

P3.1

Mole fraction: $x_{PS} = \frac{n_{PS}}{\sum n_i} = \frac{10 \text{ gmol}}{10 \text{ gmol} + 1000 \text{ gmol}} = 0.0099$

Weight fraction:

$$w_{PS} = \frac{n_{PS}M_{PS}}{\sum n_i M_i} = \frac{10 \text{ gmol}(66000 \text{ g/gmol})}{10 \text{ gmol}(66000 \text{ g/gmol}) + 1000 \text{ gmol}(78 \text{ g/gmol})} = 0.894$$

P3.2

$$\dot{m}_{w,in} = 0.95 \times 15 \text{ g/min} = 14.25 \text{ g/min}$$

$$\dot{m}_{w,out} = 0.90 \times 15 \text{ g/min} = 13.5 \text{ g/min}$$

$$\dot{m}_{s,in} = 0.05 \times 15 \text{ g/min} = 0.75 \text{ g/min}$$

$$\dot{m}_{s,out} = 0.10 \times 15 \text{ g/min} = 1.5 \text{ g/min}$$

$$\dot{m}_{in} = 15 \text{ g/min}$$

$$\dot{m}_{out} = 15 \text{ g/min}$$

$$\frac{dm_{sys}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 15 - 15 = 0 \text{ g/min}$$

$$\frac{dm_{w,sys}}{dt} = \dot{m}_{w,in} - \dot{m}_{w,out} = 14.25 - 13.5 = 0.75 \text{ g/min}$$

$$\frac{dm_{s,sys}}{dt} = \dot{m}_{s,in} - \dot{m}_{s,out} = 0.75 - 1.5 = -0.75 \text{ g/min}$$

P3.3

Use the integral mass balance, $m_{sys,f} - m_{sys,0} = \int_{t_0}^{t_f} \dot{m}_{in} dt - \int_{t_0}^{t_f} \dot{m}_{out} dt$ because we are interested in what happens over a defined time interval:

$$(a) \ m_{sys,f} = \int_0^2 (1 + 2t) dt = t + t^2 \Big|_0^2 = 6 \text{ kg}$$

$$(b) \ m_{sys,f} = \int_0^2 (3 \exp(2t)) dt = \frac{3}{2} \exp(2t) \Big|_0^2 = 81.9 - 1.5 = 80.4 \text{ kg}$$

P3.4

$$\dot{r}_{O_2} = \nu_{O_2} \dot{\xi} = -2 \times 5 = -10 \text{ gmol/min}$$

$$\dot{r}_{CO_2} = \nu_{CO_2} \dot{\xi} = 1 \times 5 = 5 \text{ gmol/min}$$

$$\dot{r}_{H_2O} = \nu_{H_2O} \dot{\xi} = 2 \times 5 = 10 \text{ gmol/min}$$

$$\dot{R}_{CH_4} = \nu_{CH_4} M_{CH_4} \dot{\xi} = -1 \times 16 \times 5 = -80 \text{ g/min}$$

$$\dot{R}_{O_2} = \nu_{O_2} M_{O_2} \dot{\xi} = -2 \times 32 \times 5 = -320 \text{ g/min}$$

$$\dot{R}_{CO_2} = \nu_{CO_2} M_{CO_2} \dot{\xi} = 1 \times 44 \times 5 = 220 \text{ g/min}$$

$$\dot{R}_{H_2O} = \nu_{H_2O} M_{H_2O} \dot{\xi} = 2 \times 18 \times 5 = 180 \text{ g/min}$$

P3.5

$$\dot{r}_{NH_3} = 45 \text{ gmol/min} = \nu_{NH_3} \dot{\xi} = 2 \dot{\xi}$$

$$\dot{\xi} = 22.5 \text{ gmol/min}$$

$$\dot{n}_{out} = \sum_i \dot{n}_{i,in} + \sum_i \nu_i \dot{\xi} = 100 + 100 + 22.5(-1 - 3 + 2) = 155 \text{ gmol/min}$$

P3.6

$$\dot{m}_{g,out} = \dot{m}_{g,in} + \dot{R}_g$$

$$0.06(100) \text{ lb/h} = 0.1(100) \text{ lb/h} + \dot{R}_g$$

$$\dot{R}_g = -4 \text{ lb/h}$$

$$\dot{\xi} = \frac{\dot{R}_g}{\nu_g M_g} = \frac{-4 \text{ lb/h}}{-1(180 \text{ lb/lbmol})} = 0.022\bar{2} \text{ lbmol/h}$$

P3.7

Apply row reduction:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

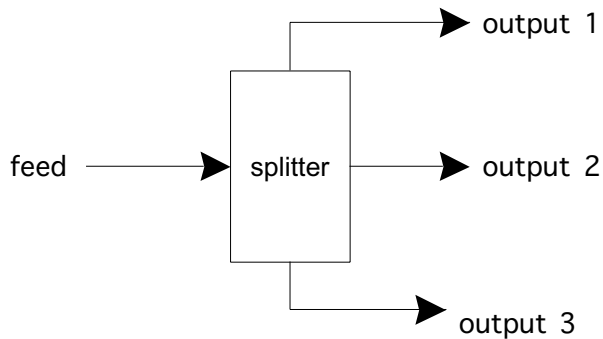
There is a row of zeros, so this system is not linearly independent.

P3.8

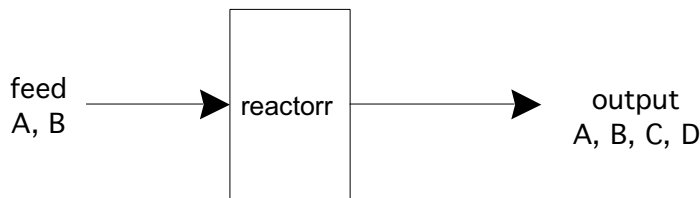
Fractional split: fraction of a stream fed to a splitter that exits in a designated outlet; ratio of flow in an outlet stream from a splitter to the flow in the inlet stream

Fractional conversion: fraction of a reactant that reacts to products, ratio of reaction rate of a reactant to the feed rate of that reactant

Fractional recovery: fraction of a component fed to a separator that exits in a designated output stream; ratio of component flow in an outlet stream to component flow in the feed stream.

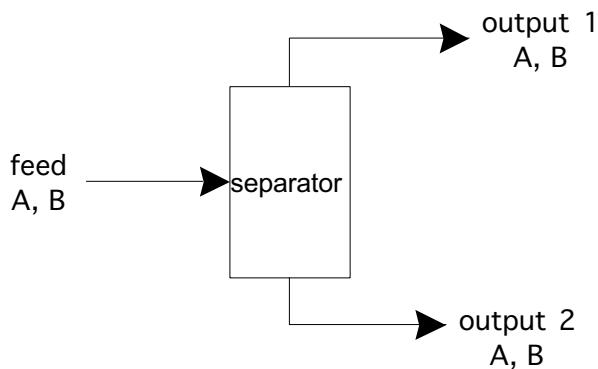


$$\text{Fractional split } f_{S1} = \frac{\text{output 1 flow rate}}{\text{feed flow rate}}$$



Fractional conversion of A:

$$f_{CA} = \frac{\text{flow rate of component A in output} - \text{flow rate of component A in feed}}{\text{flow rate of component A in feed}}$$



$$\text{Fractional recovery of A: } f_{RA1} = \frac{\text{flow rate of component A in output 1}}{\text{flow rate of component A in feed}}$$

P3.9

$$f_{S1} = \frac{2 \text{ lb/h}}{10 \text{ lb/h}} = 0.2$$

Flow rate of water = $0.2(90) = 18 \text{ lb/h}$

P3.10

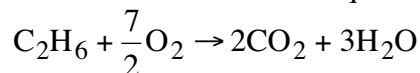
$$f_{Cs} = \frac{10 - 6}{10} = 0.4$$

P3.11

$$f_{Rg1} = \frac{0.09(50)}{0.06(100)} = 0.75$$

P3.12

The balanced chemical reaction equation is



Molar mass of ethane is 30 g/gmol, and that of oxygen is 32 g/gmol. 100 gmol/s ethane, or 3000 g/s ethane, and 400 gmol/s oxygen, or 12800 g/s O_2 , are fed to the burner. The burner outlet stream contains no ethane, but does contain some oxygen (because it is the excess reactant) along with CO_2 and H_2O .

We will use E for ethane, O for O_2 , C for carbon dioxide, and W for water in the following equations.

The component mass balance equation for ethane is:

$$\dot{m}_{E,out} = \dot{m}_{E,in} + \dot{R}_E$$

$$0 = 3000 + \dot{R}_E$$

$$\dot{R}_E = -3000 \text{ g/s} = \nu_E M_E \dot{\xi} = -1(30 \text{ g/gmol})\dot{\xi}$$

$$\dot{\xi} = 100 \text{ gmol/s}$$

The other mass balance equations are:

$$\dot{m}_{O,out} = \dot{m}_{O,in} + \dot{R}_O = \dot{m}_{O,in} + \nu_O M_O \dot{\xi} = 12800 - 3.5(32)100 = 1600 \text{ g/s}$$

$$\dot{m}_{C,out} = \dot{m}_{C,in} + \dot{R}_C = \nu_C M_C \dot{\xi} = 2(44)100 = 8800 \text{ g/s}$$

$$\dot{m}_{W,out} = \dot{m}_{W,in} + \dot{R}_W = \nu_W M_W \dot{\xi} = 3(18)100 = 5400 \text{ g/s}$$

Does total mass in = total mass out?

$$3000 + 12800 = 0 + 1600 + 8800 + 5400?$$

$$15800 = 15800 \text{ Yes!}$$

The component mole balance equations are:

$$\dot{n}_{E,out} = \dot{n}_{E,in} + \dot{r}_E = \dot{n}_{E,in} - v_E \dot{\xi} = 100 - 100 = 0 \text{ gmol/s}$$

$$\dot{n}_{O,out} = \dot{n}_{O,in} + \dot{r}_O = \dot{n}_{O,in} - \frac{7}{2} \dot{\xi} = 400 - 350 = 50 \text{ gmol/s}$$

$$\dot{n}_{C,out} = \dot{n}_{C,in} + \dot{r}_C = 2\dot{\xi} = 200 \text{ gmol/s}$$

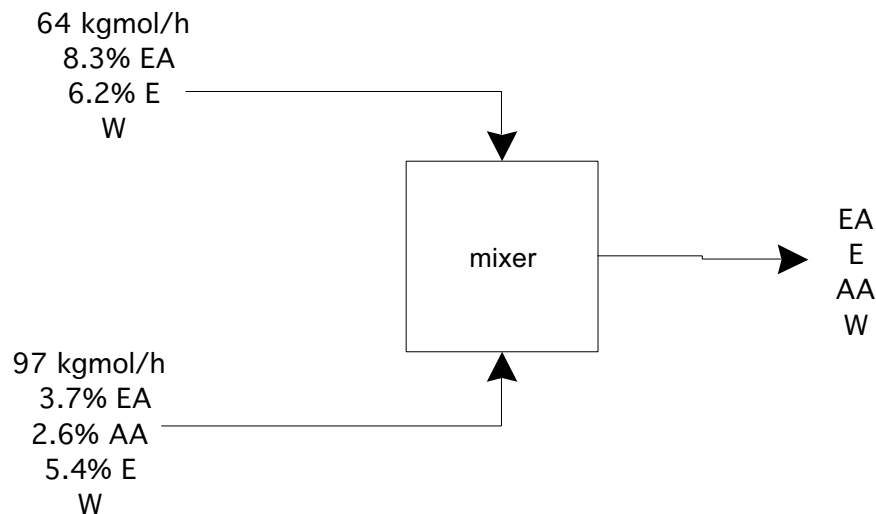
$$\dot{n}_{W,out} = \dot{n}_{W,in} + \dot{r}_C = 3\dot{\xi} = 300 \text{ gmol/s}$$

Does total moles in = total moles out?

$$100 + 400 = 0 + 50 + 200 + 300?$$

$$500 = 550 \text{ NO!}$$

P3.13



All stream compositions given in mol%, with EA = ethyl acetate, E = ethanol, AA = acetic acid, and W = water.

DOF analysis:

	No. of variables		No. of constraints
Stream variables	11	Specified flow	2
System variables	0	Specified composition	5
		Specified performance	0
		Material balances	4
Total	11		11

$$\text{DOF} = 11 - 11 = 0$$

We choose differential mole balance equation, because we are interested in what is happening at any instance of time. Mixer is at steady state.

