Chapter 11 • Turbomachinery

11.1 Describe the geometry and operation of a human peristaltic PDP which is cherished by every romantic person on earth. How do the two ventricles differ?

Solution: Clearly we are speaking of the *human heart*, driven periodically by travelling compression of the heart walls. One ventricle serves the brain and the rest of one's extremities, while the other ventricle serves the lungs and promotes oxygenation of the blood. *Ans.*

11.2 What would be the technical classification of the following turbomachines:

(a) a household fan = **an axial flow fan**. *Ans.* (a)
(b) a windmill = **an axial flow turbine**. *Ans.* (b)
(c) an aircraft propeller = **an axial flow fan**. *Ans.* (c)
(d) a fuel pump in a car = **a positive displacement pump (PDP)**. *Ans.* (d)
(e) an eductor = **a liquid-jet-pump** (special purpose). *Ans.* (e)
(f) a fluid coupling transmission = **a double-impeller energy transmission device**. *Ans.* (f)
(g) a power plant steam turbine = **an axial flow turbine**. *Ans.* (g)

11.3 A PDP can deliver almost any fluid, but there is always a limiting very-high *viscosity* for which performance will deteriorate. Can you explain the probable reason?

Solution: High-viscosity fluids take a long time to enter and fill the inlet cavity of a PDP. Thus a PDP pumping high-viscosity liquid should be run slowly to ensure filling. *Ans.*

11.4 An interesting turbomachine is the *torque converter* [58], which combines both a pump and a turbine to change torque between two shafts. Do some research on this concept and describe it, with a report, sketches and performance data, to the class.

Solution: As described, for example, in Ref. 58, the torque converter transfers torque T from a pump runner to a turbine runner such that $\omega_{pump} T_{pump} \approx \omega_{turbine} T_{turbine}$. Maximum efficiency occurs when the turbine speed $\omega_{turbine}$ is approximately one-half of the pump speed ω_{pump}. *Ans.*

11.5 What type of pump is shown in Fig. P11.5? How does it operate?

Solution: This is a *flexible-liner* pump. The rotating eccentric cylinder acts as a "squeegee." *Ans.*

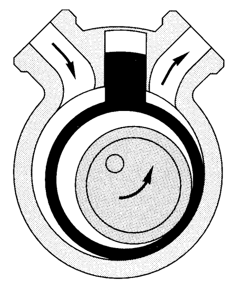


Fig. P11.5

11.6 Fig. P11.6 shows two points a half-period apart in the operation of a pump. What type of pump is this? How does it work? Sketch your best guess of flow rate versus time for a few cycles.

Solution: This is a **diaphragm** pump. As the center rod moves to the right, opening A and closing B, the check valves allow A to fill and B to discharge. Then, when the rod moves to the left, B fills and A discharges. Depending upon the exact oscillatory motion of the center rod, the flow rate is fairly steady, being higher when the rod is faster. *Ans.*

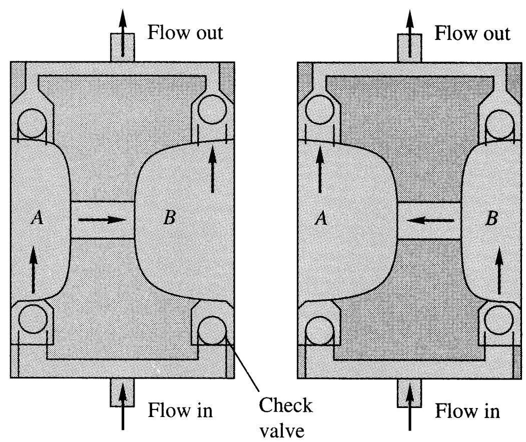


Fig. P11.6

11.7 A piston PDP has a 5-in diameter and a 2-in stroke and operates at 750 rpm with 92% volumetric efficiency. (a) What is the delivery, in gal/min? (b) If the pump delivers SAE 10W oil at 20°C against a head of 50 ft, what horsepower is required when the overall efficiency is 84%?

Solution: For SAE 10W oil, take $\rho \approx 870\ \text{kg/m}^3 \approx 1.69\ \text{slug/ft}^3$. The volume displaced is

$$\upsilon = \frac{\pi}{4}(5)^2(2) = 39.3\ \text{in}^3,$$

$$\therefore\quad Q = \left(39.3\ \frac{\text{in}^3}{\text{stroke}}\right)\left(\frac{1\ \text{gal}}{231\ \text{in}^3}\right)\left(750\ \frac{\text{strokes}}{\text{min}}\right)(0.92\ \text{efficiency})$$

or: $Q \approx \mathbf{117\ gal/min}$ *Ans.* (a)

$$\text{Power} = \frac{\rho g Q H}{\eta} = \frac{1.69(32.2)\left(\frac{117}{449}\ \text{ft}^3/\text{s}\right)(50)}{0.84} = 846\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \div 550 \approx \mathbf{1.54\ hp} \quad \textit{Ans.}\ \text{(b)}$$

11.8 A centrifugal pump delivers 550 gal/min of water at 20°C when the brake horsepower is 22 and the efficiency is 71%. (a) Estimate the head rise in ft and the pressure rise in psi. (b) Also estimate the head rise and horsepower if instead the delivery is 550 gal/min of gasoline at 20°C.

Solution: (a) For water at 20°C, take $\rho \approx 998\ \text{kg/m}^3 \approx 1.94\ \text{slug/ft}^3$. The power relation is

$$\text{Power} = 22(550) = 12100\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} = \frac{\rho g Q H}{\eta} = \frac{(62.4)\left(\frac{550}{449}\ \frac{\text{ft}^3}{\text{s}}\right)H}{0.71},$$

$$\text{or} \quad H \approx \mathbf{112\ ft} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Pressure rise}\ \ \Delta p = \rho g H = (62.4)(112) = 7011\ \text{psf} \div 144 \approx \mathbf{49\ psi} \quad \textit{Ans.}\ \text{(a)}$$

(b) For gasoline at 20°C, take $\rho \approx 680\ \text{kg/m}^3 \approx 1.32\ \text{slug/ft}^3$. If viscosity (Reynolds number) is not important, the operating conditions (flow rate, impeller size and speed) are exactly the same and hence the head is the same and the power scales with the density:

$$H \approx \mathbf{112\ ft}\ (\text{of}\ \textit{gasoline}); \quad \text{Power} = P_{\text{water}} \frac{\rho_{\text{gasoline}}}{\rho_{\text{water}}} = 22\left(\frac{680}{998}\right) \approx \mathbf{15\ hp} \quad \textit{Ans.}\ \text{(b)}$$

11.9 Figure P11.9 shows the measured performance of the Vickers Inc. Model PVQ40 piston pump when delivering SAE 10W oil at 180°F ($\rho \approx 910\ \text{kg/m}^3$). Make some general observations about these data vis-à-vis Fig. 11.2 and your intuition about PDP behavior.

Solution: The following are observed:
(a) The discharge Q is almost linearly proportional to speed Ω and slightly less for the higher heads (H or Δp).
(b) The efficiency (volumetric or overall) is nearly independent of speed Ω and again slightly less for high Δp.
(c) The power required is linearly proportional to the speed Ω and also to the head H (or Δp). *Ans.*

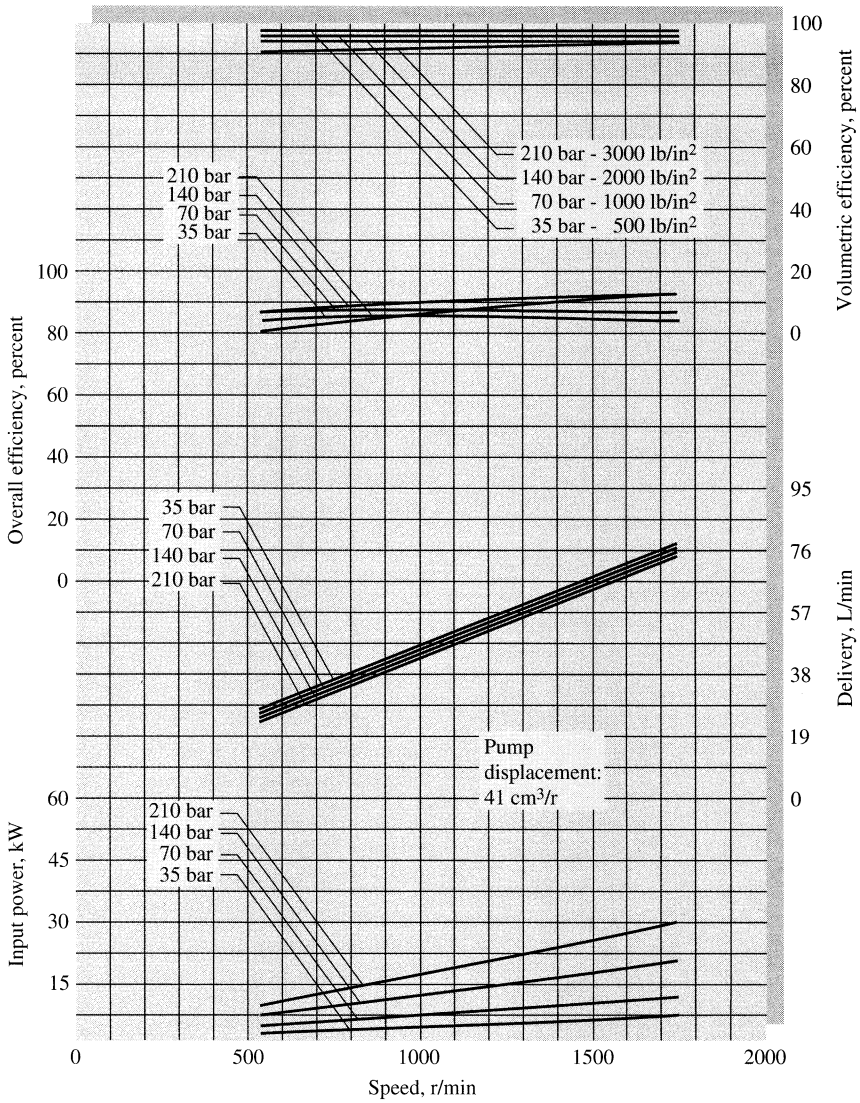


Fig. P11.9

11.10 Suppose that the pump of Fig. P11.9 is run at 1100 r/min against a pressure rise of 210 bar. (a) Using the measured displacement, estimate the theoretical delivery in gal/min. From the chart, estimate (b) the actual delivery; and (c) the overall efficiency.

Solution: (a) From Fig. P11.9, the pump displacement is 41 cm^3. The theoretical delivery is

$$Q = \left(1100\ \frac{\text{r}}{\text{min}}\right)\left(41\ \frac{\text{cm}^3}{\text{r}}\right) = 45100\ \frac{\text{cm}^3}{\text{min}} = 45\ \frac{\text{L}}{\text{min}} = \mathbf{11.9\ \frac{gal}{min}} \quad \textit{Ans.}\ \text{(a)}$$

(b) From Fig. P11.9, at 1100 r/min and Δp = 210 bar, read

$$Q \approx 47 \text{ L/min} \approx \mathbf{12\ gal/min}. \quad \textit{Ans.}\text{ (b)}$$

(c) From Fig. P11.9, at 1100 r/min and Δp = 210 bar, read $\eta_{\mathbf{overall}} \approx \mathbf{87\%}$. *Ans*. (c)

11.11 A pump delivers 1500 L/min of water at 20°C against a pressure rise of 270 kPa. Kinetic and potential energy changes are negligible. If the driving motor supplies 9 kW, what is the overall efficiency?

Solution: With pressure rise given, we don't need density. Compute "water" power:

$$P_{water} = \rho g Q H = Q\,\Delta p = \left(\frac{1.5}{60}\ \frac{\text{m}^3}{\text{s}}\right)\left(270\ \frac{\text{kN}}{\text{m}^2}\right) = 6.75 \text{ kW}, \quad \therefore\ \eta = \frac{6.75}{9.0} = \mathbf{75\%} \quad \textit{Ans.}$$

11.12 In a test of the pump in the figure, the data are: p_1 = 100 mmHg (vacuum), p_2 = 500 mmHg (gage), D_1 = 12 cm, and D_2 = 5 cm. The flow rate is 180 gal/min of light oil (SG = 0.91). Estimate (a) the head developed; and (b) the input power at 75% efficiency.

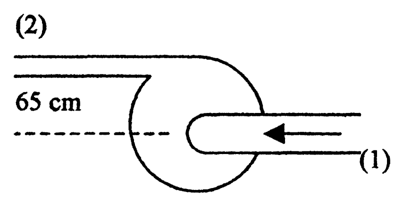


Fig. P11.12

Solution: Convert 100 mmHg = 13332 Pa, 500 mmHg = 66661 Pa, 180 gal/min = 0.01136 m^3/s. Compute $V_1 = Q/A_1 = 0.01136/[(\pi/4)(0.12)^2] = 1.00$ m/s. Also, $V_2 = Q/A_2$ = 5.79 m/s. Calculate $\gamma_{oil} = 0.91(9790) = 8909$ N/m^3. Then the head is

$$H = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - \frac{p_1}{\gamma} - \frac{V_1^2}{2g} - z_1$$

$$= \frac{66661}{8909} + \frac{(5.79)^2}{2(9.81)} + 0.65 - \frac{-13332}{8909} - \frac{(1.00)^2}{2(9.81)} - 0, \quad \text{or:} \quad \mathbf{H = 11.3\ m} \quad \textit{Ans.}\text{ (a)}$$

$$Power = \frac{\gamma Q H}{\eta} = \frac{8909(0.01136)(11.3)}{0.75} = \mathbf{1520\ W} \quad \textit{Ans.}\text{ (b)}$$

11.13 A 20-hp pump delivers 400 gal/min of gasoline at 20°C with 80% efficiency. What head and pressure rise result across the pump?

Solution: For gasoline at 20°C, take $\rho \approx 680\ \text{kg/m}^3 \approx 1.32\ \text{slug/ft}^3$. Compute the power

$$P = 20\times 550 = 11000\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} = \frac{\rho g Q H}{\eta} = \frac{1.32(32.2)\left(\frac{400}{449}\right)H}{0.80},\quad \text{solve } H \approx \mathbf{232\ ft} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Then}\quad \Delta p = \rho g H = 1.32(32.2)(232) = 9870\ \text{psf} \div 144 \approx \mathbf{69\ psi} \quad \textit{Ans.}\ \text{(b)}$$

11.14 A pump delivers gasoline at 20°C and 12 m^3/h. At the inlet, p_1 = 100 kPa, z_1 = 1 m, and V_1 = 2 m/s. At the exit p_2 = 500 kPa, z_2 = 4 m, and V_2 = 3 m/s. How much power is required if the motor efficiency is 75%?

Solution: For gasoline, take $\rho g \approx 680(9.81) = 6671\ \text{N/m}^3$. Compute head and power:

$$H = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 - \frac{p_1}{\rho g} - \frac{V_1^2}{2g} - z_1 = \frac{500000}{6671} + \frac{(3)^2}{2(9.81)} + 4 - \frac{100000}{6671} - \frac{(2)^2}{2(9.81)} - 1,$$

$$\text{or:}\quad H \approx 63.2\ \text{m},\quad \text{Power} = \frac{\rho g Q H}{\eta} = \frac{6671\left(\frac{12}{3600}\right)(63.2)}{0.75} \approx \mathbf{1870\ W} \quad \textit{Ans.}$$

11.15 A lawn sprinkler can be used as a simple turbine. As shown in Fig. P11.15, flow enters normal to the paper in the center and splits evenly into $Q/2$ and V_{rel} leaving each nozzle. The arms rotate at angular velocity ω and do work on a shaft. Draw the velocity diagram for this turbine. Neglecting friction, find an expression for the power delivered to the shaft. Find the rotation rate for which the power is a maximum.

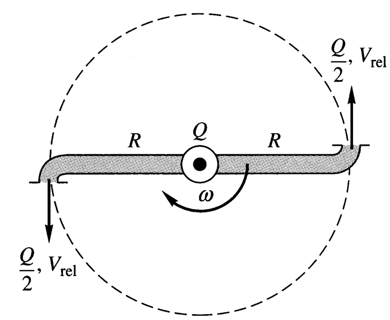
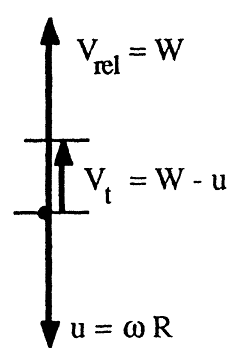


Fig. P11.15

Solution: Utilizing the velocity diagram at right, we apply the Euler turbine formula:



$$P = \rho Q(u_2 V_{t2} - u_1 V_{t1}) = \rho Q[u(W-u) - 0]$$

$$\text{or:}\quad \mathbf{P = \rho Q \omega R(V_{rel} - \omega R)} \quad \textit{Ans.}$$

$$\frac{dP}{du} = \rho Q(V_{rel} - 2u) = 0 \quad \text{if} \quad \omega = \frac{\mathbf{V_{rel}}}{\mathbf{2R}} \quad \textit{Ans.}$$

$$\text{where}\quad P_{max} = \rho Q u(2u - u) = \rho Q(\omega R)^2$$

11.16 For the "sprinkler turbine'' of Fig. P11.15, let R = 18 cm, with total flow rate of 14 m^3/h of water at 20°C. If the nozzle exit diameter is 8 mm, estimate (a) the maximum power delivered in W and (b) the appropriate rotation rate in r/min.

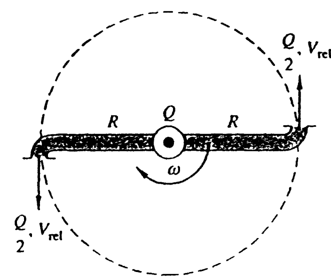


Fig. P11.15

Solution: For water at 20°C, take $\rho \approx$ 998 kg/m^3. Each arm takes 7 m^3/h:

$$V_{rel} = \frac{Q/2}{A_{exit}} = \frac{7/3600}{(\pi/4)(0.008)^2} = 38.7\ \frac{m}{s};\quad \text{at max power,}$$

$$u = \omega R = \frac{1}{2}V_{rel} = 19.34\ \frac{m}{s} = \omega(0.18\text{ m}),\quad \text{solve } \omega = 107\ \frac{rad}{s} \approx \mathbf{1030\ rpm} \quad \textit{Ans.}\text{ (b)}$$

$$P_{max} = \rho Q u^2 = 998(14/3600)(19.34)^2 \approx \mathbf{1450\ W} \quad \textit{Ans.}\text{ (a)}$$

11.17 A centrifugal pump has d_1 = 7 in, d_2 = 13 in, b_1 = 4 in, b_2 = 3 in, β_1 = 25°, and β_2 = 40° and rotates at 1160 r/min. If the fluid is gasoline at 20°C and the flow enters the blades radially, estimate the theoretical (a) flow rate in gal/min, (b) horsepower, and (c) head in ft.

Solution: For gasoline, take $\rho \approx$ 1.32 slug/ft^3. Compute ω = 1160 rpm = 121.5 rad/s.

$$u_1 = \omega r_1 = 121\left(\frac{3.5}{12}\right) \approx 35.4 \text{ ft/s}$$

$$V_{n1} = u_1 \tan\beta_1 = 35.4 \tan 25° \approx 16.5 \text{ ft/s}$$

$$Q = 2\pi r_1 b_1 V_{n1} = 2\pi\left(\frac{3.5}{12}\right)\left(\frac{4}{12}\right)(16.5)$$

$$\approx \mathbf{10\ \frac{ft^3}{s}} \quad \textit{Ans.}\text{ (a)}$$

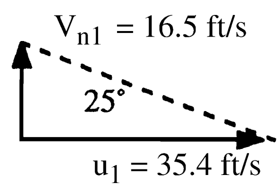
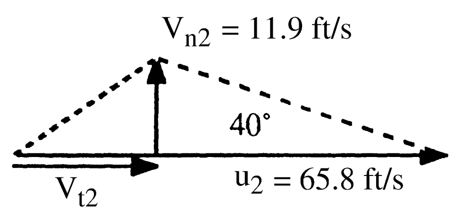


Fig. P11.17

$$V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{10.0}{2\pi\left(\frac{6.5}{12}\right)\left(\frac{3}{12}\right)} \approx 11.9\ \frac{ft}{s}$$

$$u_2 = \omega r_2 = 121(6.5/12) \approx 65.8 \text{ ft/s}$$

$$V_{t2} = u_2 - V_{n2}\cot 40° \approx 51.7 \text{ ft/s}$$



Finally, $\mathbf{P_{ideal}} = \rho Q u_2 V_{t2} = 1.32(10.0)(65.8)(51.7) = 44900 \div 550 \approx \mathbf{82\ hp}$. *Ans.* (b)
Theoretical head $\mathbf{H} = P/(\rho g Q) = 44900/[1.32(32.2)(10.0)] \approx \mathbf{106\ ft}$. *Ans.* (c)

11.18 A jet of velocity V strikes a vane which moves to the right at speed V_c, as in Fig. P11.18. The vane has a turning angle θ. Derive an expression for the power delivered to the vane by the jet. For what vane speed is the power maximum?

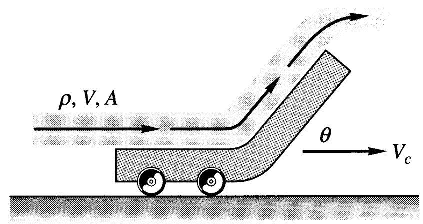


Fig. P11.18

Solution: The jet approaches the vane at relative velocity $(V - V_c)$. Then the force is

$$F = \rho A(V - V_c)^2(1 - \cos\theta),\ \text{and Power} = FV_c = \boldsymbol{\rho A V_c (V - V_c)^2 (1 - \cos\theta)} \quad \textit{Ans. (a)}$$

Maximum power occurs when $\dfrac{dP}{dV_c} = 0$,

$$\text{or:}\quad \mathbf{V_c = \frac{1}{3} V_{jet}} \quad \textit{Ans. (b)} \quad \left(P = \frac{4}{27}\rho A V^3[1 - \cos\theta]\right)$$

11.19 A centrifugal water pump has $r_2 = 9$ in, $b_2 = 2$ in, and $\beta_2 = 35°$ and rotates at 1060 r/min. If it generates a head of 180 ft, determine the theoretical (a) flow rate in gal/min and (b) horsepower. Assume near-radial entry flow.

Solution: For water take $\rho = 1.94$ slug/ft^3. Convert $\omega = 1060$ rpm $= 111$ rad/s. Then

$$u_2 = \omega r_2 = 111\left(\frac{9}{12}\right) = 83.3\ \frac{ft}{s};$$

$$\text{Power} = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right),\quad \text{and}\quad H = \frac{P}{\rho g Q} = 180\ \text{ft}$$

$$\text{or:}\quad P = 62.4QH = 1.94Q(83.3)\left[83.3 - \frac{Q}{2\pi(9/12)(2/12)}\cot 35°\right] \quad \text{with } H = 180$$

Solve for $\mathbf{Q} = 7.5\ \text{ft}^3/\text{s} \approx \mathbf{3360\ gal/min}$ *Ans.* (a)

With Q and H known, $\mathbf{P} = \rho g Q H = 62.4(7.5)(180) \div 550 \approx \mathbf{153\ hp}$. *Ans.* (b)

11.20 Suppose that Prob. 11.19 is reversed into a statement of the theoretical power P = 153 hp. Can you then compute the theoretical (a) flow rate; and (b) head? Explain and resolve the difficulty which arises.

Solution: With power known, the basic theory becomes quadratic in flow rate:

$$u_2 = 83.3\ \frac{ft}{s}, \quad P = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)$$

$$= 1.94Q(83.3)[83.3 - 1.818Q] = 153 \times 550\ \frac{ft\cdot lbf}{s}$$

$$\text{Clean up: } Q^2 - 45.8Q + 287 = 0, \quad \text{two roots: } Q_1 = \mathbf{7.5}\ \frac{\mathbf{ft^3}}{\mathbf{s}}; \quad Q_2 = \mathbf{38.3}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{These correspond to } H_1 = \mathbf{180\ ft}; \quad H_2 = \mathbf{35\ ft} \quad \textit{Ans.}\text{ (b)}$$

So the ideal pump theory admits to <u>two</u> valid combinations of Q and H which, for the given geometry and speed, give the theoretical power of 153 hp. Prob. 11.19 was solution **1**.

11.21 The centrifugal pump of Fig. P11.21 develops a flow rate of 4200 gpm with gasoline at 20°C and near-radial absolute inflow. Estimate the theoretical (a) horsepower; (b) head rise; and (c) appropriate blade angle at the inner radius.

