CHAPTER 5

5.1 The function to evaluate is

$$f(c_d) = \sqrt{\frac{gm}{c_d}}\tanh\left(\sqrt{\frac{gc_d}{m}}t\right) - v(t)$$

or substituting the given values

$$f(c_d) = \sqrt{\frac{9.81(95)}{c_d}}\tanh\left(\sqrt{\frac{9.81c_d}{95}}9\right) - 46$$

The first iteration is

$$x_r = \frac{0.2+0.5}{2} = 0.35$$
$$f(0.2)f(0.35) = 12.70647(2.3387193) = 29.71688$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_u = 0.35$. The second iteration is

$$x_r = \frac{0.35+0.5}{2} = 0.425$$
$$\varepsilon_a = \left|\frac{0.425-0.35}{0.425}\right| \times 100\% = 17.65\%$$
$$f(0.35)f(0.425) = 2.3387193(-1.2809449) = -2.99577$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 0.425$. The remainder of the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$	$f(x_l)\times f(x_r)$
1	0.2	12.70647	0.5	−4.2485678	0.35	2.3387193		29.71688
2	0.35	2.338719	0.5	−4.2485678	0.425	−1.2809449	17.65%	−2.99577
3	0.35	2.338719	0.425	−1.2809449	0.3875	0.4340883	9.68%	1.015211
4	0.3875	0.434088	0.425	−1.2809449	0.40625	−0.4452446	4.62%	−0.19328

Thus, after four iterations, we obtain a root estimate of **0.40625** with an approximate error of 4.62%.

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5.2

```
function [root,Ea,ea,n] = bisectnew(func,xl,xu,Ead,varargin)
% bisection roots zero
% [root,Ea,ea,n] = bisectnew(func,xl,xu,Ead,varargin)
%   uses bisection method to find the root of a function
%   with a fixed number of iterations to attain
%   a prespecified tolerance
% input:
%   func = name of function
%   xl, xu = lower and upper guesses
%   Ead = (optional) desired tolerance (default = 0.000001)
%   p1,p2,... = additional parameters used by func
% output:
%   root = real root
%   Ea = absolute error
%   ea = % relative error
%   n = iterations

if func(xl,varargin{:})*func(xu,varargin{:})>0
  %if guesses do not bracket a sign change
  disp('no bracket')    %display an error message
  return                %and terminate
end
% if necessary, assign default values
if nargin<4|isempty(Ead),Ead=0.000001;end %if Ead blank set to
0.000001
xr = xl;
% compute n and round up to next highest integer
n = ceil(log2((xu - xl)/Ead));
for i = 1:n
  xrold = xr;
  xr = (xl + xu)/2;
  if xr ~= 0, ea = abs((xr - xrold)/xr) * 100; end
  Ea = abs(xr - xrold);
  test = func(xl,varargin{:})*func(xr,varargin{:});
  if test < 0
    xu = xr;
  elseif test > 0
    xl = xr;
  else
    ea = 0;
  end
end
root = xr;
```

The following MATLAB script (`LastNameHmwk04Script`) uses the function to solve Prob. 5.1 with $E_{a,d} = 0.0001$.

```
xl=0.2;xu=0.5;Ead=0.0001;g=9.81;m=95;v=46;t=9;
format long
f=@(cd,g,m,v,t) sqrt(g*m/cd)*tanh(sqrt(g*cd/m)*t)-v;
[root,Ea,ea,n]=bisectnew(f,xl,xu,Ead,g,m,v,t)
```

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```
root =
   0.396655273437500
Ea =
     7.324218750004441e-05
ea =
   0.018464947374911
n =
    12
```

5.3 The derivative of the function is

$$\frac{dy}{dx} = -\frac{w}{48EI}(8x^3 - 9Lx^2 + L^3)$$

Therefore, to find the maximum deflection, we need to find the location where the derivative is zero. That is, the point of maximum deflection is the root of

$$f(x) = 0 = -\frac{w}{48EI}(8x^3 - 9Lx^2 + L^3)$$

The following script can be developed to determine the root with your bisection function ,

```
clear;clc;clf
w=4*1e3*100;L=400/100;E=52000*1e3*1e4;I=32000/100^4;
f = @(x,w,E,I,L) -w/(48*E*I)*(8*x.^3-9*L*x.^2+L^3);
d = @(xr,w,E,I,L) -w/(48*E*I)*(2*xr.^4-3*L*xr.^3+L^3*xr);

subplot(2,1,1)
xp=[0:L/16:L];
dydxp=f(xp,w,E,I,L);
plot(xp,dydxp),grid
xlabel('x(m)'),ylabel('dy/dx')
title('Slope versus distance')

subplot(2,1,2)
yp=d(xp,w,E,I,L);
plot(xp,yp),grid
xlabel('x(m)'),ylabel('y(m)')
title('Deflection versus distance')

format long, format compact
[root,Ea,ea,n] =bisectnew(f,0,0.9*L,0.0000001,w,E,I,L)
max_deflection=d(root,w,E,I,L)
```

Here is the bisection function

```
function [root,Ea,ea,n] = bisectnew(func,xl,xu,Ead,varargin)
% bisection roots zero
% [root,Ea,ea,n] = bisectnew(func,xl,xu,Ead,varargin)
%   uses bisection method to find the root of a function
%   with a fixed number of iterations to attain
```

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```
%   a prespecified tolerance
% input:
%   func = name of function
%   xl, xu = lower and upper guesses
%   Ead = (optional) desired tolerance (default = 0.000001)
%   p1,p2,... = additional parameters used by func
% output:
%   root = real root
%   Ea = absolute error
%   ea = % relative error
%   n = iterations
if func(xl,varargin{:})*func(xu,varargin{:})>0
  %if guesses do not bracket a sign change
  disp('no bracket')    %display an error message
  return                %and terminate
end
% if necessary, assign default values
if nargin<4|isempty(Ead),Ead=0.000001;end %if Ead blank set to
0.000001
xr = xl;
% compute n and round up to next highest integer
n = ceil(log2((xu - xl)/Ead));
for i = 1:n
  xrold = xr;
  xr = (xl + xu)/2;
  if xr ~= 0, ea = abs((xr - xrold)/xr) * 100; end
  Ea = abs(xr - xrold);
  test = func(xl,varargin{:})*func(xr,varargin{:});
  if test < 0
    xu = xr;
  elseif test > 0
    xl = xr;
  else
    ea = 0;
  end
end
root = xr;
```

When the script is run, the result is

```
root =
   1.686140710115433
Ea =
     5.364418020903372e-08
ea =
     3.181477078823466e-06
n =
    26
max_deflection =
  -0.003332997911279
```

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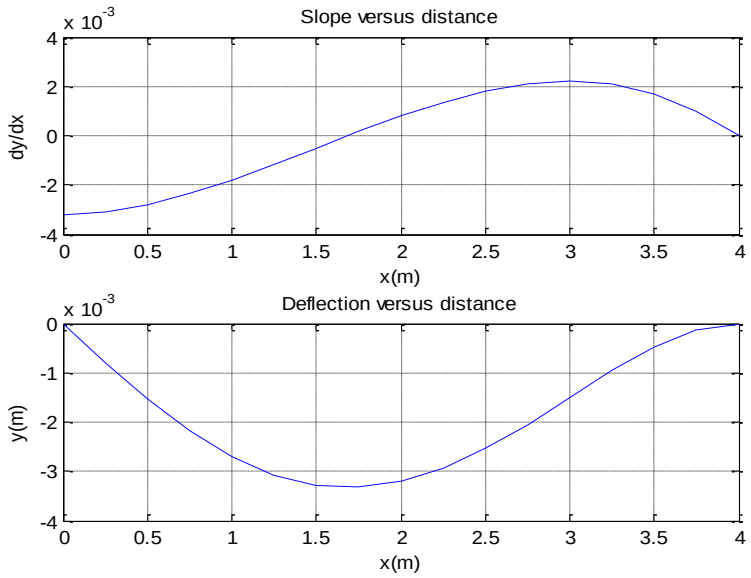

