Chapter 10 · Open Channel Flow

10.1 The formula for shallow-water wave propagation speed, Eq. (10.9) or (10.10), is independent of the physical properties of the liquid, i.e., density, viscosity, or surface tension. Does this mean that waves propagate at the same speed in water, mercury, gasoline, and glycerin? Explain.

Solution: The shallow-water wave formula, $c_o = \sqrt{(gy)}$, is valid for any fluid except for *viscosity and surface tension effects*. If the wave is very small, or "capillary" in size, its propagation may be influenced by surface tension and Weber number [Ref. 5–7]. If the fluid is very viscous, its speed may be influenced by Reynolds number. The formula is accurate for water, mercury, and gasoline but would be inaccurate for *glycerin*.

10.2 A shallow-water wave 12 cm high propagates into still water of depth 1.1 m. Compute (a) the wave speed; and (b) the induced velocity δV.

Solution: The wave is high enough to include the δy terms in Eq. (10.9):

$$c = \sqrt{gy(1+\delta y/y)(1+\delta y/2y)}$$

$$= \sqrt{9.81(1.1)(1+0.12/1.1)[1+0.12/\{2(1.1)\}]} = \mathbf{3.55\ m/s} \quad \textit{Ans.}\text{ (a)}$$

$$\delta V = \frac{c\delta y}{y+\delta y} = \frac{(3.55\text{ m/s})(0.12\text{ m})}{1.1+0.12\text{ m}} = \mathbf{0.35\ m/s} \quad \textit{Ans.}\text{ (b)}$$

10.3 Narragansett Bay is approximately 21 (statute) mi long and has an average depth of 42 ft. Tidal charts for the area indicate a time delay of 30 min between high tide at the mouth of the bay (Newport, Rhode Island) and its head (Providence, Rhode Island). Is this delay correlated with the propagation of a shallow-water tidal-crest wave through the bay? Explain.

Solution: If it *is* a simple shallow-water wave phenomenon, the time delay would be

$$\Delta t = \frac{\Delta L}{c_o} = \frac{(21\text{ mi})(5280\text{ ft/mi})}{\sqrt{32.2(42)}} \approx 3015\text{ s} \approx \mathbf{50\ min} \quad \textit{Ans.}???$$

This doesn't agree with the measured $\Delta t \approx 30$ min. In reality, tidal propagation in estuaries is a *dynamic* process, dependent on estuary shape, bottom friction, and tidal period.

10.4 The water-channel flow in Fig. P10.4 has a free surface in three places. Does it qualify as an open-channel flow? Explain. What does the dashed line represent?

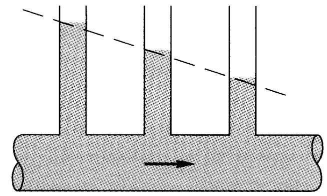


Fig. P10.4

Solution: No, this is ***not*** an open-channel flow. The open tubes are merely piezometer or pressure-measuring devices, there is no flow in them. The dashed line represents the pressure distribution in the tube, or the "hydraulic grade line" (HGL).

10.5 Water flows rapidly in a channel 25 cm deep. Piercing the surface with a pencil point creates a wedge-like wave of included angle 38°, as shown. Estimate the velocity *V* of the water flow.

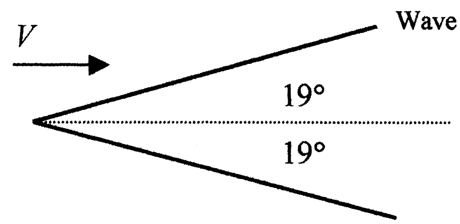


Fig. P10.5

Solution: This is a "supercritical" flow, analogous to supersonic gas flow. The Froude number is analogous to Mach number here:

$$\sin\theta = \sin(19°) = \frac{1}{Fr} = \frac{c}{V} = \frac{\sqrt{gy}}{V} = \frac{\sqrt{(9.81\text{ m/s}^2)(0.25\text{ m})}}{V},$$

$$\text{Solve} \quad V = \mathbf{4.81\ \frac{m}{s}} \quad \textit{Ans.} \ (\mathrm{Fr} = 3.1)$$

10.6 Pebbles dropped successively at the same point, into a water-channel flow of depth 42 cm, create two circular ripples, as in Fig. P10.6. From this information, estimate (a) the Froude number; and (b) the stream velocity.

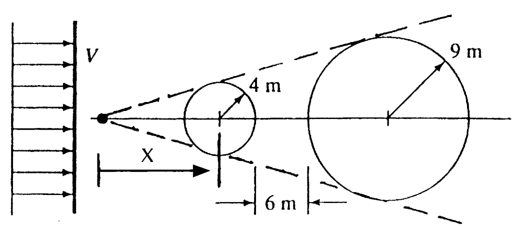


Fig. P10.6

Solution: The center of each circle moves at stream velocity V. For the small circle,

$$\text{small circle:} \quad X = \frac{4V}{c_o}; \quad \text{large circle:} \ X + 4 + 6 + 9 = \frac{9V}{c_o};$$

$$\text{Solve} \quad \mathbf{Fr} = \frac{V}{c_o} = \mathbf{3.8} \quad \textit{Ans.}\ (a)$$

$$\text{Compute } c_o = \sqrt{gh} = \sqrt{9.81(0.42)} = 2.03\ \frac{m}{s}, \quad \mathbf{V_{current}} = 3.8c_o \approx \mathbf{7.7\ \frac{m}{s}} \quad \textit{Ans.}\ (b)$$

10.7 Pebbles dropped successively at the same point, into a water-channel flow of depth 65 cm, create two circular ripples, as in Fig. P10.7. From this information, estimate (a) the Froude number; and (b) the stream velocity.

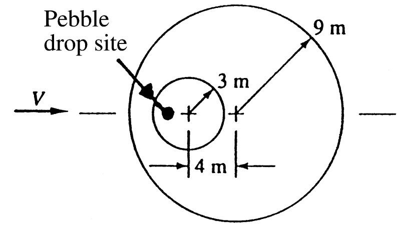


Fig. P10.7

Solution: If the pebble-drop-site is at distance X ahead of the small-circle center,

$$\text{small circle:}\quad X = \frac{3V}{c_o};\quad \text{large circle:}\ X + 4 = \frac{9V}{c_o};\quad \text{solve}\ \mathbf{Fr} = \frac{V}{c_o} = \mathbf{\frac{2}{3}}\quad \textit{Ans. (a)}$$

$$c_o = \sqrt{gh} = \sqrt{9.81(0.65)} = 2.53\ \frac{m}{s};\quad \mathbf{V_{current}} = \frac{2}{3}c_o \approx \mathbf{1.68\ \frac{m}{s}}\quad \textit{Ans. (b)}$$

10.8 An earthquake near the Kenai Peninsula, Alaska, creates a single "tidal" wave (called a 'tsunami') which propagates south across the Pacific Ocean. If the average ocean depth is 4 km and seawater density is 1025 kg/m^3, estimate the time of arrival of this tsunami in Hilo, Hawaii.

Solution: Everyone get out your Atlases, how far is it from Kenai to Hilo? Well, it's about 2800 statute miles (4480 km), and seawater *density* has nothing to do with it:

$$\Delta t_{travel} = \frac{\Delta x}{c_o} = \frac{4480E3\ m}{\sqrt{9.81(4000\ m)}} \approx 22600\ s \approx \mathbf{6.3\ hours}\quad \textit{Ans.}$$

So, given warning of an earthquake in Alaska (by a seismograph), there is plenty of time to warn the people of Hilo (which is very susceptible to tsunami damage) to take cover.

10.9 Equation (10.10) is for a single disturbance wave. For *periodic* small-amplitude surface waves of wavelength λ and period *T*, inviscid theory [5 to 9] predicts a wave propagation speed

$$c_0^2 = \frac{g\lambda}{2\pi}\tanh\frac{2\pi y}{\lambda}$$

where y is the water depth and surface tension is neglected. (a) Determine if this expression is affected by the Reynolds number, Froude number, or Weber number. Derive the limiting values of this expression for (b) $y \ll \lambda$ and (c) $y \gg \lambda$. (d) Also for what ratio y/λ is the wave speed within 1 percent of limit (c)?

Solution: (a) Obviously there is **no effect** in this theory for Reynolds number or Weber number, because viscosity and surface tension are not present in the formula. There *is* a Froude number effect, and we can rewrite it as Froude number versus dimensionless depth:

$$\mathbf{Fr_{wave}} = \frac{\mathbf{c_o}}{\sqrt{\mathbf{g}\lambda}} = \sqrt{\frac{\mathbf{1}}{\mathbf{2}\pi}\mathbf{tanh}\left(\frac{\mathbf{2}\pi\mathbf{y}}{\lambda}\right)} = \mathbf{fcn}\left(\frac{\mathbf{y}}{\lambda}\right) \quad \textit{Ans. (a)}$$

(b) $y \le\le \lambda$: $\tanh\zeta \approx \zeta$ if $\zeta \ll 1$: $c^2_{o,\text{long waves}} \approx \frac{g\lambda}{2\pi}\frac{2\pi y}{\lambda} \approx \mathbf{gy}$ (same as Eq. 10.10) *Ans.* (b)

(c) $y \gg \lambda$: $\tanh\zeta \approx 1$ if $\zeta \gg 1$: $c^2_{o,\text{short waves}} \approx \frac{\mathbf{g}\lambda}{\mathbf{2}\pi}$ (periodic **deep-water** waves) *Ans.* (c)

(d) $c_o = 0.99c_{o,deep}$ if $\tanh(2\pi y/\lambda) \approx 0.995 \approx \tanh(3)$, or $\mathbf{y/\lambda \approx 0.48}$. *Ans.* (d).

