s}} = 3302\,\text{s} = \mathbf{55.0\,min}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty} \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting, $$\frac{T(r_o) - 25}{78-25} = \left(\frac{40-25}{78-25}\right)\frac{\sin(1.9569\,\text{rad})}{1.9569} \longrightarrow T(r_o) = 32.1°\text{F}$$

which is above the freezing temperature of 31°F for oranges. Therefore, no part of the oranges will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.26\text{Btu/h.ft.°F}}{(4.6\text{ Btu/h.ft}^2.°\text{F})(1.25/12\,\text{ft})} = 0.543 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{40-25}{78-25} = 0.283 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.43 \qquad \text{Fig. (4-19}a\text{)}$$

Therefore, $$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.43)(1.25/12\text{ft})^2}{1.4\times10^{-6}\,\text{ft}^2/\text{s}} = 3333\,\text{s} = 55.6\,\text{min}$$

The lowest temperature during cooling will occur on the surface (r/r_o =1) of the oranges is determined to be

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.543 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_0 - T_\infty} = 0.45 \qquad \text{Fig. (4-19}b\text{)}$$

which gives $$T_{surface} = T_\infty + 0.45(T_0 - T_\infty) = 25 + 0.45(40-25) = 31.8°\text{F}$$

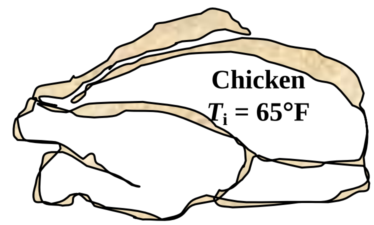
The slight difference between the two results is due to the reading error of the charts.

4-81E Whole chickens are to be cooled in the racks of a large refrigerator. Heat transfer coefficient that will enable to meet temperature constraints of the chickens while keeping the refrigeration time to a minimum is to be determined.

Assumptions **1** The chicken is a homogeneous spherical object. **2** Heat conduction in the chicken is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the chicken are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the chicken are given to be k = 0.26 Btu/h.ft.°F, ρ = 74.9 lbm/ft^3, c_p = 0.98 Btu/lbm.°F, and α = 0.0035 ft^2/h.

Analysis The radius of the chicken is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{5\text{ lbm}}{74.9\text{ lbm/ft}^3} = 0.06676\text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.06676\text{ ft}^3)}{4\pi}} = 0.2517\text{ ft}$$

From Fig. 4-18b we have

$$\left.\begin{array}{l} \dfrac{T - T_\infty}{T_0 - T_\infty} = \dfrac{35-5}{45-5} = 0.75 \\ \dfrac{x}{r_o} = \dfrac{r_o}{r_o} = 1 \end{array}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 2$$

Then the heat transfer coefficients becomes

$$h = \frac{k}{2r_o} = \frac{0.26\text{ Btu/.ft.°F}}{2(0.2517\text{ ft})} = \mathbf{0.516\ Btu/h.ft^2.°F}$$

Discussion We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the one-term solution relation for a sphere, but it would require a trial and error approach since the Bi number is not known.

Transient Heat Conduction in Semi-Infinite Solids

4-82C A thick plane wall can be treated as a semi-infinite medium if all we are interested in is the variation of temperature in a region near one of the surfaces for a time period during which the temperature in the mid section of the wall does not experience any change.

4-83C A semi-infinite medium is an idealized body which has a single exposed plane surface and extends to infinity in all directions. The earth and thick walls can be considered to be semi-infinite media.

4-84C The total amount of heat transfer from a semi-infinite solid up to a specified time t_0 can be determined by integration from

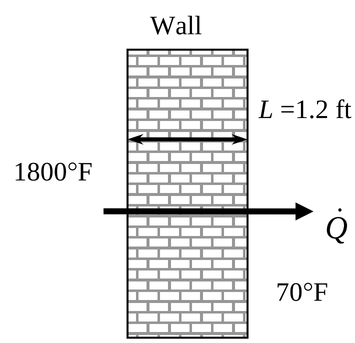
$$Q=\int_0^{t_o} Ah[T(0,t)-T_\infty]dt$$

where the surface temperature $T(0, t)$ is obtained from Eq. 4-47 by substituting $x = 0$.

4-85E The walls of a furnace made of concrete are exposed to hot gases at the inner surfaces. The time it will take for the temperature of the outer surface of the furnace to change is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 1800°F. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete are given to be $k = 0.64$ Btu/h.ft.°F and $\alpha = 0.023$ ft^2/h.



Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

But,

$$\frac{T(x,t)-T_i}{T_s-T_i}=\frac{70.1-70}{1800-70}=0.00006 \;\rightarrow\; 0.00006=erfc(2.85) \quad \text{(Table 4-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}}=2.85\longrightarrow t=\frac{x^2}{4\times(2.85)^2\alpha}=\frac{(1.2\text{ ft})^2}{4\times(2.85)^2(0.023\text{ ft}^2/\text{h})}=1.93\text{ h}=\mathbf{116\ min}$$

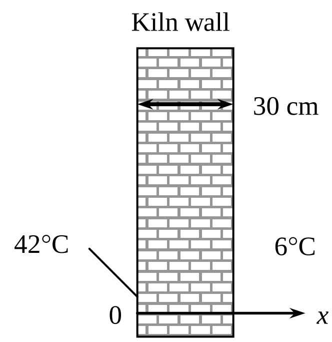
4–86 A curing kiln is heated by injecting steam into it and raising its inner surface temperature to a specified value. It is to be determined whether the temperature at the outer surfaces of the kiln changes during the curing period.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 45°C. **2** The thermal properties of the concrete wall are constant.

Properties The thermal diffusivity of the concrete wall is given to be $\alpha = 0.23\times10^{-5}$ m^2/s.

Analysis We determine the temperature at a depth of x = 0.3 m in 2.5 h using the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



Substituting,

$$\frac{T(x,t)-6}{42-6} = erfc\left(\frac{0.3\,\text{m}}{2\sqrt{(0.23\times10^{-5}\ \text{m}^2/\text{s})(2.5\,\text{h}\times3600\,\text{s/h})}}\right)$$

$$= erfc(1.043) = 0.1402$$

$$T(x,t) = \mathbf{11.0\,°C}$$

which is greater than the initial temperature of 6°C. Therefore, heat will propagate through the 0.3 m thick wall in 2.5 h, and thus it may be desirable to insulate the outer surface of the wall to save energy.

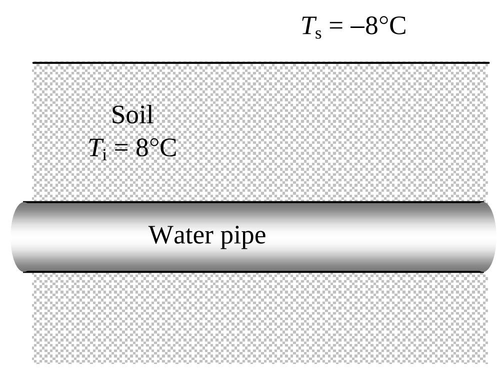
4-87 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.35 W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis The length of time the snow pack stays on the ground is

$$t = (60\text{ days})(24\text{ hr/days})(3600\text{ s/hr}) = 5.184\times10^6\text{ s}$$



The surface is kept at -8°C at all times. The depth at which freezing at 0°C occurs can be determined from the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$

$$\frac{0-8}{-8-8} = erfc\left(\frac{x}{2\sqrt{(0.15\times10^{-6}\ \text{m}^2/\text{s})(5.184\times10^6\ \text{s})}}\right)$$

$$0.5 = erfc\left(\frac{x}{1.7636}\right)$$

Then from Table 4-4 we get

$$\frac{x}{1.7636} = 0.4796 \longrightarrow x = \mathbf{0.846\,m}$$

Discussion The solution could also be determined using the chart, but it would be subject to reading error.

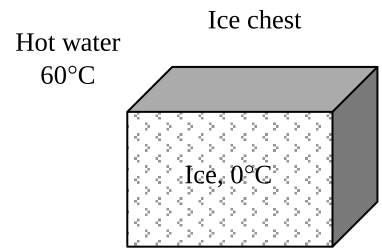
4-88 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be k = 52 W/m.°C and $\alpha = 1.70\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



But,

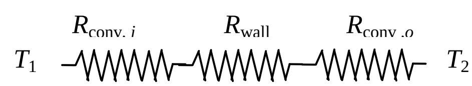
$$\frac{T(x,t)-T_i}{T_s-T_i}=\frac{0.1-0}{60-0}=0.00167\ \rightarrow\ 0.00167=erfc(2.226)\quad \text{(Table 4-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}}=2.206\longrightarrow t=\frac{x^2}{4\times(2.226)^2\alpha}=\frac{(0.05\ \text{m})^2}{4(2.226)^2(1.7\times10^{-5}\ \text{m}^2/\text{s})}=\mathbf{7.42\,s}$$

The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as

$$R_{conv,i}=\frac{1}{h_iA}=\frac{1}{(250\ \text{W/m}^2.°\text{C})(1.2\times 2\ \text{m}^2)}=0.001667°\text{C/W}$$



$$R_{wall}=\frac{L}{kA}=\frac{0.05\ \text{m}}{(52\ \text{W/m.°C})(1.2\times 2\ \text{m}^2)}=0.00040°\text{C/W}$$

$$R_{conv,o}=\frac{1}{h_oA}=\frac{1}{(\infty)(1.2\times 2\ \text{m}^2)}\cong 0°\text{C/W}$$

$$R_{total}=R_{conv,i}+R_{wall}+R_{conv,o}=0.001667+0.00040+0=0.002067°\text{C/W}$$

$$\dot{Q}=\frac{T_2-T_1}{R_{total}}=\frac{(60-0)°\text{C}}{0.002067^{\text{o}}\ \text{C/W}}=\mathbf{29{,}030\ W}$$

4-89 With the highway surface temperature maintained at 25°C, the temperature at the depth of 3 cm from surface and the heat flux transferred after 60 minutes are to be determined.

Assumptions **1** The highway is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The properties of asphalt are ρ = 2115 kg/m^3, c_p = 920 J/kg · K, and k = 0.062 W/m · K (from Table A-8).

Analysis The thermal diffusivity for asphalt is

$$\alpha = \frac{k}{\rho c_p} = \frac{0.062\ \text{W/m}\cdot\text{K}}{(2115\ \text{kg/m}^3)(920\ \text{J/kg}\cdot\text{K})} = 3.186\times 10^{-8}\ \text{m}^2\text{/s}$$

For semi-infinite solid with specified surface temperature, we have

$$\frac{T(x,t)-T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

where

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.03\ \text{m}}{2\sqrt{(3.186\times 10^{-8}\ \text{m}^2\text{/s})(60\times 60\ \text{s})}} = 1.40$$

From Table 4-4, erfc(1.40) = 0.04772. Hence the temperature at the depth of 3 cm from the highway surface after 60 minutes is

$$T(0.03\ \text{m}, 3600\ \text{s}) = (T_s - T_i)\text{erfc}(1.40) + T_i = (25\,°\text{C} - 55\,°\text{C})(0.04772) + 55\,°\text{C} = \mathbf{53.6\,°C}$$

The heat flux transferred from the highway after 60 minutes is

$$\dot{q}_s(t) = \frac{k(T_i - T_s)}{\sqrt{\pi \alpha t}} \quad \rightarrow \quad \dot{q}_s(3600\ \text{s}) = \frac{(0.062\ \text{W/m}\cdot\text{K})(55-25)\ \text{K}}{\sqrt{\pi(3.186\times 10^{-8}\ \text{m}^2\text{/s})(60\times 60\ \text{s})}} = \mathbf{98\ W/m^2}$$

Discussion Having very low thermal diffusivity, asphalt diffuses heat so slowly that even after 60 minutes of the surface maintained at 25°C, the temperature at the depth of 3 cm only drops by less than 2°C.

4-90 An aluminum block is subjected to heat flux. The surface temperature of the block is to be determined.

Assumptions **1** All heat flux is absorbed by the block. **2** Heat loss from the block is disregarded (and thus the result obtained is the maximum temperature). **3** The block is sufficiently thick to be treated as a semi-infinite solid, and the properties of the block are constant.

Properties Thermal conductivity and diffusivity of aluminum at room temperature are k = 237 kg/m^3 and α = 97.1×10^{-6} m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the surface temperature can be determined to be

$$T_s = T_i + \frac{\dot{q}_s}{k}\sqrt{\frac{4\alpha t}{\pi}} = 20°\text{C} + \frac{4000\ \text{W/m}^2}{237\ \text{W/m}\cdot°\text{C}}\sqrt{\frac{4(9.71\times 10^{-5}\ \text{m}^2\text{/s})(30\times 60\ \text{s})}{\pi}} = 28.0°\text{C}$$

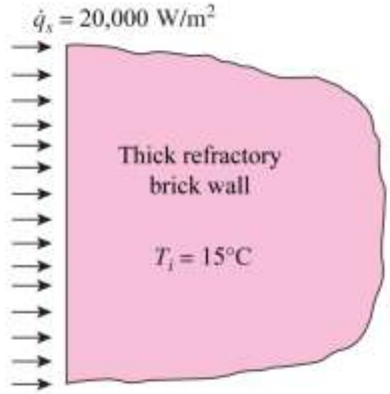
Then the temperature rise of the surface becomes

$$\Delta T_s = 28 - 20 = \mathbf{8.0°C}$$

4–91 A thick refractory brick wall is subjected to uniform heat flux. The temperature at the depth of 10 cm from the wall surface after an hour is to be determined.

Assumptions **1** The wall is thick and can be treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the wall are constant.

Properties The properties of the brick wall are given as $k = 1.0$ W/m·K and $\alpha = 5.08 \times 10^{-7}$ m^2/s.



Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the wall temperature at $x = 0.1$ m and $t = 3600$ s can be determined from

$$T(x,t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

where

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.1\,\mathrm{m}}{2\sqrt{(5.08\times10^{-7}\ \mathrm{m^2/s})(3600\,\mathrm{s})}} = 1.169$$

From Table 4-4, erfc(1.169) = 0.09832. Hence, the temperature at the depth of 10 cm from the wall surface after an hour is

$$T(x,t) = \frac{20{,}000\ \mathrm{W/m^2}}{1.0\ \mathrm{W/m\cdot K}}\left[\sqrt{\frac{4(5.08\times10^{-7}\ \mathrm{m^2/s})(3600\,\mathrm{s})}{\pi}}\exp\left[-\frac{(0.1\,\mathrm{m})^2}{4(5.08\times10^{-7}\ \mathrm{m^2/s})(3600\,\mathrm{s})}\right]\right.$$
$$\left. - (0.1\,\mathrm{m})\mathrm{erfc}\left(\frac{0.1\,\mathrm{m}}{2\sqrt{(5.08\times10^{-7}\ \mathrm{m^2/s})(3600\,\mathrm{s})}}\right)\right] + 15°\mathrm{C}$$

$$T(x,t) = \mathbf{64.5°C} \qquad \text{where} \quad x = 0.1\ \mathrm{m} \quad \text{and} \quad t = 3600\ \mathrm{s}$$

Discussion At the wall surface after one hour, the temperature is 980°C

$$T(x,t) = \frac{20{,}000\ \mathrm{W/m^2}}{1.0\ \mathrm{W/m\cdot K}}\sqrt{\frac{4(5.08\times10^{-7}\ \mathrm{m^2/s})(3600\,\mathrm{s})}{\pi}} + 15°\mathrm{C} = 980°\mathrm{C}$$

4–92 A thick refractory brick wall is subjected to uniform heat flux. The temperatures at the surface and at the depths of 1 cm and 5 cm from the wall surface as a function of heating time are to be determined.

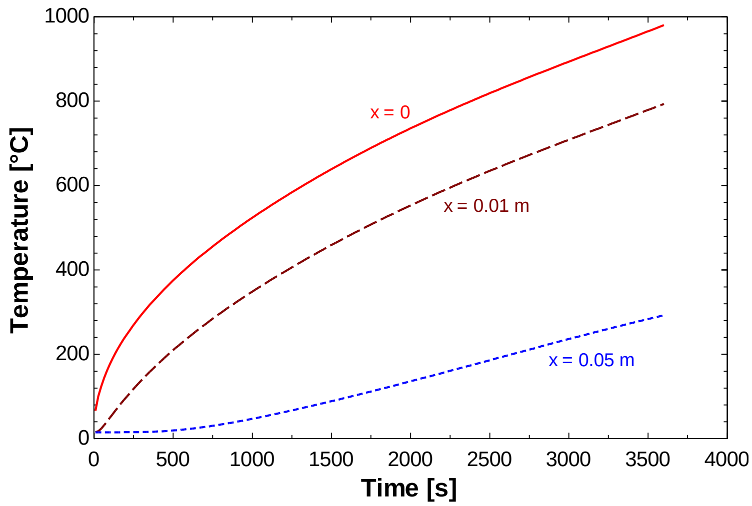
Assumptions **1** The wall is thick and can be treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the wall are constant.

Properties The properties of the brick wall are given as k = 1.0 W/m·K and $\alpha = 5.08 \times 10^{-7}$ m^2/s.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
q_dot_s=20000 [W/m^2]
T_i=15 [C]
x1=0.01 [m]
x2=0.05 [m]

"PROPERTIES"
k=1.0 [W/m-K]
alpha=5.08e-7 [m^2/s]

"ANALYSIS"
T_0-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5)
T_1cm-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5*exp(-x1^2/(4*alpha*t))-x1*erfc(x1/(2*(alpha*t)^0.5)))
T_5cm-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5*exp(-x2^2/(4*alpha*t))-x2*erfc(x2/(2*(alpha*t)^0.5)))
```

		$T(x, t)$ [°C]	
Time [s]	$x = 0$	$x = 0.01$ m	$x = 0.05$ m
10	65.9	15.0	15.0
15	77.3	15.3	15.0
30	103.1	18.1	15.0
50	128.7	25.4	15.0
75	154.3	36.9	15.0
100	175.8	49.1	15.0
200	242.5	96.3	15.0
400	336.7	175.5	16.7
800	469.9	297.7	33.3
1200	572.2	394.9	62.7
2400	803.0	619.1	175.7
3600	980.1	793.3	292.3



Discussion The temperature at the wall depth of 5 cm remained at the initial temperature and did not increase until after 200 s have elapsed.

4–93 Thick stainless steel and copper slabs are subjected to uniform heat flux. The temperatures of both slabs at the depth of 8 cm from the surface as a function of time are to be determined.

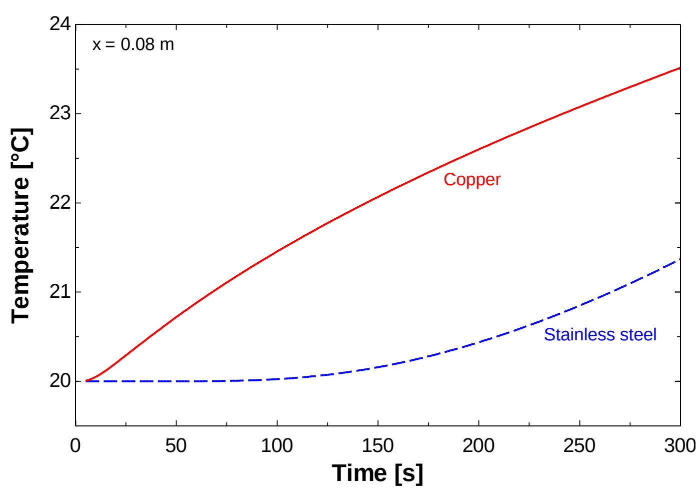
Assumptions **1** The slabs are treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the slabs are constant.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and $\alpha = 3.95\times 10^{-6}$ m^2/s; the properties of copper are given as k = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
q_dot_s=10000 [W/m^2]
T_i=20 [C]
x=0.08 [m]

"PROPERTIES"
"stainless steel"
k_ss=14.9 [W/m-K]
alpha_ss=3.95e-6 [m^2/s]
"copper"
k_cu=401 [W/m-K]
alpha_cu=117e-6 [m^2/s]

"ANALYSIS"
T_ss-T_i=q_dot_s/k_ss*((4*alpha_ss*t/pi#)^0.5*exp(-x^2/(4*alpha_ss*t))-x*erfc(x/(2*(alpha_ss*t)^0.5)))
T_cu-T_i=q_dot_s/k_cu*((4*alpha_cu*t/pi#)^0.5*exp(-x^2/(4*alpha_cu*t))-x*erfc(x/(2*(alpha_cu*t)^0.5)))
```

	$T(x, t)$ [°C]	
Time [s]	SS	Cu
5	20.0	20.0
10	20.0	20.1
20	20.0	20.2
30	20.0	20.4
40	20.0	20.6
50	20.0	20.7
60	20.0	20.9
80	20.0	21.2
100	20.0	21.5
150	20.2	22.1
200	20.4	22.6
300	21.4	23.5



Discussion The copper slab, having a much higher thermal diffusivity value, diffuses the heat energy through the medium faster than the stainless steel slab. Due to the low thermal diffusivity of stainless steel, the tempearure of the slab stays virtually constant for the first 100 seconds.