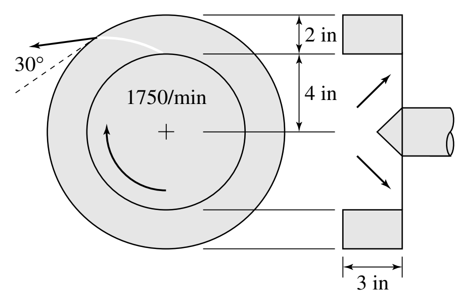


Fig. P11.21

Solution: For gasoline take $\rho \approx 1.32$ slug/ft^3. Convert Q = 4200 gal/min = 9.36 ft^3/s and ω = 1750 rpm = 183 rad/s. Note r_2 = 6 in and β_2 = 30°. The ideal power is computed as

$$P = \rho Q u_2\left(u_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right), \quad \text{where } u_2 = \omega r_2 = 183\left(\frac{6}{12}\right) \approx 91.6\text{ ft/s.} \quad \text{Plug in:}$$

$$P = 1.32(9.36)(91.6)\left[91.6 - \frac{9.36}{2\pi(6/12)(3/12)}\cot 30°\right] = 80400 \div 550 \approx \mathbf{146\ hp} \quad \textit{Ans.}\text{ (a)}$$

$$H = \frac{P}{\rho g Q} = \frac{80400}{1.32(32.2)(9.36)} \approx \mathbf{202\ ft} \quad \textit{Ans.}\text{ (b)}$$

Compute $V_{n1} = Q/[2\pi r_1 b_1] = 9.36/[2\pi(4/12)(3/12)] \approx 17.9$ ft/s, $u_1 = \omega r_1 = 183(4/12) \approx 61.1$ ft/s. For purely radial inflow, $\beta_1 = \tan^{-1}(17.9/61.1) \approx \mathbf{16°}$. *Ans.* (c)

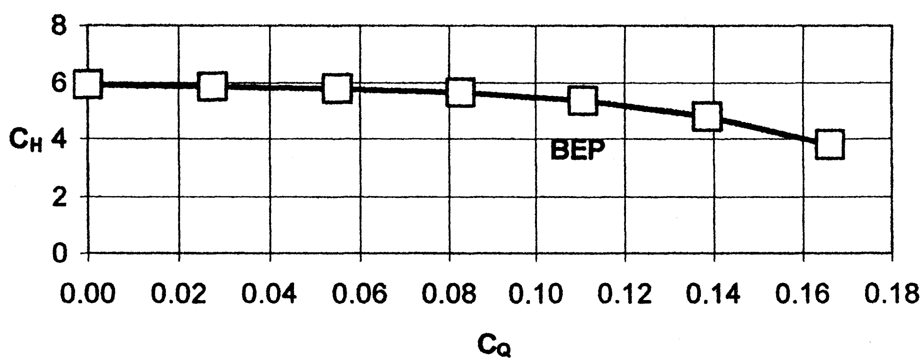
11.22 A 37-cm-diameter centrifugal pump, running at 2140 rev/min with water at 20°C produces the following performance data:

Q, m^3/s:	0.0	0.05	0.10	0.15	0.20	0.25	0.30
H, m:	105	104	102	100	95	85	67
P, kW:	100	115	135	171	202	228	249
η:	**0%**	**44%**	**74%**	**86%**	**92%**	**91%**	**79%**

(a) Determine the best efficiency point. (b) Plot C_H versus C_Q. (c) If we desire to use this same pump family to deliver 7000 gal/min of kerosene at 20°C at an input power of 400 kW, what pump speed (in rev/min) and impeller size (in cm) are needed? What head will be developed?

Solution: Efficiencies, computed by $\eta = \rho gQH/\text{Power}$, are listed above. The best efficiency point (BEP) is approximately **92% at Q = 0.2 m^3/s**. *Ans.* (a)

The dimensionless coefficients are $C_Q = Q/(nD^3)$, where $n = 2160/60 = 36$ rev/s and $D = 0.37$ m, plus $C_H = gH/(n^2D^2)$ and $C_P = P/(\rho n^3D^5)$, where $\rho_{water} = 998$ kg/m^3. BEP values are $C_Q^* = 0.111$, $C_H^* = 5.35$, and $C_P^* = 0.643$. **A plot of C_H versus C_Q is below**. The values are similar to Fig. 11.8 of the text. *Ans.* (b)



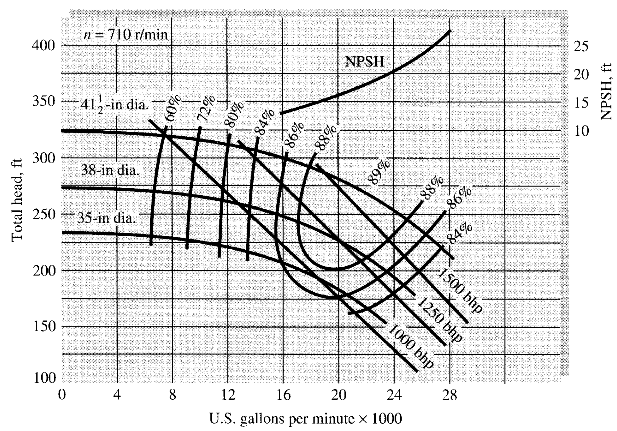
(c) For kerosene, $\rho_k = 804$ kg/m^3. Convert 7000 gal/min = 0.442 m^3/s. At BEP, we require the above values of dimensionless parameters:

$$\frac{Q}{nD^3} = \frac{0.442}{nD^3} = 0.111; \quad \frac{P}{\rho n^3 D^5} = \frac{400000}{804 n^3 D^5} = 0.643;$$

$$\text{Solve} \quad n = 26.1\ \frac{\text{rev}}{\text{s}} = \mathbf{1560}\ \frac{\textbf{rev}}{\textbf{min}}; \quad D = \mathbf{0.534\ m} \quad \textit{Ans.}\ \text{(c)}$$

$$\text{Also,} \quad H^* = C_H^*(n^2D^2)/g = 5.35(26.1)^2(0.534)^2/9.81 = \mathbf{106\ m} \quad \textit{Ans.}\ \text{(c)}$$

11.23 If the 38-in pump from Fig. 11.7(b) is used to deliver 20°C kerosene, at 850 rpm and 22000 gal/min, what (a) head; and (b) brake horsepower will result?



(b)

Fig. 11.7b

Solution: For kerosene, take ρ = 1.56 slug/ft^3 and for water ρ = 1.56 slug/ft^3. Use the scaling laws, Eq. (11.28):

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2}\left(\frac{D_1}{D_2}\right)^3 = \frac{Q_1}{22000} = \frac{710}{850}, \quad \therefore\ Q_1 \approx 18400\ \text{gpm}$$

$$\text{Read} \quad H_1 \approx 235\ \text{ft} \quad \text{and} \quad P_1 \approx 1175\ \text{bhp}$$

$$D_1 = D_2\text{:}\ H_2 = 235\left(\frac{850}{710}\right)^2 \approx \mathbf{340\ ft} \quad \textit{Ans.}\ \text{(a)}$$

$$P_2 = P_1(\rho_2/\rho_1)(n_2/n_1)^3 = 1175(1.56/1.94)(850/710)^3 \approx \mathbf{1600\ bhp} \quad \textit{Ans.}\ \text{(b)}$$

11.24 Figure P11.24 shows performance data for the Taco, Inc., model 4013 pump. Compute the ratios of measured shutoff head to the ideal value U^2/g for all seven impeller sizes. Determine the average and standard deviation of this ratio and compare it to the average for the six impellers in Fig. 11.7.

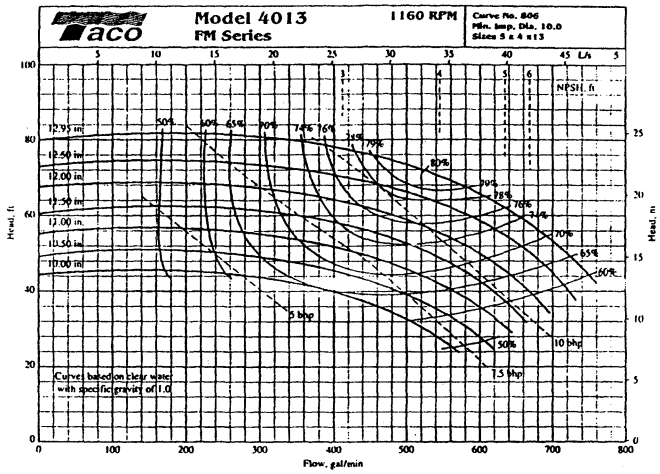


Fig. P11.24 Performance data for a centrifugal pump. (*Courtesy of Taco. Inc., Cranston, Rhode Island.*)

Solution: All seven pumps are run at 1160 rpm = 121 rad/s. The 7 diameters are given. Thus we can easily compute $U = \omega r = \omega D/2$ and construct the following table:

D, inches:	10.0	10.5	11.0	11.5	12.0	12.5	12.95
U, ft/s:	50.6	53.1	55.7	58.2	60.7	63.3	65.5
H_o, ft:	44	49	53	61	67	73	80
$H_o/(U^2/g)$:	**0.553**	**0.559**	**0.551**	**0.580**	**0.585**	**0.587**	**0.599**

The average ratio is **0.573** *Ans.* (a), and the standard deviation is **0.019**. *Ans.* (b)

For the six pumps of Fig. 11.7, the average is *0.611*, the standard deviation is *0.025*. These results are true of most centrifugal pumps: we can take an average of *0.60 ± 0.04*.

11.25 At what speed in rpm should the 35-in-diameter pump of Fig. 11.7(b) be run to produce a head of 400 ft at a discharge of 20000 gal/min? What brake horsepower will be required? *Hint*: Fit H(Q) to a formula.

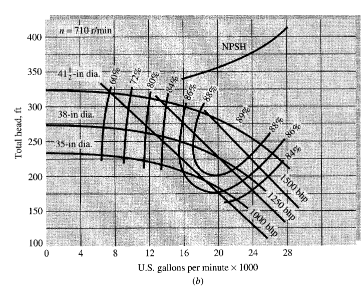


Fig. 11.7b

Solution: A curve-fit formula for H(Q) for this pump is $H(ft) \approx 235 - 0.125Q^2$, with Q in kgal/min. Then, for constant diameter, the similarity rules predict

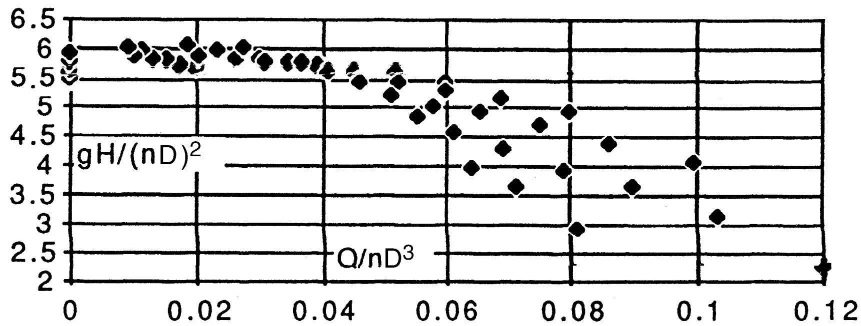
$$H_1 = H_2\left(\frac{n_1}{n_2}\right)^2 \quad \text{and} \quad Q_1 = Q_2\left(\frac{n_1}{n_2}\right), \quad \text{or:} \quad H_1 = 400\left(\frac{710}{n_2}\right)^2 \approx 235 - 0.125\left[20\left(\frac{710}{n_2}\right)\right]^2$$

$$\text{Solve for} \quad n_2 = \left(\frac{2.27E8}{235}\right)^{1/2} \approx \mathbf{980\ rpm} \quad \textit{Ans.}$$

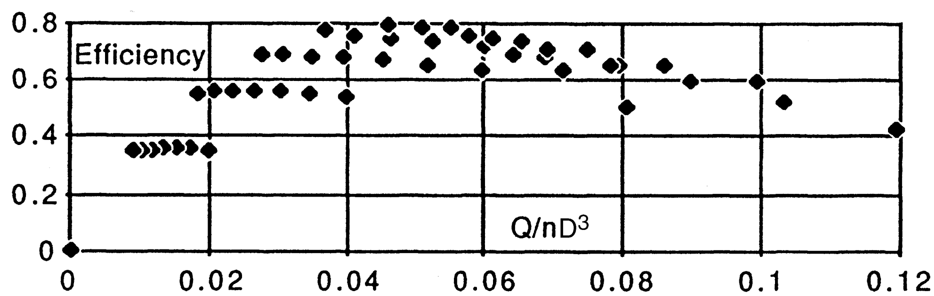
Now backtrack to compute $H_1 \approx 210$ ft and $Q_1 \approx 14500$ gal/min. From Fig. 11.7(b) we may then read the power $P_1 \approx 900$ bhp (or compute this from $\eta_1 \approx 0.85$). Then, by similarity, $P_2 = (n_2/n_1)^3 = 900(980/710)^3 \approx$ **2400 bhp**. *Ans.*

11.26 Determine if the seven Taco, Inc. pumps in Fig. 11.24 on the previous page can be collapsed into a single dimensionless chart of C_H, C_P, and η versus C_Q, as in Fig. 11.8 of the text. Comment on the results.

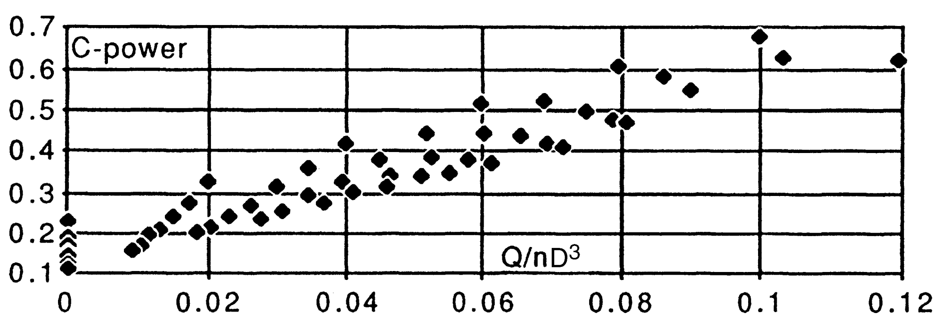
Solution: The head curves collapse *fairly well*, especially at low flow rates. Higher flow rates are poorer, as shown below. Recall that these pumps are <u>not</u> geometrically similar but rather consist of different sized impellers inside a single large housing.



Efficiencies are more scattered and rise significantly with impeller size:



The power coefficients are also rather scattered, probably due to geometric nonsimilarity:



In all these curves above, "n" is taken in revolutions per second (1160/60 ≈ 19.33 rps).

11.27 The 12-in pump of Fig. P11.24 is to be scaled up in size to provide a head of 90 ft and a flow rate of 1000 gal/min at BEP. Determine the correct (a) impeller diameter; (b) speed in rpm; and (c) horsepower required.

Solution: From the chart, at BEP, Q ≈ 480 gpm, H ≈ 60 ft, and $\eta \approx 76.5\%$, from which

$$P = \frac{\rho g Q H}{\eta} = \frac{(62.4)(480/449)(60)}{0.765} \approx 5234\ \frac{\text{ft·lbf}}{\text{s}},$$

$$C_P^* = \frac{P}{\rho n^3 D^5} = \frac{5234}{1.94(19.3)^3(1)^5} \approx 0.373$$

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{480/449}{(19.3)(1)^3} \approx 0.0553; \quad C_H^* = \frac{gH^*}{n^2D^2} = \frac{(32.2)(60)}{(19.3)^2(1)^2} \approx 5.17$$

$$\text{Larger pump, } H = 90,\ Q = 1000:\quad \frac{32.2(90)}{n^2D^2} \approx 5.17,\quad \frac{1000/449}{nD^3} \approx 0.0553$$

Solve for $D \approx 1.305$ ft ≈ **15.7 in** *Ans.* (a) and $n \approx 18.15$ rps ≈ **1090 rpm** *Ans.* (b)

$$P^* = C_P^* \rho n^3 D^5 = 0.373(1.94)(18.15)^3(1.305)^5 \approx 16300\ \frac{\text{ft·lb}}{\text{s}} \div 550 \approx \mathbf{30\ bhp} \quad \textit{Ans. (c)}$$

11.28 Tests by the Byron Jackson Co. of a 14.62-in centrifugal water pump at 2134 rpm yield the data below. What is the BEP? What is the specific speed? Estimate the max discharge.

Q, ft^3/s:	0	2	4	6	8	10
H, ft:	340	340	340	330	300	220
bhp:	135	160	205	255	330	330

Solution: The efficiencies are computed from $\eta = \rho gQH/(550\ \text{bhp})$ and are as follows:

Q:	0	2	4	**6**	8	10
η:	0	0.482	0.753	**0.881**	0.825	0.756

Thus the BEP is, even without a plot, close to **Q ≈ 6 ft^3/s**. *Ans.* The specific speed is

$$N_s \approx \frac{nQ^{*1/2}}{H^{*3/4}} = \frac{2134[(6)(449)]^{1/2}}{(330)^{3/4}} \approx \mathbf{1430} \quad \textit{Ans.}$$

For estimating Q_{max}, the last three points fit a Power-law to within ±0.5%:

$$H \approx 340 - 0.00168Q^{4.85} = 0 \quad \text{if} \quad Q \approx \mathbf{12.4\ \frac{ft^3}{s}} = Q_{max} \quad \textit{Ans.}$$

11.29 If the scaling laws are applied to the Byron Jackson pump of Prob. 11.28 for the same impeller diameter, determine (a) the speed for which the shut-off head will be 280 ft; (b) the speed for which the BEP flow rate will be 8.0 ft^3/s; and (c) the speed for which the BEP conditions will require 80 horsepower.

Solution: From the table in Prob. 11.28, the shut-off head at 2134 rpm is 340 ft. Thus

$$\text{If } D_1 = D_2, \quad n_2 = n_1(H_2/H_1)^{1/2} = 2134(280/340)^{1/2} \approx \mathbf{1940\ rpm} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{If } Q_2^* = 8\ \frac{ft^2}{s}, \quad n_2 = n_1(Q_2^*/Q_1^*) = 2134(8.0/6.0) \approx \mathbf{2850\ rpm} \quad \textit{Ans.}\ \text{(b)}$$

Finally, if $\rho_1 = \rho_2$ (water) and the diameters are the same, then Power $\propto n^3$, or, at BEP,

$$n_2 = n_1(P_2^*/P_1^*)^{1/3} = 2134(80/255)^{1/3} \approx \mathbf{1450\ rpm} \quad \textit{Ans.}\ \text{(c)}$$

11.30 A pump from the same family as Prob. 11.28 is built with D = 18 in and a BEP power of 250 bhp for *gasoline* (not water). Using the scaling laws, estimate the resulting (a) speed in rpm; (b) flow rate at BEP; and (c) shutoff head.

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3, whereas for water take $\rho \approx 1.94$ slug/ft^3.

$$\frac{P_2^*}{P_1^*} = \frac{250}{255} = \frac{\rho_2}{\rho_1}\left(\frac{n_2}{n_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{1.32}{1.94}\right)\left(\frac{n_2}{2134}\right)^3\left(\frac{18.0}{14.62}\right)^5,$$

$$\text{Solve} \quad n_2 \approx \mathbf{1700\ rpm} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Then} \quad Q_2^* = Q_1^*\left(\frac{n_2}{n_1}\right)\left(\frac{D_2}{D_1}\right)^3 = (6.0)\left(\frac{1700}{2134}\right)\left(\frac{18.0}{14.62}\right)^3 \approx \mathbf{8.9\ \frac{ft^3}{s}} \quad \textit{Ans.}\ \text{(b)}$$

Finally, with the change in speed known and an original shut-off head of 340 ft,

$$H_{o2} = H_{o1}\left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = 340\left(\frac{1700}{2134}\right)^2\left(\frac{18.0}{14.62}\right)^2 \approx \mathbf{330\ ft} \quad \textit{Ans.}\ \text{(c)}$$

11.31 A centrifugal pump with backward-curved blades has the following measured performance when tested with water at 20°C:

Q, gal/min:	0	400	800	1200	1600	2000	2400
H, ft:	123	115	108	101	93	81	62
P, hp:	30	36	40	44	47	48	46

(a) Estimate the best efficiency point and the maximum efficiency. (b) Estimate the most efficient flow rate, and the resulting head and brake horsepower, if the diameter is doubled and the rotation speed increased by 50%.

Solution: (a) Convert the data above into efficiency. For example, at $Q = 400$ gal/min,

$$\eta = \frac{\gamma QH}{P} = \frac{(62.4\ \text{lbf/ft}^3)(400/448.8\ \text{ft}^3\text{/s})(115\ \text{ft})}{(36 \times 550\ \text{ft}\cdot\text{lbf/s})} = 0.32 = 32\%$$

When converted, the efficiency table looks like this:

Q, gal/min:	0	400	800	1200	1600	2000	2400
η, %:	0	32%	55%	70%	80%	85%	82%

So maximum efficiency of **85%** occurs at ***Q* = 2000 gal/min.** *Ans.* (a)

(b) We don't know the values of C_Q^* or C_H^* or C_P^*, but we can set them equal for conditions 1 (the data above) and 2 (the performance when n and D are changed):

$$C_Q^* = \frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3} = \frac{Q_2}{(1.5n_1)(2D_1)^3},$$

$$\text{or:}\quad Q_2 = 12Q_1 = 12(2000\ \text{gpm}) = \mathbf{24,000\ \frac{gal}{min}} \quad \textit{Ans. (b)}$$

$$C_H^* = \frac{gH_1}{n_1^2 D_1^2} = \frac{gH_2}{n_2^2 D_2^2} = \frac{gH_2}{(1.5n_1)^2(2D_1)^2},$$

$$\text{or:}\quad H_2 = 9H_1 = 9(81\ \text{ft}) = \mathbf{729\ ft} \quad \textit{Ans. (b)}$$

$$C_P^* = \frac{P_1}{\rho n_1^3 D_1^5} = \frac{P_2}{\rho n_2^3 D_2^5} = \frac{P_2}{\rho(1.5n_1)^3(2D_1)^5},$$

$$\text{or:}\quad P_2 = 108P_1 = 108(48\ \text{hp}) = \mathbf{5180\ hp} \quad \textit{Ans. (b)}$$

11.32 The data of Prob. 11.31 correspond to a pump speed of 1200 r/min. (Were you able to solve Prob. 11.31 without this knowledge?) (a) Estimate the diameter of the impeller [*HINT*: See Prob. 11.24 for a clue.]. (b) Using your estimate from part (a), calculate the BEP parameters C_Q^*, C_H^*, and C_P^* and compare with Eq. (11.27). (c) For what speed of this pump would the BEP head be 280 ft?

Solution: **Yes**, we were able to solve Prob. 11.31 by simply using *ratios*.
(a) Prob. 11.24 showed that, for a wide range of centrifugal pumps, the shut-off head $H_o \approx 0.6U^2/g \pm 6\%$, where U is the impeller blade tip velocity, $U = \omega D/2$. Use this estimate with the shut-off head and speed of the pump in Prob. 11.31:

$$\omega = 2\pi(1200 \text{ rpm})/60 = 126 \text{ rad/s}, \quad H_o \approx 123 \text{ ft} = 0.6[(126D/2)^2/32.2 \text{ ft/s}^2]$$

$$\text{Solve for} \quad D \approx 1.29 \text{ ft} \approx \mathbf{15.5\ in} \quad \textit{Ans.} \text{ (a)}$$

(b) With diameter $D \approx 1.29$ ft estimated and speed $n = 1200/60 = 20$ r/s given, we can calculate:

$$C_Q^* = \frac{(2000/448.8) \text{ ft}^3/\text{s}}{(20 \text{ r/s})(1.29 \text{ ft})^3} \approx \mathbf{0.103}; \quad C_H^* = \frac{(32.2 \text{ ft/s}^2)(81 \text{ ft})}{(20 \text{ r/s})^2(1.29 \text{ ft})^2} \approx \mathbf{3.9}$$

$$C_P^* = \frac{(48)(550 \text{ ft·lbf/s})}{(1.94 \text{ slug/ft}^3)(20 \text{ r/s})^3(1.29 \text{ ft})^5} \approx \mathbf{0.47} \quad \textit{Ans.} \text{ (b)}$$

(c) Use the estimate of C_H^* to estimate the speed needed to produce 280 ft of head:

$$C_H^* \approx 3.9 = \frac{(32.2 \text{ ft/s}^2)(280 \text{ ft})}{n^2(1.29 \text{ ft})^2}, \quad \text{solve for } n = 37 \text{ r/s} \approx \mathbf{2230\ r/min} \quad \textit{Ans.} \text{ (c)}$$

11.33 For the pump family of Probs. 11.31 and 11.32, find the appropriate (a) diameter and (b) rotation speed which will deliver, at BEP, 5300 gal/min against a head of 210 ft. (c) What is the brake horsepower for this condition?

Solution: Armed with the three BEP results from Prob. 11.32, we can solve for these variables:

$$\text{(a, b)} \quad C_Q^* = 0.103 = \frac{(5300/448.8) \text{ ft}^3/\text{s}}{n_2 D_2^3}; \quad C_H^* = 3.9 = \frac{(32.2 \text{ ft/s}^2)(210 \text{ ft})}{n_2^2 D_2^2}$$

$$\text{Solve together for} \quad D_2 = 1.66 \text{ ft} \approx \mathbf{20\ in}, \quad n_2 = 25.1 \text{ r/s} \approx \mathbf{1510\ r/min} \quad \textit{Ans.} \text{ (a, b)}$$

$$C_P^* = 0.47 = \frac{P_2}{\rho n_2^3 D_2^5} = \frac{P_2}{(1.94\ \text{slug/ft}^3)(25.1\ \text{r/s})^3(1.66\ \text{ft})^5},$$

$$\text{Solve}\quad P_2 = 181400\ \frac{\text{ft·lbf}}{\text{s}} = \mathbf{330\ hp}\quad \textit{Ans.}\ \text{(c)}$$

11.34 Consider a pump geometrically similar to the 9-in-diameter pump of Fig. P11.34 to deliver 1200 gal/min of *kerosene* at 1500 rpm. Determine the appropriate (a) impeller diameter; (b) BEP horsepower; (c) shut-off head; and (d) maximum efficiency.