10.10 If surface tension Υ is included in the analysis of Prob. 10.9, the resulting wave speed is [Refs. 5 to 9]:

$$c_0^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi Y}{\rho\lambda}\right)\tanh\frac{2\pi y}{\lambda}$$

(a) Determine if this expression is affected by the Reynolds number, Froude number, or Weber number. Derive the limiting values of this expression for (b) $y \ll \lambda$ and (c) $y \gg \lambda$. (d) Finally determine the wavelength λ_{crit} for a minimum value of c_0, assuming that $y \gg \lambda$.

Solution: (a) Obviously there is **no effect** in this theory for Reynolds number, because viscosity is not present in the formula. There *are* Froude number and Weber number effects, and we can rewrite it as Froude no. versus Weber no. and dimensionless depth:

$$\mathbf{Fr_{wave}} = \frac{\mathbf{c_o}}{\sqrt{\mathbf{g}\lambda}} = \sqrt{\left(\frac{\mathbf{1}}{\mathbf{2}\pi} + \frac{\mathbf{2}\pi\mathbf{Y}}{\rho\mathbf{g}\lambda^2}\right)\mathbf{tanh}\left(\frac{\mathbf{2}\pi\mathbf{y}}{\lambda}\right)} = \mathbf{fcn}\left(\mathbf{We}, \frac{\mathbf{y}}{\lambda}\right), \quad We = \frac{\rho g\lambda^2}{Y} \quad \textit{Ans. (a)}$$

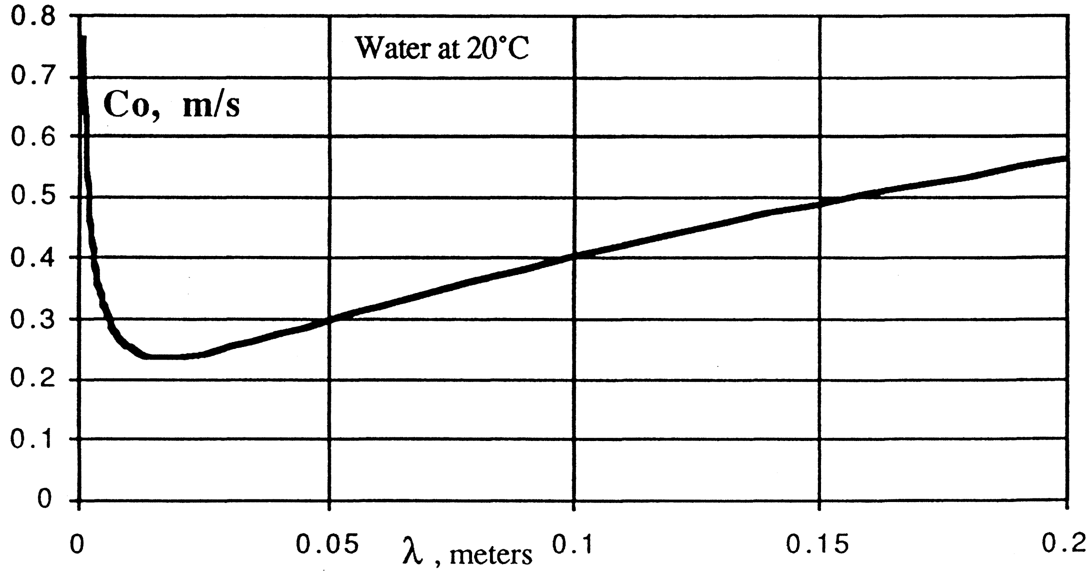
(b) $y \ll \lambda$: $\tanh\zeta \approx \zeta$ if $\zeta \ll 1$: $c^2_{o,\text{long waves}} \approx \left(\mathbf{gy} + \frac{\mathbf{4}\pi^2\mathbf{Yy}}{\rho\lambda^2}\right)$ *Ans.* (b)

(c) $y \gg \lambda$: $\tanh\zeta \approx 1$ if $\zeta \gg 1$: $c^2_{o,\text{short waves}} \approx \left(\frac{\mathbf{g}\lambda}{\mathbf{2}\pi} + \frac{\mathbf{2}\pi\mathbf{Y}}{\rho\lambda}\right)$ *Ans.* (c)

For a deep-water wave, part (c) applies, and we can differentiate with respect to λ:

$$\frac{dc_o^2}{d\lambda} = \frac{g}{2\pi} - \frac{2\pi Y}{\rho\lambda^2} = 0 \quad \text{if} \quad \boldsymbol{\lambda_{crit} = 2\pi\sqrt{\frac{Y}{\rho g}}} \quad (\text{where } c_o = c_{o,min}) \quad \textit{Ans. (d)}$$

For water at 20°C, we may compute that $\lambda_{crit} \approx 0.018$ m = 1.8 cm, as shown below.



10.11 A rectangular channel is 2 m wide and contains water 3 m deep. If the slope is 0.85° and the lining is corrugated metal, estimate the discharge for uniform flow.

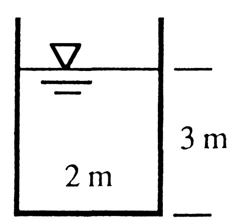


Fig. P10.11

Solution: For corrugated metal, take Manning's $n \approx 0.022$. Get the hydraulic radius:

$$R_h = \frac{A}{P} = \frac{2(3)}{3+2+3} = 0.75 \text{ m}; \quad Q \approx \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.022}(6)(0.75)^{2/3}[\tan(0.85°)]^{1/2}$$

$$\text{or:} \quad \mathbf{Q \approx 27\ m^3/s} \quad \textit{Ans.}$$

10.12 (a) For laminar draining of a wide thin sheet of water on pavement sloped at angle θ, as in Fig. P4.36, show that the flow rate is given by

$$Q = \frac{\rho g b h^3 \sin\theta}{3\mu}$$

where *b* is the sheet width and *h* its depth. (b) By (somewhat laborious) comparison with Eq. (10.13), show that this expression is compatible with a friction factor $f = 24/\text{Re}$, where $\text{Re} = V_{av}h/\nu$.

Solution: The velocity and flow rate were worked out in detail in Prob. 4.36:

$$\text{x-Mom. yields} \quad u = \frac{\rho g \sin\theta}{2\mu} y(2h - y), \quad Q = \int_0^h u b\, dy = \boldsymbol{\frac{\rho g b h^3 \sin\theta}{3\mu}} \quad \textit{Ans. (a)}$$

$$\text{for } 0 < y < h. \text{ Then} \quad V_{avg} = \frac{2}{3} u_{max} = \frac{h^2 g \sin\theta}{3\nu} \quad \text{and} \quad R_h\big|_{\text{wide channel}} \approx h$$

Interpreting "$\sin\theta$" as "S_o," the channel slope, we compare Q above with Eq. 10.13:

$$Q = \frac{gbh^3 S_o}{3\nu} = \sqrt{\frac{8g}{f}}(bh)h^{1/2}S_o^{1/2}, \quad \text{solve for } \boldsymbol{f} = \frac{72\nu^2}{gh^3 S_o} = \frac{24\nu}{V_{avg}h} = \boldsymbol{\frac{24}{\text{Re}}} \quad \textit{Ans.}$$

10.13 The laminar-draining flow from Prob. 10.12 may undergo transition to turbulence if Re > 500. If the pavement slope is 0.0045, what is the maximum sheet thickness, in mm, for which laminar flow is ensured?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Define the Reynolds number as in Prob. 10.12 above:

$$\text{Re} = \frac{V_{avg}h}{\nu}, \quad \text{where } V_{avg} = \frac{gh^2 S_o}{3\nu}, \quad \text{thus Re} < 500 \quad \text{if } \frac{gh^3 S_o}{3\nu^2} < 500,$$

$$\text{or:} \quad h^3 < \frac{3(0.001/998)^2(500)}{9.81(0.0045)} \quad \text{or: sheet depth} \quad \mathbf{h \le 0.0032\ m} \quad \textit{Ans.}$$

10.14 The Chézy formula (10.18) is independent of fluid density and viscosity. Does this mean that water, mercury, alcohol, and SAE 30 oil will all flow down a given open channel at the same rate? Explain.

Solution: The Chézy formula, $V = (1.0/n)(R_h)^{2/3}\sqrt{S_o}$, <u>appears</u> to be independent of fluid properties, with *n* only representing surface roughness, but in fact it <u>requires that the channel flow be "fully rough" and turbulent</u>, i.e., at high Reynolds number ≥ 1E6 at least. Even for low-viscosity fluids such as water, mercury, and alcohol, this requires reasonable

<u>**size**</u> for the channel, R_h of the order of 1 meter or more if the slope is small $(S_o \ll 1)$. SAE 30 oil is so viscous that it would need $R_h > 10$ m to approach the Chézy formula.

10.15 The finished-concrete channel of Fig. P10.15 is designed for a flow rate of 6 m^3/s at a normal depth of 1 m. Determine (a) the design slope of the channel and (b) the percentage of reduction in flow if the surface is asphalt.