We start with the total mole balance, which is easy because there is no reaction.

$$\dot{n}_{out} = \sum_{\text{all } j \text{ in}} \dot{n}_j = 64 + 97 = 161 \text{ kgmol/h}$$

Now the component mole balances:

$$\dot{n}_{EA,out} = \sum_{\text{all } j \text{ in}} \dot{n}_{EA,j} = (0.083)64 + (0.037)97 = 8.9 \text{ kgmol/h}$$

$$\dot{n}_{E,out} = \sum_{\text{all } j \text{ in}} \dot{n}_{E,j} = (0.062)64 + (0.054)97 = 9.21 \text{ kgmol/h}$$

$$\dot{n}_{AA,out} = \sum_{\text{all } j \text{ in}} \dot{n}_{AA,j} = (0.026)97 = 2.52 \text{ kgmol/h}$$

$$\dot{n}_{W,out} = \sum_{\text{all } j \text{ in}} \dot{n}_{W,j} = (.855)64 + (0.883)97 = 140.37 \text{ kgmol/h}$$

From these we calculate the mol% in the output stream:

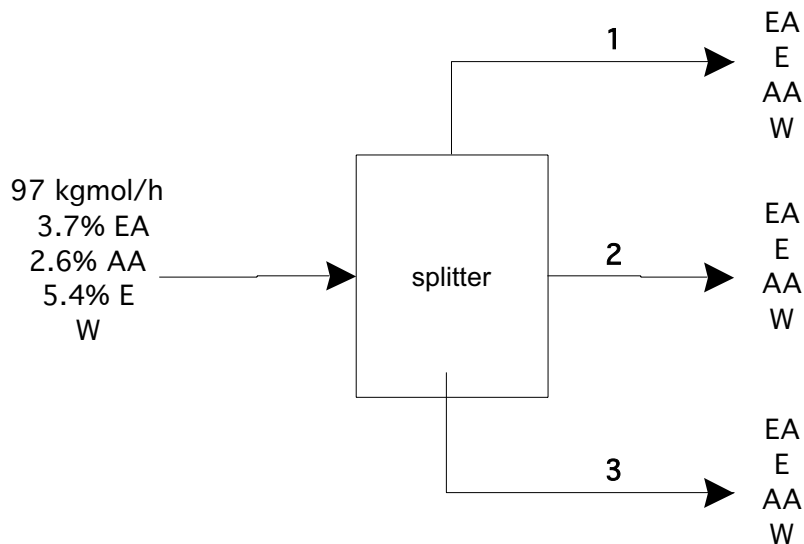
$$\frac{8.9}{161} \times 100\% = 5.5 \text{ mol\% ethyl acetate}$$

$$\frac{9.21}{161} \times 100\% = 5.7 \text{ mol\% ethanol}$$

$$\frac{2.52}{161} \times 100\% = 1.6 \text{ mol\% acetic acid}$$

$$\frac{140.37}{161} \times 100\% = 87.2 \text{ mol\% water}$$

P3.14



DOF analysis:

	No. of variables		No. of constraints
Stream variables	16	Specified flow	1
System variables	0	Specified composition	3
		Specified performance	2 (fractional splits)
		Splitter restriction	6 $((N-1)*(C-1))$
		Material balances	4
Total	16		16

$$\text{DOF} = 16 - 16 = 0$$

From fractional split specifications:

$$\dot{n}_1 = f_{S1}\dot{n}_{in} = 0.27(97) = 26.19 \text{ kgmol/h}$$

$$\dot{n}_2 = f_{S2}\dot{n}_{in} = 0.54(97) = 52.38 \text{ kgmol/h}$$

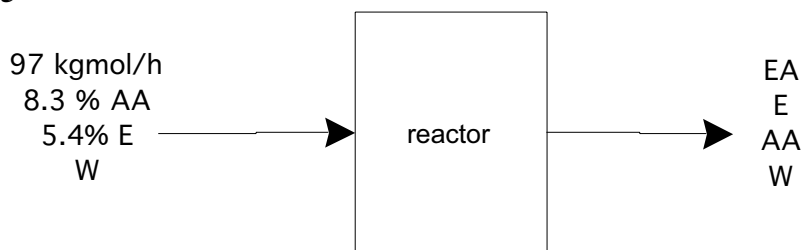
Use steady-state differential mole balance equation with no reaction:

$$\sum_{\text{all } j \text{ out}} \dot{n}_j = \dot{n}_1 + \dot{n}_2 + \dot{n}_3 = 26.19 + 52.38 + \dot{n}_3 = \dot{n}_{in} = 97 \text{ kgmol/h}$$

$$\dot{n}_3 = 18.43 \text{ kgmol/h}$$

The composition of all 3 output streams is identical to the input stream.

P3.15



DOF analysis:

	No. of variables		No. of constraints
Stream variables	7	Specified flow	1
System variables	1	Specified composition	2
		Specified performance	1 (rate of reaction)
		Material balances	4
Total	8		8

$$\text{DOF} = 8 - 8 = 0$$

Use steady-state differential component mole balance. From stoichiometry, we know that:

$$\dot{r}_E = \dot{r}_{AA} = -\dot{r}_{EA} = -\dot{r}_W = -4.8 \text{ kgmol/h}$$

$$\dot{n}_{E,out} = \dot{n}_{E,in} + \dot{r}_E = (0.054)97 - 4.8 = 0.44 \text{ kgmol/h}$$

$$\dot{n}_{AA,out} = \dot{n}_{AA,in} + \dot{r}_{AA} = (0.083)97 - 4.8 = 3.25 \text{ kgmol/h}$$

$$\dot{n}_{EA,out} = \dot{n}_{EA,in} + \dot{r}_{EA} = 4.8 \text{ kgmol/h}$$

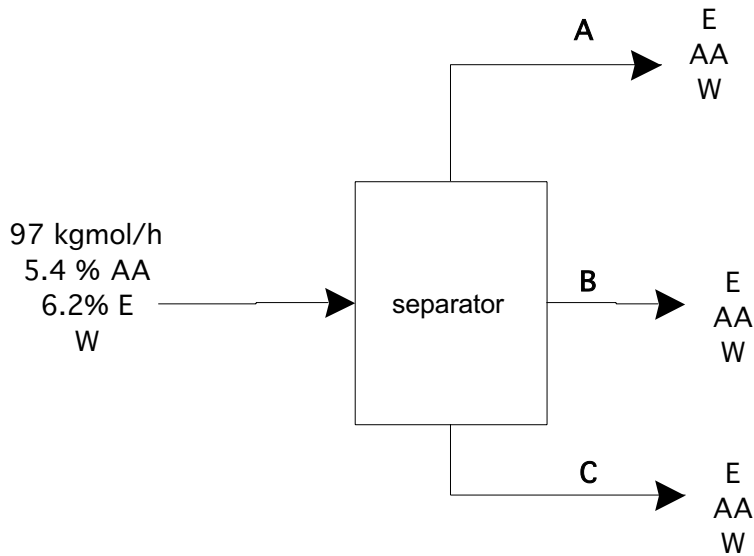
$$\dot{n}_{W,out} = \dot{n}_{W,in} + \dot{r}_W = (0.863)97 + 4.8 = 88.51 \text{ kgmol/h}$$

$$\dot{n}_{out} = \dot{n}_{E,out} + \dot{n}_{AA,out} + \dot{n}_{EA,out} + \dot{n}_{W,out} = 0.44 + 3.25 + 4.8 + 88.51 = 97 \text{ kgmol/h}$$

(Alternatively, we could have used the differential total mole balance instead to get this last result.)

Composition of output stream is calculated by dividing the component molar flow rate by the total molar flow rate: 0.45 mol% ethanol, 3.35 mol% acetic acid, 4.95 mol% ethyl acetate, and 91.25 mol% water.

P3.16



DOF analysis:

	No. of variables		No. of constraints
Stream variables	12	Specified flow	1
System variables	0	Specified composition	2
		Specified performance	6 (fractional recoveries)
		Material balances	3
Total	12		12

$$\text{DOF} = 12 - 12 = 0$$

From the specified fractional component recoveries we find:

$$\dot{n}_{E,A} = f_{RE,A} \dot{n}_{E,in} = 0.94(0.062)(97) = 5.65 \text{ kgmol/h}$$

$$\dot{n}_{E,B} = f_{RE,B} \dot{n}_{E,in} = 0.04(0.062)(97) = 0.24 \text{ kgmol/h}$$

$$\dot{n}_{AA,A} = f_{RAA,A} \dot{n}_{AA,in} = 0.10(0.054)(97) = 0.52 \text{ kgmol/h}$$

$$\dot{n}_{AA,B} = f_{RAA,B} \dot{n}_{AA,in} = 0.85(0.054)(97) = 4.45 \text{ kgmol/h}$$

$$\dot{n}_{W,A} = f_{RW,A} \dot{n}_{W,in} = 0.15(0.884)(97) = 12.86 \text{ kgmol/h}$$

$$\dot{n}_{W,B} = f_{RW,B} \dot{n}_{W,in} = 0.15(0.884)(97) = 12.86 \text{ kgmol/h}$$

Use steady-state differential component mole balance with no reaction to find remaining flows:

$$\sum_{\text{all } j \text{ out}} \dot{n}_{E,j} = \dot{n}_{E,A} + \dot{n}_{E,B} + \dot{n}_{E,C} = 5.65 + 0.24 + \dot{n}_{E,C} = \dot{n}_{E,in} = (0.062)97 \text{ kgmol/h}$$

$$\dot{n}_{E,C} = 0.12 \text{ kgmol/h}$$

$$\sum_{\text{all } j \text{ out}} \dot{n}_{AA,j} = \dot{n}_{AA,A} + \dot{n}_{AA,B} + \dot{n}_{AA,C} = 0.52 + 4.45 + \dot{n}_{AA,C} = \dot{n}_{AA,in} = (0.054)97 \text{ kgmol/h}$$

$$\dot{n}_{AA,C} = 0.27 \text{ kgmol/h}$$

$$\sum_{\text{all } j \text{ out}} \dot{n}_{W,j} = \dot{n}_{W,A} + \dot{n}_{W,B} + \dot{n}_{W,C} = 12.86 + 12.86 + \dot{n}_{W,C} = \dot{n}_{W,in} = (0.884)97 \text{ kgmol/h}$$

$$\dot{n}_{W,C} = 60.03 \text{ kgmol/h}$$

Total molar flows in each output stream are calculated by summing the component molar flows in that stream. Mol % is calculated by dividing the component molar flow rate by the total molar flow rate.

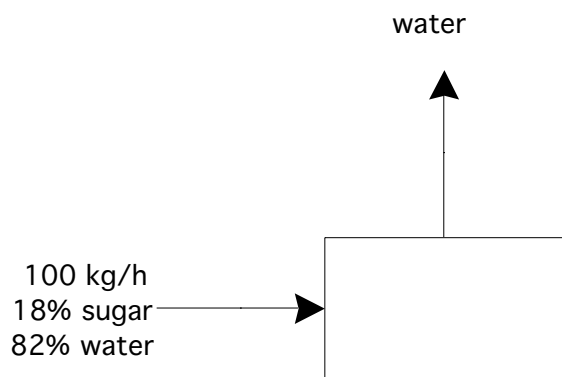
We find:

Stream A: 19 kgmol/h, 29.7 mol% ethanol, 2.7 mol% acetic acid, 67.6 mol% water

Stream B: 17.55 kgmol/h, 1.37 mol% ethanol, 25.36 mol% acetic acid, 73.27 mol% water

Stream C: 60.42 kgmol/h, 0.2 mol% ethanol, 0.45 mol% acetic acid, 99.35 mol% water.

P3.17



DOF analysis:

	No. of variables		No. of constraints
Stream variables	3	Specified flow	1
System variables	2 (accumulation)	Specified compositions	2
		Specified performance	0
		Material balances	2
Total	5		5

$$\text{DOF} = 5 - 5 = 0$$

Since we want to know a *rate* of evaporation, we use the differential mass balance equation. There is no reaction, but there is an accumulation term.

$$\frac{dm_{S,\text{sys}}}{dt} = \dot{m}_{S,\text{in}} = 0.18(100) = 18 \text{ kg/h}$$

$$\frac{dm_{W,\text{sys}}}{dt} = \dot{m}_{W,\text{in}} - \dot{m}_{W,\text{out}} = 0.82(100) - \dot{m}_{W,\text{out}} = 82 \text{ kg/h} - \dot{m}_{W,\text{out}}$$

To keep a *constant* 65 wt% sugar/ 35 wt% water in the vessel, the *ratio* of the rate of accumulation of sugar and of water must be constant and equal to 65/35. We can write this in mathematical terms as:

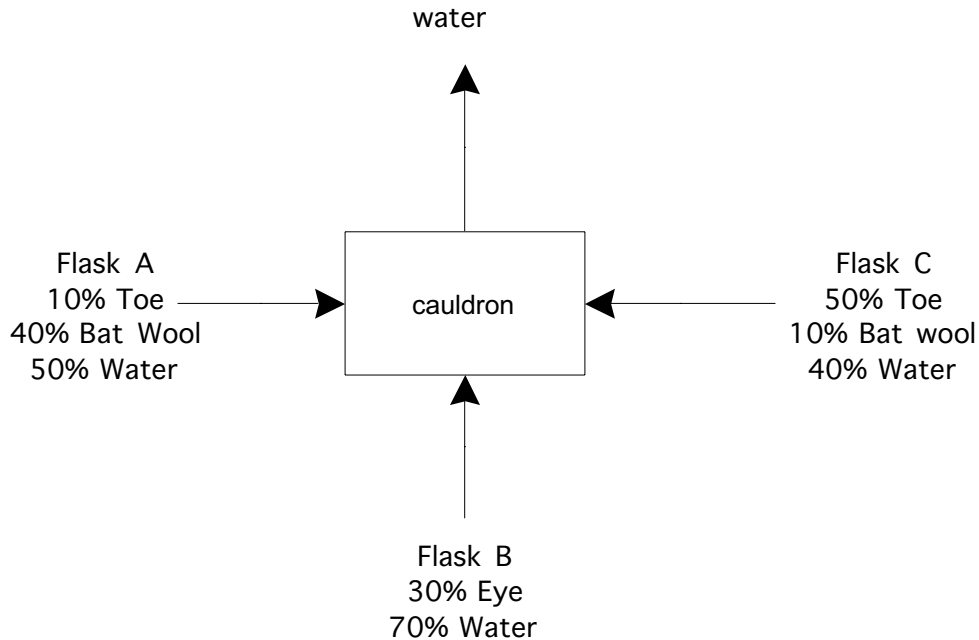
$$\frac{dm_{S,\text{sys}}}{dm_{W,\text{sys}}} = \frac{65}{35} = \frac{\frac{dm_{S,\text{sys}}}{dt}}{\frac{dm_{W,\text{sys}}}{dt}} = \frac{18 \text{ kg/h}}{82 \text{ kg/h} - \dot{m}_{W,\text{out}}}$$

We solve to find

$$\dot{m}_{W,\text{out}} = 82 - \frac{35}{65}(18) = 72.3 \text{ kg/h}$$

P3.18

The cauldron is a combined mixer and separator, as shown. We'll use t for toe of frog, bw for wool of bat, e for eye of newt, and w for water, and indicate streams by A, B, C, or evap.