5.4 We need to find the root of

$$f(H) = \sqrt{2gH}\tanh\left(\frac{\sqrt{2gH}}{2L}t\right) - v$$

The following script can be developed to determine the root with your bisection function ,

```
% Steve Chapra
% ES-55
% Homework 4 Wet Script
clear;clc;clf
g=9.81;v=5;t=2.5;L=4;
f = @(H,g,v,t,L) sqrt(2*g*H).*tanh(sqrt(2*g*H)/2/L*t)-v;
Hp=[0:0.25:4];
fHp=f(Hp,g,v,t,L);
plot(Hp,fHp),grid
xlabel('H (m)'),ylabel('f(H)')
title('Plot of f(H) versus H')
format long, format compact
[root,Ea,ea,n] =bisectnew(f,0,4,0.0000001,g,v,t,L)
```

The bisection function is identical to the one for the dry problem above. When the script is run, the result is

```
root =
   1.465894639492035
Ea =
     5.960464477539063e-08
ea =
     4.066093371897789e-06
n =
    26
```

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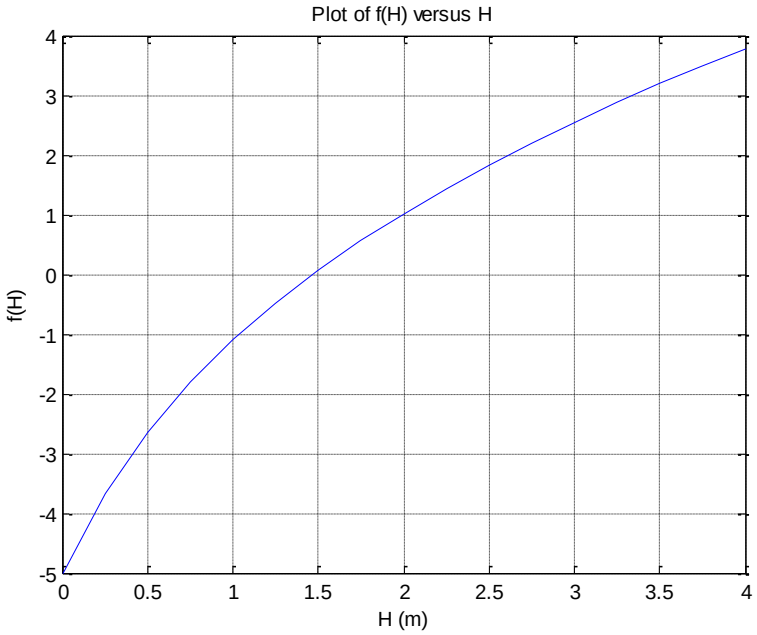

5.5 The function to evaluate is

$$f(c_d) = \sqrt{\frac{9.81(95)}{c_d}}\tanh\left(\sqrt{\frac{9.81c_d}{95}}9\right) - 46$$

The first iteration is

$$x_r = 0.5 - \frac{-4.248568(0.2-0.5)}{12.70647-(-4.248568)} = 0.4248265$$

$$f(0.2)f(0.4248265) = 12.70647(-1.2734) = -16.1804$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 0.424826471$. The remaining iterations are summarized in the following table

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a	$f(x_l)\times f(x_r)$
1	0.2	12.70647	0.5	−4.24857	0.424827	−1.2734		−16.1804
2	0.2	12.70647	0.424826	−1.2734	0.404348	−0.358067	5.06%	−4.54976
3	0.2	12.70647	0.404347	−0.35807	0.3987468	−0.098794	1.40%	−1.25532

Therefore, after only three iterations, we obtain a root estimate of 0.3987468 with an approximate error of 1.4% which is below the stopping criterion of 5%.

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5.6

```
function [root,ea,iter]=falsepos(func,xl,xu,es,maxit,varargin)
% falsepos: root location zeroes
%   [root,ea,iter]=falsepos(func,xl,xu,es,maxit,p1,p2,...):
%      uses false position to find the root of func
% input:
%   func = name of function
%   xl, xu = lower and upper guesses
%   es = desired relative error (default = 0.0001%)
%   maxit = maximum allowable iterations (default = 50)
%   p1,p2,... = additional parameters used by function
% output:
%   root = real root
%   ea = approximate relative error (%)
%   iter = number of iterations

if nargin<3,error('at least 3 input arguments required'),end
test = func(xl,varargin{:})*func(xu,varargin{:});
if test>0,error('no sign change'),end
if nargin<4|es<=0, es=0.0001;end
if nargin<5|maxit<=0, maxit=50;end
iter = 0; xr = xl;
while (1)
  xrold = xr;
  xr = (xl + xu)/2;
  fl=func(xl,varargin{:});
  fu=func(xu,varargin{:});
  xr = xu - fu*(xl - xu)/(fl - fu);
  iter = iter + 1;
  if xr ~= 0,ea = abs((xr - xrold)/xr) * 100;end
  test = fl*func(xr,varargin{:});
  if test < 0
    xu = xr;
  elseif test > 0
    xl = xr;
  else
    ea = 0;
  end
  if ea <= es | iter >= maxit,break,end
end
root = xr;
```

The following is a MATLAB session that uses the function to solve Prob. 5.1:

```
clear, clc
format long
fcd=@(cd) sqrt(9.81*95/cd)*tanh(sqrt(9.81*cd/95)*9)-46;
falsepos(fcd,0.2,0.5,5)

ans =

   0.398746796215879
```

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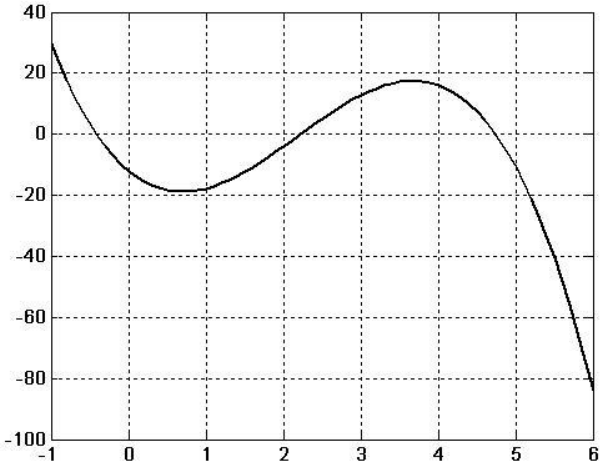
5.7 (a) The graph can be generated with MATLAB

```
>> x=[-1:0.1:6];
>> f=-12-21*x+18*x.^2-2.75*x.^3;
>> plot(x,f)
>> grid
```



This plot indicates that roots are located at about –0.4, 2.25 and 4.7.