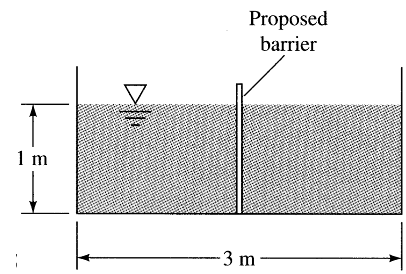


Fig. P10.15

Solution: For finished concrete, n ≈ 0.012. Evaluate the hydraulic radius and then S_o:

$$R_h = \frac{A}{P} = \frac{3\text{ m}^2}{5\text{ m}} = 0.6\text{ m},\quad \text{Chézy: } Q = 6\,\frac{\text{m}^3}{\text{s}} = \frac{1.0}{0.012}(3)(0.6)^{2/3}S_o^{1/2},$$

$$\mathbf{S_o \approx 0.00114} \quad \textit{Ans. } \text{(a)}$$

$$\text{(b) Asphalt: } n \approx 0.016,\quad \therefore\ Q = 6\frac{n_1}{n_2} = 6\left(\frac{0.012}{0.016}\right) \approx \mathbf{4.5\ \frac{m^3}{s}} \quad \text{(25\% less)} \quad \textit{Ans. } \text{(b)}$$

10.16 In Prob. 10.15, for finished concrete, determine the percentage reduction in flow if the channel is divided in the center by the proposed barrier in Fig. P10.15 above. How does your estimate change if all surfaces are clay tile?

Solution: For any given n, we are simply comparing one large to two small channels:

$$\frac{Q_{2\text{ small}}}{Q_{1\text{ large}}} = \frac{2(1/n)(1.5)(1.5/3.5)^{2/3}S_o^{1/2}}{(1/n)(3.0)(3/5)^{2/3}S_o^{1/2}} = \left[\frac{R_{h,\text{small}}}{R_{h,\text{large}}}\right]^{2/3} = \left(\frac{3/7}{3/5}\right)^{2/3} \approx \mathbf{0.80} \text{ (20\% less)} \quad \textit{Ans.}$$

Since *n* is the same for each, this result is <u>independent of the surface</u>—clay tile, etc.

10.17 The trapezoidal channel of Fig. P10.17 is made of brickwork and slopes at 1:500. Determine the flow rate if the normal depth is 80 cm.

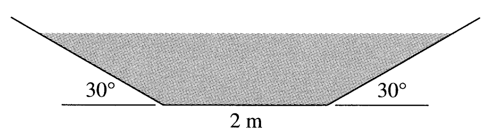


Fig. P10.17

Solution: For brickwork, $n \approx 0.015$. Evaluate the hydraulic radius with y = 0.8 m:

$$A = 2y + y^2\cot\theta = 2(0.8) + (0.8)^2\cot 30° = 2.71\text{ m}^2$$

$$P = 2 + 2(0.8)\csc 30° = 5.2\text{ m},\quad R_h = A/P = 2.71/5.2 \approx 0.521\text{ m}$$

$$Q = \frac{1.0}{n}AR_h^{2/3}S_o^{1/2} = \frac{1.0}{0.015}(2.71)(0.521)^{2/3}\left(\frac{1}{500}\right)^{1/2} \approx \mathbf{5.23\ m^3/s}\quad \textit{Ans.}$$

10.18 Modify Prob. 10.17 as follows: Determine the normal depth for which the flow rate will be 8 m^3/s.

Solution: We must iterate to find the depth *y* for this flow rate. We know y > 0.8 m:

$$Q = 8\ \frac{m^3}{s} \stackrel{?}{=} \frac{1.0}{0.015}(2y + y^2\cot 30°)\left[\frac{2y + y^2\cot 30°}{2 + 2y\csc 30°}\right]^{2/3}\left(\frac{1}{500}\right)^{1/2}$$

Guess $y = 1$ m, $Q = 8.11\ m^3/s$, drop down, converge to $\mathbf{y \approx 0.993\ m}$ *Ans.*

10.19 Modify Prob. 10.17 as follows: Let the surface be clean earth, which erodes if V exceeds 1.5 m/s. What is the maximum depth to avoid erosion?

Solution: For clean earth, $n \approx 0.022$. Guess y and iterate to find $V \approx 1.5$ m/s:

$$\text{Guess}\quad y \approx 0.8\text{ m},\ A = 2.71\ m^2,\ R_h = 0.521\text{ m},\ V = \frac{1.0}{0.022}(0.521)^{2/3}\left(\frac{1}{500}\right)^{1/2} \approx 1.32\text{ m/s}$$

Try $y \approx 1.0$ m to get $V \approx 1/48$ m/s, move up slightly to $\mathbf{y < 1.025\ m}$ *Ans.*

10.20 A circular corrugated-metal storm drain is flowing half-full over a slope of 4 ft/mile. Estimate the normal discharge if the drain diameter is 8 ft.

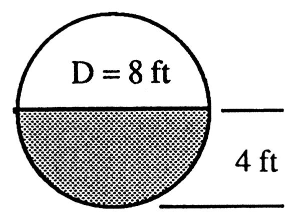


Fig. P10.20

Solution: For corrugated metal, $n \approx 0.022$. Evaluate the hydraulic radius, etc.:

$$A = (\pi/2)R^2 = 25.13\text{ ft}^2;\quad P = \pi R = 12.56\text{ ft},\quad R_h = A/P = R/2 = 2\text{ ft}$$

$$Q = \frac{1.486}{n}AR_h^{2/3}S_o^{1/2} = \frac{1.486}{0.022}(25.13)(2.0)^{2/3}\left(\frac{4}{5280}\right)^{1/2} \approx \mathbf{74\ \frac{ft^3}{s}}\quad \textit{Ans.}$$

10.21 An engineer makes careful measurements with a weir (see Sect. 10.7 later) which monitors a rectangular unfinished concrete channel laid on a slope of 1°. She finds, perhaps with surprise, that when the water depth doubles from 2 ft 2 inches to 4 ft 4 inches, the normal flow rate more than doubles, from 200 to 500 ft^3/s. (a) Is this plausible? (b) If so, estimate the channel width.

Solution: (a) **Yes**, Q always more than doubles for this situation where the depth doubles. *Ans.* (a)

(b) For unfinished concrete, take n = 0.014. Apply the normal-flow formula (10.19) to this data:

$$Q = \frac{1.486}{n} A R_h^{2/3} S_o^{1/2} = \frac{1.486}{0.014}(bh)\left(\frac{bh}{b+2h}\right)^{2/3}\sqrt{\sin 1^\circ} = 200(or\ 500)\ \frac{\text{ft}^3}{\text{s}}$$

$$\text{if}\ \ h = 2.17(or\ 4.33)\text{ft}$$

The two pieces of flow rate data give us two equations to solve for width b. It is unusual, but true, that both round-number flow rates converge to the same width $\boldsymbol{b} = \mathbf{5.72\ ft}$. *Ans.* (b)

10.22 A trapezoidal aqueduct has b = 5 m and θ = 40° and carries a normal flow of 60 m^3/s when y = 3.2 m. For clay tile surfaces, estimate the required elevation drop in m/km.

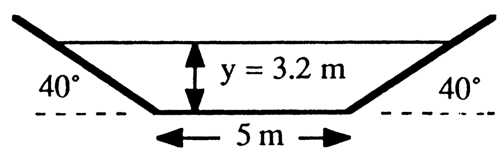


Fig. P10.22

Solution: For clay tile, take $n \approx 0.014$. The geometry leads to these values:

$$A = by + y^2 \cot\theta = 28.2\ m^2;\quad P = b + 2y\csc\theta = 14.96\ m,\quad R_h = A/P = 1.886\ m$$

$$Q = 60\ m^3/s = \frac{1.0}{0.014}(28.2)(1.886)^{2/3} S_o^{1/2},\quad \text{solve for } S_o = 0.00038 = \mathbf{0.38\ m/km}\quad \textit{Ans.}$$

10.23 It is desired to excavate a clean-earth channel as a trapezoidal cross-section with θ = 60° (see Fig. 10.7). The expected flow rate is 500 ft^3/s, and the slope is 8 ft per mile. The uniform flow depth is planned, for efficient performance, such that the flow cross-section is half a hexagon. What is the appropriate bottom width of the channel?

Solution: For clean earth, take $n = 0.022$. For a half-hexagon, from Fig. 10.7 of the text, depth $y = \sin(60°)b = 0.866b$, and

$$A = by + y^2\cot 60° = b(0.866b) + (0.866b)^2(0.577) = 1.299b^2; \quad P = 3b$$

$$Q = 500\ \frac{ft^3}{s} = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1.486}{0.022}(1.299b^2)\left(\frac{1.299b^2}{3b}\right)^{2/3}(8/5280)^{1/2} = 1.955b^{8/3}$$

Solve for **b = 8.0 ft** *Ans.*

10.24 A riveted-steel channel slopes at 1:500 and has a Vee shape with an included angle of 80°. Find the normal depth if the flow rate is 900 m^3/h.

Solution: For riveted steel take $n \approx 0.015$. From Ex. 10.5 (the same included angle),

$$Q = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{900}{3600}\frac{m^3}{s} = \frac{1}{0.015}(y^2\cot 50°)\left(\frac{y}{2}\cos 50°\right)^{2/3}(1/500)^{1/2},$$

Solve for $y^{8/3} = 0.213$, or: $\mathbf{y_n \approx 0.56\ m}$ *Ans.*

10.25 The equilateral-triangle in Fig. P10.25 has constant slope and Manning factor *n*. Find Q_{max} and V_{max}. Then, by analogy with Fig. 10.6b, plot the ratios Q/Q_{max} and V/V_{max} as a function of y/a for the range $0 < y/a < 0.866$.