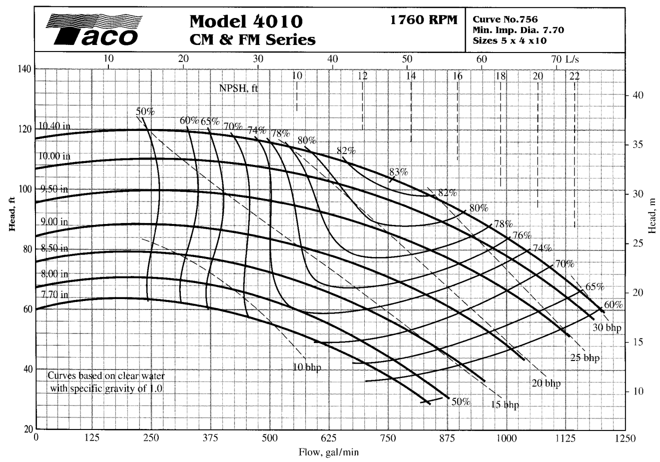


Fig. P11.34 Performance data for a family of centrifugal pump impellers. (*Courtesy of Taco, Inc., Cranston, Rhode Island.*)

Solution: For kerosene, take $\rho \approx 1.56$ slug/ft^3, whereas for water $\rho \approx 1.94$ slug/ft^3. From Fig. P11.34, at BEP, read Q* ≈ 675 gpm, H* ≈ 76 ft, and $\eta_{max} \approx 0.77$. Then

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{675/449}{(1760/60)(9/12)^3} \approx 0.122 = \frac{1200/449}{(1500/60)D^3},$$

$$\text{Solve for}\quad D_{imp} \approx 0.96\ \text{ft} \approx \mathbf{11.5\ in}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{Shut-off:}\quad \frac{gH_o}{n^2D^2} = \frac{32.2(84\ \text{ft})}{(1760/60)^2(9/12)^2} = \frac{32.2H_o}{(1500/60)^2(0.96)^2},$$

$$\text{Solve}\quad H_o \approx \mathbf{100\ ft}\quad \textit{Ans.}\ \text{(c)}$$

$$\text{Moody:}\quad \frac{1-\eta_2}{1-0.77} \approx \left(\frac{9.0}{11.5}\right)^{1/4},\quad \text{solve for}\ \eta_2 \approx \mathbf{0.784}\quad \textit{Ans.}\ \text{(d) (crude estimate)}$$

$$\text{Fig. P11.34:}\quad \text{Read H*} \approx 76\ \text{ft},\quad \text{whence}\quad \frac{32.2(76)}{(29.3)^2(0.75)^2} = \frac{32.2\ H^*_{new}}{(25)^2(0.96)^2},$$

$$\text{or}\quad H^*_{new} \approx 90.1\ \text{ft}$$

$$\text{Then}\quad P^*_{new} = \frac{1.56(32.2)(1200/449)(90.1)}{0.784} \approx 15440\ \frac{\text{ft·lbf}}{\text{s}} \div 550 \approx \mathbf{28\ bhp} \quad \textit{Ans.}\ \text{(b)}$$

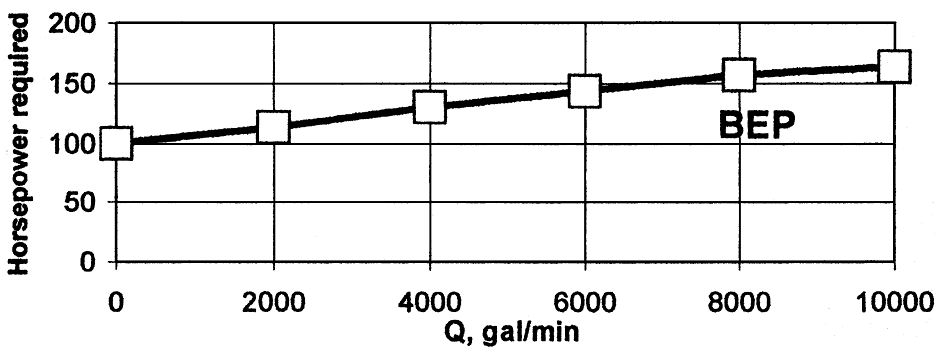
11.35 An 18-in-diameter centrifugal pump, running at 880 rev/min with water at 20°C, generates the following performance data:

Q, gal/min:	0.0	2000	4000	6000	8000	10000
H, ft:	92	89	84	78	68	50
P, hp:	100	112	130	143	156	163
η:	**0%**	**40%**	**65%**	**83%**	**88%**	**78%**

Determine (a) the BEP; (b) the maximum efficiency; and (c) the specific speed. (d) Plot the required input power versus the flow rate.

Solution: We have computed the efficiencies and listed them. The BEP is the next-to-last point: **Q = 8000 gal/min,** η_{max} **= 88%.** *Ans.* (a, b) The specific speed is $N'_s = nQ^{*1/2}/(gH^*)^{3/4} = (880/60)(8000/448.83)^{1/2}/[32.2(68)]^{3/4} \approx \mathbf{0.193}$, or $\mathbf{N_s = 3320}$ (probably a centrifugal pump). *Ans.* (c)

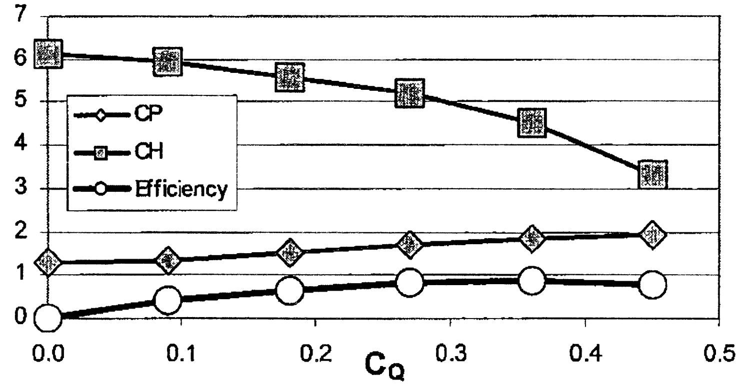
The plot of input horsepower versus flow rate is shown below—there are no surprises in this plot. *Ans.* (d)



11.36 Plot the dimensionless performance curves for the pump of Prob. P11.35 and compare with Fig. 11.8 of the text. Find the appropriate diameter in inches and the speed, in rev/min, for a geometrically similar pump to deliver 400 gal/min against a head of 200 ft. What brake horsepower would be required?

Solution: The data are plotted below in the form of C_H, C_P, and η versus C_Q. The head and power coefficients are about the same as Fig. 11.8 of the text, but the flow

coefficients are four time larger, primarily because the specific speed here is twice as large as that of Fig. 11.8.



Maximum efficiency occurs at about $C_Q^* \approx 0.36,$ for which $C_H^* \approx 4.52.$ Thus, for the proposed new conditions ($H = 200$ ft, $Q = 400$ gal/min), we obtain best efficiency at

$$C_Q^* = 0.36 = \frac{(400/448.83)\ \text{ft}^3/\text{s}}{\boldsymbol{nD^3}} \quad \text{and} \quad C_H^* = 4.52 = \frac{(32.2\ \text{ft/s}^2)(200\ \text{ft})}{\boldsymbol{n^2D^2}}$$

Solve simultaneously for $D = 0.256$ ft = **3.1 in** and $n = 147$ r/s = **8830 r/min**. *Ans.*

This is a poor result: too small and too high a speed. Better designs are available. We could retain the efficiency of 88%, or the Moody step-up formula, Eq. (11.29a), will predict a lower efficiency of 81%. The horsepower required would be

$$\boldsymbol{P} = \frac{\rho g Q H}{\eta} = \frac{(62.4\ \text{lbf/ft}^3)(400/449\ \text{ft}^3/\text{s})(200\ \text{ft})}{0.81} = 13700\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \div 550 \approx \mathbf{25\ bhp} \quad \textit{Ans.}$$

11.37 Knowing that the pump of Prob. 11.35 has BEP at Q = 8000 gal/min, use the similarity rules to find the appropriate (a) impeller diameter, (b) rotation speed, and (c) head produced for a pump of the same family delivering 1000 gal/min at 12 brake horsepower.

Solution: In Prob. 11.35 maximum efficiency of 88%, for a diameter of 1.5 ft, was found at Q = 8000 gal/min, H = 68 ft, P = 156 hp, and n = 880/60 r/s. From this compute $C_Q^* = 0.360,$ $C_H^* = 4.52,$ and $C_P^* = 1.85.$ (N_s was about 3320.) Apply the BEP coefficients to the new data:

$$C_Q^* = \frac{Q_2}{n_2 D_2^3} = 0.360 = \frac{1000/449}{n_2 D_2^3};\ C_H^* = \frac{gH_2}{n_2^2 D_2^2} = 4.52 = \frac{(32.2)H_2}{n_2^2 D_2^2}$$

$$C_P^* = \frac{P_2}{\rho n_2^3 D_2^5} = 1.85 = \frac{12(550)}{(1.94)n_2^3 D_2^5}$$

Solve simultaneously, or use EES, to obtain:

$$\text{(a) } D_2 = 0.60\text{ ft} = \mathbf{7.2\text{ in}};\quad \text{(b) } n_2 = 28.8\text{ r/s} = \mathbf{1730\text{ r/min}};\quad \text{(c) } H_2 = \mathbf{41.9\text{ ft}}$$

11.38 A 6.85-in pump, running at 3500 rpm, has the measured performance at right for water at 20°C. (a) Estimate the horsepower at BEP. If this pump is rescaled in water to provide 20 bhp at 3000 rpm, determine the appropriate (b) impeller diameter; (c) flow rate; and (d) efficiency for this new condition.

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	81	79	74

Solution: The BEP of 81% is at about Q = 350 gpm and H = 169 ft. Hence the power is

$$P^* = \frac{\rho g Q^* H^*}{\eta} = \frac{62.4(350/449)(169)}{0.81} = 10150\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \div 550 \approx \mathbf{18.5\text{ bhp}} \quad \textit{Ans.}\text{ (a)}$$

If the new conditions are 20 hp at n = 3000 rpm = 50 rps, we equate power coefficients:

$$C_P^* = \frac{10150}{1.94(3500/60)^3(6.85/12)^5} = 0.435 \stackrel{?}{=} \frac{20\times 550}{1.94(50)^3 D^5},$$

$$\text{Solve}\quad D_{imp} \approx 0.636\text{ ft} \approx \mathbf{7.64\text{ in}} \quad \textit{Ans.}\text{ (b)}$$

With diameter known, the flow rate is computed from BEP flow coefficient:

$$C_Q^* = \frac{Q^*}{nD^3} = \frac{350/449}{(3500/60)(6.85/12)^3} = 0.0719 \stackrel{?}{=} \frac{Q^*}{50(0.636)^3},$$

$$\text{Solve}\quad Q^* = 0.926\text{ ft}^3/\text{s} \approx \mathbf{415\text{ gal/min}} \quad \textit{Ans.}\text{ (c)}$$

Finally, since $D_1 \approx D_2$, we can assume the same maximum efficiency: **81%**. *Ans.* (d)

11.39 The Allis-Chalmers D30LR centrifugal compressor delivers 33,000 ft^3/min of SO_2 with a pressure change from 14.0 to 18.0 lbf/in^2 absolute using an 800-hp motor at 3550 r/min. What is the overall efficiency? What will the flow rate and Δp be at 3000 r/min? Estimate the diameter of the impeller.

Solution: For SO_2, take $M = 64.06$, hence $R = 49720/64.06 \approx 776$ ft·lbf/(slug·°R). Then

$$\Delta p = (18-14)(144) = 576 \text{ psf}, \quad \text{Power} = Q\Delta p = 576\left(\frac{33000}{60}\right) \div 550 \approx 576 \text{ hp delivered}$$

$$\text{Then} \quad \eta = P_{\text{delivered}}/P_{\text{motor}} = 576/800 \approx \mathbf{72\%} \quad \textit{Ans.} \text{ (a)}$$

$$\text{If } n_2 = 3000 \text{ rpm}, \quad Q_2 = Q_1\left(\frac{n_2}{n_1}\right) = 33000\left(\frac{3000}{3550}\right) \approx \mathbf{27900\ \frac{ft^3}{min}} \quad \textit{Ans.} \text{ (b)}$$

$$\Delta p_2 = \Delta p_1(n_2/n_1)^2 = (4\,\text{psi})\left(\frac{3000}{3550}\right)^2 \approx \mathbf{2.86\ psi} \quad \textit{Ans.} \text{ (c)}$$

To estimate impeller diameter, we have little to go on except the specific speed:

$$\rho_{\text{avg}} \approx \frac{16(144)}{776(520)} \approx 0.0057\ \frac{\text{slug}}{\text{ft}^3}, \quad H = \frac{\Delta p}{\rho g} = \frac{4(144)}{0.0057(32.2)} \approx 3133 \text{ ft},$$

$$N_s = \frac{\text{rpm(gpm)}^{1/2}}{(\text{H-ft})^{3/4}} = \frac{3550[33000(449)/60]^{1/2}}{(3133)^{3/4}} \approx 4212: \quad \text{Fig. P11.49: } C_Q^* \approx 0.45$$

$$\text{Crudely,} \quad C_Q^* \approx 0.45 = \frac{33000/60}{(3550/60)D^3}, \quad \text{solve for } D_{\text{impeller}} \approx \mathbf{2.7\ ft} \quad \textit{Ans.} \text{ (d)}$$

Clearly this last part depends upon the ingenuity and resourcefulness of the student.

11.40 The specific speed N_s, as defined by Eq. (11.30), does not contain the impeller diameter. How then should we size the pump for a given N_s? Logan [7] suggests a parameter called the *specific diameter* D_S, which is a dimensionless combination of Q, (gH), and D. (a) If D_S is proportional to D, determine its form. (b) What is the relationship, if any, of D_S to C_{Q*}, C_{H*}, and C_{P*}? (c) Estimate D_S for the two pumps of Figs. 11.8 and 11.13.

Solution: If we combine C_Q and C_H in such a way as to eliminate speed *n*, and also to make the result linearly proportional to *D*, we obtain Logan's result:

$$\text{Specific diameter} \quad \mathbf{D_s = \frac{D(gH^*)^{1/4}}{Q^{*1/2}}} \quad \textit{Ans.} \text{ (a)} \quad \mathbf{D_s = \frac{C_H^{*\,1/4}}{C_Q^{*\,1/2}}} \quad \textit{Ans.} \text{ (b)}$$

(c) For the pumps of Figs. 11.8 and 11.13, we obtain

$$D_{s\text{-}\mathbf{Fig.11.8}} = \frac{(5.0)^{1/4}}{(0.115)^{1/2}} = \mathbf{4.41}; \quad D_{s\text{-}\mathbf{Fig.11.13}} = \frac{(1.07)^{1/4}}{(0.55)^{1/2}} = \mathbf{1.37} \quad \textit{Ans.} \text{ (c)}$$

11.41 It is desired to build a centrifugal pump geometrically similar to Prob. 11.28 (data at right) to deliver 6500 gal/min of gasoline at 1060 rpm. Estimate the resulting (a) impeller diameter; (b) head; (c) brake horsepower; and (d) maximum efficiency.

Q, ft^3/s:	0	2	4	6	8	10
H, ft:	340	340	340	330	300	220
bhp:	135	160	205	255	330	330

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3. From Prob. 11.28, BEP occurs at Q* ≈ 6 ft^3/s, $\eta_{max} \approx 0.88$. The data above are for $n = 2134$ rpm = 35.6 rps and $D = 14.62$ in.

$$\text{Then}\quad C_Q^* = \frac{6.0}{35.6(14.62/12)^3} = 0.0933 \overset{?}{=} \frac{6500/449}{(1060/60)D^3},$$

$$\text{Solve for}\quad D_{imp} \approx \mathbf{2.06\ ft}\quad \textit{Ans.}\ \text{(a)}$$

$$C_H^* = \frac{32.2(330)}{(35.6)^2(14.62/12)^2} = 5.66 \overset{?}{=} \frac{32.2H}{(1060/60)^2(2.06)^2},\quad \text{solve for}\ \ H \approx \mathbf{233\ ft}\quad \textit{Ans.}\ \text{(b)}$$

Step-up the efficiency with Moody's correlation, Eq. (11.29*a*), for $D_1 = 14.62/12 \approx 1.22$ ft:

$$\frac{1-\eta_2}{1-0.88} \approx \left(\frac{D_1}{D_2}\right)^{1/4} = \left(\frac{1.22}{2.06}\right)^{1/4} = 0.877,\quad \text{solve for}\ \ \eta_2 \approx 0.895$$

$$\text{Then}\quad P_2 = \frac{\rho g Q_2 H_2}{\eta_2} = \frac{1.32(32.2)(6500/449)(233)}{0.895} = 160200 \div 550 \approx \mathbf{290\ bhp}\quad \textit{Ans.}\ \text{(c)}$$

11.42 An 8-inch model pump delivering water at 180°F at 800 gal/min and 2400 rpm begins to cavitate when the inlet pressure and velocity are 12 psia and 20 ft/s, respectively. Find the required NPSH of a prototype which is 4 times larger and runs at 1000 rpm.

Solution: For water at 180°F, take $\rho g \approx 60.6$ lbf/ft^3 and $p_v \approx 1600$ psfa. From Eq. 11.19,

$$NPSH_{model} = \frac{p_i - p_v}{\rho g} + \frac{V_i^2}{2g} = \frac{12(144)-1600}{60.6} + \frac{(20)^2}{2(32.2)} = 8.32\ \text{ft}$$

$$\text{Similarity:}\quad NPSH_{proto} = NPSH_m\left(\frac{n_p}{n_m}\right)^2\left(\frac{D_p}{D_m}\right)^2 = 8.32\left(\frac{1000}{2400}\right)^2\left(\frac{4}{1}\right)^2 \approx \mathbf{23\ ft}\quad \textit{Ans.}$$

11.43 The 28-in-diameter pump in Fig. 11.7*a* at 1170 r/min is used to pump water at 20°C through a piping system at 14,000 gal/min. (a) Determine the required brake horsepower. The average friction factor is 0.018. (b) If there is 65 ft of 12-in-diameter pipe upstream of the pump, how far below the surface should the pump inlet be placed to avoid cavitation?

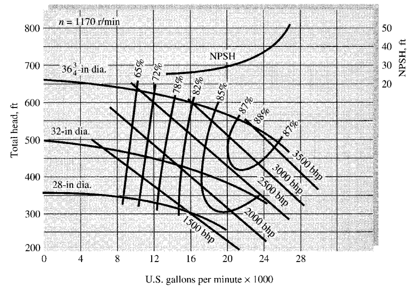


Fig. 11.7a

Solution: For water at 20°F, take $\rho g \approx 62.4$ lbf/ft^3 and $p_v \approx 49$ psfa. From Fig. 11.7a (above), at 28″ and 14000 gpm, read H ≈ 320 ft, $\eta \approx 0.81$, and P ≈ **1400** bhp. *Ans.*

$$\text{Or: Required bhp} = \frac{\rho g Q H}{\eta} = \frac{(62.4)(14000/449)(320)}{0.81} = 769000 \div 550 \approx \mathbf{1400\ bhp} \quad \textit{Ans.}$$

From the figure, at 14000 gal/min, read NPSH ≈ 25 ft. Assuming p_a = 1 atm = 2116 psf,

$$\text{Eq. 11.20:} \quad \text{NPSH} = \frac{p_a - p_v}{\rho g} - Z_i - h_{fi} = \frac{2116 - 49}{62.4} - Z_i - h_{fi} \approx 25 \text{ ft}, \quad h_{fi} = f\frac{L}{D}\frac{V^2}{2g},$$

$$V = \frac{Q}{A} = \frac{14000/449}{(\pi/4)(1\text{ ft})^2} \approx 39.7\ \frac{\text{ft}}{\text{s}},$$

$$\text{so:} \quad Z_i = 33.1 - 25 - 0.018\left(\frac{65}{1}\right)\left[\frac{(39.7)^2}{2(32.2)}\right] \approx \mathbf{-21\ ft} \quad \textit{Ans.}$$

11.44 The pump of Prob. 11.28 is scaled up to an 18-in-diameter, operating in water at BEP at 1760 rpm. The measured NPSH is 16 ft, and the friction loss between the inlet and the pump is 22 ft. Will it be sufficient to avoid cavitation if the pump inlet is placed 9 ft below the surface of a sea-level reservoir?

Solution: For water at 20°C, take ρg = 62.4 lbf/ft^3 and p_v = 49 psfa. Since the NPSH is *given*, there is no need to use the similarity laws. Merely apply Eq. 11.20:

$$\text{NPSH} \le \frac{p_a - p_v}{\rho g} - Z_i - h_{fi}, \quad \text{or:} \quad Z_i \le \frac{2116 - 49}{62.4} - 22 - 16 = -4.9 \text{ ft}, \ \mathbf{OK,}$$

$$\mathbf{Z_{actual} = -9\ ft} \quad \textit{Ans.}$$

This works. Putting the inlet 9 ft below the surface gives 4 ft of margin against cavitation.

11.45 Determine the specific speeds of the seven Taco, Inc. pump impellers in Fig. P11.24. Are they appropriate for centrifugal designs? Are they approximately equal within experimental uncertainty? If not, why not?

Solution: Read the BEP values for each impeller and make a little table for *1160* rpm:

D, inches:	10.0	10.5	11.0	11.5	12.0	12.5	12.95
Q*, gal/min:	390	420	440	460	480	510	530
H*, ft:	41	44	49	56	60	66	72
Specific speed N_S:	**1414**	**1392**	**1314**	**1215**	**1179**	**1131**	**1080**

These are well within the centrifugal-pump range ($N_S < 4000$) but they are not equal because they are not geometrically similar (7 different impellers within a single housing). *Ans.*

11.46 The answer to Prob. 11.40 is that the dimensionless "specific diameter" takes the form $D_s = D(gH^*)^{1/4}/Q^{*1/2}$, evaluated at the BEP. Data collected by the writer for 30 different pumps indicates, in Fig. P11.46, that D_s correlates well with specific speed N_s. Use this figure to estimate the appropriate impeller diameter for a pump which delivers 20,000 gal/min of water and a head of 400 ft running at 1200 rev/min. Suggest a curve-fit formula to the data (*Hint*: a hyperbola).

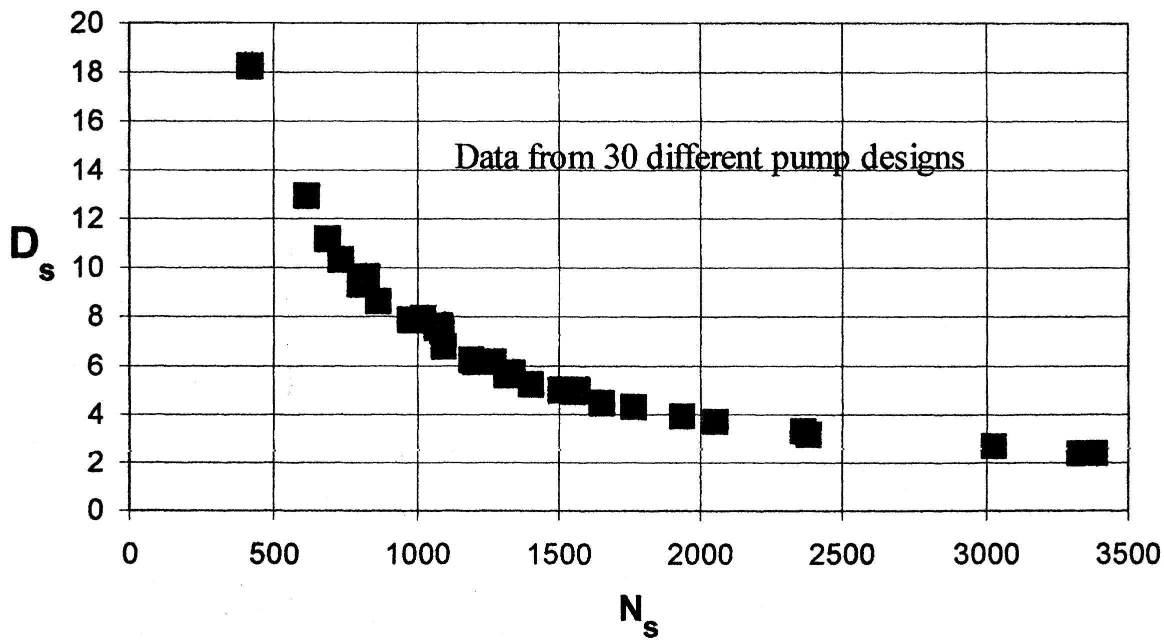


Fig. P11.46

Solution: We see that the data are very well correlated by a single curve. (NOTE: These are all *centrifugal* pumps—a slightly different correlation holds for mixed- and axial-flow pumps.) The data are well fit by a hyperbola:

$$\text{Figure P11.46:}\quad \mathbf{D_s} \approx \frac{\mathbf{Const}}{\mathbf{N_s}},\quad \text{where Const} \approx 7800 \pm 300 \quad \textit{Ans.}$$

For the given pump-data example, we compute

$$N_s = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{(\textit{Head}\text{–ft})^{3/4}} = \frac{1200(20000)^{1/2}}{(400)^{3/4}} = 1897,$$

$$\text{Hence}\quad D_s \approx \frac{7800}{1897} \approx 4.11 = \frac{D[32.2(400)]^{1/4}}{(20000/448.83)^{1/2}},\quad \text{solve } \mathbf{D \approx 2.6 \pm 0.1\ ft} \quad \textit{Ans.}$$

11.47 A typical household basement sump pump provides a discharge of 5 gal/min against a head of 15 ft. Estimate (a) the maximum efficiency; and (b) the minimum horsepower required to drive such a pump.

