Solutions Manual

for

Heat and Mass Transfer: Fundamentals & Applications

5th Edition

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McGraw-Hill, 2015

Chapter 3
STEADY HEAT CONDUCTION

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Steady Heat Conduction in Plane Walls

3-1C The temperature distribution in a plane wall will be a straight line during steady and one dimensional heat transfer with constant wall thermal conductivity.

3-2C In steady heat conduction, the rate of heat transfer into the wall is equal to the rate of heat transfer out of it. Also, the temperature at any point in the wall remains constant. Therefore, the energy content of the wall does not change during steady heat conduction. However, the temperature along the wall and thus the energy content of the wall will change during transient conduction.

3-3C The thermal resistance of a medium represents the resistance of that medium against heat transfer.

3-4C Yes. The convection resistance can be defined as the inverse of the convection heat transfer coefficient per unit surface area since it is defined as $R_{conv} = 1/(hA)$.

3-5C Convection heat transfer through the wall is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$. In steady heat transfer, heat transfer rate to the wall and from the wall are equal. Therefore at the outer surface which has convection heat transfer coefficient three times that of the inner surface will experience three times smaller temperature drop compared to the inner surface. Therefore, at the outer surface, the temperature will be closer to the surrounding air temperature.

3-6C The combined heat transfer coefficient represents the combined effects of radiation and convection heat transfers on a surface, and is defined as $h_{\text{combined}} = h_{\text{convection}} + h_{\text{radiation}}$. It offers the convenience of incorporating the effects of radiation in the convection heat transfer coefficient, and to ignore radiation in heat transfer calculations.

3-7C The convection and the radiation resistances at a surface are parallel since both the convection and radiation heat transfers occur simultaneously.

3-8C The temperature of each surface in this case can be determined from

$$\dot{Q} = (T_{\infty 1} - T_{s1})/R_{\infty 1-s1} \longrightarrow T_{s1} = T_{\infty 1} - (\dot{Q}R_{\infty 1-s1})$$
$$\dot{Q} = (T_{s2} - T_{\infty 2})/R_{s2-\infty 2} \longrightarrow T_{s2} = T_{\infty 2} + (\dot{Q}R_{s2-\infty 2})$$

where $R_{\infty-i}$ is the thermal resistance between the environment ∞ and surface i.

3-9C Yes, it is.

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3-10C The blanket will introduce additional resistance to heat transfer and slow down the heat gain of the drink wrapped in a blanket. Therefore, the drink left on a table will warm up faster.

3-11C The new design introduces the thermal resistance of the copper layer in addition to the thermal resistance of the aluminum which has the same value for both designs. Therefore, the new design will be a poorer conductor of heat.

3-12C For a surface of A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, the single equivalent heat transfer coefficient is $h_{eqv} = h_{conv} + h_{rad}$ when the medium and the surrounding surfaces are at the same temperature. Then the equivalent thermal resistance will be $R_{eqv} = 1/(h_{eqv}A)$.

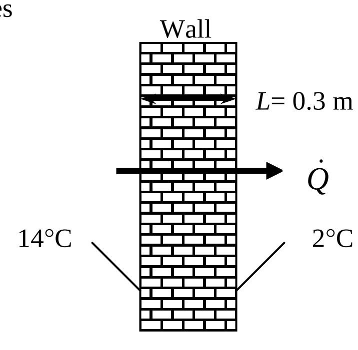
3-13C The thermal resistance network associated with a five-layer composite wall involves five single-layer resistances connected in series.

3-14C Once the rate of heat transfer $\dot{Q}$ is known, the temperature drop across any layer can be determined by multiplying heat transfer rate by the thermal resistance across that layer, $\Delta T_{layer} = \dot{Q}R_{layer}$

3-15C The window glass which consists of two 4 mm thick glass sheets pressed tightly against each other will probably have thermal contact resistance which serves as an additional thermal resistance to heat transfer through window, and thus the heat transfer rate will be smaller relative to the one which consists of a single 8 mm thick glass sheet.

3-16 The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.



Properties The thermal conductivity is given to be k = 0.8 W/m·°C.

Analysis The surface area of the wall and the rate of heat loss through the wall are

$$A = (3\,\text{m}) \times (6\,\text{m}) = 18\,\text{m}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\ \text{W/m}\cdot{}^\circ\text{C})(18\,\text{m}^2)\frac{(14-2)^\circ\text{C}}{0.3\,\text{m}} = \mathbf{576\ W}$$

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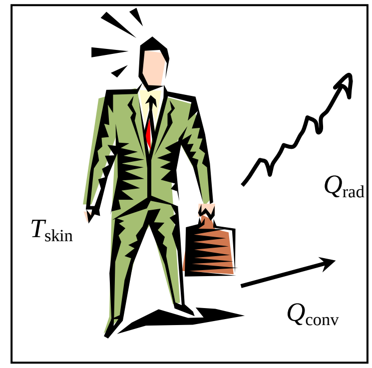
3-17 A person is dissipating heat at a rate of 150 W by natural convection and radiation to the surrounding air and surfaces. For a given deep body temperature, the outer skin temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire exposed surface of the person. **3** The surrounding surfaces are at the same temperature as the indoor air temperature. **4** Heat generation within the 0.5-cm thick outer layer of the tissue is negligible.



Properties The thermal conductivity of the tissue near the skin is given to be k = 0.3 W/m·°C.

Analysis The skin temperature can be determined directly from

$$\dot{Q} = kA\frac{T_1 - T_{skin}}{L}$$

$$T_{skin} = T_1 - \frac{\dot{Q}L}{kA} = 37°\text{C} - \frac{(150\text{ W})(0.005\text{ m})}{(0.3\text{ W/m}\cdot°\text{C})(1.7\text{ m}^2)} = \mathbf{35.5°C}$$

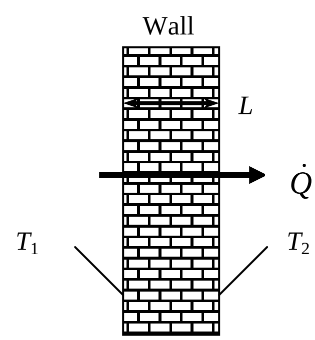
3-18E The inner and outer surfaces of the walls of an electrically heated house remain at specified temperatures during a winter day. The amount of heat lost from the house that day and its cost are to be determined.

Assumptions **1** Heat transfer through the walls is steady since the surface temperatures of the walls remain constant at the specified values during the time period considered. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the walls is constant.

Properties The thermal conductivity of the brick wall is given to be k = 0.40 Btu/h·ft·°F.

Analysis We consider heat loss through the walls only. The total heat transfer area is

$$A = 2(50\times9 + 35\times9) = 1530\text{ ft}^2$$



The rate of heat loss during the daytime is

$$\dot{Q}_{\text{day}} = kA\frac{T_1 - T_2}{L} = (0.40\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(1530\text{ ft}^2)\frac{(55-45)°\text{F}}{1\text{ ft}} = 6120\text{ Btu/h}$$

The rate of heat loss during nighttime is

$$\dot{Q}_{\text{night}} = kA\frac{T_1 - T_2}{L}$$

$$= (0.40\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(1530\text{ ft}^2)\frac{(55-35)°\text{C}}{1\text{ ft}} = 12{,}240\text{ Btu/h}$$

The amount of heat loss from the house that night will be

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow Q = \dot{Q}\Delta t = 10\dot{Q}_{day} + 14\dot{Q}_{night} = (10\text{ h})(6120\text{ Btu/h}) + (14\text{ h})(12{,}240\text{ Btu/h})$$

$$= \mathbf{232{,}560 Btu}$$

Then the cost of this heat loss for that day becomes

$$Cost = (232{,}560/3412\text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$6.13}$$

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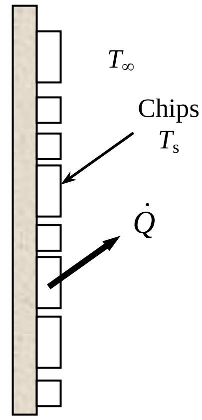
3-19 A circuit board houses 100 chips, each dissipating 0.06 W. The surface heat flux, the surface temperature of the chips, and the thermal resistance between the surface of the board and the cooling medium are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back surface of the board is negligible. **2** Heat is transferred uniformly from the entire front surface.

Analysis (*a*) The heat flux on the surface of the circuit board is

$$A_s = (0.12\,\text{m})(0.18\,\text{m}) = 0.0216\,\text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{(100\times 0.06)\text{ W}}{0.0216\,\text{m}^2} = \mathbf{278\ W/m^2}$$



(*b*) The surface temperature of the chips is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 40°\text{C} + \frac{(100\times 0.06)\text{ W}}{(10\text{ W/m}^2\cdot°\text{C})(0.0216\,\text{m}^2)} = \mathbf{67.8°C}$$

(*c*) The thermal resistance is

$$R_{conv} = \frac{1}{hA_s} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(0.0216\,\text{m}^2)} = \mathbf{4.63°C/W}$$

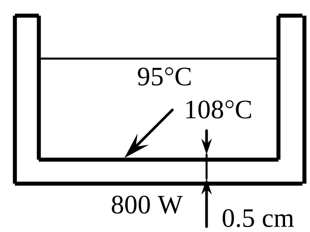
3-20 Heat is transferred steadily to the boiling water in an aluminum pan. The inner surface temperature of the bottom of the pan is given. The boiling heat transfer coefficient and the outer surface temperature of the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the thickness of the bottom of the pan is small relative to its diameter. **3** The thermal conductivity of the pan is constant.

Properties The thermal conductivity of the aluminum pan is given to be k = 237 W/m·°C.

Analysis (*a*) The boiling heat transfer coefficient is

$$A_s = \frac{\pi D^2}{4} = \frac{\pi(0.25\,\text{m})^2}{4} = 0.0491\,\text{m}^2$$



$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{800\text{ W}}{(0.0491\,\text{m}^2)(108-95)°\text{C}} = \mathbf{1254\ W/m^2.°C}$$

(*b*) The outer surface temperature of the bottom of the pan is

$$\dot{Q} = kA\frac{T_{s,outer} - T_{s,inner}}{L}$$

$$T_{s,outer} = T_{s,inner1} + \frac{\dot{Q}L}{kA} = 108°\text{C} + \frac{(800\text{ W})(0.005\,\text{m})}{(237\text{ W/m}\cdot°\text{C})(0.0491\,\text{m}^2)} = \mathbf{108.3°C}$$

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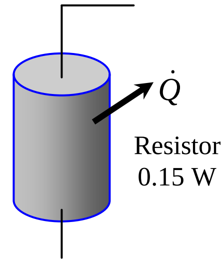
3-21 A cylindrical resistor on a circuit board dissipates 0.15 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the resistor.

Analysis (*a*) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15\ \text{W})(24\ \text{h}) = \mathbf{3.6\ Wh}$$



(*b*) The heat flux on the surface of the resistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.003\ \text{m})^2}{4} + \pi(0.003\ \text{m})(0.012\ \text{m}) = 0.000127\ \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.15\ \text{W}}{0.000127\ \text{m}^2} = \mathbf{1179\ W/m^2}$$

(*c*) The surface temperature of the resistor can be determined from

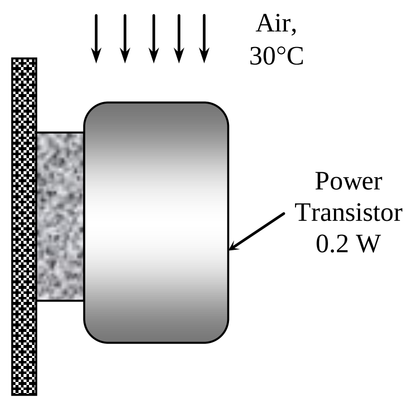
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 40°\text{C} + \frac{0.15\ \text{W}}{(9\ \text{W/m}^2\cdot°\text{C})(0.000127\ \text{m}^2)} = \mathbf{171°C}$$

3-22 A power transistor dissipates 0.2 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the transistor.

Analysis (*a*) The amount of heat this transistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.2\ \text{W})(24\ \text{h}) = 4.8\ \text{Wh} = \mathbf{0.0048\ kWh}$$



(*b*) The heat flux on the surface of the transistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL$$
$$= 2\frac{\pi(0.005\ \text{m})^2}{4} + \pi(0.005\ \text{m})(0.004\ \text{m}) = 0.0001021\ \text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.2\ \text{W}}{0.0001021\ \text{m}^2} = \mathbf{1959\ W/m^2}$$

(*c*) The surface temperature of the transistor can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 30°\text{C} + \frac{0.2\ \text{W}}{(18\ \text{W/m}^2\cdot°\text{C})(0.0001021\ \text{m}^2)} = \mathbf{139°C}$$

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3-23 A double-pane window is considered. The rate of heat loss through the window and the temperature difference across the largest thermal resistance are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer coefficients are constant.

Properties The thermal conductivities of glass and air are given to be 0.78 W/m·K and 0.025 W/m·K, respectively.

Analysis (*a*) The rate of heat transfer through the window is determined to be

$$\dot{Q} = \frac{A\Delta T}{\dfrac{1}{h_i} + \dfrac{L_g}{k_g} + \dfrac{L_a}{k_a} + \dfrac{L_g}{k_g} + \dfrac{1}{h_o}}$$

$$= \frac{(1\times 1.5\ \text{m}^2)[20-(-20)]^\circ\text{C}}{\dfrac{1}{40\ \text{W/m}^2\cdot{}^\circ\text{C}} + \dfrac{0.004\ \text{m}}{0.78\ \text{W/m}\cdot{}^\circ\text{C}} + \dfrac{0.005\ \text{m}}{0.025\ \text{W/m}\cdot{}^\circ\text{C}} + \dfrac{0.004\ \text{m}}{0.78\ \text{W/m}\cdot{}^\circ\text{C}} + \dfrac{1}{20\ \text{W/m}^2\cdot{}^\circ\text{C}}}$$

$$= \frac{(1\times 1.5\ \text{m}^2)[20-(-20)]^\circ\text{C}}{0.025 + 0.000513 + 0.2 + 0.000513 + 0.05} = \mathbf{210\ W}$$

(*b*) Noting that the largest resistance is through the air gap, the temperature difference across the air gap is determined from

$$\Delta T_a = \dot{Q}R_a = \dot{Q}\frac{L_a}{k_a A} = (210\ \text{W})\frac{0.005\ \text{m}}{(0.025\ \text{W/m}\cdot{}^\circ\text{C})(1\times 1.5\ \text{m}^2)} = \mathbf{28^\circ C}$$

3-24 The two surfaces of a window are maintained at specified temperatures. The rate of heat loss through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k = 0.78 W/m·°C.

Analysis The area of the window and the individual resistances are

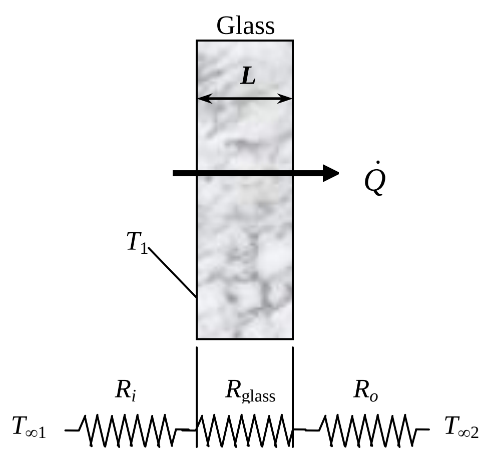
$$A = (1.2\ \text{m})\times(2.0\ \text{m}) = 2.4\ \text{m}^2$$

$$R_\text{i} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2.^\circ\text{C})(2.4\ \text{m}^2)} = 0.04167\ ^\circ\text{C/W}$$

$$R_\text{glass} = \frac{L}{k_1 A} = \frac{0.006\ \text{m}}{(0.78\ \text{W/m.}^\circ\text{C})(2.4\ \text{m}^2)} = 0.00321\ ^\circ\text{C/W}$$

$$R_\text{o} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\ \text{W/m}^2.^\circ\text{C})(2.4\ \text{m}^2)} = 0.01667\ ^\circ\text{C/W}$$

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2}$$
$$= 0.04167 + 0.00321 + 0.01667 = 0.06155\ ^\circ\text{C/W}$$



The steady rate of heat transfer through window glass is then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24-(-5)]^\circ\text{C}}{0.06155\ ^\circ\text{C/W}} = \mathbf{471 W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\text{C} - (471\ \text{W})(0.04167\ ^\circ\text{C/W}) = \mathbf{4.4^\circ C}$$

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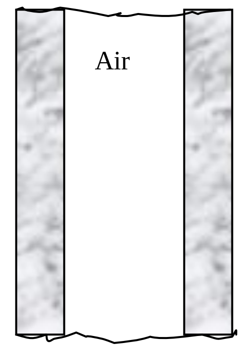
3-25 A double-pane window consists of two layers of glass separated by a stagnant air space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.



Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass and air are given to be k_{glass} = 0.78 W/m·°C and k_{air} = 0.026 W/m·°C.

Analysis The area of the window and the individual resistances are

$$T_{\infty 1} \quad R_i \quad R_1 \quad R_2 \quad R_3 \quad R_o \quad T_{\infty 2}$$

$$A = (1.2\text{ m}) \times (2\text{ m}) = 2.4\text{ m}^2$$

$$R_{\text{i}} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.°\text{C})(2.4\text{ m}^2)} = 0.04167\text{ °C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m.°C})(2.4\text{ m}^2)} = 0.00160\text{ °C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\text{ m}}{(0.026\text{ W/m.°C})(2.4\text{ m}^2)} = 0.19231\text{ °C/W}$$

$$R_{\text{o}} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^{\text{o}}\text{C})(2.4\text{ m}^2)} = 0.01667\text{ °C/W}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.04167 + 2(0.00160) + 0.19231 + 0.01667 \\ &= 0.25385\text{ °C/W} \end{aligned}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24 - (-5)]°\text{C}}{0.25385°\text{C/W}} = \mathbf{114\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24°\text{C} - (114\text{ W})(0.04167°\text{C/W}) = \mathbf{19.2°C}$$

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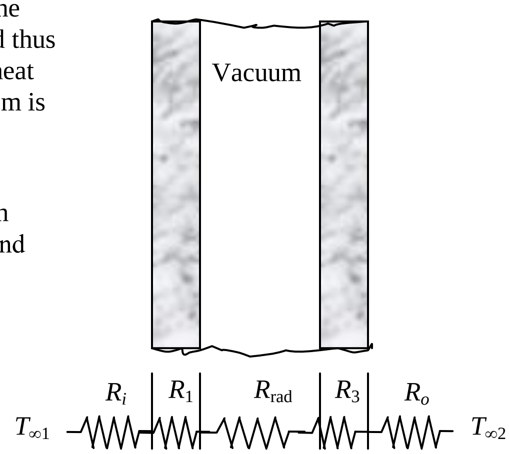
3-26 A double-pane window consists of two layers of glass separated by an evacuated space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the glass is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.78 W/m·°C.

Analysis Heat cannot be conducted through an evacuated space since the thermal conductivity of vacuum is zero (no medium to conduct heat) and thus its thermal resistance is zero. Therefore, if radiation is disregarded, the heat transfer through the window will be zero. Then the answer of this problem is **zero** since the problem states to disregard radiation.



Discussion In reality, heat will be transferred between the glasses by radiation. We do not know the inner surface temperatures of windows. In order to determine radiation heat resistance we assume them to be 5°C and 15°C, respectively, and take the emissivity to be 1. Then individual resistances are

$$A = (1.2\text{ m})\times(2\text{ m}) = 2.4\text{ m}^2$$

$$R_{\text{i}} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.°\text{C})(2.4\text{ m}^2)} = 0.04167\text{ °C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.003\text{ m}}{(0.78\text{ W/m.°C})(2.4\text{ m}^2)} = 0.00160\text{ °C/W}$$

$$\begin{aligned} R_{rad} &= \frac{1}{\varepsilon\sigma A(T_s{}^2 + T_{surr}{}^2)(T_s + T_{surr})} \\ &= \frac{1}{1(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(2.4\text{ m}^2)[288^2 + 278^2][288+278]K^3} \\ &= 0.08103\text{ °C/W} \end{aligned}$$

$$R_{\text{o}} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^{\text{o}}\text{C})(2.4\text{ m}^2)} = 0.01667\ ^{\text{o}}\text{C/W}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + 2R_1 + R_{rad} + R_{conv,2} = 0.04167 + 2(0.00160) + 0.08103 + 0.01667 \\ &= 0.14257\text{ °C/W} \end{aligned}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24-(-5)]°\text{C}}{0.14257°\text{C/W}} = \mathbf{203\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24°\text{C} - (203\text{ W})(0.04167°\text{C/W}) = \mathbf{15.5°C}$$

Similarly, the inner surface temperatures of the glasses are calculated to be 15.2 and -1.6°C (we had assumed them to be 15 and 5°C when determining the radiation resistance). We can improve the result obtained by reevaluating the radiation resistance and repeating the calculations.

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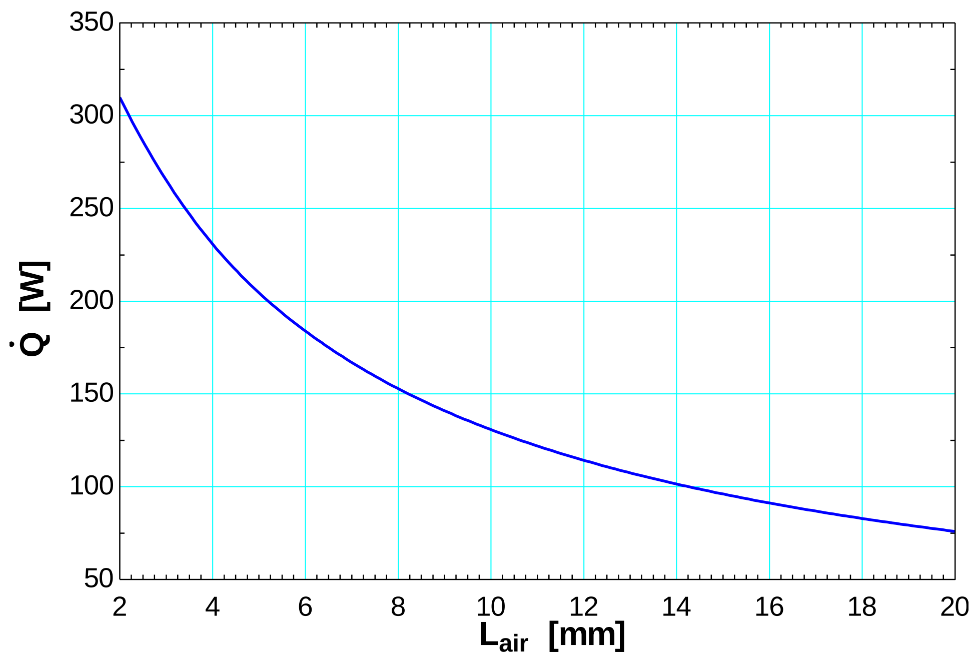
3-27 Prob. 3-26 is reconsidered. The rate of heat transfer through the window as a function of the width of air space is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
A=1.2*2 [m^2]
L_glass=3 [mm]
k_glass=0.78 [W/m-C]
L_air=12 [mm]
k_air=0.026 [W/m-C]
T_infinity_1=24 [C]
T_infinity_2=-5 [C]
h_1=10 [W/m^2-C]
h_2=25 [W/m^2-C]

"ANALYSIS"
R_conv_1=1/(h_1*A)
R_glass=(L_glass*Convert(mm, m))/(k_glass*A)
R_air=(L_air*Convert(mm, m))/(k_air*A)
R_conv_2=1/(h_2*A)
R_total=R_conv_1+2*R_glass+R_air+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
T_1=T_infinity_1-Q_dot*R_conv_1
```

L_{air} [mm]	Q [W]
2	309.9
4	230.8
6	183.9
8	152.8
10	130.8
12	114.2
14	101.4
16	91.21
18	82.86
20	75.91



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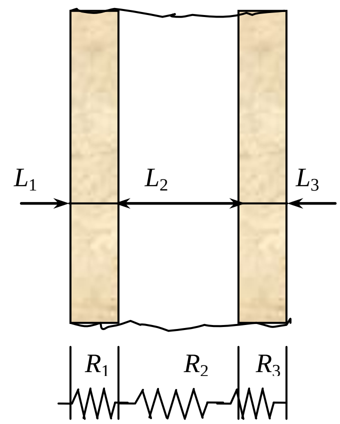
3-28E A wall is constructed of two layers of sheetrock with fiberglass insulation in between. The thermal resistance of the wall and its R-value of insulation are to be determined.

Assumptions **1** Heat transfer through the wall is one-dimensional. **2** Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k_{\text{sheetrock}}$ = 0.10 Btu/h·ft·°F and $k_{\text{insulation}}$ = 0.020 Btu/h·ft·°F.



Analysis (*a*) The surface area of the wall is not given and thus we consider a unit surface area (A = 1 ft^2). Then the *R*-value of insulation of the wall becomes equivalent to its thermal resistance, which is determined from.

$$R_{sheetrock} = R_1 = R_3 = \frac{L_1}{k_1} = \frac{0.7/12\text{ ft}}{(0.10\text{ Btu/h.ft.°F})} = 0.5833\text{ ft}^2\text{.°F.h/Btu}$$

$$R_{fiberglass} = R_2 = \frac{L_2}{k_2} = \frac{7/12\text{ ft}}{(0.020\text{ Btu/h.ft.°F})} = 29.17\text{ ft}^2\text{.°F.h/Btu}$$

$$R_{total} = 2R_1 + R_2 = 2\times 0.5833 + 29.17 = \mathbf{30.34\ ft^2.°F.h/Btu}$$

(*b*) Therefore, this is approximately a ***R*-30** wall in English units.

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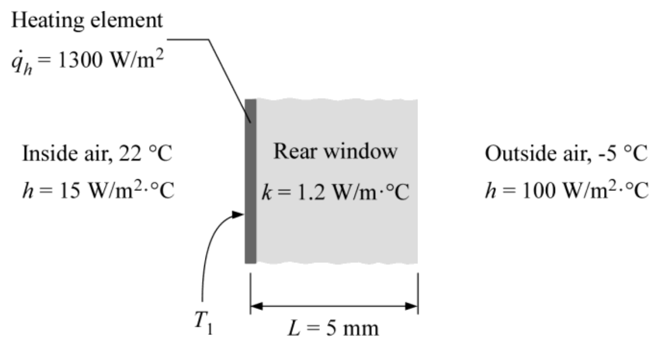
3-29 A very thin transparent heating element is attached to the inner surface of an automobile window for defogging purposes, the inside surface temperature of the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** Thermal resistance of the thin heating element is negligible.

Properties Thermal conductivity of the window is given to be k = 1.2 W/m · °C.



Analysis The thermal resistances are

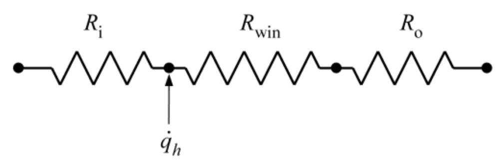
$$R_i = \frac{1}{h_i A} \qquad R_o = \frac{1}{h_o A} \qquad \text{and} \qquad R_{\text{win}} = \frac{L}{kA}$$

From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_{\infty,i} - T_1}{R_i} + \dot{q}_h A = \frac{T_1 - T_{\infty,o}}{R_{\text{win}} + R_o}$$

or

$$\frac{T_{\infty,i} - T_1}{1/(h_i A)} + \dot{q}_h A = \frac{T_1 - T_{\infty,o}}{L/(kA) + 1/(h_o A)}$$



$$\frac{T_{\infty,i} - T_1}{1/h_i} + \dot{q}_h = \frac{T_1 - T_{\infty,o}}{L/k + 1/h_o}$$

$$\frac{22\,°\text{C} - T_1}{1/15\ \text{W/m}^2\cdot°\text{C}} + 1300\ \text{W/m}^2 = \frac{T_1 - (-5\,°\text{C})}{(0.005\,\text{m}/1.2\ \text{W/m}\cdot°\text{C}) + (1/100\ \text{W/m}^2\cdot°\text{C})}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(22-T_1)/(1/15)+1300=(T_1-(-5))/(0.005/1.2+1/100)
```

Solving by EES software, the inside surface temperature of the window is

$$T_1 = \mathbf{14.9\,°C}$$

Discussion In actuality, the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. To maintain the inner surface temperature of the window, it is necessary to vary the heat flux to the heating element according to the outside condition.

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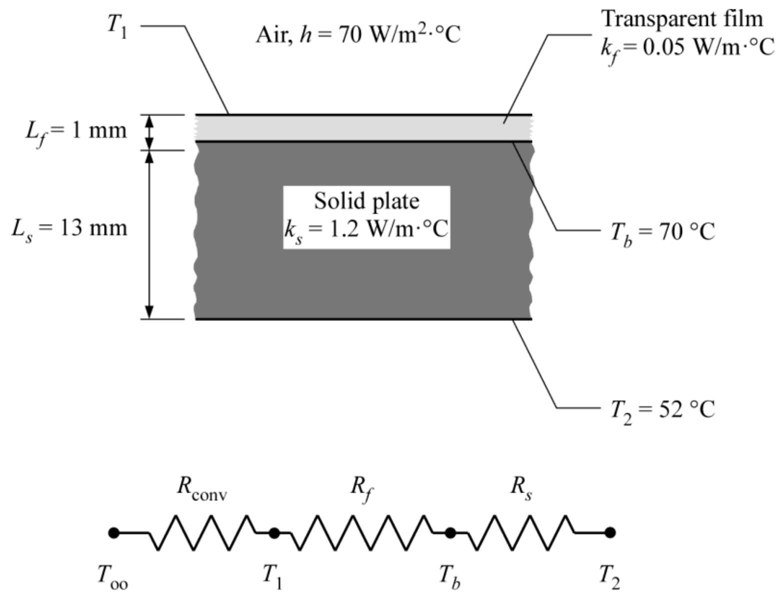
3-30 A process of bonding a transparent film onto a solid plate is taking place inside a heated chamber. The temperatures inside the heated chamber and on the transparent film surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** Thermal contact resistance is negligible.

Properties The thermal conductivities of the transparent film and the solid plate are given to be 0.05 W/m · °C and 1.2 W/m · °C, respectively.



Analysis The thermal resistances are

$$R_{\text{conv}} = \frac{1}{hA}$$

$$R_f = \frac{L_f}{k_f A}$$

and $$R_s = \frac{L_s}{k_s A}$$

Using the thermal resistance concept, the following equation is expressed:

$$\frac{T_\infty - T_b}{R_{\text{conv}} + R_f} = \frac{T_b - T_2}{R_s}$$

Rearranging and solving for the temperature inside the chamber yields

$$T_\infty = \frac{T_b - T_2}{R_s}\left(R_{\text{conv}} + R_f\right) + T_b = \frac{T_b - T_2}{L_s / k_s}\left(\frac{1}{h} + \frac{L_f}{k_f}\right) + T_b$$

$$T_\infty = \frac{(70-52)\ ^\circ\text{C}}{0.013\ \text{m}/1.2\ \text{W/m}\cdot{}^\circ\text{C}}\left(\frac{1}{70\ \text{W/m}^2\cdot{}^\circ\text{C}} + \frac{0.001\ \text{m}}{0.05\ \text{W/m}\cdot{}^\circ\text{C}}\right) + 70\ ^\circ\text{C} = \mathbf{127\ ^\circ C}$$

The surface temperature of the transparent film is

$$\frac{T_1 - T_b}{R_f} = \frac{T_b - T_2}{R_s}$$

$$T_1 = \frac{T_b - T_2}{R_s} R_f + T_b = \frac{T_b - T_2}{L_s / k_s}\left(\frac{L_f}{k_f}\right) + T_b$$

$$T_1 = \frac{(70-52)\ ^\circ\text{C}}{0.013\ \text{m}/1.2\ \text{W/m}\cdot{}^\circ\text{C}}\left(\frac{0.001\ \text{m}}{0.05\ \text{W/m}\cdot{}^\circ\text{C}}\right) + 70\ ^\circ\text{C} = \mathbf{103\ ^\circ C}$$

Discussion If a thicker transparent film were to be bonded on the solid plate, then the inside temperature of the heated chamber would have to be higher to maintain the temperature of the bond at 70 °C.

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3-31 Warm air blowing over the inner surface of an automobile windshield is used for defrosting ice accumulated on the outer surface. The convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield necessary to cause the accumulated ice to begin melting is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the windshield is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** The automobile is operating at 1 atm.

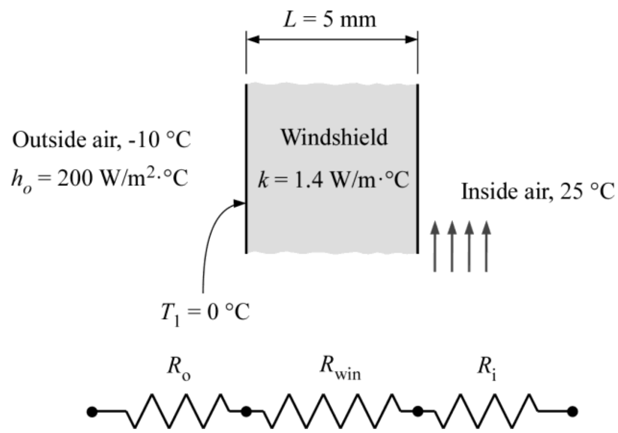
Properties Thermal conductivity of the windshield is given to be k = 1.4 W/m · °C.

Analysis The thermal resistances are

$$R_i = \frac{1}{h_i A}$$

$$R_o = \frac{1}{h_o A}$$

and $$R_{\text{win}} = \frac{L}{kA}$$



From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_{\infty,o} - T_1}{R_o} = \frac{T_1 - T_{\infty,i}}{R_{\text{win}} + R_i}$$

$$R_i = \frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1} R_o - R_{\text{win}}$$

or $$\frac{1}{h_i} = \frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1}\left(\frac{1}{h_o}\right) - \frac{L}{k}$$

For the ice to begin melting, the outer surface temperature of the windshield (T_1) should be at least 0 °C. The convection heat transfer coefficient for the warm air is

$$h_i = \left[\frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1}\left(\frac{1}{h_o}\right) - \frac{L}{k}\right]^{-1}$$

$$= \left[\frac{(0-25)\,°\text{C}}{(-10-0)\,°\text{C}}\left(\frac{1}{200\ \text{W/m}^2\cdot°\text{C}}\right) - \frac{0.005\ \text{m}}{1.4\ \text{W/m}\cdot°\text{C}}\right]^{-1}$$

$$= \mathbf{112\ W/m^2\cdot°C}$$

Discussion In practical situations, the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. Therefore the convection heat transfer coefficient of the warm air necessary to melt the ice should be varied as well. This is done by adjusting the warm air flow rate and temperature.

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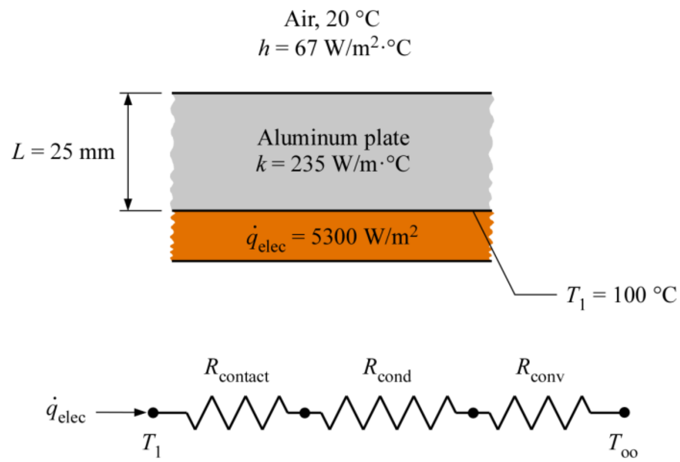
3-32 The thermal contact conductance for an aluminum plate attached on a copper plate, that is heated electrically, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the aluminum plate is given to be 235 W/m · °C.

Analysis The thermal resistances are



$$R_{\text{cond}} = \frac{L}{kA}$$

and $$R_{\text{conv}} = \frac{1}{hA}$$

From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\dot{q}_{\text{elec}} A = \frac{T_1 - T_\infty}{R_c / A + R_{\text{cond}} + R_{\text{conv}}}$$

or $$\dot{q}_{\text{elec}} A = \frac{T_1 - T_\infty}{R_c / A + L/(kA) + 1/(hA)}$$

Rearranging the equation and solving for the contact resistance yields

$$R_c = \frac{T_1 - T_\infty}{\dot{q}_{\text{elec}}} - \frac{L}{k} - \frac{1}{h}$$
$$= \frac{(100-20)\ {}^\circ\text{C}}{5300\ \text{W/m}^2} - \frac{0.025\ \text{m}}{235\ \text{W/m}\cdot{}^\circ\text{C}} - \frac{1}{67\ \text{W/m}^2\cdot{}^\circ\text{C}} = 6.258\times10^{-5}\ \text{m}^2\cdot{}^\circ\text{C/W}$$

The thermal contact conductance is

$$h_c = 1/R_c = \mathbf{16000\ W/m^2\cdot{}^\circ C}$$

Discussion By comparing the value of the thermal contact conductance with the values listed in Table 3-2, the surface conditions of the plates appear to be milled.

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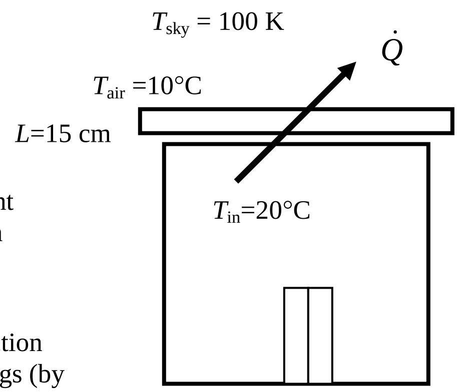
3-33 The roof of a house with a gas furnace consists of a concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.



Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be k = 2 W/m·°C. The emissivity of both surfaces of the roof is given to be 0.9.

Analysis When the surrounding surface temperature is different than the ambient temperature, the thermal resistances network approach becomes cumbersome in problems that involve radiation. Therefore, we will use a different but intuitive approach.

In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}{}^4 - T_{s,in}{}^4) = (5\text{ W/m}^2\cdot°\text{C})(300\text{ m}^2)(20 - T_{s,in})°\text{C}$$
$$+(0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(20+273\text{ K})^4 - (T_{s,in}+273\text{ K})^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\text{ W/m}\cdot°\text{C})(300\text{ m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\text{ m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4) = (12\text{ W/m}^2\cdot°\text{C})(300\text{ m}^2)(T_{s,out} - 10)°\text{C}$$
$$+(0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(T_{s,out}+273\text{ K})^4 - (100\text{ K})^4\right]$$

Solving the equations above simultaneously gives

$$\dot{Q} = \mathbf{37{,}440\ W},\ T_{s,in} = \mathbf{7.3°C},\text{ and } T_{s,\text{out}} = -2.1°\text{C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\text{ kJ/s})(14\times3600\text{ s})}{0.80}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 22.36\text{ therms}$$

Finally, the money lost through the roof during that period is

$$\text{Money lost} = (22.36\text{ therms})(\$1.20/\text{therm}) = \mathbf{\$26.8}$$

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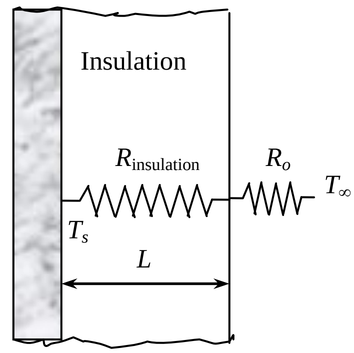
3-34 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficient accounts for the radiation effects.

Properties The thermal conductivity of the glass wool insulation is given to be $k = 0.038$ W/m·°C.

Analysis The rate of heat transfer without insulation is



$$A = (2\text{ m})(1.5\text{ m}) = 3\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2\cdot{}^\circ\text{C})(3\text{ m}^2)(80-30)^\circ\text{C} = 1500\text{ W}$$

In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 1500\text{ W} = 150\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(80-30)^\circ\text{C}}{150\text{ W}} = 0.3333\,^\circ\text{C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{\text{conv}} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2\cdot{}^\circ\text{C})(3\text{ m}^2)} + \frac{L}{(0.038\text{ W/m.}^\circ\text{C})(3\text{ m}^2)} = 0.3333\,^\circ\text{C/W}$$

$$L = 0.0342\text{ m} = \mathbf{3.42\,cm}$$

Noting that heat is saved at a rate of 0.9×1500 =1350 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(1.350\text{ kJ/s})(8760\text{ h})}{0.78}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 517.4\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (517.4\text{ therms})(\$1.10/\text{therm}) = \$569.1\,(\text{per year})$$

The insulation will pay for its cost of \$250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$569.1/\text{yr}} = \mathbf{0.439\,yr}$$

which is equal to 5.3 months.

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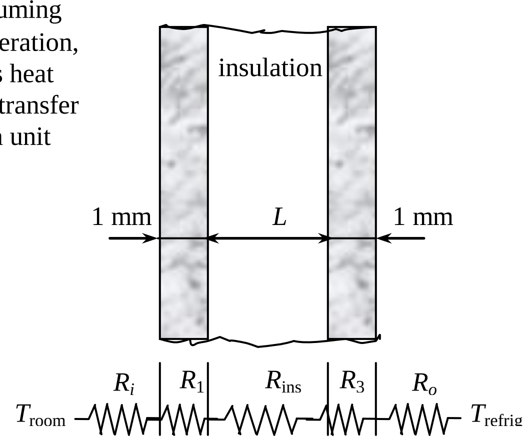
3-35 The wall of a refrigerator is constructed of fiberglass insulation sandwiched between two layers of sheet metal. The minimum thickness of insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces is to be determined.

Assumptions **1** Heat transfer through the refrigerator walls is steady since the temperatures of the food compartment and the kitchen air remain constant at the specified values. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation effects.

Properties The thermal conductivities are given to be k = 15.1 W/m·°C for sheet metal and 0.035 W/m·°C for fiberglass insulation.