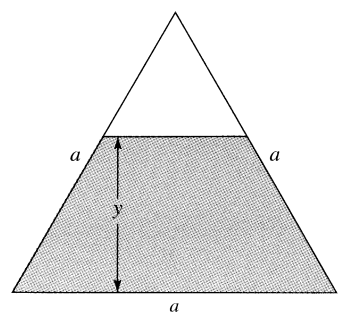


Fig. P10.25

Solution: The geometry is really not too hard, so we may compute V and Q as follows:

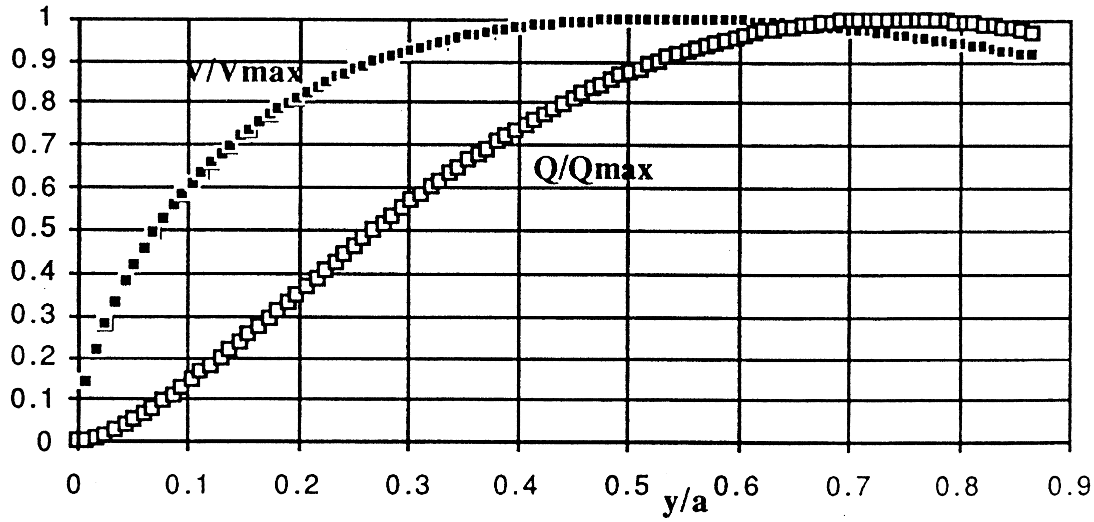
$$V = \frac{1}{n}R_h^{2/3}S_o^{1/2}, \quad \text{where } R_h = \frac{A}{P} \quad \text{and} \quad A = \frac{a^2}{2}\left[0.866 - \left(1 - \frac{y}{0.866a}\right)^2\right],$$

$$P = a + 2a\left(1 - \frac{y}{0.866a}\right) \quad \text{and, finally, } Q = VA$$

The maximum velocity and flow-rate values are

$$V_{max} \approx \mathbf{0.301}\frac{\mathbf{a^{2/3}}}{\mathbf{n}}\mathbf{S_o^{1/2}} \quad \text{at } \frac{y}{a} \approx 0.54; \quad Q_{max} \approx \mathbf{0.123}\frac{\mathbf{a^{8/3}}}{\mathbf{n}}\mathbf{S_o^{1/2}} \quad \text{at } \frac{y}{a} \approx 0.74 \quad \textit{Ans.}$$

The desired plots are shown on the following page and resemble Fig. 10.6b in the text.



10.26 In the spirit of Fig. 10.6*b*, analyze a rectangular channel in uniform flow with constant area $A = by$, constant slope, but varying width b and depth y. Plot the resulting flow rate Q, normalized by its maximum value Q_{max}, in the range $0.2 < b/y < 4.0$, and comment on whether it is crucial for discharge efficiency to have the channel flow at a depth exactly equal to half the channel width.

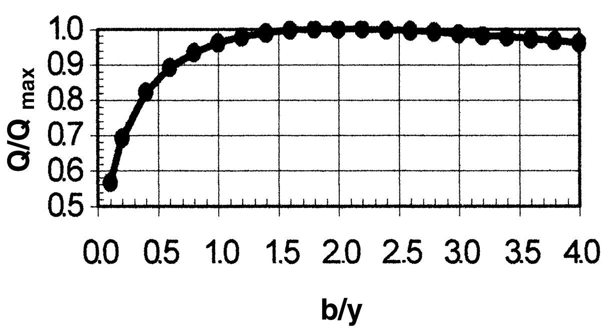
Solution: The Manning formula for a rectangular channel is:

$$Q = \frac{\alpha}{n} A R_h^{2/3} S_o^{1/2} \quad \text{where } R_h = \frac{A}{b+2y} \quad \text{and} \quad A = by$$

$$Q = Q_{max} \quad \text{when } b = 2y, \quad \textit{or}: R_h = A/2b$$

Then A cancels in the ratio $Q/Q_{max} = [2b/(b+2y)]^{2/3}$

Plot Q/Q_{max} versus (b/y) making sure that area is constant, that is, $b = A/y$. The results are shown in the graph below. The curve is very flat near $b = 2y$, so **depth is not crucial**.



Problem 10.26

10.27 A circular unfinished-cement water channel has a slope of 1:600 and a diameter of 5 ft. Estimate the normal discharge in gal/min for which the average wall shear stress is 0.18 lbf/ft^2, and compare your result to the maximum possible discharge for this channel.

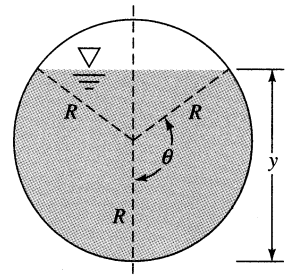


Fig. 10.6 (a)

Solution: For unfinished cement, take $n \approx 0.014$. From Prob. 10.28, we obtain

$$\tau_o = \rho g R_h S_o, \quad \text{where } R_{h,\text{circle}} = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right),$$

$$\text{or:}\quad R_h = \frac{\tau_o}{\rho g S_o} = \frac{0.15 \text{ psf}}{62.4(1/600)} \approx 1.44 \text{ ft}$$

$$\text{Solve}\quad R_h = 1.44 \text{ ft} = \frac{2.5 \text{ ft}}{2}\left[1 - \frac{\sin 2\theta}{2\theta}\right] \quad \text{for } \theta \approx \mathbf{108°}, \quad \text{for which } A \approx 13.55 \text{ ft}^2$$

This is below the point of maximum flow rate, which occurs at $\theta \approx 151°$. Compute

$$Q = \frac{1.486}{n} A R_h^{2/3} S_o^{1/2} = \frac{1.486}{0.014}(13.55)(1.44)^{2/3}\left(\frac{1}{600}\right)^{1/2} \approx 75 \frac{\text{ft}^3}{\text{s}} \approx \mathbf{33600} \ \frac{\textbf{gal}}{\textbf{min}} \quad \textit{Ans.}$$

This is about **71%** of $Q_{max} = 2.219\left(\frac{1.486}{n}\right) R^{8/3} S_o^{1/2} = 47700$ gal/min

---

10.28 Show that, for any straight, prismatic channel in uniform flow, the average wall shear stress is given by

$$\tau_{avg} \approx \rho g R_h S_o$$

Use this result in Prob. 10.27 also.

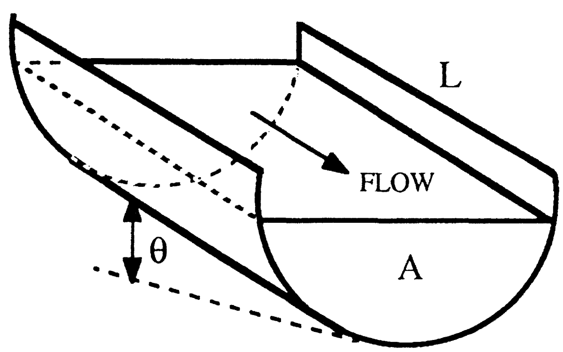


Fig. P10.28

Solution: For a control volume enclosing the fluid prism of length L as shown in the figure,

$$\sum F_{\text{along flow}} = W_{\text{fluid}} \sin\theta - \tau_{\text{avg}} A_{\text{wall}}, \quad \text{or} \quad \tau_{\text{avg}} PL = \rho g A L \sin\theta$$

But $\sin\theta = S_o$ by definition, L cancels, leaving $\boldsymbol{\tau_{avg} = \rho g R_h \sin\theta}$ *Ans.*

10.29 Suppose that the trapezoidal channel of Fig. P10.17 contains sand and silt which we wish not to erode. According to an empirical correlation by A. Shields in 1936, the average wall shear stress τ_{crit} required to erode sand particles of diameter d_p is approximated by

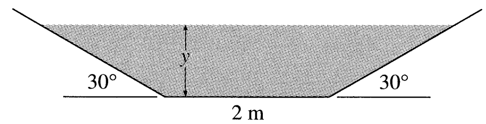


Fig. P10.17

$$\frac{\tau_{\text{crit}}}{(\rho_s - \rho) g\, d_p} \approx 0.5$$

where $\rho_s \approx 2400\ \text{kg/m}^3$ is the density of sand. If the slope of the channel in Fig. P10.17 is 1:900 and $n \approx 0.014$, determine the maximum water depth to keep from eroding particles of 1-mm diameter.