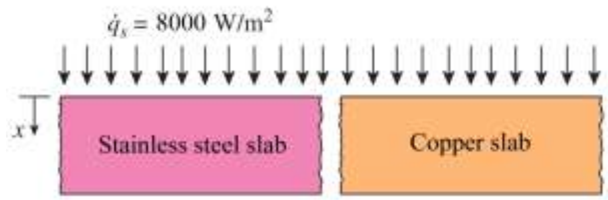
4–94 Thick stainless steel and copper slabs are subjected to uniform heat flux. The temperatures of both slabs at the depth of 1 cm from the surface after 60 s are to be determined.

Assumptions **1** The slabs are treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the slabs are constant.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and α = 3.95× 10^{-6} m^2/s; the properties of copper are given as k = 401 W/m·K and α = 117 × 10^{-6} m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the temperatures at x = 0.01 m and t = 60 s can be determined from

$$T(x,t)-T_i=\frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right)-x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$



For stainless steel,

$$\frac{x}{2\sqrt{\alpha t}}=\frac{0.01\,\mathrm{m}}{2\sqrt{(3.95\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}}=0.3248$$

From Table 4-4, erfc(0.3248) = 0.646. The temperature at the depth of 1 cm from the surface after 60 s is

$$T(x,t)=\frac{8000\ \mathrm{W/m^2}}{14.9\ \mathrm{W/m\cdot K}}\left[\sqrt{\frac{4(3.95\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}{\pi}}\exp\left[-\frac{(0.01\,\mathrm{m})^2}{4(3.95\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}\right]\right.$$
$$\left.-(0.01\,\mathrm{m})\mathrm{erfc}\left(\frac{0.01\,\mathrm{m}}{2\sqrt{(3.95\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}}\right)\right]+20^\circ\mathrm{C}$$

$T(x,t)=\mathbf{24.9^\circ C}$ (stainless steel slab) where x = 0.01 m and t = 60 s

For copper,

$$\frac{x}{2\sqrt{\alpha t}}=\frac{0.01\,\mathrm{m}}{2\sqrt{(117\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}}=0.05968$$

From Table 4-4, erfc(0.05968) = 0.9327. The temperature at the depth of 1 cm from the surface after 60 s is

$$T(x,t)=\frac{8000\ \mathrm{W/m^2}}{401\ \mathrm{W/m\cdot K}}\left[\sqrt{\frac{4(117\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}{\pi}}\exp\left[-\frac{(0.01\,\mathrm{m})^2}{4(117\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}\right]\right.$$
$$\left.-(0.01\,\mathrm{m})\mathrm{erfc}\left(\frac{0.01\,\mathrm{m}}{2\sqrt{(117\times10^{-6}\ \mathrm{m^2/s})(60\,\mathrm{s})}}\right)\right]+20^\circ\mathrm{C}$$

$T(x,t)=\mathbf{21.7^\circ C}$ (copper slab) where x = 0.01 m and t = 60 s

Discussion Aside from using Table 4-4, the complementary error function can be solved using the EES software with the following lines:

```
EF_ss=erfc(0.3248)
EF_cu=erfc(0.05968)
```

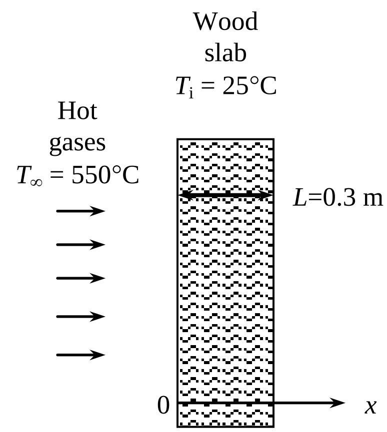
4-95 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

Assumptions **1** The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. **2** The thermal properties of the wood slab are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are k = 0.17 W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(35\text{ W/m}^2.°\text{C})\sqrt{(1.28\times10^{-7}\text{ m}^2\text{ /s})(5\times60\text{ s})}}{0.17\text{ W/m.°C}} = 1.276$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 1.276^2 = 1.628$$

Noting that x = 0 at the surface and using Table 4-4 for *erfc* values,

$$\frac{T(x,t)-25}{550-25} = erfc(0) - \exp(0+1.628)erfc(0+1.276)$$

$$= 1-(5.0937)(0.0712)$$

$$= 0.637$$

Solving for $T(x, t)$ gives

$$T(x,t) = \mathbf{360°C}$$

which is less than the ignition temperature of 450°C. Therefore, the wood will not ignite.

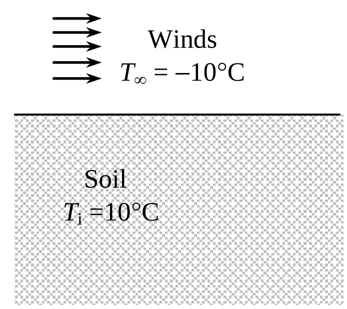
4-96 An area is subjected to cold air for a 10-h period. The soil temperatures at distances 0, 10, 20, and 50 cm from the earth's surface are to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.9 W/m.°C and $\alpha = 1.6\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the ground can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(40\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\times3600\ \text{s})}}{0.9\ \text{W/m.°C}} = 33.7$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 33.7^2 = 1138$$

Then we conclude that the last term in the temperature distribution relation above must be zero regardless of x despite the exponential term tending to infinity since (1) $erfc(\eta) \to 0$ for $\eta > 4$ (see Table 4-4) and (2) the term has to remain less than 1 to have physically meaningful solutions. That is,

$$\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right] = \exp\left(\frac{hx}{k}+1138\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+33.7\right)\right] \cong 0$$

Therefore, the temperature distribution relation simplifies to

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) \to T(x,t) = T_i + (T_\infty - T_i)erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Then the temperatures at 0, 10, 20, and 50 cm depth from the ground surface become

x = 0:

$$T(0{,}10\ \text{h}) = T_i + (T_\infty - T_i)erfc\left(\frac{0}{2\sqrt{\alpha t}}\right) = T_i + (T_\infty - T_i)erfc(0) = T_i + (T_\infty - T_i)\times 1 = T_\infty = \mathbf{-10°C}$$

x = 0.1m:

$$T(0.1\ \text{m}{,}10\ \text{h}) = 10 + (-10-10)erfc\left(\frac{0.1\ \text{m}}{2\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\ \text{h}\times3600\ \text{s/h})}}\right)$$
$$= 10 - 20erfc(0.066) = 10 - 20\times0.9257 = \mathbf{-8.5°C}$$

x = 0.2 m:

$$T(0.2\ \text{m}{,}10\ \text{h}) = 10 + (-10-10)erfc\left(\frac{0.2\ \text{m}}{2\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\ \text{h}\times3600\ \text{s/h})}}\right)$$
$$= 10 - 20erfc(0.132) = 10 - 20\times0.8519 = \mathbf{-7.0°C}$$

x = 0.5 m:

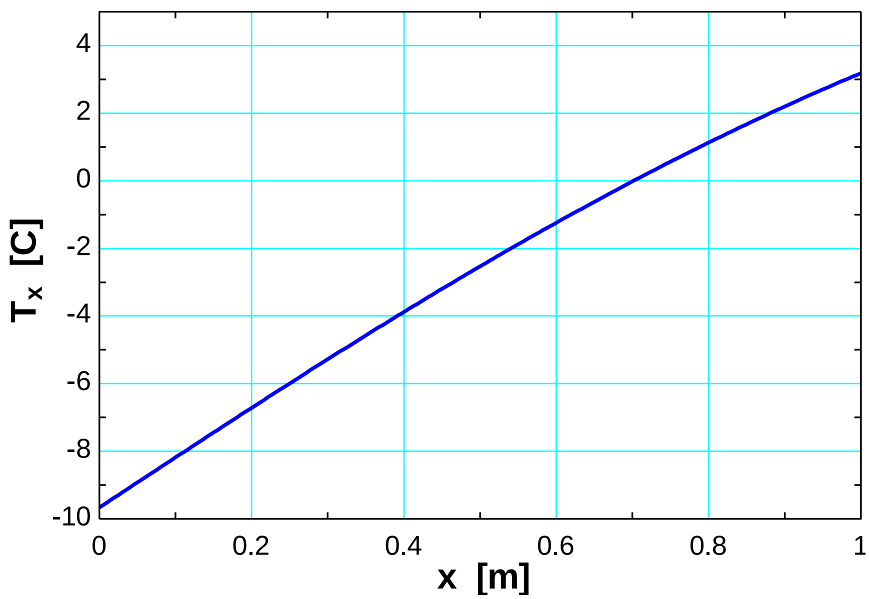
$$T(0.5\ \text{m}{,}10\ \text{h}) = 10 + (-10-10)erfc\left(\frac{0.5\ \text{m}}{2\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\ \text{h}\times3600\ \text{s/h})}}\right)$$
$$= 10 - 20erfc(0.329) = 10 - 20\times0.6418 = \mathbf{-2.8°C}$$

4-97 Prob. 4-96 is reconsidered. The soil temperature as a function of the distance from the earth's surface is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=10 [C]
T_infinity=-10 [C]
h=40 [W/m^2-C]
time=10*3600 [s]
x=0.1 [m]

"PROPERTIES"
k=0.9 [W/m-C]
alpha=1.6E-5 [m^2/s]

"ANALYSIS"
(T_x-T_i)/(T_infinity-T_i)=erfc(x/(2*sqrt(alpha*time)))-
exp((h*x)/k+(h^2*alpha*time)/k^2)*erfc(x/(2*sqrt(alpha*time))+(h*sqrt(alpha*time)/k))
```

x [m]	T_x [C]
0	-9.666
0.05	-8.923
0.1	-8.183
0.15	-7.447
0.2	-6.716
0.25	-5.993
0.3	-5.277
0.35	-4.572
0.4	-3.878
0.45	-3.197
0.5	-2.529
0.55	-1.877
0.6	-1.24
0.65	-0.6207
0.7	-0.01894
0.75	0.5643
0.8	1.128
0.85	1.672
0.9	2.196
0.95	2.7
1	3.183

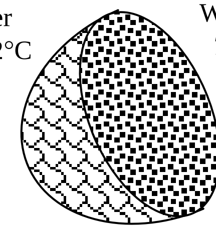


4–98 A spherical watermelon that is cut into two equal parts is put into a freezer. The time it will take for the center of the exposed cut surface to cool from 25°C to 3°C is to be determined.

Assumptions **1** The temperature of the exposed surfaces of the watermelon is affected by the convection heat transfer at those surfaces only. Therefore, the watermelon can be considered to be a semi-infinite medium **2** The thermal properties of the watermelon are constant.

Properties The thermal properties of the water is closely approximated by those of water at room temperature, k = 0.607 W/m.K and $\alpha = 0.146\times10^{-6}$ m^2/s (Table A-9).

Analysis We use the transient chart in Fig. 4-31 in this case for convenience (instead of the analytic solution),



$$\left.\begin{array}{l} 1-\dfrac{T(x,t)-T_\infty}{T_i-T_\infty}=1-\dfrac{3-(-12)}{25-(-12)}=0.595 \\ \eta=\dfrac{x}{2\sqrt{\alpha t}}=0 \end{array}\right\} \dfrac{h\sqrt{\alpha t}}{k}=1$$

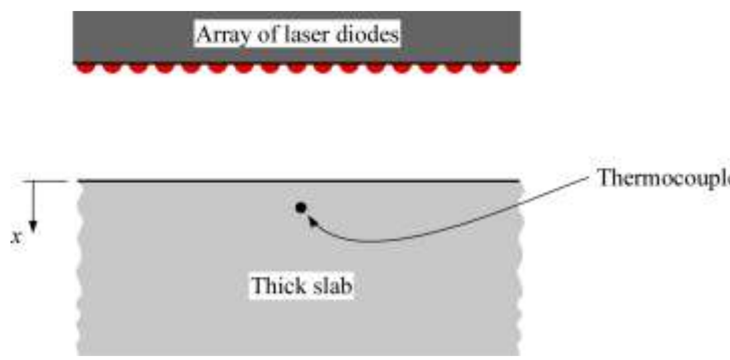
Therefore,

$$t=\frac{(1)^2k^2}{h^2\alpha}=\frac{(0.607\text{ W/m}K)^2}{(22\text{ W/m}^2.K)^2(0.146\times10^{-6}\text{ m}^2/\text{s})}=5214\text{ s}=\mathbf{86.9\,min}$$

4-99 A slab surface has been exposed to laser pulse, (a) the amount of energy per unit surface area directed on the slab surface and (b) the thermocouple reading (at x = 25 mm) after 60 s has elapsed are to be determined.

Assumptions **1** The slab is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The properties of the slab are given to be k = 63.9 W/m · K and $\alpha = 18.8\times10^{-6}$ m^2/s.



Analysis (a) For semi-infinite solid with energy pulse at surface, we have

$$T(x,t)-T_i=\frac{e_s}{k\sqrt{\pi\ t/\alpha}}\exp\left(-\frac{x^2}{4\alpha\ t}\right)$$

$$\begin{aligned} e_s &= k\sqrt{\frac{\pi\ t}{\alpha}}\exp\left(\frac{x^2}{4\alpha\ t}\right)[T(x,t)-T_i] \\ &= (63.9\text{ W/m}\cdot\text{K})\sqrt{\frac{\pi(30\text{ s})}{18.8\times10^{-6}\text{ m}^2/\text{s}}}\exp\left(\frac{(0.025\text{ m})^2}{4(18.8\times10^{-6}\text{ m}^2/\text{s})(30\text{ s})}\right)(130-20)\text{ K} \\ &= \mathbf{2.076\times10^7\ J/m^2} \end{aligned}$$

(b) After 60 s has elapsed, the thermocouple reading is

$$T(x,t)=\frac{2.076\times10^7\text{ J/m}^2}{(63.9\text{ W/m}\cdot{}^\circ\text{C})\sqrt{\dfrac{\pi(60\text{ s})}{18.8\times10^{-6}\text{ m}^2/\text{s}}}}\exp\left(-\frac{(0.025\text{ m})^2}{4(18.8\times10^{-6}\text{ m}^2/\text{s})(60\text{ s})}\right)+20\,{}^\circ\text{C}$$

$$T(0.025\text{ m},60\text{ s})=\mathbf{109\,{}^\circ C}$$

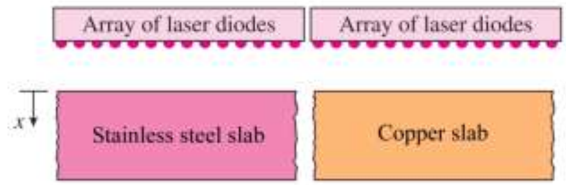
Discussion High-power laser diodes can be used in many industrial applications, such as welding, heat treatment, and cladding.

4–100 Thick stainless steel and copper slabs are subjected to an energy pulse. The temperatures of both slabs at the depth of 5 cm from the surface, after 60 s of receiving the energy pulse, are to be determined.

Assumptions **1** The slabs are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Entire energy from the pulse enters the slabs.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and $\alpha = 3.95\times 10^{-6}$ m^2/s; the properties of copper are given as k = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to an energy pulse at the surface, and the temperatures at x = 0.05 m and t = 60 s can be determined from



$$T(x,t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}} \exp\left(-\frac{x^2}{4\alpha\ t}\right)$$

For the stainless steel slab,

$$T(x,t) = \frac{5\times 10^7\ \text{J/m}^2}{(14.9\ \text{W/m}\cdot\text{K})\sqrt{\dfrac{\pi(60\,\text{s})}{3.95\times 10^{-6}\ \text{m}^2/\text{s}}}} \exp\left(-\frac{(0.05\,\text{m})^2}{4(3.95\times 10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}\right) + 20^\circ\text{C} = \mathbf{54.8^\circ C}$$

For the copper slab,

$$T(x,t) = \frac{5\times 10^7\ \text{J/m}^2}{(401\ \text{W/m}\cdot\text{K})\sqrt{\dfrac{\pi(60\,\text{s})}{117\times 10^{-6}\ \text{m}^2/\text{s}}}} \exp\left(-\frac{(0.05\,\text{m})^2}{4(117\times 10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}\right) + 20^\circ\text{C} = \mathbf{109.9^\circ C}$$

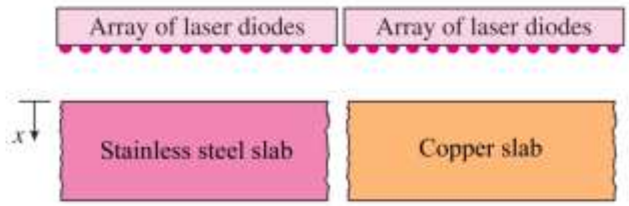
Discussion High-power laser diodes are used in many industrial applications, such as welding, heat treatment, and cladding.

4–101 Thick stainless steel and copper slabs are subjected to an energy pulse. The temperatures of both slabs at the depth of 5 cm from the surface as a function of time are to be determined.

Assumptions **1** The slabs are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Entire energy from the pulse enters the slabs.

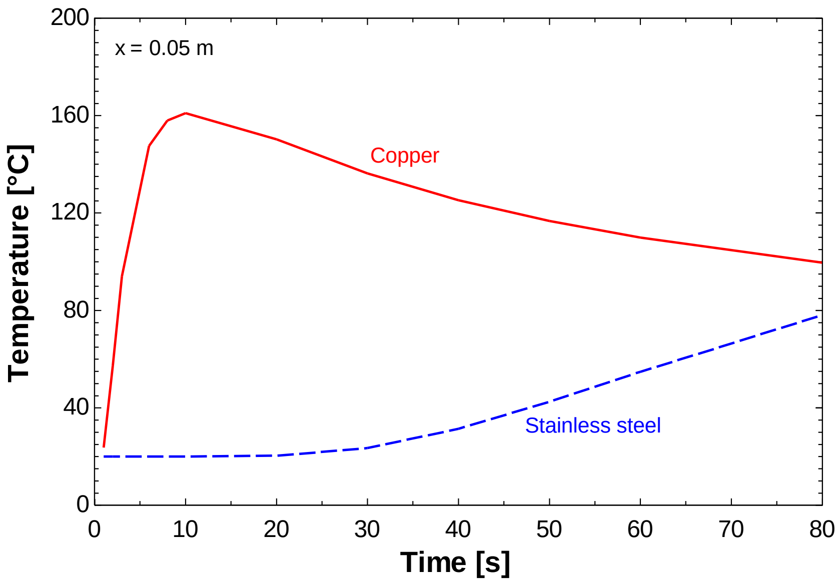
Properties The properties of stainless steel are given as k = 14.9 W/m·K and α = 3.95× 10^{-6} m^2/s; the properties of copper are given as k = 401 W/m·K and α = 117 × 10^{-6} m^2/s.

Analysis The problem is solved using EES, and the solution is given below.