Analysis The minimum thickness of insulation can be determined by assuming the outer surface temperature of the refrigerator to be 20°C. In steady operation, the rate of heat transfer through the refrigerator wall is constant, and thus heat transfer between the room and the refrigerated space is equal to the heat transfer between the room and the outer surface of the refrigerator. Considering a unit surface area,



$$\dot{Q} = h_o A(T_{room} - T_{s,out})$$
$$= (9\ \text{W/m}^2 \cdot °\text{C})(1\ \text{m}^2)(25 - 20)°\text{C} = 45\ \text{W}$$

Using the thermal resistance network, heat transfer between the room and the refrigerated space can be expressed as

$$\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}}$$

$$\dot{Q}/A = \frac{T_{room} - T_{refrig}}{\dfrac{1}{h_o} + 2\left(\dfrac{L}{k}\right)_{metal} + \left(\dfrac{L}{k}\right)_{insulation} + \dfrac{1}{h_i}}$$

Substituting,

$$45\ \text{W/m}^2 = \frac{(25-3)°\text{C}}{\dfrac{1}{9\ \text{W/m}^2 \cdot °\text{C}} + \dfrac{2 \times 0.001\ \text{m}}{15.1\ \text{W/m}^2 \cdot °\text{C}} + \dfrac{L}{0.035\ \text{W/m}^2 \cdot °\text{C}} + \dfrac{1}{4\ \text{W/m}^2 \cdot °\text{C}}}$$

Solv ing for L, the minimum thickness of insulation is determined to be

L = 0.004468 m = **0.447 cm**

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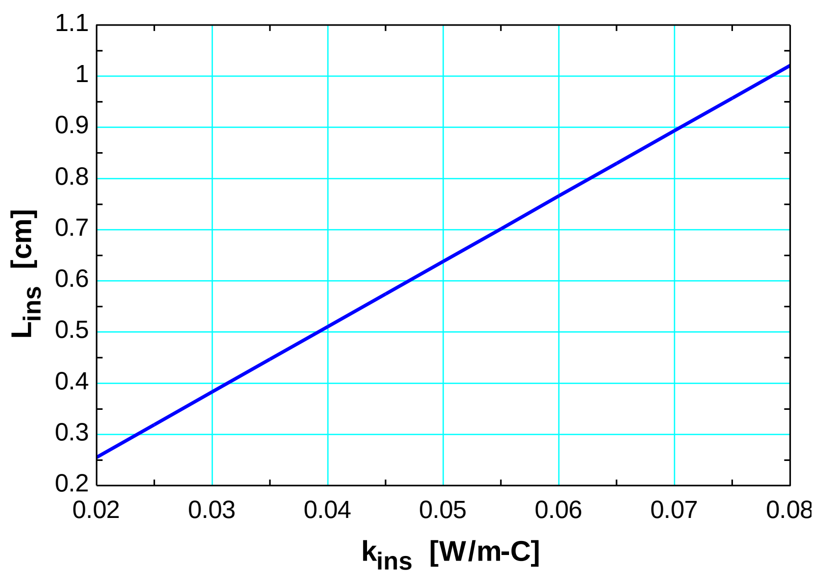
3-36 Prob. 3-35 is reconsidered. The effects of the thermal conductivities of the insulation material and the sheet metal on the thickness of the insulation is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k_ins=0.035 [W/m-C]
L_metal=0.001 [m]
k_metal=15.1 [W/m-C]
T_refrig=2 [C]
T_kitchen=24 [C]
h_i=4 [W/m^2-C]
h_o=9 [W/m^2-C]
T_s_out=20 [C]

"ANALYSIS"
A=1 [m^2] “a unit surface area is considered"
Q_dot=h_o*A*(T_kitchen-T_s_out)
Q_dot=(T_kitchen-T_refrig)/R_total
R_total=R_conv_i+2*R_metal+R_ins+R_conv_o
R_conv_i=1/(h_i*A)
R_metal=L_metal/(k_metal*A)
R_ins=(L_ins*Convert(cm, m))/(k_ins*A) "L_ins is in cm"
R_conv_o=1/(h_o*A)
```

k_{ins} [W/m.C]	L_{ins} [cm]
0.02	0.2553
0.025	0.3191
0.03	0.3829
0.035	0.4468
0.04	0.5106
0.045	0.5744
0.05	0.6382
0.055	0.702
0.06	0.7659
0.065	0.8297
0.07	0.8935
0.075	0.9573
0.08	1.021



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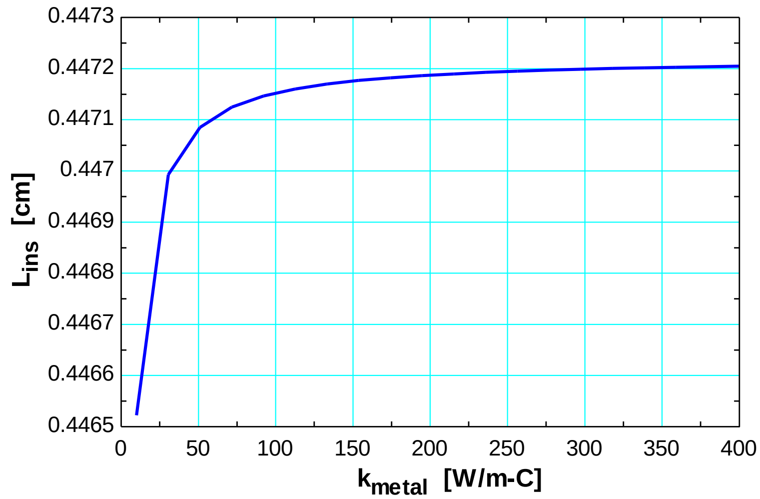
k_{metal} [W/m.C]	L_{ins} [cm]
10	0.4465
30.53	0.447
51.05	0.4471
71.58	0.4471
92.11	0.4471
112.6	0.4472
133.2	0.4472
153.7	0.4472
174.2	0.4472
194.7	0.4472
215.3	0.4472
235.8	0.4472
256.3	0.4472
276.8	0.4472
297.4	0.4472
317.9	0.4472
338.4	0.4472
358.9	0.4472
379.5	0.4472
400	0.4472



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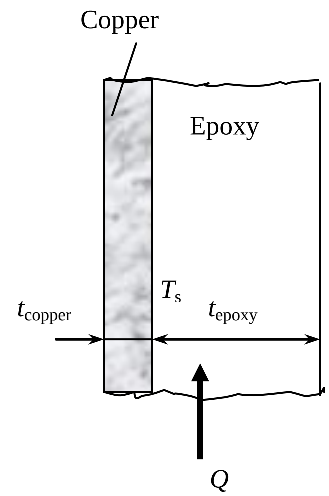
3-37 Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.



Properties The thermal conductivities are given to be k = 386 W/m·°C for copper and 0.26 W/m·°C for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}}$$
$$= \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right] w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thickness $t = t_{copper} + t_{epoxy}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}\,(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}\,(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386\text{ W/m}\cdot{}^\circ\text{C})(0.0001\text{ m}) = 0.0386\text{ W/}^\circ\text{C}$$
$$(kt)_{epoxy} = (0.26\text{ W/m}\cdot{}^\circ\text{C})(0.0012\text{ m}) = 0.000312\text{ W/}^\circ\text{C}$$
$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912\text{ W/}^\circ\text{C}$$
$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$
$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

and

$$k_{eff} = \frac{(386\times0.0001 + 0.26\times0.0012)\text{ W/}^\circ\text{C}}{(0.0001 + 0.0012)\text{ m}} = \mathbf{29.9\text{ W/m.}^\circ\text{C}}$$

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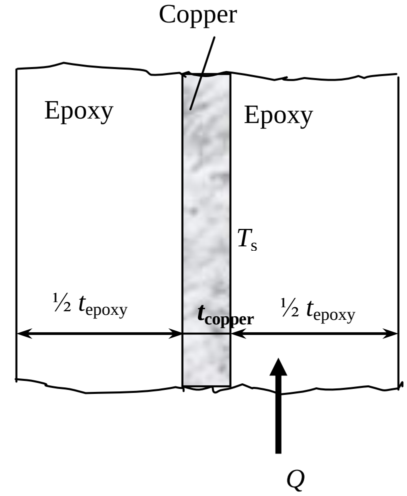
3-38E A thin copper plate is sandwiched between two layers of epoxy boards. The effective thermal conductivity of the board along its 9 in long side and the fraction of the heat conducted through copper along that side are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces are disregarded **3** Thermal conductivities are constant.



Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper and 0.15 Btu/h·ft·°F for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer plate can be expressed as (we treat the two layers of epoxy as a single layer that is twice as thick)

$$\begin{aligned}\dot{Q} &= \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} \\ &= \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}\end{aligned}$$

Heat conduction along an "equivalent" plate of thick ness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}\,(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}\,(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fraction of heat conducted along the copper layer and the effective thermal conductivity of the plate are determined to be

$$\begin{aligned}(kt)_{\text{copper}} &= (223\text{ Btu/h.ft.°F})(0.03/12\text{ ft}) = 0.5575\text{ Btu/h.°F} \\ (kt)_{\text{epoxy}} &= 2(0.15\text{ Btu/h.ft.°F})(0.15/12\text{ ft}) = 0.00375\text{ Btu/h.°F} \\ (kt)_{\text{total}} &= (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = 0.5575 + 0.00375 = 0.56125\text{ Btu/h.°F}\end{aligned}$$

and

$$\begin{aligned}k_{eff} &= \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{\text{t}_{\text{copper}} + t_{\text{epoxy}}} \\ &= \frac{0.56125\text{ Btu/h.°F}}{[(0.03/12) + 2(0.15/12)]\text{ ft}} = \mathbf{20.41\,Btu/h.ft.°F}\end{aligned}$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.5575}{0.56125} = 0.993 = \mathbf{99.3\%}$$

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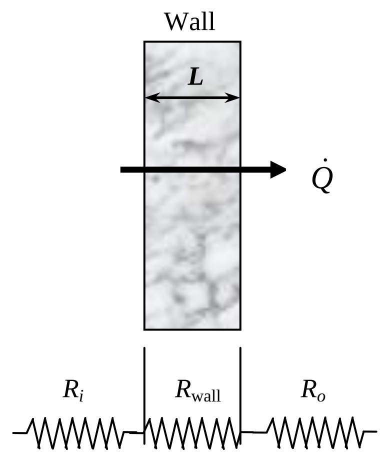
3-39 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \mathrm{W/m^2\cdot {}^\circ C})(10\times 4\ \mathrm{m^2})} = 0.003571\ \mathrm{{}^\circ C/W}$$

$$R_{\rm wall} = \frac{L_{\rm wall}}{kA} = \frac{R-value}{A} = \frac{2.31\ \mathrm{m^2\cdot {}^\circ C/W}}{(10\times 4\ \mathrm{m^2})} = 0.05775\ \mathrm{{}^\circ C/W}$$

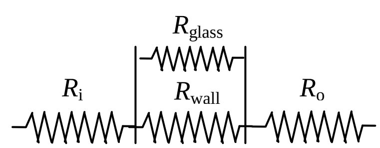
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \mathrm{W/m^2\cdot {}^\circ C})(10\times 4\ \mathrm{m^2})} = 0.001389\mathrm{{}^\circ C/W}$$

$$R_{\rm total} = R_i + R_{\rm wall} + R_o = 0.003571 + 0.05775 + 0.001389 = 0.06271\ \mathrm{{}^\circ C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(24-8)\mathrm{{}^\circ C}}{0.06271\mathrm{{}^\circ C/W}} = \mathbf{255.1\ W}$$

Wall with single pane windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \mathrm{W/m^2\cdot {}^\circ C})(20\times 4\ \mathrm{m^2})} = 0.001786\ \mathrm{{}^\circ C/W}$$

$$R_{\rm wall} = \frac{L_{\rm wall}}{kA} = \frac{R-value}{A} = \frac{2.31\ \mathrm{m^2\cdot {}^\circ C/W}}{(20\times 4) - 5(1.2\times 1.8)\ \mathrm{m^2}} = 0.033382\ \mathrm{{}^\circ C/W}$$

$$R_{\rm glass} = \frac{L_{\rm glass}}{kA} = \frac{0.005\ \mathrm{m}}{(0.78\ \mathrm{W/m^2\cdot {}^o\ C})(1.2\times 1.8)\mathrm{m^2}} = 0.002968\ \mathrm{{}^\circ C/W}$$

$$\frac{1}{R_{\rm eqv}} = \frac{1}{R_{\rm wall}} + 5\frac{1}{R_{\rm glass}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.000583\ \mathrm{{}^oC/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \mathrm{W/m^2\cdot {}^\circ C})(20\times 4\ \mathrm{m^2})} = 0.000694\ \mathrm{{}^\circ C/W}$$

$$R_{\rm total} = R_i + R_{\rm eqv} + R_o = 0.001786 + 0.000583 + 0.000694 = 0.003063\ \mathrm{{}^\circ C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\rm total}} = \frac{(24-8)\mathrm{{}^\circ C}}{0.003063\mathrm{{}^\circ C/W}} = \mathbf{5224\ W}$$

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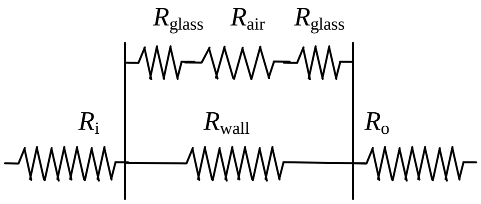
4th wall with double pane windows:



$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\cdot\text{°C/W}}{(20\times 4)-5(1.2\times 1.8)\text{m}^2} = 0.033382\,\text{°C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\text{ W/m}^2\cdot\text{°C})(1.2\times 1.8)\text{m}^2} = 0.002968\,\text{°C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\,\text{m}}{(0.026\text{ W/m}^2\cdot{}^{\text{o}}\text{ C})(1.2\times 1.8)\text{m}^2} = 0.267094\,\text{°C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303\,\text{°C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717\,\text{°C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000694 = 0.023197\,\text{°C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(24-8)\text{°C}}{0.023197\text{°C/W}} = \mathbf{690\ W}$$

The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\ \text{pane}}} - \dot{Q}_{\substack{\text{double}\\ \text{pane}}} = 5224 - 690 = 4534\text{ W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

$$Q_{save} = \dot{Q}_{save}\Delta t = (4.534\,\text{kW})(7\times 30\times 24\,\text{h}) = 22{,}851\,\text{kWh}$$

Money savings = (Energy saved)(Unit cost of energy) = (22,851 kWh)($0.08/kWh) = **$1828**

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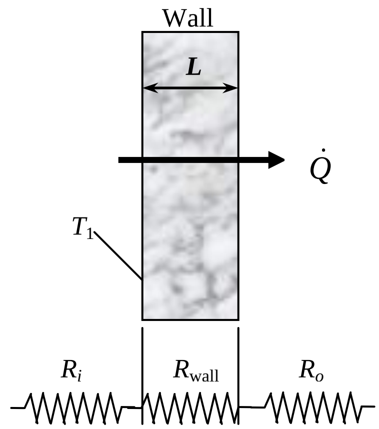
3-40E Two of the walls of a house have no windows while the other two walls have 4 windows each. The ratio of heat transfer through the walls with and without windows is to be determined.

Assumptions **1** Heat transfer through the walls and the windows is steady and one-dimensional. **2** Thermal conductivity of each wall is constant. **3** Any direct radiation gain or loss through the windows is negligible. **4** Heat transfer coefficients are constant and uniform over the entire surface.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.45 Btu/h·ft·°F. The R-value of the wall is given to be 19 h·ft^2·°F/Btu.

Analysis The thermal resistances through the wall without windows are



$$A = (12\text{ ft})(40\text{ ft}) = 480\text{ ft}^2$$

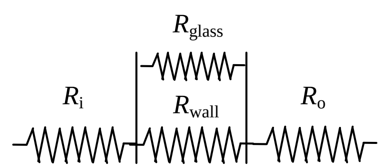
$$R_i = \frac{1}{h_i A} = \frac{1}{(2\text{ Btu/h.ft}^2\cdot°\text{F})(480\text{ ft}^2)} = 0.0010417\text{ h}\cdot°\text{F/Btu}$$

$$R_{\text{wall}} = \frac{L}{kA} = \frac{19\text{ h}\cdot\text{ft}^2\,°\text{F/Btu}}{480\text{ ft}^2} = 0.03958\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(480\text{ ft}^2)} = 0.00052\text{ h}\cdot°\text{F/Btu}$$

$$R_{total,1} = R_i + R_{wall} + R_o$$
$$= 0.0010417 + 0.03958 + 0.00052 = 0.0411417\text{ h}\cdot°\text{F/Btu}$$

The thermal resistances through the wall with windows are



$$A_{windows} = 4(3\times 5) = 60\text{ ft}^2$$

$$A_{wall} = A_{total} - A_{windows} = 480 - 60 = 420\text{ ft}^2$$

$$R_2 = R_{glass} = \frac{L}{kA} = \frac{0.25/12\text{ ft}}{(0.45\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(60\text{ ft}^2)} = 0.0007716\text{ h}\cdot°\text{F/Btu}$$

$$R_4 = R_{wall} = \frac{L}{kA} = \frac{19\text{ h}\cdot\text{ft}^2\cdot°\text{F/Btu}}{(420\text{ ft}^2)} = 0.04524\text{ h}\cdot°\text{F/Btu}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{glass}} + \frac{1}{R_{wall}} = \frac{1}{0.0007716} + \frac{1}{0.04524} \longrightarrow R_{eqv} = 0.00076\cdot\text{h°F/Btu}$$

$$R_{total,2} = R_i + R_{eqv} + R_o = 0.001047 + 0.00076 + 0.00052 = 0.002327\text{ h}\cdot°\text{F/Btu}$$

Then the ratio of the heat transfer through the walls with and without windows becomes

$$\frac{\dot{Q}_{total,2}}{\dot{Q}_{total,1}} = \frac{\Delta T / R_{total,2}}{\Delta T / R_{total,1}} = \frac{R_{total,1}}{R_{total,2}} = \frac{0.0411417}{0.002327} = \mathbf{17.7}$$

Discussion In case of heat transfer through the walls with widows, additional heat transfer may occur due to air infiltration through the cracks around the window edges. Heat transfer calculations for building heating and cooling applications account for this additional heat transfer through the 'crack' method.

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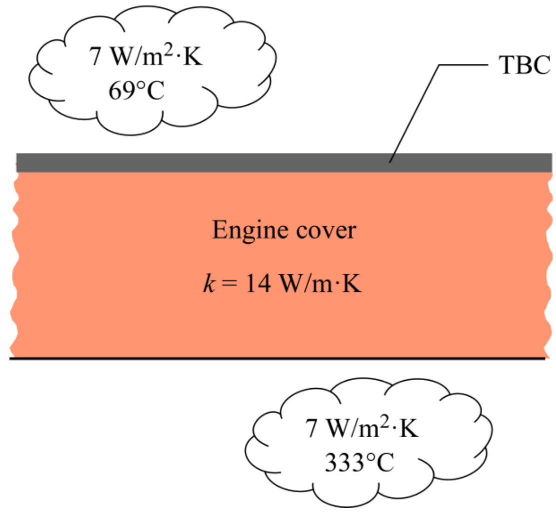
3-41 An engine cover is subjected to convection heat transfer on the inner surface and the outer surface. The thickness of a thermal barrier coating (TBC) layer applied on the engine cover outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistance at interface is negligible.

Properties The thermal conductivities of the stainless steel and the TBC are given to be k_1 = 14 W/m·K and k_2 = 1.1 W/m·K, respectively.

Analysis The thermal resistances of different layers are



$$R_{\text{conv},1} = \frac{1}{h_1 A} \quad \text{(inside surface convection resistance)}$$

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel layer resistance)}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(TBC layer resistance)}$$

$$R_{\text{conv},2} = \frac{1}{h_2 A} \quad \text{(outside surface convection resistance)}$$

Then,

$$\begin{aligned} AR_{\text{total}} &= A(R_{\text{conv},1} + R_1 + R_2 + R_{\text{conv},2}) \\ &= \frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2} \\ &= \frac{1}{7\ \text{W/m}^2\cdot\text{K}} + \frac{0.01\ \text{m}}{14\ \text{W/m}\cdot\text{K}} + \frac{0.004\ \text{m}}{1.1\ \text{W/m}\cdot\text{K}} + \frac{1}{7\ \text{W/m}^2\cdot\text{K}} \\ &= 0.2901\ \text{m}^2\cdot\text{K/W} \end{aligned}$$

and

$$AR_{\text{conv},2} = \frac{1}{h_2} = \frac{1}{7\ \text{W/m}^2\cdot\text{K}} = 0.1429\ \text{m}^2\cdot\text{K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,1} - T_{\infty,2}}{AR_{\text{total}}} = \frac{T_2 - T_{\infty,2}}{AR_{\text{conv},2}} \quad \rightarrow \quad T_2 = \frac{R_{\text{conv},2}}{R_{\text{total}}}(T_{\infty,1} - T_{\infty,2}) + T_{\infty,2}$$

$$T_2 = \frac{0.1429}{0.2901}(333 - 69)^\circ\text{C} + 69^\circ\text{C} = \mathbf{199^\circ C}$$

Yes, a TBC layer with a thickness of 4 mm will keep the engine cover surface below 200°C.

Discussion Since the calculated cover surface temperature of 199°C is very close to the required temperature of 200°C to prevent fire hazard, it is recommended to increase the TBC layer thickness and use a TBC layer of lower thermal conductivity. Doubling the TBC layer (8mm) would reduce the cover surface temperature to about 197°C, a reduction of 2°C.

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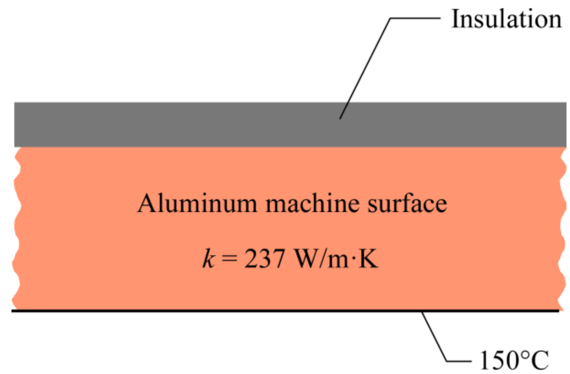
3-42 To prevent hot spots on a machine surface from causing thermal burns, the thickness of an insulation to cover the machine surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistance at interface is negligible.

Properties The thermal conductivities of the aluminum and the insulation are given to be k_1 = 237 W/m·K and k_2 = 0.06 W/m·K, respectively. The thermal contact conductance at the interface is given as 3000 W/m^2·K.

Analysis The thermal resistances of different layers are



$$R_1 = \frac{L_1}{k_1 A} \quad \text{(aluminum layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(insulation layer resistance)}$$

Then,

$$AR_{\text{total}} = A(R_1 + R_{\text{interface}} + R_2) = \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_1 - T_2}{AR_{\text{total}}} = \frac{T_1 - T_2}{\dfrac{L_1}{k_1} + \dfrac{1}{h_c} + \dfrac{L_2}{k_2}} \quad \rightarrow \quad L_2 = k_2\left[\frac{T_1 - T_2}{\dot{q}} - \left(\frac{L_1}{k_1} + \frac{1}{h_c}\right)\right]$$

$$L_2 = (0.06\ \text{W/m}\cdot\text{K})\left[\frac{(150-45)\ \text{K}}{300\ \text{W/m}^2} - \left(\frac{0.005\ \text{m}}{237\ \text{W/m}\cdot\text{K}} + \frac{1}{3000\ \text{W/m}^2\cdot\text{K}}\right)\right]$$

$$= 0.021\ \text{m}$$

$$= \mathbf{21\ mm}$$

Discussion By covering the surface of the machine with 21mm of insulation, the surface temperature can be kept below 45°C.

Thermal Contact Resistance

3-43C The resistance that an interface offers to heat transfer per unit interface area is called thermal contact resistance, R_c. The inverse of thermal contact resistance is called the thermal contact conductance.

3-44C The thermal contact resistance will be greater for rough surfaces because an interface with rough surfaces will contain more air gaps whose thermal conductivity is low.

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3-45C Thermal contact resistance can be minimized by (1) applying a thermally conducting liquid on the surfaces before they are pressed against each other, (2) by replacing the air at the interface by a better conducting gas such as helium or hydrogen, (3) by increasing the interface pressure, and (4) by inserting a soft metallic foil such as tin, silver, copper, nickel, or aluminum between the two surfaces.

3-46C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance has significance only for highly conducting materials like metals. Therefore, the thermal contact resistance can be ignored for two layers of insulation pressed against each other.

3-47C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance is significant for highly conducting materials like metals. Therefore, the thermal contact resistance must be considered for two layers of metals pressed against each other.

3-48C Heat transfer through the voids at an interface is by conduction and radiation. Evacuating the interface eliminates heat transfer by conduction, and thus increases the thermal contact resistance.

3-49 The thickness of copper plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of copper is k = 386 W/m·°C.

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is determined to be

$$R_c = \frac{1}{h_c} = \frac{1}{18{,}000\ \text{W/m}^2.°\text{C}} = 5.556\times10^{-5}\ \text{m}^2.°\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as $R = \frac{L}{k}$ where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR = kR_c = (386\ \text{W/m}\cdot°\text{C})(5.556\times10^{-5}\ \text{m}^2\cdot°\text{C/W}) = 0.0214\ \text{m} = \mathbf{2.14\ cm}$$

Therefore, the interface between the two plates offers as much resistance to heat transfer as a 2.14 cm thick copper. Note that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

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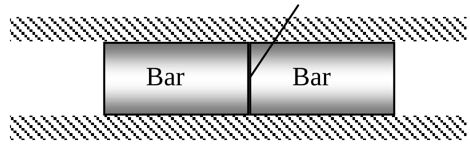
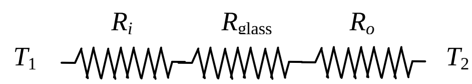
3-50 Two cylindrical aluminum bars with ground surfaces are pressed against each other in an insulation sleeve. For specified top and bottom surface temperatures, the rate of heat transfer along the cylinders and the temperature drop at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional in the axial direction since the lateral surfaces of both cylinders are well-insulated. **3** Thermal conductivities are constant.



Properties The thermal conductivity of aluminum bars is given to be k = 176 W/m·°C. The contact conductance at the interface of aluminum-aluminum plates for the case of ground surfaces and of 20 atm ≈ 2 MPa pressure is h_c = 11,400 W/m^2·°C (Table 3-2).



Analysis (*a*) The thermal resistance network in this case consists of two conduction resistance and the contact resistance, and they are determined to be

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(11{,}400\ \text{W/m}^2\cdot{}^\circ\text{C})[\pi(0.05\,\text{m})^2/4]} = 0.0447\,^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.15\,\text{m}}{(176\ \text{W/m}\cdot{}^\circ\text{C})[\pi(0.05\,\text{m})^2/4]} = 0.4341\,^\circ\text{C/W}$$

Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{\text{contact}} + 2R_{\text{bar}}} = \frac{(150-20)^\circ\text{C}}{(0.0447 + 2\times 0.4341)\ ^\circ\text{C/W}} = \mathbf{142.4\ W}$$

Therefore, the rate of heat transfer through the bars is 142.4 W.

(*b*) The temperature drop at the interface is determined to be

$$\Delta T_{\text{interface}} = \dot{Q}R_{\text{contact}} = (142.4\ \text{W})(0.0447^\circ\text{C/W}) = \mathbf{6.4^\circ C}$$

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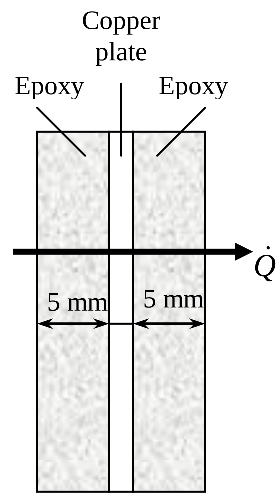
3-51 A thin copper plate is sandwiched between two epoxy boards. The error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the plate is large. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper plates and k = 0.26 W/m·°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be h_c = 6000 W/m^2·°C.

Analysis The thermal resistances of different layers for unit surface area of 1 m^2 are



$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(6000\ \text{W/m}^2\cdot{}^\circ\text{C})(1\,\text{m}^2)} = 0.00017\,^\circ\text{C/W}$$

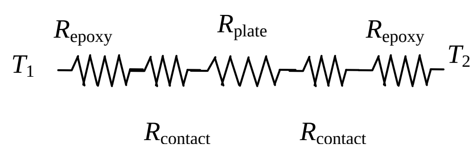
$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(386\ \text{W/m}\cdot{}^\circ\text{C})(1\,\text{m}^2)} = 2.6\times10^{-6}\ ^\circ\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.005\,\text{m}}{(0.26\ \text{W/m}\cdot{}^\circ\text{C})(1\,\text{m}^2)} = 0.01923\,^\circ\text{C/W}$$

The total thermal resistance is

$$\begin{aligned} R_{\text{total}} &= 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}} \\ &= 2\times0.00017 + 2.6\times10^{-6} + 2\times0.01923 = 0.03880\,^\circ\text{C/W} \end{aligned}$$

Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be



$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}}\times100 = \frac{2\times0.00017}{0.03880}\times100 = \mathbf{0.88\%}$$

which is negligible.

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3-52 Two identical aluminum plates are pressed against each other, where the interface is filled with glycerin. The thermal contact conductance of the glycerin is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivity is constant.

Properties The thermal conductivity of the aluminum plates is given to be k = 237 W/m·K.

Analysis The thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} \quad \text{and} \quad R_{\text{plate}} = \frac{L}{kA}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + 2R_{\text{plate}} = \frac{1}{h_c A} + \frac{2L}{kA}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{\dfrac{1}{h_c A} + \dfrac{2L}{kA}} \quad \text{or} \quad \dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta T}{1/h_c + 2L/k}$$

Thus, the thermal contact conductance of the glycerin is

$$h_c = \left(\frac{\Delta T}{\dot{q}} - \frac{2L}{k}\right)^{-1} = \left[\frac{(50-30)\text{ K}}{7800\text{ W/m}^2} - \frac{2(0.30\text{ m})}{237\text{ W/m}\cdot\text{K}}\right]^{-1} = \mathbf{30{,}810\ W/m^2\cdot K}$$

Discussion By comparing the calculated value of h_c = 30,810 W/m^2·K for glycerin with the value listed in Table 3-1 for glycerin (37,700 W/m^2·K), the calculated value is about 18% lower. The discrepancy between the calculated h_c and the value listed in Table 3-1 may be due to the surface roughness of the aluminum plates that causes imperfect contact between plate surface and glycerin.

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3-53 A two-layer wall is made of stainless steel and aluminum plates pressed together. The stainless steel surface is subjected to uniform heat flux, while the aluminum surface is subjected to convection heat transfer. The surface temperature of the stainless steel plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities of the stainless steel and aluminum plates are given to be k_1 = 14 W/m·K and k_2 = 237 W/m·K, respectively. The thermal contact conductance of the stainless steel-aluminum interface with a surface roughness of about 25 μm at an average pressure of 10 MPa is h_c = 2900 W/m^2·K (Table 3-2).

Analysis The thermal resistances of different layers are

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel plate resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(aluminum plate resistance)}$$

$$R_{\text{conv}} = \frac{1}{h_{\text{conv}} A}$$

The total thermal resistance is

$$R_{\text{total}} = R_1 + R_{\text{interface}} + R_2 + R_{\text{conv}}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_1 - T_\infty}{AR_{\text{total}}} = \frac{T_1 - T_\infty}{\dfrac{L_1}{k_1} + \dfrac{1}{h_c} + \dfrac{L_2}{k_2} + \dfrac{1}{h_{\text{conv}}}}$$

The surface temperature of the stainless steel plate is

$$\begin{aligned} T_1 &= \dot{q}\left(\frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h_{\text{conv}}}\right) + T_\infty \\ &= (800\ \text{W/m}^2)\left[\frac{0.005\ \text{m}}{14\ \text{W/m}\cdot\text{K}} + \frac{1}{2900\ \text{W/m}^2\cdot\text{K}} + \frac{0.015\ \text{m}}{237\ \text{W/m}\cdot\text{K}} + \frac{1}{12\ \text{W/m}^2\cdot\text{K}}\right] + 20°\text{C} \\ &= \mathbf{87.3°C} \end{aligned}$$

Discussion Among the thermal resistances considered in this problem, the convection resistance has the largest value.

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3-54 An aluminum plate and a stainless steel plate are pressed against each other. The impact of the plate surface roughness on the temperature drop at the interface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

Properties From Table 3-2, the thermal contact conductance of the stainless steel-aluminum interface at an average pressure of 20 MPa is $h_{c,\text{rough}}$ = 3600 W/m^2·K (for roughness = 20 μm) and $h_{c,\text{smooth}}$ = 20,800 W/m^2·K (for roughness = 2 μm).

Analysis The heat rate through the interface is

$$\dot{Q} = \frac{\Delta T_{\text{interface}}}{R_{\text{interface}}} = \frac{\Delta T_{\text{interface}}}{\dfrac{1}{h_c A}} \quad \text{or} \quad \Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}}$$

Thus,

$$\frac{(\Delta T_{\text{interface}})_{\text{rough}}}{(\Delta T_{\text{interface}})_{\text{smooth}}} = \frac{\dot{Q}(R_{\text{interface}})_{\text{rough}}}{\dot{Q}(R_{\text{interface}})_{\text{smooth}}} = \frac{\left(\dfrac{1}{h_{c,\text{rough}} A}\right)}{\left(\dfrac{1}{h_{c,\text{smooth}} A}\right)} = \frac{h_{c,\text{smooth}}}{h_{c,\text{rough}}}$$

$$\frac{(\Delta T_{\text{interface}})_{\text{rough}}}{(\Delta T_{\text{interface}})_{\text{smooth}}} = \frac{20{,}800\ \text{W/m}^2\cdot\text{K}}{3600\ \text{W/m}^2\cdot\text{K}} = 5.78$$

If the surface roughness of the plates is increased by tenfold, then the temperature drop at the interface would increase by about a factor of six

Discussion Thus, thermal contact resistance between two plates can be minimized by reducing the plate surface roughness or by increasing the contact pressure.

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3-55 A thin electronic component is cooled by dissipating heat through a heat sink attached on its top surface. There is contact resistance at the interface of the electronic component and the heat sink, and the temperature of the electronic component is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** The electronic component maintains a constant temperature.

Properties The thermal contact conductance at the electronic component/heat sink interface is given as h_c = 25,000 W/m^2·K, the combined convection and radiation thermal resistance of the heat sink is given as 1.3 K/W.

Analysis The thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} = \frac{1}{(25{,}000\ \text{W/m}^2\cdot\text{K})(950\ \text{cm}^2)(1/100\ \text{m/cm})^2} = 0.000421\ \text{K/W}$$

$$R_{\text{heat sink}} = 1.3\ \text{K/W}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{heat sink}} = 1.300421\ \text{K/W}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{total}}}$$

The temperature of the electronic component is

$$T_s = \dot{Q}R_{\text{total}} + T_\infty = (45\ \text{W})(1.300421\ \text{K/W}) + 30°\text{C} = \mathbf{88.5°C}$$

The contact resistance at the interface is only about 0.03% (0.000421/1.300421) of the total thermal resistance, thus it is negligible in this case and does not play a significant role in the heat dissipation.

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3-56 An engine cover made with two layers of metal (stainless steel and aluminum) pressed together. Both the inside and outside surface is subjected to convection heat transfer. The heat flux through the engine cover is to be determined

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities of the stainless steel and aluminum plates are given to be k_1 = 14 W/m·K and k_2 = 237 W/m·K, respectively. The thermal contact conductance of the stainless steel-aluminum interface with a surface roughness of about 23 μm and at an average pressure of 20 MPa is h_c = 3600 W/m^2·K (Table 3-2).

Analysis The thermal resistances of different layers are

$$R_{\text{conv},1} = \frac{1}{h_1 A} \quad \text{(inside surface convection resistance)}$$

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(aluminum layer resistance)}$$

$$R_{\text{conv},2} = \frac{1}{h_2 A} \quad \text{(outside surface convection resistance)}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{conv},1} + R_1 + R_{\text{interface}} + R_2 + R_{\text{conv},2}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,1} - T_{\infty,2}}{AR_{\text{total}}}$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h_2}}$$

$$= \frac{(150 - 40)\ \text{K}}{\frac{1}{10\ \text{W/m}^2\cdot\text{K}} + \frac{0.010\ \text{m}}{14\ \text{W/m}\cdot\text{K}} + \frac{1}{3600\ \text{W/m}^2\cdot\text{K}} + \frac{0.005\ \text{m}}{237\ \text{W/m}\cdot\text{K}} + \frac{1}{25\ \text{W/m}^2\cdot\text{K}}} = \mathbf{780\ W/m^2}$$

Discussion The contact resistance at the interface is about 0.2% of the total thermal resistance.

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3-57 An Inconel® plate covered with a layer of thermal barrier coating (TBC). The plate is exposed to hot combustion gases with known convection heat transfer coefficient. The temperature of the surface exposed to the hot gases is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities of the Inconel® and the thermal barrier coating are given to be k_1 = 25 W/m·K and k_2 = 1.5 W/m·K, respectively. The thermal contact conductance at the interface is given as h_c = 10,500 W/m^2·K.

Analysis The thermal resistances of different layers are

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(Inconel layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(TBC layer resistance)}$$

$$R_{\text{conv}} = \frac{1}{hA}$$

Then,

$$\begin{aligned} AR_{\text{total}} &= A(R_1 + R_{\text{interface}} + R_2 + R_{\text{conv}}) \\ &= \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h} \\ &= \frac{0.012/2\text{ m}}{25\text{ W/m}\cdot\text{K}} + \frac{1}{10{,}500\text{ W/m}^2\cdot\text{K}} + \frac{300\times10^{-6}\text{ m}}{1.5\text{ W/m}\cdot\text{K}} + \frac{1}{750\text{ W/m}^2\cdot\text{K}} \\ &= 0.001869\text{ m}^2\cdot\text{K/W} \end{aligned}$$

and

$$AR_{\text{conv}} = \frac{1}{h} = \frac{1}{750\text{ W/m}^2\cdot\text{K}} = 0.001333\text{ m}^2\cdot\text{K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_\infty - T_2}{AR_{\text{conv}}} = \frac{T_\infty - T_1}{AR_{\text{total}}}$$

Thus,

$$\begin{aligned} T_1 &= T_\infty - \frac{AR_{\text{total}}}{AR_{\text{conv}}}(T_\infty - T_2) \\ &= 1500°\text{C} - \frac{0.001869\text{ m}^2\cdot\text{K/W}}{0.001333\text{ m}^2\cdot\text{K/W}}(1500-1200)°\text{C} \\ &= \mathbf{1080°C} \end{aligned}$$

Discussion If the contact resistance is neglected in the analysis, the mid-plane temperature would be 1100°C.

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Generalized Thermal Resistance Networks

3-58C Two approaches used in development of the thermal resistance network in the x-direction for multi-dimensional problems are (1) to assume any plane wall normal to the x-axis to be isothermal and (2) to assume any plane parallel to the x-axis to be adiabatic.

3-59C The thermal resistance network approach will give adequate results for multi-dimensional heat transfer problems if heat transfer occurs predominantly in one direction.

3-60C Parallel resistances indicate simultaneous heat transfer (such as convection and radiation on a surface). Series resistances indicate sequential heat transfer (such as two homogeneous layers of a wall).

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3-61 A wall is to be constructed of 10-cm thick wood studs or with pairs of 5-cm thick wood studs nailed to each other. The rate of heat transfer through the solid stud and through a stud pair nailed to each other, as well as the effective conductivity of the nailed stud pair are to be determined.

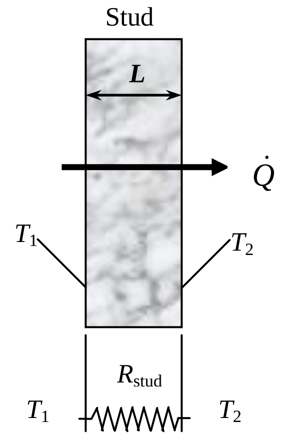
Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x* direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance between the two layers is negligible. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.11 W/m·°C for wood studs and k = 50 W/m·°C for manganese steel nails.

Analysis (*a*) The heat transfer area of the stud is A = (0.1 m)(2.5 m) = 0.25 m^2. The thermal resistance and heat transfer rate through the solid stud are

$$R_{stud} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(0.11\,\text{W/m}\cdot{}^\circ\text{C})(0.25\,\text{m}^2)} = 3.636\,^\circ\text{C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{8^\circ\text{C}}{3.636\,^\circ\text{C/W}} = \mathbf{2.2\ W}$$



(*b*) The thermal resistances of stud pair and nails are in parallel

$$A_{nails} = 50\frac{\pi D^2}{4} = 50\left[\frac{\pi(0.004\,\text{m})^2}{4}\right] = 0.000628\,\text{m}^2$$

$$R_{nails} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(50\ \text{W/m}\cdot{}^\circ\text{C})(0.000628\,\text{m}^2)} = 3.18\,^\circ\text{C/W}$$

$$R_{stud} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(0.11\,\text{W/m}\cdot{}^\circ\text{C})(0.25 - 0.000628\,\text{m}^2)} = 3.65\,^\circ\text{C/W}$$

$$\frac{1}{R_{total}} = \frac{1}{R_{stud}} + \frac{1}{R_{nails}} = \frac{1}{3.65} + \frac{1}{3.18} \longrightarrow R_{total} = 1.70\,^\circ\text{C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{8^\circ\text{C}}{1.70\,^\circ\text{C/W}} = \mathbf{4.7\ W}$$

(*c*) The effective conductivity of the nailed stud pair can be determined from

$$\dot{Q} = k_{eff}A\frac{\Delta T}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{\Delta T A} = \frac{(4.7\ \text{W})(0.1\,\text{m})}{(8^\circ\text{C})(0.25\,\text{m}^2)} = \mathbf{0.235\,W/m.{}^\circ C}$$

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3-62E The thermal resistance of an epoxy glass laminate across its thickness is to be reduced by planting cylindrical copper fillings throughout. The thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the plate is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 0.10 Btu/h·ft·°F for epoxy glass laminate and k = 223 Btu/h·ft·°F for copper fillings.

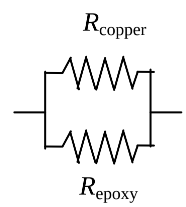
Analysis The thermal resistances of copper fillings and the epoxy board are in parallel. The number of copper fillings in the board and the area they comprise are

$$A_{total} = (6/12\text{ ft})(8/12\text{ ft}) = 0.3333\text{ ft}^2$$

$$n_{copper} = \frac{0.3333\text{ ft}^2}{(0.06/12\text{ ft})(0.06/12\text{ ft})} = 13{,}333 \text{ (number of copper fillings)}$$

$$A_{copper} = n\frac{\pi D^2}{4} = 13{,}333\frac{\pi(0.02/12\text{ ft})^2}{4} = 0.02909\text{ ft}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.3333 - 0.02909 = 0.3042\text{ ft}^2$$



The thermal resistances are evaluated to be

$$R_{copper} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(223\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.02909\text{ ft}^2)} = 0.0006423\text{ h}\cdot{}^\circ\text{F/Btu}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(0.10\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.3042\text{ ft}^2)} = 0.1370\text{ h}\cdot{}^\circ\text{F/Btu}$$

Then the thermal resistance of the entire epoxy board becomes

$$\frac{1}{R_{board}} = \frac{1}{R_{copper}} + \frac{1}{R_{epoxy}} = \frac{1}{0.0006423} + \frac{1}{0.1370} \longrightarrow R_{board} = \mathbf{0.00064\,h\cdot{}^\circ F/Btu}$$

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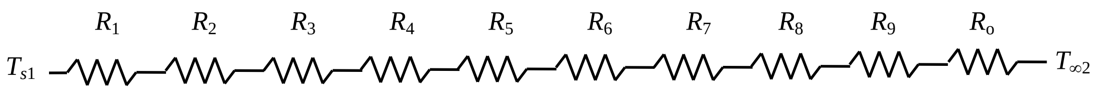
3-63 A coat is made of 5 layers of 0.1 mm thick synthetic fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.13 W/m·°C for synthetic fabric, k = 0.026 W/m·°C for air, and k = 0.035 W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are



$$R_{fabric} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.0001\,\text{m}}{(0.13\ \text{W/m}\cdot{}^\circ\text{C})(1.25\,\text{m}^2)} = 0.0006154\,{}^\circ\text{C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015\,\text{m}}{(0.026\ \text{W/m}\cdot{}^\circ\text{C})(1.25\,\text{m}^2)} = 0.0462\,{}^\circ\text{C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})(1.25\,\text{m}^2)} = 0.0320\,{}^\circ\text{C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5\times 0.0006154 + 4\times 0.0462 + 0.0320 = 0.2199\,{}^\circ\text{C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{(28-0)^\circ\text{C}}{0.2199\,{}^\circ\text{C/W}} = \mathbf{127\ W}$$

If the jacket is made of a single layer of 0.5 mm thick synthetic fabric, the rate of heat transfer would be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5\times R_{fabric} + R_o} = \frac{(28-0)^\circ\text{C}}{(5\times 0.0006154 + 0.0320)\ {}^\circ\text{C/W}} = \mathbf{798\,W}$$

The thickness of a wool fabric that has the same thermal resistance is determined from

$$R_{\text{total}} = R_{\substack{\text{wool}\\\text{fabric}}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

$$0.2199\,{}^\circ\text{C/W} = \frac{L}{(0.035\ \text{W/m}\cdot{}^\circ\text{C})(1.25\,\text{m}^2)} + 0.0320 \longrightarrow L = 0.00822\ \text{m} = \mathbf{8.22\,mm}$$

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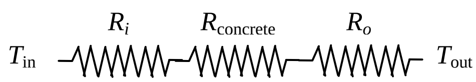
3-64 A kiln is made of 20 cm thick concrete walls and ceiling. The two ends of the kiln are made of thin sheet metal covered with 2-cm thick styrofoam. For specified indoor and outdoor temperatures, the rate of heat transfer from the kiln is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the walls and ceiling is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer. **5** Heat loss through the floor is negligible. **6** Thermal resistance of sheet metal is negligible.

Properties The thermal conductivities are given to be k = 0.9 W/m·°C for concrete and k = 0.033 W/m·°C for styrofoam insulation.

Analysis In this problem there is a question of which surface area to use. We will use the outer surface area for outer convection resistance, the inner surface area for inner convection resistance, and the average area for the conduction resistance. Or we could use the inner or the outer surface areas in the calculation of all thermal resistances with little loss in accuracy. For top and the two side surfaces:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2\cdot\text{°C})[(40\ \text{m})(13-1.2)\ \text{m}]} = 0.0071\times10^{-4}\ \text{°C/W}$$

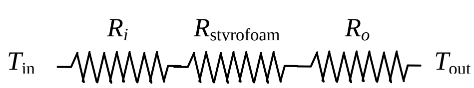
$$R_{concrete} = \frac{L}{kA_{ave}} = \frac{0.2\ \text{m}}{(0.9\ \text{W/m}\cdot\text{°C})[(40\ \text{m})(13-0.6)\ \text{m}]} = 4.480\times10^{-4}\ \text{°C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2\cdot\text{°C})[(40\ \text{m})(13\ \text{m})]} = 0.769\times10^{-4}\ \text{°C/W}$$

$$R_{total} = R_i + R_{concrete} + R_o = (0.0071 + 4.480 + 0.769)\times10^{-4} = 5.256\times10^{-4}\ \text{°C/W}$$

and $$\dot{Q}_{top+sides} = \frac{T_{in} - T_{out}}{R_{total}} = \frac{[40-(-4)]\text{°C}}{5.256\times10^{-4}\ \text{°C/W}} = 83{,}700\ \text{W}$$

Heat loss through the end surface of the kiln with styrofoam:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2\cdot\text{°C})[(4-0.4)(5-0.4)\ \text{m}^2]} = 0.201\times10^{-4}\ \text{°C/W}$$

$$R_{styrofoam} = \frac{L}{kA_{ave}} = \frac{0.02\ \text{m}}{(0.033\ \text{W/m}\cdot\text{°C})[(4-0.2)(5-0.2)\ \text{m}^2]} = 0.0332\ \text{°C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2\cdot\text{°C})[4\times5\ \text{m}^2]} = 0.0020\ \text{°C/W}$$

$$R_{total} = R_i + R_{styrpfoam} + R_o = 0.201\times10^{-4} + 0.0332 + 0.0020 = 0.0352\ \text{°C/W}$$

and $$\dot{Q}_{end\ surface} = \frac{T_{in} - T_{out}}{R_{total}} = \frac{[40-(-4)]\text{°C}}{0.0352\ \text{°C/W}} = 1250\ \text{W}$$

Then the total rate of heat transfer from the kiln becomes

$$\dot{Q}_{total} = \dot{Q}_{top+sides} + 2\dot{Q}_{side} = 83{,}700 + 2\times1250 = \mathbf{86{,}200\ W}$$

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3-65 Prob. 3-64 is reconsidered. The effects of the thickness of the wall and the convection heat transfer coefficient on the outer surface of the rate of heat loss from the kiln are to be investigated.

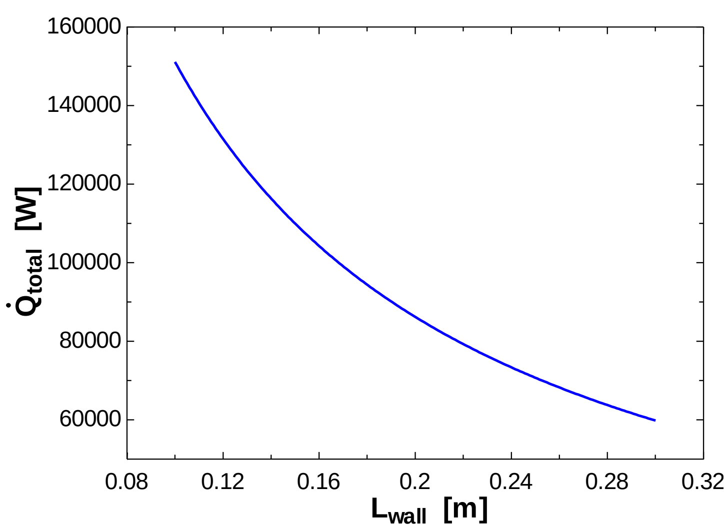
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
width=5 [m]
height=4 [m]
length=40 [m]
L_wall=0.2 [m]
k_concrete=0.9 [W/m-C]
T_in=40 [C]
T_out=-4 [C]
L_sheet=0.003 [m]
L_styrofoam=0.02 [m]
k_styrofoam=0.033 [W/m-C]
h_i=3000 [W/m^2-C]
h_o=25 [W/m^2-C]

"ANALYSIS"
R_conv_i=1/(h_i*A_1)
A_1=(2*height+width-6*L_wall)*length
R_concrete=L_wall/(k_concrete*A_2)
A_2=(2*height+width-3*L_wall)*length
R_conv_o=1/(h_o*A_3)
A_3=(2*height+width)*length
R_total_top_sides=R_conv_i+R_concrete+R_conv_o
Q_dot_top_sides=(T_in-T_out)/R_total_top_sides "Heat loss from top and the two side surfaces"

R_conv_i_end=1/(h_i*A_4)
A_4=(height-2*L_wall)*(width-2*L_wall)
R_styrofoam=L_styrofoam/(k_styrofoam*A_5)
A_5=(height-L_wall)*(width-L_wall)
R_conv_o_end=1/(h_o*A_6)
A_6=height*width
R_total_end=R_conv_i_end+R_styrofoam+R_conv_o_end
Q_dot_end=(T_in-T_out)/R_total_end "Heat loss from one end surface"
Q_dot_total=Q_dot_top_sides+2*Q_dot_end
```

L_{wall} [m]	Q_{total} [W]
0.1	151098
0.12	131499
0.14	116335
0.16	104251
0.18	94395
0.2	86201
0.22	79281
0.24	73359
0.26	68233
0.28	63751
0.3	59800



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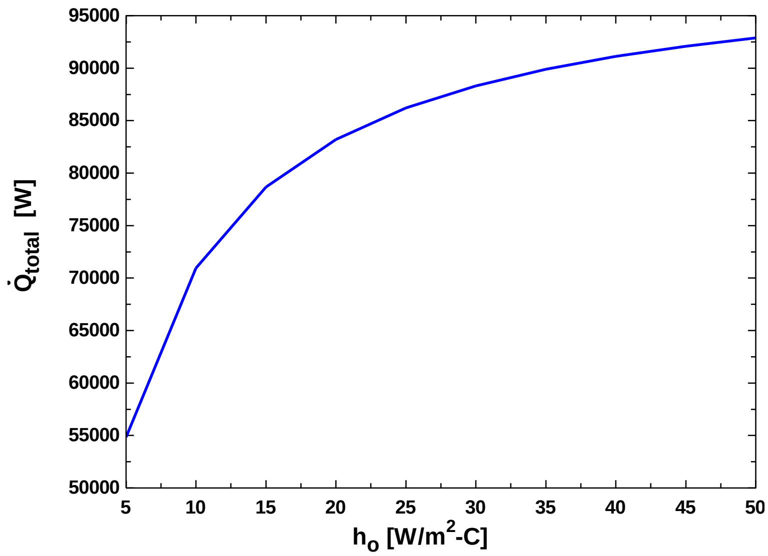
h_o [W/m^2.C]	Q_{total} [W]
5	54834
10	70939
15	78670
20	83212
25	86201
30	88318
35	89895
40	91116
45	92089
50	92882



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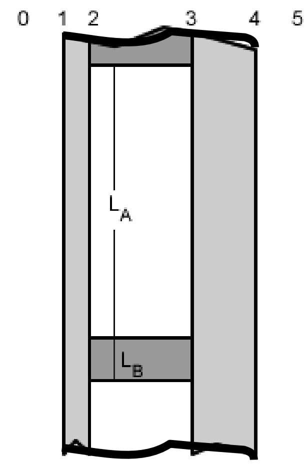
3-66 A typical section of a building wall is considered. The average heat flux through the wall is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The thermal conductivities are given to be k_{23b} = 50 W/m·K, k_{23a} = 0.03 W/m·K, k_{12} = 0.5 W/m·K, k_{34} = 1.0 W/m·K.