Solution: We relate the Shields critical shear stress to our result in Prob. 10.28 above:

$$\tau_{\text{crit}} = 0.5(2400 - 998)(9.81)(0.001\ \text{m}) \approx 6.88\ \text{Pa} = \rho g R_h S_o = (9790) R_h \left(\frac{1}{900}\right)$$

Solve for $R_{h,\text{crit}} \approx 0.632\ \text{m} = \dfrac{A}{P}$, where $A = by + y^2 \cot 30°$ and $P = b + 2y \csc 30°$

By iteration (b = 2 m), we solve for water depth **y < 1.02 m** to avoid erosion. *Ans.*

10.30 A clay tile V-shaped channel, with an included angle of 90°, is 1 km long and is laid out on a 1:400 slope. When running at a depth of 2 m, the upstream end is suddenly closed while the lower end continues to drain. Assuming quasi-steady normal discharge, find the time for the channel depth to drop to 20 cm.

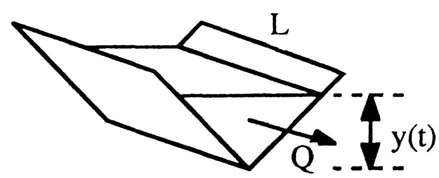


Fig. P10.30

Solution: We assume quasi-steady uniform flow at any instant. For a control volume enclosing the entire channel of length L = 1 km, we obtain

$$\frac{d}{dt}(m_{CV}) = -\dot{m}_{out}, \quad \text{or,} \quad \text{cancelling } \rho, \quad \frac{d}{dt}(LA) = -Q_{out} = -\frac{1}{n}AR_h^{2/3}S_o^{1/2}$$

$$\text{For a Vee-channel,} \quad A = y^2\cot 45° \quad \text{and} \quad R_h = \frac{y}{2}\cos 45°, \quad L = 1000\text{ m}, \quad S_o = \frac{1}{400}$$

Clean this up, separate the variables, and integrate:

$$\int_{y_o}^{y}\frac{dy}{y^{5/3}} = -\text{const}\int_0^t dt, \quad \text{or} \quad y_o^{-2/3} - y^{-2/3} = -Ct, \quad \text{where } C = \frac{(1/n)(\cos 45°/2)^{2/3}S_o^{1/2}}{(3/2)L}$$

$$\text{or:} \quad C = 0.00119 \quad \text{for our case. Set } (2.0)^{-2/3} - (0.2)^{-2/3} = -0.00119t_{drain}$$

$$\text{Solve for} \quad \mathbf{t_{drain}} = 1927\text{ sec} \approx \mathbf{32\ min} \quad \textit{Ans.}$$

10.31 A storm drain has the cross section shown in Fig. P10.31 and is laid on a slope 1.5 m/km. If it is constructed of brickwork, find the normal discharge when the water level passes through the center of the circle.

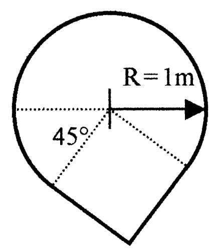


Fig. P10.31

Solution: For brickwork take n = 0.015. The section properties are:

$$A = R^2\left(1+\frac{\pi}{4}\right) = (1\text{ m})^2(1+0.785) = 1.785\text{ m}^2; \quad P = \frac{1}{4}(2\pi)(1\text{ m}) + 1\text{ m} + 1\text{ m} = 3.571\text{ m}$$

$$Q = \frac{1}{0.015}(1.785\text{ m}^2)\left(\frac{1.785\text{ m}^2}{3.571\text{ m}}\right)^{2/3}\sqrt{0.0015} = \mathbf{2.90\ m^3/s} \quad \textit{Ans.}$$

10.32 A 2-m-diameter clay tile sewer pipe runs half full on a slope of 0.25°. Compute the normal flow rate in gal/min.

Solution: For clay tile, take $n \approx 0.014$. For a half-full circle,

$$A = \frac{\pi}{2}R^2 = 1.57\text{ m}^2, \quad R_h = \frac{R}{2} = 0.5\text{ m},$$

$$Q = \frac{1}{0.014}(1.57)(0.5)^{2/3}\sqrt{\sin(0.25°)} = 4.67\text{ m}^3/\text{s} \approx \mathbf{74000\ gal/min} \quad \textit{Ans.}$$

10.33 Five of the sewer pipes from Prob. 10.32 empty into a single asphalt pipe, also laid out at 0.25°. If the large pipe is also to run half-full, what should be its diameter?

Solution: For asphalt, take $n \approx 0.016$. This time the radius is unknown:

$$Q = 5Q_{small} = 5(4.67)\ \frac{m^3}{s} = \frac{1}{0.016}\left(\frac{\pi}{2}R^2\right)\left(\frac{R}{2}\right)^{2/3}\sqrt{\sin 0.25°}, \quad \text{solve } R \approx 1.92\ m$$

$$\text{or:}\quad \mathbf{D \approx 3.84\ m} \quad \textit{Ans.}$$

10.34 A brick rectangular channel, with a slope of 0.002, is designed to carry 230 ft^3/s of water in uniform flow. There is an argument over whether the channel width should be 4 ft or 8 ft. Which design needs fewer bricks? By what percentage?

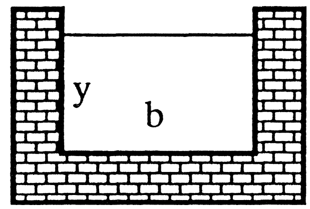


Fig. P10.34

Solution: For brick, take $n \approx 0.015$. For both designs, $A = by$ and $P = b + 2y$. Thus

$$Q = 230\frac{ft^3}{s} = \frac{1.486}{0.015}(by)\left(\frac{by}{b+2y}\right)^{2/3}(0.002)^{1/2}$$

(a) If $b = \mathbf{4}$ ft, solve for $y \approx 9.31$ ft or perimeter $P \approx$ **22.62 ft** *Ans.* (a)

(b) If $b = \mathbf{8}$ ft, solve $y \approx 4.07$ ft or $P \approx$ **16.14 ft** *Ans.* (b)

For a given channel-wall thickness, the number of bricks is proportional to the *perimeter*. Thus the 8-ft-wide channel has 16.14/22.62 = 71% as many, or **29% fewer bricks**.

10.35 In flood stage a natural channel often consists of a deep main channel plus two floodplains, as in Fig. P10.35. The floodplains are often shallow and rough. If the channel has the same slope everywhere, how would you analyze this situation for the discharge? Suppose that $y_1 = 20$ ft, $y_2 = 5$ ft, $b_1 = 40$ ft, $b_2 = 100$ ft, $n_1 = 0.020$, $n_2 = 0.040$, with a slope of 0.0002. Estimate the discharge in ft^3/s.

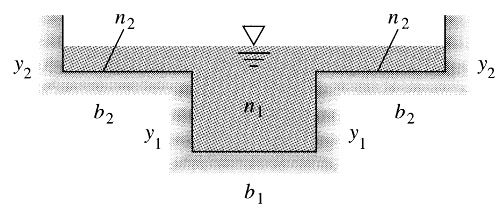


Fig. P10.35

Solution: We compute the flow rate in three pieces, with the dashed lines in the figure above serving as "water walls" which are not counted as part of the perimeter:

$$\text{(a)}\quad \text{Deep channel:}\quad Q_1 = \frac{1.486}{0.02}(25\times 40)\left(\frac{25\times 40}{20+40+20}\right)^{2/3}(0.0002)^{1/2} \approx 5659\ ft^3/s$$

(b) Flood plains: $2Q_2 = 2\left(\frac{1.486}{0.04}\right)(5\times 100)\left(\frac{5\times 100}{5+100+0}\right)^{2/3}(0.0002)^{1/2} \approx 1487\ \text{ft}^3/\text{s}$

Total discharge $\mathbf{Q} = Q_1 + 2Q_2 = \mathbf{7150\ ft^3/s}$ *Ans.*

10.36 The Blackstone River in northern Rhode Island normally flows at about 25 m^3/s and resembles Fig. P10.35 with a clean-earth center channel, $b_1 \approx 20$ m and $y_1 \approx 3$ m. The bed slope is about 2 ft/mi. The sides are heavy brush with $b_2 \approx 150$ m. During hurricane Carol in 1955, a record flow rate of 1000 m^3/s was estimated. Use this information to estimate the maximum flood depth y_2 during this event.