Solution: Typical small sump pumps run at about 1750 rpm, so we can estimate:

$$\mathrm{N_s} = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{(\text{head})^{3/4}} \approx \frac{1750(5)^{1/2}}{(15\text{ ft})^{3/4}} \approx 513.\ \text{Fig. 11.14: read } \eta_{\max} \approx \mathbf{0.27} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Then}\quad \mathrm{P_{min}} = \frac{\rho g Q H}{\eta_{\max}} = \frac{62.4(5/449)(15)}{0.27} = 39 \div 550 \approx \mathbf{0.07\ bhp} \quad \textit{Ans.}\text{ (b)}$$

11.48 When operating at 42 r/s near BEP, a pump delivers 0.06 m^3/s against a head of 100 m. (a) What is its specific speed? (b) What kind of pump is this likely to be? (c) Estimate its impeller diameter.

Solution: (a) We have to go English to calculate the traditional specific speed. Convert $Q = 0.06\ m^3/s = 951$ gal/min, $H = 100$ m = 328 ft, and n = 42 r/s = 2520 r/min. Then

$$N_S = \frac{\text{rpm}(\text{gal/min})^{1/2}}{(\textit{Head in}\ \text{ft})^{3/4}} = \frac{2520(951)^{1/2}}{(328)^{3/4}} \approx \mathbf{1000} \quad \textit{Ans.}\text{ (a)}$$

(b) This specific speed is characteristic of a **centrifugal pump**. *Ans.* (b)

(c) From Prob. 11.46, the dimensionless specific diameter $D_s = D(gH^*)^{1/4}/Q^{*1/2}$ is closely correlated with specific speed:

$$D_S \approx \frac{7800}{N_S} = \frac{7800}{1000} = 7.8 = \frac{D[9.81\text{ m/s}^2(100\text{ m})]^{1/4}}{(0.06\text{ m}^3/\text{s})^{1/2}},$$

$$\text{Solve for}\quad \mathbf{D \approx 0.34\text{ m}}\ (13\text{ in})\quad Ans.\ (c)$$

11.49 Data collected by the writer for flow coefficient at BEP for 30 different pumps are plotted at right in Fig. P11.49. Determine if the values of C_Q^* fit this correlation for the pumps of Problems P11.24, P11.28, P11.35, and P11.38. If so, suggest a curve fit formula.

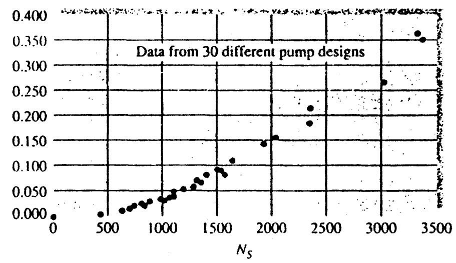


Fig. P11.49 Flow coefficient at BEP for 30 commercial pumps.

Solution: Make a table of these values:

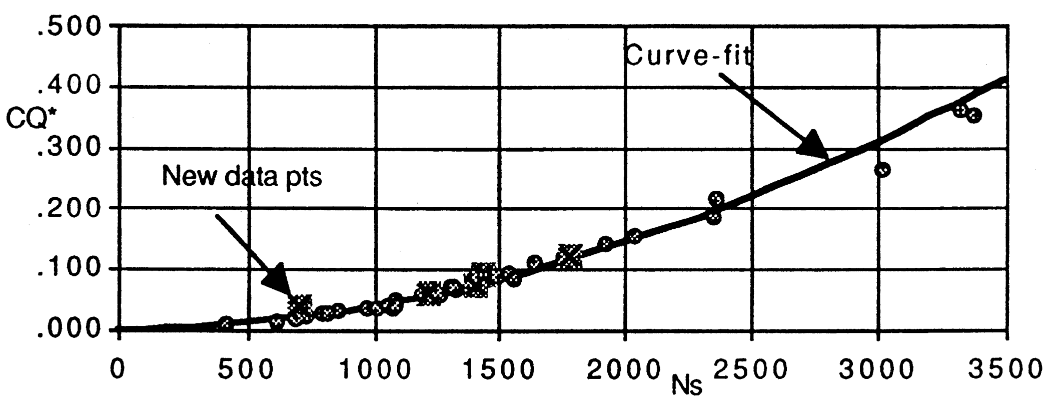
	Q*, gpm	D, inches	*n*, rpm	N_S	$C_Q^* = Q^*/(nD^3)$
Prob. 11.28:	2692	14.62	2134	**1430**	*0.0933*
Prob. 11.35:	37	5.0	1800	**700**	*0.0384*
Prob. 11.38:	350	6.85	3500	**1400**	*0.0719*
Fig. P11.24:	460	11.5	1160	**1215**	*0.0602* *Ans.*

When added to the plot shown below, all four seem to fit quite well, although the 'suspect' data point #2 (taken from Prob. 11.35) is rather high (about 75%). The data are useful for predicting general centrifugal-pump behavior and are well fit to either a 2nd-order polynomial or a single-term Power-law slightly less than parabolic:

Polynomial: $C_Q^* \approx 1.97E\text{–}5N_S + 2.58E\text{–}8N_S^2$

Power-law: $C_Q^* \approx 6.83E\text{–}8N_S^{1.914}$

(Correlation $R^2 \approx 0.99$) *Ans.*



11.50 Data collected by the writer for power coefficient at BEP for 30 different pumps are plotted at right in Fig. P11.50. Determine if the values of C_Q^* for the FOUR pumps of Prob. 11.49 above fit this correlation.

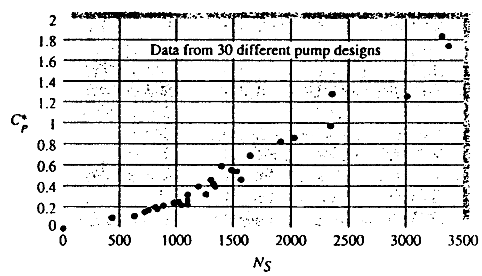


Fig. P11.50 Power coefficient at BEP for 30 commercial pumps.

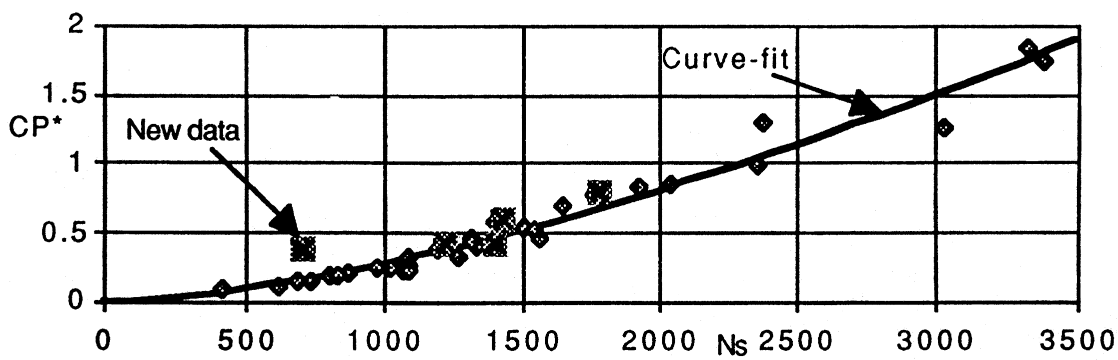
Solution: Make a table of these values, similar to Prob. 11.50:

	P*, bhp	**D, inches**	***n*, rpm**	$\mathbf{N_S}$	$\mathbf{C_P^* = P^*/(\rho n^3 D^5)}$
Prob. 11.28:	255	14.62	2134	**1430**	*0.600*
Prob. 11.35:	0.46	5.0	1800	**700**	*0.386*
Prob. 11.38:	18.5	6.85	3500	**1400**	*0.435*
Fig. P11.24:	8.7	11.5	1160	**1215**	*0.421* *Ans.*

When added to the plot shown below, three of them seem to fit reasonably well, but the ‘suspect’ data point #2 (from Prob. 11.35) is rather high (>100%). The data are moderately useful for predicting general centrifugal-pump behavior and can be fit to either a 2nd-order polynomial or a single-term Power-law:

Polynomial: $C_P^* \approx 2.12E\text{–}4N_s + 9.5E\text{–}8N_s^2$ (poorer correlation, $R^2 \approx 0.96$) *Ans.*

Power-law: $C_P^* \approx 6.78E\text{–}6N_s^{1.537}$



11.51 An axial-flow pump delivers 40 ft^3/s of air which enters at 20°C and 1 atm. The flow passage has a 10-in outer radius and an 8-in inner radius. Blade angles are $\alpha_1 = 60°$ and $\beta_2 = 70°$, and the rotor runs at 1800 rpm. For the first stage, compute (a) the head rise; and (b) the power required.

Solution: Assume an average radius of (8 + 10)/2 = 9 inches and compute the blade speed:

$$u_{avg} = \omega r_{avg} = \left(1800\frac{2\pi}{60}\right)\left(\frac{9}{12}\right) \approx 141\ \frac{ft}{s};\quad V_n = \frac{Q}{A} = \frac{40\ ft^3/s}{\pi[(10/12)^2 - (8/12)^2]} \approx 50.9\ \frac{ft}{s}$$

$$\text{Theory: } gH = u^2 - uV_n(\cot\alpha_1 + \cot\beta_2) = (141)^2 - 141(50.9)(\cot 60° + \cot 70°),$$

$$\mathbf{H \approx 410\ ft} \quad \textit{Ans. (a)}$$

$$P_{theory} = \rho gQH = \left[\frac{2116}{1717(528)}\right](32.2)(40)(410) = 1232 \div 550 \approx \mathbf{2.24\ hp} \quad \textit{Ans. (b)}$$

11.52 An axial-flow fan operates in sea-level air at 1200 r/min and has a blade-tip diameter of 1 m and a root diameter of 80 cm. The inlet angles are $\alpha_1 = 55°$ and $\beta_1 = 30°$, while at the outlet $\beta_2 = 60°$. Estimate the theoretical values of the (a) flow rate, (b) horse-power, and (c) outlet angle α_2.

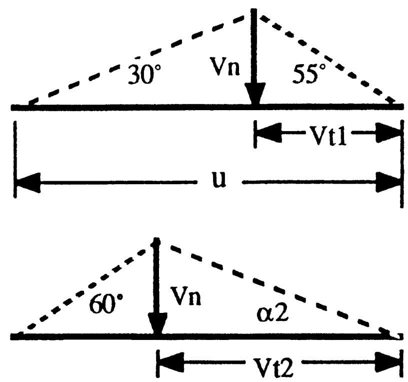


Fig. P11.52

Solution: For air, take $\rho \approx 1.205\ kg/m^3$. The average radius is 0.45 m. Thus

$$u = \omega R = \left(1200\frac{2\pi}{60}\right)(0.45) \approx 56.6\ \frac{m}{s} = V_n(\cot\alpha_1 + \cot\beta_1) = V_{n2}(\cot\alpha_2 + \cot\beta_2)$$

$$\text{Solve}\quad V_{n1} = V_{n2} = \frac{56.6}{\cot 55° + \cot 30°} \approx 23.2\ \frac{m}{s}\quad \text{and}\quad \alpha_2 \approx \mathbf{28.3°}\quad \textit{Ans. (c)}$$

$$\text{Then}\quad Q = V_n A = (23.2)[\pi\{(0.5)^2 - (0.4)^2\}] \approx \mathbf{6.56\ \frac{m^3}{s}}\quad \textit{Ans. (a)}$$

$$gH = u^2 - uV_n(\cot\alpha_1 + \cot\beta_2) = (56.6)^2 - 56.6(23.2)(\cot 55° + \cot 60°) = 1520\ m^2/s^2$$

$$\text{Finally,}\quad \mathbf{P} = \rho QgH = 1.205(6.56)(1520) = \mathbf{12{,}000\ W}\quad \textit{Ans. (b)}$$

11.53 If the axial-flow pump of Fig. 11.13 is used to deliver 70,000 gal/min of 20°C water at 1170 rpm, estimate (a) the proper impeller diameter; (b) the shut-off head; (c) the shut-off horsepower; and (d) Δp at best efficiency.

Solution: From Fig. 11.13, read $C_Q^* \approx 0.55$, $C_H^* \approx 1.07$, $C_P^* \approx 0.70$, and $\eta_{max} \approx 0.84$.

$$C_Q^* = 0.55 = \frac{Q^*}{nD^3} = \frac{70000/449}{(1170/60)D^3}, \quad \text{solve for } D_{impeller} \approx \mathbf{2.44\ ft} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Also read} \quad C_{Ho}(\text{shut-off}) \approx 2.85 = \frac{(32.2)H_o}{(1170/60)^2(2.44)^2}, \quad \text{solve } H_{shutoff} \approx \mathbf{200\ ft} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Read} \quad C_{Po}(\text{shutoff}) \approx 1.2,\ P_o = 1.2(1.94)\left(\frac{1170}{60}\right)^3(2.44)^5$$

$$C_{Po} = 1.5E6 \div 550 \approx \mathbf{2700\ hp} \quad \textit{Ans.}\ \text{(c)}$$

$$\text{Finally, } \Delta p^* = \rho g H^* = 62.4\left[1.07\frac{(1170/60)^2(2.44)^2}{32.2}\right] \approx 4697 \div 144 \approx \mathbf{33\ psi} \quad \textit{Ans.}\ \text{(d)}$$

11.54 The Colorado River Aqueduct uses Worthington Corp. pumps which deliver 200 ft^3/s of water at 450 rpm against a head of 440 ft. What kind of pumps are these? Estimate the impeller diameter.

Solution: Evaluate the specific speed to see what type of pumps we have:

$$N_s = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{(\text{head})^{3/4}} = \frac{450(200\times 449)^{1/2}}{(440)^{3/4}} \approx 1400 \quad \therefore \ \textbf{Centrifugal pumps} \quad \textit{Ans.}\ \text{(a)}$$

To estimate the diameter, use the curve-fit to the correlation we had in Fig. P11.50:

$$C_Q^* \approx (6.83E{-}8)(1400)^{1.914} \approx 0.072 = \frac{200}{(450/60)D^3}, \quad \text{solve } D_{impeller} \approx \mathbf{7.2\ ft} \quad \textit{Ans.}\ \text{(b)}$$

11.55 We want to pump 70°C water at 20,000 gal/min and 1800 rpm. Estimate the type of pump needed, the horsepower required, and the impeller diameter if the required pressure rise for one stage is (a) 170 kPa; and (b) 1350 kPa.

Solution: For water to 70°C, take $\rho \approx 978$ kg/m^3. Evaluate the specific speed:

$$\text{(a)}\ \Delta p = 170\ \text{kPa},\ H = \frac{\Delta p}{\rho g} = \frac{170000}{978(9.81)} = 17.7\ \text{m} \approx 58\ \text{ft} \quad N_s = \frac{(1800)(20000)^{1/2}}{(58)^{3/4}} \approx 12090$$

$$\therefore \quad \text{Need } \textbf{axial-flow pump}\ (\eta \approx 0.900) \quad \textit{Ans.}\ \text{(a)}$$

Then $P = \rho gQH/\eta = \left(\dfrac{978}{515}\right)(32.2)\left(\dfrac{20000}{449}\right)(58)/0.9 = 175500 \div 550 \approx \mathbf{320\ hp}$ *Ans.* (a)

Fig. 11.13 ($N_s \approx 12000$): $C_Q^* \approx 0.55 = \dfrac{20000/449}{(1800/60)D^3}$, solve $D_{impeller} \approx \mathbf{1.4\ ft}$ *Ans.* (a)

(b) $\Delta p = 1350,\ H = \dfrac{1{,}350{,}000}{978(9.81)} = 141\text{ m} \approx 462\text{ ft},\quad N_s = \dfrac{1800(20000)^{1/2}}{(462)^{3/4}} \approx 2560$

Centrifugal pump, $\eta \approx 0.92$ *Ans.* (b)

$P = (1.9)(32.2)(20000/449)(462)/0.92 = 1.37E6 \div 550 \approx \mathbf{2500\ hp}$ *Ans.* (b)

Fig. P11.49: $C_Q^* \approx 0.23 = \dfrac{20000/449}{(1800/60)D^3}$, solve $D_{impeller} \approx \mathbf{1.9\ ft}$ *Ans.* (b)

11.56 A pump is needed to deliver 40,000 gpm of gasoline at 20°C against a head of 90 ft. Find the impeller size, speed, and brake horsepower needed to use the pump families of (a) Fig. 11.8; and (b) Fig. 11.13. Which is the better design?

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3.

(a) For the *centrifugal* design, $C_Q^* \approx 0.115 = \dfrac{40000/449}{nD^3}$ and $C_H^* \approx 5.0 = \dfrac{32.2(90)}{n^2D^2}$,

Solve for $n \approx 4.24\text{ rps} \approx \mathbf{255\ rpm}$ and $D_{impeller} \approx \mathbf{5.67\ ft}$ *Ans.* (a)

$P^* = C_P^*\rho n^3D^5 = 0.65(1.32)(4.24)^3(5.67)^5 \div 550 \approx \mathbf{700\ bhp}$ *Ans.* (a)

(b) For the axial-flow design, Fig. 11.13,

$$C_Q^* = 0.55 = \frac{40000/449}{nD^3},\quad C_H^* = 1.07 = \frac{32.2(90)}{n^2D^2}$$

or: $n \approx \mathbf{1770\ rpm}$, $D \approx \mathbf{1.76\ ft}$ *Ans.* (b)

$P^* = C_P^*\rho n^3D^3 = 0.70(1.32)(29.5)^3(1.76)^5 \div 550 \approx \mathbf{740\ bhp}$ *Ans.* (b)

The **axial-flow design** (**b**) is far better for this system: smaller and faster. *Ans.*

11.57 Performance data for a 21-in-diameter air blower running at 3550 rpm are shown below. What is the specific speed? How does the performance compare with Fig. 11.13? What are C_Q^*, C_H^*, C_P^*?

Δp, in H_2O:	29	30	28	21	10
Q, ft^3/min:	500	1000	2000	3000	4000
bhp:	6	8	12	18	25

Solution: Assume 1-atm air, $\rho \approx 0.00233$ slug/ft^3. Convert the data to dimensionless form and put the results into a table:

Δp, psf:	151	156	146	109	52
Q, ft^3/min:	500	1000	2000	3000	4000
Q, gal/min:	3740	7480	14960	22440	29920
H, ft (of air):	2010	2080	1940	1455	693
C_Q:	0.0263	0.0526	**0.105**	0.158	0.210
C_H:	6.04	6.25	**5.83**	4.37	2.08
C_P:	0.417	0.555	**0.833**	1.25	1.74
η:	0.381	0.592	**0.735**	0.552	0.251

Close enough without plotting: $C_Q^* \approx$ **0.105**, $C_H^* \approx$ **5.83**, $C_P^* \approx$ **0.833** *Ans.*

$$\text{Specific speed} \quad N_s = \frac{(3550\text{ rpm})(14960\text{ gpm})^{1/2}}{(1940\text{ ft})^{3/4}} \approx \mathbf{1485} \quad \textit{Ans.}$$

This centrifugal pump is **very similar** to the dimensionless data of **Fig. 11.8**. *Ans.*

11.58 The Worthington Corp. Model A-12251 water pump, operating at maximum efficiency, produces 53 ft of head at 3500 rpm, 1.1 bhp at 3200 rpm, and 60 gal/min at 2940 rpm. What type of pump is this? What is its efficiency, and how does this compare with Fig. 11.14? Estimate the impeller diameter.

Solution: We can convert the power and flow-rate values to 3500 rpm with similarity:

$$P^*_{3500} \approx (1.1)\left(\frac{3500}{3200}\right)^3 \approx 1.44\text{ bhp}; \quad Q^*_{3500} \approx (60)\left(\frac{3500}{2940}\right) \approx 71.4\text{ gal/min}$$

$$\text{Then} \quad N_s = \frac{(3500)(71.4)^{1/2}}{(53)^{3/4}} \approx \mathbf{1510\ \ Centrifugal\ pump}. \quad \textit{Ans.}$$

$$\eta_{max} = \frac{\rho g Q H}{P} = \frac{62.4(71.4/449)(53)}{1.44(550)} \approx \mathbf{66.5\%} \quad \text{(compares well with Fig. 11.14)} \quad \textit{Ans.}$$

$$\text{Fig. P11.49:}\quad C_Q^* \approx 0.085 = \frac{71.4/449}{(3500/60)D^3} \quad \text{solve for } D_{impeller} \approx \mathbf{0.32\ ft}\ (4\text{ in}) \quad \textit{Ans.}$$

11.59 Suppose it is desired to deliver 700 ft^3/min of propane gas (molecular weight = 44.06) at 1 atm and 20°C with a single-stage pressure rise of 8.0 in H_2O. Determine the appropriate size and speed for using the pump families of (a) Prob. 11.57 and (b) Fig. 11.13. Which is the better design?

Solution: For propane, with M = 44.06, the gas constant R = 49720/44.06 ≈ 1128 ft·lbf/(slug·°R). Convert Δp = 8 inH_2O = (62.4)(8/12) = 41.6 psf. The propane density and head rise are

$$\rho_{gas} = \frac{p}{RT} = \frac{2116\text{ psf}}{1128(528)} \approx 0.00355\ \frac{\text{slug}}{\text{ft}^3},$$

$$\text{Hence}\quad H_{pump} = \frac{41.6}{0.00355(32.2)} \approx 364\text{ ft propane}$$

$$\text{(a) Prob. 11.57:}\quad C_Q^* \approx 0.105 = \frac{700/60}{nD^3} \quad \text{and} \quad C_H^* \approx 5.83 = \frac{32.2(364)}{n^2D^2}$$

Solve for $n = 28.5\text{ rps} \approx \mathbf{1710\ rpm}$ and $D \approx \mathbf{1.57\ ft}$ *Ans.* (a) (centrifugal pump)

(b) Fig. 11.13: $C_Q^* \approx 0.55$ and $C_H^* \approx 1.07$ yield

$$n \approx \mathbf{14000\ rpm},\quad D \approx \mathbf{0.45\ ft} \quad \textit{Ans.}\text{ (b)} \quad \text{(axial flow)}$$

The **centrifugal pump** (**a**) is the better design—nice size, nice speed. The axial flow pump is much smaller but runs too fast. *Ans.*

11.60 A 45-hp pump is desired to generate a head of 200 ft when running at BEP with 20°C gasoline at 1200 rpm. Using the correlations in Figs. P11.49 and P11.50, determine the appropriate (a) specific speed; (b) flow rate; and (c) impeller diameter.

Solution: For gasoline, take $\rho \approx 1.32$ slug/ft^3. The two correlations from Problems 11.49 and 11.50 are

$$\frac{Q}{nD^3} \approx 6.83\text{E–}8N_s^{1.914} \quad \text{and} \quad \frac{P}{\rho n^3D^5} \approx 6.78\text{E–}6N_s^{1.537} \quad \text{where } N_s = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{(\text{head})^{3/4}}$$

With n, ρ, P, and H known, the unknowns are the flow rate Q and diameter D. Not seeing exactly how to resolve this analytically, the writer simply ran a computer program for various diameters until the flow rates were the same for both correlations. Finally,

$$N_s \approx \mathbf{623} \quad \textit{Ans.}\text{ (a)} \qquad Q \approx \mathbf{762\ gal/min} \quad \textit{Ans.}\text{ (b)} \qquad D \approx \mathbf{1.77\ ft} \quad \textit{Ans.}\text{ (c)}$$

11.61 A mine ventilation fan delivers 500 m^3/s of sea-level air at 295 rpm and Δp = 1100 Pa. Is this fan axial, centrifugal, or mixed? Estimate its diameter in feet. If the flow rate is increased 50% for the same diameter, by what percent will Δp change?

Solution: For sea-level air, take $\rho g \approx 11.8\ N/m^3$, hence $H = \Delta p/\rho g = 1100/11.8 \approx 93$ m $\approx$ 305 ft. Convert 500 m^3/s to 7.93E6 gal/min and calculate the specific speed:

$$N_s = \frac{\text{rpm(gal/min)}^{1/2}}{(\text{head})^{3/4}} = \frac{295(7.93E6)^{1/2}}{(305)^{3/4}} \approx \mathbf{11400}\ \ (\textbf{axial-flow pump}) \quad \textit{Ans.}\text{ (a)}$$

$$\text{Estimate } C_Q^* \approx 0.55 = \frac{500\ m^3/s}{(295/60)D^3}, \quad \text{solve } D_{impeller} \approx 5.7\ m \approx \mathbf{19\ ft} \quad \textit{Ans.}\text{ (b)}$$

At constant D, Q $\propto$ n and Δp (or H) $\propto n^2$. Therefore, if Q increases 50%, so does n, and therefore Δp increases as $(1.5)^2 = 2.25$, or **a 125% increase**. *Ans.* (c)

11.62 The actual mine-ventilation fan in Prob. 11.61 had a diameter of 20 ft [Ref. 20, p. 339]. What would be the proper diameter for the pump family of Fig. 11.14 to provide 500 m^3/s at 295 rpm and BEP? What would be the resulting pressure rise in Pa?