(b) Using bisection, the first iteration is

$$x_r = \frac{-1+0}{2} = -0.5$$

$$f(-1)f(-0.5) = 29.75(3.34375) = 99.47656$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = -0.5$. The second iteration is

$$x_r = \frac{-0.5+0}{2} = -0.25$$

$$\varepsilon_a = \left|\frac{-0.25-(-0.5)}{-0.25}\right| 100\% = 100\%$$

$$f(-0.5)f(-0.25) = 3.34375(-5.5820313) = -18.66492$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = -0.25$. The remainder of the iterations are displayed in the following table:

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i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	−1	29.75	0	−12	−0.5	3.34375	
2	−0.5	3.34375	0	−12	−0.25	−5.5820313	100.00%
3	−0.5	3.34375	−0.25	−5.5820313	−0.375	−1.4487305	33.33%
4	−0.5	3.34375	−0.375	−1.4487305	−0.4375	0.8630981	14.29%
5	−0.4375	0.863098	−0.375	−1.4487305	−0.40625	−0.3136673	7.69%
6	−0.4375	0.863098	−0.40625	−0.3136673	−0.421875	0.2694712	3.70%
7	−0.42188	0.269471	−0.40625	−0.3136673	−0.414063	−0.0234052	1.89%
8	−0.42188	0.269471	−0.41406	−0.0234052	−0.417969	0.1227057	0.93%

Thus, after eight iterations, we obtain a root estimate of **–0.417969** with an approximate error of 0.93%, which is below the stopping criterion of 1%.

(c) Using false position, the first iteration is

$$x_r = 0 - \frac{-12(-1-0)}{29.75-(-12)} = -0.287425$$

$$f(-1)f(-0.287425) = 29.75(-4.4117349) = -131.2491$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = -0.287425$. The second iteration is

$$x_r = -0.287425 - \frac{-4.4117349(-1-(-0.287425))}{29.75-(-4.4117349)} = -0.3794489$$

$$\varepsilon_a = \left|\frac{-0.3794489-(-0.2874251)}{-0.3794489}\right|100\% = 24.25\%$$

$$f(-1)f(-0.3794489) = 29.75(-1.2896639) = -38.3675$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = -0.379449$. The remainder of the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	−1	29.75	0	−12	−0.287425	−4.4117349	
2	−1	29.75	−0.28743	−4.4117349	−0.379449	−1.2896639	24.25%
3	−1	29.75	−0.37945	−1.2896639	−0.405232	−0.3512929	6.36%
4	−1	29.75	−0.40523	−0.3512929	−0.412173	−0.0938358	1.68%
5	−1	29.75	−0.41217	−0.0938358	−0.414022	−0.0249338	0.45%

Therefore, after five iterations we obtain a root estimate of **–0.414022** with an approximate error of 0.45%, which is below the stopping criterion of 1%.

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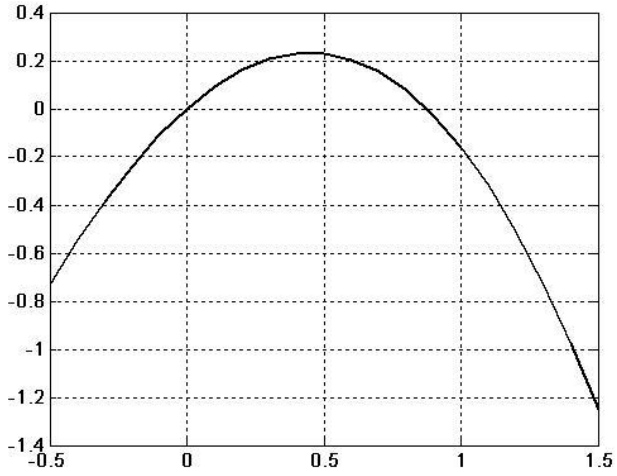
5.8 A graph of the function can be generated with MATLAB

```
>> x=[-0.5:0.1:1.5];
>> f=sin(x)-x.^2;
>> plot(x,f)
>> grid
```



This plot indicates that a nontrivial root (i.e., nonzero) is located at about 0.85.

Using bisection, the first iteration is

$$x_r = \frac{0.5+1}{2} = 0.75$$
$$f(0.5)f(0.75) = 0.229426(0.1191388) = 0.027333$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = 0.75$. The second iteration is

$$x_r = \frac{0.75+1}{2} = 0.875$$
$$\varepsilon_a = \left|\frac{0.875-0.75}{0.875}\right| 100\% = 14.29\%$$
$$f(0.75)f(0.875) = 0.119139(0.0019185) = 0.000229$$

Because the product is positive, the root is in the second interval and the lower guess is redefined as $x_l = 0.875$. The remainder of the iterations are displayed in the following table:

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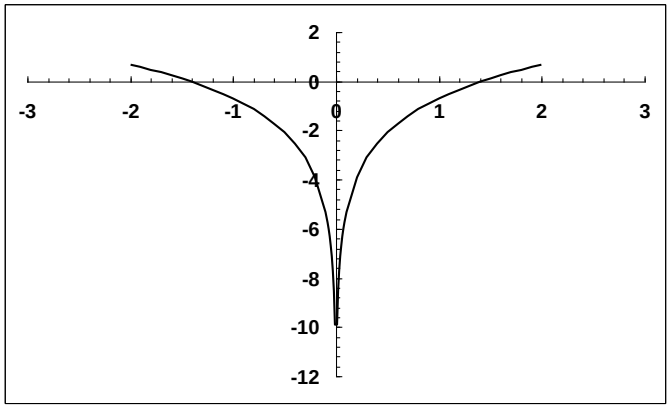
i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	0.229426	1	−0.158529	0.75	0.1191388	
2	0.75	0.119139	1	−0.158529	0.875	0.0019185	14.29%
3	0.875	0.001919	1	−0.158529	0.9375	−0.0728251	6.67%
4	0.875	0.001919	0.9375	−0.0728251	0.90625	−0.0340924	3.45%
5	0.875	0.001919	0.90625	−0.0340924	0.890625	−0.0157479	1.75%

Therefore, after five iterations we obtain a root estimate of **0.890625** with an approximate error of 1.75%, which is below the stopping criterion of 2%.

5.9 **(a)** A graph of the function indicates a positive real root at approximately $x = 1.4$.



(b) Using bisection, the first iteration is

$$x_r = \frac{0.5+2}{2} = 1.25$$
$$f(0.5)f(1.25) = -2.08629(-0.2537129) = 0.52932$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = 1.25$. The second iteration is

$$x_r = \frac{1.25+2}{2} = 1.625$$
$$\varepsilon_a = \left|\frac{1.625-1.25}{1.625}\right|100\% = 23.08\%$$
$$f(1.25)f(1.625) = -0.253713(0.2710156) = -0.06876$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.625$. The remainder of the iterations are displayed in the following table:

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i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	−2.08629	2	0.6862944	1.25	−0.2537129	
2	1.25	−0.25371	2	0.6862944	1.625	0.2710156	23.08%
3	1.25	−0.25371	1.625	0.2710156	1.4375	0.025811	13.04%

Thus after three iterations, we obtain a root estimate of **1.4375** with an approximate error of 13.04%.