```
"GIVEN"
e_s=5e7 [J/m^2]
T_i=20 [C]
x=0.05 [m]

"PROPERTIES"
"stainless steel"
k_ss=14.9 [W/m-K]
alpha_ss=3.95e-6 [m^2/s]
"copper"
k_cu=401 [W/m-K]
alpha_cu=117e-6 [m^2/s]

"ANALYSIS"
T_ss-T_i=e_s/(k_ss*(pi#*t/alpha_ss)^0.5)*exp(-x^2/(4*alpha_ss*t))
T_cu-T_i=e_s/(k_cu*(pi#*t/alpha_cu)^0.5)*exp(-x^2/(4*alpha_cu*t))
```

	T (*x*, *t*) [°C]	
Time [s]	SS	Cu
1	20.0	23.6
2	20.0	57.2
3	20.0	94.0
6	20.0	147.5
8	20.0	158.0
10	20.0	161.0
20	20.3	150.3
30	23.5	136.3
40	31.4	125.3
50	42.5	116.7
60	54.8	109.9
80	78.2	99.6



Discussion The copper slab, having a much higher thermal diffusivity value, diffuses the heat energy from the pulse much quicker than the stainless steel slab. As shown in the table and figure, the copper temperature increases sharply for the first 10 seconds from an initial temperature of 20°C to a temperature of 161°C, while the temperature of stainless steel stays constant at its initial temperature of 20°C .

4-102 The contact surface temperatures when a bare footed person steps on aluminum and wood blocks are to be determined.

Assumptions **1** Both bodies can be treated as the semi-infinite solids. **2** Heat loss from the solids is disregarded. **3** The properties of the solids are constant.

Properties The $\sqrt{k\rho c_p}$ value is 24 kJ/m^2·°C for aluminum, 0.38 kJ/m^2·°C for wood, and 1.1 kJ/m^2·°C for the human flesh.

Analysis The surface temperature is determined from Eq. 4-49 to be

$$T_s = \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{Al}}}T_{\text{Al}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{Al}}}} = \frac{(1.1\,\text{kJ/m}^2\cdot{}^\circ\text{C})(32^\circ\text{C}) + (24\,\text{kJ/m}^2\cdot{}^\circ\text{C})(20^\circ\text{C})}{(1.1\,\text{kJ/m}^2\cdot{}^\circ\text{C}) + (24\,\text{kJ/m}^2\cdot{}^\circ\text{C})} = \mathbf{20.5^\circ C}$$

In the case of wood block, we obtain

$$\begin{aligned} T_s &= \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{wood}}}T_{\text{wood}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{wood}}}} \\ &= \frac{(1.1\,\text{kJ/m}^2\cdot{}^\circ\text{C})(32^\circ\text{C}) + (0.38\,\text{kJ/m}^2\cdot{}^\circ\text{C})(20^\circ\text{C})}{(1.1\,\text{kJ/m}^2\cdot{}^\circ\text{C}) + (0.38\,\text{kJ/m}^2\cdot{}^\circ\text{C})} \\ &= \mathbf{28.9^\circ C} \end{aligned}$$

Transient Heat Conduction in Multidimensional Systems

4-103C The product solution enables us to determine the dimensionless temperature of two- or three-dimensional heat transfer problems as the product of dimensionless temperatures of one-dimensional heat transfer problems. The dimensionless temperature for a two-dimensional problem is determined by determining the dimensionless temperatures in both directions, and taking their product.

4-104C The dimensionless temperature for a three-dimensional heat transfer is determined by determining the dimensionless temperatures of one-dimensional geometries whose intersection is the three dimensional geometry, and taking their product.

4-105C This short cylinder is physically formed by the intersection of a long cylinder and a plane wall. The dimensionless temperatures at the center of plane wall and at the center of the cylinder are determined first. Their product yields the dimensionless temperature at the center of the short cylinder.

4-106C The heat transfer in this short cylinder is one-dimensional since there is no heat transfer in the axial direction. The temperature will vary in the radial direction only.

4-107 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*-directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

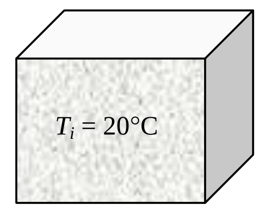
Properties The thermal properties of the granite are given to be $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m^2/s.

Analysis

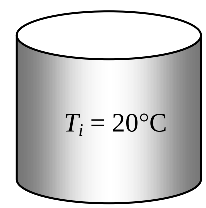
Cubic block: This cubic block can physically be formed by the intersection of three infinite plane walls of thickness 2L = 5 cm.

<u>After 10 minutes</u>: The Biot number, the corresponding constants, and the Fourier number are

5 cm × 5 cm × 5 cm



Hot gases
500°C



$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]^3$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right)^3$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^3 = 0.369$$

$$T(0,0,0,t) = \mathbf{323°C}$$

<u>After 20 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(20\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^3 = 0.115 \longrightarrow T(0,0,0,t) = \mathbf{445°C}$$

<u>After 60 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^3 = 0.00109 \longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm and a plane wall of thickness 2L = 5 cm.

<u>After 10 minutes</u>: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,t)_{block} = [\theta(0,t)_{wall}][\theta(0,t)_{cyl}]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2 \tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.352 \longrightarrow T(0,0,t) = \mathbf{331°C}$$

<u>After 20 minutes</u>

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.107 \longrightarrow T(0,0,t) = \mathbf{449°C}$$

<u>After 60 minutes</u>

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.00092 \longrightarrow T(0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

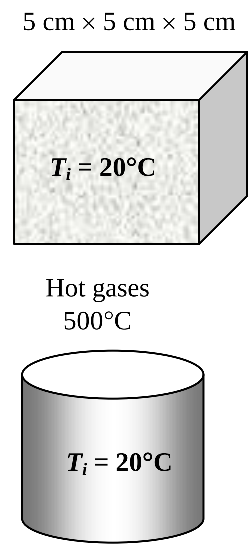
4-108 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*-directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the granite are k = 2.5 W/m.°C and $\alpha = 1.15\times10^{-6}$ m^2/s.

Analysis

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane wall of thickness $2L$ = 5 cm. Two infinite plane walls are exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.^\circ\text{C}$ and one with $h = 80\ \text{W/m}^2.^\circ\text{C}$.



After 10 minutes: The Biot number and the corresponding constants for $h = 40\ \text{W/m}^2.^\circ\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.}^\circ\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

The Biot number and the corresponding constants for $h = 80\ \text{W/m}^2.^\circ\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.}^\circ\text{C})} = 0.800$$

$$\longrightarrow \lambda_1 = 0.7910 \quad \text{and} \quad A_1 = 1.1016$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]^2\left[\theta(0,t)_{\text{wall}}\right]$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)^2\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\} = 0.284$$

$$T(0,0,0,t) = \mathbf{364^\circ C}$$

After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(20\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\} = 0.0654$$

$$\longrightarrow T(0,0,0,t) = \mathbf{469^\circ C}$$

After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^2 \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\} = 0.000186$$

$$\longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that τ > 0.2 in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.°\text{C}$ and a plane wall of thickness 2L = 5 cm exposed to the hot gases with $h = 80\ \text{W/m}^2.°\text{C}$.

<u>After 10 minutes</u>: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m}.°\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}][\theta(0,t)_{cyl}]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} \left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.271$$

$$T(0,0,t) = \mathbf{370°C}$$

<u>After 20 minutes</u>

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.06094 \longrightarrow T(0,0,t) = \mathbf{471°C}$$

<u>After 60 minutes</u>

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.0001568 \longrightarrow T(0,0,t) = \mathbf{500°C}$$

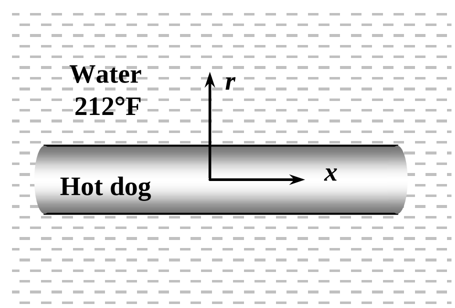
Note that τ > 0.2 in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

4-109E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and also as an infinitely long cylinder.

Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be k = 0.44 Btu/h.ft.°F, ρ = 61.2 lbm/ft^3 c_p = 0.93 Btu/lbm.°F, and α = 0.0077 ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2$ = (0.4/12) ft and a plane wall of thickness $2L$ = (5/12) ft. The distance x is measured from the midplane.



After 5 minutes

First the Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(2.5/12\ \text{ft})}{(0.44\ \text{Btu/h.ft}.°\text{F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.5421 \quad \text{and} \quad A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.015)} = 1.228$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(0.4/12\ \text{ft})}{(0.44\ \text{Btu/h.ft}.°\text{F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.228 \times 0.106 = 0.130$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.130 \longrightarrow T(0,0,t) = \mathbf{190°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} = 1.185$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.0071$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.185 \times 0.0071 = 0.0084$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.0084 \longrightarrow T(0,0,t) = \mathbf{211°F}$$

<u>After 15 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} = 1.143$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.00048$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.143 \times 0.00048 = 0.00055$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.00055 \longrightarrow T(0,0,t) = \mathbf{212\ °F}$$

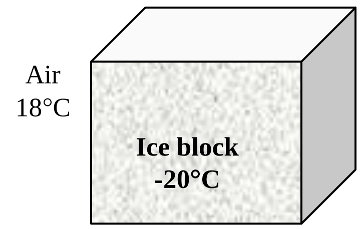
(*b*) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

4-110 A rectangular ice block is placed on a table. The time the ice block starts melting is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *y*-directions. **2** The thermal properties of the ice block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This rectangular ice block can be treated as a short rectangular block that can physically be formed by the intersection of two infinite plane wall of thickness $2L = 4$ cm and an infinite plane wall of thickness $2L = 12$ cm. We measure *x* from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 12$ cm. Since the melting starts at the corner of the top surface, we need to determine the time required to melt ice block which will happen when the temperature drops below 0°C at this location. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall},1} = \frac{hL_1}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.02\text{ m})}{(2.22\text{ W/m}.°\text{C})} = 0.1081 \longrightarrow \lambda_1 = 0.3208 \quad \text{and} \quad A_1 = 1.0173$$

$$Bi_{\text{wall},3} = \frac{hL_3}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.05\text{ m})}{(2.22\text{ W/m}.°\text{C})} = 0.2703 \longrightarrow \lambda_1 = 0.4954 \quad \text{and} \quad A_1 = 1.0409$$

The ice will start melting at the corners because of the maximum exposed surface area there. Noting that $\tau = \alpha t / L^2$ and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(L_1, L_2, L_3, t)_{block} = \theta(L_1, t)_{\text{wall},1}{}^2\, \theta(L_3, t)_{\text{wall},2}$$

$$\frac{0-18}{-20-18} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_1 / L_1)\right]^2 \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_3 / L_3)\right]$$

$$0.500 = \left\{(1.0173)\exp\left[-(0.3208)^2\frac{(0.124\times10^{-7})t}{(0.02)^2}\right]\cos(0.3208)\right\}^2$$

$$\times\left\{(1.0409)\exp\left[-(0.4954)^2\frac{(0.124\times10^{-7})t}{(0.05)^2}\right]\cos(0.4954)\right\}$$

$$\longrightarrow t = 77{,}520\text{ s} = 1292\text{ min} = \mathbf{21.5\,hours}$$

Therefore, the ice will start melting in about 20 hours.

Discussion Note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(77{,}520\text{ s})}{(0.05\text{ m})^2} = 0.384 > 0.2$$

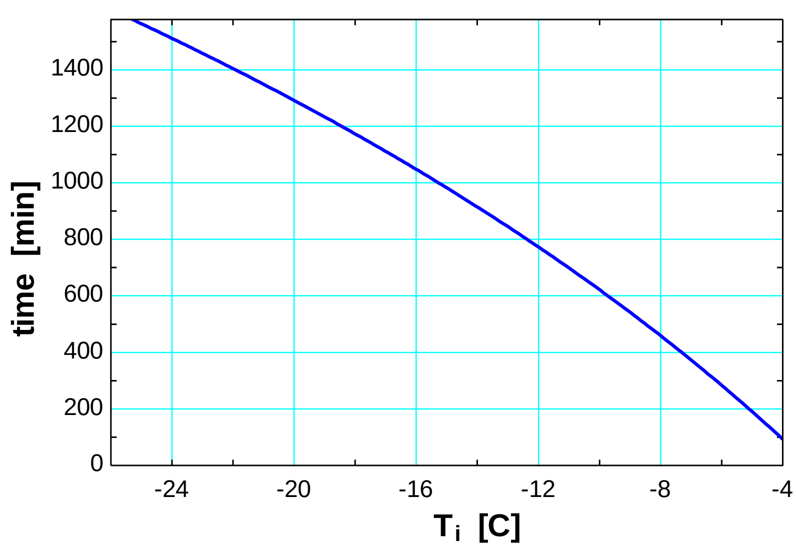
and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

4-111 Prob. 4-110 is reconsidered. The effect of the initial temperature of the ice block on the time period before the ice block starts melting is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L_1=0.04 [m]
L_2=L_1
2*L_3=0.10 [m]
"T_i=-20 [C]"
T_infinity=18 [C]
h=12 [W/m^2-C]
T_L1_L2_L3=0 [C]

"PROPERTIES"
k=2.22 [W/m-C]
alpha=0.124E-7 [m^2/s]

"ANALYSIS"
"This block can physically be formed by the intersection of two infinite plane wall of thickness 2L=4 cm and an
infinite plane wall of thickness 2L=10 cm"
"For the two plane walls"
Bi_w1=(h*L_1)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w1=0.3208 "w stands for wall"
A_1_w1=1.0173
time*Convert(min, s)=tau_w1*L_1^2/alpha
"For the third plane wall"
Bi_w3=(h*L_3)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w3=0.4954
A_1_w3=1.0409
time*Convert(min, s)=tau_w3*L_3^2/alpha
theta_L_w1=A_1_w1*exp(-lambda_1_w1^2*tau_w1)*Cos(lambda_1_w1*L_1/L_1) "theta_L_w1=(T_L_w1-
T_infinity)/(T_i-T_infinity)"
theta_L_w3=A_1_w3*exp(-lambda_1_w3^2*tau_w3)*Cos(lambda_1_w3*L_3/L_3) "theta_L_w3=(T_L_w3-
T_infinity)/(T_i-T_infinity)"
(T_L1_L2_L3-T_infinity)/(T_i-T_infinity)=theta_L_w1^2*theta_L_w3 "corner temperature"
```

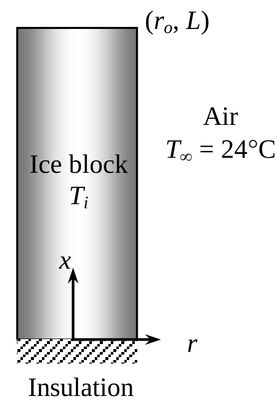
T_i [C]	time [min]
-26	1613
-24	1511
-22	1404
-20	1292
-18	1173
-16	1048
-14	914.6
-12	773
-10	621.7
-8	459.1
-6	283.5
-4	92.67



4-112 A cylindrical ice block is placed on a table. The initial temperature of the ice block to avoid melting for 2 h is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *r*-directions. **2** Heat transfer from the base of the ice block to the table is negligible. **3** The thermal properties of the ice block are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m²/s.



Analysis This cylindrical ice block can be treated as a short cylinder that can physically be formed by the intersection of a long cylinder of diameter $D = 2$ cm and an infinite plane wall of thickness $2L = 4$ cm. We measure *x* from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 4$ cm. The melting starts at the outer surfaces of the top surface when the temperature drops below 0°C at this location. The Biot numbers, the corresponding constants, and the Fourier numbers are

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(13\text{ W/m}^2.°\text{C})(0.02\text{ m})}{(2.22\text{ W/m.°C})} = 0.1171 \longrightarrow \lambda_1 = 0.3319 \quad \text{and} \quad A_1 = 1.0187$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(13\text{ W/m}^2.°\text{C})(0.01\text{ m})}{(2.22\text{ W/m.°C})} = 0.05856 \longrightarrow \lambda_1 = 0.3393 \quad \text{and} \quad A_1 = 1.0144$$

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(3\text{ h}\times3600\text{ s/h})}{(0.02\text{ m})^2} = 0.3348 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(3\text{ h}\times3600\text{ s/h})}{(0.01\text{ m})^2} = 1.3392 > 0.2$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable. The product solution for this problem can be written as

$$\theta(L, r_o, t)_{\text{block}} = \theta(L, t)_{\text{wall}}\,\theta(r_o, t)_{\text{cyl}}$$

$$\frac{0-24}{T_i-24} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L)\right]\left[A_1 e^{-\lambda_1^2\tau}J_0(\lambda_1 r_o/r_o)\right]$$

$$\frac{0-24}{T_i-24} = \left[(1.0187)e^{-(0.3319)^2(0.3348)}\cos(0.3319)\right]\left[(1.0146)e^{-(0.3393)^2(1.3392)}(0.9708)\right]$$

which gives

$$T_i = \mathbf{-6.6°C}$$

Therefore, the ice will not start melting for at least 3 hours if its initial temperature is -6.6°C or below.

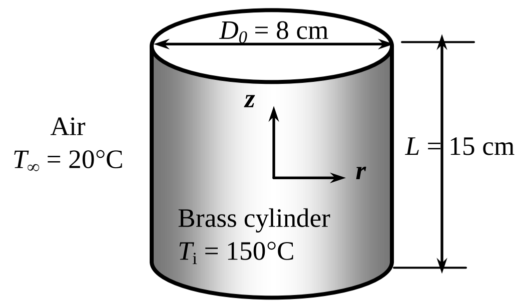
4-113 A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface as well as the total heat transfer from the cylinder for 15 min of cooling are to be determined.

Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of brass are given to be $\rho = 8530 \text{ kg/m}^3$, $c_p = 0.389 \text{ kJ/kg}\cdot°\text{C}$, $k = 110 \text{ W/m}\cdot°\text{C}$, and $\alpha = 3.39\times10^{-5} \text{ m}^2\text{/s}$.

Analysis This short cylinder can physically be formed by the intersection of a long cylinder of radius $D/2$ = 4 cm and a plane wall of thickness $2L$ = 15 cm. We measure *x* from the midplane.