Solution: For sea-level air, take $\rho g \approx 11.8\ N/m^3$. As in Prob. 11.61 above, the specific speed of this fan is **11400**, hence an *axial-flow* fan. Figure 11.14 indicates an efficiency of about 90%, and the only values we know for performance are from Fig. 11.13:

$$N_s \approx 12000\text{:}\ \ C_Q^* \approx 0.55 = \frac{500}{(295/60)D^3}, \quad \text{solve } \mathbf{D_{impeller}} \approx 5.7\ m \approx \mathbf{18.7\ ft} \quad \textit{Ans.}\text{ (a)}$$

$$C_H^* \approx 1.07, \quad H = 1.07\frac{(295/60)^2(20\times 0.3048)^2}{9.81} = 98\ m,$$

$$\Delta p = (11.8)(98) \approx \mathbf{1160\ Pa} \quad \textit{Ans.}\text{ (b)}$$

11.63 The 36.75-in pump in Fig. 11.7*a* at 1170 r/min is used to pump water at 60°F from a reservoir through 1000 ft of 12-in-ID galvanized-iron pipe to a point 200 ft above the reservoir surface. What flow rate and brake horsepower will result? If there is 40 ft of pipe upstream of the pump, how far below the surface should the pump inlet be placed to avoid cavitation?

Solution: For galvanized pipe, $\varepsilon \approx 0.0005$ ft, hence $\varepsilon/d \approx 0.0005$. Assume fully-rough flow, with $f \approx 0.0167$. The pipe head loss is thus approximately

$$h_f = \Delta z + f\frac{L}{d}\frac{[Q/(\pi d^2/4)]^2}{2g} = 200 + 0.0167\left(\frac{1000}{1}\right)\frac{[Q/(\pi(1)^2/4)]^2}{2(32.2)}$$

$$\approx 200 + 0.42Q^2 \ \text{(Q in ft}^3\text{/s)}$$

Curve-fit Fig. 11.7a, $D = 36.75''$, to a parabola: $H_p \approx 665 - 0.051Q^2$ (again Q in ft³/s)

Equate: $665 - 0.051Q^2 = 200 + 0.42Q^2$, solve $Q \approx 31.4$ ft³/s ≈ **14100 gpm** *Ans.* (a)

Figure 11.7a: $P = \rho gQH/\eta = 62.4(31.4)(615)/0.78 \div 550 \approx$ **2800 bhp**. *Ans.* (b)

Check $V = Q/A \approx 40$ ft/s, $Re_d = Vd/\nu \approx 3.71E6$, Moody chart, $f \approx \underline{0.0168}$ (OK).

(c) Figure 11.7a @ 14000 gpm: read NPSH ≈ **25** ft. Calculate the head loss upstream:

$$h_{fi} = f\frac{L}{d}\frac{V^2}{2g} = 0.0168\left(\frac{40}{1}\right)\frac{(40.0)^2}{2(32.2)} \approx 16.7 \text{ ft, use Eq. 11.20:}$$

$$\text{NPSH} = 25 \le \frac{p_a - p_v}{\rho g} - Z_i - h_{fi} = \frac{2116 - 39}{62.4} - Z_i - 16.7, \quad \text{solve } Z_i \le \mathbf{-8.5\ ft} \quad \textit{Ans.}\text{(c)}$$

11.64 A leaf blower is essentially a centrifugal impeller exiting to a tube. Suppose that the tube is smooth PVC pipe, 4 ft long, with a diameter of 2.5 in. The desired exit velocity is 73 mi/h in sea-level standard air. If we use the pump family of Eq. (11.27) to drive the blower, what approximate (a) diameter and (b) rotation speed are appropriate? (c) Is this a good design?

Solution: Recall that Eq. (11.27) gave BEP coefficients for the pumps of Fig. 11.7:

$$C_Q^* \approx 0.115; \quad C_H^* \approx 5.0; \quad C_P^* \approx 0.65$$

Apply these coefficients to the leaf-blower data. Neglect minor losses, that is, let the pump head match the pipe friction loss. For air, take $\rho = 1.2$ kg/m³ and $\mu = 1.8E{-}5$ kg/m·s.

Convert 73 mi/h = 32.6 m/s, 4 ft = 1.22 m and 2.5 in = 0.0635 m:

$$h_f = H_{pump} = 5.0\frac{n^2D^2}{9.81\text{ m/s}^2} = f\frac{L}{d_{pipe}}\frac{V_{pipe}^2}{2g} = f\left(\frac{1.22\text{ m}}{0.0635\text{ m}}\right)\left[\frac{(32.6\text{ m/s})^2}{2(9.81\text{ m/s}^2)}\right],\quad f = f(\text{Re}_d)$$

$$Q = \frac{\pi}{4}d_{pipe}^2V = \frac{\pi}{4}(0.0635\text{ m})^2(32.6\text{ m/s}) = 0.103\ \frac{\text{m}^3}{\text{s}} = \mathbf{0.115}nD^3$$

We know the Reynolds number, $\text{Re}_d = \rho Vd/\mu$ = (1.2)(32.6)(0.0635)/(1.8E–5) = 138,000, and for a smooth pipe, from the Moody chart, calculate f_{smooth} = 0.0168. Then $H = h_f$ = 17.5 m, and the two previous equations can then be solved for

$$D_{pump} \approx \mathbf{0.39\text{ m}}\ (15.4\text{ in});\quad n \approx 15\text{ r/s} = \mathbf{900\text{ r/min}}\quad \textit{Ans. (a, b)}$$

(c) This blower is **too slow and too large**, a better (mixed or axial flow) pump can be designed. *Ans.* (c)

11.65 The 38-inch pump in Fig. 11.7*b* is used in *series* to lift 20°C water 3000 ft through 4000-ft of 18-inch-diameter cast iron pipe. For most efficient operation, how many pumps in series are needed if the rotation speed is (a) 710 rpm; and (b) 1200 rpm?

Solution: (a) At BEP in Fig. 11.7*b*, D = 38″, n = 710 rpm, read H* ≈ 225 ft and Q*≈ 20000 gpm = 44.6 ft^3/s. Take ε = 0.00085 ft. Evaluate the system-head loss at BEP flow:

$$V = \frac{44.6}{(\pi/4)(1.5)^2} \approx 25.2\ \frac{\text{ft}}{\text{s}},\quad \text{Re}_d = \frac{Vd}{\nu} = \frac{25.2(1.5)}{1.08\text{E–}5} \approx 3.51\text{E}6,$$

$$\frac{\varepsilon}{d} = \frac{0.00085}{1.5},\quad f \approx \mathbf{0.0173}$$

$$\text{Then}\quad h_{system} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 3000 + 0.0173\left(\frac{4000}{1.5}\right)\frac{(25.2)^2}{2(32.2)} = 3000 + 456 \approx 3456\text{ ft}$$

Since each pump provides 225 ft, we need 3456/225 ≈ **15 pumps** @ 710 rpm. *Ans.* (a)

(b) If we increase the speed to 1200 rpm at constant diameter, both H* and Q* change:

$$H^* = 225\left(\frac{1200}{710}\right)^2 \approx 643\text{ ft},\quad Q^* = 44.6\left(\frac{1200}{710}\right) \approx 75.3\ \frac{\text{ft}^3}{\text{s}},$$

$$V = 42.6\ \frac{\text{ft}}{\text{s}},\quad \text{Re}_d \approx 5.92\text{E}6,$$

$$\text{Read } f \approx 0.0237, \quad h_{syst} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} \approx 4785 \text{ ft} \div 645 \ \frac{\text{ft}}{\text{pump}}$$

$$\approx \mathbf{8\ pumps} \text{ needed} \quad \textit{Ans. (b)}$$

11.66 It is proposed to run the pump of Prob. 11.35 at 880 rpm to pump water at 20°C through the system of Fig. P11.66. The pipe is 20-cm diameter commercial steel. What flow rate in ft^3/min results? Is this an efficient operation?

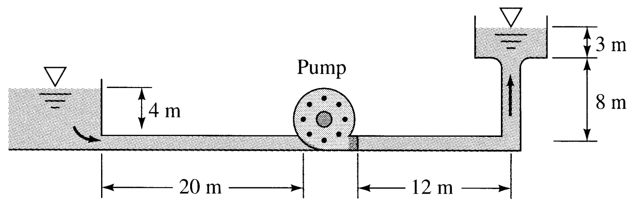


Fig. P11.66

Solution: For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For commercial steel, take ε = 0.046 mm. Write the energy equation for the system:

$$H_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 11 - 4 + f\frac{40}{0.2}\frac{[Q/(\pi(0.2)^2/4)]^2}{2(9.81)} = 7 + 10328fQ^2, \quad \left(Q \text{ in } \frac{\text{m}^3}{\text{s}}\right)$$

$$\text{Meanwhile, } H_p = fcn(Q)_{Prob.\ 11.35} \quad \text{and} \quad f = fcn\left(Re_d, \frac{\varepsilon}{d}\right)_{Moodychart}$$

where ε/d = 0.046/200 = 0.00023. If we guess f as the fully rough value of 0.0141, we find that H_p is about 60 ft and Q is about 9000 gal/min (0.57 m^3/s). To do better would require some very careful plotting and interpolating, or: **EES** is made for this job! Iteration with EES leads to the more accurate solution:

$$f = 0.0144; \quad V = 18.6\ \frac{\text{m}}{\text{s}}; \quad P = 162 \text{ hp}; \quad H_p = 58 \text{ ft}$$

$$\mathbf{Q = 9260\ \frac{gal}{min}} = 1240\ \frac{\text{ft}^3}{\text{min}} \quad \textit{Ans.}$$

The efficiency is **84%**, slightly off the maximum of 88% but not a bad system fit. *Ans.*

11.67 The pump of Prob. 11.35, running at 880 r/min, is to pump water at 20°C through 75 m of horizontal galvanized-iron pipe (ε = 0.15 mm). All other system losses are neglected. Determine the flow rate and input power for (a) pipe diameter = 20 cm; and (b) the pipe diameter yielding maximum pump efficiency.

Solution: (a) There is no elevation change, so the pump head matches the friction:

$$H_p = f\frac{L}{d}\frac{V^2}{2g} = f\frac{75}{0.2}\frac{[Q/(\pi(0.2)^2/4)]^2}{2(9.81)} = 19366fQ^2,\quad Re_d = \frac{4\rho Q}{\pi\mu d},\quad \frac{\varepsilon}{d} = \frac{0.15}{200} = 0.00075$$

But also H_p = fcn(Q) from the data in Prob. 11.35. Guessing f equal to the fully rough value of 0.0183 yields Q of about 7000 gal/min. Use **EES** to get closer:

$$f = 0.0185;\quad V = 14.4\ \frac{\text{m}}{\text{s}};\quad Re_d = 2.87E6;\quad H_p = 73\text{ ft};$$

$$\mathbf{Q = 7160\ \frac{gal}{min}} = 0.452\ \frac{\text{m}^3}{\text{s}}\quad \textit{Ans.}\text{ (a)}$$

The efficiency is 87%, not bad! (b) If we vary the diameter but hold the pump at maximum efficiency (Q* = 8000 gal/min), we obtain a best **d = 0.211 ft**. *Ans.* (b)

11.68 Suppose that we use the axial-flow pump of Fig. 11.13 to drive the leaf blower of Prob. 11.64. What approximate (a) diameter and (b) rotation speed are appropriate? (c) Is this a good design?

Solution: Recall that Fig. 11.13 gave BEP coefficients for an axial-flow pump:

$$C_Q^* \approx 0.55;\quad C_H^* \approx 1.07;\quad C_P^* \approx 0.70$$

Apply these coefficients to the leaf-blower data. Neglect minor losses, that is, let the pump head match the pipe friction loss. For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Convert 73 mi/h = 32.6 m/s, 4 ft = 1.22 m and 2.5 in = 0.0635 m:

$$h_f = H_{pump} = \mathbf{1.07}\frac{n^2D^2}{9.81\text{ m/s}^2} = f\frac{L}{d_{pipe}}\frac{V_{pipe}^2}{2g} = f\left(\frac{1.22\text{ m}}{0.0635\text{ m}}\right)\left[\frac{(32.6\text{ m/s})^2}{2(9.81\text{ m/s}^2)}\right],\quad f = f(\text{Re}_d)$$

$$Q = \frac{\pi}{4}d_{pipe}^2V = \frac{\pi}{4}(0.0635\text{ m})^2(32.6\text{ m/s}) = 0.103\ \frac{\text{m}^3}{\text{s}} = \mathbf{0.55}nD^3$$

We know the Reynolds number, $\text{Re}_d = \rho Vd/\mu$ = (1.2)(32.6)(0.0635)/(1.8E–5) = 138,000, and for a smooth pipe, from the Moody chart, calculate f_{smooth} = 0.0168. Then $H = h_f$ = 17.5 m, and the two equations above can then be solved for:

$$D_{\text{pump}} \approx \mathbf{0.122\ m}\ \ (4.8\text{ in});\quad n \approx 104\text{ r/s} = \mathbf{6250\ r/min}\quad \textit{Ans.}\text{ (a, b)}$$

This blower is **too fast and too small**, a better (mixed flow) pump can be designed. *Ans.* (c)

11.69 The pump of Prob. 11.38, running at 3500 rpm, is used to deliver water at 20°C through 600 ft of cast-iron pipe to an elevation 100 ft higher. Find (a) the proper pipe diameter for BEP operation; and (b) the flow rate which results if the pipe diameter is 3 inches.

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	81	79	74

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. (a) The data above *are* for 3500 rpm, with BEP at 350 gal/min:

$$H^* = 169\text{ ft} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 100 + f\frac{600}{d}\frac{[Q/(\pi d^2/4)]^2}{2(32.2)} = 100 + \frac{9.18f}{d^5},\quad Q = Q^* = \frac{350}{449}\frac{\text{ft}^3}{\text{s}}$$

$$\text{Iterate, converges to } Re_d = 2.87\text{E}5,\quad \frac{\varepsilon}{d} = 0.00265,\quad f = 0.0258,$$

$$\mathbf{d \approx 0.321\ ft = 3.85\ in} \quad \textit{Ans. (a)}$$

(b) If $d = 3$ inches, the above solution changes to a new flow rate:

$$\text{Curve-fit:}\quad H \approx 201 - 61Q^{2.7} = 100 + \frac{600}{0.25}\frac{[Q/(\pi(0.25)^2/4)]^2}{2(32.2)} = 100 + 15466fQ^2$$

$$\text{Iterate or EES: } f = 0.0277,\quad Re = 2.21\text{E}5,\quad H = 193\text{ ft},$$

$$\mathbf{Q} = 0.47\ \frac{\text{ft}^3}{\text{s}} = \mathbf{209\ \frac{gal}{min}} \quad \textit{Ans. (b)}$$

11.70 The pump of Prob. 11.28, operating at 2134 rpm, is used with water at 20°C in the system of Fig. P11.70. The diameter is 8 inches. (a) If it is operating at BEP, what is the proper elevation z_2? (b) If $z_2 = 225$ ft, what is the flow rate?

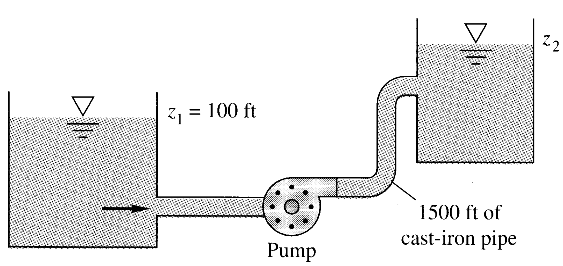


Fig. P11.70

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft, hence $\varepsilon/d \approx 0.00128$. (a) At BEP from Prob. 11.28, $Q^* \approx 6$ ft^3/s and $H^* \approx 330$ ft. Then the pipe head loss can be determined:

$$V = \frac{Q}{A} = \frac{6}{(\pi/4)(8/12)^2} = 17.2\ \frac{ft}{s},\quad Re_d = \frac{17.2(8/12)}{1.08E{-}5} = 1.06E6,\quad f_{Moody} \approx 0.0211$$

$$H_{syst} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = z_2 - 100 + 0.0211\left(\frac{1500}{8/12}\right)\frac{(17.2)^2}{2(32.2)} = z_2 - 100 + 218 \overset{?}{=} H^* = 330\text{ ft}$$

$$\text{Solve for}\quad z_2 \approx \mathbf{212\ ft}\quad \textit{Ans.}\ \text{(a)}$$

(b) If $z_2 = 225$ ft, the flow rate will be slightly lower and we will be barely off-design:

$$H = H(Q)_{Table} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 225 - 100 + f\frac{1500}{8/12}\frac{V^2}{2(32.2)},\quad V = \frac{Q}{(\pi/4)(8/12)^2}$$

$$\text{Converges to } f \approx 0.0211,\ V \approx 16.6\ \frac{ft}{s},\ H \approx 331\text{ ft},\ Q \approx \mathbf{5.8\ ft^3/s}\quad \textit{Ans.}\ \text{(b)}$$

11.71 The pump of Prob. 11.38, running at 3500 r/min, delivers water at 20°C through 7200 ft of horizontal 5-in-diameter commercial-steel pipe. There are a sharp entrance, sharp exit, four 90° elbows, and a gate valve. Estimate (a) the flow rate if the valve is wide open and (b) the valve closing percentage which causes the pump to operate at BEP. (c) If the latter condition holds continuously for 1 year, estimate the energy cost at 10 ¢/kWh.

Data at 3500 rpm:

Q, gal/min:	50	100	150	200	250	300	350	400	450
H, ft:	201	200	198	194	189	181	169	156	139
η, %:	29	50	64	72	77	80	81	79	74

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d \approx 0.00036$. The data above show BEP at 350 gal/min. The minor losses are a sharp entrance (K = 0.5), sharp exit (K = 1.0), 4 elbows (4×0.28), and an open gate valve (K = 0.1), or $\sum K \approx$ **2.72**. Pump and systems heads are equal:

$$H_p = H(Q)_{Table} = H_{syst} = \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right),\quad \text{where } V = Q/[(\pi/4)(5/12)^2],\ \frac{L}{d} \approx 17280$$

The friction factor $f \geq 0.0155$ depends slightly upon Q through the Reynolds number.

(a) Iterate on Q until both heads are equal. The result is

$$f \approx 0.0178,\ Re_d \approx 227000,\ H_p = H_{syst} \approx 167\text{ ft},\ \mathbf{Q \approx 359\ gal/min}\quad \textit{Ans.}\ \text{(a)}$$

(b) Bring Q down to BEP, ≈ 350 gpm, by increasing the gate-valve loss. The result is $f \approx 0.0179$, $Re_d \approx 221000$, $H \approx 169$ ft, $Q \approx 350$ gpm, $\mathbf{K_{valve} \approx 21}$ **(25% open)** *Ans.* (b)

(c) Continue case (b) for 1 year. What does it cost at 10¢ per kWh? Well, we know the power level is exactly BEP, so just figure the energy:

$$P = \frac{\rho g Q H}{\eta} = \frac{62.4(350/449)(169)}{0.81} \approx 10152\ \frac{\text{ft·lbf}}{\text{s}} = 13.8\text{ kW}$$

$$\text{Annual cost} = 13.8(365\text{ days*24 hours})(\$0.1/\text{kWh}) \approx \mathbf{\$12,100} \quad \textit{Ans. (c)}$$

11.72 Performance data for a small commercial pump are shown below. The pump supplies 20°C water to a horizontal 5/8-in-diameter garden hose ($\varepsilon \approx 0.01$ in) which is 50 ft long. Estimate (a) the flow rate; and (b) the hose diameter which would cause the pump to operate at BEP.

Q, gal/min:	0	10	20	30	40	50	60	70
H, ft:	75	75	74	72	68	62	47	24

Solution: For water at 20°C, take ρ = 1.94 slug/ft³ and μ = 2.09E–5 slug/ft·s. Given $\varepsilon/d = 0.01/(5/8) \approx 0.016$, so f(fully rough) ≈ 0.045. Pump and hose heads must equate, including the velocity head in the outlet jet:

$$H_p = H(Q)_{Table} = H_{syst} = \frac{V^2}{2g}\left(f\frac{L}{d}+1\right), \quad \frac{L}{d} = \frac{50}{5/8/12} \approx 960, \quad V = \frac{Q}{(\pi/4)(5/8/12)^2}$$

(a) Iterate on Q until both heads are equal. The result is:

$$f \approx 0.0456, \quad Re_d \approx 50440, \quad V \approx 10.4\ \frac{\text{ft}}{\text{s}}, \quad H \approx 75\text{ ft}, \quad \mathbf{Q \approx 10\ gal/min} \quad \textit{Ans. (a)}$$

The hose is too small.

(b) We don't know *exactly* where the BEP is, but it typically lies at about 60% of maximum flow rate (0.6 × 77 ≈ **45** gpm). Iterate on hose diameter D to make the flow rate lie between 40 and 50 gal/min. The results are:

Q = 40 gal/min,	H = 68 ft,	f ≈ 0.0362,	Re ≈ 104000,	D_{hose} ≈ 1.22 inches	
45	65	0.0354	109000	1.30 inches	*Ans.* (b)
50	62	0.0348	114000	1.38 inches	

11.73 The piston pump of Fig. P11.9 (at right) is run at 1500 rpm to deliver SAE 10W oil through 100 m of vertical 2-cm-diameter wrought-iron pipe. If other system losses are neglected, estimate (a) the flow rate; (b) the pressure rise; and (c) the power required.

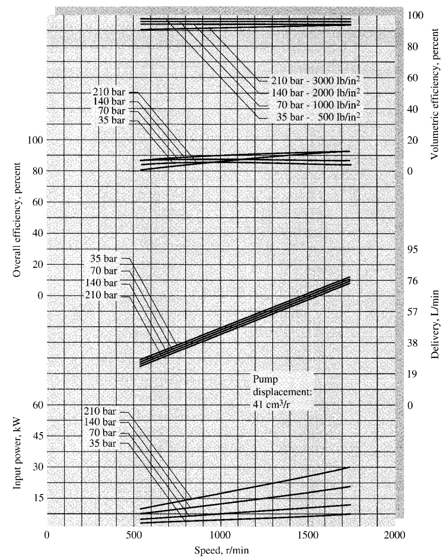


Fig. P11.9

Solution: For SAE-10W oil, take $\rho \approx 870\ \text{kg/m}^3$ and $\mu \approx 0.104$ kg/m·s. For wrought iron, $\varepsilon \approx 0.046$ m. From the figure, at 1500 rpm, the delivery is about 67 ± 2 L/min over the whole pressure range. Use this to estimate

$$Q \approx \mathbf{67}\ \frac{\mathbf{L}}{\mathbf{min}} = \mathbf{0.00112}\ \frac{\mathbf{m^3}}{\mathbf{s}},\quad V = \frac{Q}{A} = \frac{0.00112}{(\pi/4)(0.02)^2} \approx 3.55\ \frac{m}{s}$$

$$Re_d = \frac{870(3.55)(0.02)}{0.104} \approx 595\ (\textbf{laminar}),\quad H_s = \Delta z + \frac{128\mu LQ}{\pi\rho g d^4} \approx 100 + 348 \approx 448\ \text{m}$$

$$\text{Therefore}\quad \Delta p = \rho g H_s = 870(9.81)(448) \approx \mathbf{38.2\ bar}$$

All of this checks out pretty well, we accept: $Q \approx$ **67 L/min**, $\Delta p \approx$ **38 bar**. *Ans.* (a, b)
From the figure, the overall efficiency is about 84% at 1500 rpm and 35 bar. Thus

$$\text{Power} = \frac{\rho g Q H}{\eta} = \frac{870(9.81)(0.00112)(448)}{0.84} \approx \mathbf{5100\ W}\quad \textit{Ans. } (c)$$

11.74 The 32-in-diameter pump in Fig. 11.7*a* is used at 1170 rpm in a system whose head curve is $H_s(\text{ft}) \approx 100 + 1.5Q^2$, with Q in kgal/min. Find the discharge and brake horsepower required for (a) one pump; (b) two pumps in parallel; and (c) two pumps in series. Which configuration is best?

Solution: Assume plain old water, $\rho g \approx 62.4\ \text{lbf/ft}^3$. A reasonable curve-fit to the pump head is taken from Fig. 11.7*a*: $H_p(\text{ft}) \approx 500 - 0.3Q^2$, with Q in kgal/min. Try each case:
(a) **One** pump: $H_p = 500 - 0.3Q^2 = H_s = 100 + 1.5Q^2$: $Q \approx$ **14.9 kgal/min** *Ans.* (a)
(b) Two pumps in **parallel**: $500 - 0.3(Q/2)^2 = 100 + 1.5Q^2$, $Q \approx$ **15.9 kgal/min** *Ans.* (b)
(c) Two pumps in **series**: $2(500 - 0.3Q^2) = 100 + 1.5Q^2$: $Q \approx$ **20.7 kgal/min** *Ans.* (c)
Clearly **case(c) is best,** because it is very near the BEP of the pump. *Ans.*

11.75 Two 35-inch pumps from Fig. 11.7*b* are installed in parallel for the system of Fig. P11.75. Neglect minor losses. For water at 20°C, estimate the flow rate and power required if (a) both pumps are running; and (b) one pump is shut off and isolated.