(c) Using false position, the first iteration is

$$x_r = 2 - \frac{0.6862944(0.5-2)}{-2.086294-0.6862944} = 1.628707$$
$$f(0.5)f(1.628707) = -2.086294(0.2755734) = -0.574927$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.628707$. The second iteration is

$$x_r = 1.628707 - \frac{0.275573(0.5-1.628707)}{-2.086294-0.275573} = 1.4970143$$
$$\varepsilon_a = \left|\frac{1.4970143-1.6287074}{1.4970143}\right| 100\% = 8.8\%$$
$$f(0.5)f(1.4970143) = -2.086294(0.1069453) = -0.223119$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 1.497014$. The remainder of the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	−2.08629	2	0.6862944	1.6287074	0.2755734	
2	0.5	−2.08629	1.628707	0.2755734	1.4970143	0.1069453	8.80%
3	0.5	−2.08629	1.497014	0.1069453	1.4483985	0.040917	3.36%

Therefore, after three iterations we obtain a root estimate of **1.4483985** with an approximate error of 3.36%.

5.10 (a) Equation (5.6) can be used to determine the number of iterations

$$n = \log_2\left(\frac{\Delta x^0}{E_{a,d}}\right) = \log_2\left(\frac{35}{0.05}\right) = 9.4512$$

which can be rounded up to 10 iterations.

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(b) Here is an M-file that evaluates the temperature in °C using 10 iterations of bisection based on a given value of the oxygen saturation concentration in freshwater:

```
function TC = TempEval(osf)
% function to evaluate the temperature in degrees C based
% on the oxygen saturation concentration in freshwater (osf).
xl = 0 + 273.15;
xu = 35 + 273.15;
if fTa(xl,osf)*fTa(xu,osf)>0  %if guesses do not bracket
  error('no bracket')   %display an error message and
terminate
end
xr = xl;
for i = 1:10
  xrold = xr;
  xr = (xl + xu)/2;
  if xr ~= 0, ea = abs((xr - xrold)/xr) * 100; end
  test = fTa(xl,osf)*fTa(xr,osf);
  if test < 0
    xu = xr;
  elseif test > 0
    xl = xr;
  else
    ea = 0;
  end
end
TC = xr - 273.15;
end

function f = fTa(Ta, osf)
f = -139.34411 + 1.575701e5/Ta - 6.642308e7/Ta^2;
f = f + 1.2438e10/Ta^3 - 8.621949e11/Ta^4;
f = f - log(osf);
```

The function can be used to evaluate the test cases:

```
>> TempEval(8)
ans =
   26.7627

>> TempEval(10)
ans =
   15.4150

>> TempEval(14)
ans =
    1.5381
```

These results correspond to absolute approximate errors of $E_a = 0.034$. The true errors are all less than the desired value of 0.05: $E_t = 0.018$, 0.027, and 0.017, respectively.

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5.11 Solve for the reactions:

R_1=265 lbs. R_2= 285 lbs.

Write beam equations:

$0<x<3$

$$M+(16.667x^2)\frac{x}{3}-265x=0$$
$$(1)\quad M=265x-5.5555556x^3$$

$3<x<6$

$$M+100(x-3)(\frac{x-3}{2})+150(x-\frac{2}{3}(3))-265x=0$$
$$(2)\quad M=-50x^2+415x-150$$

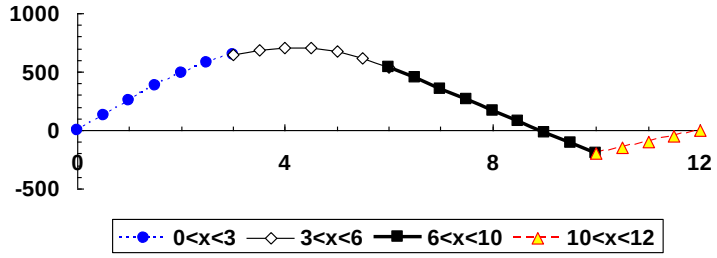
$6<x<10$

$$M=150\left(\frac{2}{3}(3)-x\right)+300(4.5-x)+265x$$
$$(3)\quad M=-185x+1650$$

$10<x<12$

$$M+100(12-x)=0$$
$$(4)\quad M=100x-1200$$

A plot of these equations can be generated:



Combining Equations:

Because the curve crosses the axis between 6 and 10, use (3).

(3) $M=-185x+1650$

Set $x_L=6; x_U=10$

$M(x_L)=540$
$M(x_U)=-200$
$M(x_R)=170 \rightarrow$ *replaces* x_L

$$x_r=\frac{x_L+x_U}{2}=8$$

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$M(x_L) = 170$
$M(x_U) = -200$
$M(x_R) = -15 \rightarrow$ *replaces* x_U
$x_r = \dfrac{8+10}{2} = 9$

$M(x_L) = 170$
$M(x_U) = -15$
$M(x_R) = 77.5 \rightarrow$ *replaces* x_L
$x_r = \dfrac{8+9}{2} = 8.5$

$M(x_L) = 77.5$
$M(x_U) = -15$
$M(x_R) = 31.25 \rightarrow$ *replaces* x_L
$x_r = \dfrac{8.5+9}{2} = 8.75$

$M(x_L) = 31.25$
$M(x_U) = -15$
$M(x_R) = 8.125 \rightarrow$ *replaces* x_L
$x_r = \dfrac{8.75+9}{2} = 8.875$

$M(x_L) = 8.125$
$M(x_U) = -15$
$M(x_R) = -3.4375 \rightarrow$ *replaces* x_U
$x_r = \dfrac{8.875+9}{2} = 8.9375$

$M(x_L) = 8.125$
$M(x_U) = -3.4375$
$M(x_R) = 2.34375 \rightarrow$ *replaces* x_L
$x_r = \dfrac{8.875+8.9375}{2} = 8.90625$

$M(x_L) = 2.34375$
$M(x_U) = -3.4375$
$M(x_R) = -0.546875 \rightarrow$ *replaces* x_U
$x_r = \dfrac{8.90625+8.9375}{2} = 8.921875$

$M(x_L) = 2.34375$
$M(x_U) = -0.546875$
$M(x_R) = 0.8984$
$x_r = \dfrac{8.90625+8.921875}{2} = 8.9140625$

Therefore, $x = 8.91$ feet

5.12 (a) The function to be evaluated is

$$f(y) = 1 - \frac{400}{9.81(3y + y^2/2)^3}(3+y)$$

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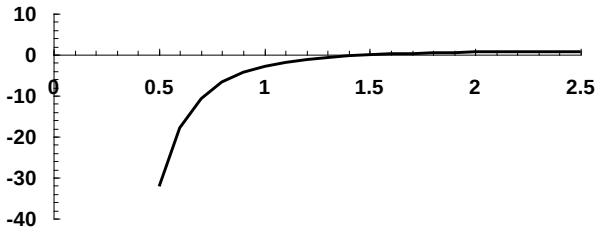
A graph of the function indicates a positive real root at approximately 1.5.



(b) Using bisection, the first iteration is

$$x_r = \frac{0.5+2.5}{2} = 1.5$$
$$f(0.5)f(1.5) = -32.2582(-0.030946) = 0.998263$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = 1.5$. The second iteration is

$$x_r = \frac{1.5+2.5}{2} = 2 \qquad \varepsilon_a = \left|\frac{2-1.5}{2}\right|100\% = 25\%$$

$$f(1.5)f(2) = -0.030946(0.601809) = -0.018624$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 2$. All the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a
1	0.5	−32.2582	2.5	0.813032	1.5	−0.030946	
2	1.5	−0.03095	2.5	0.813032	2	0.601809	25.00%
3	1.5	−0.03095	2	0.601809	1.75	0.378909	14.29%
4	1.5	−0.03095	1.75	0.378909	1.625	0.206927	7.69%
5	1.5	−0.03095	1.625	0.206927	1.5625	0.097956	4.00%
6	1.5	−0.03095	1.5625	0.097956	1.53125	0.036261	2.04%
7	1.5	−0.03095	1.53125	0.036261	1.515625	0.003383	1.03%
8	1.5	−0.03095	1.515625	0.003383	1.5078125	−0.013595	0.52%

After eight iterations, we obtain a root estimate of **1.5078125** with an approximate error of 0.52%.