(*a*) The Biot number is calculated for the plane wall to be



$$Bi = \frac{hL}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.075 \text{ m})}{(110 \text{ W/m.°C})} = 0.02727$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.1620 \quad \text{and} \quad A_1 = 1.0045$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39\times10^{-5} \text{ m}^2\text{/s})(15 \text{ min}\times 60 \text{ s/min})}{(0.075 \text{ m})^2} = 5.424 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0045)e^{-(0.1620)^2(5.424)} = 0.8712$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.04 \text{ m})}{(110 \text{ W/m.°C})} = 0.01455$$

Approximating Biot number as 0.01 for use in Table 4-2,

$$\lambda_1 = 0.1677 \quad \text{and} \quad A_1 = 1.0036$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.39\times10^{-5} \text{ m}^2\text{/s})(15\times 60 \text{ s})}{(0.04 \text{ m})^2} = 19.07 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0036)e^{-(0.1677)^2(19.07)} = 0.5870$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta_{o,wall}\times\theta_{o,cyl} = 0.8712\times0.5870 = 0.5114$$

$$\frac{T(0,0,t) - 20}{150 - 20} = 0.5114 \longrightarrow T(0,0,t) = \mathbf{86.5°C}$$

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder (r = 0), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to determine the dimensionless temperature at the surface of the wall.

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L) = (1.0045)e^{-(0.1620)^2(5.424)}\cos(0.1620) = 0.8598$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{wall} \times \theta_{o,cyl} = 0.8598 \times 0.5870 = 0.5047$$

$$\frac{T(L,0,t)-20}{150-20} = 0.5047 \longrightarrow T(L,0,t) = \mathbf{85.6°C}$$

(*c*) We first need to determine the maximum heat can be transferred from the cylinder

$$m = \rho \boldsymbol{V} = \rho \pi r_o^2 L = (8530\text{ kg/m}^3)\left[\pi(0.04\text{ m})^2 (0.15\text{ m})\right] = 6.431\text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (6.431\text{ kg})(0.389\text{ kJ/kg.°C})(150-20)\text{°C} = 325.2\text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.8712)\frac{\sin(0.1620)}{0.1620} = 0.1326$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.5870)\frac{0.08348}{0.1677} = 0.4156$$

since $J_1(0.1677) = 0.08348$ from Table 4-3. The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.1326 + (0.4156)(1-0.1326) = 0.4931$$

Then the total heat transfer from the short cylinder during the first 15 minutes of cooling becomes

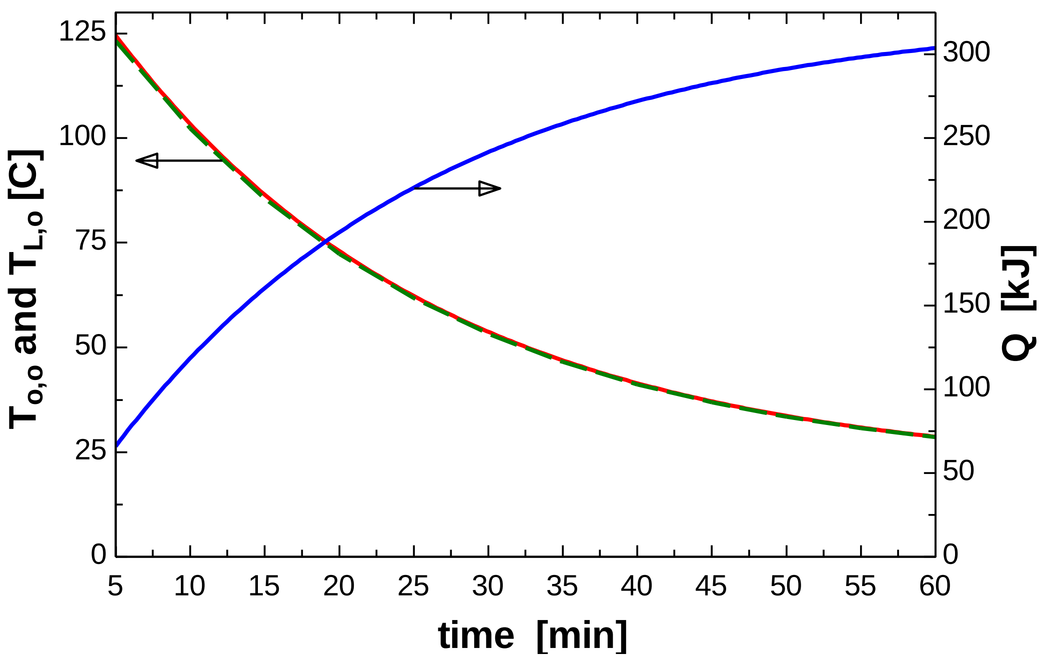
$$Q = 0.4931 Q_{\max} = (0.4931)(325.2\text{ kJ}) = \mathbf{160\,kJ}$$

4-114 Prob. 4-113 is reconsidered. The effect of the cooling time on the center temperature of the cylinder, the center temperature of the top surface of the cylinder, and the total heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.08 [m]
r_o=D/2
height=0.15 [m]
L=height/2
T_i=150 [C]
T_infinity=20 [C]
h=40 [W/m^2-C]
"time=15 [min]"

"PROPERTIES"
k=110 [W/m-C]
rho=8530 [kg/m^3]
c_p=0.389 [kJ/kg-C]
alpha=3.39E-5 [m^2/s]

"ANALYSIS"
"(a)"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o and a plane wall of
thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w=0.1620 "w stands for wall"
A_1_w=1.0045
tau_w=(alpha*time*Convert(min, s))/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_c=0.1677
A_1_c=1.0036
tau_c=(alpha*time*Convert(min, s))/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of short cylinder"
"(b)"
theta_L_w=A_1_w*exp(-lambda_1_w^2*tau_w)*Cos(lambda_1_w*L/L) "theta_L_w=(T_L_w-T_infinity)/(T_i-
T_infinity)"
(T_L_o-T_infinity)/(T_i-T_infinity)=theta_L_w*theta_o_c "center temperature of the top surface"
"(c)"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.08348 "From Table 4-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

time [min]	$T_{o,o}$ [C]	$T_{L,o}$ [C]	Q [kJ]
5	124.5	123.2	66.03
10	103.4	102.3	118.5
15	86.49	85.62	160.4
20	73.03	72.33	193.7
25	62.29	61.74	220.4
30	53.73	53.29	241.6
35	46.9	46.55	258.5
40	41.45	41.17	272
45	37.11	36.89	282.8
50	33.65	33.47	291.4
55	30.88	30.74	298.2
60	28.68	28.57	303.7



4-115 A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 5 cm from the end surface is to be determined.

Assumptions **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of aluminum are given to be $k = 237$ W/m.°C and $\alpha = 9.71\times10^{-5}$m^2/s.

Analysis This semi-infinite cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 7.5$ cm and a semi-infinite medium. The dimensionless temperature 5 cm from the surface of a semi-infinite medium is first determined from

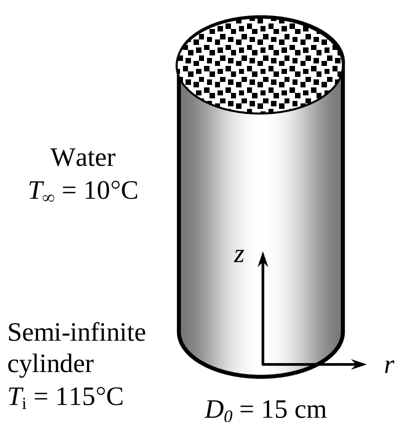
$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$= erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}\right) - \exp\left(\frac{(140)(0.05)}{237}+\frac{(140)^2(9.71\times10^{-5})(8\times60)}{(237)^2}\right)$$

$$\times\left[erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}+\frac{(140)\sqrt{(9.71\times10^{-5})(8\times60)}}{237}\right)\right]$$

$$= erfc(0.1158) - \exp(0.0458)erfc(0.2433) = 0.8699 - (1.0468)(0.7308) = 0.1049$$

$$\theta_{semi-\inf} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = 1 - 0.1049 = 0.8951$$



The Biot number is calculated for the long cylinder to be

$$Bi = \frac{hr_o}{k} = \frac{(140\text{ W/m}^2.°\text{C})(0.075\text{ m})}{237\text{ W/m.°C}} = 0.0443$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.2948 \quad \text{and} \quad A_1 = 1.0110$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71\times10^{-5}\text{ m}^2\text{/s})(8\times60\text{ s})}{(0.075\text{ m})^2} = 8.286 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0110)e^{-(0.2948)^2(8.286)} = 0.4921$$

The center temperature of the semi-infinite cylinder then becomes

$$\left[\frac{T(x,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = \theta_{\text{semi-inf}}(x,t)\times\theta_{o,cyl} = 0.8951\times0.4921 = 0.4405$$

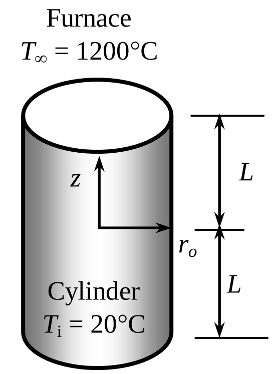
$$\left[\frac{T(x,0,t)-10}{115-10}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = 0.4405 \longrightarrow T(x,0,t) = \mathbf{56.3°C}$$

4-116 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transfer to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (it will be verified).

Properties The thermal properties of the aluminum block are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness 2L = 20 cm, and a long cylinder of radius $r_o = D/2$ = 7.5 cm. The Biot numbers and the corresponding constants are first determined to be



$$Bi = \frac{hL}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.10\text{ m})}{(236\text{ W/m.°C})} = 0.0339 \longrightarrow \lambda_1 = 0.1811 \text{ and } A_1 = 1.0056$$

$$Bi = \frac{hr_0}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.075\text{ m})}{236\text{ W/m.°C}} = 0.0254 \longrightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0056)\exp\left[-(0.1811)^2\,\frac{(9.75\times10^{-5})t}{(0.10)^2}\right]\right\}\times\left\{(1.0063)\exp\left[-(0.2217)^2\,\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time t gives

t = 241 s = **4.02 min**

We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\text{ m}^2/\text{s})(241\text{ s})}{(0.10\text{ m})^2} = 2.353 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\text{ m}^2/\text{s})(241\text{ s})}{(0.075\text{ m})^2} = 4.183 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0056)\exp\left[-(0.1811)^2(2.353)\right] = 0.9309$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(4.183)\right] = 0.8193$$

The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2(2L) = (2702\text{ kg/m}^3)\left[\pi(0.075\text{ m})^2\cdot(0.2\text{ m})\right] = 9.550\text{ kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (9.550\text{ kg})(0.896\text{ kJ/kg.°C})(20-1200)°\text{C} = 10{,}100\text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.9309)\frac{\sin(0.1811)}{0.1811} = 0.07416$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\,\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.8193)\frac{0.1101}{0.2217} = 0.1862$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right]$$

$$= 0.07416 + (0.1862)(1 - 0.07416) = 0.2466$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2466 Q_{\max} = (0.2466)(10{,}100\text{ kJ}) = \mathbf{2490\ kJ}$$

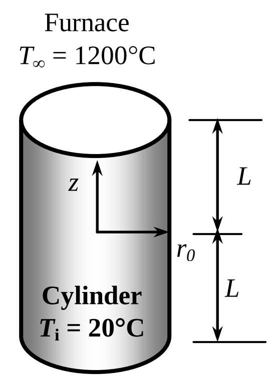
which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

4-117 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transferred to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** Heat transfer from the bottom surface of the block is negligible. **3** The thermal properties of the aluminum are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the aluminum block are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness 2L = 40 cm and a long cylinder of radius $r_o = D/2$ = 7.5 cm. Note that the height of the short cylinder represents the half thickness of the infinite plane wall where the bottom surface of the short cylinder is adiabatic. The Biot numbers and corresponding constants are first determined to be



$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.2\ \text{m})}{(236\ \text{W/m.°C})} = 0.0678 \rightarrow \lambda_1 = 0.2568 \quad \text{and} \quad A_1 = 1.0110$$

$$Bi = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.075\ \text{m})}{(236\ \text{W/m.°C})} = 0.0254 \rightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0110)\exp\left[-(0.2568)^2\frac{(9.75\times10^{-5})t}{(0.2)^2}\right]\right\}\left\{(1.0063)\exp\left[-(0.2217)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time t gives

t = 284.5 s = **4.74 min**

We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(284.5\ \text{s})}{(0.2\ \text{m})^2} = 0.6934 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(284.5\ \text{s})}{(0.075\ \text{m})^2} = 4.931 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0110)\exp\left[-(0.2568)^2(0.6934)\right] = 0.9658$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(4.931)\right] = 0.7897$$

The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2(2L) = (2702\ \text{kg/m}^3)\left[\pi(0.075\ \text{m})^2\cdot(0.2\ \text{m})\right] = 9.550\ \text{kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (9.550\ \text{kg})(0.896\ \text{kJ/kg.°C})(20-1200)°\text{C} = 10{,}100\ \text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.9658)\frac{\sin(0.2568)}{0.2568} = 0.04477$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7897)\frac{0.1101}{0.2217} = 0.2156$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{long}\\\text{cylinder}}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}}\right]$$

$$= 0.04477 + (0.2156)(1 - 0.04477) = 0.2507$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2507 Q_{\max} = (0.2507)(10{,}100\text{ kJ}) = \mathbf{2530\ kJ}$$

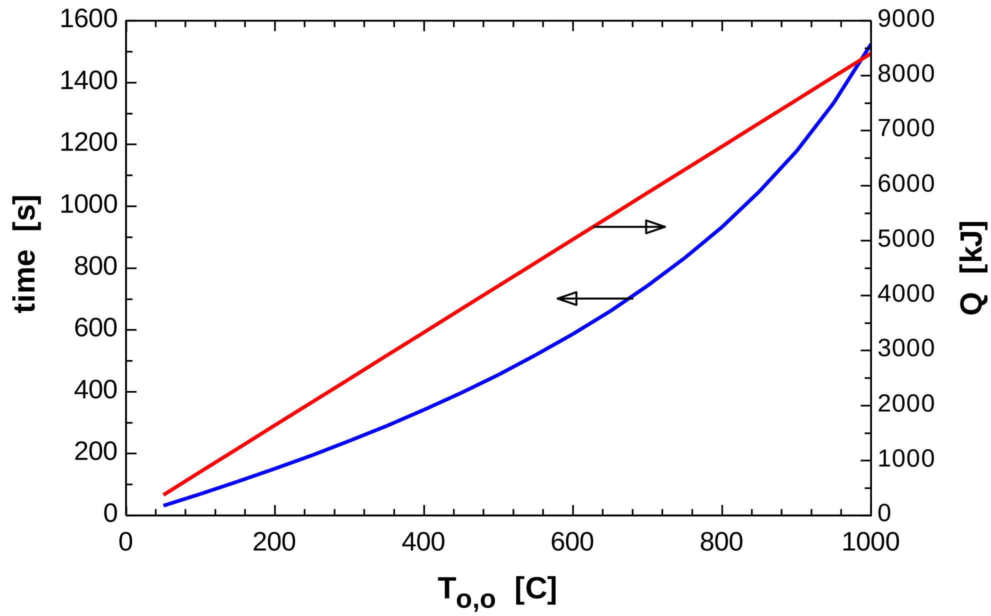
which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

4-118 Prob. 4-117 is reconsidered. The effect of the final center temperature of the block on the heating time and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L=0.20 [m]
2*r_o=0.15 [m]
T_i=20 [C]
T_infinity=1200 [C]
T_o_o=300 [C]
h=80 [W/m^2-C]

"PROPERTIES"
k=236 [W/m-C]
rho=2702 [kg/m^3]
c_p=0.896 [kJ/kg-C]
alpha=9.75E-5 [m^2/s]

"ANALYSIS"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o and a plane wall of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_w=0.1811 "w stands for wall"
A_1_w=1.0056
tau_w=(alpha*time)/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_c=0.2217
A_1_c=1.0063
tau_c=(alpha*time)/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of cylinder"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*c_p*(T_infinity-T_i)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.1101 "From Table 4-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

$T_{o,o}$ [C]	time [s]	Q [kJ]
50	32.1	376.6
100	70.04	799.2
150	109.7	1222
200	151.4	1644
250	195.2	2067
300	241.3	2490
350	290.1	2912
400	341.8	3335
450	396.9	3757
500	455.8	4180
550	519	4603
600	587.3	5025
650	661.6	5448
700	742.9	5871
750	832.9	6293
800	933.4	6716
850	1047	7138
900	1179	7561
950	1335	7984
1000	1525	8406



Special Topic: Refrigeration and Freezing of Foods

4-119C The common kinds of microorganisms are bacteria, yeasts, molds, and viruses. The undesirable changes caused by microorganisms are off-flavors and colors, slime production, changes in the texture and appearances, and the spoilage of foods.

4-120C Microorganisms are the prime cause for the spoilage of foods. Refrigeration prevents or delays the spoilage of foods by reducing the rate of growth of microorganisms. Freezing extends the storage life of foods for months by preventing the growths of microorganisms.

4-121C The environmental factors that affect of the growth rate of microorganisms are the temperature, the relative humidity, the oxygen level of the environment, and air motion.

4-122C Cooking kills the microorganisms in foods, and thus prevents spoilage of foods. It is important to raise the internal temperature of a roast in an oven above 70ºC since most microorganisms, including some that cause diseases, may survive temperatures below 70ºC.

4-123C The contamination of foods with microorganisms can be prevented or minimized by (1) preventing contamination by following strict sanitation practices such as washing hands and using fine filters in ventilation systems, (2) inhibiting growth by altering the environmental conditions, and (3) destroying the organisms by heat treatment or chemicals.

The growth of microorganisms in foods can be retarded by keeping the temperature below 4ºC and relative humidity below 60 percent. Microorganisms can be destroyed by heat treatment, chemicals, ultraviolet light, and solar radiation.

4-124C (a) High air motion retards the growth of microorganisms in foods by keeping the food surfaces dry, and creating an undesirable environment for the microorganisms. (b) Low relative humidity (dry) environments also retard the growth of microorganisms by depriving them of water that they need to grow. Moist air supplies the microorganisms with the water they need, and thus encourages their growth. Relative humidities below 60 percent prevent the growth rate of most microorganisms on food surfaces.

4-125C Cooling the carcass with refrigerated air is at -10ºC would certainly reduce the cooling time, but this proposal should be rejected since it will cause the outer parts of the carcasses to freeze, which is undesirable. Also, the refrigeration unit will consume more power to reduce the temperature to -10ºC, and thus it will have a lower efficiency.

4-126C The freezing time could be decreased by (a) lowering the temperature of the refrigerated air, (b) increasing the velocity of air, (c) increasing the capacity of the refrigeration system, and (d) decreasing the size of the meat boxes.

4-127C The rate of freezing can affect color, tenderness, and drip. Rapid freezing increases tenderness and reduces the tissue damage and the amount of drip after thawing.

4-128C This claim is reasonable since the lower the storage temperature, the longer the storage life of beef. This is because some water remains unfrozen even at subfreezing temperatures, and the lower the temperature, the smaller the unfrozen water content of the beef.

4-129C A refrigerated shipping dock is a refrigerated space where the orders are assembled and shipped out. Such docks save valuable storage space from being used for shipping purpose, and provide a more acceptable working environment for the employees. The refrigerated shipping docks are usually maintained at 1.5ºC, and therefore the air that flows into the freezer during shipping is already cooled to about 1.5ºC. This reduces the refrigeration load of the cold storage rooms.

4-130C (*a*) The heat transfer coefficient during immersion cooling is much higher, and thus the cooling time during immersion chilling is much lower than that during forced air chilling. (*b*) The cool air chilling can cause a moisture loss of 1 to 2 percent while water immersion chilling can actually cause moisture absorption of 4 to 15 percent. (*c*) The chilled water circulated during immersion cooling encourages microbial growth, and thus immersion chilling is associated with more microbial growth. The problem can be minimized by adding chloride to the water.