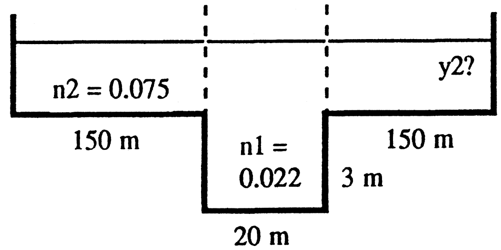


Fig. P10.36

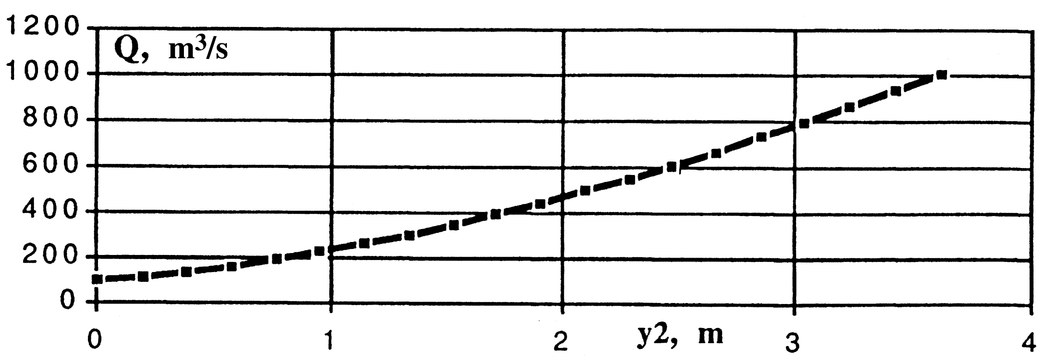
Solution: For heavy brush, $n_2 = 0.075$ and for clean earth, $n_1 = 0.022$, as shown in the figure. Use the same "zero-perimeter water-wall" scheme as in Prob. 10.35:

$$Q = Q_1 + 2Q_2 = \frac{1}{0.022}A_1R_{h1}^{2/3}S_o^{1/2} + 2\frac{1}{0.075}A_2R_{h2}^{2/3}S_o^{1/2} = \left(1000\ \frac{m^3}{s}\right)$$

where $S_o = \frac{2}{5280}$, $A_1 = (3+y_2)20$, $R_{h1} = \frac{A_1}{6+20}$, $A_2 = 150y_2$, and $R_{h2} = \frac{A_2}{y_2+150}$

Solve by iteration for $\mathbf{y_2 \approx 3.6\ m}$. *Ans.*

This heavy rainfall overflowed the flood plains and was the worst in Rhode Island history. A graph of flow rate versus flood-plain depth y_2 is shown below.



10.37 A triangular channel (see Fig. E10.6) is to be constructed of corrugated metal and will carry 8 m^3/s on a slope of 0.005. The supply of sheet metal is limited, so the engineers want to minimize the channel surface. What is (a) the best included angle θ for the channel; (b) the normal depth for part (a); and (c) the wetted perimeter for part (b).

Solution: For corrugated metal, take n = 0.022. From Ex. 10.5, for a vee-channel, recall that

$$A = y^2\tan(\theta/2);\quad P = 2y\sec(\theta/2);$$
$$R_h = 0.5y\sin(\theta/2)$$

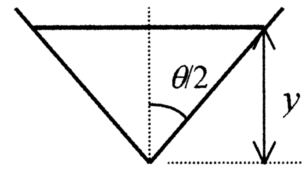


Fig. P10.37

Manning's formula (10.19) predicts that:

$$Q = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = 8\ \frac{m^3}{s} = \frac{1}{0.022}\left(y^2\tan\frac{\theta}{2}\right)\left(\frac{y}{2}\sin\frac{\theta}{2}\right)^{2/3}\sqrt{0.005}$$

(a) Eliminate y in terms of P and set $dP/d\theta = 0$. The algebra is not too bad, and the result is:

Minimum perimeter P occurs for a given flow rate at $\boldsymbol{\theta = 90°}$ *Ans.* (a)

(b) Insert $\theta = 90°$ in the formula for Q = 8 m^3/s above and solve for:

$\boldsymbol{y = 1.83\ m}$ *Ans.* (b) and $\boldsymbol{P_{min} = 5.16\ m}$. *Ans.* (c)

10.38 A rectangular channel has b = 3 m and y = 1 m. If n and S_o are the same, what is the diameter of a semicircular channel which will have the same discharge? Compare the two wetted perimeters.

Solution: The rectangular channel has A = 3 m^2 and P = 5 m. Set the flow rates equal:

$$Q_{rect} = \frac{1}{n}(3\ m^2)\left(\frac{3}{5}\ m\right)^{2/3}S_o^{1/2} \overset{?}{=} Q_{semicircle} = \frac{1}{n}\left(\frac{\pi}{2}R^2\right)(R/2)^{2/3}S_o^{1/2},$$

$$\text{or:}\quad R_{circle}^{8/3} = 2.16,\quad R \approx 1.334\ m,\quad \text{or}\quad \mathbf{D_{semicircle} \approx 2.67\ m}\quad \textit{Ans.}$$

The semicircle perimeter is $P = \pi R \approx 4.19$ m, or **16% less** than the rectangle P = 5 m.

10.39 A trapezoidal channel has n = 0.022 and S_0 = 0.0003 and is made in the shape of a half-hexagon for maximum efficiency. What should the length of the side of the hexagon be if the channel is to carry 225 ft^3/s of water? What is the discharge of a semicircular channel of the same cross-sectional area and the same S_0 and n?

Solution: The half-hexagon corresponds to Fig. 10.7 with $\theta = 60°$. Its properties are

$$A = \frac{3b^2}{2}\sin 60°, \quad R_h = \frac{b}{2}\sin 60°,$$

$$Q = 225\ \frac{ft^3}{s} = \frac{1.486}{0.022}\left(\frac{3}{2}b^2\sin 60°\right)\left(\frac{b}{2}\sin 60°\right)^{2/3}(0.0003)^{1/2}$$

$$\text{or:}\quad b^{8/3} \approx 259, \quad \mathbf{b \approx 8.03\ ft} \quad \textit{Ans.} \quad (\text{for which } A_{hexagon} \approx 83.79\ ft^2)$$

A semicircular channel of the same area has D = $[8(83.79)/\pi]^{1/2} \approx 14.6$ ft. Its hydraulic radius and flow rate are

$$R_{h,semicircle} = D/4 \approx 3.65\ ft, \quad Q = \frac{1.486}{0.022}(83.8)(3.65)^{2/3}(0.0003)^{1/2}$$

$$Q \approx 232\ ft^3/s \ (\text{about } \mathbf{3\%} \text{ more flow}) \quad \textit{Ans.}$$

10.40 Using the geometry of Fig. 10.6a, prove that the most efficient circular open channel (maximum hydraulic radius for a given flow area) is a semicircle.

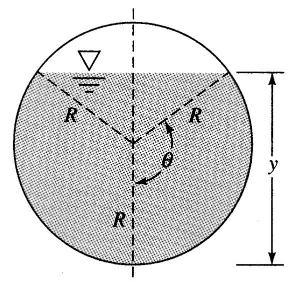


Fig. 10.6 (a)

Solution: Maximum hydraulic radius means minimum perimeter. Using Eq. 10.20,

$$A = R^2\left(\theta - \frac{1}{2}\sin 2\theta\right), \quad P = 2R\theta, \text{ eliminate R:} \quad P = \frac{2\theta\sqrt{A}}{\sqrt{\theta - \sin(2\theta)/2}}$$

$$\text{Differentiate:} \quad \left.\frac{dP}{d\theta}\right|_{\text{constant A}} = 0 \quad \text{if} \quad \boldsymbol{\theta = 90°} \quad \textit{Ans.}$$

10.41 Determine the most efficient value of θ for the vee-shaped channel of Fig. P10.41.

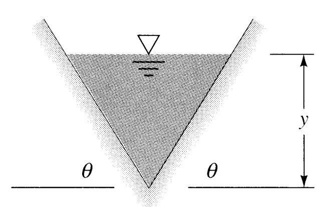


Fig. P10.41

Solution: Given the (simple) geometric properties

$$A = y^2 \cot\theta; \quad P = 2y\csc\theta; \quad \text{Eliminate y: } P = 2\csc\theta\sqrt{A\tan\theta}$$

$$\text{Set } \left.\frac{dP}{d\theta}\right|_{\text{constant A}} = 0 \quad \text{if} \quad \boldsymbol{\theta = 45°} \quad \textit{Ans.}$$

10.42 Suppose that the side angles of the trapezoidal channel in Prob. 10.39 are reduced to 15° to avoid earth slides. If the bottom flat width is 8 ft, (a) determine the normal depth and (b) compare the resulting wetted perimeter with the solution P = 24.1 ft from Prob. 10.39. (Do not reveal this answer to friends still struggling with Prob. 10.39.)

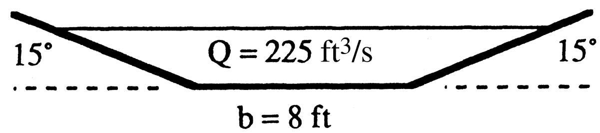


Fig. P10.42

Solution: Recall that we specified $n \approx 0.022$ and $S_o = 0.0003$. The new bottom width, b = 8 ft, is almost exactly what we found in Prob. 10.39 for the half-hexagon (8.03 ft).

$$\text{Set} \quad Q = 225\ \frac{ft^3}{s} = \frac{1.486}{0.022} A\left(\frac{A}{P}\right)^{2/3}(0.0003)^{1/2}, \quad \text{where } A = 8y + y^2\cot(15°)$$

and $P = 8 + 2y\csc(15°)$. Solve by iteration for $\mathbf{y_n \approx 4.31\ ft}$ *Ans.* (a)

Wetted perimeter $= 8 + 2(4.31)\csc(15°) \approx \mathbf{41.3\ ft}$ *Ans.* (b) (71% more)

10.43 What are the most efficient dimensions for a riveted-steel *rectangular* channel to carry 4.8 m^3/s of water at a slope of 1:900?