(c) Using false position, the first iteration is

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$$x_r = 2.5 - \frac{0.81303(0.5-2.5)}{-32.2582-0.81303} = 2.45083$$

$$f(0.5)f(2.45083) = -32.25821(0.79987) = -25.80248$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 2.45083$. The second iteration is

$$x_r = 2.45083 - \frac{0.79987(0.5-2.45083)}{-32.25821-0.79987} = 2.40363 \qquad \varepsilon_a = \left|\frac{2.40363-2.45083}{2.40363}\right|100\% = 1.96\%$$

$$f(0.5)f(2.40363) = -32.2582(0.78612) = -25.35893$$

The root is in the first interval and the upper guess is redefined as $x_u = 2.40363$. All the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a
1	0.5	−32.2582	2.50000	0.81303	2.45083	0.79987	
2	0.5	−32.2582	2.45083	0.79987	2.40363	0.78612	1.96%
3	0.5	−32.2582	2.40363	0.78612	2.35834	0.77179	1.92%
4	0.5	−32.2582	2.35834	0.77179	2.31492	0.75689	1.88%
5	0.5	−32.2582	2.31492	0.75689	2.27331	0.74145	1.83%
6	0.5	−32.2582	2.27331	0.74145	2.23347	0.72547	1.78%
7	0.5	−32.2582	2.23347	0.72547	2.19534	0.70900	1.74%
8	0.5	−32.2582	2.19534	0.70900	2.15888	0.69206	1.69%
9	0.5	−32.2582	2.15888	0.69206	2.12404	0.67469	1.64%
10	0.5	−32.2582	2.12404	0.67469	2.09077	0.65693	1.59%

After ten iterations we obtain a root estimate of **2.09077** with an approximate error of 1.59%. Thus, after ten iterations, the false position method is converging at a very slow pace and is still far from the root in the vicinity of 1.5 that we detected graphically.

Discussion: This is a classic example of a case where false position performs poorly and is inferior to bisection. Insight into these results can be gained by examining the plot that was developed in part **(a)**. This function violates the premise upon which false position was based–that is, if $f(x_u)$ is much closer to zero than $f(x_l)$, then the root is closer to x_u than to x_l (recall Figs. 5.8 and 5.9). Because of the shape of the present function, the opposite is true.

5.13 The problem amounts to determining the root of

$$f(S) = S_0 - v_m t + k_s \ln(S_0 / S) - S$$

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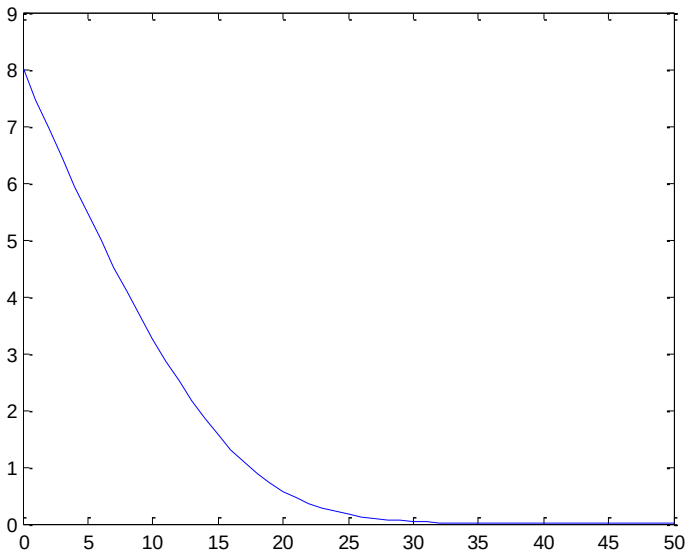
The following script calls the `bisect` function (Fig. 5.7) for various values of t in order to generate the solution.

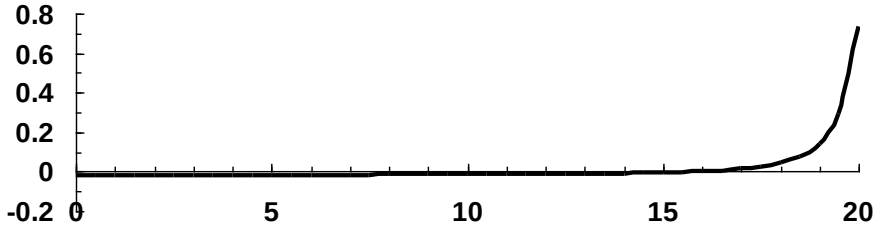
```
S0=8;vm=0.7;ks=2.5;
f = @(S,t) S0 - vm * t + ks * log(S0 / S) - S;
t=0:50;S=0:50;
n=length(t);
for i = 1:n
  S(i)=bisect(f,0.00001,10.01,1e-6,100,t(i));
end
plot(t,S)
```



5.14 The function to be solved is

$$f(x) = \frac{(4+x)}{(42-2x)^2(28-x)} - 0.016 = 0$$

(a) A plot of the function indicates a root at about $x = 16$.



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(b) The shape of the function indicates that false position would be a poor choice (recall Fig. 5.9). Bisection with initial guesses of 0 and 20 can be used to determine a root of 15.85938 after 8 iterations with $\varepsilon_a = 0.493\%$. Note that false position would have required 68 iterations to attain comparable accuracy.

i	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0	20	10	−0.01592	−0.01439	0.000229	100.000%
2	10	20	15	−0.01439	−0.00585	8.42×10^{-5}	33.333%
3	15	20	17.5	−0.00585	0.025788	−0.00015	14.286%
4	15	17.5	16.25	−0.00585	0.003096	-1.8×10^{-5}	7.692%
5	15	16.25	15.625	−0.00585	−0.00228	1.33×10^{-5}	4.000%
6	15.625	16.25	15.9375	−0.00228	0.000123	-2.8×10^{-7}	1.961%
7	15.625	15.9375	15.78125	−0.00228	−0.00114	2.59×10^{-6}	0.990%
8	15.78125	15.9375	15.85938	−0.00114	−0.00052	5.98×10^{-7}	0.493%

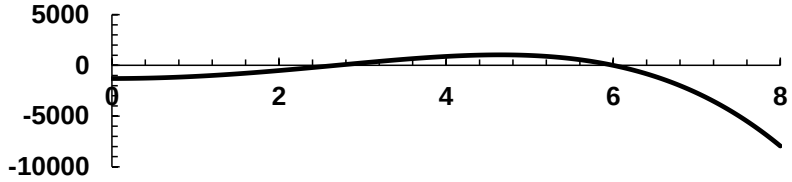
5.15 This problem can be solved by determining the root of the derivative of the elastic curve

$$\frac{dy}{dx} = 0 = \frac{w_0}{120EIL}(-5x^4 + 6L^2x^2 - L^4)$$

Therefore, after substituting the parameter values (in meter-kilogram-second units), we must determine the root of

$$f(x) = -5x^4 + 216x^2 - 1296 = 0$$

A plot of the function indicates a root at about $x = 2.6$ m.