4-131C The proper storage temperature of frozen poultry is about -18ºC or below. The primary freezing methods of poultry are the air blast tunnel freezing, cold plates, immersion freezing, and cryogenic cooling.

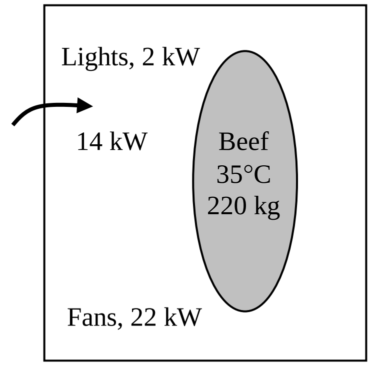
4-132C The factors, which affect the quality of frozen, fish are the condition of the fish before freezing, the freezing method, and the temperature and humidity during storage and transportation, and the length of storage time.

4-133 The chilling room of a meat plant with a capacity of 350 beef carcasses is considered. The cooling load and the air flow rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Specific heats of beef carcass and air are constant.

Properties The density and specific heat of air at 0°C are given to be 1.28 kg/m^3 and 1.0 kJ/kg·°C. The specific heat of beef carcass is given to be 3.14 kJ/kg·°C.

Analysis (*a*) The amount of beef mass that needs to be cooled per unit time is



$$\begin{aligned}\dot{m}_{beef} &= (\text{Total beef mass cooled})/(\text{cooling time})\\ &= (350\times 220\ \text{kg/carcass})/(12\ \text{h}\times 3600\ \text{s}) = 1.782\ \text{kg/s}\end{aligned}$$

The product refrigeration load can be viewed as the energy that needs to be removed from the beef carcass as it is cooled from 35 to 16ºC at a rate of 2.27 kg/s, and is determined to be

$$\begin{aligned}\dot{Q}_{beef} &= (\dot{m}c_p\Delta T)_{beef}\\ &= (1.782\ \text{kg/s})(3.14\text{kJ/kg.}^\circ\text{C})(35-16)^\text{o}\,\text{C} = 106\ \text{kW}\end{aligned}$$

Then the total refrigeration load of the chilling room becomes

$$\dot{Q}_{\text{total,chillingroom}} = \dot{Q}_{beef} + \dot{Q}_{fan} + \dot{Q}_{lights} + \dot{Q}_{heatgain} = 106 + 22 + 2 + 14 = \mathbf{144\ kW}$$

(*b*) Heat is transferred to air at the rate determined above, and the temperature of air rises from -2.2ºC to 0.5ºC as a result. Therefore, the mass flow rate of air is

$$\dot{m}_{air} = \frac{\dot{Q}_{air}}{(c_p\Delta T)_{air}} = \frac{144\ \text{kW}}{(1.0\ \text{kJ/kg.°C})[0.5-(-2.2)°\text{C}]} = 53.3\ \text{kg/s}$$

Then the volume flow rate of air becomes

$$\dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{53.3\ \text{kg/s}}{1.28\ \text{kg/m}^3} = \mathbf{41.7\ m^3/s}$$

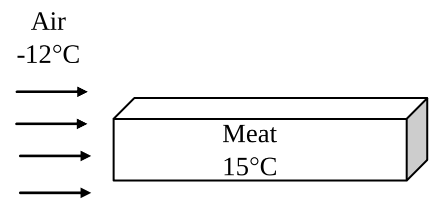
4-134 The center temperature of meat slabs is to be lowered by chilled air to below 5°C while the surface temperature remains above -1°C to avoid freezing. The average heat transfer coefficient during this cooling process is to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 5-cm. **2** Heat conduction in the meat slabs is one-dimensional because of symmetry about the centerplane. **3** The thermal properties of the meat slab are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the beef slabs are given to be ρ = 1090 kg/m^3, c_p = 3.54 kJ/kg.°C, k = 0.47 W/m.°C, and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis The lowest temperature in the steak will occur at the surfaces and the highest temperature at the center at a given time since the inner part of the steak will be last place to be cooled. In the limiting case, the surface temperature at $x = L$ = 5 cm from the center will be -1°C while the mid plane temperature is 5°C in an environment at -12°C. Then from Fig. 4-17b we obtain

$$\left.\begin{aligned}\frac{x}{L} &= \frac{5\text{ cm}}{5\text{ cm}} = 1\\ \frac{T(L,t)-T_\infty}{T_o - T_\infty} &= \frac{-1-(-12)}{5-(-12)} = 0.65\end{aligned}\right\}\quad \frac{1}{\text{Bi}} = \frac{k}{hL} = 0.95$$



which gives

$$h = \frac{k}{L}\text{Bi} = \frac{0.47\text{ W/m.°C}}{0.05\text{ m}}\left(\frac{1}{0.95}\right) = \mathbf{9.9\ W/m^2.°C}$$

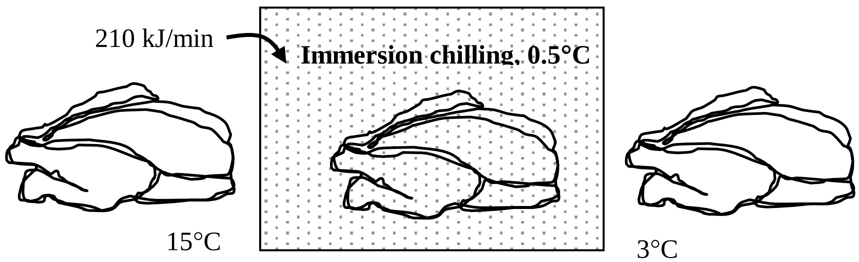
Therefore, the convection heat transfer coefficient should be kept below this value to satisfy the constraints on the temperature of the steak during refrigeration. We can also meet the constraints by using a lower heat transfer coefficient, but doing so would extend the refrigeration time unnecessarily.

Discussion We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the one-term solution relation for an infinite plane wall, but it would require a trial and error approach since the Bi number is not known.

4-135 Chickens are to be cooled by chilled water in an immersion chiller. The rate of heat removal from the chicken and the mass flow rate of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of chickens are constant.

Properties The specific heat of chicken are given to be 3.54 kJ/kg.°C. The specific heat of water is 4.18 kJ/kg.°C (Table A-9).



Analysis (*a*) Chickens are dropped into the chiller at a rate of 500 per hour. Therefore, chickens can be considered to flow steadily through the chiller at a mass flow rate of

$$\dot{m}_{\text{chicken}} = (500\ \text{chicken/h})(2.2\ \text{kg/chicken}) = 1100\ \text{kg/h} = 0.3056\text{kg/s}$$

Then the rate of heat removal from the chickens as they are cooled from 15°C to 3ºC at this rate becomes

$$\dot{Q}_{\text{chicken}} = (\dot{m}c_p\Delta T)_{\text{chicken}} = (0.3056\ \text{kg/s})(3.54\ \text{kJ/kg.°C})(15-3)^{\circ}\text{C} = \mathbf{13.0\ kW}$$

(*b*) The chiller gains heat from the surroundings as a rate of 210 kJ/min = 3.5 kJ/s. Then the total rate of heat gain by the water is

$$\dot{Q}_{\text{water}} = \dot{Q}_{\text{chicken}} + \dot{Q}_{\text{heat gain}} = 13.0 + 3.5 = 16.5\ \text{kW}$$

Noting that the temperature rise of water is not to exceed 2ºC as it flows through the chiller, the mass flow rate of water must be at least

$$\dot{m}_{\text{water}} = \frac{\dot{Q}_{\text{water}}}{(c_p\Delta T)_{\text{water}}} = \frac{16.5\text{kW}}{(4.18\ \text{kJ/kg.°C})(2^{\circ}\text{C})} = \mathbf{1.97\ kg/s}$$

If the mass flow rate of water is less than this value, then the temperature rise of water will have to be more than 2°C.

4-136E Chickens are to be frozen by refrigerated air. The cooling time of the chicken is to be determined for the cases of cooling air being at –40°F and -80°F.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of chickens are constant.

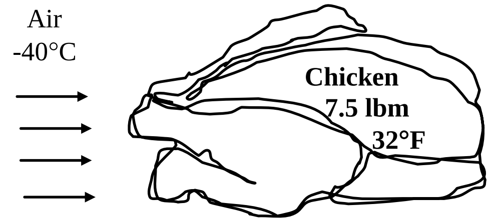
Analysis The time required to reduce the inner surface temperature of the chickens from 32ºF to 25ºF with refrigerated air at -40ºF is determined from Fig. 4-53 to be

$t \cong$ **2.3 hours**

If the air temperature were -80ºF, the freezing time would be

$t \cong$ **1.4 hours**

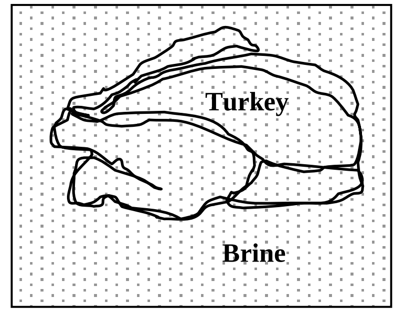
Therefore, the time required to cool the chickens to 25°F is reduced considerably when the refrigerated air temperature is decreased.



4-137 Turkeys are to be frozen by submerging them into brine at -29°C. The time it will take to reduce the temperature of turkey breast at a depth of 3.8 cm to -18°C and the amount of heat transfer per turkey are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of turkeys are constant.

Properties It is given that the specific heats of turkey are 2.98 and 1.65 kJ/kg.°C above and below the freezing point of -2.8°C, respectively, and the latent heat of fusion of turkey is 214 kJ/kg.



Analysis The time required to freeze the turkeys from 1°C to -18ºC with brine at -29ºC can be determined directly from Fig. 4-55 to be

$$t \cong 180 \text{ min.} \cong \textbf{3 hours}$$

(*a*) Assuming the entire water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from 1°C to -18°C is

Cooling to -2.8ºC:

$$Q_{\text{coolingfresh}} = (mc_p\Delta T)_{\text{fresh}} = (7\text{ kg})(2.98\text{ kJ/kg}\cdot{}^\circ\text{C})[1\text{ - }(\text{-}2.8)^\circ\text{C}] = 79.3\text{ kJ}$$

Freezing at -2.8ºC:

$$Q_{\text{freezing}} = mh_{\text{latent}} = (7\text{ kg})(214\text{ kJ/kg}) = 1498\text{ kJ}$$

Cooling -18ºC:

$$Q_{\text{coolingfrozen}} = (mc_{\text{p}}\Delta T)_{\text{frozen}} = (7\text{ kg})(1.65\text{ kJ/kg.}^\circ\text{C})[-2.8-(-18)]^\circ\text{C} = 175.6\text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{coolingfresh}} + Q_{\text{freezing}} + Q_{\text{coolingfrozen}} = 79.3+1498+175.6 \cong \textbf{1753 kJ}$$

(*b*) Assuming only 90 percent of the water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from 1°C to -18°C is

Cooling to -2.8ºC:

$$Q_{\text{coolingfresh}} = (mc_{\text{p}}\Delta T)_{\text{fresh}} = (7\text{ kg})(2.98\text{ kJ/kg}\cdot{}^\circ\text{C})[1\text{ - }(\text{-}2.98)^\circ\text{C}] = 79.3\text{ kJ}$$

Freezing at -2.8ºC:

$$Q_{\text{freezing}} = mh_{\text{latent}} = (7\times 0.9\text{ kg})(214\text{ kJ/kg}) = 1348\text{ kJ}$$

Cooling -18ºC:

$$Q_{\text{coolingfrozen}} = (mc_p\Delta T)_{\text{frozen}} = (7\times 0.9\text{ kg})(1.65\text{kJ/kg.}^\circ\text{C})[-2.8-(-18)]^\circ\text{C} = 158\text{ kJ}$$

$$Q_{\text{coolingunfrozen}} = (mc_p\Delta T)_{\text{fresh}} = (7\times 0.1\text{ kg})(2.98\text{ kJ/kg.}^\circ\text{C})[\text{-}2.8-(-18)^\circ\text{ C}] = 31.7\text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{coolingfresh}} + Q_{\text{freezing}} + Q_{\text{coolingfrozen\&unfrozen}} = 79.3+1348+158+31.7 = \textbf{1617 kJ}$$

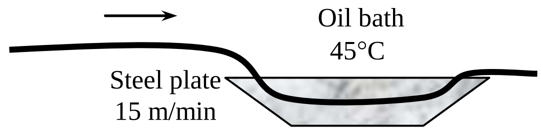
Review Problems

4-138 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The temperature of the sheet metal after quenching and the rate at which heat needs to be removed from the oil in order to keep its temperature constant are to be determined.

Assumptions **1** The thermal properties of the steel plate are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be checked).

Properties The properties of the steel plate are k = 60.5 W/m.°C, ρ = 7854 kg/m^3, and c_p = 434 J/kg.°C (Table A-3).

Analysis The characteristic length of the steel plate and the Biot number are



$$L_c = \frac{V}{A_s} = L = 0.0025 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(860 \text{ W/m}^2.°\text{C})(0.0025 \text{ m})}{60.5 \text{ W/m.°C}} = 0.036 < 0.1$$

Since $\text{Bi} < 0.1$, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{860 \text{ W/m}^2.°\text{C}}{(7854 \text{ kg/m}^3)(434 \text{ J/kg.°C})(0.0025 \text{ m})} = 0.10092 \text{ s}^{-1}$$

$$\text{time} = \frac{\text{length}}{\text{velocity}} = \frac{9 \text{ m}}{15 \text{ m/min}} = 0.6 \text{ min} = 36 \text{ s}$$

Then the temperature of the sheet metal when it leaves the oil bath is determined to be

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 45}{820 - 45} = e^{-(0.10092 \text{ s}^{-1})(36 \text{ s})} \longrightarrow T(t) = \mathbf{65.5°C}$$

The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho \dot{V} = \rho w t V = (7854 \text{ kg/m}^3)(2 \text{ m})(0.005 \text{ m})(15 \text{ m/min}) = 1178 \text{ kg/min}$$

Then the rate of heat transfer from the sheet metal to the oil bath and thus the rate at which heat needs to be removed from the oil in order to keep its temperature constant at 45°C becomes

$$\dot{Q} = \dot{m} c_p [T_i - T(t)] = (1178 \text{ kg/min})(0.434 \text{ kJ/kg.°C})(820 - 65.5)°\text{C} = 385{,}770 \text{ kJ/min} = \mathbf{6430 \text{ kW}}$$

4-139 Large steel plates are quenched in an oil reservoir. The quench time is to be determined.

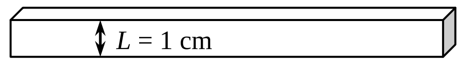
Assumptions **1** The thermal properties of the plates are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of steel plates are given to be k = 45 W/m·K, ρ = 7800 kg/m^3, and c_p = 470 J/kg·K.

Analysis For sphere, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L}{2} = \frac{0.01\,\text{m}}{2} = 0.005\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(400\ \text{W/m}^2.°\text{C})(0.005\,\text{m})}{45\ \text{W/m}.°\text{C}} = 0.044 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{400\ \text{W/m}^2.°\text{C}}{(7800\ \text{kg/m}^3)(470\ \text{J/kg}.°\text{C})(0.005\,\text{m})} = 0.02182\,\text{s}^{-1}$$

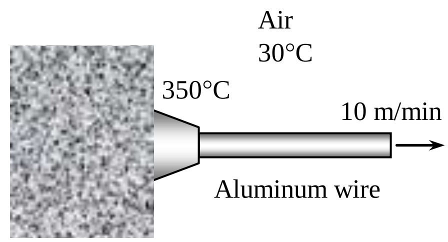
$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-30}{600-30} = e^{-(0.02182\,\text{s}^{-1})t} \longrightarrow t = 96\,\text{s} = \mathbf{1.6\,min}$$

4-140 Long aluminum wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of aluminum are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and α = 9.75×10^{-5} m^2/s.

Analysis (a) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\text{ m}}{2} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\text{ W/m}^2.°\text{C})(0.00075\text{ m})}{236\text{ W/m.°C}} = 0.00011 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35\text{ W/m}^2.°\text{C}}{(2702\text{ kg/m}^3)(896\text{ J/kg.°C})(0.00075\text{ m})} = 0.0193\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0193\text{s}^{-1})t} \longrightarrow t = \mathbf{144\text{ s}}$$

(b) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \to \text{length} = (10/60\text{ m/s})(144\text{ s}) = \mathbf{24\text{ m}}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(c) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_o^2)V = (2702\text{ kg/m}^3)\pi(0.0015\text{ m})^2(10\text{ m/min}) = 0.191\text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

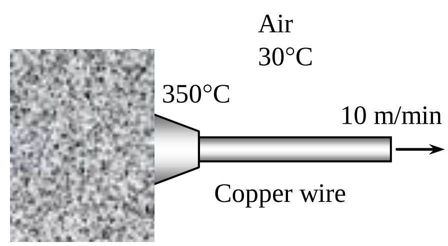
$$\dot{Q} = \dot{m}c_p[T(t)-T_\infty] = (0.191\text{ kg/min})(0.896\text{ kJ/kg.°C})(350-50)°\text{C} = 51.3\text{ kJ/min} = \mathbf{856\text{ W}}$$

4-141 Long copper wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the copper are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of copper are given to be k = 386 W/m.°C, ρ = 8950 kg/m^3, c_p = 0.383 kJ/kg.°C, and $\alpha = 1.13\times10^{-4}$ m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\text{ m}}{2} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\text{ W/m}^2.°\text{C})(0.00075\text{ m})}{386\text{ W/m.°C}} = 0.000068 < 0.1$$

Since $\text{Bi} < 0.1$ the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35\text{ W/m}^2.°\text{C}}{(8950\text{ kg/m}^3)(383\text{ J/kg.°C})(0.00075\text{ m})} = 0.0136\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0136\text{s}^{-1})t} \longrightarrow t = \mathbf{204\ s}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \longrightarrow \text{length} = \left(\frac{10\text{ m/min}}{60\text{ s/min}}\right)(204\text{ s}) = \mathbf{34\ m}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_o^2)V = (8950\text{ kg/m}^3)\pi(0.0015\text{ m})^2(10\text{ m/min}) = 0.633\text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}c_p[T(t)-T_\infty] = (0.633\text{ kg/min})(0.383\text{ kJ/kg.°C})(350-50)°\text{C} = 72.7\text{ kJ/min} = \mathbf{1212\ W}$$

4-142 Aluminum wires leaving the extruder at a specified rate are cooled in air. The necessary length of the wire is to be determined.

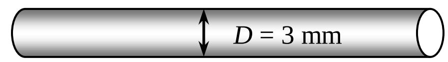
Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of aluminum are k = 237 W/m·ºC, ρ = 2702 kg/m^3, and c_p = 0.903 kJ/kg·ºC (Table A-3).

Analysis For a long cylinder, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.003\text{ m}}{4} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(50\text{ W/m}^2.\text{°C})(0.00075\text{ m})}{237\text{ W/m.°C}} = 0.000158 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{50\text{ W/m}^2.\text{°C}}{(2702\text{ kg/m}^3)(903\text{ J/kg.°C})(0.00075\text{ m})} = 0.02732\text{ s}^{-1}$$

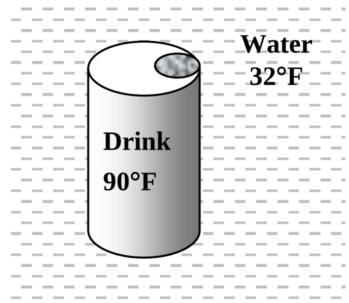
$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-25}{350-25} = e^{-(0.02732\text{ s}^{-1})t} \longrightarrow t = 93.89\text{ s}$$

Then the necessary length of the wire in the cooling section is determined to be

$$\text{Length} = \frac{t}{V} = \frac{(93.89/60)\text{ min}}{10\text{ m/min}} = \mathbf{0.156\,m}$$

4-143E A person shakes a can of drink in an iced water to cool it. The cooling time of the drink is to be determined.