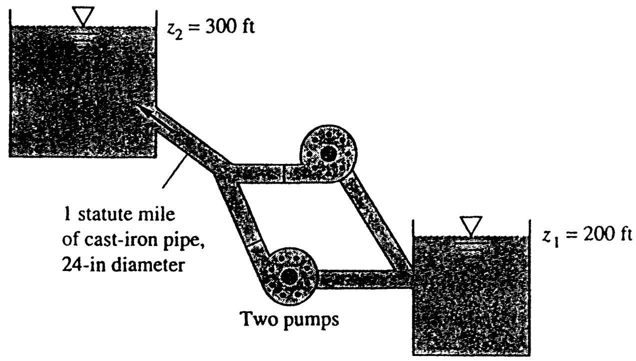


Fig. P11.75

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For cast iron, $\varepsilon \approx$ 0.00085 ft, or ε/d ≈ 0.000425. The 35-inch pump has the curve-fit head relation $H_p(\text{ft}) \approx 235 - 0.006Q^3$, with Q in kgal/min. In parallel, each pump takes Q/2:

$$H_p = \text{fcn}\left(\frac{1}{2}Q\right)_{\text{curve-fit}} = H_{\text{syst}} = \Delta z + f\frac{L}{d}\frac{V^2}{2g}, \quad \frac{L}{d} = 2640$$

$$\text{and} \quad V = \frac{Q\ (\text{ft}^3/\text{s})}{(\pi/4)(2\ \text{ft})^2}, \quad \Delta z = 100\ \text{ft}$$

(a) Two pumps: Iterate on Q (for Q/2 each) until both heads are equal. The results are:

$$f \approx 0.0164,\ Re_d \approx 2.29\text{E}6,\ H_p = H_s = 229\ \text{ft}$$

$$Q_{\text{total}} \approx \mathbf{19600\ \frac{gal}{min}}\ \text{(9800 for each pump)} \quad \textit{Ans. (a)}$$

$$P = 2P_{\text{each}} = 2\left[\frac{62.4(9800/449)(229)}{0.73}\right] = 855000 \div 550 \approx \mathbf{1550\ bhp} \quad \textit{Ans. (a)}$$

(b) One pump: Iterate on Q alone until both heads are equal. The results are:

$$f \approx 0.0164,\ Re \approx 2.3\text{E}6,\ H \approx 203\ \text{ft},\ Q \approx \mathbf{17500\ gal/min} \quad \textit{Ans. (b)}$$

$$P = 62.4(17500/449)(203)/0.87 \div 550 \approx \mathbf{1030\ bhp} \quad \textit{Ans. (b)}$$

The pumps in parallel give 12% more flow at the expense of **50%** more power.

11.76 Two 32-inch pumps are combined in parallel to deliver water at 20°C through 1500 ft of horizontal pipe. If *f* = 0.025, what pipe diameter will ensure a flow rate of 35,000 gal/min at 1170 rpm?

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. As in Prob. 11.74, a reasonable curve-fit to the pump head is taken from Fig. 11.7*a*: H_p(ft) ≈ 500 − 0.3Q^2, with Q in kgal/min. Each pump takes half the flow, 17,500 gal/min, for which

$$H_p \approx 500 - 0.3(17.5)^2 \approx 408 \text{ ft.} \quad \text{Then } Q_{pipe} = \frac{35000}{449} = 78 \ \frac{ft^3}{s} \text{ and the pipe loss is}$$

$$H_{syst} = 0.025\left(\frac{1500}{d}\right)\frac{[78/(\pi d^2/4)]^2}{2(32.2)} = \frac{5740}{d^5} = 408 \text{ ft}, \quad \text{solve for } \mathbf{d \approx 1.70\ ft} \quad \textit{Ans.}$$

11.77 Two pumps of the type tested in Prob. 11.22 are to be used at 2140 r/min to pump water at 20°C vertically upward through 100 m of commercial-steel pipe. Should they be in series or in parallel? What is the proper pipe diameter for most efficient operation?

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For commercial steel take ε = 0.046 mm. **Parallel operation is not feasible**, as the pump can barely generate 100 m of head and the friction loss must be added to this. For **series** operation, assume BEP operation, Q* = 0.2 m^3/s, H* = 95 m:

$$H_{2\,pumps} = 2(95) = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 100 + f\frac{100}{d}\frac{[0.20/(\pi d^2/4)]^2}{2(9.81)} = 100 + \frac{0.3305f}{d^5}$$

Given $Re_d = 4\rho Q/(\pi\mu d) = 4(998)(0.2)/[\pi(0.001)d] = 254000/d$ and $\varepsilon/d = 0.000046/d$, we can iterate on f until d is obtained. The **EES** result is:

f = 0.0156; Re_d = 1.8E6; V = 12.7 m/s, $\mathbf{d_{best} = 0.142\ m}$ *Ans.*

11.78 Suppose that the two pumps in Fig. P11.75 are instead arranged to be in *series*, again at 710 rpm? What pipe diameter is required for BEP operation?

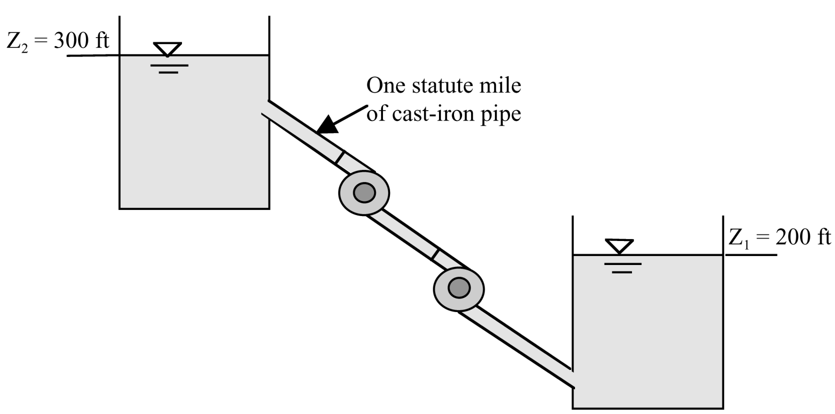


Fig. P11.75

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 35-inch pump has the BEP values Q* ≈ 18 kgal/min, H* ≈ 190 ft. In *series*, each pump takes H/2, so a BEP series operation would match

$$H_{syst} = 2H^* = 2(190) = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 100 + f\left(\frac{5280}{d}\right)\frac{[18000/449/(\pi d^2/4)]^2}{2(32.2)},$$

$$\text{or:}\quad 380 = 100 + \frac{213800f}{d^5},\quad \text{where } f \text{ depends on } Re = \frac{4\rho Q}{\pi d\mu} \text{ and } \frac{\varepsilon}{d} = \frac{0.00085}{d}$$

This converges to f ≈ 0.0169, Re ≈ 2.84E6, V ≈ 18.3 ft/s, **d ≈ 1.67 ft**. *Ans.*

$$\text{Power} = 2P^* = 2\frac{62.4(18000/449)(190)}{0.87} = 1.09E6 \div 550 \approx \mathbf{2000\ bhp}$$

We can save money on the smaller (20-inch) pipe, but putting the pumps in *series* requires twice as much power as one pump alone (Prob. 11.75, part *b*).

11.79 Two 32-inch pumps from Fig. 11.7*a* are to be used in *series* at 1170 rpm to lift water through 500 ft of vertical cast-iron pipe. What should the pipe diameter be for most efficient operation? Neglect minor losses.

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. From Fig. 11.7a, read H* ≈ 385 ft at Q* ≈ 20,000 gal/min. Equate

$$H_p = 2H^* = 2(385) = H_{syst} = \Delta z + f\frac{L}{d}\frac{V^2}{2g}$$

$$= 500 + f\left(\frac{500}{d}\right)\frac{\left[\frac{20000}{449}(\pi d^2/4)\right]^2}{2(32.2)} = 500 + \frac{24992f}{d^5}$$

Iterate, guessing f ≈ 0.02 to get *d*, then get Re_d and ε/d and repeat. The final result is

$$f \approx 0.0185,\quad V \approx 45.8 \text{ ft/s},\quad Re_d \approx 4.72E6,\quad \mathbf{d \approx 1.11\ ft}\quad \textit{Ans.}$$

11.80 It is proposed to use one 32- and one 28-in pump from Fig. 11.7*a* in parallel to deliver water at 60°F. The system-head curve is $H_s = 50 + 0.3Q^2$, with Q in thousands of gallons per minute. What will the head and delivery be if both pumps run at 1170 r/min? If the 28-in pump is reduced below 1170 r/min, at what speed will it cease to deliver?

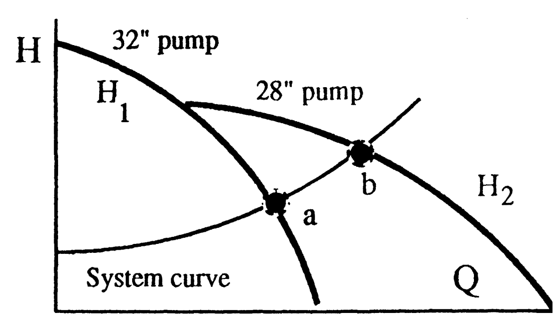


Fig. P11.80

Solution: For water at 60°F, take ρ =1.94 slug/ft^3. Use the following two curve-fits:

$$D = 32'': \ H_1 \approx 500 - 0.3Q_1^2; \quad D = 28'': \ H_2 \approx 360 - 0.24Q_2^2, \quad (Q \text{ in kgal/ min})$$

For pumps in parallel, the flow rates *add* and the pumps heads are *equal*:

$$\text{Continuity: } Q_1 + Q_2 = Q_{syst}; \quad \text{Heads: } H_1 = H_2 = H_{syst}$$

$$\therefore \quad 500 - 0.3Q_1^2 = 360 - 0.24Q_2^2 = 50 + 0.3(Q_1 + Q_2)^2$$

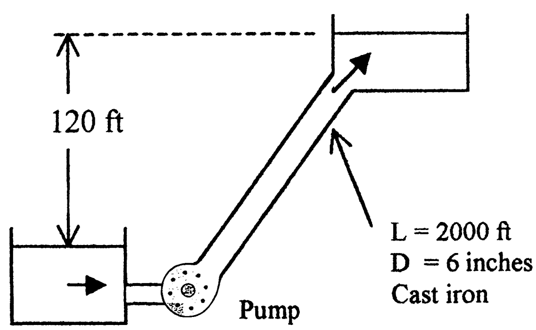
$$\text{Solve for:} \quad Q_1 \approx \mathbf{22900\ gpm}; \quad Q_2 \approx \mathbf{8400\ gpm}; \quad H \approx \mathbf{343\ ft} \quad \textit{Ans.}$$

If pump "2" reduces speed, it ceases to deliver (Q_2 = 0) when its shut-off head equals **point "a"** in the figure on previous page, where the system curve crosses pump-head "1." Thus:

$$500 - 0.3Q_1^2 \stackrel{?}{=} 50 + 0.3Q_1^2 \quad \text{or:} \quad Q_1 \approx \mathbf{27400\ gal/min} \quad \text{and} \quad H_2 = H_1 \approx \mathbf{275\ ft}.$$

$$\text{Pump-2 speed is } n_{2\text{-new}} = n_{2\text{-old}}\left(\frac{H_{2\text{-new}}}{H_{2\text{-old}}}\right)^{1/2} = 1170\left(\frac{275}{360}\right)^{1/2} \approx \mathbf{1020\ rpm} \quad \textit{Ans.}$$

11.81 Reconsider the system of Fig. P6.62. Use the Byron Jackson pump of Prob. 11.28 running at 2134 r/min, no scaling, to drive the flow. Determine the resulting flow rate between the reservoirs. What is the pump efficiency?



Solution: For water take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For cast iron take ε = 0.00085 ft, or ε/d = 0.00085/0.5 = 0.0017. The energy equation, written between reservoirs, is the same as in Prob. 6.62:

$$H_p = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\text{ ft} + f\frac{2000}{0.5}\frac{[Q/(\pi(0.5)^2/4)]^2}{2(32.2)} = 120 + 1611fQ^2$$

$$\text{where} \quad f = f_{Moody} = fcn\left(Re_d, \frac{\varepsilon}{d}\right) \quad \text{with } Re_d = \frac{4\rho Q}{\pi\mu d}$$

From the data in Prob. 11.28 for the pump, Hp = fcn(Q) and is of the order of 300 ft. Guessing $f \approx 0.02$, we can estimate a flow rate of about $\mathbf{Q \approx 2.4\ ft^3/s}$, down in the low range of the Byron-Jackson pump. Get a closer result with EES:

$$H_p = 340\text{ ft};\ f = 0.0228;\ Re_d = 579{,}000;$$

$$V = 12.5\ \frac{\text{ft}}{\text{s}};\ \text{bhp} = 169\text{ hp};\ \mathbf{Q = 2.45\ \frac{ft^3}{s}}\quad \textit{Ans.}$$

Interpolating, the pump efficiency is $\eta \approx$ **56%**. Ans. The flow rate is too low for this particular pump.

11.82 The S-shaped head-versus-flow curve in Fig. P11.82 occurs in some axial-flow pumps. Explain how a fairly *flat* system-loss curve might cause instabilities in the operation of the pump. How might we avoid instability?

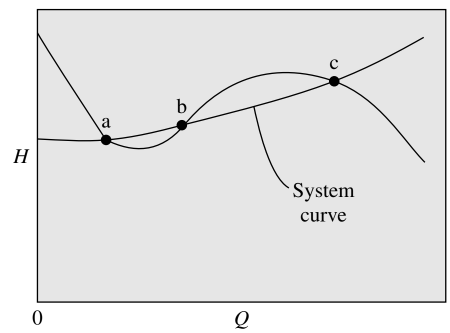


Fig. P11.82

Solution: The stability of pump operation is nicely covered in the review article by Greitzer (Ref. 41 of Chap. 11). Generally speaking, there is little danger of instability if the slope of the pump-head curve, dH/dQ, is *negative*, unless there are two such points. In Fig. P11.82 above, a flat system curve may cross the pump curve at *three* points (a, b, c). Of these 3, **point *b* is statically unstable** and cannot be maintained. Consider a small disturbance near point *b*: Suppose the flow rate drops slightly—then the system head decreases, but the pump head decreases *even more*. Then the flow rate will drop still *more*, etc., and we move away from the operating point, which therefore is *unstable*. The general rule is:

A pump operating point is statically unstable if the (positive) slope of the pump-head curve is greater than the (positive) slope of the system curve.

By this criterion, both points *a* and *c* above are statically stable. However, if the points are close together or there are large disturbances, a pump can "hunt" or oscillate between points *a* and *c*, so this could also be considered **unstable to large disturbances**.

Finally, even a steep system curve (not shown above) which crosses at only a single point *b* on the positive-slope part of the pump-head curve can be dynamically unstable, that is, it can trigger an energy-feeding oscillation which diverges from point *b*. See Greitzer's article for further details of this and other turbomachine instabilities.

11.83 The low-shutoff head-versus-flow curve in Fig. P11.83 occurs in some centrifugal pumps. Explain how a fairly flat system-loss curve might cause instabilities in the operation of the pump. What additional vexation occurs when two of these pumps are in parallel? How might we avoid instability?

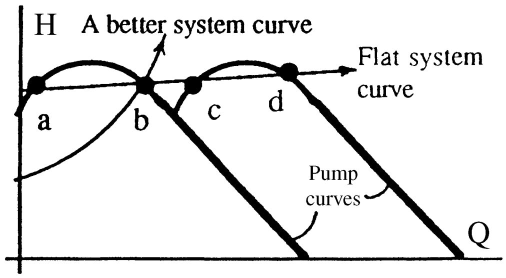


Fig. P11.83

Solution: As discussed, for one pump with a flat system curve, point *a* is statically unstable, point *b* is stable. A 'better' system curve only passes through *b*.

For two pumps in parallel, both points *a* and *c* are unstable (see above). Points *b* and *d* are stable but for large disturbances the system can 'hunt' between the two points.

11.84 Turbines are to be installed where the net head is 400 ft and the flow rate is 250,000 gal/min. Discuss the type, number, and size of turbine which might be selected if the generator selected is (a) 48-pole, 60-cycle (n =150 rpm); or (b) 8-pole (n = 900 rpm). Why are at least two turbines desirable from a planning point of view?

Solution: We select **two** turbines, of about half-flow each, so that one is still available for power generation if the other is shut down for maintenance or repairs. *Ans.*

Assume $\eta \approx 90\%$:

$$P_{total} = \eta\rho gQH \approx 0.9(62.4)\left(\frac{250000}{449}\right)(400) \div 550 \approx 22750 \text{ hp (each turbine} \approx 11375 \text{ hp)}$$

(a) n = 150 rpm:

$$N_{sp} = \frac{nP^{1/2}}{H^{5/4}} = \frac{150(11375)^{1/2}}{(400)^{5/4}} \approx \mathbf{8.9} \text{ (select two } \textbf{impulse turbines}\text{)} \quad \textit{Ans.} \text{ (a)}$$

$$\text{Estimate} \quad \phi \approx 0.47 = \frac{\pi nD}{\sqrt{2gH}} = \frac{\pi(150/60)D}{\sqrt{2(32.2)(400)}}, \quad \text{or:} \quad \mathbf{D \approx 9.6\ ft} \quad \textit{Ans.} \text{ (a)}$$

$$\text{(b) n = 900 rpm:} \quad N_{sp} = \frac{900(11375)^{1/2}}{(400)^{5/4}} \approx \mathbf{54} \text{ (select } \textbf{two Francis turbines}\text{)} \quad \textit{Ans.} \text{ (b)}$$

$$\text{Fig. 11.21:} \quad C_p^* \approx 2.6 = \frac{P^*}{\rho n^3 D^5} = \frac{11375 \times 550}{1.94\left(\frac{900}{60}\right)^3 D^5}, \quad \text{solve } \mathbf{D \approx 3.26\ ft} \quad \textit{Ans.} \text{ (b)}$$

11.85 Turbines at the Conowingo plant on the Susquehanna River each develop 54,000 bhp at 82 rpm under a head of 89 ft. What type of turbines are these? Estimate the flow rate and impeller diameter.

Solution: The turbine specific speed tells us the type and the power tells us the flow rate:

$$N_{sp} = \frac{nP^{1/2}}{H^{5/4}} = \frac{82(54000)^{1/2}}{(89)^{5/4}} \approx \mathbf{70} \text{ (These are } \textbf{Francis turbines}\text{)} \quad \textit{Ans.}$$

$$\text{Fig. 11.27: } \eta \approx 93\%, \quad P = 54000 \times 550 = 0.93(62.4)(89)Q,$$

$$\text{or: } \mathbf{Q \approx 5800 \ \frac{ft^3}{s}} \quad \textit{Ans.}$$

$$\text{Fig. 11.21: } C_P^* \approx 2.6 = \frac{P^*}{\rho n^3 D^5} = \frac{54000(550)}{1.94\left(\frac{82}{60}\right)^3 D^5}, \quad \text{or: } \mathbf{D \approx 19\ ft} \quad \textit{Ans.}$$

11.86 The Tupperware hydroelectric plant on the Blackstone River has four 36-inch-diameter turbines, each providing 447 kW at 200 rpm and 205 ft^3/s for a head of 30 ft. What type of turbines are these? How does their performance compare with Fig. 11.21?

Solution: Convert P* = 447 kW = 599 hp. Then, for D = 36″ = 3.0 ft,

$$N_{sp} = \frac{nP^{1/2}}{H^{5/4}} = \frac{200(599)^{1/2}}{(30)^{5/4}} \approx \mathbf{70} \text{ (These are } \textbf{Francis turbines}\text{)} \quad \textit{Ans.}$$

Use the given data to compute the dimensionless BEP coefficients:

$$C_Q^* = \frac{205}{(200/60)(3.0)^3} \approx \mathbf{2.3}; \quad C_P^* = \frac{599(550)}{1.94(200/60)^3(3)^5} \approx \mathbf{19} \text{ (both are } \textbf{quite different!}\text{)}$$

$$C_H^* = \frac{32.2(30)}{(200/60)^2(3)^2} \approx \mathbf{9.7} \ (\mathbf{OK}); \quad \eta_{max} = \frac{599(550)}{62.4(205)(30)} \approx \mathbf{86\%} \ (\mathbf{OK}) \quad \textit{Ans.}$$

11.87 An idealized radial turbine is shown in Fig. P11.87. The absolute flow enters at 30° and leaves radially inward. The flow rate is 3.5 m^3/s of water at 20°C. The blade thickness is constant at 10 cm. Compute the theoretical power developed at 100% efficiency.

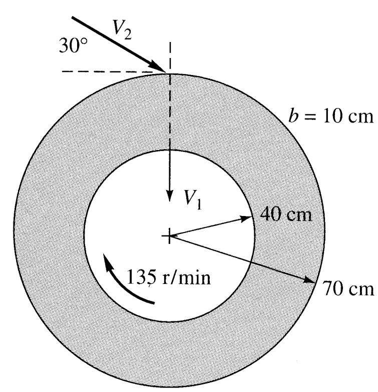


Fig. P11.87

Solution: For water, take $\rho \approx 998\ \text{kg/m}^3$. With reference to Fig. 11.22 and Eq. 11.35,

$$u_2 = \omega r_2 = 135\left(\frac{2\pi}{60}\right)(0.7) = 9.90\ \frac{m}{s},\ \alpha_2 = 30°,\ \alpha_1 = 90°,\ V_{n2} = \frac{3.5}{2\pi(0.7)(0.1)} \approx 7.96\ \frac{m}{s}$$

$$V_{t2} = \frac{V_{n2}}{\tan\alpha_2} = \frac{7.96}{\tan 30°} = 13.8\ \frac{m}{s} \quad \text{and} \quad V_{t1} = \frac{V_{n1}}{\tan 90°} = 0$$

$$\text{Thus } P_{theory} = \rho Q u_2 V_{t2} = 998(3.5)(9.90)(13.8) = 477000\ \text{W} \approx \mathbf{477\ kW} \quad \textit{Ans.}$$

11.88 Performance data for a very small (D = 8.25 cm) model water turbine, operating with an available head of 49 ft, are as follows:

Q, m^3/h:	18.7	18.7	18.5	18.3	17.6	**16.7**	15.1	11.5
rpm:	0	500	1000	1500	2000	**2500**	3000	3500
η:	0	14%	27%	38%	50%	**65%**	61%	11%

(a) What type of turbine is this likely to be? (b) What is so different about this data compared to the dimensionless performance plot in Fig. 11.21*d*? Suppose it is desired to use a geometrically similar turbine to serve where the available head and flow are 150 ft and 6.7 ft^3/s, respectively. Estimate the most efficient (c) turbine diameter; (d) rotation speed; and (e) horsepower.

Solution: (a) Convert Q = 16.7 m^3/h = 0.164 ft^3/s. Use BEP data to calculate power specific speed:

$$bhp = \rho g Q H \eta = (62.4\ \text{lbf/ft}^3)(0.164\ \text{ft}^3\text{/s})(49\ \text{ft})(0.65) = 326\ \text{ft·lbf/s} = 0.593\ \text{hp}$$

$$N_{sp} = \frac{\text{rpm(bhp)}^{1/2}}{(H\text{-ft})^{5/4}} = \frac{(2500\ \text{rpm})(0.593\ \text{bhp})^{1/2}}{(49\ \text{ft})^{5/4}} \approx 15\ (\textbf{Francis turbine}) \quad \textit{Ans.}\ \text{(a)}$$

(b) This data is different because it has ***variable speed***. Our other data is at constant speed. *Ans*. (b)

(c, d, e) First establish the BEP coefficients from the small-turbine data:

$$C_Q^* = \frac{Q}{nD^3} = \frac{(16.7/3600\ \text{m}^3\text{/h})}{(2500/60\ \text{r/s})(0.0825\ \text{m})^3} = \mathbf{0.198};$$

$$C_H^* = \frac{gH}{n^2D^2} = \frac{(9.81)(49*0.3048\ \text{m})}{(2500/60)^2(0.0825)^2} = \mathbf{12.4}$$

$$C_P^* = \frac{P}{\rho n^3 D^5} = \frac{(326*1.3558\ \text{W})}{(998\ \text{kg/m}^3)(2500/60\ \text{r/s})^3(0.0825\ \text{m})^5} = \mathbf{1.60}$$

Now enter the new data, in *English* units, to the flow and head coefficients:

$$C_Q^* = \frac{6.7\ \text{ft}^3/\text{s}}{n_2 D_2^3} = 0.198; \quad C_H^* = \frac{(32.2\ \text{ft/s}^2)(150\ \text{ft})}{n_2^2 D_2^2} = 12.4$$

$$\text{Solve for} \quad n_2 = 15.1\ \text{r/s} = \mathbf{904\ r/min}; \quad D_2 = 1.31\ \text{ft} = \mathbf{15.7\,in} \quad \textit{Ans.}\ \text{(c, d)}$$

$$\text{Then} \quad P_2 = C_P^* \rho n_2^3 D_2^5 = 1.60(1.94\ \text{slug/ft}^3)(15.1\ \text{r/s})^3(1.31\ \text{ft})^5$$

$$P_2 = 41{,}000\ \frac{\text{ft·lbf}}{\text{s}} = \mathbf{74\ hp} \quad \textit{Ans.}\ \text{(e)}$$

Actually, since the new turbine is 4.84 times larger, we could use the Moody step-up formula, Eq. (11.29a), to predict $(1 - \eta_2) \approx (1 - 0.65)/(4.84)^{1/4} = 0.236$, or $\eta_2 = 0.764 = 76.4\%$. We thus expect more power from the larger turbine:

$$P_2 = (74\ \text{hp})(76\%/65\%) \approx \mathbf{87\ hp}\ \textit{Better} \quad \textit{Ans.}\ \text{(e)}$$

11.89 A Pelton wheel of 12-ft pitch diameter operates under a new head of 2000 ft. Estimate the speed, power output, and flow rate for best efficiency if the nozzle exit diameter is 4 inches.