Solution: For riveted steel, take n ≈ 0.015. We know from Eq. (10.26) that

Best rectangle: $b = 2y$; $A = 2y^2$; $R_h = y/2$. So the flow rate is

$$Q = 4.8 = \frac{1}{0.015}(2y^2)(y/2)^{2/3}\left(\frac{1}{900}\right)^{1/2}, \quad \text{solve } \mathbf{y \approx 1.22\ m,\ b \approx 2.45\ m} \quad \textit{Ans.}$$

10.44 What are the most efficient dimensions for a *half-hexagon* cast-iron channel to carry 15000 gal/min of water at a slope of 0.16°?

Solution: For cast iron, take $n \approx 0.013$. We know from Fig. 10.7 for a half-hexagon that

$$A = \frac{3b^2}{2}\sin 60°,\ R_h = \frac{b}{2}\sin 60°, \quad \text{hence } Q = \frac{15000}{448.83}\frac{ft^3}{s} = \frac{1.486}{0.013} A R_h^{2/3}\sqrt{\sin 0.16°}$$

Solve for side length $\mathbf{b \approx 2.12\ ft}$ *Ans.*

10.45 What are the most efficient dimensions for an asphalt *trapezoidal* channel to carry 3 m^3/s of water at a slope of 0.0008?

Solution: For asphalt, take $n \approx 0.016$. We know from Fig. 10.7 for a trapezoid that

$$A = y^2[2\csc 45° - \cot 45°]; \quad R_h = \frac{1}{2}y; \quad \text{Set } Q = 3\ \frac{m^3}{s} = \frac{1}{0.016} A\, R_h^{2/3}\sqrt{0.0008}$$

or $y^{8/3} \approx 1.47$, or: $\mathbf{y_n \approx 1.16\ m}$ *Ans.* (corresponds to bottom width $b = 0.96$ m)

10.46 It is suggested that a channel which reduces erosion has a **parabolic shape**, as in Fig. P10.46. Formulas for area and perimeter of the parabolic cross-section are as follows [Ref. 7 of Chap. 10]:

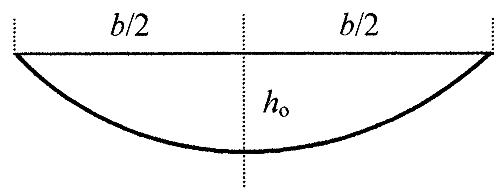


Fig. P10.46

$$A = \frac{2}{3}bh_0; \quad P = \frac{b}{2}\left[\sqrt{1+\alpha^2} + \frac{1}{\alpha}\ln\left(\alpha + \sqrt{1+\alpha^2}\right)\right], \quad \text{where } \alpha = \frac{4h_0}{b}$$

For uniform flow conditions, determine the most efficient ratio h_0/b for this channel (minimum perimeter for a given constant area).

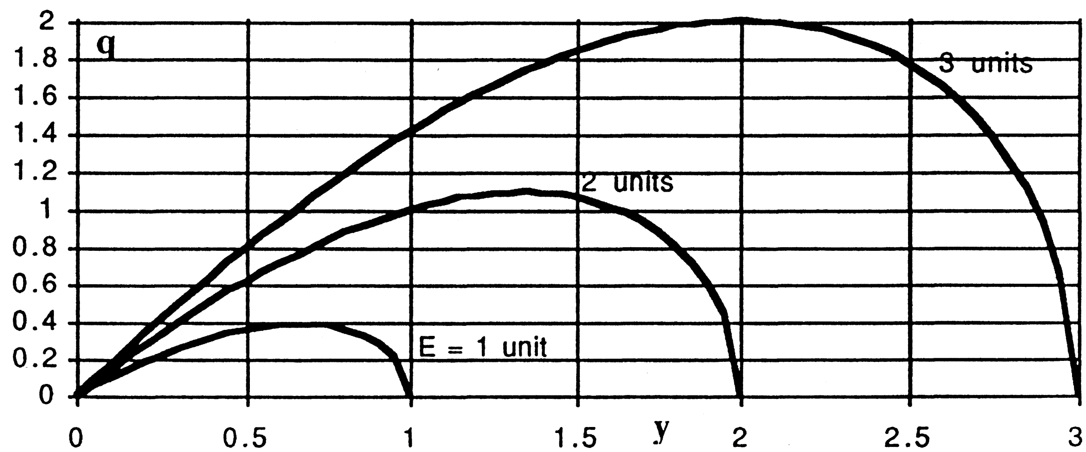
Solution: We are to minimize *P* for constant *A*, and this time, unlike Prob. 10.37, the algebra is too heavy, what with logarithms and square roots, to solve for *P* in terms of *A* and h_0/b. The writer backed off and simply used a spreadsheet (or EES) to find the minimum *P* numerically. The answer is

Minimum *P* (maximum *Q* for a given *A*) occurs at $\mathbf{h_0/b \approx 0.486}$ *Ans.*

[NOTE: For a sine-wave shape, instead of a parabola, the answer is $h_0/b \equiv 1/2$.]

10.47 Replot Fig. 10.8b in the form of *q* versus *y* for constant *E*. Does the maximum *q* occur at the critical depth?

Solution: The energy formula has the form $q^2 = 2g(Ey^2 - y^3)$, plot for constant E and g:



Differentiate to find $dq/dy|_{\text{const } E} = 0$ if $y = 2E/3$ **which indeed = y_{crit}**. *Ans.*

10.48 A wide, clean-earth river has a flow rate $q = 150$ ft³/(s·ft). What is the critical depth? If the actual depth is 12 ft, what is the Froude number of the river? Compute the critical slope by (a) Manning's formula and (b) the Moody chart.

Solution: For clean earth, take $n \approx 0.030$ and roughness $\varepsilon \approx 0.8$ ft. The critical depth is

$$y_c = (q^2/g)^{1/3} = [(150)^2/32.2]^{1/3} \approx \mathbf{8.87\ ft} \quad \textit{Ans.}$$

$$\text{If} \quad y_{actual} = 12 \text{ ft}, \quad Fr = \frac{V}{V_c} = \frac{q/y}{\sqrt{gy_c}} = \frac{150/12}{\sqrt{32.2(8.87)}} = \frac{12.5}{16.9} \approx \mathbf{0.739} \quad \textit{Ans.}$$

The critical slope is easy to compute by Manning and somewhat harder by the Moody chart:

$$\text{(a) Manning:} \quad S_c = \frac{gn^2}{\xi y_c^{1/3}} = \frac{32.2(0.030)^2}{2.208(8.87)^{1/3}} \approx \mathbf{0.00634} \text{ Manning} \quad \textit{Ans.} \text{ (a)}$$

(b) Moody: $\dfrac{1}{\sqrt{f}} \approx -2\log_{10}\left(\dfrac{0.8}{3.7(4)(8.87)}\right)$,

or $f \approx 0.0509$, $S_c = \dfrac{f}{8} \approx \mathbf{0.00637}$ Moody *Ans.* (b)

10.49 Find the critical depth of the brick channel in Prob. 10.34 for both the 4- and 8-ft widths. Are the normal flows sub- or supercritical?

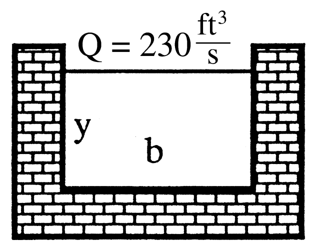


Fig. P10.49

Solution: For brick, take $n \approx 0.015$. Recall and extend our results from Prob. 10.34:

(a) $b = 4$ ft: $y_c = \left(\dfrac{Q^2}{b^2 g}\right)^{1/3} = \left[\dfrac{(230)^2}{(4)^2(32.2)}\right]^{1/3} = \mathbf{4.68\ ft}$ [$y_n = 9.31$ ft is **subcritical**]

(b) $b = 8$ ft: $y_c = \left[\dfrac{(230)^2}{(8)^2(32.2)}\right]^{1/3} = \mathbf{2.95\ ft}$ [$y_n = 4.07$ ft is **subcritical**] *Ans.* (a, b)

10.50 A pencil point piercing the surface of a rectangular channel flow creates a 25° half-angle wedgelike wave, as in Fig. P10.50. If the channel surface is painted steel and the depth is 35 cm, determine (a) the Froude no.; (b) the critical depth; and (c) the critical slope.

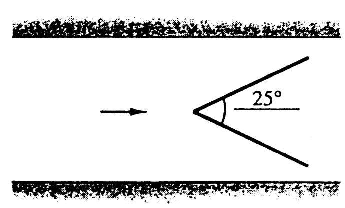


Fig. P10.50

Solution: For painted steel, take $n \approx 0.014$. The wave angle and depth give

$$Fr = \csc(25°) = \mathbf{2.37} \quad \textit{Ans. (a)} \quad \therefore\ V = Fr\,V_c = 2.37\sqrt{9.81(0.35)} = 4.38\ \frac{m}{s}$$

$$\text{Flow rate} \quad q = Vy = 4.38(0.35) = 1.53\ \frac{ft^3}{s{\cdot}ft},$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(1.53)^2}{9.81}\right]^{1/3} \approx \mathbf{0.62\ m} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Finally,} \quad S_c = \frac{gn^2}{\xi y_c^{1/3}} = \frac{9.81(0.014)^2}{(1.0)(0.62)^{1/3}} = \mathbf{0.0023}$$

10.51 An asphalt circular channel, of diameter 75 cm, is flowing half-full at an average velocity of 3.4 m/s. Estimate (a) the volume flow rate; (b) the Froude number; and (c) the critical slope.