Assumptions **1** The can containing the drink is cylindrical in shape with a radius of r_o = 1.25 in. **2** The thermal properties of the drink are taken to be the same as those of water. **3** Thermal properties of the drink are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the drink is stirred constantly, so that its temperature remains uniform at all times.



Properties The density and specific heat of water at room temperature (90°F) are ρ = 62.12 lbm/ft^3, c_p = 0.999 Btu/lbm.°F, k = 0.358 Btu/h.ft.°F (Table A-9E).

Analysis The characteristic length and Biot number for the can of drink are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi (1.25/12\text{ ft})^2 (5/12\text{ ft})}{2\pi (1.25/12\text{ ft})(5/12\text{ ft}) + 2\pi (1.25/12\text{ ft})^2} = 0.04167\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(30\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.04167\text{ ft})}{0.358\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}} = 3.49 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the canned drink to cool to 40°F

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{30\text{ Btu/h.ft}^2.{}^\circ\text{F}}{(62.22\text{ lbm/ft}^3)(0.999\text{ Btu/lbm.}^\circ\text{F})(0.04167\text{ ft})} = 11.599\text{ h}^{-1} = 0.00322\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{40-32}{90-32} = e^{-(0.00322\text{ s}^{-1})t} \longrightarrow t = \mathbf{615\text{ s}}$$

Therefore, it will take 10 minutes and 15 seconds to cool the canned drink to 40°F.

4-144 The average temperatures of aluminum and stainless steel rods, after 5 minutes elapsed time, are to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the aluminum rod are given as ρ = 2702 kg/m^3, c_p = 903 J/kg · K, and k = 237 W/m · K; the properties of the stainless steel rod are given as ρ = 8238 kg/m^3, c_p = 468 J/kg · K, and k = 13.4 W/m · K.

Analysis The characteristic length of both rod A and rod B is

$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.025\,\text{m}}{4} = 0.00625\,\text{m}$$

For rod A (aluminum), the Biot number is

$$Bi_{\text{rod A}} = \frac{hL_c}{k_{\text{rod A}}} = \frac{(20\ \text{W/m}^2\cdot\text{K})(0.00625\,\text{m})}{237\ \text{W/m}\cdot\text{K}} = 5.274\times10^{-4} < 0.1$$

Since, $Bi_{\text{rod A}}$ < 0.1, the lumped system analysis is applicable. Then the average temperature of rod A after 5 minutes elapsed time is

$$b_{\text{rod A}} = \frac{h}{\rho_{\text{rod A}} c_{p,\text{rod A}} L_c} = \frac{20\ \text{W/m}^2\cdot\text{K}}{(2702\,\text{kg/m}^3)(903\,\text{J/kg}\cdot\text{K})(0.00625\,\text{m})} = 0.001311\,\text{s}^{-1}$$

$$T(t) = (T_i - T_\infty)e^{-bt} + T_\infty$$

$$T(5\,\text{min}) = (15\,°\text{C} - 1000\,°\text{C})e^{-(0.001311)(300)} + 1000\,°\text{C} = \mathbf{335\,°C}\ \text{(rod A)}$$

For rod B (stainless steel), the Biot number is

$$Bi_{\text{rod B}} = \frac{hL_c}{k_{\text{rod B}}} = \frac{(20\ \text{W/m}^2\cdot\text{K})(0.00625\,\text{m})}{13.4\ \text{W/m}\cdot\text{K}} = 0.009328 < 0.1$$

Since $Bi_{\text{rod B}}$ < 0.1, the lumped system analysis is applicable. Then the average temperature of rod B after 5 minutes elapsed time is

$$b_{\text{rod B}} = \frac{h}{\rho_{\text{rod B}} c_{p,\text{rod B}} L_c} = \frac{20\ \text{W/m}^2\cdot\text{K}}{(8238\,\text{kg/m}^3)(468\,\text{J/kg}\cdot\text{K})(0.00625\,\text{m})} = 8.3\times10^{-4}\ \text{s}^{-1}$$

$$T(t) = (T_i - T_\infty)e^{-bt} + T_\infty$$

$$T(5\,\text{min}) = (15\,°\text{C} - 1000\,°\text{C})e^{-(8.3\times10^{-4})(300)} + 1000\,°\text{C} = \mathbf{232\,°C}\ \text{(rod B)}$$

Discussion The results indicate that it is quicker to heat the aluminum rod to a desired temperature than the stainless steel rod, because $b_{\text{rod A}} > b_{\text{rod B}}$.

4-145 Ball bearings leaving the oven at a uniform temperature of 900°C are exposed to air for a while before they are dropped into the water for quenching. The time they can stand in the air before their temperature falls below 850°C is to be determined.

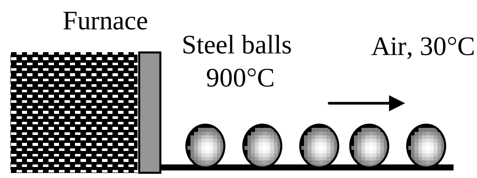
Assumptions **1** The bearings are spherical in shape with a radius of r_o = 0.6 cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be k = 15.1 W/m.°C, ρ = 8085 kg/m^3, and c_p = 0.480 kJ/kg.°C.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\text{ m}}{6} = 0.002\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(15.1\text{ W/m.°C})} = 0.0166 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{125\text{ W/m}^2.°\text{C}}{(8085\text{ kg/m}^3)(480\text{ J/kg.°C})(0.002\text{ m})} = 0.01610\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\text{s}^{-1})t} \longrightarrow t = \mathbf{3.68\text{ s}}$$

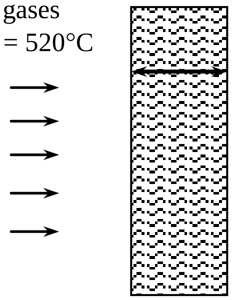
The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

4-146 The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

Assumptions **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the trunks are given to be $k = 0.17$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m^2/s.

Analysis We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes



$$Bi = \frac{hr_o}{k} = \frac{(65\text{ W/m}^2.°\text{C})(0.1\text{ m})}{(0.17\text{ W/m.°C})} = 38.24$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.28\times10^{-7}\text{ m}^2\text{/s})(5\text{ h}\times3600\text{ s/h})}{(0.1\text{ m})^2} = 0.2304$$

which is greater than 0.2. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 5 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o)$$

$$\frac{T(r_o,t)-520}{30-520} = (1.5989)e^{-(2.3420)^2(0.2304)}(0.0332) = 0.01500 \longrightarrow T(r_o,t) = \mathbf{513°C} > 410°\text{C}$$

Therefore, the trees will ignite. (Note: J_0 is read from Table 4-3).

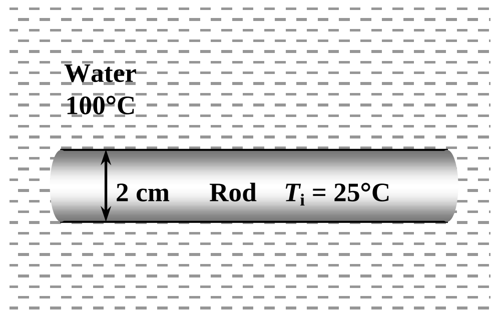
4-147 A cylindrical rod is dropped into boiling water. The thermal diffusivity and the thermal conductivity of the rod are to be determined.

Assumptions **1** Heat conduction in the rod is one-dimensional since the rod is sufficiently long, and thus temperature varies in the radial direction only. **2** The thermal properties of the rod are constant.

Properties The thermal properties of the rod available are given to be ρ = 3700 kg/m^3 and c_p = 920 J/kg.°C.

Analysis From Fig. 4-18b we have

$$\left.\begin{array}{l}\dfrac{T-T_\infty}{T_0-T_\infty}=\dfrac{93-100}{75-100}=0.28\\ \dfrac{x}{r_o}=\dfrac{r_o}{r_o}=1\end{array}\right\}\dfrac{1}{Bi}=\dfrac{k}{hr_o}=0.25$$



From Fig. 4-18a we have

$$\left.\begin{array}{l}\dfrac{1}{Bi}=\dfrac{k}{hr_o}=0.25\\ \dfrac{T_o-T_\infty}{T_i-T_\infty}=\dfrac{75-100}{25-100}=0.33\end{array}\right\}\tau=\dfrac{\alpha t}{r_o^{\,2}}=0.40$$

Then the thermal diffusivity and the thermal conductivity of the material become

$$\alpha=\frac{0.40r_o^2}{t}=\frac{(0.40)(0.01\,\text{m})^2}{3\,\text{min}\times 60\,\text{s/min}}=\mathbf{2.22\times 10^{-7}\ m^2/s}$$

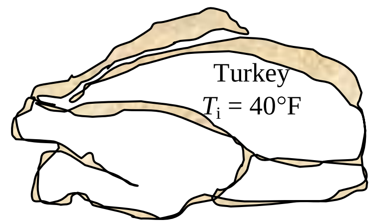
$$\alpha=\frac{k}{\alpha c_p}\longrightarrow k=\alpha\rho c_p=(2.22\times 10^{-7}\ \text{m}^2/\text{s})(3700\ \text{kg/m}^3)(920\ \text{J/kg.°C})=\mathbf{0.756\ W/m.°C}$$

4-148E A stuffed turkey is cooked in an oven. The average heat transfer coefficient at the surface of the turkey, the temperature of the skin of the turkey in the oven and the total amount of heat transferred to the turkey in the oven are to be determined.

Assumptions **1** The turkey is a homogeneous spherical object. **2** Heat conduction in the turkey is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the turkey are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the turkey are given to be k = 0.26 Btu/h.ft.°F, ρ = 75 lbm/ft^3, c_p = 0.98 Btu/lbm.°F, and α = 0.0035 ft^2/h.

Analysis (*a*) Assuming the turkey to be spherical in shape, its radius is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{14\text{ lbm}}{75\text{ lbm/ft}^3} = 0.1867\text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.1867\text{ ft}^3)}{4\pi}} = 0.3545\text{ ft}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.5\times10^{-3}\text{ ft}^2\text{/h})(5\text{ h})}{(0.3545\text{ ft})^2} = 0.1392$$

which is close to 0.2 but a little below it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the one-term solution formulation at one-third the radius from the center of the turkey can be expressed as

$$\theta(x,t)_{sph} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

$$\frac{185-325}{40-325} = 0.491 = A_1 e^{-\lambda_1^2(0.14)}\frac{\sin(0.333\lambda_1)}{0.333\lambda_1}$$

By trial and error, it is determined from Table 4-2 that the equation above is satisfied when Bi = 20 corresponding to $\lambda_1 = 2.9857$ and $A_1 = 1.9781$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.26\text{ Btu/h.ft.°F})(20)}{(0.3545\text{ ft})} = \mathbf{14.7\ Btu/h.ft^2.°F}$$

(*b*) The temperature at the surface of the turkey is

$$\frac{T(r_o,t)-325}{40-325} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9781)e^{-(2.9857)^2(0.14)}\frac{\sin(2.9857)}{2.9857} = 0.02953$$

$$\longrightarrow T(r_o,t) = \mathbf{317\text{ °F}}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (14\text{ lbm})(0.98\text{ Btu/lbm.°F})(325-40)\text{°F} = 3910\text{ Btu}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.491)\frac{\sin(2.9857)-(2.9857)\cos(2.9857)}{(2.9857)^3} = 0.828$$

$$Q = 0.828Q_{\max} = (0.828)(3910\text{ Btu}) = \mathbf{3240\ Btu}$$

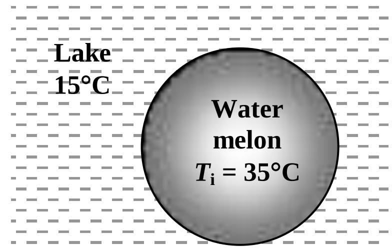
Discussion The temperature of the outer parts of the turkey will be greater than that of the inner parts when the turkey is taken out of the oven. Then heat will continue to be transferred from the outer parts of the turkey to the inner as a result of temperature difference. Therefore, after 5 minutes, the thermometer reading will probably be more than 185°F.

4-149 A watermelon is placed into a lake to cool it. The heat transfer coefficient at the surface of the watermelon and the temperature of the outer surface of the watermelon are to be determined.

Assumptions **1** The watermelon is a homogeneous spherical object. **2** Heat conduction in the watermelon is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the watermelon are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the watermelon are given to be $k = 0.618$ W/m.°C, $\alpha = 0.15\times10^{-6}$ m^2/s, $\rho = 995$ kg/m^3 and $c_p = 4.18$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.15\times10^{-6}\ \text{m}^2/\text{s})\left[(4\times60+40\ \text{min})\times60\,\text{s/min}\right]}{(0.10\,\text{m})^2} = 0.252$$

which is greater than 0.2. Then the one-term solution can be written in the form

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{20-15}{35-15} = 0.25 = A_1 e^{-\lambda_1^2(0.252)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when $Bi = 10$, which corresponds to $\lambda_1 = 2.8363$ and $A_1 = 1.9249$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.618\ \text{W/m.°C})(10)}{(0.10\,\text{m})} = \mathbf{61.8\ W/m^2.°C}$$

The temperature at the surface of the watermelon is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\,\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9249)e^{-(2.8363)^2(0.252)}\,\frac{\sin(2.8363\,\text{rad})}{2.8363}$$

$$\frac{T(r_o,t)-15}{35-15} = 0.0269 \longrightarrow T(r_o,t) = \mathbf{15.5\,°C}$$

4-150 The temperature at the center of a spherical glass bead after 3 minutes of cooling is to be determined using (a) Table 4-2 and (b) the Heisler chart (Figure 4-19).

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of glass are given to be ρ = 2800 kg/m^3, c_p = 750 J/kg · K, and k = 0.7 W/m · K.

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(28\ \text{W/m}^2\cdot\text{K})(0.005\ \text{m})}{0.7\ \text{W/m}\cdot\text{K}} = 0.2$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = \frac{(0.7\ \text{W/m}\cdot\text{K})(3\times 60\ \text{s})}{(2800\ \text{kg/m}^3)(750\ \text{J/kg}\cdot\text{K})(0.005\ \text{m})^2} = 2.4$$

(a) From Table 4-2 with Bi = 0.2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.7593 \quad \text{and} \quad A_1 = 1.0592$$

For a sphere, we have

$$\theta_{0,\,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}$$

The temperature at the center of the glass bead is

$$T_0 = (T_i - T_\infty)A_1 e^{-\lambda_1^2\tau} + T_\infty = (400\,^\circ\text{C} - 30\,^\circ\text{C})(1.0592)e^{-(0.7593)^2(2.4)} + 30\,^\circ\text{C} = \mathbf{128\,^\circ C}$$

(b) From Figure 4-19a with

$$\frac{1}{Bi} = \frac{1}{0.2} = 5 \quad \text{and} \quad \tau = 2.4$$

we get $\theta_0 \approx 0.27$. Hence, the temperature at the center of the glass bead is

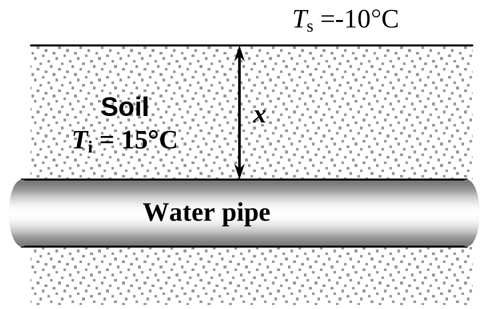
$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \quad \rightarrow \quad T_0 = 0.27(400\,^\circ\text{C} - 30\,^\circ\text{C}) + 30\,^\circ\text{C} = \mathbf{130\,^\circ C}$$

Discussion The results for part (a) and (b) are in comparable agreement. The result from part (b) is approximately 1.6% larger than the result from part (a).

4-151 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature of -10°C. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.7 W/m.°C and $\alpha = 1.4\times10^{-5}$ m^2/s.



Analysis The depth at which the temperature drops to 0°C in 75 days is determined using the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting and using Table 4-4, we obtain

$$\frac{0-15}{-10-15} = erfc\left(\frac{x}{2\sqrt{(1.4\times10^{-5}\ \text{m}^2/\text{s})(75\ \text{day}\times24\ \text{h/day}\times3600\ \text{s/h})}}\right)$$

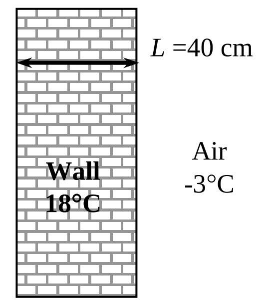
$$\longrightarrow x = \mathbf{7.05\,m}$$

Therefore, the pipes must be buried at a depth of at least 7.05 m.

4-152 A thick wall is exposed to cold outside air. The wall temperatures at distances 15, 30, and 40 cm from the outer surface at the end of 2-hour cooling period are to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only. Therefore, the wall can be considered to be a semi-infinite medium **2** The thermal properties of the wall are constant.

Properties The thermal properties of the brick are given to be $k = 0.72$ W/m.°C and $\alpha = 1.6\times10^{-7}$ m^2/s.



Analysis For a 15 cm distance from the outer surface, from Fig. 4-31 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98\\ \eta &= \frac{x}{2\sqrt{\alpha t}}\frac{0.15\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 0.70\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.25$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.25 \longrightarrow T = \mathbf{12.8°C}$$

For a 30 cm distance from the outer surface, from Fig. 4-31 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98\\ \eta &= \frac{x}{2\sqrt{\alpha t}}\frac{0.3\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 1.40\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.038$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.038 \longrightarrow T = \mathbf{17.2°C}$$

For a 40 cm distance from the outer surface, that is for the inner surface, from Fig. 4-31 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98\\ \eta &= \frac{x}{2\sqrt{\alpha t}}\frac{0.4\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 1.87\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0$$

$$1-\frac{T-(-3)}{18-(-3)} = 0 \longrightarrow T = \mathbf{18.0°C}$$

Discussion This last result shows that the semi-infinite medium assumption is a valid one.

4-153 The surface temperature and heat flux with lava flow on the ground are to be determined.

Assumptions **1** The ground is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is constant. **4** Heat transfer by radiation is negligible.

Properties The properties of the ground (dry soil) are ρ = 1500 kg/m^3, c_p = 1900 J/kg · K, and k = 1.0 W/m · K (from Table A-8).