DOF analysis:

	No. of variables		No. of constraints
Stream variables	9	Specified flow	1 (system contents at end)
System variables	4 (accumulation)	Specified compositions	8 (includes composition of system!)
		Specified performance	0
		Material balances	4
Total	13		13

$$\text{DOF} = 13 - 13 = 0$$

We are interested in what happens over a specified time interval (although we don't yet know exactly what the time interval is – that is what we want to find), so we use the integral balance equation. Everything is in mass units.

$$\text{Total mass: } m_{\text{sys},f} - m_{\text{sys},0} = \int_0^{t_f} \dot{m}_A dt + \int_0^{t_f} \dot{m}_B dt + \int_0^{t_f} \dot{m}_C dt - \int_0^{t_f} \dot{m}_{\text{evap}} dt$$

The mass of the system at the end should be 100 g, initially the cauldron is empty, and the water evaporation rate is known as a function of t. Substituting this information, we find:

$$100 \text{ g} = \int_0^{t_f} \dot{m}_A dt + \int_0^{t_f} \dot{m}_B dt + \int_0^{t_f} \dot{m}_C dt - \int_0^{t_f} (30 - 2t) dt$$

For the components (only 3 equations are independent), the grams of each component added equals the total amount of each flask times the mass fraction of that component in the flask:

$$(0.27)100 \text{ g} = \int_0^{t_f} \dot{m}_{tA} dt + \int_0^{t_f} \dot{m}_{tC} dt = 0.10 \int_0^{t_f} \dot{m}_A dt + 0.5 \int_0^{t_f} \dot{m}_C dt$$

$$(0.22)100 \text{ g} = \int_0^{t_f} \dot{m}_{eB} dt = 0.3 \int_0^{t_f} \dot{m}_B dt$$

$$(0.11)100 \text{ g} = \int_0^{t_f} \dot{m}_{bwA} dt + \int_0^{t_f} \dot{m}_{bwC} dt = 0.40 \int_0^{t_f} \dot{m}_A dt + 0.1 \int_0^{t_f} \dot{m}_C dt$$

We can't solve for the rate of addition of each flask's contents, but we can find the total quantity of flask added to the cauldron (which is all we want to know anyway):

$$\int_0^{t_f} \dot{m}_B dt = \frac{(0.22)100}{0.3} = 73.3 \text{ g (from eye of newt balance)}$$

$$\int_0^{t_f} \dot{m}_A dt = 14.7 \text{ g (from simultaneous solution of the toe and wool balances)}$$

$$\int_0^{t_f} \dot{m}_C dt = 51.1 \text{ g}$$

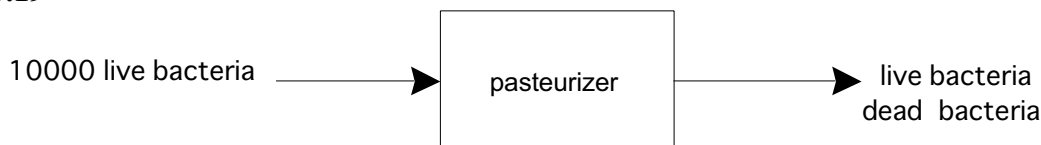
Now we go back to the total balance equation:

$$100 \text{ g} = \int_0^{t_f} \dot{m}_A dt + \int_0^{t_f} \dot{m}_B dt + \int_0^{t_f} \dot{m}_C dt - \int_0^{t_f} (30 - 2t) dt = 14.7 + 73.3 + 51.1 - \int_0^{t_f} (30 - 2t) dt$$

$$39.1 = \int_0^{t_f} (30 - 2t) dt = 30t_f - t_f^2$$

This is a quadratic equation; there are two mathematically correct solutions but only one makes physical sense: $t_f = 1.36$ min or about 82 seconds. (What is the other solution, and why is it unreasonable?)

P3.19



DOF analysis:

	No. of variables		No. of constraints
Stream variables	2	Specified flow	1
System variables	1	Specified compositions	0
		Specified performance	1 (99% conversion)
		Material balances	1
Total	3		3

$$\text{DOF} = 3 - 3 = 0$$

We'll model as a batch process, with the 10000 live bacteria in the pasteurizer at t_0 , and 99% "conversion" to dead bacteria at t_f . We are interested in what happens over a finite interval of time, so we use an integral balance equation. We consider only the live bacteria and assume they are "consumed" in a degradation reaction.

The integral equation is

$$n_{live,sys,f} - n_{live,sys,0} = 0.01(10000) - 10000 = -\int_0^{t_f} \dot{r}_{kill} dt = -10000 \int_0^{t_f} e^{-0.3t} dt$$

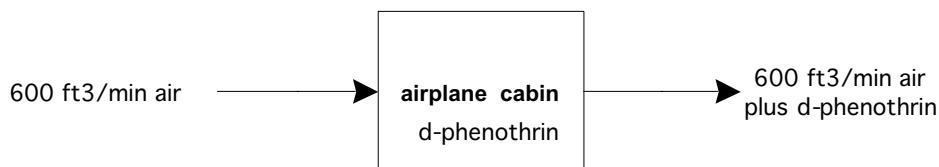
$$0.99 = \int_0^{t_f} e^{-0.3t} dt = -\frac{1}{0.3} \left(e^{-0.3t_f} - e^0 \right)$$

$$0.297 = 1 - e^{-0.3t_f}$$

$$t_f = 1.17 \text{ s}$$

Pasteurization at high temperature for short time kills many bacteria without "cooking" the milk.

P3.20



The cabin is the system, and d-phenothrin (“P”) is our component of interest.

DOF analysis:

	No. of variables		No. of constraints
Stream variables	1	Specified flow	0
System variables	1 (accumulation)	Specified compositions	1 (change in system)
		Specified performance	0
		Material balances	1
Total	2		2

$$\text{DOF} = 2 - 2 = 0$$

At $t = 0$, the cabin contains

$$V_{P,\text{sys},0} = \frac{10^4 \text{ parts d-phenothrin}}{10^6 \text{ parts air}} \times 30,000 \text{ ft}^3 \text{ air} = 300 \text{ ft}^3 \text{ d-phenothrin (assuming}$$

that a “part” is a volumetric measure.) We could convert to moles by using the ideal gas law, but since the d-phenothrin is so dilute, and volume scales with moles, we’ll just leave the units as cu. ft.

At $t = t_f$, the cabin needs to contain at most

$$V_{P,\text{sys},f} = \frac{100 \text{ parts d-phenothrin}}{10^6 \text{ parts air}} \times 30,000 \text{ ft}^3 \text{ air} = 3 \text{ ft}^3 \text{ d-phenothrin}$$

We want to find out how long it takes to reduce the d-phenothrin concentration to an acceptable level. However, we need to first figure out at what rate the d-phenothrin leaves the cabin.

Since the cabin is well-mixed, we can assume that the volume fraction of d-phenothrin in the air leaving the cabin is equal to that inside the cabin. Thus, the rate of flow of d-phenothrin exiting the cabin will change with time. The total flow rate of air is 600 ft³/min, and the volume fraction of d-phenothrin at any time t equals the total volume of d-phenothrin in the system divided by the volume of the cabin. In other words:

$$\dot{V}_{P,out} = \frac{V_{P,sys} \text{ ft}^3}{30000 \text{ ft}^3} \times 600 \text{ ft}^3/\text{min} = 0.02V_{P,sys} \text{ ft}^3/\text{min}$$

From the differential material balance equation,

$$\frac{dn_{P,sys}}{dt} = -\dot{n}_{P,out}$$

If we multiply each term in the equation by RT/P , and assume the ideal gas law applies, so $V = nRT/P$, then

$$\frac{dV_{P,sys}}{dt} = -\dot{V}_{P,out} = -0.02V_{P,sys}$$

Rearranging and integrating with the initial condition given above, we find that at any time t ,

$$V_{P,sys} = V_{P,sys,0} \exp(-0.02t) = 300 \exp(-0.02t)$$

If we now solve for the time t_f at which $V_{P,sys}$ is reduced to 3 ft³ (our target), we find
 $t_f = 230 \text{ min}$

It takes nearly 4 hours to clear the cabin.

(Alternatively, we can find that $\dot{V}_{P,out} = 0.02V_{P,sys} = 0.02V_{P,sys,0} \exp(-0.02t)$ and use this expression in the integral material balance equation. You should get the same answer.)

P3.21

The system is the body, and caffeine is the component. DOF analysis is similar to P3.19. First we find an expression for the flow rate of caffeine out:

$$\frac{dm_{c,sys}}{dt} = -\dot{m}_{c,out} = -0.116m_{c,sys}$$

where the rate is given in mg caffeine/h.

Rearranging gives

$$\frac{dm_{c,sys}}{m_{c,sys}} = -0.116dt \quad (\text{Eq. 1})$$

A general equation describing the caffeine in the body at any time is obtained by integrating with the initial condition that $m_{c,sys} = m_{c,sys,0}$ at $t = 0$:

$$m_{c,sys} = m_{c,sys,0} \exp(-0.116t) \quad (\text{Eq. 2})$$

First we want to find out how long it takes for body caffeine content to drop to 100 mg after drinking a 200 mg cup. We integrate Eq. 1 within specific limits:

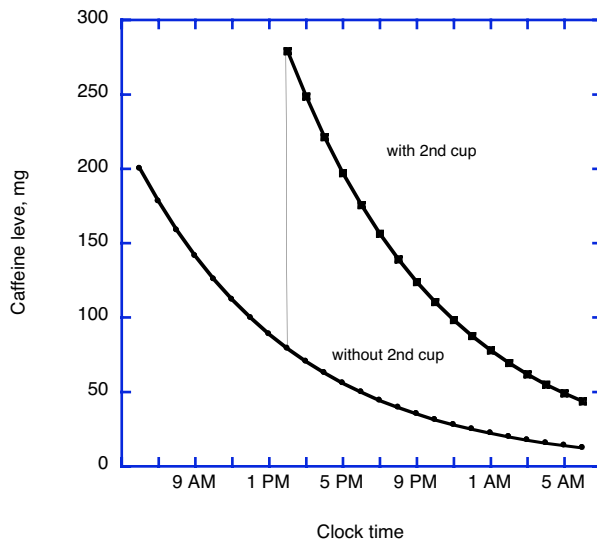
$$\int_{200}^{100} \frac{dm_{c,sys}}{m_{c,sys}} = \int_0^{t_f} -0.116dt$$

$$\ln\left(\frac{100}{200}\right) = -0.116t_f$$

$$t_f = 5.97$$

It takes about 6 h for the caffeine level to drop from 200 to 100 mg.

To plot the caffeine in the body as a function of time, with two cups at 6 AM and at 2 PM, we divide the problem into two time intervals. In the first time interval, which lasts 6 hr, we assume that $m_{c,sys,0} = 200$ mg. In the second interval (2 PM to 11 PM, or 9 h), we assume that $m_{c,sys,0} = 200 \text{ mg} + m_{c,sys,f}$ from the first time interval. We use Eq. 2 to calculate $m_{c,sys}$ at any time.



The plot shows how the caffeine level in the body changes as a function of time. Cutting out the second cup of coffee would decrease the caffeine content in the body at 11 PM from about 100 mg to about 25 mg. If 100 mg is keeping you awake, cutting out the second cup, or moving it earlier in the day, might help.

P3.22

We will divide this problem into time intervals. There are 3 components: titanium dioxide (T), zinc oxide (Z) and silicon dioxide (S). The system is the mixer. DOF analysis is similar to previous two problems.

Interval 1: (from 7 AM to 10 AM)

At 7 AM (t_0), $m_{T,sys,0} = 28 \times 50 = 1400$ kg, $m_{Z,sys,0} = 100$ kg, $m_{S,sys,0} = 500$ kg
 $m_{sys,0} = 1400 + 100 + 500 = 2000$ kg

Note that the proportion is correct: 70% TiO_2 , 5 % ZnO , 25% SiO_2 .

There is no additional input, but there is a steady flow out of 500 kg/h, so

$$\dot{m}_{out} = 500 \text{ kg/h}$$

The integral material balance (total mass) is:

$$m_{sys,f} - m_{sys,0} = - \int_{t_0}^{t_f} \dot{m}_{out} dt$$

$$m_{sys,f} = 2000 - \int_{7AM}^{10AM} 500 dt = 2000 - 1500 = 500 \text{ kg}$$

Interval 2: (from 10 AM to 12 PM)

At 10 AM, there is 500 kg in the mixer and he adds several more sacks:

$$m_{sys,0} = 500 + 14(50) + 1(50) + 5(50) = 1500 \text{ kg (Again, the proportions are correct.)}$$

There is no additional input, but there is a steady flow out of 500 kg/h, so

$$\dot{m}_{out} = 500 \text{ kg/h}$$

The integral material balance (total mass) is:

$$m_{sys,f} = 1500 - \int_{10AM}^{12PM} 500 dt = 1500 - 1000 = 500 \text{ kg}$$

Interval 3: (from 12 PM to 3 PM)

At 12 PM, there is 500 kg in the mixer and he adds several more sacks:

$$m_{sys,0} = 500 + 14(50) + 1(50) + 5(50) = 1500 \text{ kg (Again, the proportions are correct.)}$$

There is no additional input, but there is a steady flow out of 500 kg/h, so

$$\dot{m}_{out} = 500 \text{ kg/h}$$

The integral material balance (total mass) is:

$$m_{sys,f} = 1500 - \int_{12PM}^{3PM} 500 dt = 1500 - 1500 = 0 \text{ kg}$$

The tank never runs out until the end of the day, at 3 PM.

P3.23

The system is the controlled release device, and the component is the drug.

DOF analysis:

	No. of variables		No. of constraints
Stream variables	1 (drug out)	Specified flow	1 (release rate)
System variables	2 (accumulation, reaction)	Specified compositions	1 (initial quantity)
		Specified performance	1 (degradation rate)
		Material balances	1
Total	3		4

DOF = 3 - 4 = -1. Overspecified??

For the total mass left in the device at any time, start the differential balance:

$$\frac{dm_{D,sys}}{dt} = -\dot{m}_{D,out} - \dot{R}_{deg} = -8 \exp(-0.1t) - 1.1$$

Now integrate, with the initial condition that $m_{D,sys} = 100 \mu\text{g}$ at $t = 0$

$$\int_{100}^{m_{D,sys}} dm_{D,sys} = - \int_0^t (8 \exp(-0.1t) + 1.1) dt$$

$$m_{D,sys} = 20 + 80 \exp(-0.1t) - 1.1t$$

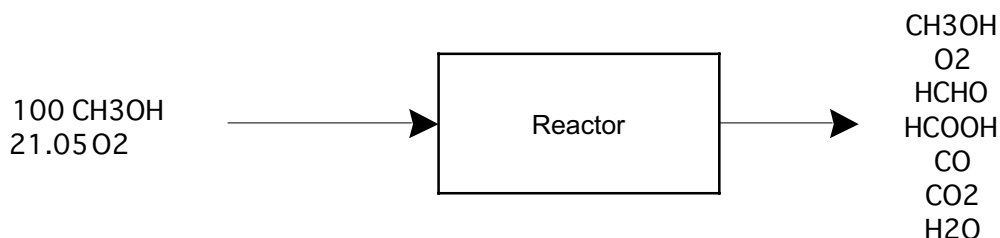
We can use this expression, with $t = 24 \text{ h}$, to find

$$m_{D,sys} = 20 + 80 \exp(-0.1 \times 24) - 1.1 \times 24 = 100 - 72.7 - 26.4 = 0.9$$

The fraction released is therefore $(100-0.9)/100 = 0.991$. This includes the material lost by degradation ((26.4% of the total initial charge) as well as that released into the body (~72.7% of the total initial charge).

The problem appeared to be overspecified, but was still solvable. This is because we have additional information, that allows us not only to determine the amount of material left in the device after 24 h, but also the relative importance of the two loss mechanisms.