Analysis We consider 1 m^2 of wall area. The thermal resistances are



$$R_{12} = \frac{t_{12}}{k_{12}} = \frac{0.01\,\text{m}}{(0.5\ \text{W/m}\cdot{}^\circ\text{C})} = 0.02\ \text{m}^2\cdot{}^\circ\text{C/W}$$

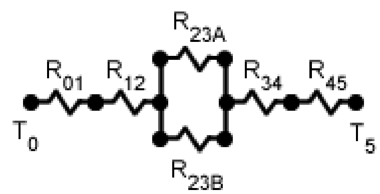
$$R_{23a} = t_{23}\frac{L_\text{a}}{k_{23\text{a}}(L_a + L_b)}$$

$$= (0.08\,\text{m})\frac{0.6\,\text{m}}{(0.03\ \text{W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 2.645\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_{23b} = t_{23}\frac{L_\text{b}}{k_{23\text{b}}(L_a + L_b)}$$

$$= (0.08\,\text{m})\frac{0.005\,\text{m}}{(50\ \text{W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 1.32\times10^{-5}\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_{34} = \frac{t_{34}}{k_{34}} = \frac{0.1\,\text{m}}{(1.0\ \text{W/m}\cdot{}^\circ\text{C})} = 0.1\ \text{m}^2\cdot{}^\circ\text{C/W}$$



The total thermal resistance and the rate of heat transfer are

$$R_\text{total} = R_{12} + \left(\frac{R_{23a}R_{23b}}{R_{23a}+R_{23b}}\right) + R_{34}$$

$$= 0.02 + 2.645\left(\frac{1.32\times10^{-5}}{2.645 + 1.32\times10^{-5}}\right) + 0.1 = 0.120\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$\dot{q} = \frac{T_4 - T_1}{R_\text{total}} = \frac{(35-20)^\circ\text{C}}{0.120\,\text{m}^2\cdot\text{C/W}} = \mathbf{125\ W/m^2}$$

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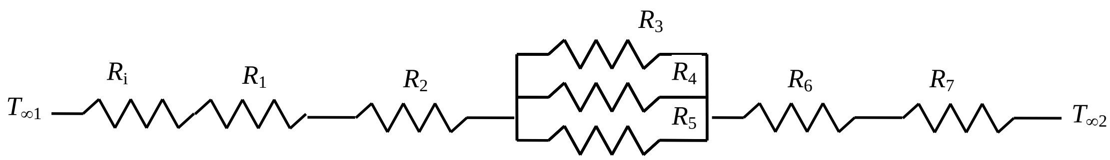
3-67 A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.72 W/m·°C for bricks, k = 0.22 W/m·°C for plaster layers, and k = 0.026 W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.33 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.3030\ {}^\circ\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.026\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 2.331\ {}^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\substack{plaster\\side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.2755\ {}^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\substack{plaster\\center}} = \frac{L}{h_o A} = \frac{0.15\ \text{m}}{(0.22\ \text{W/m}\cdot{}^\circ\text{C})(0.015\times 1\ \text{m}^2)} = 45.45{}^\circ\text{C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.18\ \text{m}}{(0.72\ \text{W/m}\cdot{}^\circ\text{C})(0.30\times 1\ \text{m}^2)} = 0.8333\ {}^\circ\text{C/W}$$

$$R_\text{o} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20\ \text{W/m}\cdot{}^\circ\text{C})(0.33\times 1\ \text{m}^2)} = 0.1515\ {}^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{45.45} + \frac{1}{0.8333} + \frac{1}{45.45} \longrightarrow R_{mid} = 0.804\ {}^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.3030 + 2.331 + 2(0.2755) + 0.804 + 0.1515 = 4.140\ {}^\circ\text{C/W}$$

The steady rate of heat transfer through the wall per $0.33\ \text{m}^2$ is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[(22-(-4)]{}^\circ\text{C}}{4.140{}^\circ\text{C/W}} = 6.280\ \text{W}$$

Then steady rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (6.280\ \text{W})\frac{(4\times 6)\text{m}^2}{0.33\ \text{m}^2} = \mathbf{457\ W}$$

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3-68 Prob. 3-67 is reconsidered. The rate of heat transfer through the wall as a function of the thickness of the rigid foam is to be plotted.

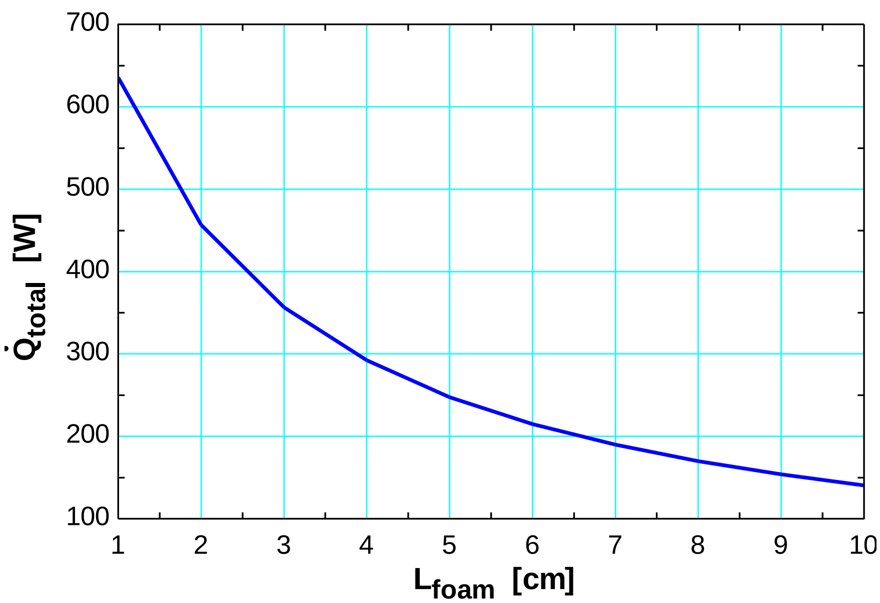
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
A=4*6 [m^2]
L_brick=0.18 [m]
L_plaster_center=0.15 [m]
L_plaster_side=0.02 [m]
L_foam=2 [cm]
k_brick=0.72 [W/m-C]
k_plaster=0.22 [W/m-C]
k_foam=0.026 [W/m-C]
T_infinity_1=22 [C]
T_infinity_2=-4 [C]
h_1=10 [W/m^2-C]
h_2=20 [W/m^2-C]

A_1=0.33*1 [m^2]
A_2=0.30*1 [m^2]
A_3=0.015*1 [m^2]

"ANALYSIS"
R_conv_1=1/(h_1*A_1)
R_foam=(L_foam*Convert(cm, m))/(k_foam*A_1) "L_foam is in cm"
R_plaster_side=L_plaster_side/(k_plaster*A_1)
R_plaster_center=L_plaster_center/(k_plaster*A_3)
R_brick=L_brick/(k_brick*A_2)
R_conv_2=1/(h_2*A_1)
1/R_mid=2*1/R_plaster_center+1/R_brick
R_total=R_conv_1+R_foam+2*R_plaster_side+R_mid+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=Q_dot*A/A_1
```

L_{foam} [cm]	Q_{total} [W]
1	635.6
2	456.7
3	356.4
4	292.2
5	247.6
6	214.8
7	189.7
8	169.8
9	153.7
10	140.4



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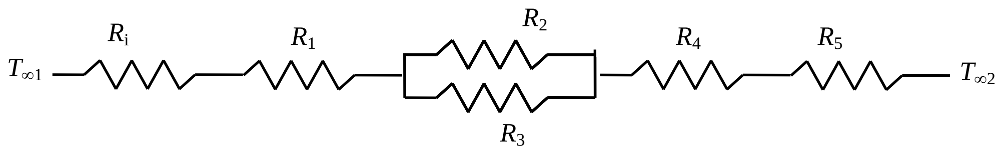
3-69 A wall is constructed of two layers of sheetrock spaced by 5 cm × 16 cm wood studs. The space between the studs is filled with fiberglass insulation. The thermal resistance of the wall and the rate of heat transfer through the wall are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.17 W/m·°C for sheetrock, k = 0.11 W/m·°C for wood studs, and k = 0.034 W/m·°C for fiberglass insulation.

Analysis (*a*) The representative surface area is $A = 1\times 0.65 = 0.65\,\text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_i = \frac{1}{h_i A} = \frac{1}{(8.3\ \text{W/m}^2\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.185\,{}^\circ\text{C/W}$$

$$R_1 = R_4 = R_{sheetrock} = \frac{L}{kA} = \frac{0.01\,\text{m}}{(0.17\ \text{W/m}\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.090\,{}^\circ\text{C/W}$$

$$R_2 = R_{stud} = \frac{L}{kA} = \frac{0.16\,\text{m}}{(0.11\ \text{W/m}\cdot{}^\circ\text{C})(0.05\,\text{m}^2)} = 29.091\,{}^\circ\text{C/W}$$

$$R_3 = R_{fiberglass} = \frac{L}{kA} = \frac{0.16\,\text{m}}{(0.034\ \text{W/m}\cdot{}^\circ\text{C})(0.60\,\text{m}^2)} = 7.843\,{}^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(34\ \text{W/m}^2\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.045\,{}^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{29.091} + \frac{1}{7.843} \longrightarrow R_{mid} = 6.178\,{}^\circ\text{C/W}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_4 + R_o$$
$$= 0.185 + 0.090 + 6.178 + 0.090 + 0.045 = \mathbf{6.588\,{}^\circ C/W}\ \text{(for a } 1\,\text{m}\times 0.65\,\text{m section)}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20-(-9)]^\circ\text{C}}{6.588\,{}^\circ\text{C/W}} = 4.40\ \text{W}$$

(*b*) Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (4.40\ \text{W})\frac{(12\,\text{m})(5\,\text{m})}{0.65\,\text{m}^2} = \mathbf{406\,W}$$

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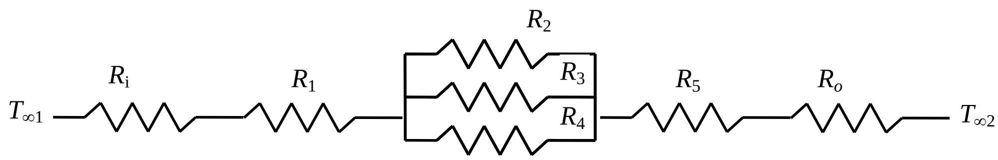
3-70E A wall is to be constructed using solid bricks or identical size bricks with 9 square air holes. There is a 0.5 in thick sheetrock layer between two adjacent bricks on all four sides, and on both sides of the wall. The rates of heat transfer through the wall constructed of solid bricks and of bricks with air holes are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.40 Btu/h·ft·°F for bricks, k = 0.015 Btu/h·ft·°F for air, and k = 0.10 Btu/h·ft·°F for sheetrock.

Analysis (*a*) The representative surface area is $A = (7.5/12)(7.5/12) = 0.3906\ \text{ft}^2$. The thermal resistance network and the individual thermal resistances if the wall is constructed of solid bricks are



$$R_i = \frac{1}{h_i A} = \frac{1}{(1.5\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.3906\ \text{ft}^2)} = 1.7068\ \text{h}^\circ\text{F/Btu}$$

$$R_1 = R_5 = R_{plaster} = \frac{L}{kA} = \frac{0.5/12\ \text{ft}}{(0.10\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.3906\ \text{ft}^2)} = 1.0667\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_2 = R_{plaster} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.10\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})[(7.5/12)\times(0.5/12)]\text{ft}^2} = 288\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_3 = R_{plaster} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.10\ \text{Btu/h}\cdot\text{ft}\cdot{}^\text{o}\ \text{F})[(7/12)\times(0.5/12)]\text{ft}^2} = 308.57\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.40\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})[(7/12)\times(7/12)]\text{ft}^2} = 5.51\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.3906\ \text{ft}^2)} = 0.6400\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{5.51} \longrightarrow R_{mid} = 5.3135\ \text{h}\cdot{}^\circ\text{F/Btu}$$

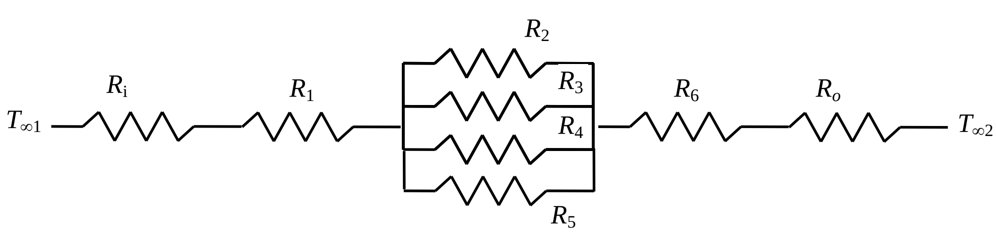
$$R_{total} = R_i + R_1 + R_{mid} + R_5 + R_o = 1.7068 + 1.0667 + 5.3135 + 1.0667 + 0.6400 = 9.7937\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-30)^\circ\text{F}}{9.7937\ \text{h}\cdot{}^\circ\text{F/Btu}} = 5.105\ \text{Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (5.105\ \text{Btu/h})\frac{(30\ \text{ft})(10\ \text{ft})}{0.3906\ \text{m}^2} = \mathbf{3921\ Btu/h}$$

(*b*) The thermal resistance network and the individual thermal resistances if the wall is constructed of bricks with air holes are



$$A_{airholes} = 9(1.5/12)\times(1.5/12) = 0.1406\ \text{ft}^2$$

$$A_{bricks} = (7/12\ \text{ft})^2 - 0.1406 = 0.1997\ \text{ft}^2$$

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$$R_4 = R_{airholes} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.015 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(0.1406 \text{ ft}^2)} = 355.62 \text{ h}\cdot°\text{F/Btu}$$

$$R_5 = R_{brick} = \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.40 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(0.1997 \text{ ft}^2)} = 9.389 \text{ h}\cdot°\text{F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{355.62} + \frac{1}{9.389} \longrightarrow R_{mid} = 8.618 \text{ h}\cdot°\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_6 + R_o = 1.7068 + 1.0667 + 8.618 + 1.0667 + 0.6400 = 13.098 \text{ h}\cdot°\text{F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-30)°\text{F}}{13.098 \text{ h}\cdot°\text{F/Btu}} = 3.817 \text{ Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (3.817 \text{ Btu/h})\frac{(30 \text{ ft})(10 \text{ ft})}{0.3906 \text{ ft}^2} = \mathbf{2932 \text{ Btu/h}}$$

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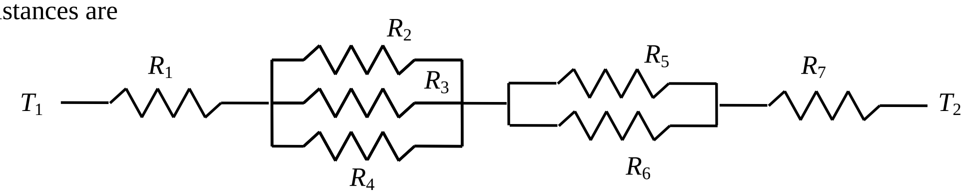
3-71 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfaces are disregarded.

Properties The thermal conductivities are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35$ W/m·°C.

Analysis (*a*) The representative surface area is $A = 0.12 \times 1 = 0.12\ \text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\ \text{m}}{(2\ \text{W/m}\cdot°\text{C})(0.12\ \text{m}^2)} = 0.04\ °\text{C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\ \text{m}}{(20\ \text{W/m}\cdot°\text{C})(0.04\ \text{m}^2)} = 0.06\ °\text{C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\ \text{m}}{(8\ \text{W/m}\cdot°\text{C})(0.04\ \text{m}^2)} = 0.16\ °\text{C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\ \text{m}}{(15\ \text{W/m}\cdot^\text{o}\text{C})(0.06\ \text{m}^2)} = 0.11\ °\text{C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\ \text{m}}{(35\ \text{W/m}\cdot°\text{C})(0.06\ \text{m}^2)} = 0.05\ ^\text{o}\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\ \text{m}}{(2\ \text{W/m}\cdot°\text{C})(0.12\ \text{m}^2)} = 0.25\ °\text{C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{mid,1} = 0.025\ °\text{C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ °\text{C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 = 0.04 + 0.025 + 0.034 + 0.25 = 0.349\ °\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300 - 100)°\text{C}}{0.349\ °\text{C/W}} = 572\ \text{W}\ \ (\text{for a } 0.12\ \text{m} \times 1\ \text{m section})$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (572\ \text{W})\frac{(5\ \text{m})(8\ \text{m})}{0.12\ \text{m}^2} = \mathbf{1.91 \times 10^5\ \ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065\ °\text{C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300°\text{C} - (572\ \text{W})(0.065°\text{C/W}) = \mathbf{263°C}$$

(*c*) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (572\ \text{W})(0.25°\text{C/W}) = \mathbf{143°C}$$

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3-72 In an experiment, the convection heat transfer coefficients of (a) air and (b) water flowing over the metal foil are to be determined.

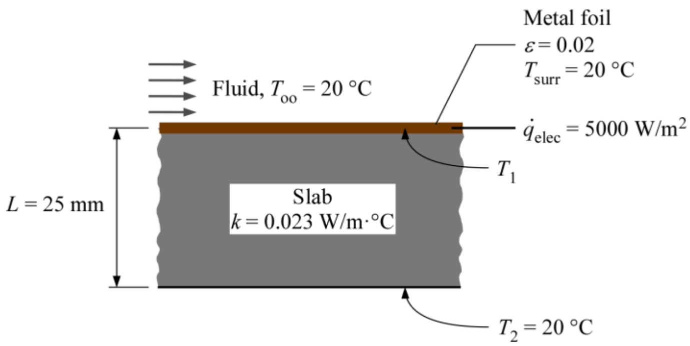
Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Thermal resistance of the thin metal foil is negligible.

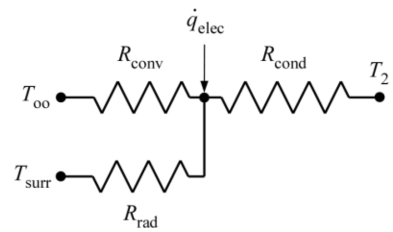
Properties Thermal conductivity of the slab is given to be k = 0.023 W/m · K and the emissivity of the metal foil is 0.02.

Analysis The thermal resistances are

$$R_{\text{cond}} = \frac{L}{kA} \qquad R_{\text{conv}} = \frac{1}{hA}$$

and
$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A}$$





From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_\infty - T_1}{R_{\text{conv}}} + \frac{T_{\text{surr}} - T_1}{R_{\text{rad}}} + \dot{q}_{\text{elec}} A = \frac{T_1 - T_2}{R_{\text{cond}}}$$

or
$$\frac{1}{R_{\text{conv}}} = \left(\frac{T_1 - T_2}{R_{\text{cond}}} - \frac{T_{\text{surr}} - T_1}{R_{\text{rad}}} - \dot{q}_{\text{elec}} A \right) \frac{1}{T_\infty - T_1}$$

$$h = \left(\frac{T_1 - T_2}{L/k} - \frac{T_{\text{surr}} - T_1}{1/h_{\text{rad}}} - \dot{q}_{\text{elec}} \right) \frac{1}{T_\infty - T_1}$$

(a) For air flowing over the metal foil, the radiation heat transfer coefficient is

$$\begin{aligned} h_{\text{rad}} &= \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \\ &= (0.02)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(423^2 + 293^2)\text{ K}^2(423 + 293)\text{ K} \\ &= 0.215 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

The convection heat transfer coefficient for air flowing over the metal foil is

$$\begin{aligned} h &= \left[\frac{(150 - 20)\text{ K}}{0.025\text{ m}/0.023\text{ W/m}\cdot\text{K}} - \frac{(20 - 150)\text{ K}}{1/0.215\text{ W/m}^2\cdot K} - 5000\text{ W/m}^2 \right] \frac{1}{(20 - 150)\text{ K}} \\ &= \mathbf{37.3\ W/m^2 \cdot K} \end{aligned}$$

(b) For water flowing over the metal foil, the radiation heat transfer coefficient is

$$\begin{aligned} h_{\text{rad}} &= \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \\ &= (0.02)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(303^2 + 293^2)\text{ K}^2(303 + 293)\text{ K} \\ &= 0.1201 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

The convection heat transfer coefficient for water flowing over the metal foil is

$$\begin{aligned} h &= \left[\frac{(30 - 20)\text{ K}}{0.025\text{ m}/0.023\text{ W/m}\cdot\text{K}} - \frac{(20 - 30)\text{ K}}{1/0.1201\text{ W/m}^2\cdot K} - 5000\text{ W/m}^2 \right] \frac{1}{(20 - 30)\text{ K}} \\ &= \mathbf{499\ W/m^2 \cdot K} \end{aligned}$$

Discussion If heat transfer by conduction through the slab and radiation on the metal foil surface is neglected, the convection heat transfer coefficient for the case with air flow would deviate by 3.2% from the result in part (a), while the convection heat transfer coefficient for the case with water flow would deviate by 0.2% from the result in part (b).

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Heat Conduction in Cylinders and Spheres

3-73C When the diameter of cylinder is very small compared to its length, it can be treated as an infinitely long cylinder. Cylindrical rods can also be treated as being infinitely long when dealing with heat transfer at locations far from the top or bottom surfaces. However, it is not proper to use this model when finding temperatures near the bottom and the top of the cylinder.

3-74C No. In steady-operation the temperature of a solid cylinder or sphere does not change in radial direction (unless there is heat generation).

3-75C Heat transfer in this short cylinder is one-dimensional since there will be no heat transfer in the axial and tangential directions.

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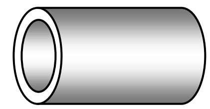
3-76 A steam pipe covered with 3-cm thick glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

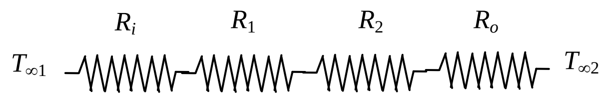
Properties The thermal conductivities are given to be k = 15 W/m·°C for steel and k = 0.038 W/m·°C for glass wool insulation

Analysis The inner and the outer surface areas of the insulated pipe per unit length are



$$A_i = \pi D_i L = \pi(0.05\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.055 + 0.06\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$

The individual thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\text{ W/m}^2\cdot{}^\circ\text{C})(0.157\text{ m}^2)} = 0.08\,^\circ\text{C/W}$$
$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 0.00101\,^\circ\text{C/W}$$
$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 3.089\,^\circ\text{C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(15\text{ W/m}^2\cdot{}^\circ\text{C})(0.361\text{ m}^2)} = 0.1845\,^\circ\text{C/W}$$
$$R_{total} = R_i + R_1 + R_2 + R_o = 0.08 + 0.00101 + 3.089 + 0.1845 = 3.354\,^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam per m. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320-5)^\circ\text{C}}{3.354\,^\circ\text{C/W}} = \mathbf{93.9\ W}$$

The temperature drops across the pipe and the insulation are

$$\Delta T_{pipe} = \dot{Q}R_{pipe} = (93.9\text{ W})(0.00101\,^\circ\text{C/W}) = \mathbf{0.095^\circ C}$$
$$\Delta T_{insulation} = \dot{Q}R_{insulation} = (93.9\text{ W})(3.089\,^\circ\text{C/W}) = \mathbf{290.1^\circ C}$$

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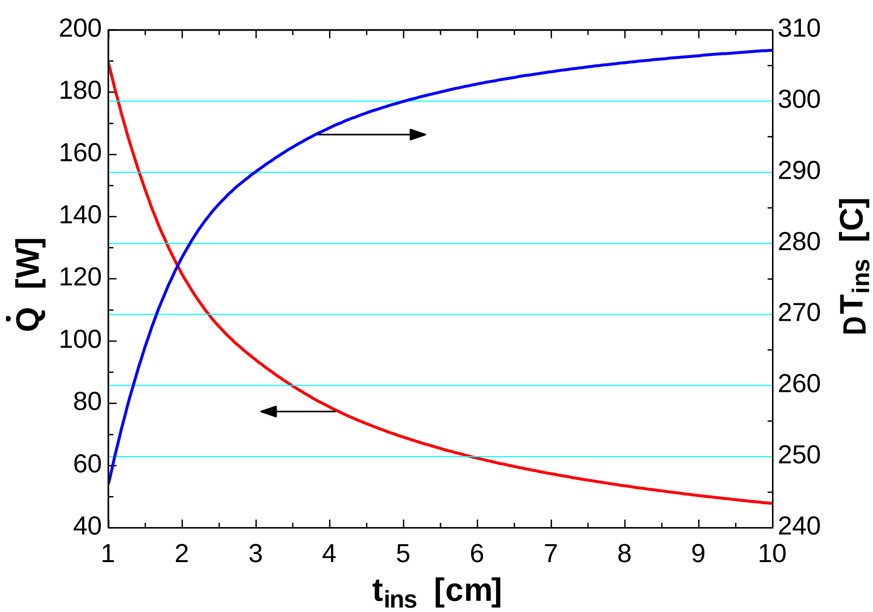
3-77 Prob. 3-76 is reconsidered. The effect of the thickness of the insulation on the rate of heat loss from the steam and the temperature drop across the insulation layer are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity_1=320 [C]
T_infinity_2=5 [C]
k_steel=15 [W/m-C]
D_i=0.05 [m]
D_o=0.055 [m]
r_1=D_i/2
r_2=D_o/2
t_ins=3 [cm]
k_ins=0.038 [W/m-C]
h_o=15 [W/m^2-C]
h_i=80 [W/m^2-C]
L=1 [m]

"ANALYSIS"
A_i=pi*D_i*L
A_o=pi*(D_o+2*t_ins*Convert(cm, m))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_steel*L)
R_ins=ln(r_3/r_2)/(2*pi*k_ins*L)
r_3=r_2+t_ins*Convert(cm, m) "t_ins is in cm"
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_ins+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
DELTAT_pipe=Q_dot*R_pipe
DELTAT_ins=Q_dot*R_ins
```

t_{ins} [cm]	Q [W]	ΔT_{ins} [C]
1	189.5	246.1
2	121.5	278.1
3	93.91	290.1
4	78.78	296.3
5	69.13	300
6	62.38	302.4
7	57.37	304.1
8	53.49	305.4
9	50.37	306.4
10	47.81	307.2



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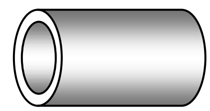
3-78 A 50-m long section of a steam pipe passes through an open space at 15°C. The rate of heat loss from the steam pipe, the annual cost of this heat loss, and the thickness of fiberglass insulation needed to save 90 percent of the heat lost are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivity is constant. **4** The thermal contact resistance at the interface is negligible. **5** The pipe temperature remains constant at about 150 °C with or without insulation. **6** The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.

Analysis (a) The rate of heat loss from the steam pipe is



$$A_o = \pi DL = \pi(0.1\,\text{m})(50\,\text{m}) = 15.71\,\text{m}^2$$

$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\ \text{W/m}^2\cdot{}^\circ\text{C})(15.71\,\text{m}^2)(150-15)^\circ\text{C} = \mathbf{42{,}412\ W}$$

(b) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (42.412\,\text{kJ/s})(365\times 24\times 3600\,\text{s/yr}) = 1.337\times 10^9\ \text{kJ/yr}$$

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

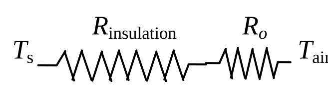
$$Q_{gas} = \frac{1.337\times 10^9\ \text{kJ/yr}}{0.75}\left(\frac{1\ \text{therm}}{105{,}500\,\text{kJ}}\right) = 16{,}903\ \text{therms/yr}$$

The annual cost of this energy lost is

$$\begin{aligned}\text{Energy cost} &= (\text{Energy used})(\text{Unit cost of energy})\\ &= (16{,}903\ \text{therms/yr})(\$0.52\,/\,\text{therm}) = \mathbf{\$8790/yr}\end{aligned}$$

(c) In order to save 90% of the heat loss and thus to reduce it to 0.1×42,412 = 4241 W, the thickness of insulation needed is determined from

$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2/r_1)}{2\pi kL}}$$



Substituting and solving for r_2, we get

$$4241\ \text{W} = \frac{(150-15)^\circ\text{C}}{\dfrac{1}{(20\ \text{W/m}^2\cdot{}^\circ\text{C})[(2\pi r_2(50\,\text{m})]} + \dfrac{\ln(r_2/0.05)}{2\pi(0.035\ \text{W/m}\cdot{}^\circ\text{C})(50\,\text{m})}} \longrightarrow r_2 = 0.0692\,\text{m}$$

Then the thickness of insulation becomes

$$t_{insulation} = r_2 - r_1 = 6.92 - 5 = \mathbf{1.92\,cm}$$

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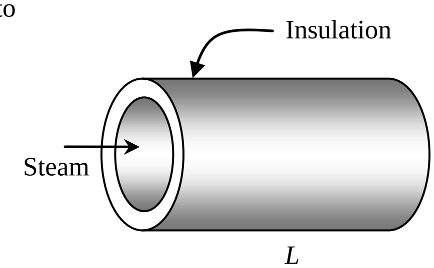
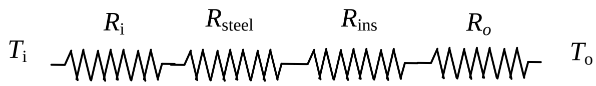
3-79 Steam flows in a steel pipe, which is insulated by gypsum plaster. The rate of heat transfer from the steam and the temperature on the outside surface of the insulation are be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties (*a*) The thermal conductivities of steel and gypsum plaster are given to be 50 and 0.5 W/m·°C, respectively.



Analysis The thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(800\ \text{W/m}^2\cdot°\text{C})\pi(0.06\ \text{m})(20\ \text{m})} = 0.0003316°\text{C/W}$$

$$R_{\text{steel}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{steel}} L} = \frac{\ln(8/6)}{2\pi(50\ \text{W/m}\cdot°\text{C})(20\ \text{m})} = 0.0000458°\text{C/W}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} = \frac{\ln(16/8)}{2\pi(0.5\ \text{W/m}\cdot°\text{C})(20\ \text{m})} = 0.011032°\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(200\ \text{W/m}^2\cdot°\text{C})\pi(0.16\ \text{m})(20\ \text{m})} = 0.0004974°\text{C/W}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{steel}} + R_{\text{ins}} + R_o = 0.0003316 + 0.0000458 + 0.011032 + 0.0004974 = 0.011907°\text{C/W}$$

$$\dot{Q} = \frac{T_i - T_o}{R_{\text{total}}} = \frac{(200-10)°\text{C}}{0.011907\ \text{m}^2\cdot\text{C/W}} = \mathbf{15,957\ W}$$

(*b*) The temperature at the outer surface of the insulation is determined from

$$\dot{Q} = \frac{T_s - T_o}{R_o} \longrightarrow 15,957\ \text{W} = \frac{(T_s - 10)°\text{C}}{0.0004974\ \text{m}^2\cdot°\text{C/W}} \longrightarrow T_s = \mathbf{17.9°C}$$

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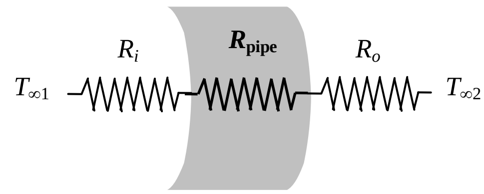
3-80E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients, the length of the tube required to condense steam at a rate of 120 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivity of copper tube is given to be k = 223 Btu/h·ft·°F. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.4/12\text{ ft})(1\text{ ft}) = 0.1047\text{ ft}^2$$
$$A_o = \pi D_o L = \pi(0.6/12\text{ ft})(1\text{ ft}) = 0.1571\text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35\text{ Btu/h.ft}^2.\text{°F})(0.1047\text{ ft}^2)} = 0.27284\text{ h°F/Btu}$$
$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.000289\text{ h°F/Btu}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500\text{ Btu/h.ft}^2.\text{°F})(0.157\text{ ft}^2)} = 0.004244\text{ h°F/Btu}$$
$$R_{total} = R_i + R_{pipe} + R_o = 0.27284 + 0.000289 + 0.004244 = 0.27737\text{ h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100-70)\text{°F}}{0.27737\text{ °F/Btu}} = 108.2\text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 120 lbm/h and the length of the tube required is determined to be

$$\dot{Q}_{total} = \dot{m}h_{fg} = (120\text{ lbm/h})(1037\text{ Btu/lbm}) = 124{,}440\text{ Btu/h}$$
$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{124{,}440}{108.2} = \mathbf{1150\ ft}$$

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3-81E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients and 0.01-in thick scale build up on the inner surface, the length of the tube required to condense steam at a rate of 120 lbm/h is to be determined.

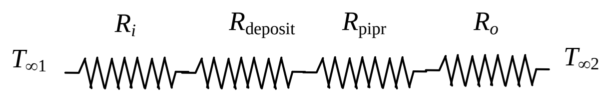
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tube and be k = 0.5 Btu/h·ft·°F for the mineral deposit. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis When a 0.01-in thick layer of deposit forms on the inner surface of the pipe, the inner diameter of the pipe will reduce from 0.4 in to 0.38 in. The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.38/12\ \text{ft})(1\ \text{ft}) = 0.09948\ \text{ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12\ \text{ft})(1\ \text{ft}) = 0.1571\ \text{ft}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35\ \text{Btu/h.ft}^2.°\text{F})(0.09948\ \text{ft}^2)} = 0.28720\ \text{h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 0.000289\ \text{h°F/Btu}$$

$$R_{deposit} = \frac{\ln(r_1/r_{dep})}{2\pi k_2 L} = \frac{\ln(0.2/0.19)}{2\pi(0.5\ \text{Btu/h.ft.°F})(1\ \text{ft})} = 0.01633\ \text{h.°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(1500\ \text{Btu/h.ft}^2.°\text{F})(0.1571\ \text{ft}^2)} = 0.004244\ \text{h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_{deposit} + R_o = 0.28720 + 0.000289 + 0.01633 + 0.004244 = 0.30806\ \text{h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100-70)°\text{F}}{0.30806\ °\text{F/Btu}} = 97.38\ \text{Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 120 lbm/h and the length of the tube required can be determined to be

$$\dot{Q}_{total} = \dot{m}h_{fg} = (120\ \text{lbm/h})(1037\ \text{Btu/lbm}) = 124{,}440\ \text{Btu/h}$$

$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{124{,}440}{97.38} = \mathbf{1278\ ft}$$

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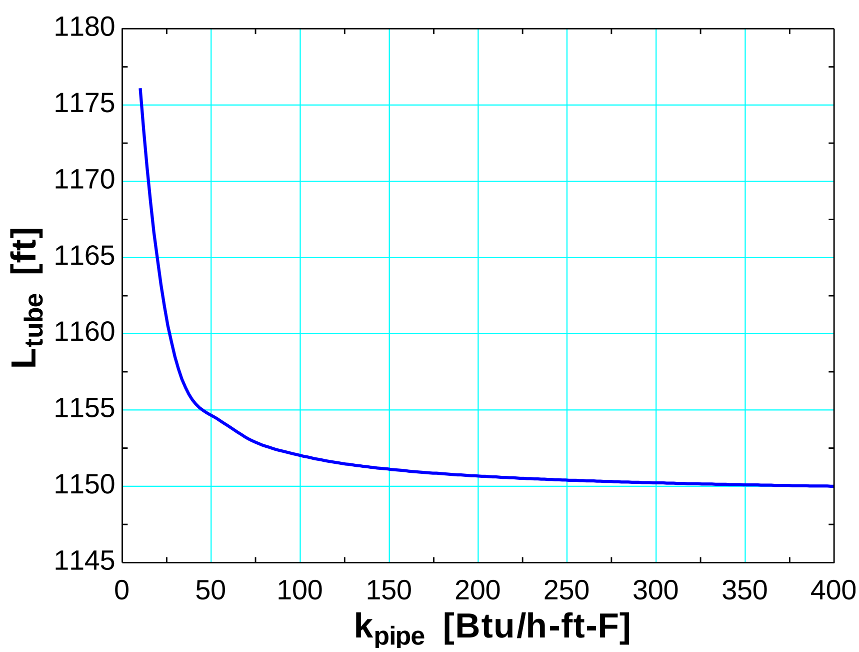
3-82E Prob. 3-81E is reconsidered. The effects of the thermal conductivity of the pipe material and the outer diameter of the pipe on the length of the tube required are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity_1=100 [F]
T_infinity_2=70 [F]
k_pipe=223 [Btu/h-ft-F]
D_i=0.4 [in]
D_o=0.6 [in]
r_1=D_i/2
r_2=D_o/2
h_fg=1037 [Btu/lbm]
h_o=1500 [Btu/h-ft^2-F]
h_i=35 [Btu/h-ft^2-F]
m_dot=120 [lbm/h]

"ANALYSIS"
L=1 [ft] “for 1 ft length of the tube"
A_i=pi*(D_i*Convert(in, ft))*L
A_o=pi*(D_o*Convert(in, ft))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_pipe*L)
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=m_dot*h_fg
L_tube=Q_dot_total/Q_dot
```

k_{pipe} [Btu/h.ft.F]	L_{tube} [ft]
10	1176
30.53	1158
51.05	1155
71.58	1153
92.11	1152
112.6	1152
133.2	1151
153.7	1151
174.2	1151
194.7	1151
215.3	1151
235.8	1150
256.3	1150
276.8	1150
297.4	1150
317.9	1150
338.4	1150
358.9	1150
379.5	1150
400	1150



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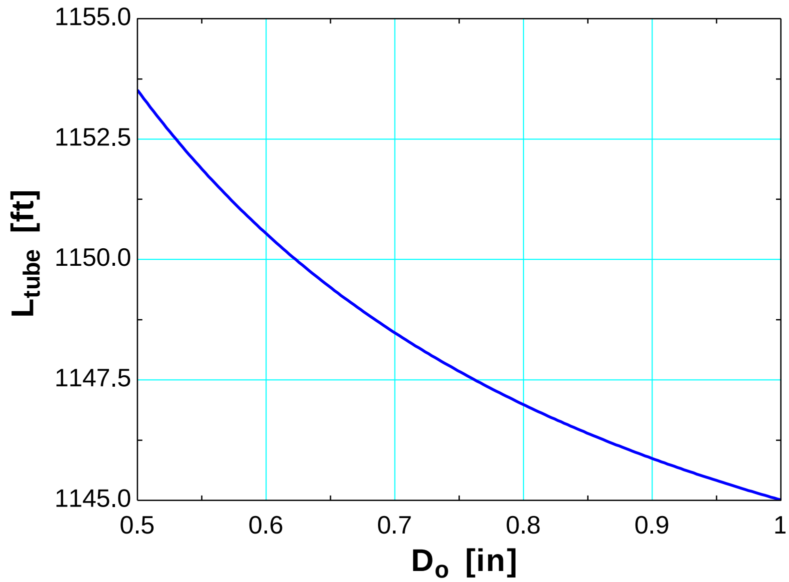
D_o [in]	L_{tube} [ft]
0.5	1154
0.525	1153
0.55	1152
0.575	1151
0.6	1151
0.625	1150
0.65	1149
0.675	1149
0.7	1148
0.725	1148
0.75	1148
0.775	1147
0.8	1147
0.825	1147
0.85	1146
0.875	1146
0.9	1146
0.925	1146
0.95	1145
0.975	1145
1	1145



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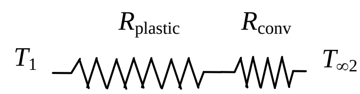
3-83 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be k = 0.15 W/m·°C.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = \mathbf{V}I = (8\text{ V})(13\text{ A}) = 104\text{ W}$$



The total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24\text{ W/m}^2.°\text{C})[\pi(0.0042\text{ m})(10\text{ m})]} = 0.3158\text{ °C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.1/1.1)}{2\pi(0.15\text{ W/m.°C})(10\text{ m})} = 0.0686\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.3158 + 0.0686 = 0.3844\text{ °C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 30°\text{C} + (104\text{ W})(0.3844\text{ °C/W}) = \mathbf{70.0°C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{24\text{ W/m}^2.°\text{C}} = 0.00625\text{ m} = 6.25\text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

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3-84 An electric hot water tank is made of two concentric cylindrical metal sheets with foam insulation in between. The fraction of the hot water cost that is due to the heat loss from the tank and the payback period of the do-it-yourself insulation kit are to be determined.

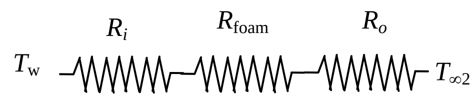
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal resistances of the water tank and the outer thin sheet metal shell are negligible. **5** Heat loss from the top and bottom surfaces is negligible.

Properties The thermal conductivities are given to be k = 0.03 W/m·°C for foam insulation and k = 0.035 W/m·°C for fiber glass insulation

Analysis We consider only the side surfaces of the water heater for simplicity, and disregard the top and bottom surfaces (it will make difference of about 10 percent). The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.40\text{ m})(2\text{ m}) = 2.513\text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(50\text{ W/m}^2.°\text{C})(2.513\text{ m}^2)} = 0.007958\text{ °C/W}$$



$$A_o = \pi D_o L = \pi(0.46\text{ m})(2\text{ m}) = 2.890\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.°\text{C})(2.890\text{ m}^2)} = 0.02883\text{ °C/W}$$

$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot°\text{C})(2\text{ m})} = 0.3707\text{ °C/W}$$

$$R_{total} = R_i + R_o + R_{foam} = 0.007958 + 0.02883 + 0.3707 = 0.4075\text{ °C/W}$$

The rate of heat loss from the hot water tank is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)°\text{C}}{0.4075\text{ °C/W}} = 68.71\text{ W}$$

The amount and cost of heat loss per year are

$$Q = \dot{Q}\Delta t = (0.06871\text{ kW})(365\times 24\text{ h/yr}) = 601.9\text{ kWh/yr}$$

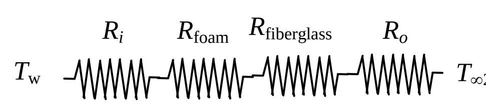
$$\text{Cost of Energy} = (\text{Amount of energy})(\text{Unit cost}) = (601.9\text{ kWh})(\$0.08/\text{kWh}) = \$48.15$$

$$f = \frac{\$48.15}{\$280} = 0.172 = \mathbf{17.2\%}$$

If 3 cm thick fiber glass insulation is used to wrap the entire tank, the individual resistances becomes

$$A_o = \pi D_o L = \pi(0.52\text{ m})(2\text{ m}) = 3.267\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(3.267\text{ m}^2)} = 0.02551°\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot°\text{C})(2\text{ m})} = 0.3707\text{ °C/W}$$

$$R_{fiberglass} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(26/23)}{2\pi(0.035\text{ W/m}^2\cdot°\text{C})(2\text{ m})} = 0.2788\text{ °C/W}$$

$$R_{total} = R_i + R_o + R_{foam} + R_{fiberglass} = 0.007958 + 0.02551 + 0.3707 + 0.2788 = 0.6830\text{ °C/W}$$

The rate of heat loss from the hot water heater in this case is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(55-27)°\text{C}}{0.6830\text{ °C/W}} = 41.00\text{ W}$$

The energy saving is

saving = 68.71 - 41.00 = 27.71 W

The time necessary for this additional insulation to pay for its cost of $30 is then determined to be

$$\text{Cost} = (0.02771\text{ kW})(\text{Time period})(\$0.08/\text{kWh}) = \$30$$

$$\longrightarrow \text{Time period} = 13{,}533\text{ hours} = 564\text{ days} \approx \mathbf{19\ months}$$

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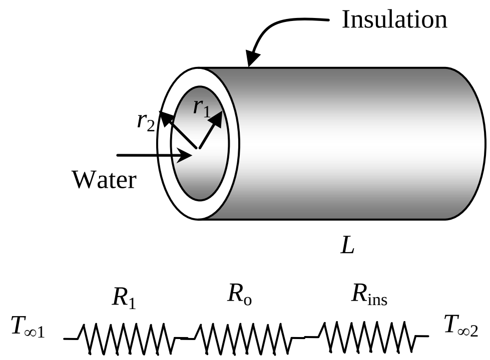
3-85 Chilled water is flowing inside a pipe. The thickness of the insulation needed to reduce the temperature rise of water to one-fourth of the original value is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity is given to be k = 0.05 W/m·°C for insulation.



Analysis The rate of heat transfer without the insulation is

$$\dot{Q}_{\text{old}} = \dot{m}c_p\Delta T = (0.98\text{ kg/s})(4180\text{ J/kg}\cdot{}^\circ\text{C})(8-7)^\circ\text{C} = 4096\text{ W}$$

The total resistance in this case is

$$\dot{Q}_{\text{old}} = \frac{T_\infty - T_w}{R_{\text{total}}}$$

$$4096\text{ W} = \frac{(30-7.5)^\circ\text{C}}{R_{\text{total}}} \longrightarrow R_{\text{total}} = 0.005493^\circ\text{C/W}$$

The convection resistance on the outer surface is

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(9\text{ W/m}^2\cdot{}^\circ\text{C})\pi(0.05\text{ m})(150\text{ m})} = 0.004716\,^\circ\text{C/W}$$

The rest of thermal resistances are due to convection resistance on the inner surface and the resistance of the pipe and it is determined from

$$R_1 = R_{\text{total}} - R_{\text{o}} = 0.005493 - 0.004716 = 0.000777\,^\circ\text{C/W}$$

The rate of heat transfer with the insulation is

$$\dot{Q}_{\text{new}} = \dot{m}c_p\Delta T = (0.98\text{ kg/s})(4180\text{ J/kg}\cdot{}^\circ\text{C})(0.25^\circ\text{C}) = 1024\text{ W}$$

The total thermal resistance with the insulation is

$$\dot{Q}_{\text{new}} = \frac{T_\infty - T_w}{R_{\text{total,new}}} \longrightarrow 1024\text{ W} = \frac{[30-(7+7.25)/2)]^\circ\text{C}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = 0.02234^\circ\text{C/W}$$

It is expressed by

$$R_{\text{total,new}} = R_1 + R_{\text{o,new}} + R_{\text{ins}} = R_1 + \frac{1}{h_o A_o} + \frac{\ln(D_2/D_1)}{2\pi k_{\text{ins}} L}$$

$$0.02234^\circ\text{C/W} = 0.000777 + \frac{1}{(9\text{ W/m}^2\cdot{}^\circ\text{C})\pi D_2(150\text{ m})} + \frac{\ln(D_2/0.05)}{2\pi(0.05\text{ W/m}\cdot{}^\circ\text{C})(150\text{ m})}$$

Solving this equation by trial-error or by using an equation solver such as EES, we obtain

$$D_2 = 0.1265\text{ m}$$

The following line in EES is used:

```
0.02234=0.000777+1/(9*pi*D2*150)+ln(D2/0.05)/(2*pi*0.05*200)
```

Then the required thickness of the insulation becomes

$$t_{\text{ins}} = (D_2 - D_1)/2 = (0.1265-0.05)/2 = 0.0383\text{ m} = \mathbf{3.83\,cm}$$

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3-86E A steam pipe covered with 2-in thick fiberglass insulation is subjected to convection on its surfaces. The rate of heat loss from the steam per unit length and the error involved in neglecting the thermal resistance of the steel pipe in calculations are to be determined.

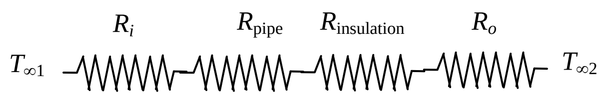
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 8.7 Btu/h·ft·°F for steel and k = 0.020 Btu/h·ft·°F for fiberglass insulation.

Analysis The inner and outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(3.5/12\text{ ft})(1\text{ ft}) = 0.916\text{ ft}^2$$

$$A_o = \pi D_o L = \pi(8/12\text{ ft})(1\text{ ft}) = 2.094\text{ ft}^2$$



The individual resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(30\text{ Btu/h.ft}^2.°\text{F})(0.916\text{ ft}^2)} = 0.036\text{ h}\cdot°\text{F/Btu}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.002\text{ h}\cdot°\text{F/Btu}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(4/2)}{2\pi(0.020\text{ Btu/h.ft.°F})(1\text{ ft})} = 5.516\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(5\text{ Btu/h.ft}^2.°\text{F})(2.094\text{ ft}^2)} = 0.096\text{ h}\cdot°\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.036 + 0.002 + 5.516 + 0.096 = 5.65\text{ h}\cdot°\text{F/Btu}$$

Then the steady rate of heat loss from the steam per ft. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(450-55)°\text{F}}{5.65\text{ h}\cdot°\text{F/Btu}} = \mathbf{69.91\text{ Btu/h}}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_i + R_2 + R_o = 0.036 + 5.516 + 0.096 = 5.648\text{ h°F/Btu}$$

Then the percentage error involved in calculations becomes

$$error\% = \frac{(5.65 - 5.648)\text{h°F/Btu}}{5.65\text{ h°F/Btu}} \times 100 = \mathbf{0.035\%}$$

which is insignificant.

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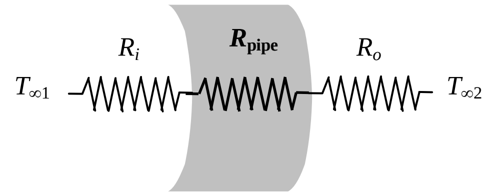
3-87 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of cast iron are given to be k = 52 W/m·°C and ε = 0.7.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.04\text{ m})(15\text{ m}) = 1.885\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.046\text{ m})(15\text{ m}) = 2.168\text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120\text{ W/m}^2.°\text{C})(1.885\text{ m}^2)} = 0.00442\text{ °C/W}$$
$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.3/2)}{2\pi(52\text{ W/m.°C})(15\text{ m})} = 0.00003\text{ °C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 60°C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$h_{rad} = \varepsilon\sigma(T_2{}^2 + T_{surr}{}^2)(T_2 + T_{surr})$$
$$= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(333\text{ K})^2 + (283\text{ K})^2](333+283) = 4.669\text{ W/m}^2.\text{K}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 4.669 + 15 = 19.67\text{ W/m}^2.°\text{C}$$
$$R_o = \frac{1}{h_{combined}A_o} = \frac{1}{(19.67\text{ W/m}^2.°\text{C})(2.168\text{ m}^2)} = 0.02345\text{ °C/W}$$
$$R_{total} = R_i + R_{pipe} + R_o = 0.00442 + 0.00003 + 0.02345 = 0.02790\text{ °C/W}$$

The rate of heat loss from the hot water pipe then becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(70-10)°\text{C}}{0.02790\text{ °C/W}} = \mathbf{2151\ W}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m}c_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{c_p\Delta T} = \frac{2151\text{ J/s}}{(4180\text{ J/kg.°C})(3\text{ °C})} = 0.1715\text{ kg/s}$$
$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.1715\text{ kg/s}}{(1000\text{ kg/m}^3)\dfrac{\pi(0.04\text{ m})^2}{4}} = \mathbf{0.136 m/s}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty1} - T_s}{R_i + R_{pipe}} \rightarrow 2151\text{ W} = \frac{(70 - T_s)°\text{C}}{(0.00442 + 0.00003)°\text{C/W}} \rightarrow T_s = 60.4°\text{C}$$

which is very close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

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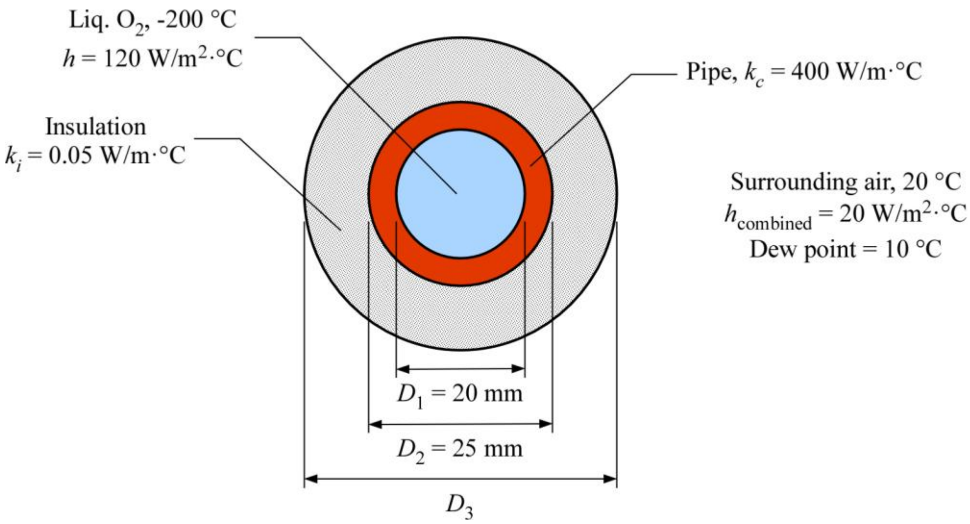
3-88 To avoid condensation on the outer surface, the necessary thickness of the insulation around a copper pipe that carries liquid oxygen is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Thermal contact resistance is negligible.

Properties The thermal conductivities of the copper pipe and the insulation are given to be 400 W/m · °C and 0.05 W/m · °C, respectively.



Analysis From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_{\infty,o}-T_{\infty,i}}{R_{\text{combined}}+R_{\text{cond,i}}+R_{\text{cond,c}}+R_{\text{conv}}}=\frac{T_{\infty,o}-T_s}{R_{\text{combined}}}$$

$$\frac{T_{\infty,o}-T_{\infty,i}}{\dfrac{1}{h_{\text{combined}}A}+\dfrac{\ln(D_3/D_2)}{2\pi k_i L}+\dfrac{\ln(D_2/D_1)}{2\pi k_c L}+\dfrac{1}{hA}}=\frac{T_{\infty,o}-T_s}{\dfrac{1}{h_{\text{combined}}A}}$$

$$\frac{T_{\infty,o}-T_{\infty,i}}{\dfrac{1}{h_{\text{combined}}\pi D_3 L}+\dfrac{\ln(D_3/D_2)}{2\pi k_i L}+\dfrac{\ln(D_2/D_1)}{2\pi k_c L}+\dfrac{1}{h\pi D_1 L}}=\frac{T_{\infty,o}-T_s}{\dfrac{1}{h_{\text{combined}}\pi D_3 L}}$$

Rearranging yields

$$\frac{T_{\infty,o}-T_{\infty,i}}{T_{\infty,o}-T_s}=1+h_{\text{combined}}D_3\left[\frac{\ln(D_3/D_2)}{2k_i}+\frac{\ln(D_2/D_1)}{2k_c}+\frac{1}{hD_1}\right]$$

$$\frac{(20+200)\,^\circ\text{C}}{(20-10)\,^\circ\text{C}}=1+(20\text{ W/m}^2\cdot{}^\circ\text{C})D_3\left[\frac{\ln(D_3/0.025\text{ m})}{2(0.05\text{ W/m}\cdot{}^\circ\text{C})}\right.$$

$$\left.+\frac{\ln(0.025\text{ m}/0.020\text{ m})}{2(400\text{ W/m}\cdot{}^\circ\text{C})}+\frac{1}{(120\text{ W/m}^2\cdot{}^\circ\text{C})(0.020\text{ m})}\right]$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(20+200)/(20-10)=1+20*D_3*(ln(D_3/25e-3)/(2*0.05)+ln(25/20)/(2*400)+1/(120*20e-3))
```

Solving by EES software, the outer diameter of the insulation is

$$D_3 = 0.0839\text{ m}$$

The thickness of the insulation necessary to avoid condensation on the outer surface is

$$t>\frac{D_3-D_2}{2}=\frac{0.0839\text{ m}-0.025\text{ m}}{2}=\mathbf{0.0295\,m}$$

Discussion If the insulation thickness is less than 29.5 mm, the outer surface temperature would decrease to the dew point at 10 °C where condensation would occur.

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3-89 Liquid H_2 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid H_2 temperature below −300°C is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 23 W/m·K and k_{ins} = 0.6 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \qquad \text{(liq. H}_2\text{ convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2 / D_1)}{2\pi k_{\text{pipe}} L} \qquad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3 / D_2)}{2\pi k_{\text{ins}} L} \qquad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \qquad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o \qquad \text{and} \qquad \dot{Q} = \frac{T_o - T_i}{R_{\text{total}}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.051\,\text{m} = \mathbf{5.1\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=200 [W/m^2-K]   "liq. H2 convection heat transfer coefficient"
h_o=50 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=23 [W/m-K]   "pipe thermal conductivity"
k_ins=0.6 [W/m-K]   "insulation thermal conductivity"
L=20 [m]   "pipe length"
D_1=0.03 [m]   "inner pipe diameter"
D_2=0.04 [m]   "outer pipe diameter"
T_3=5 [C]   "outer insulation surface temperature"
T_i=-300 [C]   "liq. NH3 temperature"
T_o=40 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi#*D_1*L)   "liq. H2 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi#*D_3*L)   "ambient air convection resistance"
R_total=R_i+R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_i)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the liquid H_2 below −300°C, the pipe insulation thickness must be at least 5.1 cm thick.

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3-90 Liquid NH_3 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid NH_3 temperature below −35°C is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 25 W/m·K and k_{ins} = 0.75 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \qquad \text{(liq. NH}_3\text{ convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{pipe}} L} \qquad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} \qquad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \qquad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o \qquad \text{and} \qquad \dot{Q} = \frac{T_o - T_i}{R_{\text{total}}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.063\,\text{m} = \mathbf{6.3\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=100 [W/m^2-K]   "liq. NH3 convection heat transfer coefficient"
h_o=20 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=25 [W/m-K]   "pipe thermal conductivity"
k_ins=0.75 [W/m-K]   "insulation thermal conductivity"
L=10 [m]   "pipe length"
D_1=0.025 [m]   "inner pipe diameter"
D_2=0.04 [m]   "outer pipe diameter"
T_3=10 [C]   "outer insulation surface temperature"
T_i=-35 [C]   "liq. NH3 temperature"
T_o=20 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi#*D_1*L)   "liq. NH3 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi#*D_3*L)   "ambient air convection resistance"
R_total=R_i+R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_i)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the liquid NH_3 below −35°C, the pipe insulation thickness must be at least 6.3 cm thick.

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3-91 A mixture of chemicals undergoing exothermic reaction is being transported in a pipe. The insulation thermal conductivity on the pipe that is necessary to keep the outer surface at 45°C or lower is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe is given to be k_{pipe} = 14 W/m·K.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \quad \text{(mixture convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2 / D_1)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3 / D_2)}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

where, $D_3 = 2t + D_2$

the rate of heat transfer is

$$\dot{Q} = \frac{T_i - T_3}{R_i + R_{\text{pipe}} + R_{\text{ins}}} = \frac{T_3 - T_O}{R_o}$$

Solving for the insulation thermal conductivity yields $k_{\text{ins}} = \mathbf{0.321\ W/m \cdot K}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=150 [W/m^2-K]   "mix. convection heat transfer coefficient"
h_o=25 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=14 [W/m-K]   "pipe thermal conductivity"
L=10 [m]   "pipe length"
D_1=0.025 [m]   "inner pipe diameter"
D_2=0.03 [m]   "outer pipe diameter"
D_3=0.08 [m]
T_3=45 [C]   "outer insulation surface temperature"
T_i=135 [C]   "avg. mix. temperature"
T_o=20 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi#*D_1*L)   "mix. convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi#*D_3*L)   "ambient air convection resistance"
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_i-T_3)/(R_i+R_pipe+R_ins)
Q_dot=(T_3-T_o)/(R_o)
```

Discussion To keep the outer surface below 45°C, the thermal conductivity of the pipe insulation must be 0.321 W/m·K or lower.

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3-92 Ice slurry is being transported in an insulated pipe. The insulation thickness on the pipe that is necessary to prevent condensation on the outer surface is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 15 W/m·K and k_{ins} = 0.95 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_{pipe} = \frac{\ln(D_2 / D_1)}{2\pi k_{pipe} L} \quad \text{(pipe layer resistance)}$$

$$R_{ins} = \frac{\ln(D_3 / D_2)}{2\pi k_{ins} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{total} = R_{pipe} + R_{ins} + R_o \quad \text{and} \quad \dot{Q} = \frac{T_o - T_1}{R_{total}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.0496\ \text{m} = \mathbf{5.0\ cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_o=10 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"
L=5 [m]   "pipe length"
D_1=0.025 [m]   "inner pipe diameter"
D_2=0.03 [m]   "outer pipe diameter"
T_1=0 [C]   "inner pipe surface temperature"
T_3=10 [C]   "outer insulation surface temperature"
T_o=20 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
"R_i=1/(h_i*pi#*D_1*L)"   "liq. NH3 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi#*D_3*L)   "ambient air convection resistance"
R_total=R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_1)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the outer surface temperature from dropping below the dew point at 10°C, the insulation thickness needs to be 5 cm or more.

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3-93 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7 \text{ kJ/kg}$. The outer surface of the tank is black and thus its emissivity is $\varepsilon = 1$.

Analysis (*a*) The inner and the outer surface areas of sphere are

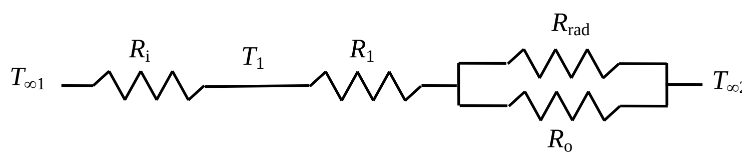
$$A_i = \pi D_i^2 = \pi(8\text{ m})^2 = 201.06\text{ m}^2$$

$$A_o = \pi D_o^2 = \pi(8.03\text{ m})^2 = 202.57\text{ m}^2$$

We assume the outer surface temperature T_2 to be 5°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr}) \\ &= 1(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(273+5\text{ K})^2 + (273+25\text{ K})^2][(273+25\text{ K})(273+5\text{ K})] \\ &= 5.424\text{ W/m}^2.\text{K} \end{aligned}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2\cdot°\text{C})(201.06\text{ m}^2)} = 0.000062\text{ °C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(4.015-4.0)\text{ m}}{4\pi(15\text{ W/m}\cdot°\text{C})(4.015\text{ m})(4.0\text{ m})} = 0.000005\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000494\text{ °C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000910\text{ °C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.000494} + \frac{1}{0.000910} \longrightarrow R_{eqv} = 0.000320\text{ °C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000062 + 0.000005 + 0.000320 = 0.000387\text{ °C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(25-0)°\text{C}}{0.000387\text{ °C/W}} = \mathbf{64{,}600\ W}$$

(*b*) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (64.600\text{ kJ/s})(24\times3600\text{ s}) = 5.581\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{5.581\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{16{,}730\ kg}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s)$$

$$\rightarrow T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 25°\text{C} - \frac{64{,}600\text{ W}}{(10+5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 4.3°\text{C}$$

which is very close to the assumed temperature of 5°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.

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3-94 A spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid nitrogen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

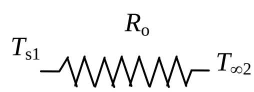
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the nitrogen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\,\text{m})^2 = 28.27\,\text{m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(28.27\,\text{m}^2)} = 0.00101\,°\text{C/W}$$

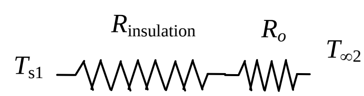


$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-196)]°\text{C}}{0.00101°\text{C/W}} = 208{,}910\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{208.910\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{1.055\,kg/s}$$

(*b*) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\,\text{m})^2 = 30.19\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(30.19\,\text{m}^2)} = 0.000946\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\ \text{m}}{4\pi(0.035\ \text{W/m.°C})(1.55\,\text{m})(1.5\,\text{m})} = 0.0489\,°\text{C/W}$$

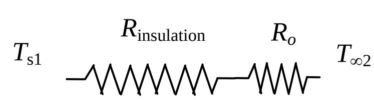
$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{0.0498\,°\text{C/W}} = 4233\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4.233\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{0.0214\,kg/s}$$

(*c*) The heat transfer rate and the rate of evaporation of the liquid with 2-cm thick layer of superinsulation is

$$A = \pi D^2 = \pi(3.04\,\text{m})^2 = 29.03\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(29.03\,\text{m}^2)} = 0.000984\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\ \text{m}}{4\pi(0.00005\ \text{W/m.°C})(1.52\,\text{m})(1.5\,\text{m})} = 13.96\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{13.96\,°\text{C/W}} = 15.11\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01511\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{0.000076 kg/s}$$

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3-95 A spherical tank filled with liquid oxygen at 1 atm and -183°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid oxygen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

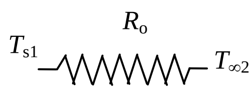
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the oxygen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid oxygen at 1 atm are given to be 213 kJ/kg and 1140 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (a) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\,\text{m})^2 = 28.27\,\text{m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(28.27\,\text{m}^2)} = 0.00101\,°\text{C/W}$$

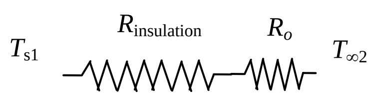


$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-183)]°\text{C}}{0.00101°\text{C/W}} = 196{,}040\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{196.040\ \text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.920\,kg/s}$$

(b) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\,\text{m})^2 = 30.19\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(30.19\,\text{m}^2)} = 0.000946\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\ \text{m}}{4\pi(0.035\ \text{W/m.°C})(1.55\,\text{m})(1.5\,\text{m})} = 0.0489\,°\text{C/W}$$

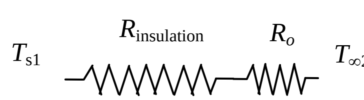
$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]°\text{C}}{0.0498\,°\text{C/W}} = 3976\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{3.976\ \text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.0187\,kg/s}$$

(c) The heat transfer rate and the rate of evaporation of the liquid with a 2-cm superinsulation is

$$A = \pi D^2 = \pi(3.04\,\text{m})^2 = 29.03\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(29.03\,\text{m}^2)} = 0.000984\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\ \text{m}}{4\pi(0.00005\ \text{W/m.°C})(1.52\,\text{m})(1.5\,\text{m})} = 13.96\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]°\text{C}}{13.96\,°\text{C/W}} = 14.18\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01418\ \text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.000067kg/s}$$

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Critical Radius of Insulation

3-96C In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of insulation, but decreases the convection resistance of the surface because of the increase in the outer surface area. Due to these opposite effects, a critical radius of insulation is defined as the outer radius that provides maximum rate of heat transfer. For a cylindrical layer, it is defined as $r_{cr} = k/h$ where k is the thermal conductivity of insulation and h is the external convection heat transfer coefficient.

3-97C For a cylindrical pipe, the critical radius of insulation is defined as $r_{cr} = k/h$. On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.

3-98C Yes, the measurements can be right. If the radius of insulation is less than critical radius of insulation of the pipe, the rate of heat loss will increase.

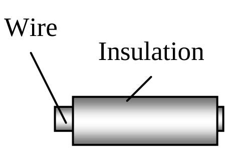
3-99C No.

3-100C It will decrease.

3-101E An electrical wire is covered with 0.02-in thick plastic insulation. It is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.



Analysis The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.075\,\text{Btu/h.ft.°F}}{2.5\,\text{Btu/h.ft}^2\text{.°F}} = 0.03\,\text{ft} = 0.36\,\text{in} > r_2 (= 0.0615\,\text{in})$$

Since the outer radius of the wire with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

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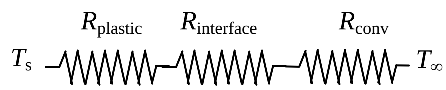
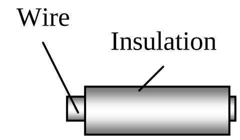
3-102E An electrical wire is covered with 0.02-in thick plastic insulation. By considering the effect of thermal contact resistance, it is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.

Analysis Without insulation, the total thermal resistance is (per ft length of the wire)





$$R_{\text{tot}} = R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\text{ Btu/h.ft}^2.°\text{F})[\pi(0.083/12\text{ ft})(1\text{ ft})]} = 18.4\text{ h.°F/Btu}$$

With insulation, the total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(2.5\text{ Btu/h.ft}^2.°\text{F})[\pi(0.123/12\text{ ft})(1\text{ ft})]} = 12.42\text{ h.°F/Btu}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(0.123/0.083)}{2\pi(0.075\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.835\text{ h.°F/Btu}$$

$$R_{\text{interface}} = \frac{h_c}{A_c} = \frac{0.001\text{ h.ft}^2.°\text{F/Btu}}{[\pi(0.083/12\text{ ft})(1\text{ ft})]} = 0.046\text{ h.°F/Btu}$$

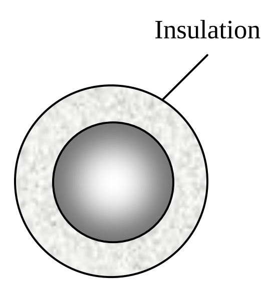
$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} + R_{\text{interface}} = 12.42 + 0.835 + 0.046 = 13.30\text{ h.°F/Btu}$$

Since the total thermal resistance decreases after insulation, plastic insulation **will increase** heat transfer from the wire. The thermal contact resistance appears to have negligible effect in this case.

3-103 A spherical ball is covered with 1-mm thick plastic insulation. It is to be determined if the plastic insulation on the ball will increase or decrease heat transfer from it.

Assumptions **1** Heat transfer from the ball is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.13 W/m·°C.



Analysis The critical radius of plastic insulation for the spherical ball is

$$r_{cr} = \frac{2k}{h} = \frac{2(0.13\text{ W/m.°C})}{20\text{ W/m}^2.°\text{C}} = 0.013\text{ m} = 13\text{ mm} > r_2 (= 3\text{ mm})$$

Since the outer radius of the ball with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

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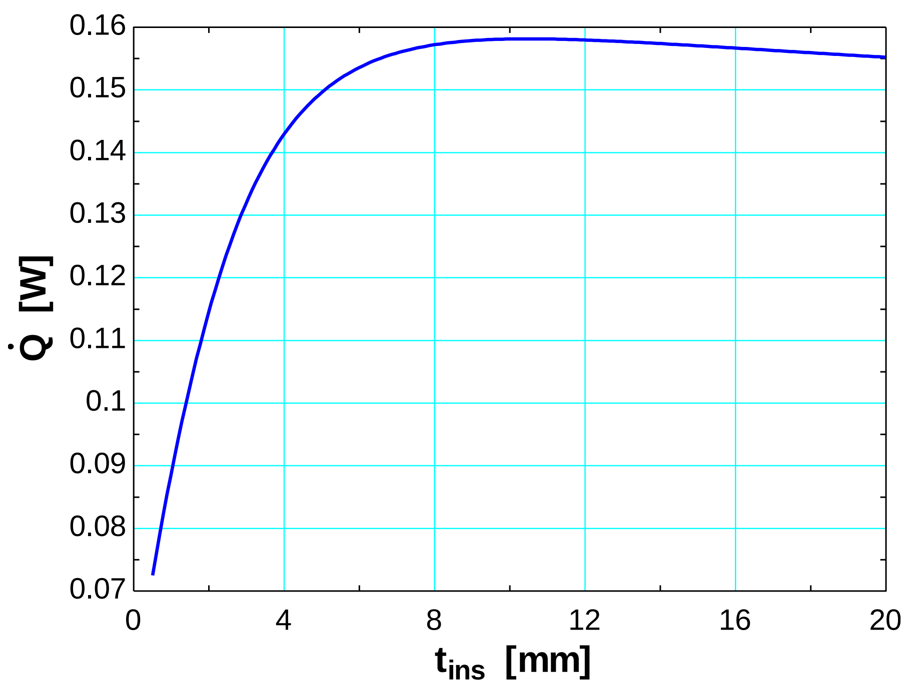
3-104 Prob. 3-103 is reconsidered. The rate of heat transfer from the ball as a function of the plastic insulation thickness is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_1=0.005 [m]
t_ins=1 [mm]
k_ins=0.13 [W/m-C]
T_ball=50 [C]
T_infinity=15 [C]
h_o=20 [W/m^2-C]

"ANALYSIS"
D_2=D_1+2*t_ins*Convert(mm, m)
A_o=pi*D_2^2
R_conv_o=1/(h_o*A_o)
R_ins=(r_2-r_1)/(4*pi*r_1*r_2*k_ins)
r_1=D_1/2
r_2=D_2/2
R_total=R_conv_o+R_ins
Q_dot=(T_ball-T_infinity)/R_total
```

t_{ins} [mm]	Q [W]
0.5	0.07248
1.526	0.1035
2.553	0.1252
3.579	0.139
4.605	0.1474
5.632	0.1523
6.658	0.1552
7.684	0.1569
8.711	0.1577
9.737	0.1581
10.76	0.1581
11.79	0.158
12.82	0.1578
13.84	0.1574
14.87	0.1571
15.89	0.1567
16.92	0.1563
17.95	0.1559
18.97	0.1556
20	0.1552



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Heat Transfer from Finned Surfaces

3-105C Fins should be attached to the outside since the heat transfer coefficient inside the tube will be higher due to forced convection. Fins should be added to both sides of the tubes when the convection coefficients at the inner and outer surfaces are comparable in magnitude.

3-106C Increasing the rate of heat transfer from a surface by increasing the heat transfer surface area.

3-107C The fin efficiency is defined as the ratio of actual heat transfer rate from the fin to the ideal heat transfer rate from the fin if the entire fin were at base temperature, and its value is between 0 and 1. Fin effectiveness is defined as the ratio of heat transfer rate from a finned surface to the heat transfer rate from the same surface if there were no fins, and its value is expected to be greater than 1.

3-108C Heat transfer rate will decrease since a fin effectiveness smaller than 1 indicates that the fin acts as insulation.

3-109C Fins enhance heat transfer from a surface by increasing heat transfer surface area for convection heat transfer. However, adding too many fins on a surface can suffocate the fluid and retard convection, and thus it may cause the overall heat transfer coefficient and heat transfer to decrease.

3-110C Effectiveness of a single fin is the ratio of the heat transfer rate from the entire exposed surface of the fin to the heat transfer rate from the fin base area. The overall effectiveness of a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

3-111C Fins should be attached on the air side since the convection heat transfer coefficient is lower on the air side than it is on the water side.

3-112C Welding or tight fitting introduces thermal contact resistance at the interface, and thus retards heat transfer. Therefore, the fins formed by casting or extrusion will provide greater enhancement in heat transfer.

3-113C If the fin is too long, the temperature of the fin tip will approach the surrounding temperature and we can neglect heat transfer from the fin tip. Also, if the surface area of the fin tip is very small compared to the total surface area of the fin, heat transfer from the tip can again be neglected.

3-114C Increasing the length of a fin decreases its efficiency but increases its effectiveness.

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3-115C Increasing the diameter of a fin increases its efficiency but decreases its effectiveness.

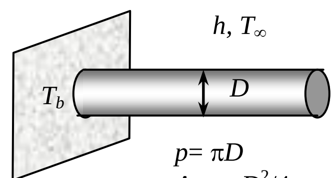
3-116C The thicker fin has higher efficiency; the thinner one has higher effectiveness.

3-117C The fin with the lower heat transfer coefficient has the higher efficiency and the higher effectiveness.

3-118 A relation is to be obtained for the fin efficiency for a fin of constant cross-sectional area A_c, perimeter *p*, length *L*, and thermal conductivity *k* exposed to convection to a medium at T_∞ with a heat transfer coefficient *h*. The relation is to be simplified for circular fin of diameter *D* and for a rectangular fin of thickness *t*.

Assumptions **1** The fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. **2** Heat transfer from the fin tips is negligible.

Analysis Taking the temperature of the fin at the base to be T_b and using the heat transfer relation for a long fin, fin efficiency for long fins can be expressed as



$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{l}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}}$$

$$= \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}}$$

This relation can be simplified for a circular fin of diameter *D* and rectangular fin of thickness t and width w to be

$$\eta_{\text{fin,circular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L}\sqrt{\frac{kD}{h}}$$

$$\eta_{\text{fin,rectangular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(wt)}{2(w+t)h}} \cong \frac{1}{L}\sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L}\sqrt{\frac{kt}{2h}}$$

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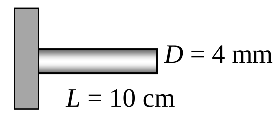
3-119 A fin is attached to a surface. The percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum fin is given to be k = 237 W/m·°C.

Analysis The expressions for the heat transfer from a fin under infinitely long fin and adiabatic fin tip assumptions are

$$\dot{Q}_{\text{longfin}} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$
$$\dot{Q}_{\text{ins.tip}} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)$$



The percent error in using long fin assumption can be expressed as

$$\%\,\text{Error} = \frac{\dot{Q}_{\text{longfin}} - \dot{Q}_{\text{ins.tip}}}{\dot{Q}_{\text{ins.tip}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty) - \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)}{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)} = \frac{1}{\tanh(mL)} - 1$$

where

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(12\text{ W/m}^2.°\text{C})\pi(0.004\text{ m})}{(237\text{ W/m.°C})\pi(0.004\text{ m})^2/4}} = 7.116\text{ m}^{-1}$$

Substituting,

$$\%\,\text{Error} = \frac{1}{\tanh(mL)} - 1 = \frac{1}{\tanh\left[(7.116\text{ m}^{-1})(0.10\text{ m})\right]} - 1 = 0.635 = \mathbf{63.5\%}$$

This result shows that using infinitely long fin assumption may yield results grossly in error.

3-120 A very long fin is attached to a flat surface. The fin temperature at a certain distance from the base and the rate of heat loss from the entire fin are to be determined.

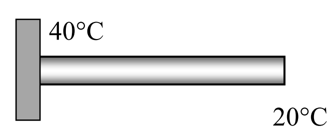
Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the fin is given to be k = 200 W/m·°C.

Analysis The fin temperature at a distance of 5 cm from the base is determined from

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(20\text{ W/m}^2.°\text{C})(2\times0.05+2\times0.001)\text{m}}{(200\text{ W/m.°C})(0.05\times0.001)\text{m}^2}} = 14.3\text{ m}^{-1}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \longrightarrow \frac{T - 20}{40 - 20} = e^{-(14.3)(0.05)} \longrightarrow T = \mathbf{29.8°C}$$



The rate of heat loss from this very long fin is

$$\begin{aligned}\dot{Q}_{\text{longfin}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\\ &= \sqrt{(20)(2\times0.05+2\times0.001)(200(0.05\times0.001)}\,(40-20)\\ &= \mathbf{2.9\ W}\end{aligned}$$

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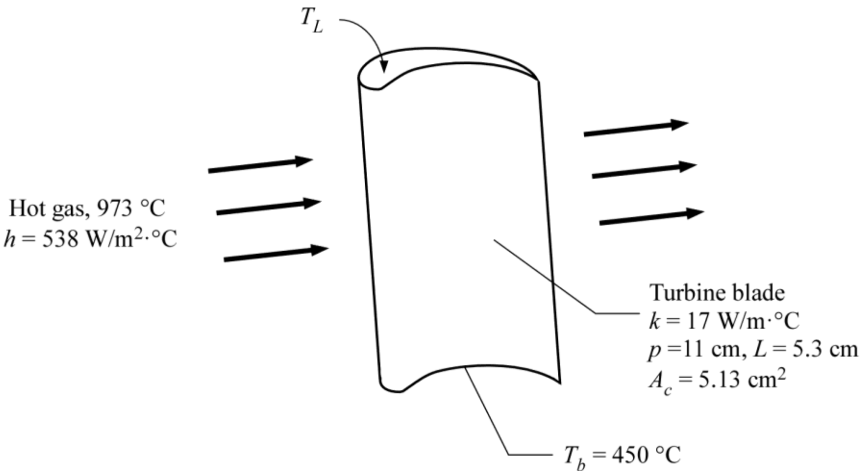
3-121 A turbine blade is exposed to hot gas from the combustion chamber. The heat transfer rate to the turbine blade and the temperature at the tip are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The cross-sectional area of the turbine blade is uniform.

Properties The thermal conductivity of the turbine blade is given as 17 W/m · °C.



Analysis The turbine blade can be treated as a uniform cross section fin with adiabatic tip. The heat transfer rate to the turbine blade can be expressed as

$$\dot{Q}_{\text{blade}} = \sqrt{hpkA_c}\,(T_\infty - T_b)\tanh mL$$

where

$$mL = \left(\frac{hp}{kA_c}\right)^{0.5} L = \left[\frac{(538\text{ W/m}^2\cdot{}^\circ\text{C})(0.11\text{ m})}{(17\text{ W/m}\cdot{}^\circ\text{C})(5.13\times10^{-4}\text{ m}^2)}\right]^{0.5}(0.053\text{ m}) = 4.366$$

$$\sqrt{hpkA_c} = \sqrt{(538\text{ W/m}^2\cdot{}^\circ\text{C})(0.11\text{ m})(17\text{ W/m}\cdot{}^\circ\text{C})(5.13\times10^{-4}\text{ m}^2)} = 0.7184\text{ W/}^\circ\text{C}$$

The heat transfer rate to the turbine blade is

$$\dot{Q}_{\text{blade}} = (0.7184\text{ W/}^\circ\text{C})(973-450)\,^\circ\text{C}(\tanh 4.366) = \mathbf{376\ W}$$

For adiabatic tip, the temperature distribution is expressed as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

The temperature at the tip of the turbine blade is

$$T_L = \frac{T_b - T_\infty}{\cosh mL} + T_\infty = \frac{(450-973)\,^\circ\text{C}}{\cosh 4.366} + 973\,^\circ\text{C} = \mathbf{960\,^\circ C}$$

Discussion The tolerance of the turbine blade to high temperature can be increased by applying Zirconia based thermal barrier coatings (TBCs) on the blade surface.

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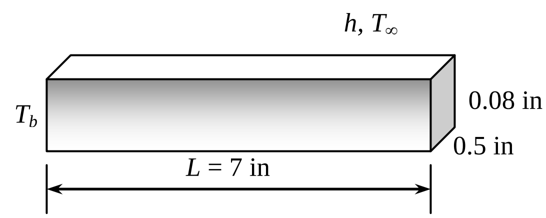
3-122E The handle of a stainless steel spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon.

Properties The thermal conductivity of the spoon is given to be k = 8.7 Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)}{\cosh mL}$$



where

$$p=2(0.5/12\text{ ft}+0.08/12\text{ ft})=0.0967\text{ ft}$$

$$A_c=(0.5/12\text{ ft})(0.08/12\text{ ft})=0.000278\text{ ft}^2$$

$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3\text{ Btu/h.ft}^2.°\text{F})(0.0967\text{ ft})}{(8.7\text{ Btu/h.ft.°F})(0.000278\text{ ft}^2)}}=10.95\text{ ft}^{-1}$$

Noting that $x = L$ = 7/12=0.583 ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$\begin{aligned}T(L)&=T_\infty+(T_b-T_\infty)\frac{\cosh m(L-L)}{\cosh mL}\\&=75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(10.95\times0.583)}\\&=75°\text{F}+(200-75)\frac{1}{296}\\&=75.4°\text{F}\end{aligned}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

$$\Delta T=T_b-T_{\text{tip}}=(200-75.4)°\text{F}=\mathbf{124.6°F}$$

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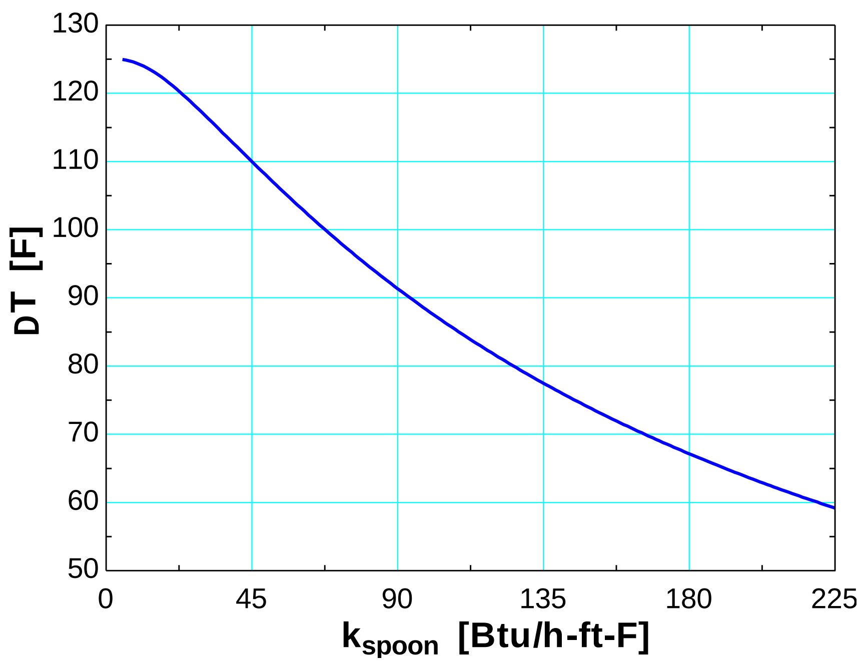
3-123E Prob. 3-122E is reconsidered. The effects of the thermal conductivity of the spoon material and the length of its extension in the air on the temperature difference across the exposed surface of the spoon handle are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k_spoon=8.7 [Btu/h-ft-F]
T_w=200 [F]
T_infinity=75 [F]
A_c=(0.08/12*0.5/12) [ft^2]
L=7 [in]
h=3 [Btu/h-ft^2-F]