Analysis (a) For semi-infinite solid with convection on the surface, the temperature of the ground surface (x = 0) can be determined with

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \text{erfc}(0) - \exp\left(\frac{h^2 \alpha t}{k^2}\right)\text{erfc}\left(\frac{h\sqrt{\alpha t}}{k}\right)$$

where

$$\alpha = \frac{k}{\rho c_p} = \frac{1.0\text{ W/m}\cdot\text{K}}{(1500\text{ kg/m}^3)(1900\text{ J/kg}\cdot\text{K})} = 3.509\times 10^{-7}\text{ m}^2\text{/s}$$

$$\frac{h^2\alpha t}{k^2} = \frac{(3500\text{ W/m}^2\cdot\text{K})^2(3.509\times 10^{-7}\text{ m}^2\text{/s})(2\text{ s})}{(1.0\text{ W/m}\cdot\text{K})^2} = 8.597$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(3500\text{ W/m}^2\cdot\text{K})\sqrt{(3.509\times 10^{-7}\text{ m}^2\text{/s})(2\text{ s})}}{1.0\text{ W/m}\cdot\text{K}} = 2.932$$

Hence

$$T(0, 2\text{ s}) = (1200\,°\text{C} - 15\,°\text{C})\left[1 - \exp(8.597)\text{erfc}(\,2.932)\right] + 15\,°\text{C}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
T=(1200-15)*(1-exp(8.597)*erfc(2.932))+15
```

Solving by EES software, the temperature of ground surface after 2 s of lava flowing on it is

$$T(0, 2\text{ s}) = \mathbf{983\,°C}$$

(b) The heat flux from the lava flow to the ground surface at t = 2 s is

$$\dot{q}_s(2\text{ s}) = h\left[T_\infty - T(0, 2\text{ s})\right] = (3500\text{ W/m}^2\cdot\text{K})(1200 - 983)\text{ K} = \mathbf{7.595\times 10^5\ W/m^2}$$

Discussion The surface temperature of the ground can also be determined using Figure 4-31:

At $\eta = 0$ and $h\sqrt{\alpha t}/k \approx 2.9$, Figure 4-31 gives

$$\frac{T(x,t)-T_i}{T_\infty - T_i} \approx 0.81 \quad \rightarrow \quad T(0, 2\text{ s}) = \mathbf{975\,°C}$$

The result determined using Figure 4-31 is about 0.8% lower than the result obtained for part (a).

4-154 The temperature at the edge of a steel block after 10 minutes of cooling is to be determined.

Assumptions **1** Two-dimensional heat conduction in x and y directions. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is constant. **4** Heat transfer by radiation is negligible.

Properties The properties of steel are (ρ = 7832 kg/m^3, c_p = 434 J/kg · K, k = 63.9 W/m · K, and α = 18.8 × 10^{-6} m^2/s.

Analysis For a quarter-infinite medium, at the edge of the steel block (x = y = 0), we have

$$\theta(0,0,t) = \theta_{\text{semi-inf}}(0,t)\theta_{\text{semi-inf}}(0,t) = [\theta_{\text{semi-inf}}(0,t)]^2$$

where

$$1-\theta_{\text{semi-inf}}(0,t) = \frac{T(0,t)-T_i}{T_\infty - T_i} = \text{erfc}(0) - \exp\left(\frac{h^2\alpha t}{k^2}\right)\text{erfc}\left(\frac{h\sqrt{\alpha t}}{k}\right)$$

At t = 10 minutes, we have

$$\frac{h^2\alpha t}{k^2} = \frac{(25\ \text{W/m}^2\cdot\text{K})^2(18.8\times10^{-6}\ \text{m}^2\text{/s})(10\times60\ \text{s})}{(63.9\ \text{W/m}\cdot\text{K})^2} = 1.727\times10^{-3}$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(25\ \text{W/m}^2\cdot\text{K})\sqrt{(18.8\times10^{-6}\ \text{m}^2\text{/s})(10\times60\ \text{s})}}{63.9\ \text{W/m}\cdot\text{K}} = 0.04155$$

Hence

$$1-\theta_{\text{semi-inf}}(0, 600\ \text{s}) = \text{erfc}(0) - \exp(1.727\times10^{-3})\text{erfc}(0.04155)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
1-theta=erfc(0)-exp(1.727e-3)*erfc(0.04155)
```

Solving by EES software, we get

$$\theta_{\text{semi-inf}}(0, 600\ \text{s}) = 0.9548$$

The temperature at the edge of the steel block after 10 minutes of cooling is

$$\theta(0,0,600\ \text{s}) = \frac{T(0,0,600\ \text{s})-T_i}{T_\infty - T_i} = [\theta_{\text{semi-inf}}(0,600\ \text{s})]^2 = 0.9548^2$$

$$T(0,0,600\ \text{s}) = (T_\infty - T_i)0.9548^2 + T_i = (450\,°\text{C} - 25\,°\text{C})0.9548^2 + 25\,°\text{C} = \mathbf{412\,°C}$$

Discussion The temperature at the steel block edge can also be determined using Figure 4-31:

At $\eta = 0$ and $h\sqrt{\alpha t}/k \approx 0.04$, Figure 4-31 gives

$$1-\frac{T(x,t)-T_\infty}{T_i - T_\infty} \approx 0.04 \quad\rightarrow\quad \theta_{\text{semi-inf}}(0,600\ \text{s}) = 0.96$$

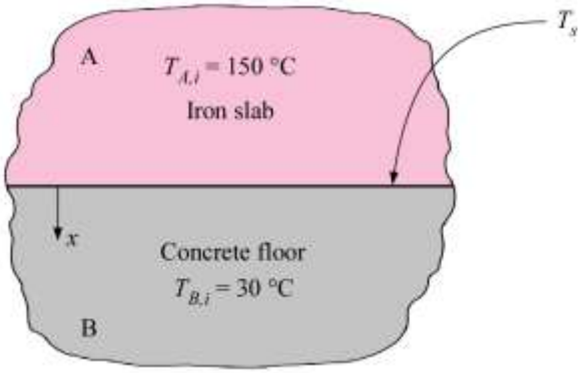
$$T(0,0,600\ \text{s}) = (450\,°\text{C} - 25\,°\text{C})0.96^2 + 25\,°\text{C} = \mathbf{417\,°C}$$

The result determined using Figure 4-31 is about 1.2% higher than the result obtained using the EES software.

4-155 A heated large iron slab is placed on a concrete floor; (a) the surface temperature and (b) the temperature of the concrete floor at the depth of 25 mm are to be determined.

Assumptions **1** The iron slab and concrete floor are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Contact resistance is negligible.

Properties The properties of iron slab are given to be ρ = 7870 kg/m^3, c_p = 447 J/kg · K, and k = 80.2 W/m · K; the properties of concrete floor are given to be ρ = 1600 kg/m^3, c_p = 840 J/kg · K, and k = 0.79 W/m · K.



Analysis (a) For contact of two semi-infinite solids, the surface temperature is

$$T_s = \frac{\sqrt{(k\rho c_p)_A}T_{A,i} + \sqrt{(k\rho c_p)_B}T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}} = \frac{(16797)(150\,°\text{C}) + (1030)(30\,°\text{C})}{16797 + 1030} = \mathbf{143\,°C}$$

where

$$\sqrt{(k\rho c_p)_A} = \sqrt{(80.2\text{ W/m}\cdot\text{K})(7870\text{ kg/m}^3)(447\text{ J/kg}\cdot\text{K})} = 16797$$

$$\sqrt{(k\rho c_p)_B} = \sqrt{(0.79\text{ W/m}\cdot\text{K})(1600\text{ kg/m}^3)(840\text{ J/kg}\cdot\text{K})} = 1030$$

(b) For semi-infinite solid with specified surface temperature, we have

$$\frac{T(x,t) - T_{B,i}}{T_s - T_{B,i}} = \text{erfc}\left[\frac{x}{2\sqrt{kt/(\rho c_p)}}\right]$$

At t = 15 minutes and x = 25 mm with T_s = 143°C,

$$\frac{T(x,t) - 30\,°\text{C}}{143\,°\text{C} - 30\,°\text{C}} = \text{erfc}\left(\frac{0.025\text{ m}}{2\sqrt{\dfrac{(0.79\text{ W/m}\cdot\text{K})(15\times 60\text{ s})}{(1600\text{ kg/m}^3)(840\text{ J/kg}\cdot\text{K})}}}\right)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(T-30)/(143-30)=erfc(0.025/(2*sqrt(0.79*15*60/(1600*840))))
```

Solving by EES software, the temperature of the concrete floor at x = 25 mm and t = 15 minutes is

$$T(0.025\text{ m}, 900\text{ s}) = \mathbf{80\,°C}$$

Discussion Depending on surface condition of the concrete floor, contact resistance may be significant and cannot be neglected.

4-156 A hot dog is to be cooked by dropping it into boiling water. The time of cooking is to be determined.

Assumptions **1** Heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the hot dog are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be k = 0.76 W/m.°C, ρ = 980 kg/m^3, c_p = 3.9 kJ/kg.°C, and α = 2×10^{-7} m^2/s.

Analysis This hot dog can physically be formed by the intersection of an infinite plane wall of thickness $2L$ = 12 cm, and a long cylinder of radius $r_o = D/2$ = 1 cm. The Biot numbers and corresponding constants are first determined to be

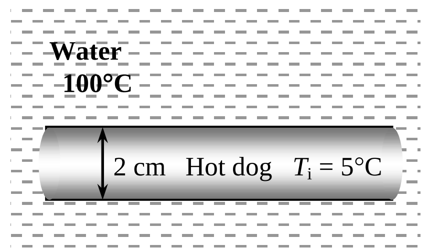
$$Bi = \frac{hL}{k} = \frac{(600\ \text{W/m}^2.^\circ\text{C})(0.06\ \text{m})}{(0.76\ \text{W/m.}^\circ\text{C})} = 47.37 \longrightarrow \lambda_1 = 1.5380 \quad \text{and} \quad A_1 = 1.2726$$

$$Bi = \frac{hr_o}{k} = \frac{(600\ \text{W/m}^2.^\circ\text{C})(0.01\ \text{m})}{(0.76\ \text{W/m.}^\circ\text{C})} = 7.895 \longrightarrow \lambda_1 = 2.1249 \quad \text{and} \quad A_1 = 1.5514$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{block} = \theta(0,t)_{wall}\,\theta(0,t)_{cyl} = \left(A_1 e^{-\lambda_1^2\tau}\right)\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{80-100}{5-100} = \left\{(1.2726)\exp\left[-(1.5380)^2\frac{(2\times10^{-7})t}{(0.06)^2}\right]\right\}$$
$$\times\left\{(1.5514)\exp\left[-(2.1249)^2\frac{(2\times10^{-7})t}{(0.01)^2}\right]\right\} = 0.2105$$



which gives

$$t = \mathbf{244\ s} = \mathbf{4.1\ min}$$

Therefore, it will take about 4.1 min for the hot dog to cook. Note that

$$\tau_{cyl} = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-7}\ \text{m}^2/\text{s})(244\ \text{s})}{(0.01\ \text{m})^2} = 0.49 > 0.2$$

and thus the assumption $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

Discussion This problem could also be solved by treating the hot dog as an infinite cylinder since heat transfer through the end surfaces will have little effect on the mid section temperature because of the large distance.

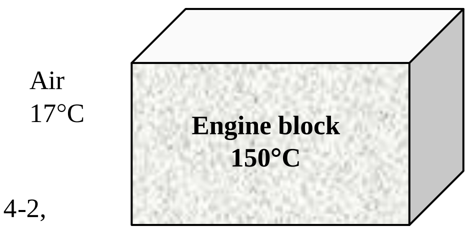
4-157 The engine block of a car is allowed to cool in atmospheric air. The temperatures at the center of the top surface and at the corner after a specified period of cooling are to be determined.

Assumptions **1** Heat conduction in the block is three-dimensional, and thus the temperature varies in all three directions. **2** The thermal properties of the block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of cast iron are given to be k = 52 W/m.°C and $\alpha = 1.7\times10^{-5}$ m^2/s.

Analysis This rectangular block can physically be formed by the intersection of two infinite plane walls of thickness 2L = 40 cm (call planes A and B) and an infinite plane wall of thickness 2L = 80 cm (call plane C). We measure x from the center of the block.

(*a*) The Biot number is calculated for each of the plane wall to be



$$Bi_{\rm A} = Bi_{\rm B} = \frac{hL}{k} = \frac{(6\ {\rm W/m^2.°C})(0.2\ {\rm m})}{(52\ {\rm W/m.°C})} = 0.0231$$

$$Bi_{\rm C} = \frac{hL}{k} = \frac{(6\ {\rm W/m^2.°C})(0.4\ {\rm m})}{(52\ {\rm W/m.°C})} = 0.0462$$

The constants λ_1 and A_1 corresponding to these Biot numbers are, from Table 4-2,

$$\lambda_{1({\rm A,B})} = 0.150 \quad \text{and} \quad A_{1({\rm A,B})} = 1.0038$$

$$\lambda_{1({\rm C})} = 0.212 \quad \text{and} \quad A_{1({\rm C})} = 1.0076$$

The Fourier numbers are

$$\tau_{\rm A,B} = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\ {\rm m^2/s})(45\ {\rm min}\times 60\ {\rm s/min})}{(0.2\ {\rm m})^2} = 1.1475 > 0.2$$

$$\tau_{\rm C} = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\ {\rm m^2/s})(45\ {\rm min}\times 60\ {\rm s/min})}{(0.4\ {\rm m})^2} = 0.2869 > 0.2$$

The center of the top surface of the block (whose sides are 80 cm and 40 cm) is at the center of the plane wall with 2L = 80 cm, at the center of the plane wall with 2L = 40 cm, and at the surface of the plane wall with 2L = 40 cm. The dimensionless temperatures are

$$\theta_{\rm o,wall\,(A)} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0038)e^{-(0.150)^2(1.1475)} = 0.9782$$

$$\theta(L,t)_{\rm wall\,(B)} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L) = (1.0038)e^{-(0.150)^2(1.1475)}\cos(0.150) = 0.9672$$

$$\theta_{\rm o,wall\,(C)} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0076)e^{-(0.212)^2(0.2869)} = 0.9947$$

Then the center temperature of the top surface of the block becomes

$$\left[\frac{T(L,0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{\rm wall\,(B)} \times \theta_{\rm o,wall\,(A)} \times \theta_{\rm o,wall\,(C)} = 0.9672\times0.9782\times0.9947 = 0.9411$$

$$\frac{T(L,0,0,t) - 17}{150 - 17} = 0.9411 \longrightarrow T(L,0,0,t) = \mathbf{142.2°C}$$

(*b*) The corner of the block is at the surface of each plane wall. The dimensionless temperature for the surface of the plane walls with 2L = 40 cm is determined in part (*a*). The dimensionless temperature for the surface of the plane wall with 2L = 80 cm is determined from

$$\theta(L,t)_{\rm wall\,(C)} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L) = (1.0076)e^{-(0.212)^2(0.2869)}\cos(0.212) = 0.9724$$

Then the corner temperature of the block becomes

$$\left[\frac{T(L,L,L,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{\rm wall,C} \times \theta(L,t)_{\rm wall,B} \times \theta(L,t)_{\rm wall,A} = 0.9724\times0.9672\times0.9672 = 0.9097$$

$$\frac{T(L,L,L,t) - 17}{150 - 17} = 0.9097 \longrightarrow T(L,L,L,t) = \mathbf{138.0°C}$$

4-158 A man is found dead in a room. The time passed since his death is to be estimated.

Assumptions **1** Heat conduction in the body is two-dimensional, and thus the temperature varies in both radial *r*- and *x*-directions. **2** The thermal properties of the body are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The human body is modeled as a cylinder. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

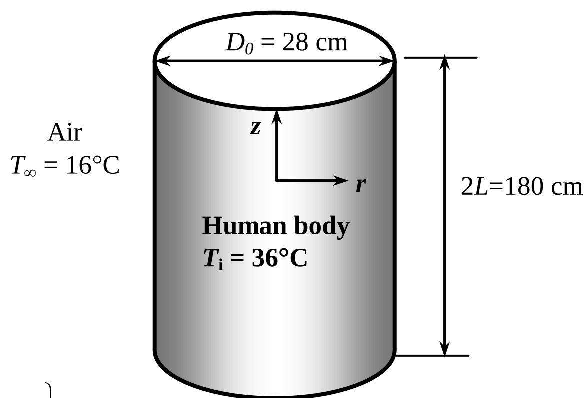
Properties The thermal properties of body are given to be $k = 0.62$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis A short cylinder can be formed by the intersection of a long cylinder of radius $D/2 = 14$ cm and a plane wall of thickness $2L = 180$ cm. We measure x from the midplane. The temperature of the body is specified at a point that is at the center of the plane wall but at the surface of the cylinder. The Biot numbers and the corresponding constants are first determined to be

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.90\ \text{m})}{(0.62\ \text{W/m.°C})} = 13.06 \longrightarrow \lambda_1 = 1.4495 \quad \text{and} \quad A_1 = 1.2644$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.14\ \text{m})}{(0.62\ \text{W/m.°C})} = 2.03 \longrightarrow \lambda_1 = 1.6052 \quad \text{and} \quad A_1 = 1.3408$$

Noting that $\tau = \alpha t / L^2$ for the plane wall and $\tau = \alpha t / r_o^2$ for cylinder and $J_0(1.6052)=0.4524$ from Table 4-3, and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as



$$\theta(0, r_0, t)_{block} = \theta(0,t)_{\text{wall}}\,\theta(r_0,t)_{\text{cyl}}$$

$$\frac{23-16}{36-16} = (A_1 e^{-\lambda_1^2\tau})\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r / r_0)\right]$$

$$0.3500 = \left\{(1.2644)\exp\left[-(1.4495)^2\,\frac{(0.15\times10^{-6})t}{(0.90)^2}\right]\right\}$$

$$\times\left\{(1.3408)\exp\left[-(1.6052)^2\,\frac{(0.15\times10^{-6})t}{(0.14)^2}\right](0.4524)\right\}$$

$$\longrightarrow t = 39{,}013\ \text{s} = \mathbf{10.8\ hours}$$

Fundamentals of Engineering (FE) Exam Problems

4-159 The Biot number can be thought of as the ratio of

(a) the conduction thermal resistance to the convective thermal resistance
(b) the convective thermal resistance to the conduction thermal resistance
(c) the thermal energy storage capacity to the conduction thermal resistance
(d) the thermal energy storage capacity to the convection thermal resistance

(e) None of the above

Answer (a) the conduction thermal resistance to the convective thermal resistance

4-160 Lumped system analysis of transient heat conduction situations is valid when the Biot number is

(a) very small (b) approximately one (c) very large

(d) any real number (e) cannot say unless the Fourier number is also known.

Answer (a) very small

4-161 Polyvinylchloride automotive body panels (k = 0.092 W/m·K, c_p = 1.05 kJ/kg·K, ρ = 1714 kg/m^3), 3-mm thick, emerge from an injection molder at 120°C. They need to be cooled to 40°C by exposing both sides of the panels to 20°C air before they can be handled. If the convective heat transfer coefficient is 30 W/m^2·K and radiation is not considered, the time that the panels must be exposed to air before they can be handled is

(a) 1.6 min (b) 2.4 min (c) 2.8 min (d) 3.5 min (e) 4.2 min

Answer (b) 2.4 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=40 [C]
Ti=120 [C]
Ta=20 [C]
r=1714 [kg/m^3]
k=0.092 [W/m-K]
c=1050 [J/kg-K]
h=30 [W/m^2-K]
L=0.003 [m]
Lc=L/2
b=h/(r*c*Lc)
(T-Ta)/(Ti-Ta)=exp(-b*time)
```

4-162 A steel casting cools to 90 percent of the original temperature difference in 30 min in still air. The time it takes to cool this same casting to 90 percent of the original temperature difference in a moving air stream whose convective heat transfer coefficient is 5 times that of still air is