P3.24



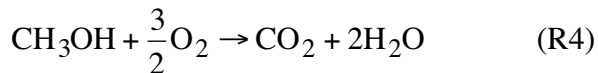
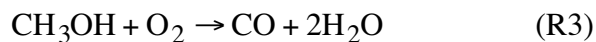
DOF analysis:

	No. of variables		No. of constraints
Stream variables	9	Specified flow	2
System variables	4 (reactions, because there are 7 compounds and 3 elements)	Specified compositions	0
		Specified performance	2 (fractional conversions)
		Material balances	7
Total	13		11

DOF = 13 - 11 = 2. underspecified??

Let M = methanol, O = O₂, F = formaldehyde, FA = formic acid, CM = carbon monoxide, CD = carbon dioxide, W = water. The system is the reactor, and we'll assume it is steady state. All flows in kgmol/h.

A set of 4 independent balanced chemical reactions that describe this system is:



From the reactor performance specifications:

$$\dot{n}_{M,out} = (1 - f_{C,M})\dot{n}_{M,in} = (1 - 0.4)100 = 60$$

$$\dot{n}_{O,out} = (1 - f_{C,O})\dot{n}_{O,in} = (1 - 0.95)21.05 = 1.05$$

From material balance equations:

$$\dot{n}_{M,out} = \dot{n}_{M,in} + \sum v_{ik}\dot{\xi}_k = 60 = 100 - (\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \dot{\xi}_4)$$

$$\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \dot{\xi}_4 = 40$$

$$\dot{n}_{O,out} = \dot{n}_{O,in} + \sum v_{ik}\dot{\xi}_k = 1.05 = 21.05 - \left(\frac{1}{2}\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \frac{3}{2}\dot{\xi}_4\right)$$

$$\frac{1}{2}\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \frac{3}{2}\dot{\xi}_4 = 20$$

We have 2 equations in 4 unknowns. Can we get anywhere? Perhaps. Suppose we try subtracting the two equations we derived from material balances;

$$\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \dot{\xi}_4 - \left(\frac{1}{2}\dot{\xi}_1 + \dot{\xi}_2 + \dot{\xi}_3 + \frac{3}{2}\dot{\xi}_4\right) = 40 - 20$$

$$\frac{1}{2}\dot{\xi}_1 - \frac{1}{2}\dot{\xi}_4 = 20$$

$$\dot{\xi}_1 - \dot{\xi}_4 = 40$$

Now we note that all extents or reaction must be positive, because only methanol and oxygen are fed to the reactor. This is an additional constraint on the solution that was not explicitly considered in the DOF analysis. Although there are an infinite number of mathematical solutions, there is only one physically reasonable solution:

$$\dot{\xi}_1 = 40, \dot{\xi}_4 = \dot{\xi}_3 = \dot{\xi}_2 = 0$$

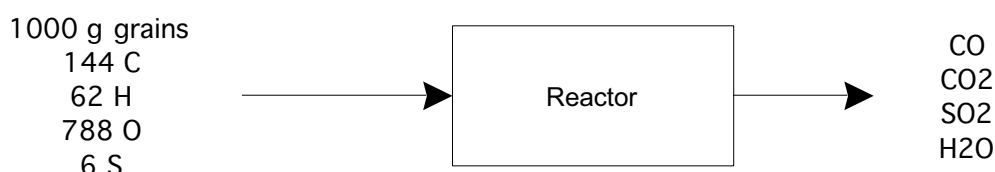
The material balances on the other components become:

$$\begin{aligned}\dot{n}_{F,out} &= \sum v_{ik} \dot{\xi}_k = \dot{\xi}_1 = 40 \\ \dot{n}_{W,out} &= \sum v_{ik} \dot{\xi}_k = \dot{\xi}_1 = 40 \\ \dot{n}_{FA,out} &= \dot{n}_{CM,out} = \dot{n}_{CD,out} = 0\end{aligned}$$

The reactor outlet flow is 60 kgmol/h methanol, 40 kgmol/h formaldehyde, 40 kgmol/h water, and 1.05 kgmol/h oxygen. In terms of mol% and total flow: 141.05 kgmol/h at 42.5% methanol, 28.4% each formaldehyde and water, and 0.7% oxygen.

P3.25

This is a semi-batch reactor.



We'll treat C, H, O, and S as components, as well as CO, CO₂, SO₂, and H₂O. There are 4 independent reactions.

DOF analysis:

	No. of variables		No. of constraints
Stream variables	8	Specified flow	1
System variables	4	Specified compositions	3
		Specified performance	0
		Material balances	8
Total	12		12

$$\text{DOF} = 12 - 12 = 0.$$

This is easiest to do as element balances, we will convert to gmol for the balance equations. We will use the integral material balance equation, considering the grains as an initial charge to the reactor at $t = 0$, and the mass of the exit gases as the integral of the flow out of the reactor over the entire time interval. (It is not necessary for us to know how long the reaction was or the flow rates, as we care only about the total mass leaving the reactor. We will simplify the notation, letting n_i = the total moles of i leaving the reactor.)

$$\text{H balance:} \quad 62 \text{ g H} \left(\frac{\text{gmol H}}{1 \text{ g H}} \right) = 62 \text{ gmol} = 2n_{H_2O}$$

$$\text{S balance: } 6 \text{ g S} \left(\frac{\text{gmol S}}{32 \text{ g S}} \right) = 0.1875 \text{ gmol} = n_{\text{SO}_2}$$

$$\text{C balance: } 144 \text{ g C} \left(\frac{\text{gmol C}}{12 \text{ g C}} \right) = 12 \text{ gmol} = n_{\text{CO}} + n_{\text{CO}_2}$$

$$\text{O balance: } 788 \text{ g O} \left(\frac{\text{gmol O}}{16 \text{ g O}} \right) = 49.25 \text{ gmol} = n_{\text{CO}} + 2n_{\text{CO}_2} + 2n_{\text{SO}_2} + n_{\text{H}_2\text{O}}$$

Solving this system of 4 equations in 4 unknowns, we find:

$$n_{\text{H}_2\text{O}} = 31 \text{ gmol}$$

$$n_{\text{SO}_2} = 0.1875 \text{ gmol}$$

$$n_{\text{CO}} = 6.125 \text{ gmol}$$

$$n_{\text{CO}_2} = 5.875 \text{ gmol}$$

The total moles of exit gas is 43.1875 gmol, of which 0.43 mol% is SO_2 .

The ideal gas law is used to calculate the standard liters of gas:

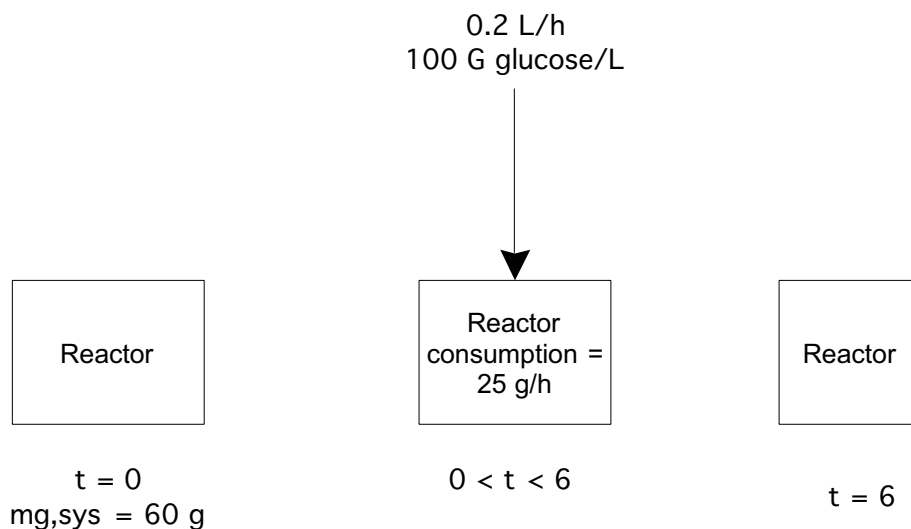
$$V = 43.1875 \text{ gmol} \times 22.4 \text{ liters/gmol} = 967.4 \text{ liters}$$

To convert CO to CO_2 , we require 1/2 gmol O_2 per gmol CO:

$$n_{\text{O}_2} = 6.125 \text{ gmol CO} \times \frac{1 \text{ gmol O}_2}{2 \text{ gmol CO}} = 3.06 \text{ gmol O}_2, \text{ or } 98 \text{ g O}_2 \text{ per kg grain.}$$

P3.26

A semibatch system, with a single component of interest: glucose.



DOF analysis:

	No. of variables		No. of constraints
Stream variables	1 (glucose in)	Specified flow	1
System variables	2 (reaction, accumulation)	Specified compositions	1 (initial glucose)
		Specified performance	0
		Material balances	1
Total	3		3

$$\text{DOF} = 3 - 3 = 0.$$

We use an integral material balance, with units in g and h.

$$m_{g,\text{sys},f} - m_{g,\text{sys},0} = \int_{t_0}^{t_f} \dot{m}_{g,\text{in}} dt - \int_{t_0}^{t_f} \dot{m}_{g,\text{out}} dt + \int_{t_0}^{t_f} \dot{R}_g dt$$

$$m_{g,\text{sys},f} - 60 = \int_0^6 20 dt - 0 + \int_0^6 -25 dt = 120 - 150$$

$$m_{g,\text{sys},f} = 30 \text{ g glucose}$$

To calculate the concentration, we need to do a total mass balance, recalling that 1 mL broth = 1 g.

$$m_{\text{sys},f} - m_{\text{sys},0} = \int_{t_0}^{t_f} \dot{m}_{\text{in}} dt - \int_{t_0}^{t_f} \dot{m}_{\text{out}} dt$$

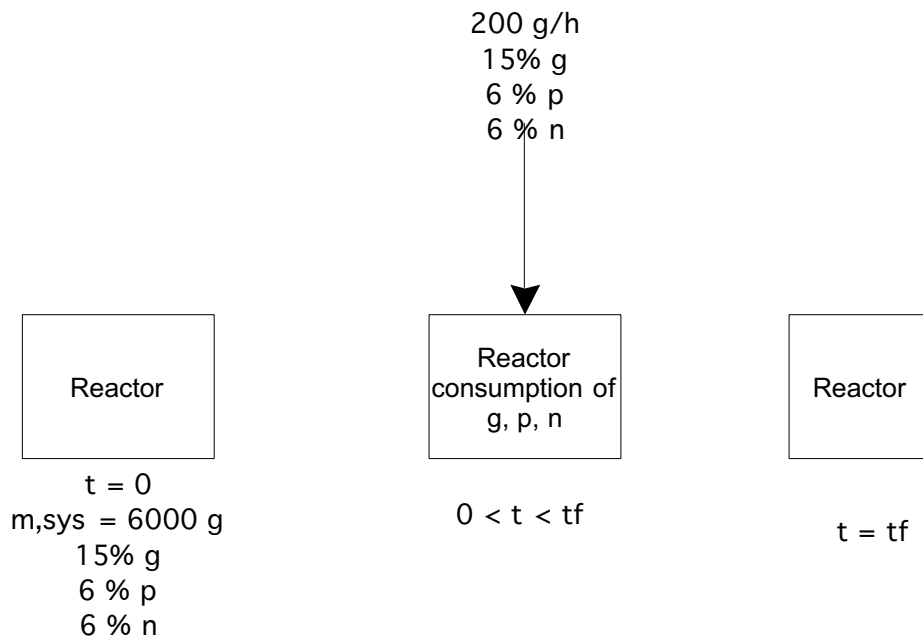
$$m_{\text{sys},f} - 600 = \int_0^6 200 dt = 1200$$

$$m_{\text{sys},f} = 1800 \text{ g} = 1.8 \text{ L}$$

Final concentration = 30 g glucose/1.8 L or 16.67 g/L.

P3.27

There are 3 components of interest in this semibatch operation.



DOF analysis:

	No. of variables		No. of constraints
Stream variables	4	Specified flow	1
System variables	7 (reaction, accumulation)	Specified compositions	6 (initial + input flow)
		Specified performance	0
		Material balances	4
Total	11		11

$$\text{DOF} = 11 - 11 = 0.$$

(We count glucose, phosphate and nitrates as components, plus one additional component which is everything else – water, trace nutrients...which can accumulate but do not react.)

We use an integral material balance, with units in g and h. We will solve for the time at which the mass of each nutrient reaches zero; that nutrient with the shortest time is the limiting nutrient.

$$\begin{aligned} \text{Glucose:} \quad 0 - 900 &= \int_0^{t_f} 30 dt - \int_0^{t_f} 35 dt = -900 = -5t_f \\ t_f &= 180 \text{ h} \end{aligned}$$

$$\text{Phosphate: } 0 - 360 = \int_0^{t_f} 12 dt - \int_0^{t_f} 13 dt = -1t_f$$

$$t_f = 360 \text{ h}$$

$$\text{Nitrates: } 0 - 360 = \int_0^{t_f} 12 dt - \int_0^{t_f} 12 dt = -1t_f$$

$$t_f \rightarrow \infty$$

Glucose runs out first, at 180 h. We can now calculate the concentration of phosphate and nitrate at 180 h

$$m_{p,sys,f} = 360 + \int_0^{180} 12 dt - \int_0^{180} 13 dt = 180 \text{ g}$$

$$m_{n,sys,f} = 360 + \int_0^{180} 12 dt - \int_0^{180} 12 dt = 360 \text{ g}$$

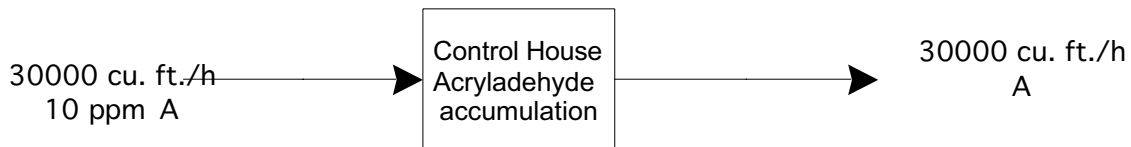
$$m_{sys,f} = 6000 + \int_0^{180} 200 dt = 42000 \text{ g or 42 L}$$

Concentrations at the end of the fermentation are: $180 \text{ g}/42 \text{ L} = 4.28 \text{ g/L}$ phosphate, and $360/42$ or 8.57 g/L nitrate.

(In this solution we assumed the broth density is constant, at 1 mg/mL , independent of composition, and we assume that there is no loss of mass to gases.)

P3.28

The control house is the system, and acrylaldehyde is the component of interest. It is unsteady state, with accumulation but no reaction.