Solution: First get the jet velocity and then assume BEP at $\phi \approx 0.47$:

$$C_v \approx 0.94, \quad \text{so:} \quad V_{jet} = C_v\sqrt{2gH} = 0.94\sqrt{2(32.2)(2000)} \approx 337\ \text{ft/s}$$

$$\text{BEP at} \quad \phi \approx 0.47 = \frac{\pi nD}{\sqrt{2gH}} = \frac{\pi n(12.0)}{\sqrt{2(32.2)(2000)}},$$

$$\text{Solve} \quad n = 4.47\ \frac{r}{s} \times 60 \approx \mathbf{268\,rpm} \quad \textit{Ans.}$$

$$Q = V_{jet} A_{nozzle} = 337\left(\frac{\pi}{4}\right)\left(\frac{4}{12}\right)^2 = 29.4\ \frac{\text{ft}^3}{\text{s}} \approx \mathbf{13200}\ \frac{\textbf{gal}}{\textbf{min}} \quad \textit{Ans.}$$

$$u_{best} = V_{jet}/2 \approx 169\ \text{ft/s}$$

$$P_{theory} = \rho Qu(V_j - u)(1 - \cos\beta)$$
$$\approx 1.94(29.4)(169)(337 - 169)(1 - \cos 165°) \div 550 \approx 5800\ \text{hp}_{100\%}$$

$$P_{turb} \approx 4500\ \text{bhp}, \quad N_{sp} \approx \frac{268(4500)^{1/2}}{(2000)^{5/4}} \approx 1.34: \quad \text{Fig. 11.27, read } \eta_{max} < 80\%,\ \text{say, } 75\%$$

$$\text{Actual power output} \approx \eta P_{100\%} = 0.75(5800) \approx \mathbf{4350\ bhp} \quad \textit{Ans.}$$

11.90 An idealized radial turbine is shown in Fig. P11.90. The absolute flow enters at 25° with the blade angles as shown. The flow rate is 8 m^3/s of water at 20°C. The blade thickness is constant at 20 cm. Compute the theoretical power developed at 100% efficiency.

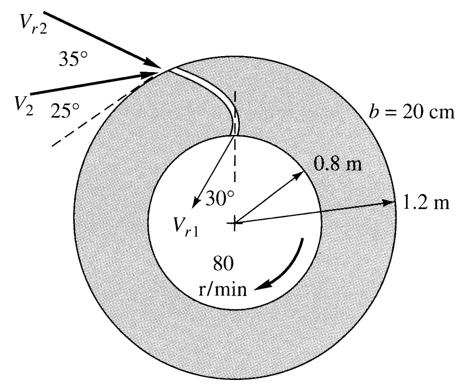
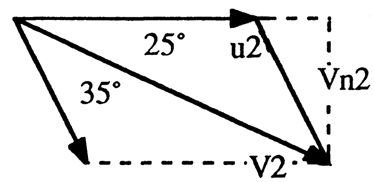
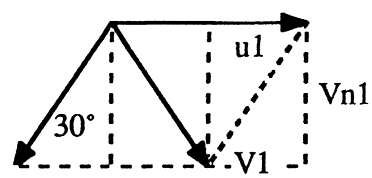


Fig. P11.90

Solution: The inlet (2) and outlet (1) velocity vector diagrams are shown at right. The normal velocities are





$$V_{n2} = Q/A_2 = \frac{8.0}{2\pi(1.2)(0.2)} = 5.31 \text{ m/s}$$

$$V_{n1} = Q/A_1 = \frac{8.0}{2\pi(0.8)(0.2)} = 7.96 \text{ m/s}$$

From these we can compute the tangential velocities at each section:

$$u_2 = \omega r_2 = 80\left(\frac{2\pi}{60}\right)(1.2) = 10.1 \text{ m/s};$$

$$u_1 = \omega r_1 = 80\left(\frac{2\pi}{60}\right)(0.8) = 6.70 \text{ m/s}$$

$$V_{t2} = V_{n2}\cot 25° = 11.4 \text{ m/s}; \quad V_{t1} = u_1 - V_{n1}\tan 30° = 2.11 \text{ m/s}$$

$$P_{theory} = \rho Q(u_2V_{t2} - u_1V_{t1}) = 998(8)[10.1(11.4) - 6.7(2.11)] \approx \mathbf{800{,}000\ W} \quad \textit{Ans.}$$

11.91 The flow through an axial-flow *turbine* can be idealized by modifying the stator-rotor diagrams of Fig. 11.12 for energy absorption. Sketch a suitable blade and flow arrangement and the associated velocity vector diagrams. For further details, see Chap. 8 of Ref. 25.

Solution: Some typical velocity diagrams are shown on the next page, where u = ωr = blade speed. The power delivered to the turbine, at 100% ideal shock-free flow, is

$$P_{ideal} = \rho Q(u_1V_{t1} - u_2V_{t2})$$

where $V_{t1,2}$ are the tangential components of $V_{1,2}$

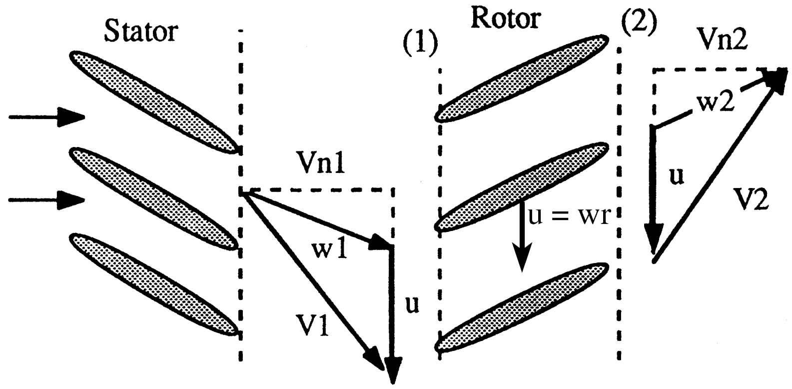


Fig. P11.91

11.92 A dam on a river is being sited for a hydraulic turbine. The flow rate is 1500 m^3/h, the available head is 24 m, and the turbine speed is to be 480 r/min. Discuss the estimated turbine size and feasibility for (a) a Francis turbine; and (b) a Pelton wheel.

Solution: Assume $\eta \approx 89\%$, as in Fig. 11.21*d*. The power generated by the turbine would be $P = \eta\gamma QH$ = (0.89)(62.4 lbf/ft^3)(14.7 ft^3/s)(78.7 ft) = 64,300 ft-lbf/s = 117 hp. Now compute $\mathbf{N_{sp}}$ = (480 rpm)(117 hp)$^{1/2}$/(78.7 ft)$^{5/4}$ ≈ **22**, appropriate for a Francis turbine. (a) A Francis turbine, similar to Fig. 11.21*d*, would have $C_Q^* \approx 0.34$ = (14.7 ft^3/s)/[(480/60 r/s)D^3]. Solve for a turbine diameter of about **1.8 ft**, which would be excellent for the task. *Ans.* (a)
(b) A Pelton wheel at best efficiency (half the jet velocity) would only be **18 inches** in diameter, with a huge nozzle, $d \approx 6$ inches, which is too large for the wheel. We conclude that a Pelton wheel would be a poor design. *Ans.* (b)

11.93 Figure P11.93 shown on the following page, shows a *crossflow* or “Banki” turbine [Ref. 55], which resembles a squirrel cage with slotted curved blades. The flow enters at about 2 o’clock, passes through the center and then again through the blades, leaving at about 8 o’clock.

Report to the class on the operation and advantages of this design, including idealized velocity vector diagrams.

Brief Discussion (not a “Solution”):
The crossflow turbine is ideal for small dam owners, because of its simple, inexpensive design. It can easily be constructed by a novice (such as the writer) from wood and plastic.

It is not especially efficient (≈ 60%) but makes good, inexpensive use of a small stream to produce electric power. For details, see Ref. 55 or the paper "Design and Testing of an Inexpensive Crossflow Turbine," by W. Johnson et al., ASME Symposium on Small Hydropower Fluid Machinery, Phoenix, AZ, Nov. 1982, ASME vol. H00233, pp. 129–133.

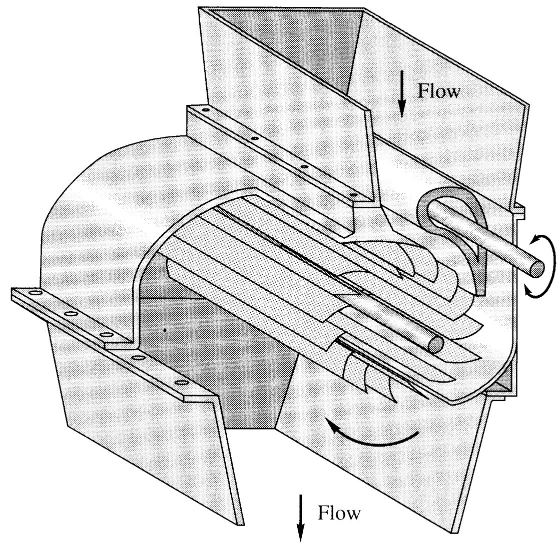


Fig. P11.93

11.94 A simple crossflow turbine, Fig. P11.93 above, was constructed and tested at the University of Rhode Island. The blades were made of PVC pipe cut lengthwise into three 120°-arc pieces. When tested in water at a heads of 5.3 ft and a flow rate of 630 gal/min, the measured power output was 0.6 hp. Estimate (a) the efficiency; and (b) the power specific speed if $n \approx 200$ rpm.

Solution: We have sufficient information to compute the available water power:

$$P_{avail} = \rho gQH = (62.4)\left(\frac{630}{449}\right)(5.3) = 464 \div 550 = 0.844 \text{ hp}, \quad \therefore\ \eta = \frac{0.6}{0.844} \approx \mathbf{71\%} \quad \textit{Ans.} \text{ (a)}$$

$$\text{At 200 rpm,} \quad N_{sp} = \frac{\text{rpm(hp)}^{1/2}}{(\text{head})^{5/4}} = \frac{200(0.6)^{1/2}}{(5.3)^{5/4}} \approx \mathbf{19} \quad \textit{Ans.} \text{ (b)}$$

11.95 One can make a theoretical estimate of the proper diameter for a penstock in an impulse turbine installation, as in Fig. P11.95. Let L and H be known, and let the turbine performance be idealized by Eqs. (11.38) and (11.39). Account for friction loss h_f in the penstock, but neglect minor losses. Show that (a) the maximum power is generated when $h_f = H/3$, (b) the optimum jet velocity is $(4gH/3)^{1/2}$, and (c) the best nozzle diameter is $D_j = [D^5/(2fL)]^{1/4}$, where f is the pipe-friction factor.

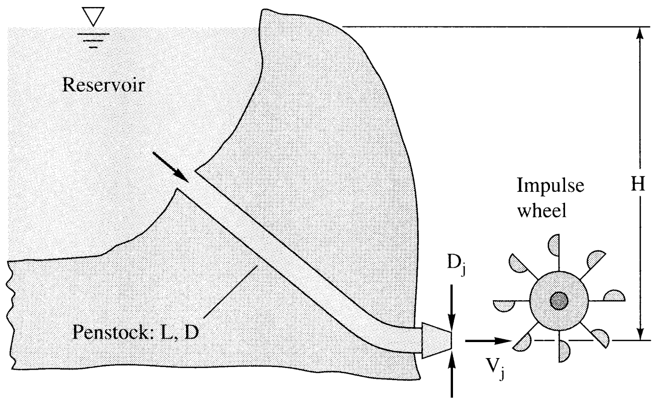


Fig. P11.95

Solution: From Eqs. 11.38 and 39, maximum power is obtained when $u = V_j/2$, or:

$$P_{max} = \rho Q\frac{V_j}{2}\left(V_j - \frac{V_j}{2}\right)(1-\cos\beta) = \rho A_j\left(\frac{1-\cos\beta}{4}\right)V_j^3 = CV_j^2V_j, \quad C = \text{constant}$$

Now apply the steady-flow energy equation between the reservoir and the outlet jet:

$$\Delta z = H = f\frac{L}{D}\frac{V_{pipe}^2}{2g} + \frac{V_j^2}{2g}, \quad \text{or:} \quad V_j^2 = 2gH - f\frac{L}{D}\left(\frac{D_j}{D}\right)^4 V_j^2 \quad \text{since } V_j\frac{\pi}{4}D_j^2 = V_p\frac{\pi}{4}D^2$$

$$\text{Thus} \quad P_{max} = C\left[2gHV_j - f\frac{L}{D}\left(\frac{D_j}{D}\right)^4 V_j^3\right]; \quad \text{Differentiate: } \frac{dP_{max}}{dV_j} = 0 \quad \text{if } 2gH = 3f\frac{L}{D}V_p^2$$

$$\text{or if:} \quad H = 3h_{f,pipe}! \quad \text{The pipe head loss} = \mathbf{H/3} \quad \textit{Ans. (a)}$$

$$\text{Continuing,} \quad V_j^2\Big|_{optimum} = 2g(H - h_f) = 2g(H - H/3), \quad \text{or:} \quad V_{j,optimum} = \sqrt{\frac{\mathbf{4}}{\mathbf{3}}\mathbf{gH}} \quad \textit{Ans. (b)}$$

Then the correct pipe flow speed is obtained by back-substitution:

$$f\frac{L}{D}\frac{V_{pipe}^2}{2g} = \frac{H}{3}, \quad \text{or:} \quad V_{pipe} = \sqrt{\frac{\mathbf{2gH}}{\mathbf{3fL/D}}}$$

$$\text{Continuing,} \quad V_p^2 = V_j^2\left(\frac{D_j}{D}\right)^4 = \frac{2gH}{3fL/D}, \quad \text{solve for } D_{jet} = \left(\frac{\mathbf{D_p^5}}{\mathbf{2fL}}\right)^{\mathbf{1/4}} \quad \textit{Ans. (c)}$$

11.96 Apply the results of Prob. 11.95 to determining the optimum (a) penstock diameter, and (b) nozzle diameter for the data of Prob. 11.92, with a commercial-steel penstock of length 1500 ft. [H = 800 ft, Q = 40,000 gal/min.]

Solution: For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For commercial steel, $\varepsilon \approx$ 0.00015 ft. We can't find f_{Moody} until we know D, so iteration is required. We can immediately compute $V_{jet} = (4gH/3)^{1/2} \approx 185$ ft/s, but this wasn't asked! Anyway,

$$h_f = H/3 = 267\text{ ft} = f\frac{L}{D}\frac{V_p^2}{2g} = f\frac{1500}{D}\frac{V_p^2}{2(32.2)},\quad \text{where } V_{pipe} = \frac{40000/449}{(\pi/4)D^2}\ (\text{ft/s})$$

Iterate to find $f \approx 0.0120$, $Re_D \approx 6.24\text{E}6$, and $D = D_{penstock} \approx \mathbf{1.68\ ft}$ *Ans.* (a)

$$\text{Then}\quad D_{jet} = \left(\frac{D^5}{2fL}\right)^{1/4} = \left[\frac{(1.68)^5}{2(0.0120)(1500)}\right]^{1/4} \approx \mathbf{0.78\ ft = 9.4\ inches}\quad \textit{Ans. (b)}$$

11.97 Consider the following non-optimum version of Prob. 11.95: H = 450 m, L = 5 km, D = 1.2 m, D_j = 20 cm. The penstock is concrete, ε = 1 mm. The impulse wheel diameter is 3.2 m. Estimate (a) the power generated by the wheel at 80% efficiency; and (b) the best speed of the wheel in r/min. Neglect minor losses.

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. This is a non-optimum condition, so we simply make a standard energy and continuity analysis. Refer to the figure on the next page for the notation:

$$\Delta z = H = h_f + \frac{V_j^2}{2g},\quad V_j D_j^2 = V_{pipe}D^2,\quad \text{combine and solve for jet velocity:}$$

$$V_j^2\left[1 + f\frac{L}{D}\left(\frac{D_j}{D}\right)^4\right] = 2gH,\quad \text{where}\quad f = fcn\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right),\quad \frac{\varepsilon}{D} = \frac{0.001}{1.2} = 8.33\text{E–4}$$

For example, guessing $f \approx 0.02$, we estimate $V_j \approx 91.2$ m/s. Using **EES** yields

$$f = 0.0189,\quad Re_d = 3.03\text{E}6,\quad V_{jet} = 91.23\ \frac{\text{m}}{\text{s}},\quad \boldsymbol{Q = 2.87\ \frac{m^3}{s}}$$

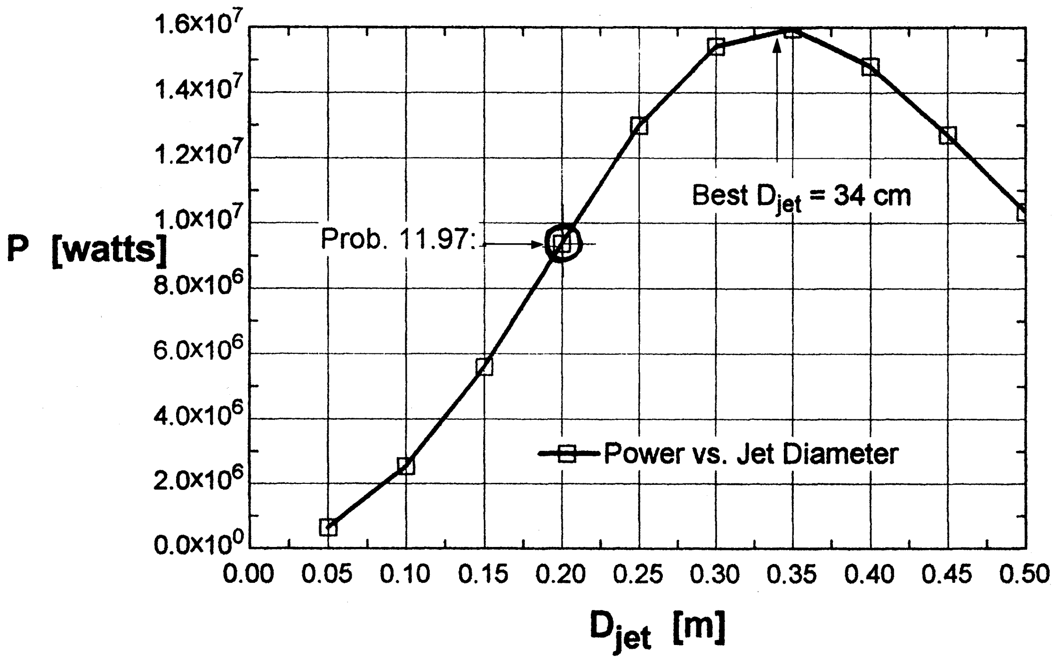
The power generated (at 80% efficiency) and best wheel speed are

$$Power = \rho Q u_{wheel}(V_j - u_{wheel})(1-\cos\beta)(0.8),\quad u_{best} = \frac{1}{2}V_{jet} = \Omega_{wheel}\frac{D_{wheel}}{2},\quad \beta \approx 165°$$

$$\textit{If}\quad D_{wheel} = 3.2\text{ m},\quad \text{solve for}\quad \Omega_{wheel} = 28.5\ \frac{\text{rad}}{\text{s}} = \mathbf{272\ \frac{rev}{min}},$$

$$Power = \mathbf{9.36\ MW}\quad \textit{Ans. (a, b)}$$

As shown on the figure below, which varies D_j, the optimum jet diameter is *34 cm*, not 20 cm, and the optimum power would be *16 MW*, or 70% more!



11.98 Francis and Kaplan (enclosed) turbines are often provided with *draft tubes*, which lead the exit flow into the tailwater region, as in Fig. P11.98. Explain at least two advantages to using a draft tube.

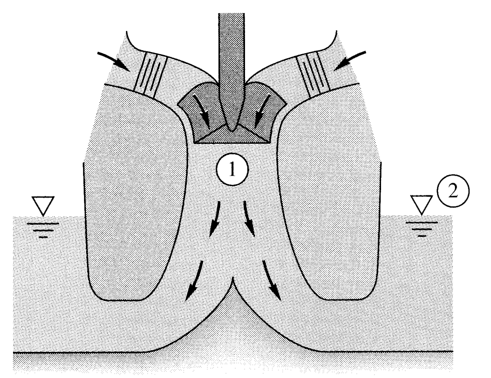


Fig. P11.98

Solution: Draft tubes have two big advantages:
(1) They reduce the *exit loss*, since a draft tube is essentially a diffuser, as in Fig. 6.23 of the text, so more of the water head is converted to power.
(2) They reduce total losses *downstream of the turbine*, so that the turbine runner can be placed higher up without the danger of cavitation.

11.99 Like pumps, turbines can also cavitate when the pressure at point 1 in Fig. P11.98 drops too low. With NPSH defined by Eq. 11.20, the empirical criterion given by Wislicenus [Ref. 4] for cavitation is

$$N_{ss} = \frac{(\text{rpm})(\text{gal/min})^{1/2}}{[\text{NPSH(ft)}]^{3/4}} \geq 11{,}000$$

Use this criterion to compute how high ($z_1 - z_2$) the impeller eye in Fig. P11.98 can be placed for a Francis turbine, with a head of 300 ft, $N_{sp} = 40$, and $p_a = 14$ psia, before cavitation occurs in 60°F water.

Solution: For water at 60°F, take $\rho g = 62.4\ \text{lbf/ft}^3$ and $p_v \approx 37$ psfa = 0.25 psia. Then

$$\text{Eq. 11.20: } \text{NPSH} = \frac{p_a - p_v}{\rho g} - \Delta z - h_{fi} = \frac{(14.0 - 0.25)(144)}{62.4} - \Delta z - 0 = 31.7\text{ ft} - \Delta z$$

Now we need the NPSH, which we find between the two specific-speed criteria:

$$N_{sp} = 40 = \frac{n(P\text{ in hp})^{1/2}}{(300\text{ ft})^{5/4}} \quad \text{with } P = \eta\rho g(Q/449)(300\text{ ft}) \div 550$$

$$N_{ss} = 11000 = \frac{n(Q\text{ in gpm})^{1/2}}{(\text{NPSH})^{3/4}}. \quad \text{Iterate to solve by assuming } \eta \approx 90\%$$

(We can't find n or Q, only $nQ^{1/2}$). Result: NPSH ≈ 45.0 ft, $\Delta z = 31.7 - 45 \approx$ **–13.3 ft**. *Ans.*

11.100 One of the largest wind generators in operation today is the ERDA/NASA two-blade propeller HAWT in Sandusky, Ohio. The blades are 125 ft in diameter and reach maximum power in 19 mi/h winds. For this condition estimate (a) the power generated in kW, (b) the rotor speed in r/min, and (c) the velocity V_2 behind the rotor.

Solution: For air in Ohio (?), take $\rho \approx 0.0023\ \text{slug/ft}^3$. Convert 19 mi/h = 27.9 ft/s. From Fig. 11.34 for the propeller HAWT, read optimum power coefficient and speed ratio:

$$C_{P,max} \approx 0.46 = \frac{P}{(1/2)\rho A V_1^3} = \frac{P_{max}}{(1/2)(0.0023)(\pi/4)(125)^2(27.9)^3}$$

$$\text{Solve for } P_{max} \approx 1.4\text{E}6\ \text{ft·lbf/s} \approx \mathbf{190\ kW} \quad \textit{Ans. (a)}$$

$$\left.\frac{\omega r}{V_1}\right|_{optimum} \approx 5.7 = \frac{\omega(125/2)}{27.9}, \quad \text{or} \quad \omega \approx 2.54\ \frac{\text{rad}}{\text{s}} \times \frac{60}{2\pi} \approx \mathbf{24\ rpm} \quad \textit{Ans. (b)}$$

$$\text{From ideal-windmill theory,} \quad V_{behind} = V_2 = \frac{1}{3}V_1 = \frac{27.9}{3} \approx \mathbf{9.3\ ft/s} \quad \textit{Ans. (c)}$$

11.101 A Darrieus VAWT in operation in Lumsden, Saskatchewan, that is 32 ft high and 20 ft in diameter sweeps out an area of 432 ft^2. Estimate (a) the maximum power and (b) the rotor speed if it is operating in 16 mi/h winds.