(a) 3 min (b) 6 min (c) 9 min (d) 12 min (e) 15 min

Answer (b) 6 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
t1=30 [min]
per=0.9
a=ln(per)/t1
t2=ln(per)/(5*a)
```

4-163 An 18-cm-long, 16-cm-wide, and 12-cm-high hot iron block (ρ = 7870 kg/m^3, c_p = 447 J/kg·ºC) initially at 20ºC is placed in an oven for heat treatment. The heat transfer coefficient on the surface of the block is 100 W/m^2·ºC. If it is required that the temperature of the block rises to 750ºC in a 25-min period, the oven must be maintained at

(a) 750ºC (b) 830ºC (c) 875ºC (d) 910ºC (e) 1000ºC

Answer (d) 910ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Length=0.18 [m]
Width=0.16 [m]
Height=0.12 [m]
rho=7870 [kg/m^3]
c_p=447 [J/kg-C]
T_i=20 [C]
T_f=750 [C]
h=100 [W/m^2-C]
t=25*60 [s]
A_s=2*Length*Width+2*Length*Height+2*Width*Height
V=Length*Width*Height
b=(h*A_s)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*t)
```

4-164 A 10-cm-inner diameter, 30-cm long can filled with water initially at 25ºC is put into a household refrigerator at 3ºC. The heat transfer coefficient on the surface of the can is 14 $W/m^2 \cdot ºC$. Assuming that the temperature of the water remains uniform during the cooling process, the time it takes for the water temperature to drop to 5ºC is

(a) 0.55 h (b) 1.17 h (c) 2.09 h (d) 3.60 h (e) 4.97 h

Answer (e) 4.97 h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.10 [m]
L=0.30 [m]
T_i=25 [C]
T_infinity=3 [C]
T_f=5 [C]
h=14 [W/m^2-C]
A_s=pi*D*L
V=pi*D^2/4*L
rho=1000 [kg/m^3]
c_p=4180 [J/kg-C]
b=(h*A_s)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*t)
t_hour=t*Convert(s, h)
```

7-165 A 6-cm-diameter 13-cm-high canned drink (ρ = 977 kg/m^3, k = 0.607 W/m·°C, c_p = 4180 J/kg·°C) initially at 25°C is to be cooled to 5°C by dropping it into iced water at 0°C. Total surface area and volume of the drink are A_s = 301.6 cm^2 and V= 367.6 cm^3. If the heat transfer coefficient is 120 W/m^2·°C, determine how long it will take for the drink to cool to 5°C. Assume the can is agitated in water and thus the temperature of the drink changes uniformly with time.

(a) 1.5 min (b) 8.7 min (c) 11.1 min (d) 26.6 min (e) 6.7 min

Answer (c) 11.1 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.06 [m]
L=0.13 [m]
Cp=4180 [J/kg-K]
rho= 977 [kg/m^3]
k=0.607 [W/m-K]
V=pi*L*D^2/4
A=2*pi*D^2/4+pi*D*L
m=rho*V
h=120 [W/m^2-C]
Ti=25 [C]
Tinf=0 [C]
T=5 [C]
b=h*A/(rho*V*Cp)

"Lumped system analysis is applicable. Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)
t_min=time/60

"Some Wrong Solutions with Common Mistakes:"
(T-0)/(Ti-0)=exp(-b*W1_time); W1_t=W1_time/60 "Tinf is ignored"
(T-Tinf)/(Ti-Tinf)=exp(-b*W2_time); W2_t=W2_time/60 "Sign error"
(T-Ti)/(Tinf-Ti)=exp(-b*W3_time); W3_t=W3_time/60 "Switching Ti and Tinf"
(T-Tinf)/(Ti-Tinf)=exp(-b*W4_time) "Using seconds instead of minutes"
```

4-166 Copper balls (ρ = 8933 kg/m^3, k = 401 W/m·°C, c_p = 385 J/kg·°C, α = 1.166×10^{-4} m^2/s) initially at 200°C are allowed to cool in air at 30°C for a period of 2 minutes. If the balls have a diameter of 2 cm and the heat transfer coefficient is 80 W/m^2·°C, the center temperature of the balls at the end of cooling is

(a) 104°C (b) 87°C (c) 198°C (d) 126°C (e) 152°C

Answer (a) 104°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.02 [m]
Cp=385 [J/kg-K]
rho= 8933 [kg/m^3]
k=401 [W/m-K]
V=pi*D^3/6
A=pi*D^2
m=rho*V
h=80 [W/m^2-C]
Ti=200 [C]
Tinf=30 [C]
b=h*A/(rho*V*Cp)
time=2*60 [s]
Bi=h*(V/A)/k

"Lumped system analysis is applicable. Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)

“Some Wrong Solutions with Common Mistakes:”
(W1_T-0)/(Ti-0)=exp(-b*time) “Tinf is ignored”
(-W2_T+Tinf)/(Ti-Tinf)=exp(-b*time) “Sign error”
(W3_T-Ti)/(Tinf-Ti)=exp(-b*time) “Switching Ti and Tinf”
(W4_T-Tinf)/(Ti-Tinf)=exp(-b*time/60) “Using minutes instead of seconds”
```

4-167 Carbon steel balls (ρ = 7830 kg/m^3, k = 64 W/m·°C, c_p = 434 J/kg·°C) initially at 150°C are quenched in an oil bath at 20°C for a period of 3 minutes. If the balls have a diameter of 5 cm and the convection heat transfer coefficient is 450 W/m^2·°C, the center temperature of the balls after quenching will be (Hint: Check the Biot number).

(a) 27.4°C (b) 143°C (c) 12.7°C (d) 48.2°C (e) 76.9°C

Answer (a) 27.4°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.05 [m]
Cp=434 [J/kg-K]
rho= 7830 [kg/m^3]
k=64 [W/m-K]
V=pi*D^3/6
A=pi*D^2
m=rho*V
h=450 [W/m^2-C]
Ti=150 [C]
Tinf=20 [C]
b=h*A/(rho*V*Cp)
time=3*60 [s]
Bi=h*(V/A)/k

"Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)

“Some Wrong Solutions with Common Mistakes:”
(W1_T-0)/(Ti-0)=exp(-b*time) “Tinf is ignored”
(-W2_T+Tinf)/(Ti-Tinf)=exp(-b*time) “Sign error”
(W3_T-Ti)/(Tinf-Ti)=exp(-b*time) “Switching Ti and Tinf”
(W4_T-Tinf)/(Ti-Tinf)=exp(-b*time/60) “Using minutes instead of seconds”
```

4-168 In a production facility, large plates made of stainless steel (k = 15 W/m·ºC, $\alpha = 3.91\times10^{-6}$ m^2/s) of 40 cm thickness are taken out of an oven at a uniform temperature of 750ºC. The plates are placed in a water bath that is kept at a constant temperature of 20ºC with a heat transfer coefficient of 600 W/m^2·ºC. The time it takes for the surface temperature of the plates to drop to 100ºC is

(a) 0.28 h (b) 0.99 h (c) 2.05 h (d) 3.55 h (e) 5.33 h

Answer (d) 3.55 h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=15 [W/m-C]
alpha=3.91E-6 [m^2/s]
2*L=0.4 [m]
T_i=750 [C]
T_infinity=20 [C]
h=600 [W/m^2-C]
T_s=100 [C]

Bi=(h*L)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi number of 8 are obtained from
Table 4-2 of the text as"
lambda_1=1.3978
A_1=1.2570
tau=(alpha*t)/L^2
(T_s-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*cos(lambda_1)

"Some Wrong Solutions with Common Mistakes"
tau_1=(alpha*W1_t)/L^2
(T_s-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau_1) "Using the relation for center temperature"
```

4-169 A long 18-cm-diameter bar made of hardwood (k = 0.159 W/m·ºC, α = 1.75×10^{-7} m^2/s) is exposed to air at 30ºC with a heat transfer coefficient of 8.83 W/m^2·ºC. If the center temperature of the bar is measured to be 15ºC after a period of 3-hours, the initial temperature of the bar is

(a) 11.9ºC (b) 4.9ºC (c) 1.7ºC (d) 0ºC (e) -9.2ºC

Answer (b) 4.9ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.18 [m]
k=0.159 [W/m-C]
alpha=1.75E-7 [m^2/s]
T_infinity=30 [C]
h=8.83 [W/m^2-C]
T_0=15 [C]
t=3*3600 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 5 are obtained from Table
4-2 of the text as"
lambda_1=1.9898
A_1=1.5029
tau=(alpha*t)/r_0^2
(T_0-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

"Some Wrong Solutions with Common Mistakes"
lambda_1a=1.3138
A_1a=1.2403
(T_0-T_infinity)/(W1_T_i-T_infinity)=A_1a*exp(-lambda_1a^2*tau) "Using coefficients for plane wall in Table 4-2"
 lambda_1b=2.5704
A_1b=1.7870
(T_0-T_infinity)/(W2_T_i-T_infinity)=A_1b*exp(-lambda_1b^2*tau) "Using coefficients for sphere in Table 4-2"
```

4-170 Consider a 7.6-cm-long and 3-cm-diameter cylindrical lamb meat chunk (ρ = 1030 kg/m^3, c_p = 3.49 kJ/kg·ºC, k = 0.456 W/m·ºC, α = 1.3×10^{-7} m^2/s). Such a meat chunk initially at 2ºC is dropped into boiling water at 95ºC with a heat transfer coefficient of 1200 W/m^2·ºC. The amount of heat transfer during the first 8 minutes of cooking is

(a) 71 kJ (b) 227 kJ (c) 238 kJ (d) 269 kJ (e) 307 kJ

Answer (c) 269 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
2*L=0.076 [m]
D=0.03 [m]
n=15
rho=1030 [kg/m^3]
c_p=3490 [J/kg-C]
k=0.456 [W/m-C]
alpha=1.3E-7 [m^2/s]
T_i=2 [C]
T_infinity=95 [C]
h=1200 [W/m^2-C]
t=8*60 [s]
Bi_wall=(h*L)/k
lambda_1_wall=1.5552 "for Bi_wall = 100 from Table 4-2"
A_1_wall=1.2731
tau_wall=(alpha*t)/L^2
theta_wall=A_1_wall*exp(-lambda_1_wall^2*tau_wall)
Q\Q_max_wall=1-theta_wall*sin(lambda_1_wall)/lambda_1_wall
r_0=D/2
Bi_cyl=(h*r_0)/k
lambda_1_cyl=2.3455 "for Bi_cyl = 40 from Table 4-2"
A_1_cyl=1.5993
tau_cyl=(alpha*t)/r_0^2
theta_cyl=A_1_cyl*exp(-lambda_1_cyl^2*tau_cyl)
J_1=0.5309 "For xi = lambda_a_cyl = 2.3455 from Table 4-2"
Q\Q_max_cyl=1-2*theta_cyl*J_1/lambda_1_cyl
V=pi*D^2/4*(2*L)
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q\Q_max=Q\Q_max_wall+Q\Q_max_cyl*(1-Q\Q_max_wall)
Q=Q_max*Q\Q_max

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*Q\Q_max_wall "Considering large plane wall only"
W3_Q=Q_max*Q\Q_max_cyl "Considering long cylinder only"
```

4-171 Consider a 7.6-cm-long and 3-cm-diameter cylindrical lamb meat chunk (ρ = 1030 kg/m^3, c_p = 3.49 kJ/kg·ºC, k = 0.456 W/m·ºC, $\alpha = 1.3\times10^{-7}$ m^2/s). Such a meat chunk initially at 2ºC is dropped into boiling water at 95ºC with a heat transfer coefficient of 1200 W/m^2·ºC. The time it takes for the center temperature of the meat chunk to rise to 75ºC is

(a) 136 min (b) 21.2 min (c) 13.6 min (d) 11.0 min (e) 8.5 min

Answer (d) 11.0 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
2*L=0.076 [m]
D=0.03 [m]
rho=1030 [kg/m^3]
c_p=3490 [J/kg-C]
k=0.456 [W/m-C]
alpha=1.3E-7 [m^2/s]
T_i=2 [C]
T_infinity=95 [C]
h=1200 [W/m^2-C]
T_0=75 [C]
Bi_wall=(h*L)/k
lambda_1_wall=1.5552 "for Bi_wall = 100 from Table 4-2"
A_1_wall=1.2731
r_0=D/2
Bi_cyl=(h*r_0)/k
lambda_1_cyl=2.3455 "for Bi_cyl = 40 from Table 4-2"
A_1_cyl=1.5993
tau_wall=(alpha*t)/L^2
theta_wall=A_1_wall*exp(-lambda_1_wall^2*tau_wall)
tau_cyl=(alpha*t)/r_0^2
theta_cyl=A_1_cyl*exp(-lambda_1_cyl^2*tau_cyl)
theta=theta_wall*theta_cyl
theta=(T_0-T_infinity)/(T_i-T_infinity)

"Some Wrong Solutions with Common Mistakes"
tau_wall_w=(alpha*W1_t)/L^2
theta_wall_w=A_1_wall*exp(-lambda_1_wall^2*tau_wall_w)
theta_wall_w=(T_0-T_infinity)/(T_i-T_infinity) "Considering only large plane wall solution"
tau_cyl_w=(alpha*W2_t)/r_0^2
theta_cyl_w=A_1_wall*exp(-lambda_1_wall^2*tau_cyl_w)
theta_cyl_w=(T_0-T_infinity)/(T_i-T_infinity) "Considering only long cylinder solution"
```

4-172 A potato that may be approximated as a 5.7-cm-diameter solid sphere with the properties ρ = 910 kg/m^3, c_p = 4.25 kJ/kg·ºC, k = 0.68 W/m·ºC, and $\alpha = 1.76\times10^{-7}$ m^2/s. Twelve such potatoes initially at 25ºC are to be cooked by placing them in an oven maintained at 250ºC with a heat transfer coefficient of 95 W/m^2·ºC. The amount of heat transfer to the potatoes by the time the center temperature reaches 100ºC is

(a) 56 kJ (b) 666 kJ (c) 838 kJ (d) 940 kJ (e) 1088 kJ

Answer (b) 666 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.057 [m]
rho=910 [kg/m^3]
c_p=4250 [J/kg-C]
k=0.68 [W/m-C]
alpha=1.76E-7 [m^2/s]
n=12
T_i=25 [C]
T_infinity=250 [C]
h=95 [W/m^2-C]
T_0=100 [C]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 4 are obtained from Table 4-2 of the text as"
lambda_1=2.4556
A_1=1.7202
Theta_0=(T_0-T_infinity)/(T_i-T_infinity)
V=pi*D^3/6
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q=Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3)

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*(1-Theta_0*(sin(lambda_1))/lambda_1) "Using the relation for plane wall"
W3_Q_max=rho*V*c_p*(T_infinity-T_i)
W3_Q=W3_Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3) "Not multiplying with the number of potatoes"
```

4-173 A small chicken (k = 0.45 W/m·K, $\alpha = 0.15\times10^{-6}$ m^2/s) can be approximated as an 11.25-cm-diameter solid sphere. The chicken is initially at a uniform temperature of 8ºC and is to be cooked in an oven maintained at 220ºC with a heat transfer coefficient of 80 W/m^2·K. With this idealization, the temperature at the center of the chicken after a 90-min period is

(a) 25ºC (b) 61ºC (c) 89ºC (d) 122ºC (e) 168ºC

Answer (e) 168ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.45 [W/m-C]
alpha=0.15E-6 [m^2/s]
D=0.1125 [m]
T_i=8 [C]
T_infinity=220 [C]
h=80 [W/m^2-C]
t=90*60 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi number of 10 are obtained from Table 4-2 of the text as"
lambda_1=2.8363
A_1=1.9249
tau=(alpha*t)/r_0^2
(T_0-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

"Some Wrong Solutions with Common Mistakes"
lambda_1a=1.4289
A_1a=1.2620
(W1_T_0-T_infinity)/(T_i-T_infinity)=A_1a*exp(-lambda_1a^2*tau) "Using coefficients for plane wall in Table 4-2"
lambda_1b=2.1795
A_1b=1.5677
(W2_T_0-T_infinity)/(T_i-T_infinity)=A_1b*exp(-lambda_1b^2*tau) "Using coefficients for cylinder in Table 4-2"
```

4-174 A potato may be approximated as a 5.7-cm-diameter solid sphere with the properties ρ = 910 kg/m^3, c_p = 4.25 kJ/kg·K, k = 0.68 W/m·K, and α = 1.76×10^{-7} m^2/s. Twelve such potatoes initially at 25ºC are to be cooked by placing them in an oven maintained at 250ºC with a heat transfer coefficient of 95 W/m^2·K. The amount of heat transfer to the potatoes during a 30-min period is

(a) 77 kJ (b) 483 kJ (c) 927 kJ (d) 970 kJ (e) 1012 kJ

Answer (c) 927 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.057 [m]
rho=910 [kg/m^3]
c_p=4250 [J/kg-C]
k=0.68 [W/m-C]
alpha=1.76E-7 [m^2/s]
n=12
T_i=25 [C]
T_infinity=250 [C]
h=95 [W/m^2-C]
t=30*60 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 4 are obtained from Table 4-2 of the text as"
lambda_1=2.4556
A_1=1.7202
tau=(alpha*t)/r_0^2
Theta_0=A_1*exp(-lambda_1^2*tau)
V=pi*D^3/6
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q=Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3)

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*(1-Theta_0*(sin(lambda_1))/lambda_1) "Using the relation for plane wall"
W2_Q_max=rho*V*c_p*(T_infinity-T_i)
W3_Q=W2_Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3) "Not multiplying with the number of potatoes"
```

4-175 When water, as in a pond or lake, is heated by warm air above it, it remains stable, does not move, and forms a warm layer of water on top of a cold layer. Consider a deep lake (k = 0.6 W/m·K, c_p = 4.179 kJ/kg·K) that is initially at a uniform temperature of 2^{o}C and has its surface temperature suddenly increased to 20^{o}C by a spring weather front. The temperature of the water 1 m below the surface 400 hours after this change is

(a) 2.1^{o}C (b) 4.2^{o}C (c) 6.3^{o}C (d) 8.4^{o}C (e) 10.2^{o}C

Answer (b) 4.2^{o}C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.6 [W/m-C]
c=4179 [J/kg-C]
rho=1000 [kg/m^3]
T_i=2 [C]
T_s=20 [C]
x=1 [m]
time=400*3600 [s]
alpha=k/(rho*c)
xi=x/(2*sqrt(alpha*time))
(T-T_i)/(T_s-T_i)=erfc(xi)
```

4-176 A large chunk of tissue at 35°C with a thermal diffusivity of 1×10^{-7} m^2/s is dropped into iced water. The water is well-stirred so that the surface temperature of the tissue drops to 0°C at time zero and remains at 0°C at all times. The temperature of the tissue after 4 minutes at a depth of 1 cm is

(a) 5°C (b) 30°C (c) 25°C (d) 20°C (e) 10°C

Answer (a) 30°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
X=0.01 [m]
Alpha=1E-7 [m^2/s]
Ti=35 [C]
Ts=0 [C]
time=4*60 [s]
a=0.5*x/sqrt(alpha*time)
b=erfc(a)
(T-Ti)/(Ts-Ti)=b
```

4-177 The 40-cm-thick roof of a large room made of concrete (k = 0.79 W/m·ºC, α = 5.88×10^{-7} m^2/s) is initially at a uniform temperature of 15ºC. After a heavy snow storm, the outer surface of the roof remains covered with snow at -5ºC. The roof temperature at 18.2 cm distance from the outer surface after a period of 2 hours is

(a) 14ºC (b) 12.5ºC (c) 7.8ºC (d) 0ºC (e) -5ºC

Answer (a) 14ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Thickness=0.40 [m]
k=0.79 [W/m-C]
alpha=5.88E-7 [m^2/s]
T_i=15 [C]
T_s=-5 [C]
x=0.182 [m]
time=2*3600 [s]
xi=x/(2*sqrt(alpha*time))
(T-T_i)/(T_s-T_i)=erfc(xi)
```

4-178 ⋯ 4-181 Design and Essay Problems