DOF analysis:

	No. of variables		No. of constraints
Stream variables	2 (A in, A out)	Specified flow	1 (A in)

System variables	1 (accumulation)	Specified compositions	1 (change in A)
		Specified performance	0
		Material balances	1
Total	3		3

$$\text{DOF} = 3 - 3 = 0.$$

We need to convert to useful units. Assuming the ppm is a mass ratio, that the air is at standard temperature and pressure (in the absence of any better information) and that air is 21% oxygen, 79% nitrogen, we calculate

$$\dot{m}_{A,in} = \frac{10 \text{ g A}}{10^6 \text{ g air}} \times \frac{30000 \text{ cu.ft air}}{\text{h}} \times \frac{28.317 \text{ L}}{\text{cu.ft}} \times \frac{1 \text{ gmol}}{22.4 \text{ L}} \times \frac{28.8 \text{ g air}}{\text{gmol air}} = 10.9 \text{ g A/h}$$

Since the air in the control house is well-mixed, the concentration of A in the air leaving the control house equals the concentration in the house.

$$\dot{m}_{A,out} = \frac{m_{A,sys} \text{ g A}}{10000 \text{ cu.ft}} \times \frac{30000 \text{ cu.ft.}}{\text{h}} = 3 m_{A,sys} \text{ (in units of g/h)}$$

At the danger point, the concentration in the control house is 4 ppm or

$$m_{A,sys,f} = \frac{4 \text{ g A}}{10^6 \text{ g air}} \times 10000 \text{ cu.ft air} \times \frac{28.317 \text{ L}}{\text{cu.ft}} \times \frac{1 \text{ gmol}}{22.4 \text{ L}} \times \frac{28.8 \text{ g air}}{\text{gmol air}} = 1.46 \text{ g A}$$

$m_{A,sys}$ changes with time. To find an expression that is independent of time, we go to the differential material balance equation;

$$\frac{dm_{A,sys}}{dt} = \dot{m}_{A,in} - \dot{m}_{A,out} = 10.9 - 3m_{A,sys}$$

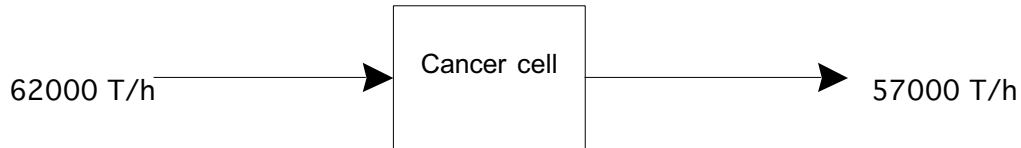
Integration from initial time, where there is no A in the house, to final time, where A = 1.46 g, gives:

$$\int_0^{1.46} \frac{dm_{A,sys}}{10.9 - 3m_{A,sys}} = \int_0^{t_f} dt$$

$$t_f = -\frac{1}{3} \ln \left(\frac{10.9 - 3(1.46)}{10.9 - 3(0)} \right) = 0.171 \text{ h} = 10.3 \text{ min}$$

P3.29

The immunotoxin (T) is the component of interest, and the cancer cell is the system. Inside the system, both accumulation and reaction occur



DOF analysis:

	No. of variables		No. of constraints
Stream variables	2 (T in, T out)	Specified flow	2
System variables	2 (accumulation, reaction)	Specified compositions	0
		Specified performance	1 (reaction rate)
		Material balances	1
Total	4		4

$$\text{DOF} = 4 - 4 = 0.$$

An integral balance is appropriate, since we are interested in accumulation after a finite time interval.

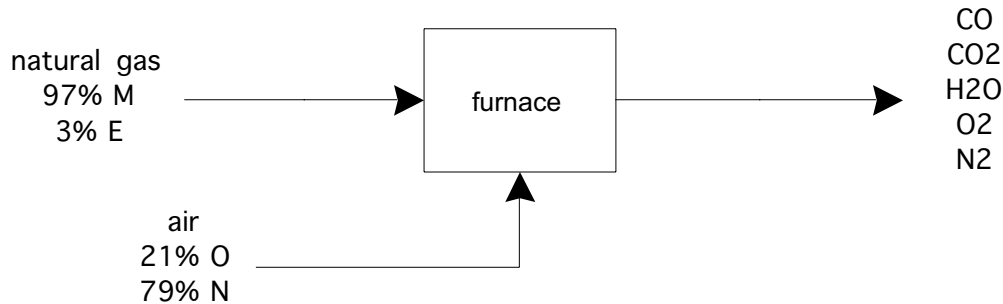
$$m_{T,sys,f} - 0 = \int_0^8 62000 dt - \int_0^8 57000 dt + \int_0^8 -2700(1 - e^{-0.3t}) dt$$

$$m_{T,sys,f} - 0 = 40000 - \left[2700(8) + \frac{2700}{0.3} \left(e^{-0.3(8)} - e^0 \right) \right] = 40000 - 21600 + 816 - 9000 = 26,580$$

The product comes close but doesn't quite achieve the goal of 30,000 immunotoxin molecules accumulated in the cell over 8 h. If you believe that the technology could be improved a bit, though, it looks like a pretty good investment.

P3.30

Assume steady state operation and that air is 21 mol% O₂, 79 mol% N₂.



DOF analysis:

	No. of variables		No. of constraints
Stream variables	9	Specified flow	0
System variables	2 (reactions)	Specified compositions	2
		Specified performance	1 (oxygen conversion)
		Material balances	7
Total	11		10

DOF = 11 - 10 = 1. Choosing a basis will fully specify this system.

We'll choose 100 gmols natural gas fed per h as the basis. The problem can be solved either using compounds or elements as components. We'll use elements in this solution.

$$\text{C: } \dot{n}_{M,in} + 2\dot{n}_{E,in} = 97 + 2(3) = 103 = \dot{n}_{CO,out} + \dot{n}_{CO_2,out}$$

$$\text{H: } 4\dot{n}_{M,in} + 6\dot{n}_{E,in} = 4(97) + 6(3) = 406 = 2\dot{n}_{H_2O,out}$$

$$\text{O: } 2\dot{n}_{O,in} = \dot{n}_{CO,out} + 2\dot{n}_{CO_2,out} + \dot{n}_{H_2O,out} + 2\dot{n}_{O_2,out}$$

$$\text{N: } 2\dot{n}_{N,in} = 2\dot{n}_{N,out}$$

From system performance specification:

$$n_{O_2,out} = (1 - f_{c,O})n_{O_2,in} = 0.1n_{O_2,in}$$

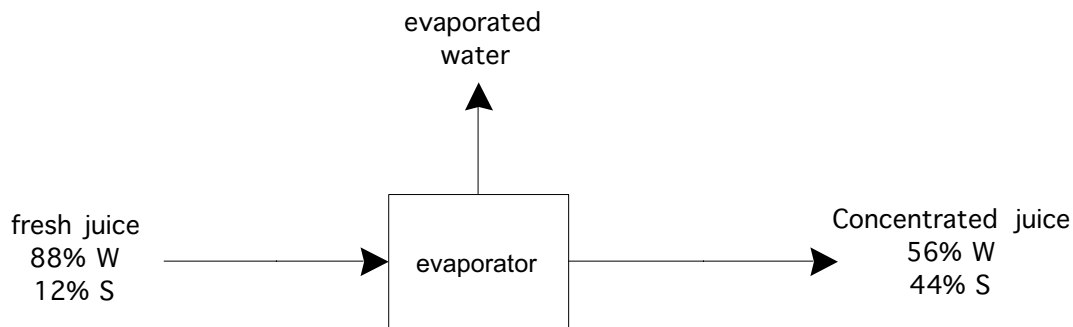
From stream composition specifications:

$$\frac{n_{O_2,in}}{n_{N_2,in}} = \frac{79}{21}$$

$$\frac{n_{CO,out}}{n_{CO_2,out}} = \frac{1}{5}$$

These equations can be solved to find, per 100 gmol natural gas fed, 217.7 gmol oxygen and 819 gmol nitrogen are fed, and that the flue gas contains 17.2 gmol CO, 85.8 gmol CO₂, 203 gmol H₂O, 21.8 gmol O₂ and 819 gmol N₂. Total flue gas out is 1147 gmol (1.5 mol% CO, 7.5 mol% CO₂, 17.7 mol% H₂O, 1.9 mol% O₂, 71.4 mol% N₂.)

P3.31



DOF analysis:

	No. of variables		No. of constraints
Stream variables	5	Specified flow	1 (water evap)
System variables	0	Specified compositions	2
		Specified performance	0
		Material balances	2
Total	5		5

$$\text{DOF} = 5 - 5 = 0.$$

The water evaporation rate starts at 1770 lb/day and then slowly decreases over time, as:

$$\dot{m}_{W, \text{evap}} = 1770(0.9)^{d-1}$$

where d = day of the week, $d = 1$ for day 1, etc. The material balance equations on any given day (with all numbers given in units of lb/day and ff = fresh juice, cj = concentration juice) are

$$0.88\dot{m}_{ff, \text{in}} = \dot{m}_{\text{evap}} + 0.56\dot{m}_{cj, \text{out}} \quad (\text{water balance})$$

$$\dot{m}_{ff, \text{in}} = \dot{m}_{\text{evap}} + \dot{m}_{cj, \text{out}} \quad (\text{total mass balance})$$

Combining these equations we find

$$\dot{m}_{ff,in} = \frac{1}{0.88} (\dot{m}_{evap} + 0.56(\dot{m}_{ff,in} - \dot{m}_{evap})) = \frac{1}{0.88} (0.56\dot{m}_{ff,in} + 0.44\dot{m}_{evap})$$

$$\dot{m}_{ff,in} = \frac{0.44/0.88}{1 - (0.56/0.88)} \dot{m}_{evap} = 1.375\dot{m}_{evap} = 1.375(1770)(0.9)^{d-1} = 2433.75(0.9)^{d-1}$$

We can use this equation to calculate the amount of fresh juice to be fed every day, then add up over 7 days to calculate the total weekly fresh juice required.

Day	1	2	3	4	5	6	7
Fresh juice	2433.75	2190.375	1971.34	1774.2	1596.78	1437.1	1293.39

Total = 12697 lb fresh juice per week.

P3.32



The tank initial contains 5000 L, or 6280 kg solution (calculated from the solution density). At $t = 0$, the leak rate is 5 L/min (or 6.28 kg/min); the leak rate increases linearly such that at $t = 10$ min, the leak rate is 69.08 kg/min. We write this as a linear equation:

$$\dot{m}_{leak} = 6.28 + 6.28t$$

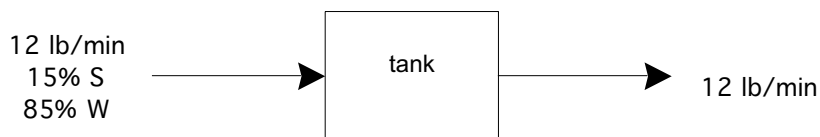
where t is in minutes. Since we are interested in what happens over a finite interval of time, we use an integral balance;

$$m_f - m_0 = - \int_0^{20} \dot{m}_{leak} dt = - \int_0^{20} (6.28 + 6.28t) dt = - \left[6.28(20) + \frac{6.28}{2} (20^2) \right] = -1381.6 \text{ kg}$$

This is the total loss of material from the tank, since the material is 40 wt% nitric acid, there have been 552.6 kg nitric acid spilled onto the floor in 20 minutes.

P3.33

Initially, the tank contains 50 lb Na_2SO_4 (“S”) and 50 lb water (“W”). Then material is pumped in and out of the tank for 10 minutes



This is an unsteady-state process. Since the tank is well-mixed, the mass fraction of S in the stream leaving the tank equals the mass fraction in the tank. We will work in units of lb and h. The total mass in the tank remains constant at 100 lb.

First we will derive a general expression for the mass in the system at any given time.

$$\frac{dm_{S,sys}}{dt} = \dot{m}_{S,in} - \dot{m}_{S,out} = 12(0.15) - 12x_{S,sys} = 1.8 - 12\left(\frac{m_{S,sys}}{100}\right)$$

(because the mass fraction of salt in the outlet stream equals the mass fraction in the system, which equals the mass of salt in the system divided by the total mass in the system.)

We re-arrange and integrate, using the initial conditions that at $t = 0$, $m_{S,sys} = 50$ lb:

$$\int_{50}^{m_{S,sys}} \frac{dm_{S,sys}}{1.8 - 0.12m_{S,sys}} = \int_0^t dt$$

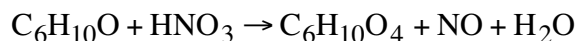
$$-\frac{1}{0.12} \ln \left| \frac{1.8 - 0.12m_{S,sys}}{-4.2} \right| = t$$

$$m_{S,sys} = 15 + 35e^{-0.12t}$$

Evaluating at $t = 10$ minutes, we find that the tank contains 25.5 lbs sodium sulfate, or the mass fraction is 0.255 lb/lb solution.

P3.34

The unbalanced reaction is



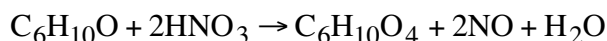
Setting this up as a matrix, with cyclohexanone as the basis compound, and writing the molecular formulas in the order C, H, O, N, we find

$$\begin{bmatrix} 0 & 6 & 0 & 0 \\ 1 & 10 & 0 & 2 \\ 3 & 4 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 1 \\ 0 \end{bmatrix}$$

Finding the solution on my TI-83+ is straightforward:

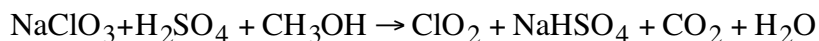
$$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

The balanced equation is



P3.35

The unbalanced reaction is



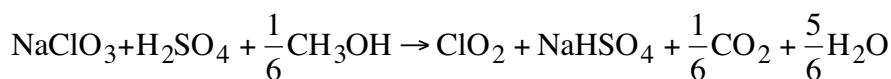
Choosing sodium chlorite as the basis compound, and writing the elements in the order C, H, O, S, Na, Cl, we set up the matrix equation as:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 0 & 1 & 0 & 2 \\ 4 & 1 & 2 & 4 & 2 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Finding the solution on my TI-83+ is straightforward:

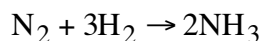
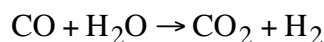
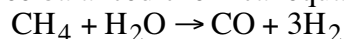
$$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} -1 \\ -1/6 \\ 1 \\ 1 \\ 1/6 \\ 5/6 \end{bmatrix}$$

The balanced equation is



P3.36

The three balanced chemical equations are:



Our goal is to combine these equations so there is no net generation or consumption of CO or hydrogen. Following the steps in Section 3.3.3:

1. There are 7 rows (7 compounds) and 3 columns (3 reactions).

$$\begin{array}{l} \text{CH}_4 \\ \text{H}_2\text{O} \\ \text{CO} \\ \text{H}_2 \\ \text{CO}_2 \\ \text{N}_2 \\ \text{NH}_3 \end{array} \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & -1 & 0 \\ 3 & 1 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

We cross out rows 1, 5, 6, and 7 because they have only a single nonzero entry;

$$\begin{bmatrix} -1 & -1 & 0 \\ 1 & -1 & 0 \\ 3 & 1 & -3 \end{bmatrix}$$

We have the same number of rows as columns. The first row is water; this is an acceptable byproduct, so we'll cross it out.