"ANALYSIS"
p=2*(0.08/12+0.5/12)
a=sqrt((h*p)/(k_spoon*A_c))
(T_tip-T_infinity)/(T_w-T_infinity)=cosh(a*(L-x)*Convert(in, ft))/cosh(a*L*Convert(in, ft))
x=L "for tip temperature"
DELTAT=T_w-T_tip
```

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	124.9
16.58	122.6
28.16	117.8
39.74	112.5
51.32	107.1
62.89	102
74.47	97.21
86.05	92.78
97.63	88.69
109.2	84.91
120.8	81.42
132.4	78.19
143.9	75.19
155.5	72.41
167.1	69.82
178.7	67.4
190.3	65.14
201.8	63.02
213.4	61.04
225	59.17



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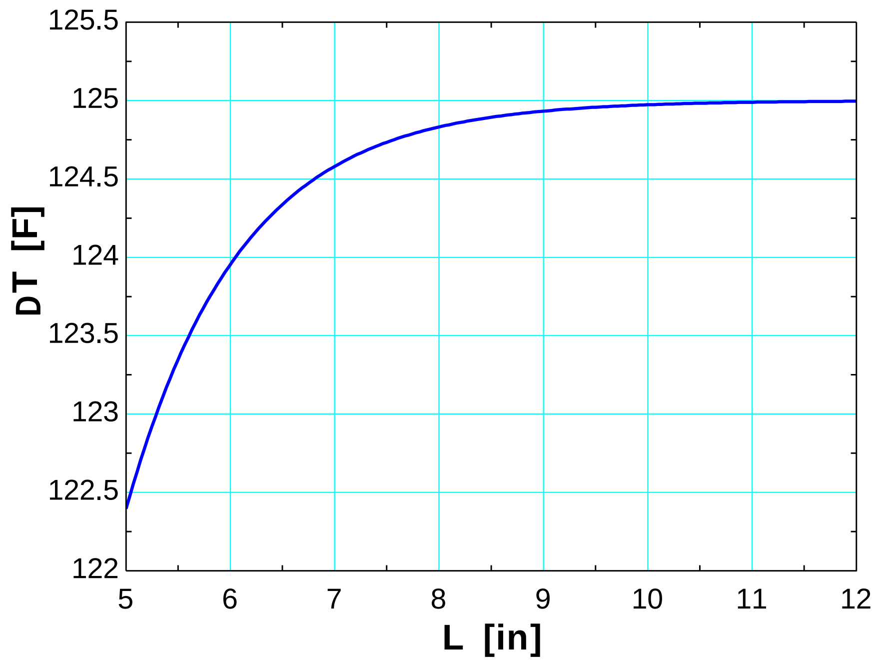
L [in]	ΔT [F]
5	122.4
5.5	123.4
6	124
6.5	124.3
7	124.6
7.5	124.7
8	124.8
8.5	124.9
9	124.9
9.5	125
10	125
10.5	125
11	125
11.5	125
12	125



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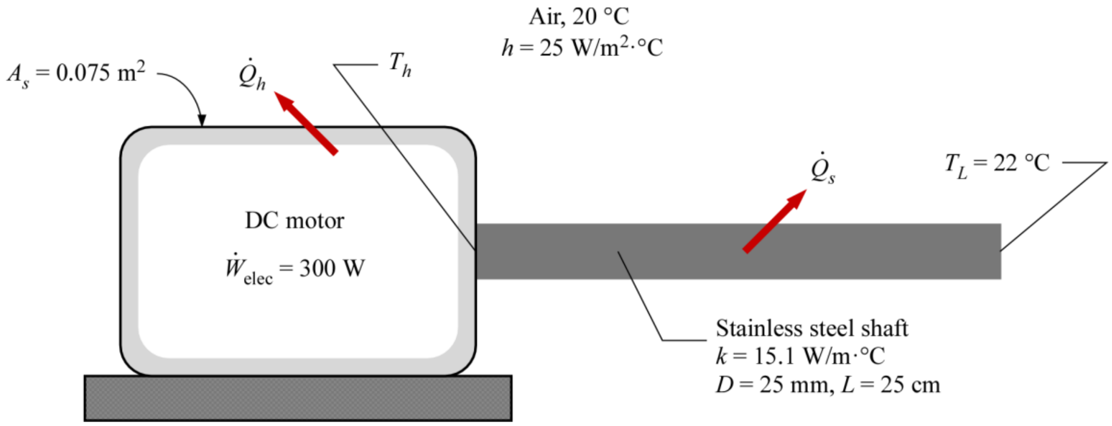
3-124 A DC motor draws electrical power and delivers mechanical power to rotate a stainless steel shaft. The surface temperature of the motor housing is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The surface temperature of the motor housing is uniform. **5** The base temperature of the shaft is equal to the surface temperature of the motor housing.

Properties The thermal conductivity of the stainless steel shaft is given as 15.1 W/m · °C.



Analysis From energy balance, the following equation is expressed:

$$\dot{W}_{\text{elec}} = \dot{W}_{\text{mech}} + \dot{Q}_h + \dot{Q}_s \qquad \text{or} \qquad \dot{W}_{\text{elec}} = 0.55\dot{W}_{\text{elec}} + \dot{Q}_h + \dot{Q}_s$$

The heat transfer rate from the motor housing surface is

$$\dot{Q}_h = hA_s(T_h - T_\infty)$$

The motor shaft can be treated as a circular fin with a specified fin tip temperature. The heat transfer rate from the motor shaft can be written as

$$\dot{Q}_s = \sqrt{hpkA_c}\,(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

$$= \sqrt{hkD^3\frac{\pi^2}{4}}(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

where

$$mL = \left(\frac{hp}{kA_c}\right)^{0.5} L = \left(\frac{4h}{kD}\right)^{0.5} L = \left[\frac{4(25\ \text{W/m}^2\cdot{}^\circ\text{C})}{(15.1\ \text{W/m}\cdot{}^\circ\text{C})(0.025\ \text{m})}\right]^{0.5}(0.25\ \text{m}) = 4.069$$

$$\sqrt{hk\frac{\pi^2}{4}D^3} = \sqrt{(25\ \text{W/m}^2\cdot{}^\circ\text{C})(15.1\ \text{W/m}\cdot{}^\circ\text{C})(0.025\ \text{m})^3\frac{\pi^2}{4}} = 0.1206\ \text{W/}^\circ\text{C}$$

Substituting the listed terms into the energy balance equation we get

$$0.45\dot{W}_{\text{elec}} = hA_s(T_h - T_\infty) + \sqrt{hk\frac{\pi^2}{4}D^3}\,(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

Rearranging the equation, the surface temperature of the motor housing is

$$T_h = T_\infty + \frac{0.45\dot{W}_{\text{elec}} + \sqrt{hk\frac{\pi^2}{4}D^3}\,\dfrac{(T_L - T_\infty)}{\sinh mL}}{hA_s + \sqrt{hk\frac{\pi^2}{4}D^3}\left(\dfrac{\cosh mL}{\sinh mL}\right)} = 20\,{}^\circ\text{C} + \frac{0.45(300\ \text{W}) + (0.1206\ \text{W/}^\circ\text{C})\dfrac{(22-20)\,{}^\circ\text{C}}{\sinh 4.069}}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})(0.075\ \text{m}^2) + (0.1206\ \text{W/}^\circ\text{C})\left(\dfrac{\cosh 4.069}{\sinh 4.069}\right)} = \mathbf{87.7\,{}^\circ C}$$

Discussion If the surface of the motor housing has a high emissivity, heat transfer by radiation from the motor housing would decrease the surface temperature.

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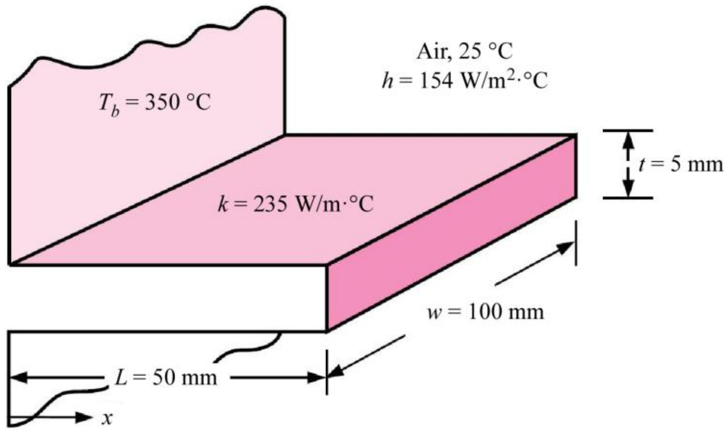
3-125 Using Table 3-3 and Figure 3-43, the efficiency, heat transfer rate, and effectiveness of a straight rectangular fin are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 235 W/m · °C.



Analysis (a) From Table 3-3, for straight rectangular fins, we have

$$m = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(154\ \text{W/m}^2\cdot°\text{C})}{(235\ \text{W/m}\cdot°\text{C})(0.005\ \text{m})}} = 16.19\ \text{m}^{-1}$$

$$L_c = L + t/2 = (0.05\ \text{m}) + (0.005\ \text{m})/2 = 0.0525\ \text{m}$$

$$A_{\text{fin}} = 2wL_c = 2(0.1\ \text{m})(0.0525\ \text{m}) = 0.0105\ \text{m}^2$$

The fin efficiency is

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c} = \frac{\tanh\left[(16.19\ \text{m}^{-1})(0.0525\ \text{m})\right]}{(16.19\ \text{m}^{-1})(0.0525\ \text{m})} = \mathbf{0.813}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.813)(154\ \text{W/m}^2\cdot°\text{C})(0.0105\ \text{m}^2)(350-25)\ °\text{C} = \mathbf{427\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{427\ \text{W}}{(154\ \text{W/m}^2\cdot°\text{C})(0.005\ \text{m})(0.1\ \text{m})(350-25)\ °\text{C}} = \mathbf{17.1}$$

(b) To use Figure 3-43, we need

$$L_c = 0.0525\ \text{m} \quad \text{and} \quad A_p = L_c t$$

Hence,

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.0525\ \text{m})^{3/2}\left[\frac{154\ \text{W/m}^2\cdot°\text{C}}{(235\ \text{W/m}\cdot°\text{C})(0.0525\ \text{m})(0.005\ \text{m})}\right]^{1/2} \approx 0.60$$

Using Figure 3-43, the fin efficiency is

$$\eta_f \approx \mathbf{0.81}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.81)(154\ \text{W/m}^2\cdot°\text{C})(0.0105\ \text{m}^2)(350-25)\ °\text{C} = \mathbf{426\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{426\ \text{W}}{(154\ \text{W/m}^2\cdot°\text{C})(0.005\ \text{m})(0.1\ \text{m})(350-25)\ °\text{C}} = \mathbf{17.0}$$

Discussion The results determined using Table 3-3 and Figure 3-43 are very comparable. However, it should be noted that results determined using Table 3-3 are more accurate.

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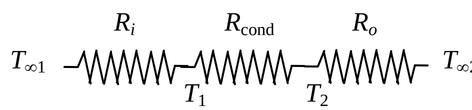
3-126 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be k = 52 W/m·°C.

Analysis (*a*) We treat the flanges as fins. The individual thermal resistances are



$$A_i = \pi D_i L = \pi(0.092\text{ m})(6\text{ m}) = 1.734\text{ m}^2$$

$$A_o = \pi D_o L = \pi(0.1\text{ m})(6\text{ m}) = 1.885\text{ m}^2$$

$$R_{\text{i}} = \frac{1}{h_i A_i} = \frac{1}{(180\text{ W/m}^2.°\text{C})(1.734\text{ m}^2)} = 0.003204\ °\text{C/W}$$

$$R_{\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\text{ W/m.°C})(6\text{ m})} = 0.000043\ °\text{C/W}$$

$$R_{\text{o}} = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(1.885\text{ m}^2)} = 0.021220\ °\text{C/W}$$

$$R_{\text{total}} = R_{\text{i}} + R_{\text{cond}} + R_{\text{o}} = 0.003204 + 0.000043 + 0.021220 = 0.02447\ °\text{C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total}}} = \frac{(200-12)°\text{C}}{0.02447\ °\text{C}} = 7684\text{ W}$$

$$\dot{Q} = \frac{T_2 - T_{\infty2}}{R_{\text{o}}} \longrightarrow T_2 = T_{\infty2} + \dot{Q}R_{\text{o}} = 12\ °\text{C} + (7684\text{ W})(0.021220\ °\text{C/W}) = \mathbf{175.0°C}$$

(*b*) The fin efficiency can be determined from (Fig. 3-44)

$$\left.\begin{aligned} &\frac{r_2 + \frac{t}{2}}{r_1} = \frac{0.1 + \frac{0.02}{2}}{0.05} = 2.2 \\ &\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.05\text{ m} + \frac{0.02}{2}\text{ m}\right)\sqrt{\frac{25\text{ W/m}^2\,°\text{C}}{(52\text{ W/m°C})(0.02\text{ m})}} = 0.29 \end{aligned}\right\}\eta_{\text{fin}} = \mathbf{0.93}$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.1\text{ m})^2 - (0.05\text{ m})^2] + 2\pi(0.1\text{ m})(0.02\text{ m}) = 0.05969\text{ m}^2$$

The heat transfer rate from the flanges is

$$\begin{aligned} \dot{Q}_{\text{finned}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= 0.93(25\text{ W/m}^2.°\text{C})(0.05969\text{ m}^2)(175.0-12)°\text{C} = \mathbf{226\ W} \end{aligned}$$

(*c*) A 6-m long section of the steam pipe is losing heat at a rate of 7684 W or 7684/6 = 1281 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent length} = \frac{226\text{ W}}{1281\text{ W/m}} = 0.176\text{ m} = \mathbf{17.6\ cm}$$

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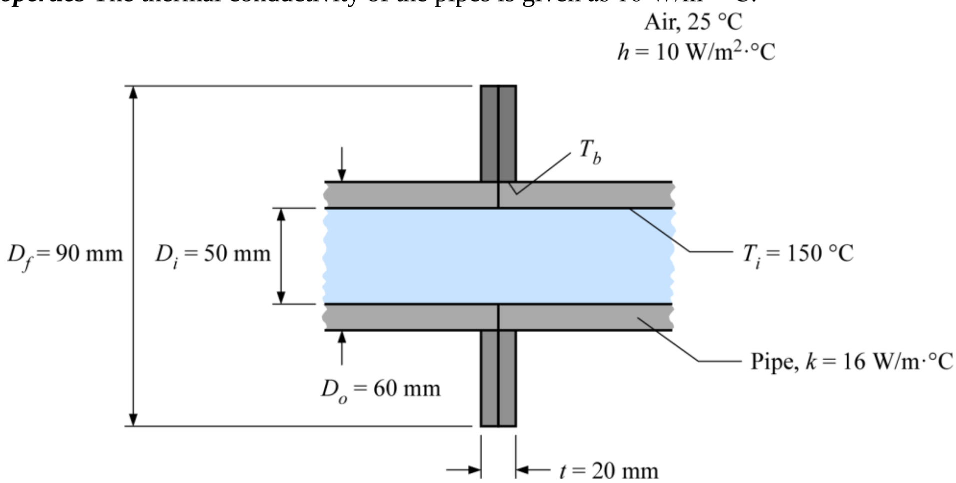
3-127 Pipes used for transporting superheated vapor are connected together by flanges. The temperature at the base of the flange and the rate of heat loss through the flange are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The flanges profile is similar to circular fins of rectangular profile.

Properties The thermal conductivity of the pipes is given as 16 W/m · °C.



Analysis The heat transfer rate through the pipe wall is equal to the heat transfer rate through the flanges:

$$\dot{Q}_{\text{pipe}} = \dot{Q}_f \qquad \text{or} \qquad 2tk\pi \frac{T_i - T_b}{\ln(D_o / D_i)} = \eta_f hA_f (T_b - T_\infty)$$

Rearranging the equation yields

$$T_b = \frac{\eta_f hA_f T_\infty + \dfrac{2tk\pi}{\ln(D_o / D_i)} T_i}{\eta_f hA_f + \dfrac{2tk\pi}{\ln(D_o / D_i)}}$$

From Table 3-3, for circular fins of rectangular profile we have

$$r_{2c} = r_2 + t/2 = \frac{0.09\text{ m}}{2} + \frac{0.02\text{ m}}{2} = 0.055\text{ m}$$

$$A_f = 2\pi(r_{2c}^2 - r_1^2) = 2\pi[(0.055\text{ m})^2 - (0.05/2\text{ m})^2] = 0.0151\text{ m}^2$$

$$L_c = L + t/2 = \frac{0.09\text{ m} - 0.06\text{ m}}{2} + \frac{0.02\text{ m}}{2} = 0.025\text{ m}$$

$$A_p = L_c t = (0.025\text{ m})(0.02\text{ m}) = 0.0005\text{ m}^2$$

Hence,

$$\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.025\text{ m})^{3/2}\left[\frac{10\text{ W/m}^2\cdot\text{°C}}{(16\text{ W/m}\cdot\text{°C})(0.0005\text{ m}^2)}\right]^{1/2} = 0.1398$$

$$r_{2c}/r_1 = \frac{0.055\text{ m}}{0.030\text{ m}} = 1.83$$

Using Figure 3-44, the fin efficiency is $\eta_f \approx 0.97$. The temperature at the base of the flange is

$$T_b = \frac{(0.97)(10\text{ W/m}^2\cdot\text{°C})(0.0151\text{ m}^2)(25\text{ °C}) + \dfrac{2(0.02\text{ m})(16\text{ W/m}\cdot\text{°C})\pi}{\ln(60/50)}(150\text{ °C})}{(0.97)(10\text{ W/m}^2\cdot\text{°C})(0.0151\text{ m}^2) + \dfrac{2(0.02\text{ m})(16\text{ W/m}\cdot\text{°C})\pi}{\ln(60/50)}} = \mathbf{148\text{ °C}}$$

The rate of heat loss through the flange is

$$\dot{Q}_f = \eta_f hA_f (T_b - T_\infty) = (0.97)(10\text{ W/m}^2\cdot\text{°C})(0.0151\text{ m}^2)(148 - 25)\text{ °C} = \mathbf{18\text{ W}}$$

Discussion The flanges act as extended surfaces, which enhanced heat transfer from the pipes.

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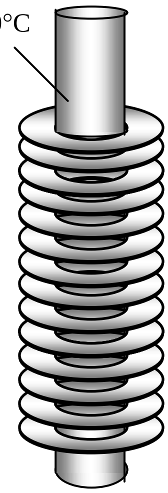
3-128 Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 186 W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is



$$A_{\text{no fin}} = \pi D_1 L = \pi(0.05\,\text{m})(1\,\text{m}) = 0.1571\,\text{m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (40\ \text{W/m}^2.°\text{C})(0.1571\,\text{m}^2)(180-25)°\text{C} = 974\ \text{W}$$

The efficiency of these circular fins is, from the efficiency curve, Fig. 3-43

$$\left.\begin{aligned} L &= (D_2 - D_1)/2 = (0.06-0.05)/2 = 0.005\,\text{m} \\ \frac{r_2+(t/2)}{r_1} &= \frac{0.03+(0.001/2)}{0.025} = 1.22 \\ L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} &= \left(L+\frac{t}{2}\right)\sqrt{\frac{h}{kt}} \\ &= \left(0.005+\frac{0.001}{2}\right)\sqrt{\frac{40\ \text{W/m}^2\,{}^\circ\text{C}}{(186\ \text{W/m}^\circ\text{C})(0.001\,\text{m})}} = 0.08 \end{aligned}\right\}\eta_{\text{fin}} = 0.97$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.001916\,\text{m}^2$$
$$\begin{aligned}\dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= 0.97(40\ \text{W/m}^2.°\text{C})(0.001916\,\text{m}^2)(180-25)°\text{C} \\ &= 11.52\ \text{W}\end{aligned}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.05\,\text{m})(0.003\,\text{m}) = 0.0004712\,\text{m}^2$$
$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (40\ \text{W/m}^2.°\text{C})(0.0004712\,\text{m}^2)(180-25)°\text{C} = 2.92\ \text{W}$$

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(11.52 + 2.92) = 3610\ \text{W}$$

Therefore the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 3610 - 974 = \mathbf{2636\ W}$$

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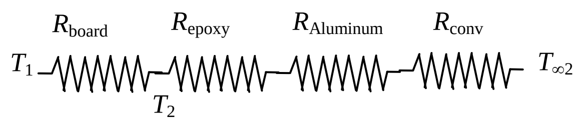
3-129 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 aluminum pin fins on the back surface.

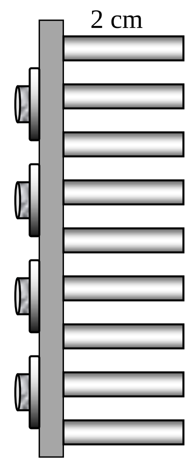
Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 30 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (a) The total rate of heat transfer dissipated by the chips is

$$\dot{Q} = 80 \times (0.04\text{ W}) = 3.2\text{ W}$$





The individual resistances are

$$A = (0.12\text{ m})(0.18\text{ m}) = 0.0216\text{ m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.003\text{ m}}{(30\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00463\text{ °C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(40\text{ W/m}^2\text{.°C})(0.0216\text{ m}^2)} = 1.15741\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00463 + 1.15741 = 1.1620\text{ °C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2\text{ W})(1.1620\text{ °C/W}) = 43.72°\text{C} \cong \mathbf{43.7°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 43.72°\text{C} - (3.2\text{ W})(0.00463\text{ °C/W}) = 43.71°\text{C} \cong \mathbf{43.7°C}$$

Therefore, the board is nearly isothermal.

(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(40\text{ W/m}^2\text{.°C})}{(237\text{ W/m.°C})(0.0025\text{ m})}} = 16.43\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh mL}{mL} = \frac{\tanh(16.43\text{ m}^{-1} \times 0.02\text{ m})}{16.43\text{ m}^{-1} \times 0.02\text{ m}} = 0.9655$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.9655. Then the various thermal resistances are

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002\text{ m}}{(1.8\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00514\text{ °C/W}$$

$$R_{\text{Al}} = \frac{L}{kA} = \frac{0.002\text{ m}}{(237\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00039\text{ °C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}} n\pi DL = 0.9655 \times 864\pi(0.0025\text{ m})(0.02\text{ m}) = 0.1310\text{ m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864\frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi(0.0025)^2}{4} = 0.0174\text{ m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.1310 + 0.0174 = 0.1484\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_{\text{total,with fins}}} = \frac{1}{(40\text{ W/m}^2\text{.°C})(0.1484\text{ m}^2)} = 0.1685\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{aluminum}} + R_{\text{conv}} = 0.00463 + 0.00514 + 0.00039 + 0.1685 = 0.1787\text{ °C/W}$$

Then the temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40°\text{C} + (3.2\text{ W})(0.1787\text{ °C/W}) = 40.57°\text{C} \cong \mathbf{40.6°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 40.57°\text{C} - (3.2\text{ W})(0.00463\text{ °C/W}) = 40.56 \cong \mathbf{40.6°C}$$

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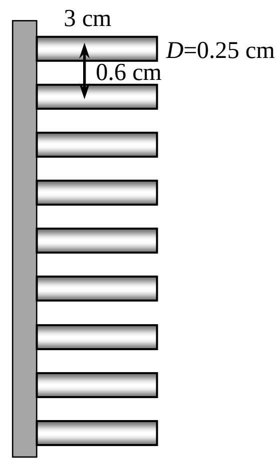
3-130 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum plate and fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be



$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{h\pi D}{k\pi D^2/4}}=\sqrt{\frac{4h}{kD}}=\sqrt{\frac{4(35\text{ W/m}^2.°\text{C})}{(237\text{ W/m.°C})(0.0025\text{ m})}}=15.37\text{ m}^{-1}$$

$$\eta_{\text{fin}}=\frac{\tanh mL}{mL}=\frac{\tanh(15.37\text{ m}^{-1}\times 0.03\text{ m})}{15.37\text{ m}^{-1}\times 0.03\text{ m}}=0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n=\frac{1\text{ m}^2}{(0.006\text{ m})(0.006\text{ m})}=27{,}777$$

$$A_{\text{fin}}=27777\left[\pi DL+\frac{\pi D^2}{4}\right]=27777\left[\pi(0.0025)(0.03)+\frac{\pi(0.0025)^2}{4}\right]$$
$$=6.68\text{ m}^2$$

$$A_{\text{unfinned}}=1-27777\left(\frac{\pi D^2}{4}\right)=1-27777\left[\frac{\pi(0.0025)^2}{4}\right]=0.86\text{ m}^2$$

$$\dot{Q}_{\text{finned}}=\eta_{\text{fin}}\dot{Q}_{\text{fin,max}}=\eta_{\text{fin}}hA_{\text{fin}}(T_b-T_\infty)$$
$$=0.935(35\text{ W/m}^2.°\text{C})(6.68\text{ m}^2)(100-30)°\text{C}$$
$$=15{,}300\text{ W}$$

$$\dot{Q}_{\text{unfinned}}=hA_{\text{unfinned}}(T_b-T_\infty)=(35\text{ W/m}^2.°\text{C})(0.86\text{ m}^2)(100-30)°\text{C}$$
$$=2107\text{ W}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}}=\dot{Q}_{\text{finned}}+\dot{Q}_{\text{unfinned}}=15{,}300+2107=1.74\times 10^4\text{ W}=\mathbf{17.4\,kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$A_{\text{no fin}}=(1\text{ m})(1\text{ m})=1\text{ m}^2$$
$$\dot{Q}_{\text{no fin}}=hA_{\text{no fin}}(T_b-T_\infty)=(35\text{ W/m}^2.°\text{C})(1\text{ m}^2)(100-30)°\text{C}=2450\text{ W}$$

Then the fin effectiveness becomes

$$\varepsilon_{\text{fin}}=\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}=\frac{17{,}400}{2450}=\mathbf{7.10}$$

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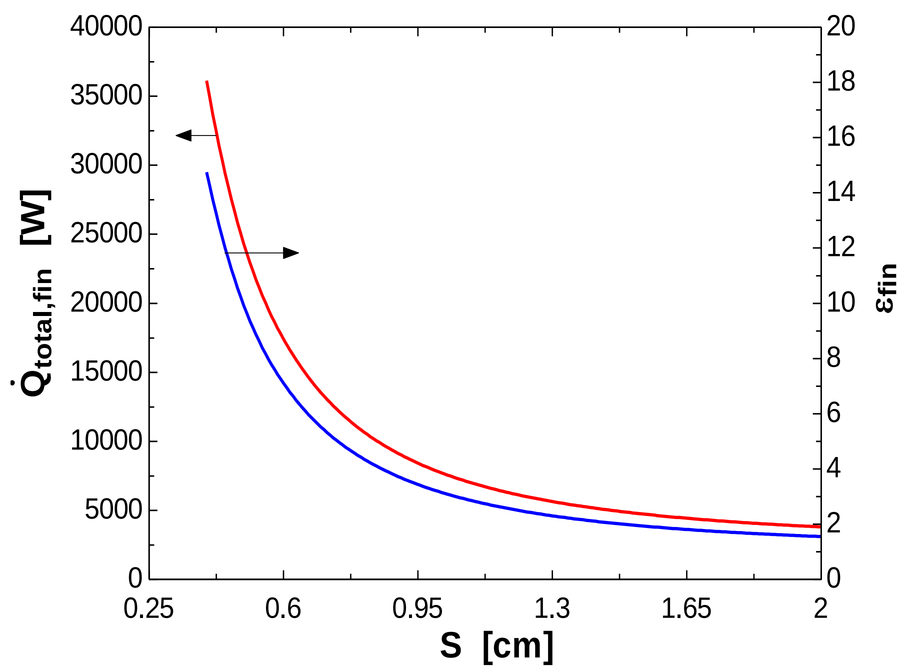
3-131 Prob. 3-130 is reconsidered. The effect of the center-to center distance of the fins on the rate of heat transfer from the surface and the overall effectiveness of the fins is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_b=100 [C]
L=0.03 [m]
D=0.0025 [m]
k=237 [W/m-C]
S=0.6 [cm]
T_infinity=30 [C]
h=35 [W/m^2-C]
A_surface=1*1 [m^2]

"ANALYSIS"
p=pi*D
A_c=pi*D^2/4
a=sqrt((h*p)/(k*A_c))
eta_fin=tanh(a*L)/(a*L)
n=A_surface/(S^2*Convert(cm^2, m^2)) "number of fins"
A_fin=n*(pi*D*L+pi*D^2/4)
A_unfinned=A_surface-n*(pi*D^2/4)
Q_dot_finned=eta_fin*h*A_fin*(T_b-T_infinity)
Q_dot_unfinned=h*A_unfinned*(T_b-T_infinity)
Q_dot_total_fin=Q_dot_finned+Q_dot_unfinned
Q_dot_nofin=h*A_surface*(T_b-T_infinity)
epsilon_fin=Q_dot_total_fin/Q_dot_nofin
```

S [cm]	$Q_{total\ fin}$ [W]	ε_{fin}
0.4	36123	14.74
0.5	24001	9.796
0.6	17416	7.108
0.7	13445	5.488
0.8	10868	4.436
0.9	9101	3.715
1	7838	3.199
1.1	6903	2.817
1.2	6191	2.527
1.3	5638	2.301
1.4	5199	2.122
1.5	4845	1.977
1.6	4555	1.859
1.7	4314	1.761
1.8	4113	1.679
1.9	3942	1.609
2	3797	1.55



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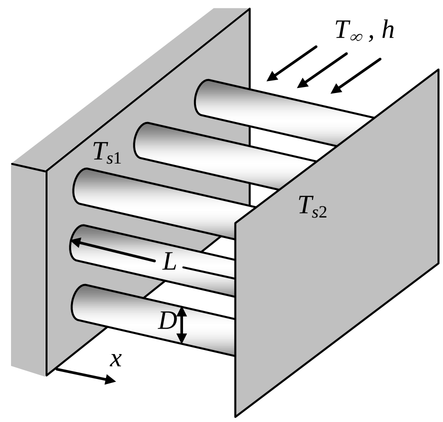
3-132 Circular fins made of copper are considered. The function $\theta(x) = T(x) - T_\infty$ along a fin is to be expressed and the temperature at the middle is to be determined. Also, the rate of heat transfer from each fin, the fin effectiveness, and the total rate of heat transfer from the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire finned and unfinned wall surfaces. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the copper fin is given to be k = 400 W/m·°C.

Analysis (*a*)



For $x = L/2$:

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(100)\pi(0.001)}{(400)\pi(0.001)^2/4}} = 31.62\ \text{m}^{-1}$$

Noting that $T_b - T_\infty = T_{s1}$ and $T_L - T_\infty = 0$,

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\left(\dfrac{T_L - T_\infty}{T_b - T_\infty}\right)\sinh(mx) + \sinh m(L-x)}{\sinh mL} = \frac{\sinh[m(L-x)]}{\sinh mL}$$

$$\frac{T(L/2)-0}{132-0} = \frac{\sinh[m(L-x)]}{\sinh mL}$$

$$T(L/2) = 132\frac{\sinh[31.62(0.0254-0.015)]}{\sinh(31.62\times 0.0254)} = \mathbf{49.5^\circ C}$$

(*b*) The rate of heat transfer from a single fin is

$$\dot{Q}_{\text{onefin}} = (T_b - T_\infty)\sqrt{hpkA_c}\ \frac{\cosh(mL) - \left(\dfrac{T_L - T_\infty}{T_b - T_\infty}\right)}{\sinh(mL)}$$

$$= (132-0)\sqrt{(100)\pi(0.001)(400)\pi(0.001)^2/4}\ \frac{\cosh(31.62\times 0.0254)-0}{\sinh(31.62\times 0.0254)}$$

$$= \mathbf{1.970\ W}$$

The effectiveness of the fin is

$$\varepsilon = \frac{\dot{Q}}{hA_c(T_b - T_\infty)} = \frac{1.970}{(100)0.25\pi(0.001)^2(132-0)} = \mathbf{190}$$

Since ε >> 2, the fins are well justified.

(*c*) The total rate of heat transfer is

$$\begin{aligned}\dot{Q}_{\text{total}} &= \dot{Q}_{\text{fins}} + \dot{Q}_{\text{base}}\\ &= n_{\text{fin}}\dot{Q}_{\text{onefin}} + (A_{\text{wall}} - n_{\text{fin}}A_c)h(T_b - T_\infty)\\ &= (625)(1.970) + [0.1\times 0.1 - 625\times 0.25\pi(0.001)^2](100)(132)\\ &= \mathbf{1357\ W}\end{aligned}$$

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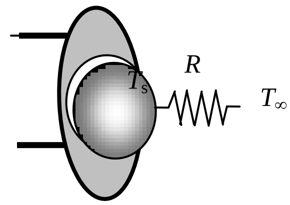
3-133 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 90°C in an environment at 20°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.



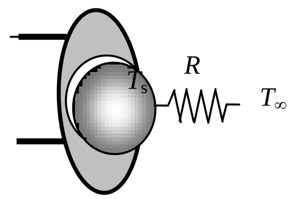
Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(90-20)°\text{C}}{40 \text{ W}} = \mathbf{1.75\ °C/W}$$

The thermal resistance of the heat sink must be below 1.75°C/W. Table 3-6 reveals that HS6071 in vertical position, HS5030 and HS6115 in both horizontal and vertical position can be selected.

3-134 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 55°C in an environment at 18°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 55°C. **3** The contact resistance between the transistor and the heat sink is negligible.



Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} \longrightarrow R_{\text{case-ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(55-18)°\text{C}}{25 \text{ W}} = \mathbf{1.5\ °C/W}$$

The thermal resistance of the heat sink must be below 1.5°C/W. Table 3-6 reveals that HS5030 in both horizontal and vertical positions, HS6071 in vertical position, and HS6115 in both horizontal and vertical positions can be selected.

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Bioheat Transfer Equation

3-135 The different human body types with three different thicknesses of skin/fat layes are subjected to the varying ambient temperature. The rate of metabolic heat generation is to be determined so as to maintain the skin temperarue constant at 34°C.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible.

Propertie: Constant thermophysical properties for skin and blood. Ambient temperature is varied between -20 and 20°C. L_{sf} = 0.075, 0.005 and 0.003 m for three different body types.

Analysis Solve the bioheat transfer differential equation along with the appropriate boundary conditions to develop an expression for the interface temperature (T_i) between the muscle and the outer skin/fat layer. The bioheat transfer differential equation is given by Eq. 3-88.

$$\frac{d^2\theta}{dx^2} - B^2\theta = 0$$

where $B^2 = \dot{p}\rho_b c_b / k$ has units of (1/m) and $\theta = T - T_a - \dot{e}_m / \dot{p}\rho_b c_b$. The boundary conditions for the problem in terms of temperature excess θ are:

$$\theta(0) = T_c - T_a - \dot{e}_m / \dot{p}\rho_b c_b = \theta_c \quad \text{and} \quad \theta(L_m) = T_i - T_a - \dot{e}_m / \dot{p}\rho_b c_b = \theta_i$$

The solution to Eq. 3-88 with the two specified temperature boundary conditions θ_c and θ_i, is given by Eq. 3-67 developed for fins (case 3 – specified temperature). For our case Eq. 3-67 becomes

$$\frac{\theta}{\theta_c} = \frac{(\theta_i/\theta_c)\sinh Bx + \sinh B(L_m - x)}{\sinh BL_m}$$

Using the Fourier's law of heat conduction, the rate of heat transfer that leaves the muscle at $x = L_m$ and enters the skin/fat layer is

$$\dot{Q}_{specified\ temp.} = -k_m A \left.\frac{dT}{dx}\right|_{x=L_m} = -k_m A \left.\frac{d\theta}{dx}\right|_{x=L_m} = -k_m AB\theta_c \frac{(\theta_i/\theta_c)\cosh BL_m - 1}{\sinh BL_m}$$

The rate at which heat is transferred through the skin/fat layer and into the environment is obtained by using the thermal resistance network concept. The total rate of heat transfer through the skin/fat layer and into the environment (the rate of heat loss from the body) is,

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} = \frac{T_i - T_s}{R_{sf}} = \left(T_s - T_\infty\right)\frac{R_{conv} + R_{rad}}{R_{conv} \times R_{rad}}$$

where the total resistance is

$$R_{total} = R_{sf} + R_{conv-rad} = R_{sf} + \frac{R_{conv} R_{rad}}{R_{conv} + R_{rad}}$$

and the individual resistances are

$$R_{sf} = \frac{L_{sf}}{k_{sf}\,A}$$

$$R_{conv} = \frac{1}{h_{conv}\,A} = \frac{1}{2(W/m^2\cdot K)\times 1.8m^2} = 0.2778\ \text{K/W}$$

$$R_{rad} = \frac{1}{h_{rad}\,A} = \frac{1}{5.9(W/m^2\cdot K)\times 1.8m^2} = 0.0941\ \text{K/W}$$

From the above heat balance, for the known values of skin temperature and the environment temperature, we find interface temperature as,

$$T_i = T_s + \dot{Q}_b R_{sf}$$

Equating the rate of heat transfer that leaves the muscle at $x = L_m$ and enters the skin/fat layer with the rate at which heat is transferred through the skin/fat layer and into the environment yields

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$$-k_m AB\,\theta_c \frac{(\theta_i/\theta_c)\cosh BL_m - 1}{\sinh BL_m} = \frac{T_i - T_\infty}{R_{total}}$$

The rearrangement of above equation by replacing θ_i and θ_c with appropriate equations results in the equation to determine the metabolic heat generation rate,

$$\dot{e}_m = \dot{p}\rho_b c_b \left(\frac{(T_i - T_\infty)\sinh(BL_m)}{k_m ABR_{tot}\cosh(BL_m)} + (T_i - T_a)\cosh(BL_m) \right) \times \frac{1}{\cosh(BL_m)+1}$$

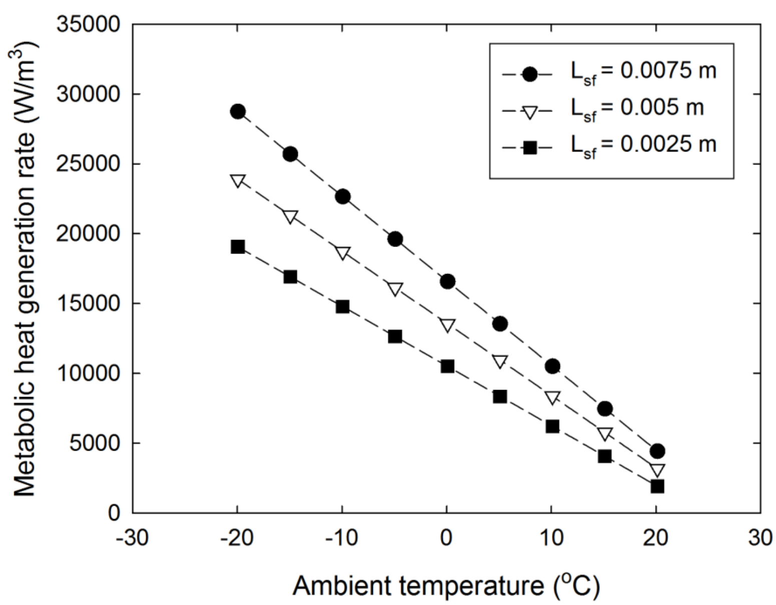
where, $B = \sqrt{\dfrac{\dot{p}\rho_b c_b}{k_m}} = 60\ \text{m}^{-1}$, $\sinh(BL_m) = 2.94$, $\cosh(BL_m) = 3.107$, $k_m = 0.5$ W/m·K and $A = 1.8\,\text{m}^2$.

The muscle and skin/fat interface temperature is a variable that will change with the change in the ambient temperature and the thickness of skin/fat layer. For all other known values, the metabolic heat generation rate ($\dot{e}_m$) is calculated as shown in table below.

Metabolic heat generation rate as a function of ambient air temperature and skin/fat layer thickness

	Metabolic heat generation rate $\dot{e}_m$ (W/m^3)		
T_∞ (°C)	L_{sf} = 0.0075 m	L_{sf} = 0.005 m	L_{sf} = 0.0025 m
-20	28750	23908	19065
-15	25709	21316	16922
-10	22669	18724	14778
-5	19629	16132	12634
0	16588	13540	10491
5	13548	10948	8347
10	10508	8356	6203
15	7467	5763	4060
20	4427	3171	1916

Variation of the metabolic heat generation rate with change in the ambient temperature for three different skin/fat layer thicknesses is shown graphically in the figure below.



Discussion It is clear that the metabolic heat generation rate required to keep a skin temperature increases with drop in the ambient temperature. The effect of increase in the skin/fat layer thickness on the metabolic heat generation rate is also evident. This also implies that, in oder to maintain the interface and core body temperature within a comfortable zone, a human being with lesser skin/fat layer thickness will have to have higher metabolism.

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3-136 The metabolic heat generation rate within human body is to be determined so as to maintain the skin temperature at 34°C.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible. **6. S**ame heat transfer everywhere throughout the different parts of body.

Properties The convective heat transfer coefficients at T_∞ = 15°C for air and water are 2 W/m²·K and 20 W/m²·K, respectively. All other properties remain same as that of the example problem 3-14 in the text.

Analysis For this problem, the properties of human body parameters remain the same as that considered in example problem 3-14 in the text. However, the ambient and surrounding temperature is lowered to 15°C and has different convective heat transfer coefficient when the human body is exposed to air and water.

The thermal resistance due to convection is

For air:
$$R_{conv} = \frac{1}{h_{conv}A} = \frac{1}{2\ \text{W/m}^2\cdot\text{K}\times 1.8\,\text{m}^2} = 0.277\ \text{K/W}$$

For water:
$$R_{conv} = \frac{1}{h_{conv}A} = \frac{1}{20\ \text{W/m}^2\cdot\text{K}\times 1.8\,\text{m}^2} = 0.0277\ \text{K/W}$$

The thermal resistance due to radiation remains unchanged and is same as that in the example problem 3-14 in the text,

$$R_{rad} = \frac{1}{h_{rad}A} = 0.09416\ \text{K/W}$$

Since we know the environment and skin temperatures, we find the rate of heat loss from the skin surface to the environment as a combined effect of convection and radiation.

$$\dot{Q}_b = (T_s - T_\infty)\frac{R_{conv}+R_{rad}}{R_{conv}\times R_{rad}} = 270.2\ \text{W}$$

From of heat transfer calculated above we can then find the skin and muscle/fat layer interface temperature using heat balance.

$$\dot{Q}_b = k_{sf}A\frac{T_i - T_s}{L_{sf}} \quad \text{and} \quad T_i = T_s + \frac{\dot{Q}_b k_{sf} A}{L_{sf}} = 34°\text{C} + \frac{270.2\ \text{W}\times 0.3\ \text{W/m}\cdot\text{K}\times 1.8\,\text{m}^2}{0.003\,\text{m}} = 35.5°\text{C}$$

We also know that, the muscle and skin/fat layer interface temperature T_i can be calculated using

$$T_i = \frac{T_\infty \sinh BL_m + k_m ABR_{total}\left[\theta_c + \left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)\cosh BL_m\right]}{\sinh BL_m + k_m ABR_{total}\cosh BL_m}$$

Rearrangement of this equation to find unknown metabolic heat generation rate yields

$$\dot{e}_m = \dot{p}\rho_b c_b\left(\frac{(T_i - T_\infty)\sinh(BL_m)}{k_m ABR_{tot}\cosh(BL_m)} + (T_i - T_a)\cosh(BL_m)\right)\times\frac{1}{\cosh(BL_m)+1}$$

For the given conditions,

$$B = \sqrt{\frac{\dot{p}\rho_b c_b}{k_m}} = \sqrt{\frac{0.0005\,(\text{s}^{-1})\times 1000\,(\text{kg/m}^3)\times 3600\,(\text{J/kg}\cdot\text{K})}{0.5\,(\text{W/m}\cdot\text{K})}} = 60\ \text{m}^{-1}$$

$$\sinh(BL_m) = 2.94,\ \cosh(BL_m) = 3.107$$

Putting these values in the equation for metabolic heat generation rate gives,

$$\dot{e}_m = 4401\ \text{W/m}^3\ \text{(air environment)}.$$

Similarly using the convective heat transfer coefficient of 20 W/m²·K for water, we find,

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$$\dot{Q}_b = (T_s - T_\infty)\frac{R_{conv} + R_{rad}}{R_{conv} \times R_{rad}} = (34 - 15)°\text{C} \times \left(\frac{0.02778 \times 0.09416}{0.02778 + 0.09416}\right) = 885.8\ \text{W}$$

and $$T_i = T_s + \frac{\dot{Q}_b k_{sf} A}{L_{sf}} = 37.66°\text{C}$$

Using the equation for metabolic heat generation rate as above we get,

$$\dot{e}_m = 23937\ \text{W/m}^3 \text{ (water environment).}$$

Discussion The human body is adaptable to adjust with the surrounding thermal environment. For instance, if the environmental conditions are cold, human body will adjust itself through the involuntarily motion of shivering. In this case, for lower ambient temperature, metabolic heat generation achieved through shivering is about 6 times higher than that given in example problem 3-14 in the text. In case of the water as surrounding fluid with about 10 times higher convective heat transfer coefficient, the metabolic heat generation rate required to maintain the skin temperature at 34 ^{o}C is about 35 times higher compared to that given in the example problem. This indicates that, for such a case only shivering motion may not be sufficient to raise the body temperature and the human body may have to perform certain physical activity to increase the metabolic heat generation rate.

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3-137 The metabolic heat generation rate in a person's body increases from 700 to 7000 W/m^3 due to rigorous exercise over a period of time. The perspiration rate in lit/s is to be determined to maintain the skin temperature at 34 °C.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible. **6. S**ame heat transfer everywhere throughout the different parts of body.

Properties Constant thermophysical properties for skin and blood. The perfusion rate stays constant at 0.0005 s^{-1}.

Analysis Since the convective heat transfer coefficient, ambient and surrounding temperatures, skin and blood thermophysical properties and the perfusion rate are same as that in the example problem 3-14 in the text, the values of B, R_{sf}, R_{conv}, R_{rad} and R_{total} remain unchanged. Thus, B = 60 m^{-1}, $\rho_b = 1000 \text{kg/m}^3$, $c_b = 3600 \text{J/kg}\cdot\text{K}$, $L_m = 0.03\text{m}$, R_{sf} = 0.00555 K/W, R_{conv} = 0.2778 K/W, R_{rad} = 0.0941 K/W and R_{total} = 0.07588 K/W.

With the increase in metabolic rate from 700 to 7000 W/m^3 and for the ambient temperature of 30°C, the skin and muscle/fat layer interface temperature is calculated using,

$$T_i = \frac{T_\infty \sinh BL_m + k_m ABR_{total}\left[\theta_c + \left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)\cosh BL_m\right]}{\sinh BL_m + k_m ABR_{total}\cosh BL_m}$$

where

$$\sinh(BL_m) = 2.94,\ \cosh(BL_m) = 3.107 \text{ and } \theta_c = T_c - T_a - \frac{\dot{e}_m}{\dot{p}\rho_b c_b} = -3.11°\text{C}$$

$$T_i = \frac{30°\text{C}\times 2.94 + 0.5\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}^2\times 60\,\text{m}^{-1}\times 0.0758\,\text{K/W}\,[-3.11°\text{C} + (37°\text{C} + 3.11°\text{C})\times 3.107]}{2.94 + (0.5\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}^2\times 60\,\text{m}^{-1}\times 0.0758\,\text{K/W}\times 3.107)}$$

$$\therefore T_i = 37.4°\text{C}.$$

From the value of interface temperature, the total heat loss from the body to the environment is calculated as,

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} = \frac{37.4°\text{C} - 30°\text{C}}{0.07588\,\text{K/W}} = 97.52\,\text{W}$$

Now, the skin temperature is calculated using Fourier's law of heat conduction to the skin/fat layer i.e.,

$$\dot{Q}_b = k_{sf} A \frac{T_i - T_s}{L_{sf}}$$

Thus the skin temperature is calculated as,

$$T_{skin} = T_i - \frac{\dot{Q}_b k_{sf} A}{L_{sf}} = 36.86°\text{C}.$$

The skin temperature is 2.86 °C higher than the desired temperature. If the skin temperature is to be maintained at 34 °C, then the excess heat generated within the body due to increased metabolic heat generation rate has to be removed through perspiration. The heat removal rate from the body when the skin temperature is 36.86°C is 97.52 W. However in order to maintain the skin temperature at 34 °C, the amount of heat to be removed is

$$\dot{Q}_b = \frac{T_i - 34}{R_{sf}} = 612\,\text{W}$$

Thus the excess amount of heat to be removed from body is 514.5W. If this heat has to be removed through perspiration then,

$$\dot{Q}_p = \dot{m}_p h_{fg}$$

Given that the perspiration properties are same as that of water evaluated at a skin surface temperature of 35.5°C, from Table A-9, we have $\rho_p = 994\,\text{kg/m}^3$ and $h_{fg} = 2417\,\text{kJ/kg}$.

Thus the volume of perspiration in lit/s is determined as,

$$\dot{V}_p = \frac{\dot{m}_p}{\rho_p} = \frac{\dot{Q}_p}{\rho_p h_{fg}} = \frac{514.5\,\text{J/s}}{2417\times 10^3\,\text{J/kg}\times 994\,\text{kg/m}^3} = 2.141\times 10^{-7}\,\text{m}^3/\text{s} = 2.141\times 10^{-4}\,\text{L/s}$$

Discussion The volumetric perspiration rate of $2.136\times 10^{-4}\,(\text{L/s}) = 0.76(\text{L/h})$ is moderate perspiration rate expected during rigorous exercise. If the perspiration rate exceeds about 2 L/h, human body may get dehydrated quickly resulting in sudden collapse. It was assumed in this problem that the perfusion rate stays constant. However, in reality, the body will automatically increase the perfusion rate that will increase the temperature at the muscle and skin/fat layer interface. Thus the body heat is rejected to the environment in form of convection and radiation and hence may result into less perspiration.

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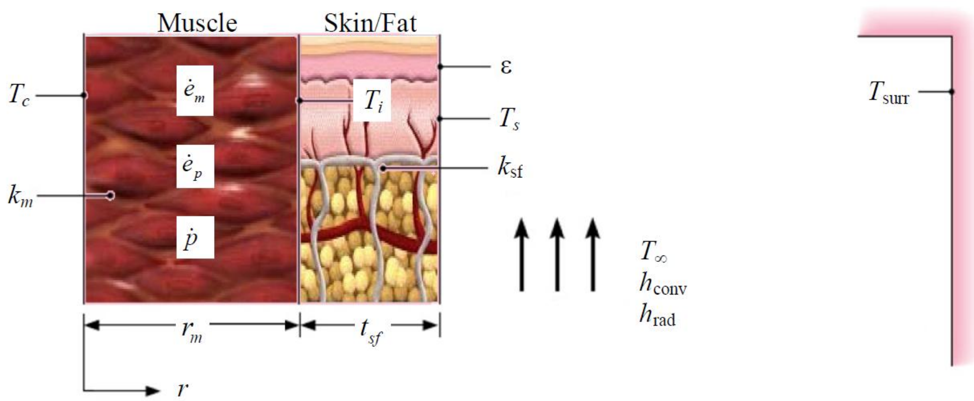
3-138 For a forearm we have been given the dimensions and thermal conductivities of a muscle layer and a skin/fat layer, metabolic heat generation and perfusion rate within the muscle layer, arterial temperature, blood density, specific heat, and ambient conditions. The mathematical formulation, the temperature at the outer surface of the muscle, and the maximum temperature in the forearm are to be determined.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D cylinder. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, metabolic heat generation rate, and perfusion rate are all constant. **4** Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature. **5** Solar radiation is negligible.