Using initial guesses of 0 and 5 m, bisection can be used to determine the root. Here are the first few iterations:

i	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0	5	2.5	−1296.00	−141.31	183141	
2	2.5	5	3.75	−141.31	752.73	−106370	33.33%
3	2.5	3.75	3.125	−141.31	336.54	−47557	20.00%
4	2.5	3.125	2.8125	−141.31	99.74	−14095	11.11%
5	2.5	2.8125	2.65625	−141.31	−20.89	2952	5.88%

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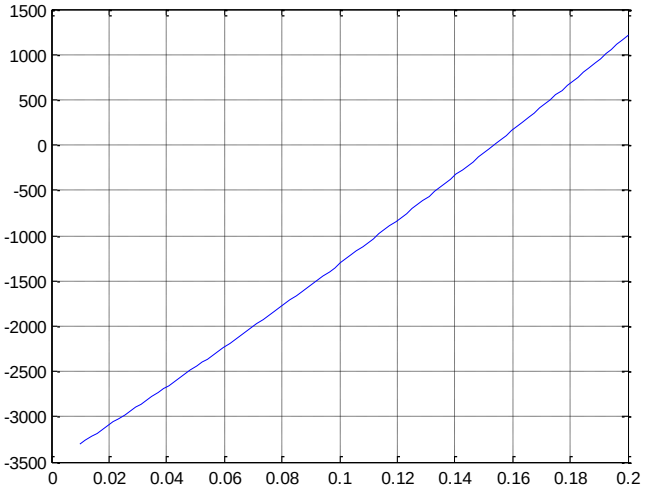
After 22 iterations, the root is determined as $x = 2.683282852172852$ m. This value can be substituted into Eq. (P5.15) to compute the maximum deflection as –0.005151900620158 m.

5.16 The solution can be formulated as

$$f(i) = 35{,}000\frac{i(1+i)^7}{(1+i)^7 - 1} - 8{,}500$$

A script can be developed to plot the function:

```
clc
format short g
P=35000;A=8500;n=7;
f = @(irate) P*irate.*(1+irate).^n./((1+irate).^n-1)-A;
iplot=linspace(0.01,.2)';
fplot=f(iplot);
plot(iplot,fplot)
grid
```



This plot suggests a root at about 0.155. The bisect function can then be employed to obtain the solution:

```
>> [xr,fx,ea,iter]=bisect(f,0.01,.3,0.00005)

xr =
      0.15346
fx =
  -0.00026251
ea =
   4.5056e-05
iter =
    22
```

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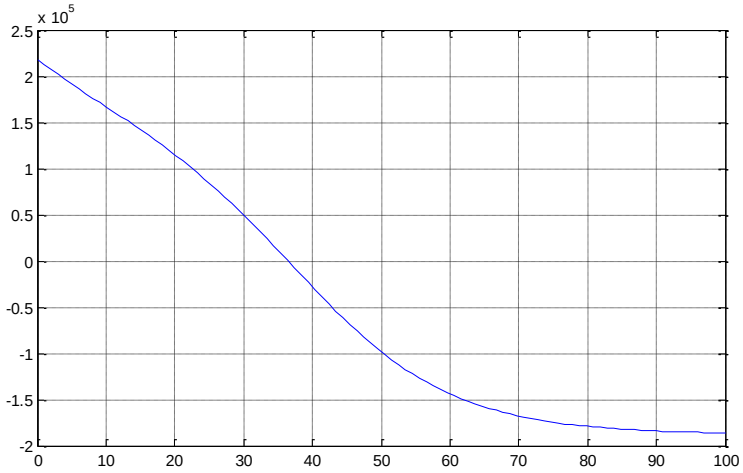
5.17 (a) The solution can be formulated as

$$f(t) = 1.2(80{,}000e^{-0.05t} + 110{,}000) - \frac{320{,}000}{1 + 31e^{-0.09t}}$$

A plot of this function can be generated with a script to suggest a root at about 36 years:

```
clc
format short g
Pumax=80000;ku=0.05;Pumin=110000;
Psmax=320000;P0=10000;ks=0.09;
% Pumax=75000;ku=0.045;Pumin=110000;
% Psmax=320000;P0=10000;ks=0.09;
f = @(t) 1.2*(Pumax*exp(-ku*t)+Pumin)-Psmax./(1+(Psmax/P0-
1)*exp(-ks*t));
iplot=linspace(0,100)';
fplot=f(iplot);
plot(iplot,fplot)
grid
```



(b) The false-position method can be implemented with the results summarized as

i	t_l	t_u	$f(t_l)$	$f(t_u)$	t_r	$f(t_r)$	$f(t_l)\times f(t_r)$	ε_a
1	0	100.0000	218000	−186134	53.9426	−119280	−2.600E+10	
2	0	53.9426	218000	−119280	34.8656	12309.95	2.684E+09	54.716%
3	34.8656	53.9426	12310	−119280	36.6502	−1817.89	−2.238E+07	4.869%
4	34.8656	36.6502	12310	−1817.89	36.4206	4.081795	5.025E+04	0.631%
5	36.4206	36.6502	4	−1817.89	36.4211	0.000548	2.236E−03	0.001%

The root is determined to be $t = 36.4211$. At this time, the ratio of the suburban to the urban population is $147{,}538/122{,}948 = 1.2$.

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5.18 The solution can be formulated as

$$f(N) = 0 = \frac{2}{q(N + \sqrt{N^2 + 4n_i^2})\mu} - \rho$$

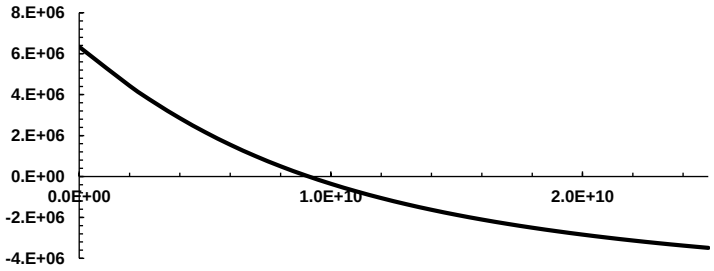
where

$$\mu = 1360\left(\frac{1000}{300}\right)^{-2.42} = 73.81971$$

Substituting this value along with the other parameters gives

$$f(N) = 0 = \frac{2}{1.2549\times10^{-17}(N + \sqrt{N^2 + 1.54256\times10^{20}})} - 6.5\times10^6$$

A plot of this function indicates a root at about $N = 9\times10^9$.



(a) The bisection method can be implemented with the results for the first 5 iterations summarized as

i	N_l	N_u	N_r	$f(N_l)$	$f(N_r)$	$f(N_l)\times f(N_r)$	ε_a
1	0.000×10^0	2.500×10^{10}	1.250×10^{10}	6.33×10^6	-1.21×10^6	-7.7×10^{12}	100.000%
2	0.000×10^0	1.250×10^{10}	6.250×10^9	6.33×10^6	1.41×10^6	8.91×10^{12}	100.000%
3	6.250×10^9	1.250×10^{10}	9.375×10^9	1.41×10^6	-1.09×10^5	-1.5×10^{11}	33.333%
4	6.250×10^9	9.375×10^9	7.813×10^9	1.41×10^6	5.88×10^5	8.27×10^{11}	20.000%
5	7.813×10^9	9.375×10^9	8.594×10^9	5.88×10^5	2.25×10^5	1.32×10^{11}	9.091%

After 16 iterations, the root is 9.114×10^9 with a relative error of 0.004%.