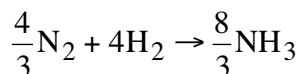
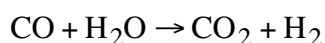
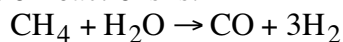
$$\begin{bmatrix} 1 & -1 & 0 \\ 3 & 1 & -3 \end{bmatrix}$$

We'll choose arbitrarily the first reaction as our basis, so that becomes vector b. Then we write, and solve, the matrix equation:

$$\begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} \chi_2 \\ \chi_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} \chi_2 \\ \chi_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4/3 \end{bmatrix}$$

The set of reactions is:



P3.37

There are 6 compounds and 3 elements, so there are at most 3 independent chemical reactions.

P3.38

Start by writing matrix of the 8 compounds (8 columns) in the order listed, and 3 rows (H, O, N).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 2 & 3 & 1 \\ 0 & 2 & 1 & 2 & 0 & 1 & 0 & 3 \\ 2 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Reduce the matrix

$$\begin{bmatrix} 1 & 0 & 0.5 & 0.5 & 0 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0.5 & 1 & 0 & 0.5 & 0 & 1.5 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1.5 & 0.5 \end{bmatrix}$$

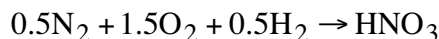
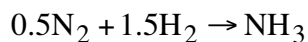
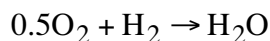
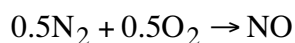
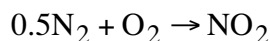
This isn't in the right form, but it will be if we switch columns 3 and 5 (and compounds NO and H₂).

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 & 0.5 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0 & 1 & 0.5 & 0.5 & 0 & 1.5 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1.5 & 0.5 \end{bmatrix}$$

Now we erase the identity matrix, multiply all remaining entries by -1, and add an identity matrix at the bottom:

$$\begin{bmatrix} -0.5 & -0.5 & 0 & -0.5 & -0.5 \\ -1 & -0.5 & -0.5 & 0 & -1.5 \\ 0 & 0 & -1 & -1.5 & -0.5 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we can read the balanced reactions down, with the compounds ordered as N_2 , O_2 , H_2 , NO_2 , NO , H_2O , NH_3 and HNO_3 .



P3.39

There are 5 compounds and 3 elements, so at most 2 independent reactions. We set up the matrix of 3 rows (C, H, O, and five columns (C_2H_4 , O_2 , CO_2 , H_2O , $\text{C}_2\text{H}_4\text{O}$).

$$\begin{bmatrix} 2 & 0 & 1 & 0 & 2 \\ 4 & 0 & 0 & 2 & 4 \\ 0 & 2 & 2 & 1 & 1 \end{bmatrix}$$

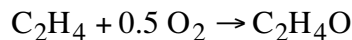
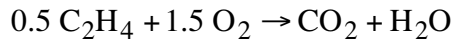
The reduced-row matrix is obtained on the TI-83+

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 & 1 \\ 0 & 1 & 0 & 1.5 & 0.5 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Now we erase the old identity matrix, multiply remaining numbers by -1 and add the new identity matrix at the bottom. Once that's done, we can read down the reactions.

$$\begin{bmatrix} -0.5 & -1 \\ -1.5 & -0.5 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reactions are



The reactor process flow calculations can now be completed. The material balances are (using E for ethylene, O for oxygen, W for water, EO for ethylene oxide, and C for CO₂). All flows are in kgmol/h; the differential material balance equation is used assuming steady-state operation.

$$\dot{n}_{E,out} = \dot{n}_{E,in} - 0.5\dot{\xi}_1 - \dot{\xi}_2 = 20000 - 0.5\dot{\xi}_1 - \dot{\xi}_2$$

$$\dot{n}_{O,out} = \dot{n}_{O,in} - 1.5\dot{\xi}_1 - 0.5\dot{\xi}_2 = 11000 - 1.5\dot{\xi}_1 - 0.5\dot{\xi}_2$$

$$\dot{n}_{EO,out} = \dot{\xi}_2$$

$$\dot{n}_{C,out} = \dot{\xi}_1$$

$$\dot{n}_{W,out} = \dot{\xi}_1$$

There are two system performance equations in which fractional conversion of the reactants are specified.

$$\dot{n}_{E,out} = (1 - f_{C,E})\dot{n}_{E,in} = 0.75 \times 20000 = 15000$$

$$\dot{n}_{O,out} = (1 - f_{C,O})\dot{n}_{O,in} = 0.091 \times 11000 = 1001$$

Using these values along with the material balance equations, we find that the extents of reaction are 6000 kgmol/h and 2000 kgmol/h, respectively, that the total flow rate out of the reactor is 30,000 kgmol/h, and that the reactor effluent composition is 50% ethylene, 3.3% oxygen, 6.7% ethylene oxide, 20% carbon dioxide, and 20% water (all mol%).

P3.40

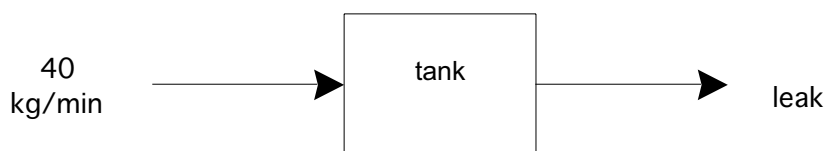
Solution is given, with stream numbers listed in first column and compounds listed in first row. All flows given in lb/day. Obtaining answers proceeds by writing down all the material balance equations along with specifications, as usual.

	B	CN	AN	H	HD	CH	O	CK	W	NA	AA	NO	N66
1				3500									
2		23900											
3	23900												
4										55700			
5						37100							
6							14100						
7	23900	23900											
8			47800										
9			47800	3500									

10					51300								
11						37100	14100						
12								43245	7955				
13								43245					
14									7955				
15								43245		55700			
16									7955		64490	26500	
17											64490		
18									7955			26500	
19					51300						64490		
20									15790				100000
21									15790				
22													100000

P3.41

The system (the tank) is unsteady state. Initially, there is a flow into the tank but none out. Later, there is both flow in and flow out, and the rate of the flow out changes with time, as the system fills.



We can either do a balance on compound X, or on total mass, because the composition doesn't change. We'll use compound X.

From $t = 0$ to $t = t_{leak}$, there is no flow out, and the tank is initially empty. At the point of the corroded spot, the tank volume is $\frac{\pi(1 \text{ m}^2)}{4} \times 1 \text{ m} = 0.785 \text{ m}^3$. Given the density, this is a mass of 785 kg (78.5 kg compound X). From the integral mass balance on compound X, we find:

$$m_{X,sys,f} - m_{X,sys,0} = \int_0^{t_{leak}} \dot{m}_{X,in} dt$$

$$78.5 = 4.0(t_{leak} - 0)$$

$$t_{leak} = 19.6 \text{ min}$$

After 19.6 min, there is an outlet flow that scales with the height of the fluid in the tank h (in meters). The flow rate of compound X out of the tank (kg/min) is given as:

$$\dot{m}_{X,out} = 0.4\sqrt{(h-1)}$$

The height of the mass in the tank depends on the flow rate in and out! We can write the differential mass balance equation, taking advantage of the known density and the dimensions of the cylindrical tank:

$$\frac{dm_{X,sys}}{dt} = (0.1 \text{ kg X/kg}) \left(\frac{\pi}{4} \text{ m}^2 \right) \times 1000 \text{ kg/m}^3 \times \frac{dh}{dt} = \dot{m}_{X,in} - \dot{m}_{X,out} = 4 - 0.4\sqrt{h-1}$$

Simplifying, we get:

$$78.5 \frac{dh}{dt} = (4 - 0.4\sqrt{h-1})$$

Now we can rearrange, and write as an integral, from $t = 19.6$ min to $t = 40$ min, and from $h = 1$ m to $h = h_f$:

$$78.5 \int_1^{h_f} \frac{dh}{(4 - 0.4\sqrt{h-1})} = \int_{19.6}^{40} dt = 20.4 \text{ min}$$

We have a couple of options. If we remember our calculus, we can derive an analytical solution. If not, we can use the numerical integration function on our calculator (or an equation solver), and just guess values of h_f till we find one that is consistent with the above equation. We know that h_f has to be greater than 1 m and less than 3 m.

Guess: 2 m: integral = 0.268

Guess: 1.95 m: integral = 0.254

Guess: 1.96 m: integral = 0.256

Guess: 1.975 m: integral = 0.261 (close enough)

The total amount of compound X pumped into the tank from 19.6 min to 40 min is simply $(0.1 \text{ kg X/kg soln})(40 \text{ kg soln/min})(40 - 19.6 \text{ min}) = 81.6 \text{ kg}$. The total amount of compound X added to the tank from 1 m high to 1.975 m high is $(1.975 - 1 \text{ m})(\pi/4)(1 \text{ m})^2(1000 \text{ kg/m}^3)(0.1 \text{ kg X/kg soln}) = 76.5 \text{ kg}$

Therefore, the amount of compound X on the floor is $81.6 - 76.6$ or about 5.1 kg. (You will get slightly different answers depending on the accuracy to which you solved the integral.)

(Another way to look at this follows: If there were no leak, we'd have

$$\frac{dm_{X,sys}}{dt} = (0.1 \text{ kg X/kg}) \left(\frac{\pi}{4} \text{ m}^2 \right) \times 1000 \text{ kg/m}^3 \times \frac{dh}{dt} = \dot{m}_{X,in} - \dot{m}_{X,out} = 4 - 0 = 4 \text{ kg/min}$$

or, rearranging and integrating,

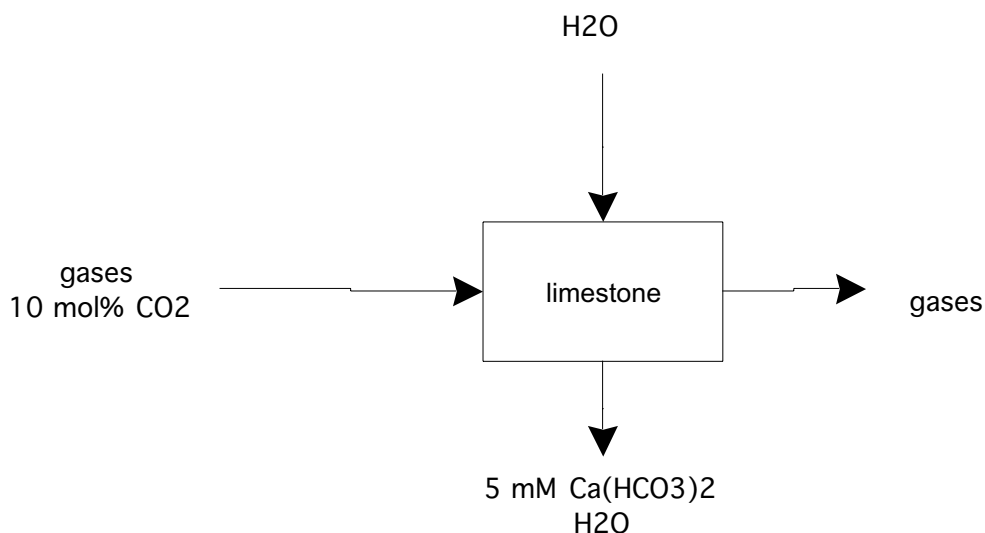
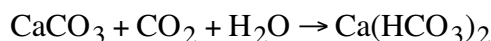
$$19.63 \int_1^{h_f} dh = \int_{19.6}^{40} dt = 20.4 \text{ min}$$

Solving, we find $h_f = 2.04$. The difference in height with and without the leak corresponds to the mass of material leaked onto the floor. We calculate this as

$$(2.04 - 1.975 \text{ m})(\pi/4)(1 \text{ m})^2(1000 \text{ kg/m}^3)(0.1 \text{ kg X/kg soln}) = 5.1 \text{ kg}$$

P3.42

The reaction between the limestone in the tank and the CO_2 in the gas and the liquid water is



Choosing as a basis 1 ton CO_2 per day, (0.0227 tonmoles CO_2 , at 44 tons/tonmol), and given the molar mass of CaCO_3 (100 tons/tonmole), we calculate that 2.27 tons CaCO_3 is required per ton CO_2 .

There is one tonmol calcium bicarbonate produced per tonmole CO_2 reacted, so the outlet flow rate of $\text{Ca}(\text{HCO}_3)_2$ is 0.0227 tonmoles per day. This is dissolved in water, and we assume that the calcium bicarbonate reaches its solubility limit (it can't go beyond that!), which is 5×10^{-3} gmoles/liter water. The flow rate of water required is therefore:

$$\frac{1 \text{ L water}}{5 \times 10^{-3} \text{ gmol Ca(HCO}_3)_2} \times \frac{908000 \text{ gmol}}{\text{tonmol}} \times 0.02273 \text{ tonmol Ca(HCO}_3)_2 \times \frac{1 \text{ kg water}}{\text{L water}} \times \frac{\text{ton}}{908 \text{ kg}} \\ = 4545 \text{ tons water/day}$$

Cost per ton CO₂ is

$$\text{Cost per ton CO}_2 = \frac{\$1.45}{\text{ton limestone}} \times 2.27 \text{ tons} + \frac{\$0.04}{\text{ton - km}} \times 2.27 \text{ tons} \times N_t (\text{km}) \\ + \frac{\$0.24}{1000 \text{ tons - meter}} \times 4545 \text{ tons water} \times N_p (\text{m}) \\ + \frac{\$0.06}{\text{ton - km}} \times 1 \text{ tons CO}_2 \times N_c (\text{km})$$

This cost must be \$90-\$180 to be competitive. Let's pick \$100 for examination:

$$100 = 3.29 + 0.091N_t + 1.09N_p + 0.06N_c$$

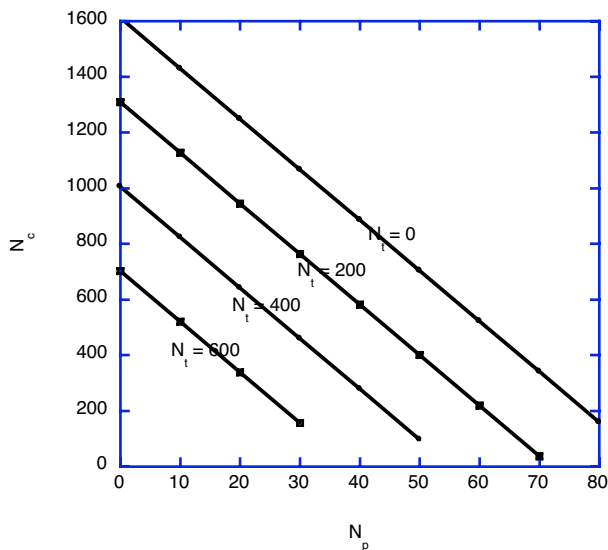
This is a linear equation in 3 variables. Some limiting cases:

If $N_p = N_c = 0$, maximum $N_t = 1063 \text{ km}$

If $N_t = N_c = 0$, maximum $N_p = 1612 \text{ km}$

If $N_t = N_c = 0$, maximum $N_p = 89 \text{ km}$

The plot below shows a series of solutions to the equation, and indicates possible combinations of the 3 variables that would meet the cost constraint.