Properties Muscle thermal conductivity k_m = 0.5 W/m·K, skin/fat layer thermal conductivity k_{sf} = 0.3 W/m·K, blood density ρ_b = 1000 kg/m^3 and blood specific heat c_b = 3600 J/kg·K.

Analysis



(a) The bioheat transfer differential equation in cylindrical coordinates with constant properties is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_m+\dot{e}_P}{k}=0$$

where $\dot{e}_m$ and $\dot{e}_p$ are the *metabolic* and *perfusion* heat source terms (W/m^3). The expression proposed by Pennes for the exchange of thermal energy between flowing blood and the surrounding tissue (perfusion) is as

$$\dot{e}_p = \dot{p}\,\rho_b\,c_b\,(T_a - T)$$

Substituting the Pennes' perfusion heat source term expression, into the differential equation in cylindrical coordinates, results in

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_m+\dot{p}\,\rho_b\,c_b\,(T_a-T)}{k}=0$$

With the following boundary conditions:

$$\left.\frac{dT}{dr}\right|_{r=0}=0 \qquad \text{and} \qquad T(r_m) = T_i$$

Assuming constant $\dot{e}_m, \dot{p}, \rho_b, c_b$ and T_a, the above differential equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)-B^2\theta=0 \quad \text{or} \quad \frac{d^2T}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-B^2\theta=0$$

where $B^2=\dot{p}\,\rho_b\,c_b/k$ has units of (1/m) and $\theta = T - T_a - \dot{e}_m / \dot{p}\,\rho_b\,c_b$. The boundary conditions for the problem in terms of temperature excess θ are:

$$@\ r = 0 \quad \left.\frac{d\theta}{dr}\right|_{r=0}=0$$

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@ r = r$_m$ $\theta(r_m) = T_i - T_a - \dot{e}_m \,/\, \dot{p}\,\rho_b c_b = \theta_i$

The differential equation in terms of temperature excess θ is a *modified Bessel equation* of order zero, and its general solution is of the form

$$\theta(r) = C_1 I_0 (Br) + C_2 K_0 (Br)$$

where I_0 and K_0 are modified, zero-order Bessel functions of the first and second kinds, respectively.

Applying the boundary condition at r = 0: $\left.\dfrac{d\theta}{dr}\right|_{r=0} = C_1 B I_1(0) + C_2 B K_1(0) = 0$

Since $K_1(0)$ is infinite, we must have $C_2 = 0$.

Applying the boundary condition at r = r$_m$: $\theta(r_m) = T_i - T_a - \dot{e}_m \,/\, \dot{p}\,\rho_b c_b = \theta_i = C_1 I_0 (Br_m)$

Solving for C_1 we get $C_1 = \theta_i / I_0(Br_m)$

The complete solution for $\theta(r)$ *is*

$$\theta = \frac{\theta_i I_0(Br)}{I_0(Br_m)} = \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\, c_b}\right)\frac{I_0(Br)}{I_0(Br_m)} \qquad (1)$$

(b) In order to find T_i, use the above equation (note that T_i that appears in θ_i in Eq. (1) above is unknown). Follow the general procedure used in the example problem on the application of the bioheat transfer equation. Use Eq. (1) to calculate the rate at which heat leaves the muscle and enters the skin/fat layer at $r = r_m$ and equate it with the rate at which heat is transferred through the skin/fat layer and into the environment.

Using the Fourier's law of heat conduction, the rate of heat transfer that leaves the muscle at $r = r_m$ and enters the skin/fat layer is

$$\dot{Q}_{specified temp.} = -k_m A_r \left.\frac{dT}{dr}\right|_{r=r_m} = -k_m A_r \left.\frac{d\theta}{dr}\right|_{r=r_m} = -k_m(2\pi r_m)B\theta_i \frac{I_1(Br_m)}{I_0(Br_m)} \qquad (2)$$

The rate at which heat is transferred through the skin/fat layer and into the environment is obtained by using the thermal resistance network concept. In this case the thermal resistance is a combined series-parallel arrangement. Heat is transferred through the skin/fat layer by conduction in series and is in parallel with heat transfer by convection and radiation. The total rate of heat transfer through the skin/fat layer and into the environment (the rate of heat loss from the forearm) is

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} \qquad (3)$$

where the total resistance is $R_{total} = R_{sf} + R_{conv-rad} = R_{sf} + \dfrac{R_{conv} R_{rad}}{R_{conv} + R_{rad}}$

and the individual resistances assuming unit length for the cylinder are

$$R_{sf} = \frac{ln\left(\dfrac{r_m + t_{sf}}{r_m}\right)}{2\pi k_{sf}}, \quad R_{conv} = \frac{1}{2\pi(r_m + t_{sf})h_{conv}} \quad \text{and} \quad R_{rad} = \frac{1}{2\pi(r_m + t_{sf})h_{rad}}$$

Equating the rate of heat transfer that leaves the muscle at $r = r_m$ and enters the skin/fat layer, Eq. (2), with the rate at which heat is transferred through the skin/fat layer and into the environment, Eq. (3), yields

$$-k_m(2\pi r_m)B\theta_i \frac{I_1(Br_m)}{I_0(Br_m)} = \frac{T_i - T_\infty}{R_{total}}$$

The above equation can be solved for T_i, the final expression is

$$T_i = \frac{T_\infty I_0(Br_m) + k_m(2\pi r_m)BR_{total}\left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)I_1(Br_m)}{I_0(Br_m) + k_m(2\pi r_m)BR_{total}\, I_1(Br_m)} \qquad (4)$$

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(c) Using the data given in the problem statement and the expression for the interface temperature (T_i) between the muscle and the outer skin/fat layer, Eq. (4), the interface temperature between the muscle and the outer skin/fat layer is

T_i = **34.2ºC**

The maximum temperature in the forearm (T_{max}) occurs at the center of the forearm (r = 0). Thus from Eq. (1), with $I_0\,(Br) = I_0\,(0) = 1$, we have

$$\theta_{max} = \left(T_{max} - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b} \right) = \frac{\theta_i\, I_0(0)}{I_0(Br_m)} = \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b} \right) \frac{1}{I_0(Br_m)}$$

or

$$T_{max} = T_a + \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b} + \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b} \right) \frac{1}{I_0(Br_m)} \qquad (5)$$

Using the data given in the problem statement and the expression for the maximum temperature (T_{max}) in the forearm, Eq. (5), we get

T_{max} = **36.7ºC**

Discussion The core body temperature is 37ºC. The maximum temperature is very close to the core body temperature which appears to be very reasonable.

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Heat Transfer in Common Configurations

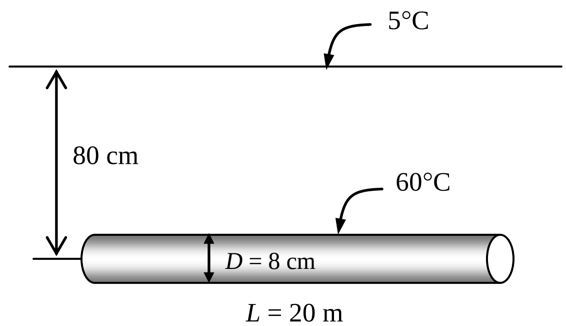
3-139C Under steady conditions, the rate of heat transfer between two surfaces is expressed as $\dot{Q} = Sk(T_1 - T_2)$ where S is the conduction shape factor. It is related to the thermal resistance by $S=1/(kR)$.

3-140C It provides an easy way of calculating the steady rate of heat transfer between two isothermal surfaces in common configurations.

3-141 The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.9 W/m·°C.



Analysis Since z >1.5D, the shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\ln(4z/D)} = \frac{2\pi(20\text{ m})}{\ln[4(0.8\text{ m})/(0.08\text{ m})]} = 34.07\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (34.07\text{ m})(0.9\text{ W/m.}^\circ\text{C})(60-5)^\circ\text{C} = \mathbf{1686\ W}$$

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3-142 A thin-walled cylindrical container, filled with chemicals undergoing exothermic reaction, is buried in fresh snow. The reaction provides a uniform heat generation. The snow surface is maintained at a specified temperature. The container surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** Isothermal container surface.

Properties The thermal conductivity of fresh snow is k = 0.60 W/m·K (Table A-8).

Analysis The shape factor for this configuration is given in Table 3-7 (Case 1) to be

$$S = \frac{2\pi L}{\ln(4z/D)} = \frac{2\pi(1.5\text{ m})}{\ln[4(0.30\text{ m})/(0.10\text{ m})]} = 3.793\text{ m}$$

The heat transfer rate from the cylindrical container is

$$\dot{Q} = \dot{e}_{\text{gen}}V = kS(T_1 - T_2)$$

Thus, the surface temperature of the container is

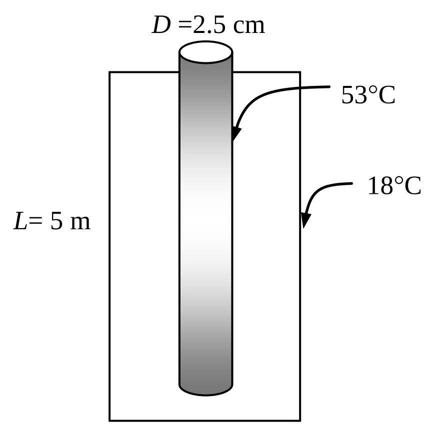
$$T_1 = \frac{\dot{e}_{\text{gen}}V}{kS} + T_2 = \frac{\dot{e}_{\text{gen}}\pi D^2 L}{4kS} + T_2 = \frac{(900\text{ W/m}^3)\pi(0.10\text{ m})^2(1.5\text{ m})}{4(0.60\text{ W/m}\cdot\text{K})(3.793\text{ m})} + (-5°\text{C}) = \mathbf{-0.34°C}$$

Discussion The surface temperature of the container is below the freezing point of water, therefore the snow around the container will not melt.

3-143 Hot water flows through a 5-m long section of a thin walled hot water pipe that passes through the center of a 14-cm thick wall filled with fiberglass insulation. The rate of heat transfer from the pipe to the air in the rooms and the temperature drop of the hot water as it flows through the pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the fiberglass insulation is constant. **4** The pipe is at the same temperature as the hot water.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.



Analysis (*a*) The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\ln\left(\frac{8z}{\pi D}\right)} = \frac{2\pi(5\text{ m})}{\ln\left[\frac{8(0.07\text{ m})}{\pi(0.025\text{ m})}\right]} = 16\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (16\text{ m})(0.035\text{ W/m.°C})(53-18)°\text{C} = \mathbf{19.6\text{ W}}$$

(*b*) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 5-m section of the wall becomes

$$\dot{Q} = \dot{m}c_p\Delta T$$

$$\Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{\dot{Q}}{\rho\dot{V}c_p} = \frac{\dot{Q}}{\rho VA_c c_p} = \frac{19.6\text{ J/s}}{(1000\text{ kg/m}^3)(0.4\text{ m/s})\left[\frac{\pi(0.025\text{ m})^2}{4}\right](4180\text{ J/kg.°C})} = \mathbf{0.024°C}$$

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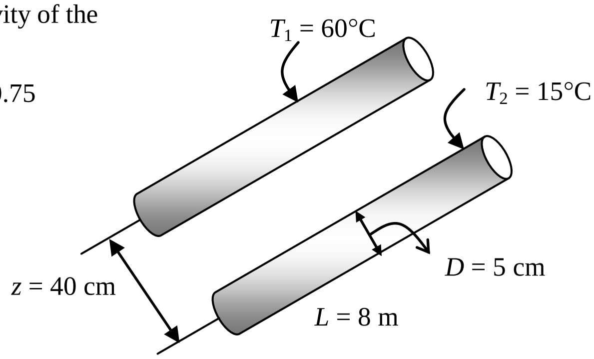
3-144 Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.



Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1 D_2}\right)}$$

$$= \frac{2\pi(8\text{ m})}{\cosh^{-1}\left(\dfrac{4(0.4\text{ m})^2 - (0.05\text{ m})^2 - (0.05\text{ m})^2}{2(0.05\text{ m})(0.05\text{ m})}\right)} = 9.078\text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

$$\dot{Q} = Sk(T_1 - T_2) = (9.078\text{ m})(0.75\text{ W/m.°C})(60 - 15)\text{°C} = \mathbf{306\text{ W}}$$

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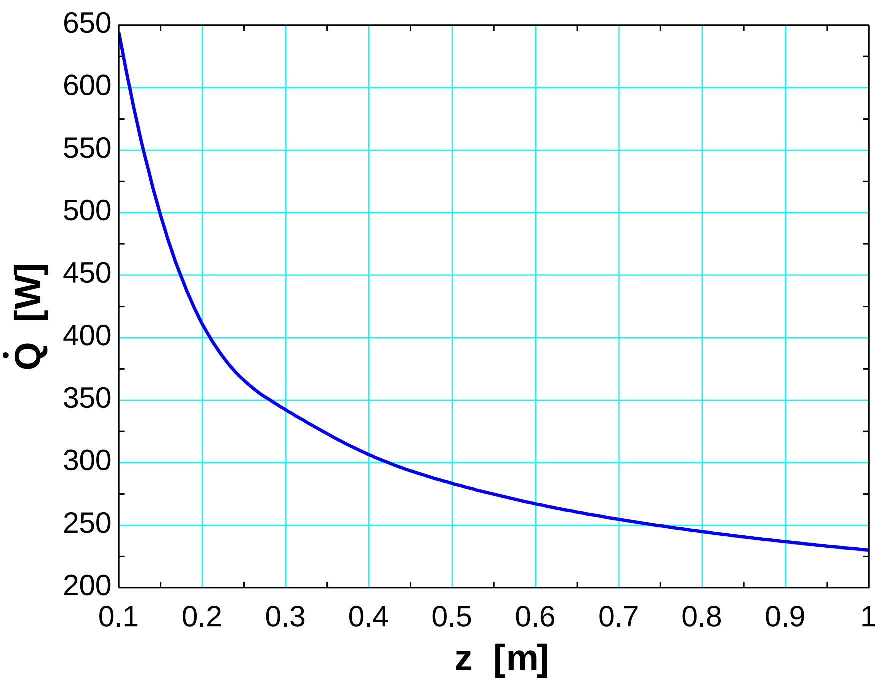
3-145  Prob. 3-144 is reconsidered. The rate of heat transfer between the pipes as a function of the distance between the centerlines of the pipes is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=8 [m]
D_1=0.05 [m]
D_2=D_1
z=0.40 [m]
T_1=60 [C]
T_2=15 [C]
k=0.75 [W/m-C]