(b) The false position method can be implemented with the results for the first 5 iterations summarized as

i	N_l	$f(N_l)$	N_u	$f(N_u)$	N_r	$f(N_r)$	$f(N_l)\times f(N_r)$	ε_a

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1	0.000×10^{0}	6.33×10^{6}	2.500×10^{10}	-3.49×10^{6}	1.612×10^{10}	-2.13×10^{6}	-1.3×10^{13}	
2	0.000×10^{0}	6.33×10^{6}	1.612×10^{10}	-2.13×10^{6}	1.206×10^{10}	-1.07×10^{6}	-6.8×10^{12}	33.639%
3	0.000×10^{0}	6.33×10^{6}	1.206×10^{10}	-1.07×10^{6}	1.031×10^{10}	-4.76×10^{5}	-3×10^{12}	16.973%
4	0.000×10^{0}	6.33×10^{6}	1.031×10^{10}	-4.76×10^{6}	9.591×10^{9}	-1.97×10^{5}	-1.2×10^{12}	7.513%
5	0.000×10^{0}	6.33×10^{6}	9.591×10^{9}	-1.97×10^{6}	9.302×10^{9}	-7.89×10^{4}	-5×10^{11}	3.106%

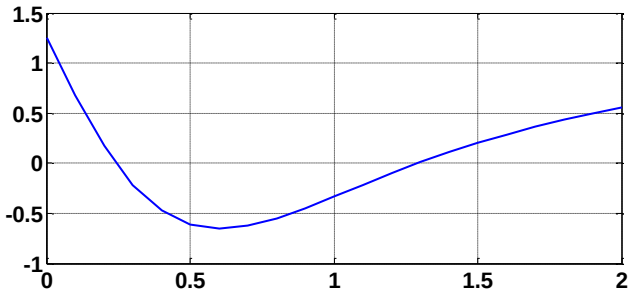
After 12 iterations, the root is 9.114×10^{9} with a relative error of 0.005%.

5.19 Using the given values, the roots problem to be solved is

$$f(x)=0=1.25-3.576516\frac{x}{(x^2+0.7225)^{3/2}}$$

A script can be developed to generate a plot:

```
format short g
e0=8.9e-12;F=1.25;q=2e-5;Q=2e-5;a=0.85;
f = @(x) F-1/(4*pi*e0)*q*Q*x./(x.^2+a^2).^(3/2);
xx=[0:0.1:2];
ff=f(xx);
plot(xx,ff),grid
```



The plot indicates roots at about 0.25 and 1.3. The bisect function can be used to determine these roots as

```
>> [x,fx,ea,iter]=bisect(f,0,0.5)

x =
      0.24104
fx =
 -4.6805e-007
ea =
  9.8911e-005
iter =
    21

>> [x,fx,ea,iter]=bisect(f,0.5,2)
```

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```
x =
       1.2913
fx =
 -3.7927e-009
ea =
  5.5391e-005
iter =
    21
```

Therefore, the roots are 0.24104 and 1.2913.

5.20 The solution can be formulated as

$$f(f) = 4\log_{10}(\mathrm{Re}\sqrt{f}) - 0.4 - \frac{1}{\sqrt{f}}$$

We want our program to work for Reynolds numbers between 2,500 and 1,000,000. Therefore, we must determine the friction factors corresponding to these limits. This can be done with any root location method to yield 0.011525 and 0.002913. Therefore, we can set our initial guesses as $x_l = 0.0028$ and $x_u = 0.012$. Equation (5.6) can be used to determine the number of bisection iterations required to attain an absolute error less than 0.000005,

$$n = \log_2\left(\frac{\Delta x^0}{E_{a,d}}\right) = \log_2\left(\frac{0.012 - 0.0028}{0.000005}\right) = 10.8454$$

which can be rounded up to 11 iterations. Here is a MATLAB function that is set up to implement 11 iterations of bisection to solve the problem.

```
function f=Fanning(func,xl,xu,varargin)
test = func(xl,varargin{:})*func(xu,varargin{:});
if test>0,error('no sign change'),end
for i = 1:11
  xr = (xl + xu)/2;
  test = func(xl,varargin{:})*func(xr,varargin{:});
  if test < 0
    xu = xr;
  elseif test > 0
    xl = xr;
  else
    break
  end
end
f=xr;
```

This can be implemented in MATLAB. For example,

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```
>> vk=@(f,Re) 4*log10(Re*sqrt(f))-0.4-1/sqrt(f);
>> format long
>> f=Fanning(vk,0.0028,0.012,2500)

f =
   0.01152832031250
```

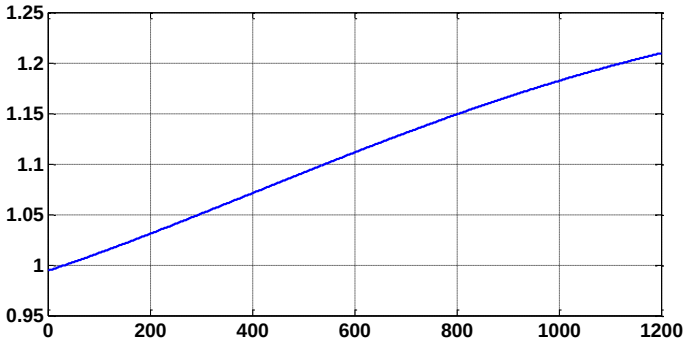
Here are additional results for a number of values within the desired range. We have included the true value and the resulting error to verify that the results are within the desired error criterion of $E_a < 5\times10^{-6}$.

Re	Root	Truth	E_t
2500	0.0115283203125	0.0115247638118	3.56×10^{-6}
3000	0.0108904296875	0.0108902285840	2.01×10^{-7}
10000	0.0077279296875	0.0077271274071	8.02×10^{-7}
30000	0.0058771484375	0.0058750482511	2.10×10^{-6}
100000	0.0045025390625	0.0045003757287	2.16×10^{-6}
300000	0.0036220703125	0.0036178949673	4.18×10^{-6}
1000000	0.0029123046875	0.0029128191460	5.14×10^{-7}

5.21 A script can be developed to generate a plot:

```
format short g
cp = @(T) 1.952e-14*T.^4-9.5838e-11*T.^3+9.7215e-8*T.^2+1.671e-
4*T+0.99403;
TT=[0:1200];
cpp=cp(TT);
plot(TT,cpp),grid
```

The results indicate a root at about $T = 550$.



The book MATLAB function bisect can be used to determine the root

```
>> format long
>> cpr = @(T) 1.952e-14*T^4-9.5838e-11*T^3+9.7215e-8*T^2 ...
```