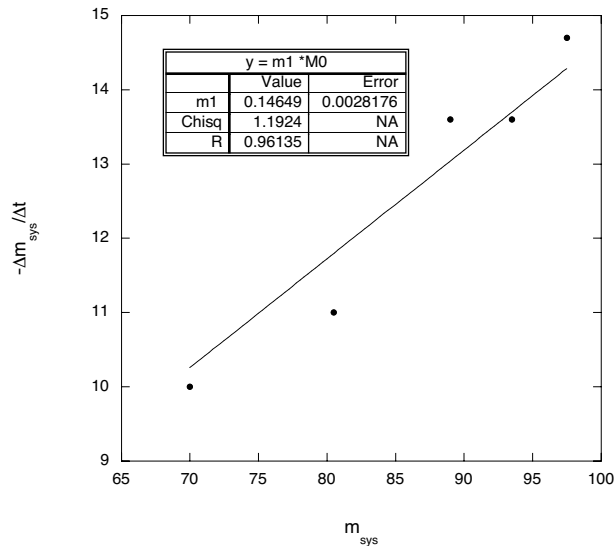


P3.43

The rate of release is simply calculated from the data at each interval as $\Delta m / \Delta t$. Total amount left in the particles is simply $100 - \text{mass released}$. These numbers are summarized in the table below

Time (hr)	Total mass in beaker (units)	Total protein left in system (units)	Interval no.	Rate of release (units/h)	Average mass left in system (units)
0	0	100			
			1	14.7	97.5
0.34	5	95			
			2	13.6	93.5
0.56	8	92			
			3	13.6	89
1.0	14	86			
			4	11	80.5
2.0	25	75			
			5	10	70
3.0	35	65			

The plot of the data is shown, along with a fit of the model equation (see below) to the data



We can model the rate of release simply from the material balance equation:

$$\frac{dm_{sys}}{dt} = -\dot{m}_{out}$$

Furthermore, from the plot we can see that there is a linear relationship between the rate of change of mass in the system and the average mass in the system, represented by the equation (obtained by linear regression fit to the data)

$$\frac{dm_{sys}}{dt} = -0.146m_{sys}$$

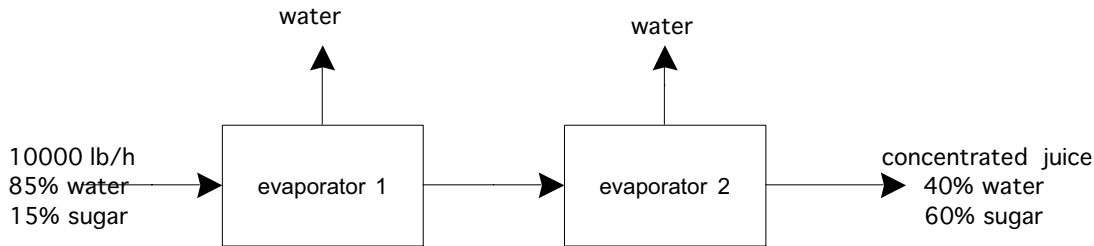
Integrating from $t = 0$ to t , with $m_{sys} = 100$ at $t = 0$, we find:

$$m_{sys} = 100e^{-0.146t}$$

If 90% of the protein C is released, then $m_{sys} = (1-0.9)100$ or 10 units. We use the above equation to find that this happens at $t = 15.8$ h.

P3.44

With two evaporators, the flow diagram is:



Let S = sugar, W = water, and denote the evaporated water stream as e1, e2, from evaporator 1 and evaporator 2, respectively and the juice as j1, j2.

First, balances using sugar as the component are simple:

$$0.15(10000) = 1500 = \dot{m}_{S,j1} = \dot{m}_{S,j2}$$

Since the juice leaving evaporator 2 is 60 wt% sugar and 40 wt% water, then

$$\dot{m}_{W,j2} = \frac{40}{60}(1500) = 1000$$

The fraction of water recovered in the evaporated stream is the same in each evaporator:

$$f_{Rw} = \frac{\dot{m}_{W,e1}}{0.85(10,000)} = \frac{\dot{m}_{W,e2}}{\dot{m}_{W,j1}}$$

or, considering the juice stream:

$$\dot{m}_{W,j1} = (1 - f_{R,w})0.85(10000)$$

$$\dot{m}_{W,j2} = (1 - f_{R,w})\dot{m}_{W,j1} = 1000$$

Combining

$$\dot{m}_{W,j1} = (1 - f_{R,w})8500$$

$$1000 = (1 - f_{R,w})^2 8500$$

We can solve to find that

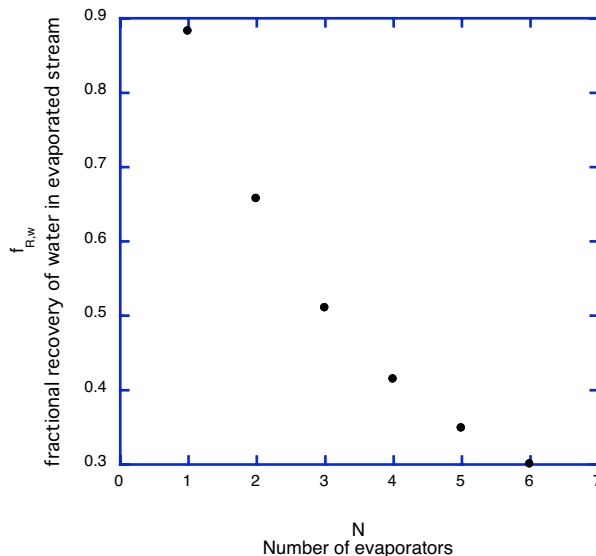
$$f_{R,w} = 0.657$$

For a system with N evaporators, each with the same fractional recovery of water in the evaporated water stream, the equation is:

$$1000 = (1 - f_{R,w})^N 8500$$

$$\text{or } f_{R,w} = 1 - \sqrt[N]{\frac{1000}{8500}}$$

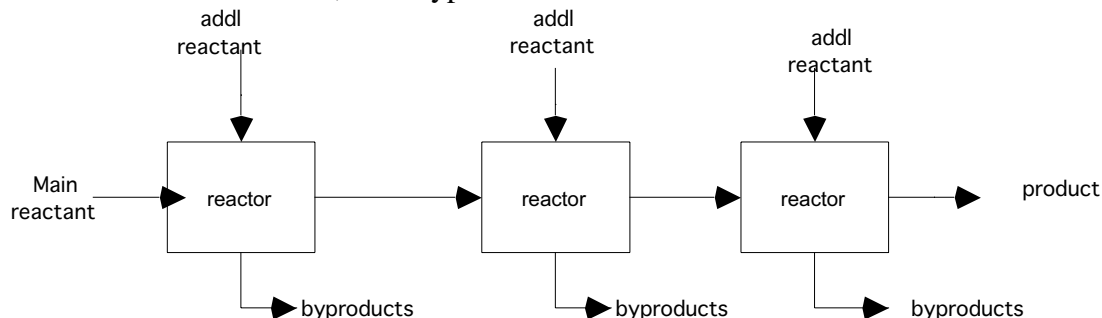
The plotted function is shown:



The job that each evaporator needs to do decreases as the number of evaporators increases. (I don't "connect the dots" because the number of evaporators is a discrete (integer) value, not a continuous function.)

P3.45

In a typical multistep synthesis, a main reactant is modified in a series of reactors by addition of other reactants, with byproducts removed.



If the fractional conversion in each reactor is f_c , then

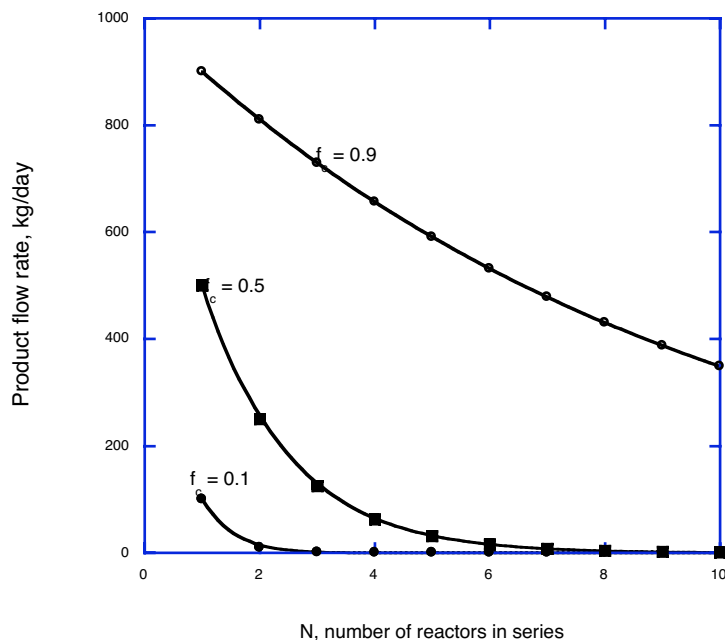
$$\dot{m}_{Rj,out} = f_C \dot{m}_{Rj,in}$$

where $\dot{m}_{Rj,out}$ is the mass flow of the main reactant out reactor R_j , and $\dot{m}_{Rj,in}$ is the mass flow of the main reactant into reactor R_j .

We write the above expression for each reactor in a series, and, recognizing that the flow out of reactor R_j equals the flow into reactor R_{j+1} , then

$$\dot{m}_{out} = f_C^N \dot{m}_{in}$$

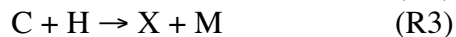
where N is the total number of reactors in series, and \dot{m}_{out} and \dot{m}_{in} are the flows out and in, respectively, of the entire process. For the case where the flow rate in is 1000 kg/day, we can plot the flow of product out as a function of fractional conversion and number of reactors:



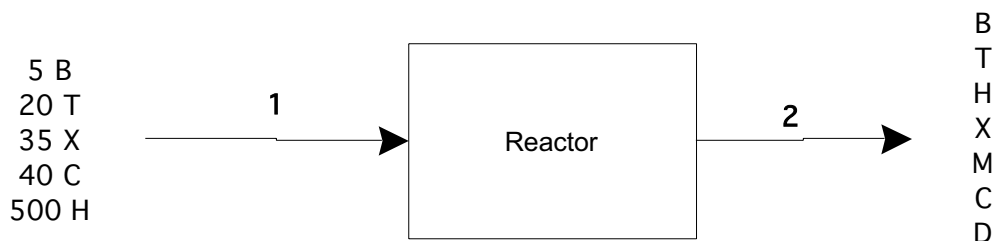
(The curves are drawn simply to aid the viewer.)

P3.46

In shorthand, using T for toluene, H for hydrogen, B for benzene, M for methane, X for xylene, C for pseudocumene, and D for diphenyl, the 4 reactions are;



The flow diagram is:



DOF analysis:

	No. of variables		No. of constraints
Stream variables	12	Specified flow	2
System variables	4 (reactions)	Specified compositions	3
		Specified performance	4 (conversions)

		Material balances	7
Total	16		16

DOF = 16 – 16 = 0. Problem is completely specified.

Using the template shown in Fig 3.6, we find the matrix equation to be:

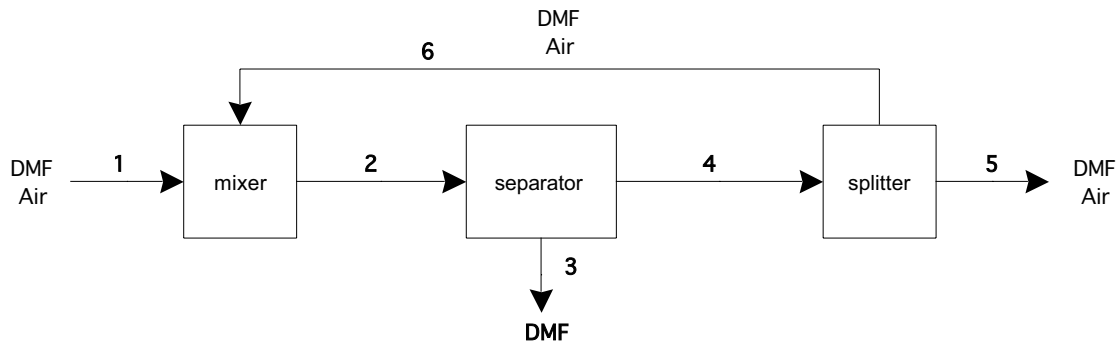
$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & -1 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & -1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
 \end{bmatrix}
 \begin{bmatrix}
 \dot{n}_{B2} \\
 \dot{n}_{T2} \\
 \dot{n}_{X2} \\
 \dot{n}_{C2} \\
 \dot{n}_{H2} \\
 \dot{n}_{M2} \\
 \dot{n}_{D2} \\
 \dot{\xi}_1 \\
 \dot{\xi}_2 \\
 \dot{\xi}_3 \\
 \dot{\xi}_4
 \end{bmatrix}
 =
 \begin{bmatrix}
 5 \\
 20 \\
 35 \\
 40 \\
 500 \\
 0 \\
 0 \\
 0.20 \times 5 \\
 0.75 \times 20 \\
 0.17 \times 35 \\
 0.70 \times 40
 \end{bmatrix}$$

(Notice that, for the lower right corner of matrix A, we write the stoichiometric coefficients for the 4 components, B, T, X and C, for which fractional conversion specifications are given. Hydrogen is not included in this list because its conversion is not specified.)