"ANALYSIS"
S=(2*pi*L)/(arccosh((4*z^2-D_1^2-D_2^2)/(2*D_1*D_2)))
Q_dot=S*k*(T_1-T_2)
```

z [m]	Q [W]
0.1	644.1
0.2	411.1
0.3	342.3
0.4	306.4
0.5	283.4
0.6	267
0.7	254.7
0.8	244.8
0.9	236.8
1	230



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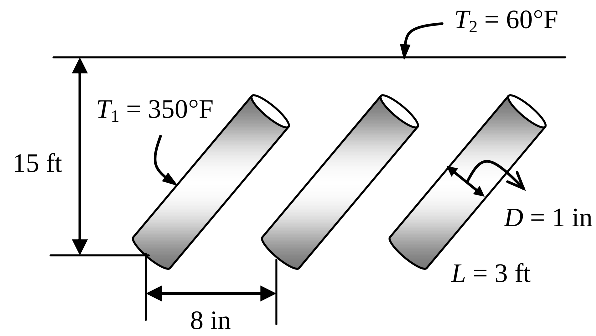
3-146E A row of used uranium fuel rods are buried in the ground parallel to each other. The rate of heat transfer from the fuel rods to the atmosphere through the soil is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.6 Btu/h·ft·°F.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S_{\text{total}} = 4\times\frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

$$= 4\times\frac{2\pi(3\text{ ft})}{\ln\left(\dfrac{2(8/12\text{ ft})}{\pi(1/12\text{ ft})}\sinh\dfrac{2\pi(15\text{ ft})}{(8/12\text{ ft})}\right)} = 0.5298\text{ ft}$$

Then the steady rate of heat transfer from the fuel rods becomes

$$\dot{Q} = S_{\text{total}}k(T_1 - T_2) = (0.5298\text{ ft})(0.6\text{ Btu/h.ft.°F})(350-60)\text{°F} = \mathbf{92.2\,Btu/h}$$

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3-147 Hot water is flowing through a pipe that extends 2 m in the ambient air and continues in the ground before it enters the next building. The surface of the ground is covered with snow at 0°C. The total rate of heat loss from the hot water and the temperature drop of the hot water in the pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** The pipe is at the same temperature as the hot water.

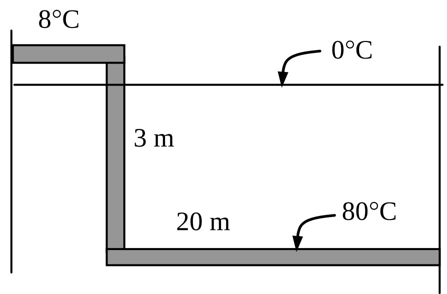
Properties The thermal conductivity of the ground is given to be k = 1.5 W/m·°C.

Analysis (a) We assume that the surface temperature of the tube is equal to the temperature of the water. Then the heat loss from the part of the tube that is on the ground is

$$A_s = \pi DL = \pi(0.05\text{ m})(2\text{ m}) = 0.3142\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$= (22\text{ W/m}^2.°\text{C})(0.3142\text{ m}^2)(80-8)°\text{C} = 498\text{ W}$$



Considering the shape factor, the heat loss for vertical part of the tube can be determined from

$$S = \frac{2\pi L}{\ln\left(\frac{4L}{D}\right)} = \frac{2\pi(3\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 3.44\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (3.44\text{ m})(1.5\text{ W/m.°C})(80-0)°\text{C} = 413\text{ W}$$

The shape factor, and the rate of heat loss on the horizontal part that is in the ground are

$$S = \frac{2\pi L}{\ln\left(\frac{4z}{D}\right)} = \frac{2\pi(20\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 22.9\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (22.9\text{ m})(1.5\text{ W/m.°C})(80-0)°\text{C} = 2748\text{ W}$$

and the total rate of heat loss from the hot water becomes

$$\dot{Q}_{\text{total}} = 498 + 413 + 2748 = \mathbf{3659\ W}$$

(b) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 25-m section of the wall becomes

$$\dot{Q} = \dot{m}c_p\Delta T$$

$$\Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{\dot{Q}}{(\rho\dot{V})c_p} = \frac{\dot{Q}}{(\rho VA_c)c_p} = \frac{3659\text{ J/s}}{(1000\text{ kg/m}^3)(1.5\text{ m/s})\left[\frac{\pi(0.05\text{ m})^2}{4}\right](4180\text{ J/kg.°C})} = \mathbf{0.30°C}$$

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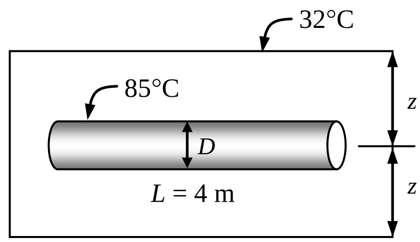
3-148 Hot water passes through a row of 8 parallel pipes placed vertically in the middle of a concrete wall whose surfaces are exposed to a medium at 32°C with a heat transfer coefficient of 8 W/m^2.°C. The rate of heat loss from the hot water, and the surface temperature of the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S = \frac{2\pi L}{\ln\left(\frac{8z}{\pi D}\right)} = \frac{2\pi(4\text{ m})}{\ln\left(\frac{8(0.075\text{ m})}{\pi(0.03\text{ m})}\right)} = 13.58\text{ m}$$

Then rate of heat loss from the hot water in 8 parallel pipes becomes

$$\dot{Q} = 8Sk(T_1 - T_2) = 8(13.58\text{ m})(0.75\text{ W/m.°C})(85-32)\text{°C} = \mathbf{4318\ W}$$

The surface temperature of the wall can be determined from

$$A_s = 2(4\text{ m})(8\text{ m}) = 64\text{ m}^2 \quad \text{(from both sides)}$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 32\text{°C} + \frac{4318\text{ W}}{(12\text{ W/m}^2.\text{°C})(64\text{ m}^2)} = \mathbf{37.6°C}$$

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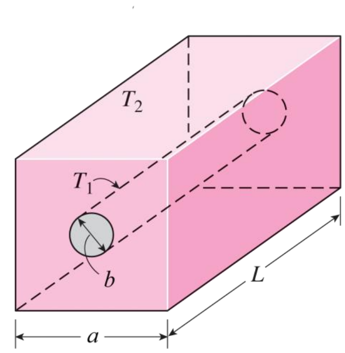
3-149 Two flow passages of the same length but of different cross-sectional shapes. Each flow passage is centered in a square solid bar of the same length. The configuration that has the higher rate of heat transfer through the square solid bar is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivities are constant. **4** Isothermal surfaces.

Analysis The shape factor of a circular flow passage centered in a square solid bar is given in Table 3-7 (Case 6) to be

$$S_{\text{cir}} = \frac{2\pi L}{\ln(1.08w/D)} = \frac{2\pi L}{\ln(1.08a/b)}$$



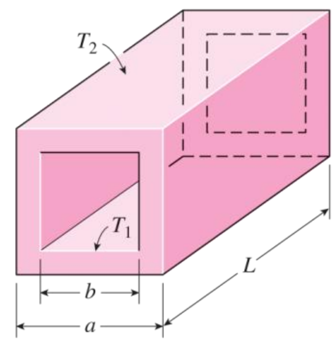
The shape factor of a square flow passage centered in a square solid bar is given in Table 3-7 (Case 10), for $a/b > 1.4$, to be

$$S_{\text{sq}} = \frac{2\pi L}{0.785\ln(a/b)} \qquad \text{(for } a/b < 1.4\text{)}$$

and

$$S_{\text{sq}} = \frac{2\pi L}{0.93\ln(0.948a/b)} \qquad \text{(for } a/b > 1.4\text{)}$$

The rate of heat transfer for both configurations can be expressed as



$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{kS_{\text{sq}}(T_1 - T_2)}{kS_{\text{cir}}(T_1 - T_2)} = \frac{S_{\text{sq}}}{S_{\text{cir}}}$$

(a) For $a = 1.2b$ ($a/b < 1.4$),

$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{S_{\text{sq}}}{S_{\text{cir}}} = \frac{\ln(1.08a/b)}{0.785\ln(a/b)} = \frac{\ln(1.08\times 1.2)}{0.785\ln(1.2)} = 1.812$$

(b) For $a = 2b$ ($a/b > 1.4$),

$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{S_{\text{sq}}}{S_{\text{cir}}} = \frac{\ln(1.08a/b)}{0.93\ln(0.948a/b)} = \frac{\ln(1.08\times 2)}{0.93\ln(0.948\times 2)} = 1.294$$

Discussion For both cases, the square flow passage has higher rate of heat transfer through the square solid bar than the circular flow passage. For $a/b < 1.4$, the square flow passage has 81.2% higher heat transfer rate than the circular flow passage. For $a/b > 1.4$, the square flow passage has 29.4% higher heat transfer rate than the circular flow passage.

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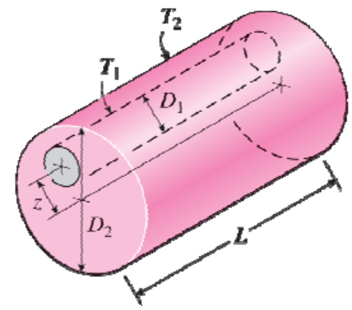
3-150 A tube transporting steam is inserted eccentrically in a cylindrically insulation. The rate of heat transfer per unit length is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity is constant. **4** Isothermal surfaces.

Properties The thermal conductivity of the insulation is given as k = 0.73 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 7) to be



$$\frac{S}{L}=\frac{2\pi}{\cosh^{-1}\left(\dfrac{D_1^2+D_2^2-4z^2}{2D_1D_2}\right)}=\frac{2\pi}{\cosh^{-1}\left[\dfrac{(0.02\text{ m})^2+(0.04\text{ m})^2-4(0.005\text{ m})^2}{2(0.02\text{ m})(0.04\text{ m})}\right]}=11.26$$

The heat transfer rate per unit length through the insulation is

$$\frac{\dot{Q}}{L}=k\frac{S}{L}(T_1-T_2)=(0.73\text{ W/m}\cdot\text{K})(11.26)(100-30)(\text{K})=\mathbf{575\ W/m}$$

Discussion To reduce heat loss from the steam, the tube should be centered in the insulation.

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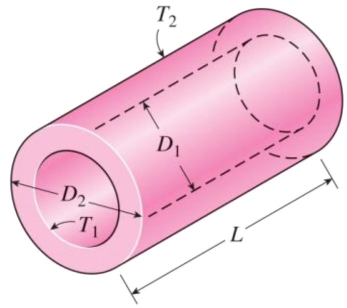
3-151 Two circular tubes, one is properly centered in a cylindrical insulation material but the other is not. The configuration that has the higher rate of heat transfer through the insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity is constant. **4** Isothermal surfaces.

Analysis The shape factor for a tube centered in a cylindrical insulation is given in Table 3-7 (Case 9) to be

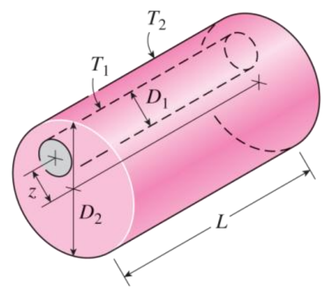
$$S_{\text{cen}} = \frac{2\pi L}{\ln(D_2/D_1)}$$



The shape factor for a tube that is eccentric in a cylindrical insulation is given in Table 3-7 (Case 7) to be

$$S_{\text{eccen}} = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1 D_2}\right)}$$

The rate of heat transfer for both configurations can be expressed as



$$\begin{aligned}
\frac{\dot{Q}_{\text{eccen}}}{\dot{Q}_{\text{cen}}} &= \frac{kS_{\text{eccen}}(T_1 - T_2)}{kS_{\text{cen}}(T_1 - T_2)} = \frac{S_{\text{eccen}}}{S_{\text{cen}}} \\
&= \frac{\ln(D_2/D_1)}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1 D_2}\right)} \\
&= \frac{\ln(0.04\text{ m}/0.02\text{ m})}{\cosh^{-1}\left[\dfrac{(0.02\text{ m})^2 + (0.04\text{ m})^2 - 4(0.005\text{ m})^2}{2(0.02\text{ m})(0.04\text{ m})}\right]} \\
&= 1.149
\end{aligned}$$

The tube that is eccentric in the cylindrical insulation has a 14.9% higher rate of heat transfer through the insulation than the one that is properly centered.

Discussion Thus, to limit heat loss the tube should be centered in the insulation.

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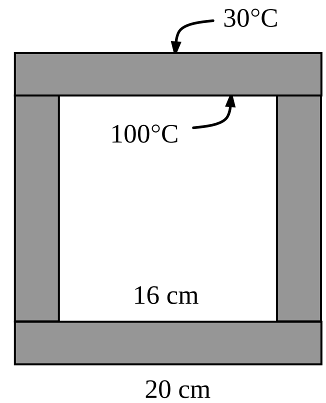
3-152 The inner and outer surfaces of a long thick-walled concrete duct are maintained at specified temperatures. The rate of heat transfer through the walls of the duct is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be

$$\frac{a}{b}=\frac{20}{16}=1.25<1.41\longrightarrow S=\frac{2\pi L}{0.785\ln\left(\frac{a}{b}\right)}=\frac{2\pi(25\text{ m})}{0.785\ln 1.25}=896.7\text{ m}$$

Then the steady rate of heat transfer through the walls of the duct becomes

$$\dot{Q}=Sk(T_1-T_2)=(896.7\text{ m})(0.75\text{ W/m.°C})(100-30)\text{°C}=4.71\times10^4\text{ W}=\mathbf{47.1kW}$$

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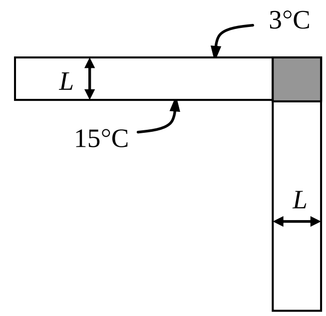
3-153 The walls and the roof of the house are made of 20-cm thick concrete, and the inner and outer surfaces of the house are maintained at specified temperatures. The rate of heat loss from the house through its walls and the roof is to be determined, and the error involved in ignoring the edge and corner effects is to be assessed.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer at the edges and corners is two-or three-dimensional. **3** Thermal conductivity of the concrete is constant. **4** The edge effects of adjoining surfaces on heat transfer are to be considered.

Properties The thermal conductivity of the concrete is given to be k = 0.75 W/m·°C.

Analysis The rate of heat transfer excluding the edges and corners is first determined to be



$$A_{\text{total}} = (12-0.4)(12-0.4)+4(12-0.4)(6-0.2) = 403.7\ \text{m}^2$$

$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1-T_2) = \frac{(0.75\ \text{W/m.°C})(403.7\ \text{m}^2)}{0.2\ \text{m}}(15-3)\text{°C} = 18{,}167\ \text{W}$$

The heat transfer rate through the edges can be determined using the shape factor relations in Table 3-7,

$$S_{\text{corners+edges}} = 4\times\text{corners} + 4\times\text{edges} = 4\times 0.15L + 4\times 0.54w$$
$$= 4\times 0.15(0.2\ \text{m}) + 4\times 0.54(12\ \text{m}) = 26.04\ \text{m}$$
$$\dot{Q}_{\text{corners+edges}} = S_{\text{corners+edges}}k(T_1-T_2) = (26.04\ \text{m})(0.75\ \text{W/m.°C})(15-3)\text{°C} = 234\ \text{W}$$

and $\dot{Q}_{\text{total}} = 18{,}167 + 234 = 1.840\times 10^4\ \text{W} = \mathbf{18.4\,kW}$

Ignoring the edge effects of adjoining surfaces, the rate of heat transfer is determined from

$$A_{\text{total}} = (12)(12)+4(12)(6) = 432\ \text{m}^2$$
$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1-T_2) = \frac{(0.75\ \text{W/m.°C})(432\ \text{m}^2)}{0.2\ \text{m}}(15-3)\text{°C} = 1.94\times 10^4 = 19.4\ \text{kW}$$

The percentage error involved in ignoring the effects of the edges then becomes

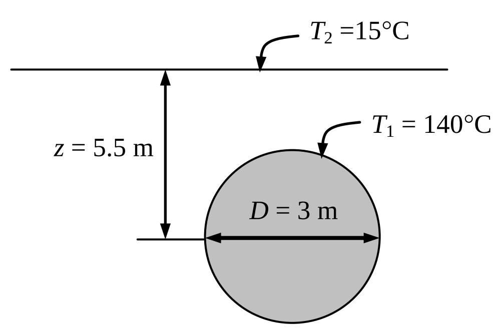
$$\%\text{error} = \frac{19.4-18.4}{18.4}\times 100 = \mathbf{5.4\%}$$

3-154 A spherical tank containing some radioactive material is buried in the ground. The tank and the ground surface are maintained at specified temperatures. The rate of heat transfer from the tank is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant.

Properties The thermal conductivity of the ground is given to be k = 1.4 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S = \frac{2\pi D}{1-0.25\dfrac{D}{z}} = \frac{2\pi(3\ \text{m})}{1-0.25\dfrac{3\ \text{m}}{5.5\ \text{m}}} = 21.83\ \text{m}$$

Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1-T_2) = (21.83\ \text{m})(1.4\ \text{W/m.°C})(140-15)\text{°C} = \mathbf{3820\,W}$$

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3-155 Radioactive material is stored in a spherical vessel that is buried underground. The ground surface temperature directly above the vessel is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** Isothermal surfaces.

Properties The thermal conductivity of the ground is given to be k = 2.0 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 15) to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(3.5\,\text{m})}{1 - 0.25\dfrac{3.5\,\text{m}}{10\,\text{m}}} = 24.1\,\text{m}$$

The heat transfer rate from the spherical container is

$$\dot{Q} = \dot{e}_{\text{gen}}V = kS(T_1 - T_2)$$

Thus, the surface temperature of the ground is

$$T_2 = T_1 - \frac{\dot{e}_{\text{gen}}V}{kS} = T_1 - \frac{\dot{e}_{\text{gen}}}{kS}\frac{\pi}{6}D^3$$

$$= 480°\text{C} - \frac{(1000\,\text{W/m}^3)(3.5\,\text{m})^3\pi}{(2.0\,\text{W/m}\cdot\text{K})(24.1\,\text{m})6} = \mathbf{14.2°C}$$

Since the ground surface directly above the vessel is at a temperature above freezing, snow will not cover that area but will be melted away.

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Special Topic: Heat Transfer through the Walls and Roofs

3-156C The *R*-value of a wall is the thermal resistance of the wall per unit surface area. It is the same as the unit thermal resistance of the wall. It is the inverse of the *U*-factor of the wall, $R = 1/U$.

3-157C The effective emissivity for a plane-parallel air space is the "equivalent" emissivity of one surface for use in the relation $\dot{Q}_{\text{rad}} = \varepsilon_{\text{effective}} \sigma A_s (T_2^4 - T_1^4)$ that results in the same rate of radiation heat transfer between the two surfaces across the air space. It is determined from

$$\frac{1}{\varepsilon_{\text{effective}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. When the effective emissivity is known, the radiation heat transfer through the air space is determined from the $\dot{Q}_{\text{rad}}$ relation above.

3-158C The unit thermal resistances (*R*-value) of both 40-mm and 90-mm vertical air spaces are given to be the same, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. This is not surprising since the convection currents that set in in the thicker air space offset any additional resistance due to a thicker air space.

3-159C Radiant barriers are highly reflective materials that minimize the radiation heat transfer between surfaces. Highly reflective materials such as aluminum foil or aluminum coated paper are suitable for use as radiant barriers. Yes, it is worthwhile to use radiant barriers in the attics of homes by covering at least one side of the attic (the roof or the ceiling side) since they reduce radiation heat transfer between the ceiling and the roof considerably.

3-160C The roof of a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times will still have an effect on heat transfer through the ceiling since the roof in this case will act as a radiation shield, and reduce heat transfer by radiation.

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3-161 The *R*-value and the *U*-factor of a wood frame wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

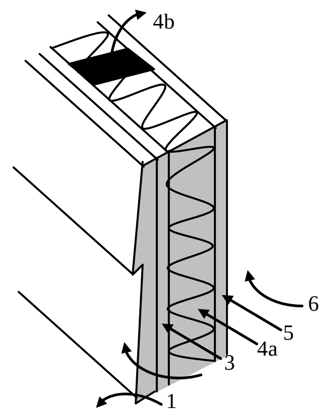
Properties The R-values of different materials are given in Table 3-8.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{insulation} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3-8 and calculating others, the total *R*-values for each section is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 13 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R* (in m^2.°C/W)	4.309	1.593
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{area,i} U_i = 0.80\times0.232+0.20\times0.628$	**0.311 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**3.213 m^2.°C/W**	

Therefore, the *R*-value and *U*-factor of the wall are $R = 3.213$ m^2.°C/W and $U = 0.311$ W/m^2.°C.

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3-162 The change in the *R*-value of a wood frame wall due to replacing fiberwood sheathing in the wall by rigid foam sheathing is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

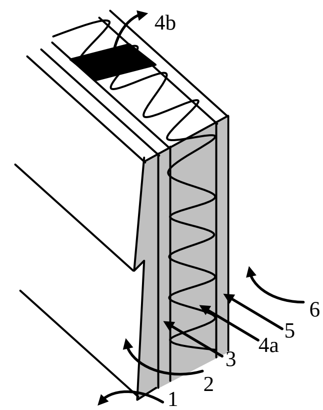
Properties The *R*-values of different materials are given in Table 3-8.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3-6 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Rigid foam, 25 mm	0.98	0.98
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R* (in m^2.°C/W)	5.059	2.343
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.198	0.426
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.80×0.232+0.20×0.628	0.2436 W/m^2.°C	
Overall unit thermal resistance, $R = 1/U$	4.105 m^2.°C/W	

The *R*-value of the existing wall is R = 3.213 m^2.°C/W. Then the change in the *R*-value becomes

$$\%\,\text{Change} = \frac{\Delta R - \text{value}}{R - \text{value, old}} = \frac{4.105 - 3.213}{4.105} = 0.217 \quad (\text{or } \mathbf{21.7\%})$$

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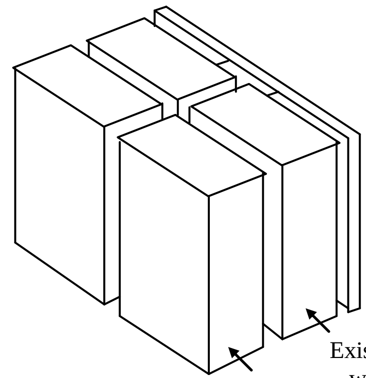
3-163 The *U*-value of a wall is given. A layer of face brick is added to the outside of a wall, leaving a 20-mm air space between the wall and the bricks. The new *U*-value of the wall and the rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The *U*-value of a wall is given to be U = 2.25 W/m^2.°C. The R - values of 100-mm face brick and a 20-mm air space between the wall and the bricks various layers are 0.075 and 0.170 m^2.°C/W, respectively.

Analysis The *R*-value of the existing wall for the winter conditions is



$$R_{\text{existing wall}} = 1/U_{\text{existing wall}} = 1/2.25 = 0.444\ \text{m}^2\cdot°\text{C/W}$$

Noting that the added thermal resistances are in series, the overall *R*-value of the wall becomes

$$R_{\text{modified wall}} = R_{\text{existing wall}} + R_{\text{brick}} + R_{\text{air layer}}$$
$$= 0.44 + 0.075 + 0.170 = 0.689\ \text{m}^2\cdot°\text{C/W}$$

Then the *U*-value of the wall after modification becomes

$$R_{\text{modified wall}} = 1/U_{\text{modified wall}} = 1/0.689 = \mathbf{1.45\,m^2}\cdot°\mathbf{C/W}$$

The rate of heat transfer through the modified wall is

$$\dot{Q}_{\text{wall}} = (UA)_{\text{wall}}(T_i - T_o) = (1.45\ \text{W/m}^2\cdot°\text{C})(3\times 7\ \text{m}^2)[22-(-25)°\text{C}] = \mathbf{1431\ W}$$

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3-164 The winter *R*-value and the *U*-factor of a flat ceiling with an air space are to be determined for the cases of air space with reflective and nonreflective surfaces.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values are given in Table 3-8 for different materials, and in Table 3-11 for air layers.

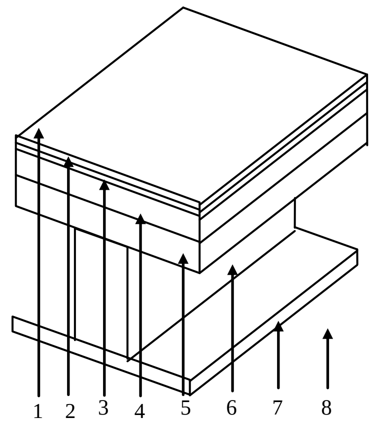
Analysis The schematic of the ceiling as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{overall} = 1/U_{overall} \quad \text{where} \quad U_{overall} = (Uf_{area})_{air\ space} + (Uf_{area})_{stud}$$

and the value of the area fraction f_{area} is 0.82 for air space and 0.18 for stud section since the headers which constitute a small part of the wall are to be treated as studs.

(*a*) Nonreflective surfaces, $\varepsilon_1 = \varepsilon_2 = 0.9$ and thus $\varepsilon_{effective} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.9 + 1/0.9 - 1} = 0.82$

Construction	*R*-value, m^2.°C/W Between studs	*R*-value, m^2.°C/W At studs
1. Still air above ceiling	0.12	0.044
2. Linoleum (R = 0.009 m^2.°C/W)	0.009	0.14
3. Felt (R = 0.011 m^2.°C/W)	0.011	0.23
4. Plywood, 13 mm	0.11	
5. Wood subfloor (R = 0.166 m^2.°C/W)	0.166	
6a. Air space, 90 mm, nonreflective	0.16	---
6b. Wood stud, 38 mm by 90 mm	---	0.63
7. Gypsum wallboard, 13 mm	0.079	0.079
8. Still air below ceiling	0.12	0.12



Total unit thermal resistance of each section, R (in m^2.°C/W)	0.775	1.243
The U-factor of each section, $U = 1/R$, in W/m^2.°C	1.290	0.805
Area fraction of each section, f_{area}	0.82	0.18
Overall U-factor, $U = \Sigma f_{area,i} U_i$ = 0.82×1.290+0.18×0.805	**1.203 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**0.831 m^2.°C/W**	

(*b*) One-reflective surface, $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9 \rightarrow \varepsilon_{effective} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.9 - 1} = 0.05$

In this case we replace item 6a from 0.16 to 0.47 m^2.°C/W. It gives R = 1.085 m^2.°C/W and U = 0.922 W/ m^2.°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{area,i} U_i$ = 0.82×1.085+0.18×0.805	**1.035 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.967 m^2.°C/W**

(*c*) Two-reflective surface, $\varepsilon_1 = \varepsilon_2 = 0.05 \rightarrow \varepsilon_{effective} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.05 - 1} = 0.03$

In this case we replace item 6a from 0.16 to 0.49 m^2.°C/W. It gives R = 1.105 m^2.°C/W and U = 0.905 W/ m^2.°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{area,i} U_i$ = 0.82×1.105+0.18×0.805	**1.051 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.951 m^2.°C/W**

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3-165 The winter R-value and the U-factor of a masonry cavity wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

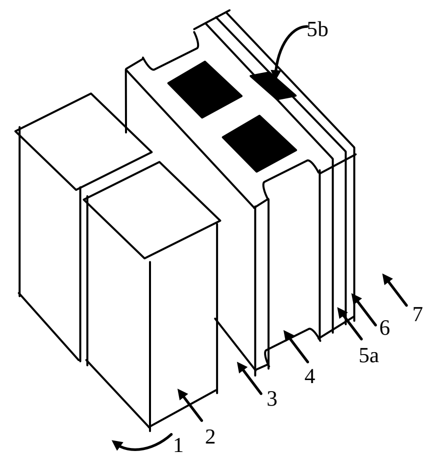
Properties The R-values of different materials are given in Table 3-8.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the U-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. Using the available R-values from Tables 3-8 and 3-11 and calculating others, the total R-values for each section of the existing wall is determined in the table below.

Construction	R-value, m^2.°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, nonreflective	0.16	0.16
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, nonreflective	0.17	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, R	0.949	1.719
The U-factor of each section, $U = 1/R$, in W/m^2.°C	1.054	0.582
Area fraction of each section, f_{area}	0.84	0.16
Overall U-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.84\times1.054+0.16\times0.582$	**0.978 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**1.02 m^2.°C/W**	

Therefore, the overall unit thermal resistance of the wall is R = 1.02 m^2.°C/W and the overall U-factor is U = 0.978 W/m^2.°C. These values account for the effects of the vertical ferring.

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3-166 The winter *R*-value and the *U*-factor of a masonry cavity wall with a reflective surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8. The *R*-values of air spaces are given in Table 3-11.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

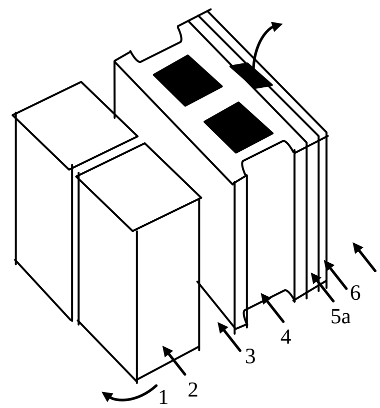
$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. For an air space with one-reflective surface, we have $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9$, and thus

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.05 + 1/0.9 - 1} = 0.05$$

Using the available *R*-values from Tables 3-8 and 3-11 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, reflective with ε = 0.05	0.45	0.45
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, reflective with ε =0.05	0.49	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R*	1.559	2.009
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.641	0.498
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.84 \times 0.641 + 0.16 \times 0.498$	**0.618 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**1.62 m^2.°C/W**	

Therefore, the overall unit thermal resistance of the wall is R = 1.62 m^2.°C/W and the overall U-factor is U = 0.618 W/m^2.°C. These values account for the effects of the vertical ferring.

Discussion The change in the *U*-value as a result of adding reflective surfaces is

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value, nonreflective}} = \frac{0.978 - 0.618}{0.978} = 0.368$$

Therefore, the rate of heat transfer through the wall will decrease by 36.8% as a result of adding a reflective surface.

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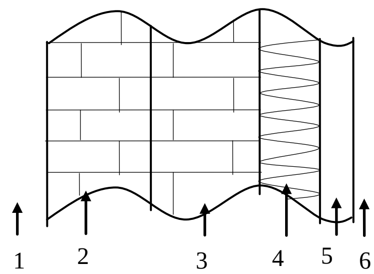
3-167 The winter *R*-value and the *U*-factor of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8.

Analysis Using the available *R*-values from Tables 3-8, the total *R*-value of the wall is determined in the table below.

Construction	*R*-value, $m^2.°C/W$
1. Outside surface, 24 km/h	0.030
2. Face brick, 100 mm	0.075
3. Common brick, 100 mm	0.12
4. Urethane foam insulation, 25-mm	0.98
5. Gypsum wallboard, 13 mm	0.079
6. Inside surface, still air	0.12



Total unit thermal resistance of each section, *R*	**1.404 m^2.°C/W**
The *U*-factor of each section, *U* = 1/*R*	**0.712 W/m^2.°C**

Therefore, the overall unit thermal resistance of the wall is *R* = 1.404 m^2.°C/W and the overall U-factor is *U* = 0.712 W/m^2.°C.

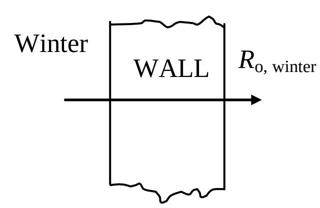
3-168 The *U*-value of a wall under winter design conditions is given. The *U*-value of the wall under summer design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (12 km/h winds) and winter (24 km/h winds) conditions are given in Table 3-8 to be $R_{\text{o, summer}} = 0.044\ \text{m}^2.°\text{C/W}$ and $R_{\text{o, winter}} = 0.030\ \text{m}^2.°\text{C/W}$.

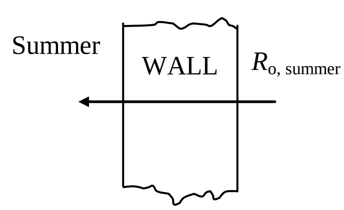
Analysis The *R*-value of the existing wall is

$$R_{\text{winter}} = 1/U_{\text{winter}} = 1/1.40 = 0.714\ \text{m}^2 \cdot °\text{C/W}$$



Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall under summer conditions becomes

$$\begin{aligned} R_{\text{summer}} &= R_{\text{winter}} - R_{\text{o,winter}} + R_{\text{o,summer}} \\ &= 0.714 - 0.030 + 0.044 \\ &= 0.728\ \text{m}^2 \cdot °\text{C/W} \end{aligned}$$



Then the summer *U*-value of the wall becomes

$$R_{\text{summer}} = 1/U_{\text{summer}} = 1/0.728 = \mathbf{1.37\ m^2 \cdot °C/W}$$

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3-169E The *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

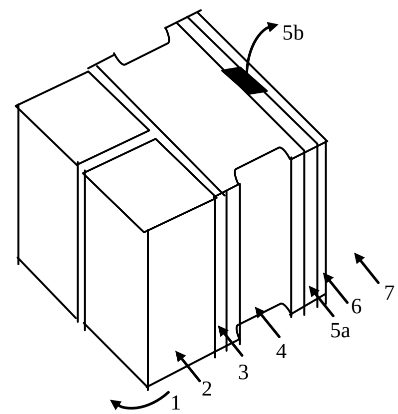
Properties The R-values of different materials are given in Table 3-8.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \quad \text{where} \quad U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for air space and 0.20 for the ferrings and similar structures. Using the available *R*-values from Table 3-8 and calculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, h.ft^2.°F/Btu	
	Between furring	At furring
1. Outside surface, 15 mph wind	0.17	0.17
2. Face brick, 4 in	0.43	0.43
3. Cement mortar, 0.5 in	0.10	0.10
4. Concrete block, 4-in	1.51	1.51
5a. Air space, 3/4-in, nonreflective	2.91	--
5b. Nominal 1 × 3 vertical furring	--	0.94
6. Gypsum wallboard, 0.5 in	0.45	0.45
7. Inside surface, still air	0.68	0.68



Total unit thermal resistance of each section, *R*	6.25	4.28
The *U*-factor of each section, $U = 1/R$, in Btu/h.ft^2.°F	0.160	0.234
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.80 \times 0.160 + 0.20 \times 0.234$	**0.175 Btu/h.ft^2.°F**	
Overall unit thermal resistance, $R = 1/U$	**5.72 h.ft^2.°F/Btu**	

Therefore, the overall unit thermal resistance of the wall is R = 5.72 h.ft^2.°F/Btu and the overall U-factor is U = 0.175 Btu/h.ft^2.°F. These values account for the effects of the vertical ferring.

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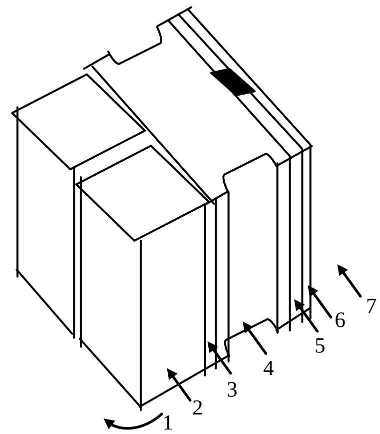
3-170 The summer and winter *R*-values of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant. **4** The air cavity does not have any reflecting surfaces.

Properties The R-values of different materials are given in Table 3-8.

Analysis Using the available *R*-values from Tables 3-8, the total *R*-value of the wall is determined in the table below.

Construction	*R* -value, m^2.°C/W Summer	*R* -value, m^2.°C/W Winter
1a. Outside surface, 24 km/h (winter)	---	0.030
1b. Outside surface, 12 km/h (summer)	0.044	---
2. Face brick, 100 mm	0.075	0.075
3. Cement mortar, 13 mm	0.018	0.018
4. Concrete block, lightweight, 100 mm	0.27	0.27
5. Air space, nonreflecting, 40-mm	0.16	0.16
5. Plaster board, 20 mm	0.122	0.122
6. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section (the *R*-value) , m^2.°C/W	**0.809**	**0.795**

Therefore, the overall unit thermal resistance of the wall is R = 0.809 m^2.°C/W in summer and R = 0.795 m^2.°C/W in winter.

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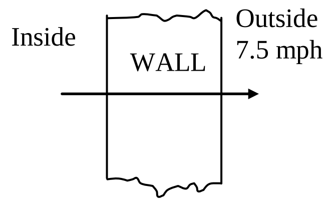
3-171E The U-value of a wall for 7.5 mph winds outside are given. The U-value of the wall for the case of 15 mph winds outside is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (7.5 mph winds) and winter (15 mph winds) conditions are given in Table 3-8 to be



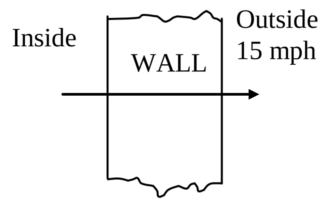
$$R_{o,\,7.5\text{ mph}} = R_{o,\text{ summer}} = 0.25\text{ h.ft}^2.°\text{F/Btu}$$

and $$R_{o,\,15\text{ mph}} = R_{o,\text{ winter}} = 0.17\text{ h.ft}^2.°\text{F/Btu}$$

Analysis The R-value of the wall at 7.5 mph winds (summer) is

$$R_{\text{wall},\,7.5\text{mph}} = 1/U_{\text{wall},\,7.5\text{mph}} = 1/0.075 = 13.33\text{ h.ft}^2\cdot°\text{F/Btu}$$

Noting that the added and removed thermal resistances are in series, the overall R-value of the wall at 15 mph (winter) conditions is obtained by replacing the summer value of outer convection resistance by the winter value,



$$\begin{aligned}R_{\text{wall},15\text{mph}} &= R_{\text{wall},\,7.5\text{mph}} - R_{o,7.5\text{mph}} + R_{o,15\text{mph}}\\ &= 13.33 - 0.25 + 0.17 = 13.25\text{ h.ft}^2\cdot°\text{F/Btu}\end{aligned}$$

Then the U-value of the wall at 15 mph winds becomes

$$R_{\text{wall},15\text{mph}} = 1/U_{\text{wal},15\text{mph}} = 1/13.25 = \mathbf{0.0755\,Btu/h.ft^2\cdot°F}$$

Discussion Note that the effect of doubling the wind velocity on the U-value of the wall is less than 1 percent since

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value}} = \frac{0.0755 - 0.075}{0.075} = 0.0067 \quad \text{(or 0.67\%)}$$

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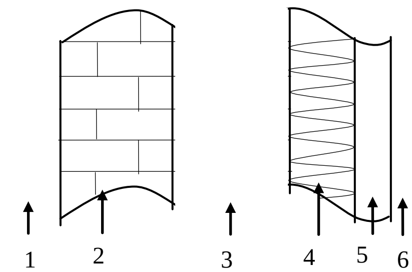
3-172 Two homes are identical, except that their walls are constructed differently. The house that is more energy efficient is to be determined.

Assumptions **1** The homes are identical, except that their walls are constructed differently. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8.

Analysis Using the available *R*-values from Tables 3-8, the total *R*-value of the masonry wall is determined in the table below.

Construction	*R* -value, m^2.°C/W
1. Outside surface, 24 km/h (winter)	0.030
2. Concrete block, light weight, 200 mm	2×0.27=0.54
3. Air space, nonreflecting, 20 mm	0.17
5. Plasterboard, 20 mm	0.12
6. Inside surface, still air	0.12



Total unit thermal resistance (the *R*-value)	**0.98 m^2.°C/W**

which is less than 2.4 m^2.°C/W. Therefore, the standard *R*-2.4 m^2.°C/W wall is better insulated and thus it is more energy efficient.

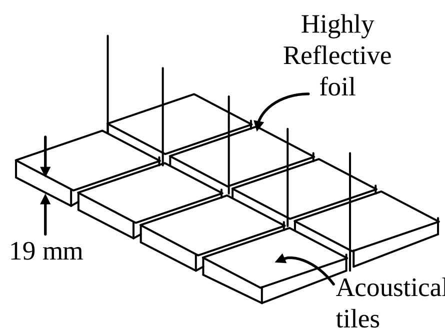
3-173 A ceiling consists of a layer of reflective acoustical tiles. The *R*-value of the ceiling is to be determined for winter conditions.

Assumptions **1** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Tables 3-8 and 3-9.

Analysis Using the available *R*-values, the total *R*-value of the ceiling is determined in the table below.

Construction	*R* -value, m^2.°C/W
1. Still air, reflective horizontal surface facing up	R = 1/h = 1/4.32 = 0.23
2. Acoustic tile, 19 mm	0.32
3. Still air, horizontal surface, facing down	R = 1/h = 1/9.26 = 0.11



Total unit thermal resistance (the *R*-value)	**0.66 m^2.°C/W**

Therefore, the R-value of the hanging ceiling is 0.66 m^2.°C/W.

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Review Problems

3-174 Two persons are wearing different clothes made of different materials with different surface areas. The fractions of heat lost from each person's body by perspiration are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient. **5** The human body is assumed to be cylindrical in shape for heat transfer purposes.

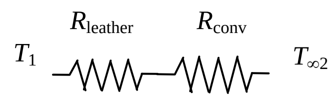
Properties The thermal conductivities of the leather and synthetic fabric are given to be k = 0.159 W/m·°C and k = 0.13 W/m·°C, respectively.

Analysis The surface area of each body is first determined from

$$A_1 = \pi DL/2 = \pi(0.25\text{ m})(1.7\text{ m})/2 = 0.6675\text{ m}^2$$
$$A_2 = 2A_1 = 2\times 0.6675 = 1.335\text{ m}^2$$

The sensible heat lost from the first person's body is

$$R_{\text{leather}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(0.159\text{ W/m.°C})(0.6675\text{ m}^2)} = 0.00942\text{ °C/W}$$
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\text{ W/m}^2\text{.°C})(0.6675\text{ m}^2)} = 0.09988\text{ °C/W}$$
$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00942 + 0.09988 = 0.10930\text{ °C/W}$$



The total sensible heat transfer is the sum of heat transferred through the clothes and the skin

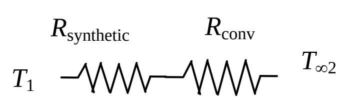
$$\dot{Q}_{\text{clothes}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.10930\text{°C/W}} = 18.3\text{ W}$$
$$\dot{Q}_{\text{skin}} = \frac{T_1 - T_{\infty 2}}{R_{\text{conv}}} = \frac{(32-30)\text{°C}}{0.09988\text{°C/W}} = 20.0\text{ W}$$
$$\dot{Q}_{\text{sensible}} = \dot{Q}_{\text{clothes}} + \dot{Q}_{\text{skin}} = 18.3 + 20 = 38.3\text{ W}$$

Then the fraction of heat lost by respiration becomes

$$f = \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-38.3}{60} = \mathbf{0.362}$$

Repeating similar calculations for the second person's body

$$R_{\text{synthetic}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(0.13\text{ W/m.°C})(1.335\text{ m}^2)} = 0.00576\text{ °C/W}$$
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\text{ W/m}^2\text{.°C})(1.335\text{ m}^2)} = 0.04994\text{ °C/W}$$
$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00576 + 0.04994 = 0.05570\text{ °C/W}$$



$$\dot{Q}_{\text{sensible}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.05570\text{°C/W}} = 35.9\text{ W}$$

$$f = \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-35.9}{60} = \mathbf{0.402}$$

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3-175 Cold conditioned air is flowing inside a duct of square cross-section. The maximum length of the duct for a specified temperature increase in the duct is to be determined.

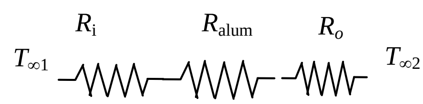
Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Steady one-dimensional heat conduction relations can be used due to small thickness of the duct wall. **5** When calculating the conduction thermal resistance of aluminum, the average of inner and outer surface areas will be used.

Properties The thermal conductivity of aluminum is given to be 237 W/m·°C. The specific heat of air at the given temperature is c_p = 1006 J/kg·°C (Table A-15).

Analysis The inner and the outer surface areas of the duct per unit length and the individual thermal resistances are

$$A_1 = 4a_1 L = 4(0.22\text{ m})(1\text{ m}) = 0.88\text{ m}^2$$

$$A_2 = 4a_2 L = 4(0.25\text{ m})(1\text{ m}) = 1.0\text{ m}^2$$



$$R_i = \frac{1}{h_1 A} = \frac{1}{(75\text{ W/m}^2.°\text{C})(0.88\text{ m}^2)} = 0.01515°\text{C/W}$$

$$R_{alum} = \frac{L}{kA} = \frac{0.015\text{ m}}{(237\text{ W/m.°C})\left[(0.88+1)/2\right]\text{m}^2} = 0.00007\ °\text{C/W}$$

$$R_o = \frac{1}{h_2 A} = \frac{1}{(13\text{ W/m}^2.°\text{C})(1.0\text{ m}^2)} = 0.07692°\text{C/W}$$

$$R_{total} = R_i + R_{alum} + R_o = 0.01515 + 0.00007 + 0.07692 = 0.09214\ °\text{C/W}$$

The rate of heat loss from the air inside the duct is

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(33-12)°\text{C}}{0.09214°\text{C/W}} = 228\text{ W}$$

For a temperature rise of 1°C, the air inside the duct should gain heat at a rate of

$$\dot{Q}_{total} = \dot{m}c_p\Delta T = (0.8\text{ kg/s})(1006\text{ J/kg.°C})(1°\text{C}) = 805\text{ W}$$

Then the maximum length of the duct becomes

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{805\text{ W}}{228\text{ W}} = \mathbf{3.53\ m}$$

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3-176 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement, and it experiences a 3°C-temperature drop. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any significant change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no significant variation in the axial direction. **3** Thermal properties are constant.

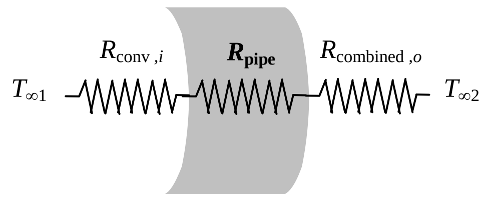
Properties The thermal conductivity of cast iron is given to be k = 52 W/m·°C.

Analysis Using water properties at room temperature, the mass flow rate of water and rate of heat transfer from the water are determined to be

$$\dot{m} = \rho \dot{\boldsymbol{V}}_c = \rho V A_c = (1000\ \text{kg/m}^3)(1.5\ \text{m/s})\left[\pi(0.03)^2/4\right]\text{m}^2 = 1.06\ \text{kg/s}$$

$$\dot{Q} = \dot{m} c_p \Delta T = (1.06\ \text{kg/s})(4180\ \text{J/kg.°C})(70-67)\text{°C} = 13{,}296\ \text{W}$$

The thermal resistances for convection in the pipe and the pipe itself are



$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k L}$$
$$= \frac{\ln(1.75/1.5)}{2\pi(52\ \text{W/m.°C})(15\ \text{m})} = 0.000031\ \text{°C/W}$$

$$R_{\text{conv},i} = \frac{1}{h_i A_i} = \frac{1}{(400\ \text{W/m}^2.\text{°C})[\pi(0.03)(15)]\text{m}^2} = 0.001768\ \text{°C/W}$$

Using arithmetic mean temperature (70+67)/2 = 68.5°C for water, the heat transfer can be expressed as

$$\dot{Q} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{total}}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{conv},i} + R_{\text{pipe}} + R_{\text{combined},o}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{conv},i} + R_{\text{pipe}} + \dfrac{1}{h_{\text{combined}} A_o}}$$

Substituting,

$$13{,}296\ \text{W} = \frac{(68.5-15)\text{°C}}{(0.000031\ \text{°C/W}) + (0.001768\ \text{°C/W}) + \dfrac{1}{h_{\text{combined}}[\pi(0.035)(15)]\text{m}^2}}$$

Solving for the combined heat transfer coefficient gives

$$h_{\text{combined}} = \mathbf{272.5\ W/m^2.°C}$$

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3-177 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be k = 0.16 W/m·°C. The density and latent heat of fusion of water at 0°C are ρ = 1000 kg/m^3 and h_{if} = 333.7 kJ/kg (Table A-9).

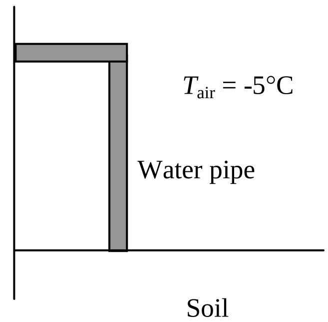
Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are

$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16\ \text{W/m.°C})(0.5\ \text{m})} = 0.3627\ \text{°C/W}$$

$$R_{\text{conv}o} = \frac{1}{h_o A} = \frac{1}{(40\ \text{W/m}^2.\text{°C})[\pi(0.024\ \text{m})(0.5\ \text{m})]} = 0.6631\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{conv}o} = 0.3627 + 0.6631 = 1.0258\ \text{°C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[0-(-5)]\text{°C}}{1.0258\ \text{°C/W}} = 4.874\ \text{W}$$



The total amount of heat lost by the water during a 14-h period that night is

$$Q = \dot{Q}\Delta t = (4.874\ \text{J/s})(14\times 3600\ \text{s}) = 245.7\ \text{kJ}$$

The amount of heat required to freeze the water in the pipe completely is

$$m = \rho \boldsymbol{V} = \rho\pi r^2 L = (1000\ \text{kg/m}^3)\pi(0.01\ \text{m})^2(0.5\ \text{m}) = 0.157\ \text{kg}$$

$$Q = mh_{fg} = (0.157\ \text{kg})(333.7\ \text{kJ/kg}) = 52.4\ \text{kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely $(245.7 > 52.4)$.

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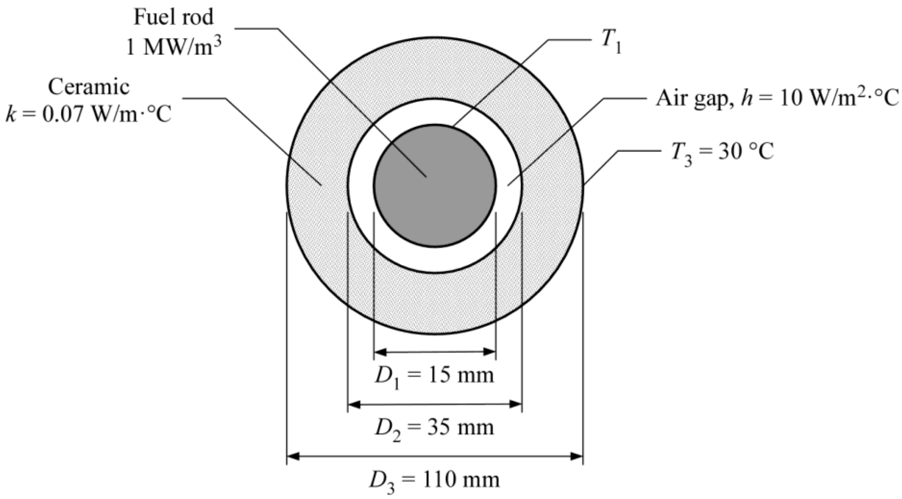
3-178 A nuclear fuel rod is encased in a concentric hollow ceramic cylinder, which created an air gap between the rod and the hollow cylinder. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat generation in the fuel rod is uniform. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of ceramic is given to be 0.07 W/m · °C.



Analysis The combined thermal resistance between the nuclear fuel rod surface and the outer surface of the ceramic cylinder is

$$R_{\text{combined}} = R_{\text{conv,rod}} + R_{\text{conv,cyl}} + R_{\text{cond,cyl}}$$
$$= \frac{1}{\pi D_1 L h} + \frac{1}{\pi D_2 L h} + \frac{\ln(D_3/D_2)}{2\pi L k}$$

or

$$R_{\text{combined}}L = \frac{1}{\pi D_1 h} + \frac{1}{\pi D_2 h} + \frac{\ln(D_3/D_2)}{2\pi k}$$
$$= \frac{1}{\pi(0.015\text{ m})(10\text{ W/m}^2\cdot{}^\circ\text{C})} + \frac{1}{\pi(0.035\text{ m})(10\text{ W/m}^2\cdot{}^\circ\text{C})} + \frac{\ln(0.110/0.035)}{2\pi(0.07\text{ W/m}\cdot{}^\circ\text{C})}$$
$$= 5.635\text{ m}\cdot{}^\circ\text{C/W}$$

The heat generated by the fuel rod is dissipated through the air gap and the ceramic cylinder, and can be expressed as

$$\dot{Q}_{\text{gen}} = \frac{T_1 - T_3}{R_{\text{combined}}} \quad \text{or} \quad \frac{\dot{Q}_{\text{gen}}}{L} = \frac{T_1 - T_3}{R_{\text{combined}}L}$$

The surface temperature of the fuel rod is

$$T_1 = \left(\frac{\dot{Q}_{\text{gen}}}{L}\right) R_{\text{combined}}L + T_3$$

$$T_1 = (1\times10^6\text{ W/m}^3)\frac{\pi}{4}(0.015\text{ m})^2(5.635\text{ m}\cdot{}^\circ\text{C/W}) + 30\,^\circ\text{C} = \mathbf{1026\,^\circ C}$$

Discussion The air gap between the fuel rod and the hollow ceramic cylinder contributed about 54% to the combined thermal resistance between the nuclear fuel rod surface and the outer surface of the ceramic cylinder.

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3-179 Steam is flowing inside a steel pipe. The thickness of the insulation needed to reduce the heat loss by 95 percent and the thickness of the insulation needed to reduce outer surface temperature to 40°C are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

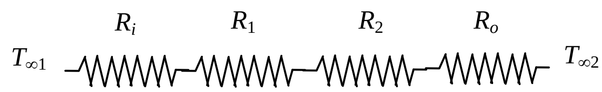
Properties The thermal conductivities are given to be k = 61 W/m·°C for steel and k = 0.038 W/m·°C for insulation.

Analysis (a) Considering a unit length of the pipe, the inner and the outer surface areas of the pipe and the insulation are

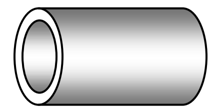
$$A_1 = \pi D_i L = \pi(0.10\text{ m})(1\text{ m}) = 0.3142\text{ m}^2$$

$$A_2 = \pi D_o L = \pi(0.12\text{ m})(1\text{ m}) = 0.3770\text{ m}^2$$

$$A_3 = \pi D_3 L = \pi D_3(1\text{ m}) = 3.1416 D_3\text{ m}^2$$



The individual thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(105\text{ W/m}^2.°\text{C})(0.3142\text{ m}^2)} = 0.03031\text{ °C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(6/5)}{2\pi(61\text{ W/m.°C})(1\text{ m})} = 0.00048\text{ °C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(D_3/0.12)}{2\pi(0.038\text{ W/m.°C})(1\text{ m})} = \frac{\ln(D_3/0.12)}{0.23876}\text{ °C/W}$$

$$R_{\text{o,steel}} = \frac{1}{h_o A_o} = \frac{1}{(14\text{ W/m}^2.°\text{C})(0.3770\text{ m}^2)} = 0.18947\text{ °C/W}$$

$$R_{\text{o,insulation}} = \frac{1}{h_o A_o} = \frac{1}{(14\text{ W/m}^2.°\text{C})(3.1416 D_3\text{ m}^2)} = \frac{0.02274}{D_3}\text{ °C/W}$$

$$R_{\text{total,noinsulation}} = R_i + R_1 + R_{\text{o,steel}} = 0.03031 + 0.00048 + 0.18947 = 0.22026\text{ °C/W}$$

$$R_{\text{total,insulation}} = R_i + R_1 + R_2 + R_{\text{o,insulation}} = 0.03031 + 0.00048 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}$$

$$= 0.03079 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\text{ °C/W}$$

Then the steady rate of heat loss from the steam per meter pipe length for the case of no insulation becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(235-20)°\text{C}}{0.22026\text{ °C/W}} = 976.1\text{ W}$$

The thickness of the insulation needed in order to save 95 percent of this heat loss can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} \longrightarrow (0.05 \times 976.1)\text{ W} = \frac{(235-20)°\text{C}}{\left(0.03079 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\right)°\text{C/W}}$$

whose solution is

$$D_3 = 0.3355\text{ m} \longrightarrow \text{thickness} = \frac{D_3 - D_2}{2} = \frac{33.55-12}{2} = \mathbf{10.8\,cm}$$

(b) The thickness of the insulation needed that would maintain the outer surface of the insulation at a maximum temperature of 40°C can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{o,insulation}}} \longrightarrow \frac{(235-20)°\text{C}}{\left(0.03079 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\right)°\text{C/W}} = \frac{(40-20)°\text{C}}{\frac{0.02274}{D_3}\text{ °C/W}}$$

whose solution is

$$D_3 = 0.1644\text{ m} \longrightarrow \text{thickness} = \frac{D_3 - D_2}{2} = \frac{16.44-12}{2} = \mathbf{2.22\,cm}$$

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3-180 A spherical vessel is used to store a fluid. The thermal resistances, the rate of heat transfer, and the temperature difference across the insulation layer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

Properties The thermal conductivity of the insulation is given to be 0.20 W/m·K.

Analysis (*a*) The thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(40\ \text{W/m}^2\cdot\text{K})\pi(3\,\text{m})^2} = \mathbf{8.84\times10^{-4}\ K/W}$$

$$R_{ins} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{(1.55-1.5)\text{m}}{4\pi(1.5\,\text{m})(1.55\,\text{m})(0.2\ \text{W/m}\cdot\text{K})} = \mathbf{8.56\times10^{-3}\ K/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})\pi(3.10\,\text{m})^2} = \mathbf{3.31\times10^{-3}\ K/W}$$

(*b*) The rate of heat transfer is

$$\dot{Q} = \frac{\Delta T}{R_i + R_{ins} + R_o} = \frac{(22-0)\ \text{K}}{(8.84\times10^{-4} + 8.56\times10^{-3} + 3.31\times10^{-3})\ \text{K/W}} = \mathbf{1725\,W}$$

(*c*) The temperature difference across the insulation layer is

$$\dot{Q} = \frac{\Delta T_{ins}}{R_{ins}} \longrightarrow 1725\ \text{W} = \frac{\Delta T_{ins}}{8.56\times10^{-3}\ \text{K/W}} \longrightarrow \Delta T_{ins} = \mathbf{14.8K}$$

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3-181 One wall of a refrigerated warehouse is made of three layers. The rates of heat transfer across the warehouse without and with the metal bolts, and the percent change in the rate of heat transfer across the wall due to metal bolts are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer coefficients are constant.

Properties The thermal conductivities are given to be k_{Al} = 200 W/m·K, $k_{\text{fiberglass}}$ = 0.038 W/m·K, k_{gypsum} = 0.48 W/m·K, and k_{bolts} = 43 W/m·K.

Analysis (*a*) The rate of heat transfer through the warehouse is

$$U_1 = \frac{1}{\dfrac{1}{h_i} + \dfrac{L_{Al}}{k_{Al}} + \dfrac{L_{fg}}{k_{fg}} + \dfrac{L_{gy}}{k_{gy}} + \dfrac{1}{h_o}}$$

$$= \frac{1}{\dfrac{1}{40\text{ W/m}^2\cdot°\text{C}} + \dfrac{0.01\text{ m}}{200\text{ W/m}\cdot°\text{C}} + \dfrac{0.08\text{ m}}{0.038\text{ W/m}\cdot°\text{C}} + \dfrac{0.03\text{ m}}{0.48\text{ W/m}\cdot°\text{C}} + \dfrac{1}{40\text{ W/m}^2\cdot°\text{C}}} = 0.451\text{ W/m}^2\cdot°\text{C}$$

$$\dot{Q}_1 = U_1 A(T_o - T_i) = (0.451\text{ W/m}^2\cdot°\text{C})(5\times 10\text{ m}^2)\left[20-(-10)°\text{C}\right] = \mathbf{676\ W}$$

(*b*) The rate of heat transfer with the consideration of metal bolts is

$$\dot{Q}_1 = U_1 A_1(T_o - T_i) = (0.451)\left[10\times 5 - 400\times 0.25\pi(0.02)^2\right]\left[20-(-10)\right] = 674.8\text{ W}$$

$$U_2 = \frac{1}{\dfrac{1}{h_i} + \dfrac{L_{bolts}}{k_{bolts}} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{40\text{ W/m}^2\cdot°\text{C}} + \dfrac{0.12\text{ m}}{43\text{ W/m}\cdot°\text{C}} + \dfrac{1}{40\text{ W/m}^2\cdot°\text{C}}} = 18.94\text{ W/m}^2\cdot°\text{C}$$

$$\dot{Q}_2 = U_2 A_2(T_o - T_i) = (18.94\text{ W/m}^2\cdot°\text{C})[400\times 0.25\pi(0.02)^2\text{ m}^2]\left[20-(-10)°\text{C}\right] = 71.4\text{ W}$$

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = 674.8 + 71.4 = \mathbf{746\ W}$$

(*c*) The percent change in the rate of heat transfer across the wall due to metal bolts is

$$\%\text{ change} = \frac{746-676}{676} = 0.103 = \mathbf{10.3\%}$$

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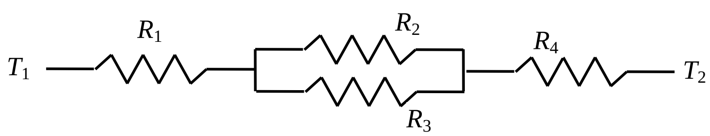
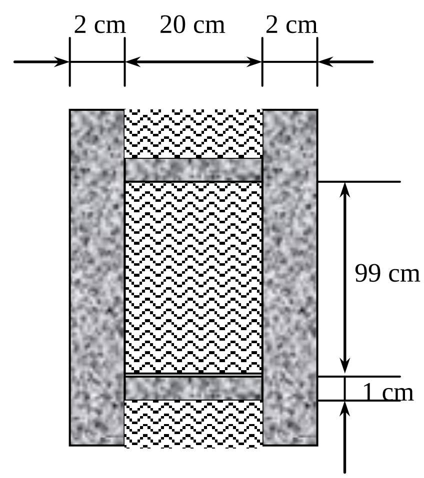
3-182 A wall is constructed of two large steel plates separated by 1-cm thick steel bars placed 99 cm apart. The remaining space between the steel plates is filled with fiberglass insulation. The rate of heat transfer through the wall is to be determined, and it is to be assessed if the steel bars between the plates can be ignored in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall can be approximated to be one-dimensional. **3** Thermal conductivities are constant. **4** The surfaces of the wall are maintained at constant temperatures.

Properties The thermal conductivities are given to be k = 15 W/m·°C for steel plates and k = 0.035 W/m·°C for fiberglass insulation.

Analysis We consider 1 m high and 1 m wide portion of the wall which is representative of entire wall.
Thermal resistance network and individual resistances are





$$R_1 = R_4 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.02\text{ m}}{(15\text{ W/m.°C})(1\text{ m}^2)} = 0.00133\text{ °C/W}$$

$$R_2 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.20\text{ m}}{(15\text{ W/m.°C})(0.01\text{ m}^2)} = 1.33333\text{ °C/W}$$

$$R_3 = R_{\text{insulation}} = \frac{L}{kA} = \frac{0.20\text{ m}}{(0.035\text{ W/m.°C})(0.99\text{ m}^2)} = 5.77201\text{ °C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.33333} + \frac{1}{5.77201} \longrightarrow R_{eqv} = 1.08313\text{ °C/W}$$

$$R_{\text{total}} = R_1 + R_{\text{eqv}} + R_4 = 0.00133 + 1.08313 + 0.00133 = 1.0858\text{ °C/W}$$

The rate of heat transfer per m^2 surface area of the wall is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{22\text{ °C}}{1.0858\text{ °C/W}} = 20.26\text{ W}$$

The total rate of heat transfer through the entire wall is then determined to be

$$\dot{Q}_{\text{total}} = (4\times 6)\dot{Q} = 24(20.26\text{ W}) = \mathbf{486.3\text{ W}}$$

If the steel bars were ignored since they constitute only 1% of the wall section, the R_{equiv} would simply be equal to the thermal resistance of the insulation, and the heat transfer rate in this case would be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_1 + R_{\text{insulation}} + R_4} = \frac{22\text{ °C}}{(0.00133 + 5.77201 + 0.00133)\text{°C/W}} = 3.81\text{ W}$$

which is mush less than 20.26 W obtained earlier. Therefore, (20.26-3.81)/20.26 = 81.2% of the heat transfer occurs through the steel bars across the wall despite the negligible space that they occupy, and obviously their effect cannot be neglected. The connecting bars are serving as "thermal bridges."

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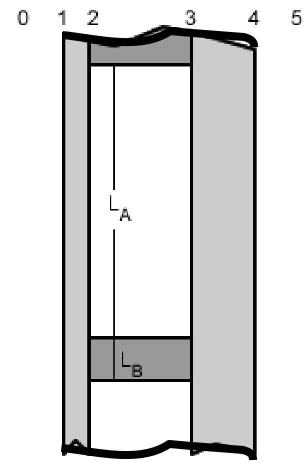
3-183 A typical section of a building wall is considered. The temperature on the interior brick surface is to be determined.

Assumptions 1 Steady operating conditions exist.

Properties The thermal conductivities are given to be k_{23b} = 50 W/m·K, k_{23a} = 0.03 W/m·K, k_{12} = 0.5 W/m·K, k_{34} = 1.0 W/m·K.

Analysis We consider 1 m^2 of wall area. The thermal resistances are



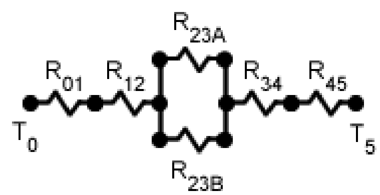
$$R_{12} = \frac{t_{12}}{k_{12}} = \frac{0.01\,\text{m}}{(0.5\ \text{W/m}\cdot{}^\circ\text{C})} = 0.02\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$\begin{aligned} R_{23a} &= t_{23}\frac{L_a}{k_{23a}(L_a+L_b)} \\ &= (0.08\,\text{m})\frac{0.6\,\text{m}}{(0.03\ \text{W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 2.645\ \text{m}^2\cdot{}^\circ\text{C/W} \end{aligned}$$

$$\begin{aligned} R_{23b} &= t_{23}\frac{L_b}{k_{23b}(L_a+L_b)} \\ &= (0.08\,\text{m})\frac{0.005\,\text{m}}{(50\ \text{W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 1.32\times10^{-5}\ \text{m}^2\cdot{}^\circ\text{C/W} \end{aligned}$$

$$R_{34} = \frac{t_{34}}{k_{34}} = \frac{0.1\,\text{m}}{(1.0\ \text{W/m}\cdot{}^\circ\text{C})} = 0.1\ \text{m}^2\cdot{}^\circ\text{C/W}$$

The total thermal resistance and the rate of heat transfer are



$$\begin{aligned} R_{\text{total}} &= R_{12} + \left(\frac{R_{23a}R_{23b}}{R_{23a}+R_{23b}}\right) + R_{34} \\ &= 0.02 + 2.645\left(\frac{(2.645)(1.32\times10^{-5})}{2.645+1.32\times10^{-5}}\right) + 0.1 = 0.120\ \text{m}^2\cdot{}^\circ\text{C/W} \end{aligned}$$

$$\dot{q} = \frac{T_4 - T_1}{R_{\text{total}}} = \frac{(35-20)^\circ\text{C}}{0.120\,\text{m}^2\cdot\text{C/W}} = 125\ \text{W/m}^2$$

The temperature on the interior brick surface is

$$\dot{q} = \frac{T_4 - T_3}{R_{34}} \longrightarrow 125\ \text{W/m}^2 = \frac{(35-T_3)^\circ\text{C}}{0.1\text{m}^2\cdot\text{C/W}} \longrightarrow T_3 = \mathbf{22.5^\circ C}$$

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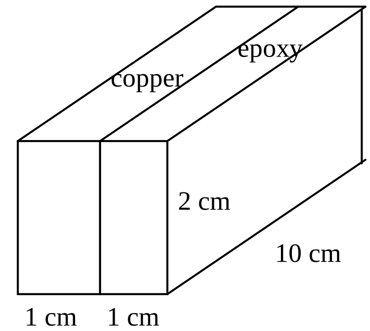
3-184 A square cross-section bar consists of a copper layer and an epoxy layer. The rates of heat transfer in different directions are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

Properties The thermal conductivities of copper and epoxy are given to be 400 and 0.4 W/m·K, respectively.

Analysis (*a*) Noting that the resistances in this case are in parallel, the heat transfer from front to back is

$$R = \left[\left(\frac{kA}{L}\right)_{Cu} + \left(\frac{kA}{L}\right)_{Ep}\right]^{-1}$$

$$= \left[\left(\frac{(400\text{ W/m}\cdot\text{K})(0.02\times 0.01)\text{ m}^2}{0.10\text{ m}}\right) + \left(\frac{(0.4\text{ W/m}\cdot\text{K})(0.02\times 0.01)\text{ m}^2}{0.10\text{ m}}\right)\right]^{-1}$$

$$= 1.249\text{ K/W}$$

$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\text{ K}}{1.249\text{ K/W}} = \mathbf{40.0\text{ W}}$$

(*b*) Noting that the resistances in this case are in series, the heat transfer from left to right is

$$R = R_{Cu} + R_{Ep} = \left(\frac{L}{kA}\right)_{Cu} + \left(\frac{L}{kA}\right)_{Ep}$$

$$= \left(\frac{0.01\text{ m}}{(400\text{ W/m}\cdot\text{K})(0.02\times 0.10)\text{ m}^2}\right) + \left(\frac{0.01\text{ m}}{(0.4\text{ W/m}\cdot\text{K})(0.02\times 0.10)\text{ m}^2}\right) = 12.51\text{ K/W}$$

$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\text{ K}}{12.51\text{ K/W}} = \mathbf{4.0\text{ W}}$$

(*c*) Noting that the resistances in this case are in parallel, the heat transfer from top to bottom is

$$R = \left[\left(\frac{kA}{L}\right)_{Cu} + \left(\frac{kA}{L}\right)_{Ep}\right]^{-1}$$

$$= \left[\left(\frac{(400\text{ W/m}\cdot\text{K})(0.01\times 0.10)\text{ m}^2}{0.02\text{ m}}\right) + \left(\frac{(0.4\text{ W/m}\cdot\text{K})(0.01\times 0.10)\text{ m}^2}{0.02\text{ m}}\right)\right]^{-1} = 0.04995\text{ K/W}$$

$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\text{ K}}{0.04995\text{ K/W}} = \mathbf{1001\text{ W}}$$

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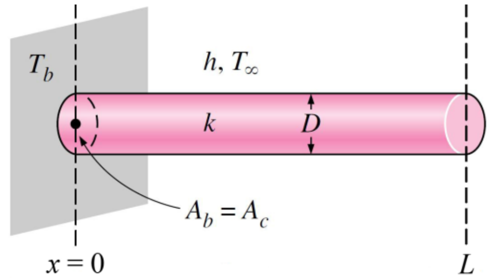
3-185 The heat transfer rates are to be determined and the temperature variations are to be plotted for infinitely long fin, adiabatic fin tip, fin tip with temperature of 250 °C, and convection from the fin tip.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m · °C.

Analysis For a circular fin with uniform cross section, the perimeter and cross section area are



$$p = \pi D = \pi(0.01\,\text{m}) = 0.03142\,\text{m}$$

and $$A_c = \frac{\pi D^2}{4} = \frac{\pi(0.01\,\text{m})^2}{4} = 7.854\times10^{-5}\ \text{m}^2$$

Also, we have

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(250\ \text{W/m}^2\cdot{}^\circ\text{C})(0.03142\ \text{m})}{(240\ \text{W/m}\cdot{}^\circ\text{C})(7.854\times10^{-5}\ \text{m}^2)}} = 20.41\,\text{m}^{-1}$$

$$\sqrt{hpkA_c} = \sqrt{(250\ \text{W/m}^2\cdot{}^\circ\text{C})(0.03142\ \text{m})(240\ \text{W/m}\cdot{}^\circ\text{C})(7.854\times10^{-5}\ \text{m}^2)} = 0.3848\ \text{W/}^\circ\text{C}$$

(a) For an infinitely long fin, the heat transfer rate can be calculated as

$$\dot{Q}_{\text{longfin}} = \sqrt{hpkA_c}\,(T_b - T_\infty) = (0.3848\ \text{W/}^\circ\text{C})(350-25)\,{}^\circ\text{C} = \mathbf{125\ W}$$

The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = e^{-mx}$$

(b) For an adiabatic fin tip, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{adiabatic tip}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,{}^\circ\text{C} - 25\,{}^\circ\text{C})\tanh\left[(20.41\,\text{m}^{-1})(0.050\ \text{m})\right]\\ &= \mathbf{96.3\ W}\end{aligned}$$

The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

(c) For fin with tip temperature of 250 °C, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{specified temp}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_b - T_\infty)}{\sinh mL}\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,{}^\circ\text{C} - 25\,{}^\circ\text{C})(0.7250)\\ &= \mathbf{90.7\ W}\end{aligned}$$

The temperature variation along the fin is as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{(T_L - T_\infty)/(T_b - T_\infty)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

(d) For fin with convection from the tip, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{conv tip}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,{}^\circ\text{C} - 25\,{}^\circ\text{C})(0.7901)\\ &= \mathbf{98.8\ W}\end{aligned}$$

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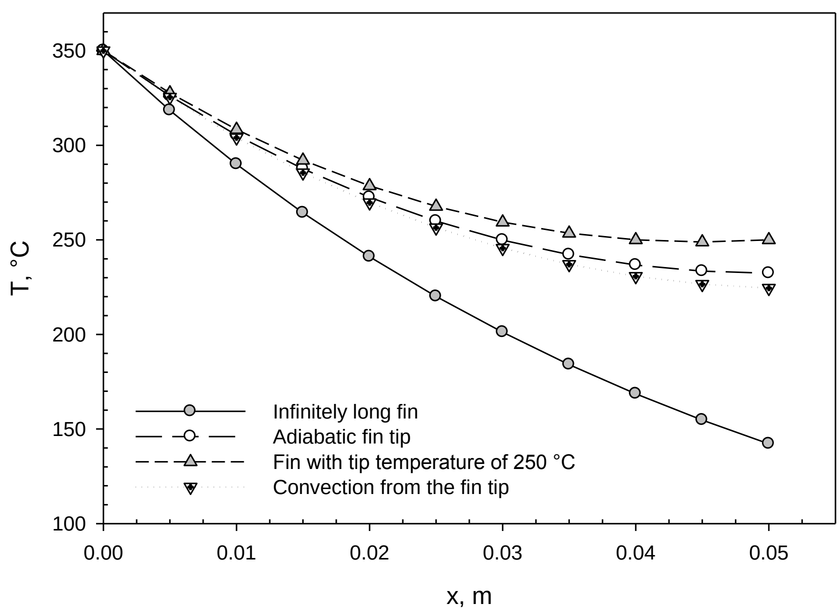
The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$$

The values for the temperature variations for parts (a) to (d) are tabulated in the following table:

L, m	T(x), °C			
	Part (a)	Part (b)	Part (c)	Part (d)
0	350	350	350	350
0.005	318	326	328	325
0.010	290	305	308	304
0.015	264	288	292	285
0.020	241	272	279	270
0.025	220	260	268	256
0.030	201	250	259	246
0.035	184	242	253	237
0.040	169	237	250	231
0.045	155	233	249	227
0.050	142	232	250	224

The temperature variations for parts (a) to (d) are plotted in the following figure:



Discussion The differences in the temperature variations show that applying the proper boundary condition is very important in order to perform the analysis correctly.

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3-186 Ten rectangular aluminum fins are placed on the outside surface of an electronic device. The rate of heat loss from the electronic device to the surrounding air and the fin effectiveness are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum fin is given to be k = 203 W/m·K.

Analysis The fin efficiency is to be determined using Fig. 3-43 in the text.

$$\xi = L_c^{3/2}\sqrt{h/(kA_p)} = (L + t/2)\sqrt{h/(kt)} = (0.020 + 0.004/2)\sqrt{\frac{100}{(203)(0.004)}} = 0.244 \longrightarrow \eta_{\text{fin}} = 0.93$$

The rate of heat loss can be determined as follows

$$A_{\text{fin}} = 2\times 10(0.020\times 0.100 + 0.004\times 0.020) = 0.0416\ \text{m}^2$$

$$A_{\text{base}} = 10(0.100\times 0.004) = 0.004\ \text{m}^2$$

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\dot{Q}_{\text{fin}}}{hA_{\text{fin}}(T_b - T_\infty)} \longrightarrow 0.93 = \frac{\dot{Q}_{\text{fin}}}{(100)(0.0416)(60-20)} \longrightarrow \dot{Q}_{\text{fin}} = 155\ \text{W}$$

$$\dot{Q}_{\text{base}} = hA_{\text{base}}(T_b - T_\infty) = (100)(0.004)(60-20) = 16\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin}} + \dot{Q}_{\text{base}} = 155 + 16 = \mathbf{171\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_{\text{base, no fin}}(T_b - T_\infty)} = \frac{171}{(100)(0.080\times 0.100)(60-20)} = \mathbf{5.34}$$

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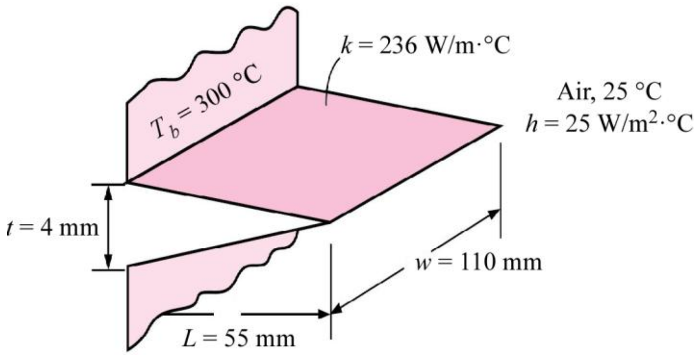
3-187 Using Table 3-3, the efficiency, heat transfer rate, and effectiveness of a straight triangular fin are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 236 W/m · °C.



Analysis From Table 3-3, for straight triangular fins, we have

$$mL = \sqrt{\frac{2h}{kt}}L = \sqrt{\frac{2(25\text{ W/m}^2\cdot°\text{C})}{(236\text{ W/m}\cdot°\text{C})(0.004\text{ m})}}(0.055\text{ m}) = 0.4$$

$$A_{\text{fin}} = 2w\sqrt{L^2 + (t/2)^2} = 2(0.110\text{ m})\sqrt{(0.055\text{ m})^2 + (0.004\text{ m}/2)^2} = 0.01211\text{ m}^2$$

$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

From Table 3-4, the modified Bessel functions are

$$e^{-2mL}I_0(2mL) = e^{-0.8}I_0(0.8) = 0.5241 \quad \text{or} \quad I_0(0.8) = 1.166$$

$$e^{-2mL}I_1(2mL) = e^{-0.8}I_1(0.8) = 0.1945 \quad \text{or} \quad I_1(0.8) = 0.4329$$

Hence, the fin efficiency is

$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{1}{0.4}\left(\frac{0.4329}{1.166}\right) = \mathbf{0.928}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.928)(25\text{ W/m}^2\cdot°\text{C})(0.01211\text{ m}^2)(300-25)\,°\text{C} = \mathbf{77.3\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{77.3\text{ W}}{(25\text{ W/m}^2\cdot°\text{C})(0.004\text{ m})(0.11\text{ m})(300-25)\,°\text{C}} = \mathbf{25.5}$$

Discussion The fin efficiency can also be determined using the EES with the following line:

```
eta_fin=1/0.4*Bessel_I1(0.8)/Bessel_I0(0.8)
```

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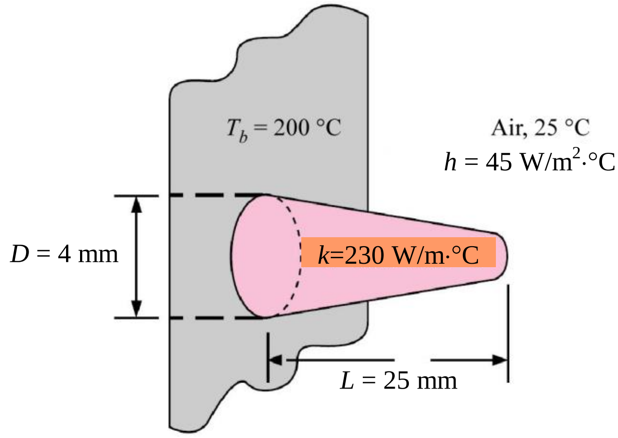
3-188 Aluminum pin fins of parabolic profile with blunt tips are attached to a plane surrface. The heat transfer rate from a single fin and the increase in the heat transfer as a result of attaching fins are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 230 W/m · °C.



Analysis From Table 3-3, for pin fins of parabolic profile (blunt tip), we have

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4(45\text{ W/m}^2\cdot{}^\circ\text{C})}{(230\text{ W/m}\cdot{}^\circ\text{C})(0.004\text{ m})}}(0.025\text{ m}) = 0.3497$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{\left[16\left(\frac{L}{D}\right)^2+1\right]^{3/2}-1\right\} = \frac{\pi(0.004\text{ m})^4}{96(0.025\text{ m})^2}\left\{\left[16\left(\frac{0.025\text{ m}}{0.004\text{ m}}\right)^2+1\right]^{3/2}-1\right\}$$

$$= 2.099\times10^{-4}\text{ m}^2$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)} = \frac{3}{2(0.3497)}\frac{I_1[4(0.3497)/3]}{I_0[4(0.3497)/3]}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
eta_fin=3/(2*0.3497)*Bessel_I1(4*0.3497/3)/Bessel_I0(4*0.3497/3)
```

Solving by EES software, the fin efficiency is

$$\eta_{\text{fin}} = 0.9738$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.9738)(45\text{ W/m}^2\cdot{}^\circ\text{C})(2.099\times10^{-4}\text{ m}^2)(200-25)\,{}^\circ\text{C} = \mathbf{1.610\,W}$$

Heat transfer from 100 fins is

$$\dot{Q}_{\text{fin,total}} = (100)(1.610\text{ W}) = 161\text{ W}$$

The surface area of the unfinned portion is

$$A_{\text{unfin}} = (1\times1)\text{ m}^2 - 100(\pi D^2/4) = 1 - 100\pi(0.004\text{ m})^2/4 = 0.9987\text{ m}^2$$

The heat transfer from the unfinned portion is

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (0.9987\text{ m}^2)(45\text{ W/m}^2\cdot{}^\circ\text{C})(200-25)\,{}^\circ\text{C} = 7865\text{ W}$$

The total heat transfer from the surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin,total}} + \dot{Q}_{\text{unfin}} = 161 + 7865 = 8026\text{ W}$$

If there was no fin at the surface,

$$\dot{Q}_{\text{nofin}} = hA_{\text{unfin}}(T_b - T_\infty) = (1\text{ m}^2)(45\text{ W/m}^2\cdot{}^\circ\text{C})(200-25)\,{}^\circ\text{C} = 7875\text{ W}$$

The increase in heat transfer as a result of attaching fins is then

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total}} - \dot{Q}_{\text{nofin}} = 8026 - 7875 = \mathbf{151\,W}$$

Discussion The values for the Bessel functions may also be approximated using Table 3-4.