Solution: For air in Saskatchewan (?), take $\rho \approx 0.0023$ slug/ft^3. Convert 16 mi/h = 23.5 ft/s. From Fig. 11.34 for the Darrieus VAWT, read optimum C_P and speed ratio:

$$C_{P,max} \approx 0.42 = \frac{P}{(1/2)\rho A V_1^3} = \frac{P_{max}}{(1/2)(0.0023)(432)(23.5)^3}$$

$$\text{Solve for} \quad P_{max} \approx 2696 \text{ ft·lbf/s} \approx \mathbf{3.7\ kW} \quad \textit{Ans. (a)}$$

$$\text{At } P_{max},\ \frac{\omega r}{V_1} \approx 4.1 = \frac{\omega(20/2)}{23.5}, \quad \text{or:} \quad \omega \approx 9.62\ \frac{\text{rad}}{\text{s}} \times \frac{60}{2\pi} \approx \mathbf{92\ rpm} \quad \textit{Ans. (b)}$$

11.102 An American 6-ft diameter multiblade HAWT is used to pump water to a height of 10 ft through 3-in-diameter cast-iron pipe. If the winds are 12 mi/h, estimate the rate of water flow in gal/min.

Solution: For air in America (?), take $\rho \approx 0.0023$ slug/ft^3. Convert 12 mi/h = 17.6 ft/s. For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. From Fig. 11.34 for the American multiblade HAWT, read optimum C_P and speed ratio:

$$C_{P,max} \approx 0.29 \text{ at } \frac{\omega r}{V_1} \approx 0.9: \quad P_{max} \approx 0.29\left(\frac{1}{2}\right)(0.0023)\frac{\pi}{4}(6)^2(17.6)^3 \approx 51.4\ \frac{\text{ft·lbf}}{\text{s}}$$

$$\text{If } \eta_{pump} \approx 80\%, \quad P_{pump} \approx 0.8(51.4) = \rho_{water} g Q H_{syst} = 62.4Q\left(\Delta z + f\frac{L}{D}\frac{V_{pipe}^2}{2g}\right)$$

$$\text{where} \quad V_{pipe} = \frac{Q}{(\pi/4)(3/12)^2}, \quad \frac{L}{D} = \frac{10}{3/12} = 40, \quad \frac{\varepsilon}{D} = \frac{0.00085}{3/12} \approx 0.0034$$

$$\text{Clean up to:} \quad 0.659 = Q(10 + 258 f_{Moody} Q^2), \text{ with Q in ft}^3\text{/s.} \quad \text{Iterate to obtain}$$

$$f \approx 0.0305,\ V_{pipe} \approx 1.34\ \frac{\text{ft}}{\text{s}},\ Re \approx 31000,\ Q \approx 0.0657\ \frac{\text{ft}^3}{\text{s}} \approx \mathbf{29\ gal/min} \quad \textit{Ans.}$$

11.103 A very large Darrieus VAWT was constructed by the U.S. Department of Energy near Sandia, New Mexico. It is 60 ft high and 30 ft in diameter, with a swept area of 1200 ft^2. If the turbine is constrained to rotate at 90 r/min, use Fig. 11.34 to plot the predicted power output in kW versus wind speed in the range V = 5 to 40 mi/h.

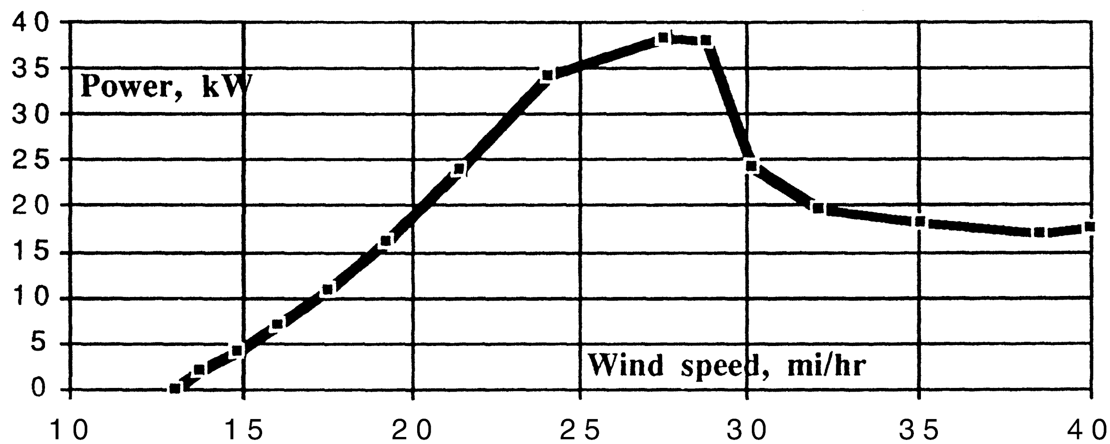
Solution: For air in New Mexico (?), take $\rho \approx 0.0023$ slug/ft^3. Given r = 15 ft and ω = 90 rpm = 9.43 rad/s, we can select wind speed V, hence $\omega r/V$ is known, read C_p, hence the power can be computed. For example,

$$\text{Select } \frac{\omega r}{V_1} = \mathbf{3.0} = \frac{9.43(15)}{V_1}, \quad \text{hence } V_1 = 47.1 \text{ ft/s} = 32.1 \text{ mi/h}$$

$$\text{Read } C_P \approx \mathbf{1.0} = \frac{P}{(1/2)\rho A V_1^3} = \frac{P}{(1/2)(0.0023)(1200)(47.1)^3},$$

$$\therefore \quad P \approx 14400 \ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \approx \mathbf{19.6\ kW}$$

Continue this across the entire "Darrieus VAWT" curve in Fig. 11.34 until you span the entire wind-speed range of 5 to 40 mi/h. The curve obtained by the writer is shown below. The rotor gives zero power for V ≤ 13 mi/h and gives power in the range of 18 to 38 kW for 20 < V < 40 mi/h. *Ans.*



COMPREHENSIVE PROBLEMS

C11.1 The net head of a little aquarium pump is given by the manufacturer as a function of volume flow rate as listed:

Q, m^3/s:	0	1E–6	2E–6	3E–6	4E–6	5E–6
H, mmH_2O:	1.10	1.00	0.80	0.60	0.35	0.0

What is the maximum achievable flow rate if you use this pump to pump water from the lower reservoir to the upper reservoir as shown in the figure?
NOTE: The tubing is smooth, with an inner diameter of 5 mm and a total length of 29.8 m. The water is at room temperature and pressure, and minor losses are neglected.

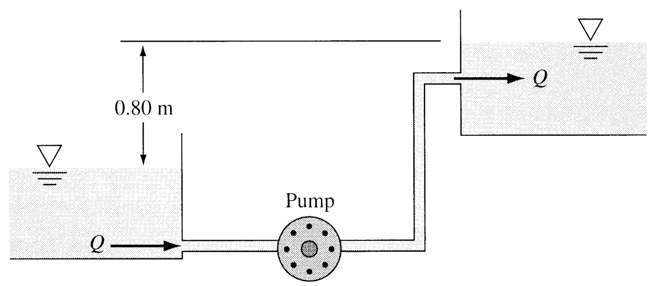


Fig. C11.1

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. NOTE: The tubing is so small that the flow is *laminar*, even at the highest pump flow rate:

$$Re_{max} = \frac{4\rho Q_{max}}{\pi\mu d} = \frac{4(998)(5E\text{–}6)}{\pi(0.001)(0.005)} = 1270 < 2000 \quad \therefore \ \textbf{Laminar}$$

The energy equation shows that the pump must fight both friction and elevation:

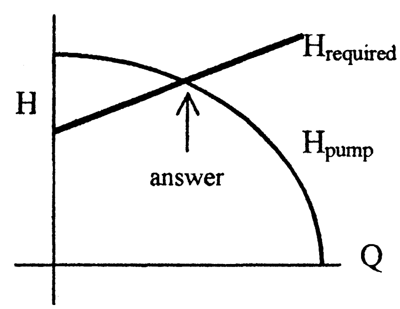
$$H_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = \Delta z + h_{f,lam} = \Delta z + \frac{128\mu LQ}{\pi d^4 \rho g} = 0.8 + \frac{128(.001)(29.8)Q}{\pi(0.005)^4(998)(9.81)},$$

$$\text{or:} \quad H_p = 0.8 + 198400Q = H_p(Q) \quad \text{from the pump data above}$$

One can plot the two relations, as at right, or use EES with a look-up table to get the final result for flow rate and head:

$$H_p = 1.00 \text{ m}$$

$$\boldsymbol{Q = 1.0E\text{–}6\ m^3/s} \quad \textit{Ans.}$$



The EES print-out gives the results $Re_d = 255$, $H = 0.999$ m, $Q = 1.004E\text{–}6$ m^3/s.

C11.2 Reconsider Prob. 6.62 as an exercise in pump selection. Select an impeller size and rotational speed from the Byron Jackson pump family of Prob. 11.28 which will deliver a flow rate of 3 ft^3/s to the system of Fig. P6.62 at minimum input power. Calculate the horsepower required.

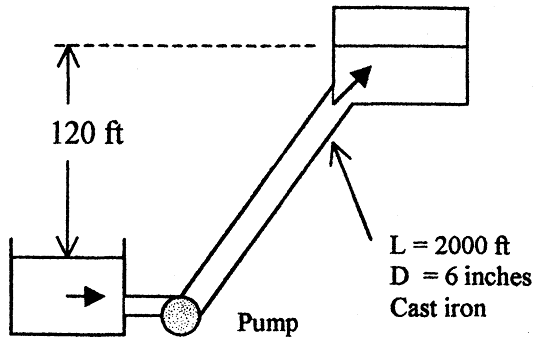


Fig. C11.2

Solution: For water take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron take $\varepsilon = 0.00085$ ft, or $\varepsilon/d = 0.00085/0.5 = 0.0017$. The energy equation, written between reservoirs, is the same as in Prob. 6.62:

$$H_p = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\text{ ft} + f\frac{2000}{0.5}\frac{[Q/(\pi/4)/(0.5)^2]^2}{2(32.2)} = 120 + 1611fQ^2$$

$$\text{where}\quad f = f_{\text{Moody}} = fcn\left(Re_d, \frac{\varepsilon}{d}\right) \quad \text{with } Re_d = \frac{4\rho Q}{\pi\mu d}$$

If, as given, Q = **3 ft^3/s**, then, from above, $f = 0.0227$ and H_p = **449.6 ft**.

Now we have to *optimize*: From Prob. 11.28, Q* = 6 ft^3/s, H* = 330 ft, and P* = 255 bhp when n = 2134 rpm (35.57 rps) and D = 14.62 inches (1.218 ft). Thus, at BEP:

$$C_Q^* = \frac{Q^*}{nD^3} = 0.0933;\quad C_H^* = \frac{gH^*}{n^2D^2} = 5.66;\quad C_P^* = \frac{P^*}{\rho n^3 D^5} = 0.599$$

For the system above, $(3.0)/[nD^3] = 0.0932$, or $nD^3 = 32.15$, and $(32.2)(449.6)/n^2D^2 = 5.66$, or $n^2D^2 = 2558$. Solve simultaneously:

$$n = 63.4\text{ rps} = 3800\ \frac{\text{rev}}{\text{min}};\quad D_p = 0.798\text{ ft};\quad Power = 0.599\rho n^3 D_p^5 = \mathbf{174\ hp}\quad \textit{Ans.}$$

C11.3 Reconsider Prob. 6.77 as an exercise in turbine selection. Select an impeller size and rotational speed from the Francis turbine family of Fig. 11.21*d* which will deliver maximum power generated by the turbine. Calculate the water turbine power output and remark on the practicality of your design.

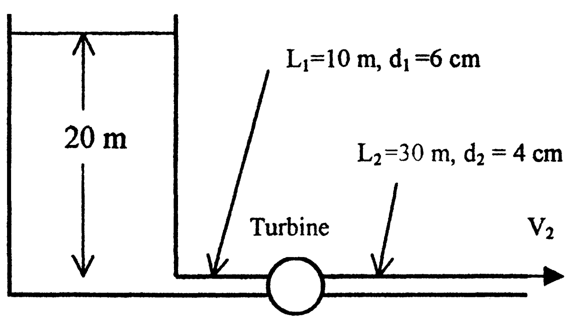


Fig. C11.3

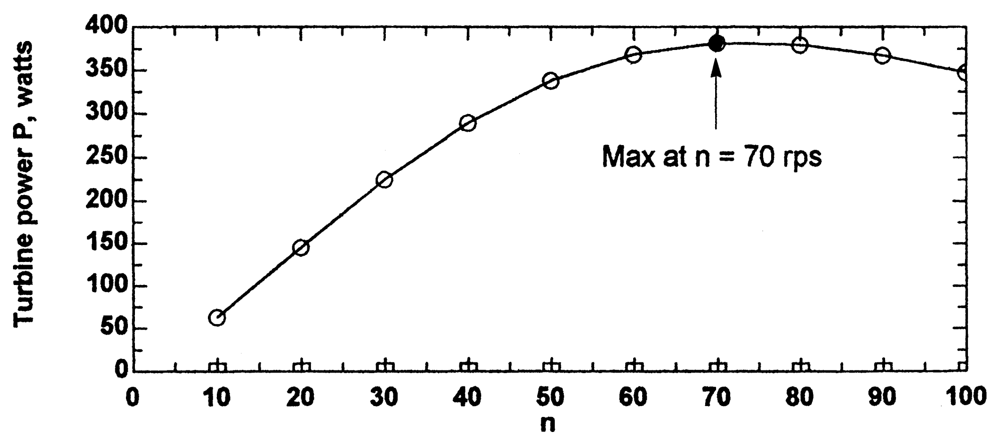
Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron take $\varepsilon = 0.046$ m, or $\varepsilon/d_1 = 0.046/60 = 0.000767$ and $\varepsilon/d_2 = 0.046/40 = 0.00115$. The energy and continuity equations yields

$$\Delta z = 20\text{ m} = \frac{V_2^2}{2g} + f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} + f_2\frac{L_2}{d_2}\frac{V_2^2}{2g} + h_{turbine};\quad V_2 = V_1\left(\frac{d_1}{d_2}\right)^2 = 2.25V_1$$

with the friction factors to be determined by the respective Reynolds numbers and roughness ratios. We use the BEP coefficients from Eq. (11.36) for this turbine:

$$C_Q^* = \frac{Q^*}{nD_t^3} = 0.34;\quad C_H^* = \frac{gH^*}{n^2D_t^2} = 9.03;\quad C_P^* = \frac{P^*}{\rho n^3 D_t^5} = 2.70$$

We know from Prob. 6.76 that the friction factors are approximately 0.022. With only *one* energy equation (above) and *two* unknowns (n and D_t), we vary, say, n from 0 to 100 rev/s and find the resulting turbine diameter and power extracted. The power is shown in the graph below, with a maximum of **381 watts** at n = **70 rev/s**, with a resulting turbine diameter $\boldsymbol{D_t}$ **= 5.3 cm**. This is a fast, small turbine! *Ans.*



C11.4 The system of Fig. C11.4 is designed to deliver water at 20°C from a sea-level reservoir to another through new cast iron pipe of diameter 38 cm. Minor losses are $\sum K_1 = 0.5$ before the pump entrance and $\sum K_2 = 7.2$ after the pump exit. (a) Select a pump from either Figs. 11.7*a* or 11.7*b*, running at the given speeds, which can perform this task at maximum efficiency. Determine (b) the resulting flow rate; (c) the brake horsepower; and (d) whether the pump as presently situated is safe from cavitation.

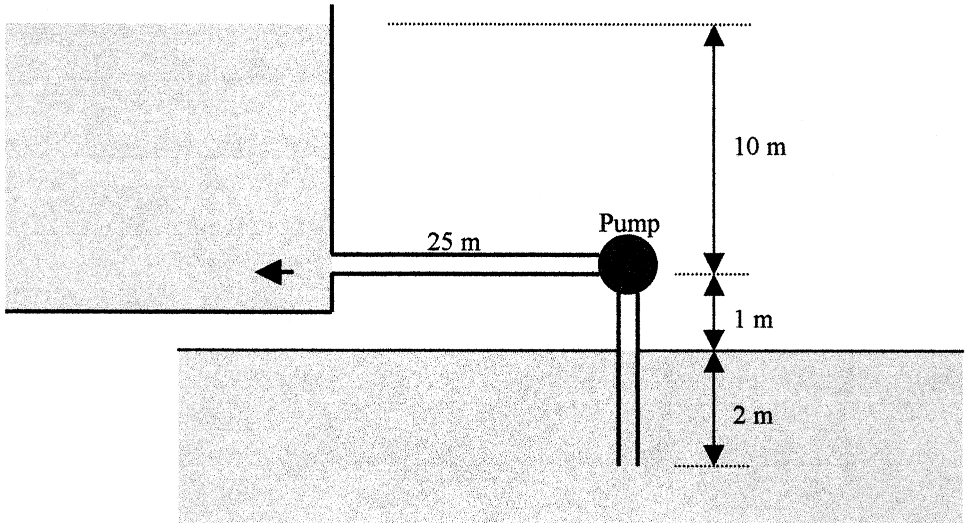


Fig. C11.4

Solution: For water take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. First establish the *system curve* of head loss versus flow rate. For cast iron take $\varepsilon = 0.26$ mm, hence $\varepsilon/d = 0.26/380 = 0.000684$. The pumps of Figs. 11.7*a*,*b* deliver flows of 4000 to 28000 gal/min, no doubt turbulent flow. The energy equation, written from the lower free surface to the upper surface, gives

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = 0+0+0 = \frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f-H_{pump} = 0+0+11\text{ m}+\frac{V_{pipe}^2}{2g}\left(f\frac{L}{d}\sum K\right)$$

$$\text{Where}\quad f^{-1/2} = -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7}+\frac{2.51}{\mathrm{Re}_d\sqrt{f}}\right),\quad \mathrm{Re}_d = \frac{\rho V_{pipe} d}{\mu}\quad\text{and}\quad V_{pipe} = \frac{Q}{(\pi/4)d^2}$$

Take, for example, $Q = 22{,}000$ gal/min $= 1.39$ m^3/s. Then $V_{\text{pipe}} = 12.24$ m/s, $\mathrm{Re_d} = 4.64\text{E}6$, $f = 0.0180$, hence the head loss becomes

$$h_f = \frac{(12.24)^2}{2(9.81)}\left[(0.0180)\left(\frac{28\text{ m}}{0.38\text{ m}}\right)+0.5+7.2\right] = 68.9\text{ m} = 226\text{ ft},$$

$$\text{Thus a match requires}\quad H = h_f + z_2 = 68.9\text{ m}+11\text{ m} = 79.9\text{ m} = 262\text{ ft}$$

No pump in Fig. 11.7 exactly matches this, but the 28-inch pump in Fig. 11.7*a* and the 41.5-inch pump in Fig. 11.7*b* are pretty close, especially the latter. We can continue the calculations:

Q, gal/min:	4000	8000	12000	16000	**20000**	22000	24000	28000
h_f, ft:	44	66	103	156	**223**	262	305	402

(a) The best match seems to be **the 38-inch pump of Fig. 11.7*b* at a flow rate of 20,000 gal/min**, near maximum efficiency of 88%. *Ans*. (a)
(b, c) The appropriate flow rate is **20,000 gal/min**. *Ans*. (b)
The horsepower is **1250 bhp**. *Ans*. (c)
(d) Use Eq. (11.20) to check the NPSH for cavitation. For water at 20°C, p_v = 2337 Pa. The velocity in the pipe is $V = Q/A$ = 11.13 m/s. The theoretical minimum net positive suction head is:

$$NPSH = \frac{p_a}{\gamma} - z_i - h_{fi} - \frac{p_v}{\gamma} = \frac{101350\text{ Pa}}{9790\text{ N/m}^3} - 1\text{ m} - \frac{(11.13\text{ m/s})^2}{2(9.81\text{ m/s}^2)}(1+0.5) - \frac{2337\text{ Pa}}{9790\text{ N/m}^3}$$

$$NPSH = \mathbf{-0.36\ m}$$

In Fig. 11.7b, at Q = 20,000 gal/min, read NPSH ≈ 16 m $\gg -0.36$ m! So this pump, when placed in its present position, **will surely cavitate**! *Ans*. (d) A new pump placement is needed.

C11.5 In Prob. 11.23, estimate the efficiency of the pump in two ways: (a) read it directly from Fig. 11.7*b* (for the dynamically similar pump); (b) calculate it from Eq. (11.5) for the actual kerosene flow. Compare your results and discuss any discrepancies.

Solution: Problem 11.23 used the 38-inch-pump in Fig. 11.7b to deliver 22000 gal/min of kerosene at 850 rpm. (a) The problem showed that the *dynamically similar water* pump, at 710 rpm, had a flow rate of 18,400 gal/min.

(a) Figure 11.7*b*: Read $\eta \approx$ **88.5%** *Ans.* (a)

(b) For kerosene, take ρ = 804 kg/m^3 = 1.56 slug/ft^3. The solution to Prob. 11.23 gave a kerosene head of 340 ft and a brake horsepower of 1600 hp. Together with the known flow rate, we can calculate the kerosene efficiency by definition:

$$\eta_{kerosene} = \frac{\rho_{kerosene} gQH}{Power} = \frac{(1.56)(32.2)(22000/448.83)(340)}{(1600)(550)\text{ ft·lbf/s}} = 0.95 \quad \text{or} \quad \mathbf{95\%} \quad \textit{Ans.}\text{ (b)}$$

This is significantly different from 88.5% in part (a) above. The main reason (the author thinks) is the **difficulty of reading bhp from Fig. 11.7b**. The actual kerosene bhp for Prob. 11.23 is probably about *1700*, not 1600. *Ans.*

C11.6 An interesting turbomachine [58] is the *fluid coupling* of Fig. C11.6, which delivers fluid from a primary pump rotor into a secondary turbine on a separate shaft. Both rotors have radial blades. Couplings are common in all types of vehicle and machine transmissions and drives. The *slip* of the coupling is defined as the dimensionless difference between shaft rotation rates, $s = 1 - \omega_s/\omega_p$. For a given percentage of fluid filling, the torque T transmitted is a function of s, ρ, ω_p, and impeller diameter D. (a) Non-dimensionalize this function into two pi groups, with one pi proportional to T. Tests on a 1-ft-diameter coupling at 2500 r/min, filled with hydraulic fluid of density 56 lbm/ft^3, yield the following torque versus slip data:

Slip, s:	0%	5%	10%	15%	20%	25%
Torque T, ft·lbf:	0	90	275	440	580	680

(b) If this coupling is run at 3600 r/min, at what slip value will it transmit a torque of 900 ft·lbf?
(c) What is the proper diameter for a geometrically similar coupling to run at 3000 r/min and 5% slip and transmit 600 ft·lbf of torque?

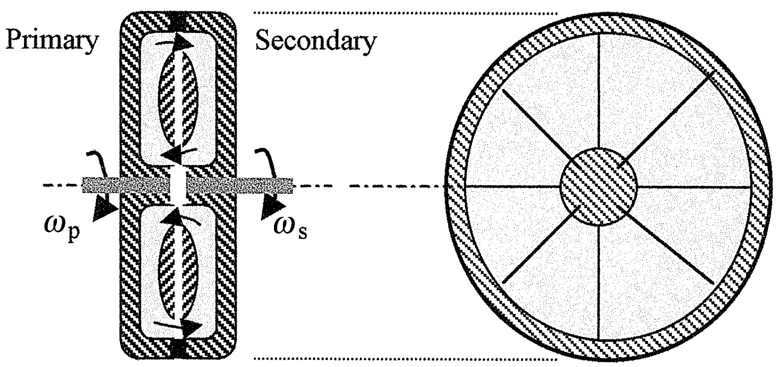


Fig. C11.6

Solution: (a) List the dimensions of the five variables, from Table 5.1:

Variable:	T	s	ρ	ω_p	D
Dimension:	$\{ML^2/T^2\}$	$\{1\}$	$\{M/L^3\}$	$\{1/T\}$	$\{L\}$

Since s is already dimensionless, the other four must form a single pi group. The result is:

$$\frac{\boldsymbol{T}}{\boldsymbol{\rho\omega_p^2 D^5}} = \mathbf{fcn}(\boldsymbol{s}) \quad \textit{Ans. (a)}$$

Now, to work parts (b) and (c), put the data above into this dimensionless form. Convert ρ_{oil} = 56 lbm/ft^3 = 1.74 slug/ft^3. Convert ω_p = 2500 r/min = 41.7 r/s. The results are:

Slip, s:	0%	5%	10%	15%	20%	25%
$T/(\rho\omega_p^2 D^5)$:	0	0.0298	0.0911	0.146	0.192	0.225

(b) With $D = 1$ ft and $\omega_p = 3500$ r/min $= 58.3$ r/s and $T = 900$ ft·lbf,

$$\frac{T}{\rho\omega_p^2 D^5} = \frac{900\ \text{ft·lbf}}{(1.74\ \text{slug/ft}^3)(58.3\ \text{r/s})^2(1\ \text{ft})^5} = 0.152, \quad \text{Estimate}\ \ \mathbf{s \approx 15\%} \quad \textit{Ans.}\ \text{(b)}$$

(c) With D unknown and $s = 5\%$ and $\omega_p = 3000$ r/min $= 50$ r/s and $T = 600$ ft·lbf,

$$\frac{T}{\rho\omega_p^2 D^5} = \frac{600\ \text{ft·lbf}}{(1.74\ \text{slug/ft}^3)(50\ \text{r/s})^2(D)^5} = 0.0298, \quad \text{Solve}\ \ \mathbf{D \approx 1.36\ ft} \quad \textit{Ans.}\ \text{(c)}$$