The solution to this linear model is:

$$\begin{bmatrix}
 \dot{n}_{B2} \\
 \dot{n}_{T2} \\
 \dot{n}_{X2} \\
 \dot{n}_{C2} \\
 \dot{n}_{H2} \\
 \dot{n}_{M2} \\
 \dot{n}_{D2} \\
 \dot{\xi}_1 \\
 \dot{\xi}_2 \\
 \dot{\xi}_3 \\
 \dot{\xi}_4
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 5 \\
 29.05 \\
 12 \\
 414.075 \\
 110.9 \\
 24.975 \\
 48.95 \\
 33.95 \\
 28 \\
 24.975
 \end{bmatrix}$$

P3.47



DOF analysis:

	No. of variables		No. of constraints
Stream variables	11	Specified flow	0
System variables	0	Specified compositions	1 (55% DMF in feed)
		Specified performance	0
		Splitter restriction	1
		Material balances	6
Total	11		8

$$\text{DOF} = 11 - 8 = 3.$$

Need to specify (1) a basis and (2) two other pieces of information, such as the exit gas mol% DMF the fractional split, and/or the fractional separation.

We'll use two components: air (A) (which we can treat as a single component) and DMF (D).

From a material balance with the entire process as the system, we find simply;

$$\dot{n}_{A1} = \dot{n}_{A5}$$

We'll first define fractional recovery f_{RD3} in the separator as the fraction of DMF fed to the separator that is recovered in stream 3 and fractional split f_{S6} in the splitter as the fraction of the feed to the splitter that leaves in stream 6.

Material balances:

$$\dot{n}_{D2} = \dot{n}_{D1} + \dot{n}_{D6} = \dot{n}_{D3} + \dot{n}_{D4}$$

$$\dot{n}_{D4} = \dot{n}_{D5} + \dot{n}_{D6}$$

We want to combine these with performance specifications to relate the DMF in the outlet stream (stream 5) to the DMF in the inlet stream (stream 1):

$$\dot{n}_{D5} = (1 - f_{S6})\dot{n}_{D4} = (1 - f_{S6})(1 - f_{RD3})\dot{n}_{D2} = (1 - f_{S6})(1 - f_{RD3})(\dot{n}_{D1} + \dot{n}_{D6})$$

We are almost there – we only need to get rid of \dot{n}_{D6} :

$$\dot{n}_{D6} = f_{S6}\dot{n}_{D4} = f_{S6}(1 - f_{RD3})\dot{n}_{D2} = f_{S6}(1 - f_{RD3})(\dot{n}_{D1} + \dot{n}_{D6})$$

Rearranging we find:

$$\dot{n}_{D6} = \frac{f_{S6}(1 - f_{RD3})}{1 - f_{S6}(1 - f_{RD3})} \dot{n}_{D1}$$

Substituting in we can now find the expression we want:

$$\dot{n}_{D5} = (1 - f_{S6})(1 - f_{RD3})(\dot{n}_{D1} + \dot{n}_{D6}) = (1 - f_{S6})(1 - f_{RD3}) \left[1 + \frac{f_{S6}(1 - f_{RD3})}{1 - f_{S6}(1 - f_{RD3})} \right] \dot{n}_{D1}$$

This provides a general formula (a linear model) relating the output stream to the feed as a function of the performance specifications for the process.

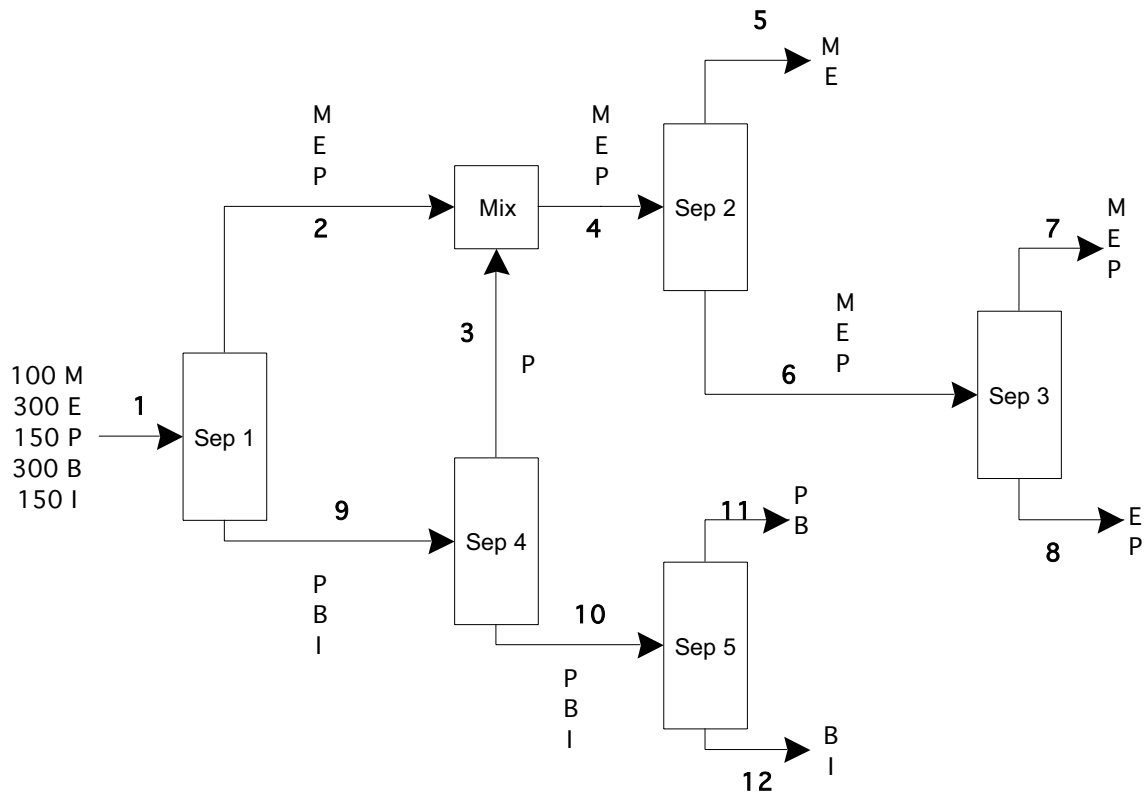
For the particular case of a feed stream containing 55 mol% DMF, let's choose a basis of 1000 gmol/h, so $\dot{n}_{A1} = 450$ gmol/h and $\dot{n}_{D1} = 550$ gmol/h. From the material balance equation for air, we know $\dot{n}_{A5} = 450$ gmol/h and we are given an outlet composition of 10% DMF, from which we calculate $\dot{n}_{D5} = 50$ gmol/h. Also, we are given $f_{RD3} = 0.67$. Plugging into our linear model of the process:

$$50 = (1 - f_{S6})(1 - 0.67) \left[1 + \frac{f_{S6}(1 - 0.67)}{1 - f_{S6}(1 - 0.67)} \right] 550$$

Solving, we find $f_{S6} = 0.8$.

P3.48

The flow diagram is a diverging tree.



There are 32 stream variables and no system variables. Specified information includes 1 flow, 4 compositions, and 7 system performance. There are 20 material balance equations. $\text{DOF} = 32 - 32 = 0$.

The linear model is simply the collection of these equations. We can write in matrix form, or simply as the list of 32 equations. (Flows in kgmol/h.)

Separator 1:

$$\dot{n}_{M1} = \dot{n}_{M2}$$

$$\dot{n}_{E1} = \dot{n}_{E2}$$

$$\dot{n}_{P1} = \dot{n}_{P2} + \dot{n}_{P9}$$

$$\dot{n}_{B1} = \dot{n}_{B9}$$

$$\dot{n}_{I1} = \dot{n}_{I9}$$

Separator 3:

$$\dot{n}_{M6} = \dot{n}_{M7}$$

$$\dot{n}_{E6} = \dot{n}_{E7} + \dot{n}_{E8}$$

$$\dot{n}_{P6} = \dot{n}_{P7} + \dot{n}_{P8}$$

Specified

flows/compositions:

Mixer:

$$\dot{n}_{M2} = \dot{n}_{M4}$$

$$\dot{n}_{E2} = \dot{n}_{E4}$$

$$\dot{n}_{P2} + \dot{n}_{P3} = \dot{n}_{P4}$$

Separator 4:

$$\dot{n}_{P9} = \dot{n}_{P3} + \dot{n}_{P10}$$

$$\dot{n}_{B9} = \dot{n}_{B10}$$

$$\dot{n}_{I9} = \dot{n}_{I10}$$

Specified performance:

Separator 2:

$$\dot{n}_{M4} = \dot{n}_{M5} + \dot{n}_{M6}$$

$$\dot{n}_{E4} = \dot{n}_{E5} + \dot{n}_{E6}$$

$$\dot{n}_{P4} = \dot{n}_{P6}$$

Separator 5:

$$\dot{n}_{P10} = \dot{n}_{P11}$$

$$\dot{n}_{B10} = \dot{n}_{B11} + \dot{n}_{B12}$$

$$\dot{n}_{I10} = \dot{n}_{I12}$$

$$\begin{aligned}
\dot{n}_{M1} &= 100 & \dot{n}_{P2} &= 0.446\dot{n}_{P1} \\
\dot{n}_{E1} &= 300 & \dot{n}_{M5} &= 0.995\dot{n}_{M4} \\
\dot{n}_{P1} &= 150 & \dot{n}_{E6} &= 0.9983\dot{n}_{E4} \\
\dot{n}_{B1} &= 300 & \dot{n}_{E7} &= 0.995\dot{n}_{E6} \\
\dot{n}_{I1} &= 150 & \dot{n}_{P8} &= 0.958\dot{n}_{P6} \\
& & \dot{n}_{P3} &= 0.964\dot{n}_{P9} \\
& & \dot{n}_{B11} &= 0.99\dot{n}_{B10}
\end{aligned}$$

The solution is summarized in a stream table:

	1	2	3	4	5	6	7	8	9	10	11	12
M	100	100		100	99.5	0.5	0.5					
E	300	300		300	0.5	299.5	298	1.5				
P	150	66.9	80.1	147		147	6.2	140.8	83.1	3	3	
B	300								300	300	297	3
I	150								150	150		150
sum	1000	466.9	80.1	547	100	447	304.7	142.3	533.1	453	300	153

The product streams are:

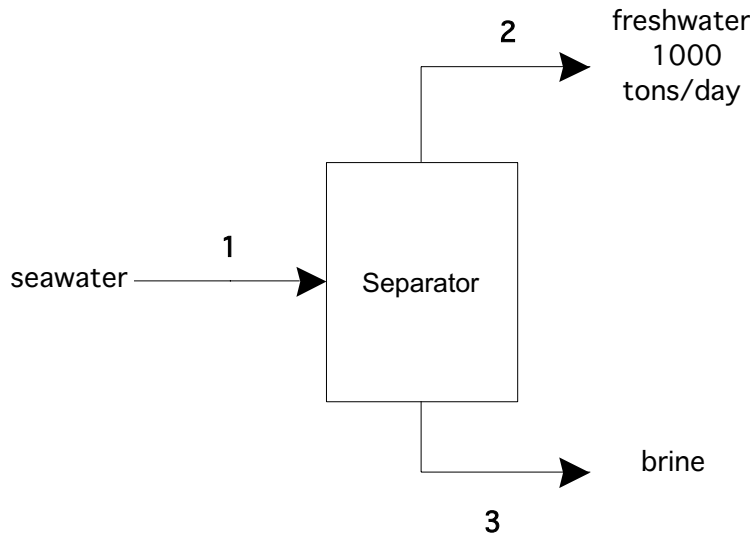
Stream 5: 100 kgmol/h: 99.5 mol% M, 0.5 mol% E

Stream 7: 304.7 kgmol/h; 0.16 mol% M, 97.8 mol% E, 2.03 mol% P

Stream 8: 142.3 kgmol/h; 1.1 mol% E, 98.9 mol% P

Stream 11: 300 kgmol/h; 1 mol% P, 99 mol% B

Stream 12: 153 kgmol/h; 2 mol% B, 98 mol% I

P3.49

All 3 streams may contain salt (S) and water (W).

The material balance equations are:

$$\dot{m}_{S1} = \dot{m}_{S2} + \dot{m}_{S3}$$

$$\dot{m}_{W1} = \dot{m}_{W2} + \dot{m}_{W3}$$

The specified flow is the freshwater production rate:

$$\dot{m}_{S2} + \dot{m}_{W2} = 1000$$

The specified system performance is:

$$\frac{\dot{m}_{W2}}{\dot{m}_{W1}} = 0.3$$

There are 6 stream variables but only 4 equations. Two more specifications are required.

(a)

For this case, the sea water composition is fixed:

$$\frac{\dot{m}_{S1}}{\dot{m}_{W1}} = \frac{3.5}{96.5}$$

At 10,000 ppm salt in the freshwater product,

$$\frac{\dot{m}_{S2}}{\dot{m}_{S2} + \dot{m}_{W2}} = \frac{10,000}{1,000,000} = 0.01$$

Combined with the freshwater flow specification:

$$\dot{m}_{S2} = 10 \text{ tons/day, and } \dot{m}_{W2} = 990 \text{ tons/day.}$$

Combined with the fractional recovery specification:

$$\dot{m}_{W1} = 3300 \text{ tons/day}$$

From this number and the salt concentration, we find: $\dot{m}_{S1} = 119.7$ tons/day and the total seawater feed rate is 3420 tons/day (rounding off). From material balance, we find that the briny byproduct flow rate is 2420 tons/day and it contains 4.5 wt% salt.

If the freshwater contains 0 ppm salt, similar calculations show that we would feed 3454 tons/day seawater, and produce 2455 tons/day of brine at 4.9 wt% salt.

(b) For this case, we fix the freshwater salt content at 1000 ppm (or 1 ton salt/day).

If the feed contains 1 wt% salt, the seawater feed rate is 3364 tons/day, the briny byproduct is 2364 tons/day and contains 1.4 wt% salt.

If the feed contains 10 wt% salt, the seawater feed rate is 3700 tons/day, the briny byproduct is 2700 tons/day and contains 13.7 wt% salt.

We summarize our results in table form:

%salt in feed	Ppm salt in freshwater	Seawater feed rate	Brine flow rate	%salt in brine
3.5	0	3454	2455	4.9
3.5	10000	3420	2420	4.5
1	1000	3364	2364	1.4
10	1000	3700	2700	13.7

The % salt in the feed has a much bigger effect on the process flow calculations than does the freshwater quality. Therefore, it would be a fairly reasonable approximation to assume that the freshwater has no salt in it, for the purposes of calculating briny byproduct and seawater feed rate required.