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```
              + 1.671e-4*T-0.10597;
>> [root,fx,ea,iter] = bisect(cpr,0,1200)

root =
       544.09
fx =
 -4.4688e-008
ea =
  5.2584e-005
iter =
    22
```

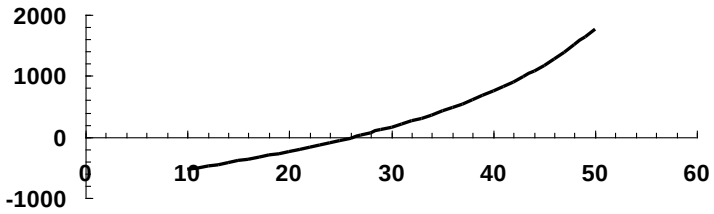
5.22 The solution can be formulated as

$$f(t) = u \ln \frac{m_0}{m_0 - qt} - gt - v$$

Substituting the parameter values gives

$$f(t) = 1{,}800 \ln \frac{160{,}000}{160{,}000 - 2{,}600t} - 9.81t - 750$$

A plot of this function indicates a root at about $t = 21$.



Because two initial guesses are given, a bracketing method like bisection can be used to determine the root,

i	t_l	t_u	t_r	$f(t_l)$	$f(t_r)$	$f(t_l)\times f(t_r)$	ε_a
1	10	50	30	−528.899	158.9182	−84051.7	
2	10	30	20	−528.899	−238.723	126260.5	50.00%
3	20	30	25	−238.723	−56.9155	13587.07	20.00%
4	25	30	27.5	−56.9155	46.13327	−2625.7	9.09%
5	25	27.5	26.25	−56.9155	−6.52111	371.1524	4.76%
6	26.25	27.5	26.875	−6.52111	19.51345	−127.249	2.33%
7	26.25	26.875	26.5625	−6.52111	6.424319	−41.8937	1.18%
8	26.25	26.5625	26.40625	−6.52111	−0.0662	0.431679	0.59%

Thus after 8 iterations, the approximate error falls below 1% with a result of $t = 26.40625$. Note that if the computation is continued, the root can be determined as 26.40784796.

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5.23 Eqs. (5.11), (5.14) and (5.16) can be substituted into Eq. (5.13) to yield

$$0 = \frac{K_1}{10^6[\mathrm{H}^+]} K_H p_{CO_2} + 2\frac{K_2 K_1}{10^6[\mathrm{H}^+]^2} K_H p_{CO_2} + \frac{K_w}{[\mathrm{H}^+]} - [\mathrm{H}^+] - Alk$$

The function then becomes

```
function f = fpHAlk(pH,pCO2,Alk)
K1=10^-6.3;K2=10^-10.3;Kw=10^-14;
KH=10^-1.46;
H=10^-pH;
f=K1/(1e6*H)*KH*pCO2+2*K2*K1/(1e6*H^2)*KH*pCO2+Kw/H-H-Alk;
```

For 2008, the resulting pH can be determined with the following script

```
clear, clc, format compact
[pH2008 fx ea iter]=bisect(@fpHAlk,2,12,1e-8,50,386,0.4e-3)

pH2008 =
    7.7723
fx =
   2.3224e-14
ea =
   7.4891e-09
iter =
    34
```

Notice how the presence of alkalinity has significantly raised the pH.

5.24 Archimedes principle says that

$$\rho_s V_s g = \rho_w V_w g$$

where V_s = the total sphere volume (m^3) and V_w = the below-water volume (m^3). The below-water volume is equal to the total volume minus the above-water volume,

$$V_w = \frac{4\pi r^3}{3} - \frac{\pi h^2}{3}(3r - h)$$

Therefore, the *Archimedes principle* amounts to finding the root of

$$0 = \rho_w \left[\frac{4\pi r^3}{3} - \frac{\pi h^2}{3}(3r - h)\right] g - \rho_s \frac{4\pi r^3}{3} g$$

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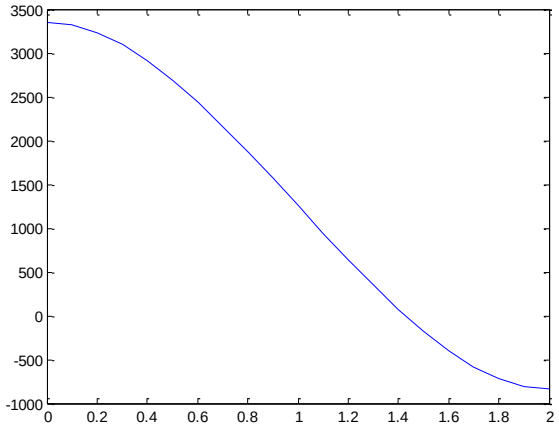
or collecting terms

$$0 = \left[\frac{4\pi r^3}{3}(\rho_w - \rho_s) - \rho_w \frac{\pi h^2}{3}(3r - h)\right]$$

Given the problem parameters, the following script can be employed to determine the root with the `bisect` program from the text (Fig. 5.7):

```
r=1;rhos=200;rhow=1000;
fh=@(h) 4/3*r^3*pi*(rhow-rhos)-rhow*pi*h.^2/3.*(3*r-h);
h=[0:2*r/20:2*r];fhh=fh(h);
plot(h,fhh)
[height f ea iter]=bisect(fh,0,2*r)
```

The result is




```
height =
    1.4257
f =
   -0.0018
ea =
  6.6891e-005
iter =
    21
```

5.25 Archimedes principle says that

$$\rho_f V_f g = \rho_w V_w g$$

where V_f = the total frustum volume (m^3) and V_w = the below-water volume (m^3). The total frustum volume is simply

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$$V_f = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$$

In order to determine the below-water volume, we must first relate the radius to the height above water as in

$$r = r_1 + \frac{r_2 - r_1}{h} h_1$$

The below- water volume is

$$V_w = \frac{\pi(h - h_1)}{3}\left[r_2^2 + \left(r_1 + \frac{r_2 - r_1}{h} h_1\right)^2 + r_2\left(r_1 + \frac{r_2 - r_1}{h} h_1\right)\right]$$

Therefore, the solution amounts to determining the value of h_1 that satisfies

$$\rho_f\left(\frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)\right) - \rho_w\left(\frac{\pi(h - h_1)}{3}\left[\left(r_1 + \frac{r_2 - r_1}{h} h_1\right)^2 + r_2^2 + \left(r_1 + \frac{r_2 - r_1}{h} h_1\right) r_2\right]\right) = 0$$

Given the problem parameters, the following script can be employed to determine the root with the `bisect` program from the text (Fig. 5.7):

```
r1=0.5;r2=1;h=1;rhof=200;rhow=1000;
fh1=@(h1) rhof*pi*h/3*(r1^2+r2^2+r1*r2)-...
    rhow*pi*(h-h1)/3.*((r1+(r2-r1)/h*h1).^2+r2^2+(r1+(r2-
r1)/h*h1)*r2);
h1=[0:h/20:h];fhh=fh1(h1);
plot(h1,fhh), grid
[height f ea iter]=bisect(fh1,0,1)
```

The result is

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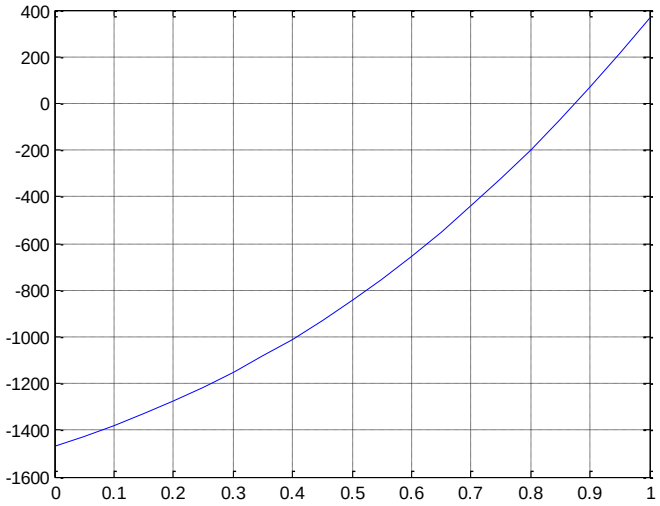


```
>> prob0525

height =
    0.8758
f =
  7.3518e-004
ea =
  5.4447e-005
iter =
    21
```

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