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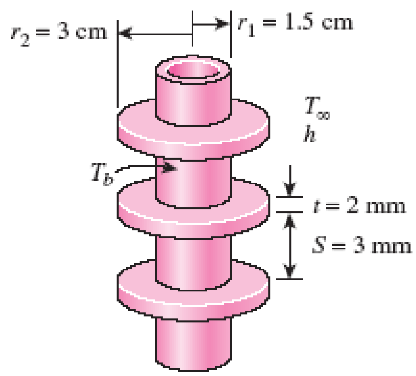
3-189 Circular aluminum alloy fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 180 W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\text{ m})(1\text{ m}) = 0.0942\text{ m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (60\text{ W/m}^2.°\text{C})(0.0942\text{ m}^2)(120-25)°\text{C} = 537\text{ W}$$



The efficiency of these circular fins is, from the efficiency curve, Fig. 3-44

$$\left.\begin{aligned} L &= (D_2 - D_1)/2 = (0.06-0.03)/2 = 0.015\text{ m} \\ \frac{r_2 + (t/2)}{r_1} &= \frac{0.03 + (0.002/2)}{0.015} = 2.07 \\ L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} &= \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} \\ &= \left(0.015 + \frac{0.002}{2}\right)\sqrt{\frac{60\text{ W/m}^2\,^\circ\text{C}}{(180\text{ W/m}^\circ\text{C})(0.002\text{ m})}} = 0.207 \end{aligned}\right\}\eta_{\text{fin}} = 0.96$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.015^2) + 2\pi(0.03)(0.002) = 0.004624\text{ m}^2$$
$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= 0.96(60\text{ W/m}^2.°\text{C})(0.004624\text{ m}^2)(120-25)°\text{C} \\ &= 25.3\text{ W} \end{aligned}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.03\text{ m})(0.003\text{ m}) = 0.000283\text{ m}^2$$
$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (60\text{ W/m}^2.°\text{C})(0.000283\text{ m}^2)(120-25)°\text{C} = 1.6\text{ W}$$

There are 200 fins and thus 200 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.3 + 1.6) = 5380\text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 5380 - 537 = \mathbf{4843\text{ W}}$$

Discussion The The overall effectiveness of the finned tube is 5380/537 = 10. That is, the rate of heat transfer from the steam tube increases by a factor of 10 as a result of adding fins. This explains the widespread use of finned surfaces.

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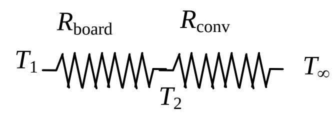
3-190 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 aluminum fins of rectangular profile on the backside.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 12 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (a) The thermal resistance of the board and the convection resistance on the backside of the board are

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.002\text{ m}}{(12\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.011\text{ °C/W}$$



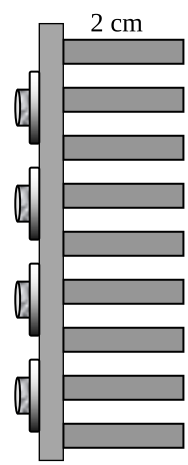
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(45\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 1.481\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.011 + 1.481 = 1.492\text{ °C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 37\text{°C} + (15\text{ W})(1.492\text{°C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 59.4\text{°C} - (15\text{ W})(0.011\text{°C/W}) = \mathbf{59.2°C}$$



(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be

$$m = \sqrt{\frac{hp}{kA_c}} \cong \sqrt{\frac{h(2w)}{k(tw)}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(45\text{ W/m}^2.\text{°C})}{(237\text{ W/m.°C})(0.002\text{ m})}} = 13.78\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh mL}{mL} = \frac{\tanh(13.78\text{ m}^{-1} \times 0.02\text{ m})}{13.78\text{ m}^{-1} \times 0.02\text{ m}} = 0.975$$

The finned and unfinned surface areas are

$$A_{\text{finned}} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\text{ m}^2$$

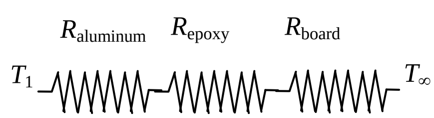
$$A_{\text{unfinned}} = (0.1)(0.15) - 20(0.002)(0.15) = 0.0090\text{ m}^2$$

Then,

$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{unfinned}} + \dot{Q}_{\text{finned}} = h(T_{\text{base}} - T_\infty)(\eta_{\text{fin}} A_{\text{fin}} + A_{\text{unfinned}})$$



Substituting, the base temperature of the finned surfaces is determined to be

$$T_{\text{base}} = T_\infty + \frac{\dot{Q}_{\text{total}}}{h(\eta_{\text{fin}} A_{\text{fin}} + A_{\text{unfinned}})} = 37\text{°C} + \frac{15\text{ W}}{(45\text{ W/m}^2.\text{°C})[(0.975)(0.126\text{ m}^2) + (0.0090\text{ m}^2)]} = \mathbf{39.5°C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{aluminum}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(237\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00028\text{ °C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0003\text{ m}}{(1.8\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.01111\text{ °C/W}$$

$$\dot{Q} = \frac{T_1 - T_{\text{base}}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)\text{°C}}{(0.00028 + 0.01111 + 0.011)\text{ °C/W}} \longrightarrow T_1 = 39.5\text{°C} + (15\text{ W})(0.02239\text{ °C/W}) = \mathbf{39.8°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 39.8\text{°C} - (15\text{ W})(0.011\text{°C/W}) = \mathbf{39.6°C}$$

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3-191 PtD An electronic device is cooled by dissipating heat through a heat sink attached on its top surface. There is contact resistance at the interface of the electronic component and the heat sink. The surface temperature of the electronic device is to be determined whether it is below 85°C or not.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** The electronic device maintains a constant surface temperature.

Properties The thermal contact conductance for aluminum plates with air at the interface, a roughness of about 10 μm and an average interface pressure of 1 atm is h_c = 3640 W/m^2·K (Table 3-1), the combined thermal resistance of an HS 5030 heat sink, attached horizontally, is 1.2 K/W (Table 3-6).

Analysis The thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} = \frac{1}{(3640\ \text{W/m}^2\cdot\text{K})(0.100\ \text{m})(0.040\ \text{m})} = 0.06868\ \text{K/W}$$

$$R_{\text{heat sink}} = 1.2\ \text{K/W}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{heat sink}} = 1.26868\ \text{K/W}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{total}}}$$

The surface temperature of the electronic device is

$$T_s = \dot{Q}R_{\text{total}} + T_\infty = (45\ \text{W})(1.26868\ \text{K/W}) + 30°\text{C} = \mathbf{87.1°C} > 85°\text{C}$$

Since the surface temperature of the electronic device is above 85°C, there is a risk of overheating. To reduce the surface temperature, the total thermal resistance needs to be reduced to promote more heat dissipation through the heat sink. One way to solve this problem is by reducing the contact resistance at the interface. This can be achieved by filling the interface with a fluid having higher thermal contact conductance than air.

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{\dfrac{1}{h_c A} + R_{\text{heat sink}}} \quad \rightarrow \quad h_c = \left[A\left(\frac{T_s - T_\infty}{\dot{Q}} - R_{\text{heat sink}}\right)\right]^{-1}$$

$$h_c = \left[(0.100\ \text{m})(0.040\ \text{m})\left(\frac{85-30}{45} - 1.2\right)\left(\frac{\text{K}}{\text{W}}\right)\right]^{-1} = 11{,}250\ \text{W/m}^2\cdot\text{K}$$

Thus, the surface temperature of the electronic device can be reduced to below 85°C by filling the interface with a fluid having a thermal contact conductance value higher than 11,250 W/m^2·K. From Table 3-1 hydrogen, silicone oil and glycerin all have thermal contact conductance greater than 11,250 W/m^2·K.

Discussion In practice, the interfaces between electronic devices and heat sinks are filled with thermally conductive epoxy adhesives to reduce thermal contact resistance.

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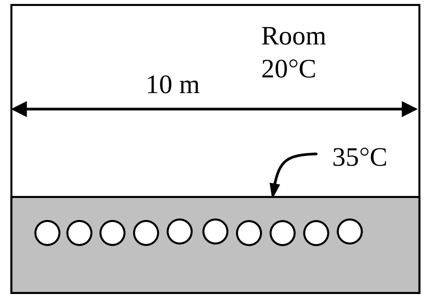
3-192 Steam passes through a row of 10 parallel pipes placed horizontally in a concrete floor exposed to room air at 20°C with a heat transfer coefficient of 12 W/m^2.°C. If the surface temperature of the concrete floor is not to exceed 35°C, the minimum burial depth of the steam pipes below the floor surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis In steady operation, the rate of heat loss from the steam through the concrete floor by conduction must be equal to the rate of heat transfer from the concrete floor to the room by combined convection and radiation, which is determined to be



$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$= (12\ \text{W/m}^2.°\text{C})[(10\ \text{m})(5\ \text{m})](35-20)°\text{C} = 9000\ \text{W}$$

Then the depth the steam pipes should be buried can be determined with the aid of shape factor for this configuration from Table 3-7 to be

$$\dot{Q} = nSk(T_1 - T_2) \longrightarrow S = \frac{\dot{Q}}{nk(T_1 - T_2)} = \frac{9000\ \text{W}}{10(0.75\ \text{W/m.°C})(145-35)°\text{C}} = 10.91\ \text{m}\ \ \text{(per pipe)}$$

$$w = \frac{a}{n} = \frac{10\ \text{m}}{10} = 1\ \text{m (center - to - center distance of pipes)}$$

$$S = \frac{2\pi L}{\ln\left(\frac{2w}{\pi D}\sinh\frac{2\pi z}{w}\right)}$$

$$10.91\ \text{m} = \frac{2\pi(5\ \text{m})}{\ln\left[\frac{2(1\ \text{m})}{\pi(0.06\ \text{m})}\sinh\frac{2\pi z}{(1\ \text{m})}\right]} \longrightarrow z = 0.205\ \text{m} = \mathbf{20.5\ cm}$$

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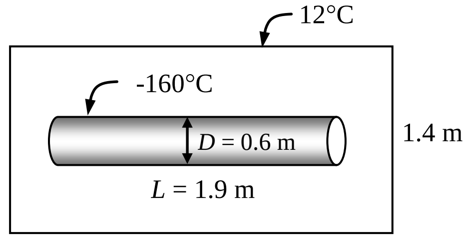
3-193 A cylindrical tank containing liquefied natural gas (LNG) is placed at the center of a square solid bar. The rate of heat transfer to the tank and the LNG temperature at the end of a one-month period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the bar is constant. **4** The tank surface is at the same temperature as the LNG.

Properties The thermal conductivity of the bar is given to be k = 0.0002 W/m·°C. The density and the specific heat of LNG are given to be 425 kg/m^3 and 3.475 kJ/kg·°C, respectively,



Analysis The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\ln\left(\frac{1.08w}{D}\right)} = \frac{2\pi(1.9\text{ m})}{\ln\left(1.08\frac{1.4\text{ m}}{0.6\text{ m}}\right)} = 12.92\text{ m}$$

Then the steady rate of heat transfer to the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (12.92\text{ m})(0.0002\text{ W/m.°C})[12-(-160)]\text{°C} = \mathbf{0.4444\ W}$$

The mass of LNG is

$$m = \rho \boldsymbol{V} = \rho\pi\frac{D^3}{6} = (425\text{ kg/m}^3)\pi\frac{(0.6\text{ m})^3}{6} = 48.07\text{ kg}$$

The amount heat transfer to the tank for a one-month period is

$$Q = \dot{Q}\Delta t = (0.4444\text{ W})(30\times 24\times 3600\text{ s}) = 1.152\times 10^6\text{ J}$$

Then the temperature of LNG at the end of the month becomes

$$Q = mc_p(T_1 - T_2)$$
$$1.152\times 10^6\text{ J} = (48.07\text{ kg})(3475\text{ J/kg.°C})[(-160) - T_2]\text{°C}$$
$$T_2 = \mathbf{-153.1°C}$$

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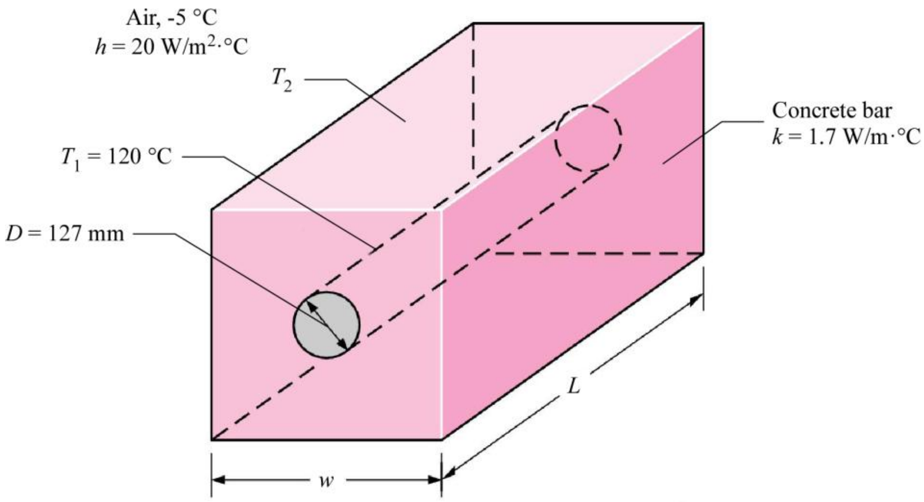
3-194 A tube carrying hot steam is centered at a square cross-section concrete bar. The width of the square concrete bar and the rate of heat loss in (W/m) are to be determined for the temperature difference between the outer surface of the square concrete bar and the ambient air to be maintained at 5 °C.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Heat conduction through the tube wall is negligible. **5** Thermal contact resistance between the tube and the concrete bar is negligible.

Properties The thermal conductivity of the concrete is given as 1.7 W/m · °C.



Analysis Using Table 3-7 (Case 6), the shape factor is given to be

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$

From energy balance, we have

$$kS(T_1 - T_2) = hA_s(T_2 - T_\infty)$$

or $$\frac{2\pi kL}{\ln(1.08w/D)}(T_1 - T_2) = 4hwL(T_2 - T_\infty)$$

Rearrange to get

$$w\ln\left(\frac{1.08w}{D}\right) = \frac{T_1 - T_2}{T_2 - T_\infty}\left(\frac{\pi k}{2h}\right)$$

$$w\ln\left(\frac{1.08w}{0.127\text{ m}}\right) = \frac{(120-0)\,°\text{C}}{5\,°\text{C}}\left[\frac{\pi(1.7\text{ W/m}\cdot°\text{C})}{2(20\text{ W/m}^2\cdot°\text{C})}\right]$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
w*ln(1.08*w/0.127)=120/5*(3.1416*1.7)/(2*20)
```

Solving by EES software, the width of the square concrete bar is

$$w = \mathbf{1.324\,m}$$

The heat loss to the ambient air is

$$\dot{Q}/L = 4hw(T_2 - T_\infty) = 4(20\text{ W/m}^2\cdot°\text{C})(1.324\text{ m})(5\,°\text{C}) = \mathbf{530\ W/m}$$

Discussion If the width of the concrete bar were less than 1.324 m, then the temperature difference between the outer surface of the concrete bar and the ambient air would be greater than 5 °C. This would mean more heat loss to the ambient air.

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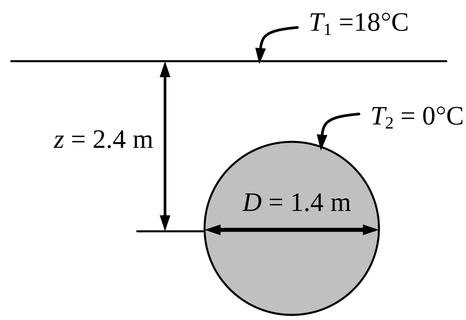
3-195 A spherical tank containing iced water is buried underground. The rate of heat transfer to the tank is to be determined for the insulated and uninsulated ground surface cases.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant. **4** The tank surface is assumed to be at the same temperature as the iced water because of negligible resistance through the steel.

Properties The thermal conductivity of the soil is given to be k = 0.55 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(1.4\text{ m})}{1 - 0.25\dfrac{1.4\text{ m}}{2.4\text{ m}}} = 10.30\text{ m}$$



Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (10.30\text{ m})(0.55\text{ W/m.°C})(18-0)\text{°C} = \mathbf{102\ W}$$

If the ground surface is insulated,

$$S = \frac{2\pi D}{1 + 0.25\dfrac{D}{z}} = \frac{2\pi(1.4\text{ m})}{1 + 0.25\dfrac{1.4\text{ m}}{2.4\text{ m}}} = 7.677\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (7.677\text{ m})(0.55\text{ W/m.°C})(18-0)\text{°C} = \mathbf{76.0\ W}$$

3-196 A thin-walled spherical tank, filled with chemicals undergoing exothermic reaction, is buried in the ground. The reaction provides uniform heat flux on the tank inner surface and the ground surface is maintained at a specified temperature. The tank surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** Isothermal tank surface.

Properties The thermal conductivity of the ground is given to be k = 1.3 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 15) to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(1.5\text{ m})}{1 - 0.25\dfrac{1.5\text{ m}}{3\text{ m}}} = 10.77\text{ m}$$

The heat transfer rate from the spherical tank is

$$\dot{Q}_{\text{sph}} = \dot{q}A_s = kS(T_1 - T_2)$$

Thus, the surface temperature of the tank is

$$T_1 = \frac{\dot{q}A_s}{kS} + T_2 = \frac{\dot{q}\pi D^2}{kS} + T_2 = \frac{(1000\text{ W/m}^2)\pi(1.5\text{ m})^2}{(1.3\text{ W/m}\cdot\text{K})(10.77\text{ m})} + 10\text{°C} = \mathbf{515°C}$$

Discussion The deeper the tank is buried in the ground, the higher its surface temperature will be. This is because the ground depth acts as a thermal resistance, limiting heat transfer between the tank surface and the ground surface.

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Fundamentals of Engineering (FE) Exam Problems

3-197 Heat is lost at a rate of 275 W per m^2 area of a 15-cm-thick wall with a thermal conductivity of k=1.1 W/m·ºC. The temperature drop across the wall is

(a) 37.5ºC (b) 27.5ºC (c) 16.0ºC (d) 8.0ºC (e) 4.0ºC

Answer (a) 37.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.15 [m]
k=1.1 [W/m-C]
q=275 [W/m^2]
q=k*DELTAT/L
```

3-198 Consider a wall that consists of two layers, *A* and *B*, with the following values: k_A = 0.8 W/m·ºC, L_A = 8 cm, k_B = 0.2 W/m·ºC, L_B = 5 cm. If the temperature drop across the wall is 18ºC, the rate of heat transfer through the wall per unit area of the wall is

(a) 180 W/m^2 (b) 153 W/m^2 (c) 89.6 W/m^2 (d) 72 W/m^2 (e) 51.4 W/m^2

Answer (a) 51.4 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k_A=0.8 [W/m-C]
L_A=0.08 [m]
k_B=0.2 [W/m-C]
L_B=0.05 [m]
DELTAT=18 [C]
R_total=L_A/k_A+L_B/k_B
q_dot=DELTAT/R_total
"Some Wrong Solutions with Common Mistakes"
W1_q_dot=DELTAT/(L_A/k_A) "Considering layer A only"
W2_q_dot=DELTAT/(L_B/k_B) "Considering layer B only"
```

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3-199 Heat is generated steadily in a 3-cm-diameter spherical ball. The ball is exposed to ambient air at 26ºC with a heat transfer coefficient of 7.5 W/m^2·ºC. The ball is to be covered with a material of thermal conductivity 0.15 W/m·ºC. The thickness of the covering material that will maximize heat generation within the ball while maintaining ball surface temperature constant is

(a) 0.5 cm (b) 1.0 cm (c) 1.5 cm (d) 2.0 cm (e) 2.5 cm

Answer (e) 2.5 cm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.03 [m]
r=D/2
T_infinity=26 [C]
h=7.5 [W/m^2-C]
k=0.15 [W/m-C]
r_cr=(2*k)/h r_cr=(2*k)/h "critical radius of insulation for a sphere"
thickness=r_cr-r
"Some Wrong Solutions with Common Mistakes"
W_r_cr=k/h
W1_thickness=W_r_cr-r "Using the equation for cylinder"
```

3-200 Consider a 1.5-m-high and 2-m-wide triple pane window. The thickness of each glass layer (k = 0.80 W/m.°C) is 0.5 cm, and the thickness of each air space (k = 0.025 W/m.°C) is 1 cm. If the inner and outer surface temperatures of the window are 10°C and 0°C, respectively, the rate of heat loss through the window is

(a) 75 W (b) 12 W (c) 46 W (d) 25 W (e) 37 W

Answer: (e) 37 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"Using the thermal resistances per unit area, Q can be expressed as Q=A*DeltaT/R_total”
Lglass=0.005 {m}
kglass=0.80 {W/mC}
Rglass=Lglass/kglass
Lair=0.01 {m}
kair=0.025 {W/mC}
Rair=Lair/kair
A=1.5*2
T1=10
T2=0
Q=A*(T1-T2)/(3*Rglass+2*Rair)
“Some Wrong Solutions with Common Mistakes:”
W1_Q=(T1-T2)/(3*Rglass+2*Rair) “Not using area”
W2_Q=A*(T1-T2)*(3*Rglass+2*Rair) “Multiplying resistance instead of dividing”
W3_Q=A*(T1-T2)/(Rglass+Rair) “Using one layer only”
W4_Q=(T1-T2)/(3*Rglass+2*Rair)/A “Dividing by area instead of multiplying”
```

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3-201 Consider two metal plates pressed against each other. Other things being equal, which of the measures below will cause the thermal contact resistance to increase?

(a) Cleaning the surfaces to make them shinier

(b) Pressing the plates against each other with a greater force

(c) Filling the gab with a conducting fluid

(d) Using softer metals

(e) Coating the contact surfaces with a thin layer of soft metal such as tin

Answer (a) Cleaning the surfaces to make them shinier

3-202 A 10-m-long, 5-cm-outer-radius cylindrical steam pipe is covered with 3-cm thick cylindrical insulation with a thermal conductivity of 0.05 W/m.°C. If the rate of heat loss from the pipe is 1000 W, the temperature drop across the insulation is

(a) 163°C (b) 600°C (c) 48°C (d) 79°C (e) 251°C

Answer (e) 251°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.05
S=0.03
R2=0.11
L=10
K=0.05
Q=1000
R=ln(r2/r1)/(2*pi*L*k)
dT=Q*R
“Some Wrong Solutions with Common Mistakes:”
W1_T=Q/k "Wrong relation"
RR1=ln(s/r1)/(2*pi*L*k)
W2_T=Q*RR1"Wrong radius"
RR2=s/k
W3_T=Q*RR2"Wrong radius"
```

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3-203 A 6-m diameter spherical tank is filled with liquid oxygen (ρ = 1141 kg/m^3, c_p = 1.71 kJ/kg·ºC) at -184ºC. It is observed that the temperature of oxygen increases to -183ºC in a 144-hour period. The average rate of heat transfer to the tank is

(a) 249 W (b) 426 W (c) 570 W (d) 1640 W (e) 2207 W

Answer (b) 426 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=6 [m]
rho=1141 [kg/m^3]
c_p=1710 [J/kg-C]
T1=-184 [C]
T2=-183 [C]
time=144*3600 [s]
V=pi*D^3/6
m=rho*V
Q=m*c_p*(T2-T1)
Q_dot=Q/time
"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=Q "Using amount of heat transfer as the answer"
Q1=m*(T2-T1)
W2_Q_dot=Q1/time "Not using specific heat in the equation"
```

3-204 A 2.5-m-high, 4-m-wide, and 20-cm-thick wall of a house has a thermal resistance of 0.0125ºC/W. The thermal conductivity of the wall is

(a) 0.72 W/m·ºC (b) 1.1 W/m·ºC (c) 1.6 W/m·ºC (d) 16 W/m·ºC (e) 32 W/m·ºC

Answer (c) 1.6 W/m·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Height=2.5 [m]
Width=4 [m]
L=0.20 [m]
R_wall=0.0125 [C/W]
A=Height*Width
R_wall=L/(k*A)
"Some Wrong Solutions with Common Mistakes"
R_wall=L/W1_k "Not using area in the equation"
```

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3-205 Consider two walls, *A* and *B*, with the same surface areas and the same temperature drops across their thicknesses. The ratio of thermal conductivities is $k_A/k_B = 4$ and the ratio of the wall thicknesses is $L_A/L_B = 2$. The ratio of heat transfer rates through the walls $\dot{Q}_A / \dot{Q}_B$ is

(a) 0.5 (b) 1 (c) 2 (d) 4 (e) 8

Answer (c) 2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k_A\k_B=4
L_A\L_B=2
Q_dot_A\Q_dot_B=k_A\k_B*(1/L_A\L_B) "From Fourier's Law of Heat Conduction"
```

3-206 A hot plane surface at 100°C is exposed to air at 25°C with a combined heat transfer coefficient of 20 W/m^2·°C. The heat loss from the surface is to be reduced by half by covering it with sufficient insulation with a thermal conductivity of 0.10 W/m·°C. Assuming the heat transfer coefficient to remain constant, the required thickness of insulation is

(a) 0.1 cm (b) 0.5 cm (c) 1.0 cm (d) 2.0 cm (e) 5 cm

Answer (b) 0.5 cm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Ts=100
Tinf=25
h=20
k=0.1
Rconv=1/h
Rins=L/k
Rtotal=Rconv+Rins
Q1=h*(Ts-Tinf)
Q2=(Ts-Tinf)/(Rconv+Rins)
Q2=Q1/2
```

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3-207 A room at 20°C air temperature is loosing heat to the outdoor air at 0°C at a rate of 1000 W through a 2.5-m-high and 4-m-long wall. Now the wall is insulated with 2-cm-thick insulation with a conductivity of 0.02 W/m.°C. Determine the rate of heat loss from the room through this wall after insulation. Assume the heat transfer coefficients on the inner and outer surface of the wall, the room air temperature, and the outdoor air temperature to remain unchanged. Also, disregard radiation.

(a) 20 W (b) 561 W (c) 388 W (d) 167 W (e) 200 W

Answer (d) 167 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Tin=20
Tout=0
Q=1000
A=2.5*4
L=0.02
k=0.02
Rins=L/(k*A)
Q=(Tin-Tout)/R
Qnew=(Tin-Tout)/(R+Rins)
"Some Wrong Solutions with Common Mistakes:"
W1_Q=(Tin-Tout)/Rins "Disregarding original resistance"
W2_Q=(Tin-Tout)*(R+L/k) "Disregarding area"
W3_Q=(Tin-Tout)*(R+Rins)"Multiplying by resistances"
```

3-208 A 1-cm-diameter, 30-cm long fin made of aluminum (k = 237 W/m·ºC) is attached to a surface at 80ºC. The surface is exposed to ambient air at 22ºC with a heat transfer coefficient of 11 W/m^2·ºC. If the fin can be assumed to be very long, the rate of heat transfer from the fin is

(a) 2.2 W (b) 3.0 W (c) 3.7 W (d) 4.0 W (e) 4.7 W

Answer (e) 4.7 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.01 [m]
L=0.30 [m]
k=237 [W/m-C]
T_b=80 [C]
T_infinity=22 [C]
h=11 [W/m^2-C]
p=pi*D
A_c=pi*D^2/4
Q_dot=sqrt(h*p*k*A_c)*(T_b-T_infinity)
"Some Wrong Solutions with Common Mistakes"
a=sqrt((h*p)/(k*A_c))
W1_Q_dot=sqrt(h*p*k*A_c)*(T_b-T_infinity)*tanh(a*L) "Using the relation for insulated fin tip"
```

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3-209 A 1-cm-diameter, 30-cm-long fin made of aluminum (k = 237 W/m·ºC) is attached to a surface at 80ºC. The surface is exposed to ambient air at 22ºC with a heat transfer coefficient of 11 W/m^2·ºC. If the fin can be assumed to be very long, its efficiency is

(a) 0.60 (b) 0.67 (c) 0.72 (d) 0.77 (e) 0.88

Answer (d) 0.77

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.01 [m]
L=0.30 [m]
k=237 [W/m-C]
T_b=80 [C]
T_infinity=22 [C]
h=11 [W/m^2-C]
p=pi*D
A_c=pi*D^2/4
a=sqrt((h*p)/(k*A_c))
eta_fin=1/(a*L)
"Some Wrong Solutions with Common Mistakes"
W1_eta_fin=tanh(a*L)/(a*L) "Using the relation for insulated fin tip"
```

3-210 A hot surface at 80°C in air at 20°C is to be cooled by attaching 10-cm-long and 1-cm-diameter cylindrical fins. The combined heat transfer coefficient is 30 W/m^2·°C, and heat transfer from the fin tip is negligible. If the fin efficiency is 0.75, the rate of heat loss from 100 fins is

(a) 325 W (b) 707 W (c) 566 W (d) 424 W (e) 754 W

Answer (d) 424 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
N=100
Ts=80
Tinf=20
L=0.1
D=0.01
h=30
Eff=0.75
A=N*pi*D*L
Q=Eff*h*A*(Ts-Tinf)
“Some Wrong Solutions with Common Mistakes:”
W1_Q= h*A*(Ts-Tinf) "Using Qmax"
W2_Q= h*A*(Ts-Tinf)/Eff "Dividing by fin efficiency"
W3_Q= Eff*h*A*(Ts+Tinf) "Adding temperatures"
W4_Q= Eff*h*(pi*D^2/4)*L*N*(Ts-Tinf) "Wrong area"
```

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3-211 A cylindrical pin fin of diameter 0.6 cm and length of 3 cm with negligible heat loss from the tip has an efficiency of 0.7. The effectiveness of this fin is

(a) 0.3 (b) 0.7 (c) 2 (d) 8 (e) 14

Answer (e) 14

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"The relation between fin efficiency and fin effectiveness is effect = (A_fin/A_base)*Efficiency"
D=0.6 {cm}
L=3 {cm}
Effici=0.7
Effect=(pi*D*L/(pi*D^2/4))*Effici
"Some Wrong Solutions with Common Mistakes:"
W1_Effect= Effici "Taking it equal to efficiency"
W2_Effect= (D/L)*Effici "Using wrong ratio"
W3_Effect= 1-Effici "Using wrong relation"
W4_Effect= (pi*D*L/(pi*D))*Effici "Using area over perimeter"
```

3-212 A 3-cm-long, 2 mm × 2 mm rectangular cross-section aluminum fin (k = 237 W/m·ºC) is attached to a surface. If the fin efficiency is 65 percent, the effectiveness of this single fin is

(a) 39 (b) 30 (c) 24 (d) 18 (e) 7

Answer (a) 39

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.03 [m]
s=0.002 [m]
k=237 [W/m-C]
eta_fin=0.65
A_fin=4*s*L
A_b=s*s
epsilon_fin=A_fin/A_b*eta_fin
```

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3-213 Two finned surfaces with long fins are identical, except that the convection heat transfer coefficient for the first finned surface is twice that of the second one. What statement below is accurate for the efficiency and effectiveness of the first finned surface relative to the second one?

(a) higher efficiency and higher effectiveness

(b) higher efficiency but lower effectiveness

(c) lower efficiency but higher effectiveness

(d) lower efficiency and lower effectiveness

(e) equal efficiency and equal effectiveness

Answer (d) lower efficiency and lower effectiveness

Solution The efficiency of long fin is given by $\eta = \sqrt{kA_c / hp} / L$, which is inversely proportional to convection coefficient *h*. Therefore, efficiency of first finned surface with higher *h* will be lower. This is also the case for effectiveness since effectiveness is proportional to efficiency, $\varepsilon = \eta (A_{fin} / A_{base})$.

3-214 A 20-cm-diameter hot sphere at 120°C is buried in the ground with a thermal conductivity of 1.2 W/m·°C. The distance between the center of the sphere and the ground surface is 0.8 m, and the ground surface temperature is 15°C. The rate of heat loss from the sphere is

(a) 169 W (b) 20 W (c) 217 W (d) 312 W (e) 1.8 W

Answer (a) 169 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.2
T1=120
T2=15
K=1.2
Z=0.8
S=2*pi*D/(1-0.25*D/z)
Q=S*k*(T1-T2)
“Some Wrong Solutions with Common Mistakes:”
A=pi*D^2
W1_Q=2*pi*z/ln(4*z/D) "Using the relation for cylinder"
W2_Q=k*A*(T1-T2)/z "Using wrong relation"
W3_Q= S*k*(T1+T2) "Adding temperatures"
W4_Q= S*k*A*(T1-T2) "Multiplying vy area also"
```

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3-215 A 25-cm-diameter, 2.4-m-long vertical cylinder containing ice at 0ºC is buried right under the ground. The cylinder is thin-shelled and is made of a high thermal conductivity material. The surface temperature and the thermal conductivity of the ground are 18ºC and 0.85 W/m·ºC, respectively. The rate of heat transfer to the cylinder is

(a) 37.2 W (b) 63.2 W (c) 158 W (d) 480 W (e) 1210 W

Answer (b) 63.2 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.25 [m]
L=2.4 [m]
T_ice=0 [C]
T_ground=18 [C]
k=0.85 [W/m-C]
S=(2*pi*L)/ln((4*L)/D)
Q_dot=S*k*(T_ground-T_ice)
```

3-216 Hot water (c = 4.179 kJ/kg·K) flows through a 200 m long PVC (k = 0.092 W/m·K) pipe whose inner diameter is 2 cm and outer diameter is 2.5 cm at a rate of 1 kg/s, entering at 40°C. If the entire interior surface of this pipe is maintained at 35^{o}C and the entire exterior surface at 20^{o}C, the outlet temperature of water is

(a) 39^{o}C (b) 38^{o}C (c) 37^{o}C (d) 36^{o}C (e) 35°C

Answer (b) 38^{o}C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
do=2.5 [cm]
di=2.0 [cm]
k=0.092 [W/m-C]
T2=35 [C]
T1=20 [C]
Q=2*pi*k*l*(T2-T1)/LN(do/di)
Tin=40 [C]
c=4179 [J/kg-K]
m=1 [kg/s]
l=200 [m]
Q=m*c*(Tin-Tout)
```

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3-217 The walls of a food storage facility are made of a 2-cm-thick layer of wood (k = 0.1 W/m·K) in contact with a 5-cm-thick layer of polyurethane foam (k = 0.03 W/m·K). If the temperature of the surface of the wood is -10°C and the temperature of the surface of the polyurethane foam is 20°C, the temperature of the surface where the two layers are in contact is

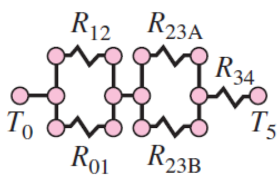
(a) -7°C (b) -2°C (c) 3°C (d) 8°C (e) 11°C

Answer (a) -7°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
kw=0.1 [W/m-C]
tkw=0.02 [m]
Tw=-10 [C]
kf=0.03 [W/m-C]
tkf=0.05 [W/m-C]
Tf=20 [C]
T=((kw*Tw/tkw)+(kf*Tf/tkf))/((kw/tkw)+(kf/tkf))
```

3-218 The equivalent thermal resistance for the thermal circuit shown here is



(a) $R_{12}R_{01} + R_{23A}R_{23B} + R_{34}$

(b) $R_{12}R_{01} + \left(\dfrac{R_{23A}R_{23B}}{R_{23A} + R_{23B}}\right) + R_{34}$

(c) $\left(\dfrac{R_{12}R_{01}}{R_{12} + R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A} + R_{23B}}\right) + \dfrac{1}{R_{34}}$

(d) $\left(\dfrac{R_{12}R_{01}}{R_{12} + R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A} + R_{23B}}\right) + R_{34}$

(e) None of them

Answer (d) $\left(\dfrac{R_{12}R_{01}}{R_{12} + R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A} + R_{23B}}\right) + R_{34}$

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3-219 The 700 m^2 ceiling of a building has a thermal resistance of 0.52 $m^2 \cdot K/W$. The rate at which heat is lost through this ceiling on a cold winter day when the ambient temperature is -10°C and the interior is at 20°C is

(a) 23.1 kW (b) 40.4 kW (c) 55.6 kW (d) 68.1 kW (e) 88.6 kW

Answer (b) 40.4 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R=0.52 [m^2-C/W]
A=700 [m^2]
T_1=20 [C]
T_2=-10 [C]
Q=A*(T_2-T_1)/R
```

3-220 A 1 m-inner diameter liquid oxygen storage tank at a hospital keeps the liquid oxygen at 90 K. This tank consists of a 0.5-cm thick aluminum (k = 170 W/m·K) shell whose exterior is covered with a 10-cm-thick layer of insulation (k = 0.02 W/m·K). The insulation is exposed to the ambient air at 20°C and the heat transfer coefficient on the exterior side of the insulation is 5 $W/m^2 \cdot K$. The rate at which the liquid oxygen gains heat is

(a) 141 W (b) 176 W (c) 181 W (d) 201 W (e) 221 W

Answer (b) 176 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.5 [m]
R2=0.55 [m]
R3=0.65 [m]
k1=170 [W/m-K]
k2=0.02 [W/m-K]
h=5[W/m^2-K]
T2=293 [K]
T1=90 [K]
R12=(R2-R1)/(4*pi*k1*R1*R2)
R23=(R3-R2)/(4*pi*k2*R2*R3)
R45=1/(h*4*pi*R3^2)
Re=R12+R23+R45
Q=(T2-T1)/Re
```

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3-221 A 1-m-inner diameter liquid oxygen storage tank at a hospital keeps the liquid oxygen at 90 K. This tank consists of a 0.5-cm-thick aluminum (k = 170 W/m·K) shell whose exterior is covered with a 10-cm-thick layer of insulation (k = 0.02 W/m·K). The insulation is exposed to the ambient air at 20°C and the heat transfer coefficient on the exterior side of the insulation is 5 W/m^2·K. The temperature of the exterior surface of the insulation is

(a) 13°C (b) 9°C (c) 2°C (d) -3°C (e) -12°C

Answer (a) 13°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.5 [m]
R2=0.55 [m]
R3=0.65 [m]
k1=170 [W/m-K]
k2=0.02 [W/m-K]
h=5[W/m^2-K]
T2=293 [K]
T1=90 [K]
R12=(R2-R1)/(4*pi*k1*R1*R2)
R23=(R3-R2)/(4*pi*k2*R2*R3)
R45=1/(h*4*pi*R3^2)
Re=R12+R23+R45
Q=(T2-T1)/Re
Q=(T2-T3)/R45
```

3-222 The fin efficiency is defined as the ratio of the actual heat transfer from the fin to

(a) The heat transfer from the same fin with an adiabatic tip

(b) The heat transfer from an equivalent fin which is infinitely long

(c) The heat transfer from the same fin if the temperature along the entire length of the fin is the same as the base temperature

(d) The heat transfer through the base area of the same fin

(e) None of the above

Answer: (c) The heat transfer from the same fin if the temperature along the entire length of the fin is the same as the base temperature

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3-223 Computer memory chips are mounted on a finned metallic mount to protect them from overheating. A 512 MB memory chip dissipates 5 W of heat to air at 25°C. If the temperature of this chip is not exceed 50°C, the overall heat transfer coefficient – area product of the finned metal mount must be at least

(a) 0.2 W/°C (b) 0.3 W/°C (c) 0.4 W/°C (d) 0.5 W/°C (e) 0.6 W/°C

Answer (a) 0.2 W/°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_1=50 [C]
T_2=25 [C]
Q=5 [W]
Q=UA*(T_1-T_2)
```

3-224 In the United States, building insulation is specified by the R-value (thermal resistance in h·ft^2·°F/Btu units). A home owner decides to save on the cost of heating the home by adding additional insulation in the attic. If the total R-value is increased from 15 to 25, the home owner can expect the heat loss through the ceiling to be reduced by

(a) 25% (b) 40% (c) 50% (d) 60% (e) 75%

Answer (b) 40%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R_1=15
R_2=25
DeltaT=1 "Any value can be chosen"
Q1=DeltaT/R_1
Q2=DeltaT/R_2
Reduction%=100*(Q1-Q2)/Q1
```

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3-225 A triangular shaped fin on a motorcycle engine is 0.5-cm thick at its base and 3-cm long (normal distance between the base and the tip of the triangle), and is made of aluminum (k = 150 W/m·K). This fin is exposed to air with a convective heat transfer coefficient of 30 W/m^2·K acting on its surfaces. The efficiency of the fin is 50 percent. If the fin base temperature is 130^{o}C and the air temperature is 25^{o}C, the heat transfer from this fin per unit width is

(a) 32 W/m (b) 47 W/m (c) 68 W/m (d) 82 W/m (e) 95 W/m

Answer (e) 95 W/m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=30 [W/m-K]
b=0.005 [m]
l=0.03 [m]
eff=0.50
Ta=25 [C]
Tb=130 [C]
A=2*(l^2+(b/2)^2)^0.5
Qideal=h*A*(Tb-Ta)
Q=eff*Qideal
```

3-226 A plane brick wall (k = 0.7 W/m·K) is 10 cm thick. The thermal resistance of this wall per unit of wall area is

(a) 0.143 m^2·K/W (b) 0.250 m^2·K/W (c) 0.327 m^2·K/W (d) 0.448 m^2·K/W (e) 0.524 m^2·K/W

Answer (a) 0.143 m^2·K/W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.7 [W/m-K]
t=0.1 [m]
R=t/k
```

3-227 ··· 3-233 Design and Essay Problems

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