Solution: For an asphalt channel, take $n = 0.016$. For a half-full channel, $A = \pi R^2/2$, $P = \pi R$, $R_h = R/2$, and $b_o = 2R$. The volume flow is easy, and Froude number and critical slope are not hard either:

$$Q = VA = \left(3.4\ \frac{m}{s}\right)[\pi(0.375\ m)^2/2] = \mathbf{0.75\ m^3/s} \quad \textit{Ans.}\ \text{(a)}$$

$$V_c = \sqrt{\frac{gA_c}{b_o}} = \sqrt{\frac{(9.81\ m/s^2)[\pi(0.375\ m)^2/2]}{0.75\ m}} = 1.70\ \frac{m}{s}, \quad \mathbf{Fr} = \frac{3.4\ m/s}{1.7\ m/s} = \mathbf{2.00} \quad \textit{Ans.}\ \text{(b)}$$

$$S_c = \frac{n^2 V_c^2}{\alpha^2 R_{hc}^{4/3}} = \frac{(0.016)^2 (1.7\ m/s)^2}{(1)^2 (0.1875\ m)^{4/3}} = \mathbf{0.0069} \quad \textit{Ans.}\ \text{(c)}$$

10.52 Water flows full in an asphalt half-hexagon channel of bottom width *W*. The flow rate is 12 m^3/s. Estimate *W* if the Froude number is exactly 0.6.

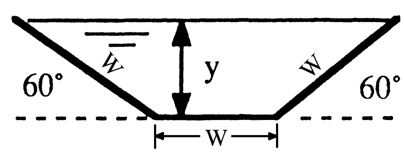


Fig. P10.52

Solution: For asphalt, $n = 0.016$, but we don't need n because *critical* flow is independent of roughness. Work out the properties of a half-hexagon:

$$y = W\sin 60°, \quad A = Wy + y^2\cot 60° = 1.299W^2, \quad b_o = W + 2W\cos 60° = 2W$$

$$V_c = \sqrt{\frac{gA}{b_o}} = \sqrt{\frac{9.81(1.299W^2)}{2W}} = 2.524W^{1/2}, \quad V = FrV_c = 0.6V_c = 1.515W^{1/2}$$

$$Q = 12\ \frac{\text{m}^3}{\text{s}} = AV = (1.299W^2)(1.515W^{1/2}) = 1.967W^{2.5}, \quad \text{solve } \mathbf{W = 2.06\ m} \quad \textit{Ans.}$$

10.53 For the river flow of Prob. 10.48, find the depth y_2 which has the same specific energy as the given depth $y_1 = 12$ ft. These are called *conjugate depths*. What is Fr_2?

Solution: Recall from Prob. 10.48 that the flow rate is $q = 150\ \text{ft}^3/(\text{s}\cdot\text{ft})$. Hence

$$E = y_1 + \frac{V_1^2}{2g} = 12 + \frac{(150/12)^2}{2(32.2)} = 14.43\ \text{ft} \stackrel{?}{=} y_2 + \frac{V_2^2}{2g} = y_2 + \frac{(150/y_2)^2}{2(32.2)}$$

This is a cubic equation which has only one realistic solution: $\mathbf{y_2 \approx 6.74\ ft}$ *Ans.*

$$V_2 = \frac{150}{6.74} = 22.2\ \frac{\text{ft}}{\text{s}}, \quad Fr_2 = \frac{V_2}{V_{c2}} = \frac{22.2}{\sqrt{32.2(6.74)}} \approx \mathbf{1.51} \quad \textit{Ans.} \text{ (compared to } Fr_1 = 0.74)$$

10.54 A clay tile V-shaped channel has an included angle of 70° and carries 8.5 m^3/s. Compute (a) the critical depth, (b) the critical velocity, and (c) the critical slope for uniform flow.

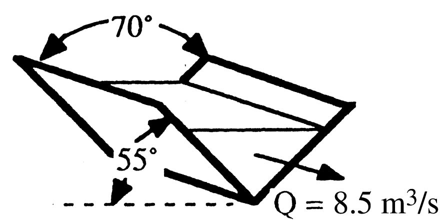


Fig. P10.54

Solution: For clay tile, take $n \approx 0.014$. The cross-section properties are

$$P = 2y\csc 55°; \quad A = y^2\cot 55°; \quad R_h = \frac{y}{2}\cos 55°; \quad b_o = 2y\cot 55°$$

$$A_c = \left(\frac{b_o Q^2}{g}\right)^{1/3} = \left[\frac{2y_c(\cot 55°)(8.5)^2}{9.81}\right]^{1/3} = y_c^2\cot 55°, \quad \text{solve for } \mathbf{y_c \approx 1.975\ m} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Compute} \quad A_c = 2.731\ \text{m}^2, \quad b_{oc} = 2.766\ \text{m}, \quad V_c = \left[\frac{9.81(2.731)}{2.766}\right]^{1/2} \approx \mathbf{3.11\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(b)}$$

$$\text{Compute} \quad R_h = 0.566\ \text{m}, \quad S_c = \frac{n^2 g A_c}{\alpha^2 b_o R_h^{4/3}} = \frac{(0.014)^2(9.81)(2.731)}{(1)(2.766)(0.566)^{4/3}} \approx \mathbf{0.00405} \quad \textit{Ans.}\ \text{(c)}$$

10.55 A trapezoidal channel resembles Fig. 10.7 with b = 1 m and $\theta = 50°$. The water depth is 2 m and Q = 32 m^3/s. If you stick your fingernail in the surface, as in Fig. P10.50, what half-angle wave might appear?

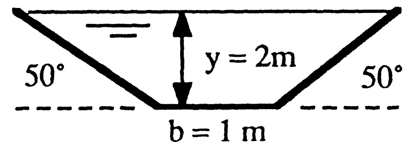


Fig. P10.55

Solution: The cross-section properties are

$$A = by + y^2 \cot 50° = 5.36\ \text{m}^2, \quad \text{hence}\ \ V = Q/A = \frac{32}{5.36} \approx 5.97\ \text{m/s}$$

$$b_o = b + 2y\cot 50° = 4.36\ \text{m}, \quad \text{hence}\ \ V_c = (gA/b_o)^{1/2} = \left[\frac{9.81(5.36)}{4.36}\right]^{1/2} = 3.47\ \text{m/s}$$

$$\text{Thus}\quad Fr = V/V_c = \frac{5.97}{3.47} = 1.72 = \csc\theta, \quad \text{or:}\quad \boldsymbol{\theta_{wave} \approx 35.5°}\quad \textit{Ans.}$$

The flow is definitely supercritical, and a 'fingernail wave' will indeed appear.

10.56 A riveted-steel triangular duct flows partly full as in Fig. P10.56. If the critical depth is 50 cm, compute (a) the critical flow rate; and (b) the critical slope.

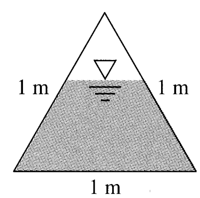


Fig. P10.56

Solution: For riveted steel, take $n \approx 0.015$. Then,

$$\text{If}\ \ y_c = 0.5\ \text{m},\ b_o = 0.423\ \text{m}, \quad A_c = 0.356\ \text{m}^2 = \left[\frac{0.423Q^2}{9.81}\right]^{1/3},$$

$$\text{Solve}\quad Q \approx \mathbf{1.02}\ \frac{\mathbf{m^3}}{\mathbf{s}}\quad \textit{Ans.}\ \text{(a)}$$

$$P = 2.15\ \text{m}, \quad R_h = 0.165\ \text{m}; \quad S_c = \frac{n^2 g A_c}{\alpha^2 b_o R_h^{4/3}} = \frac{(0.015)^2(9.81)(0.356)}{(1)(0.423)(0.165)^{4/3}} \approx \mathbf{0.0205}\quad \textit{Ans.}\ \text{(b)}$$

10.57 For the triangular duct of Fig. P10.56, if the critical flow rate is 1.0 m^3/s, compute (a) the critical depth; and (b) the critical slope.

Solution: We were almost there in Prob. 10.56, Q ≈ 1.02 m^3/s—drop *y* a little bit, to 49 cm. This is too low, Q ≈ 0.99 m^3/s. So interpolate to $\mathbf{y_c \approx 0.493\ m}$. *Ans.* (a).

$$\text{For}\quad y = 0.493\text{ m},\quad S_c = \frac{n^2 g A_c}{\alpha^2 b_o R_h^{4/3}} = \frac{(0.015)^2(9.81)(0.350)}{(1)(0.446)(0.164)^{4/3}} \approx \mathbf{0.0194}\quad \textit{Ans. (b)}$$

10.58 A circular corrugated-metal water channel is half-full and in uniform flow when laid on a slope of 0.0118. The average shear stress on the channel walls is 29 Pa. Estimate (a) the channel diameter; (b) the Froude number; and (c) the volume flow rate.

Solution: For corrugated metal take *n* = 0.022. This problem relates to Prob. 10.28, which showed that:

$$\tau_{avg} = \rho g R_h S_o = 29\text{ Pa} = \left(998\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)R_h(0.0118),\quad \text{solve for}\ \ R_h = 0.251\text{ m}$$

$$\text{Then}\quad \mathbf{D_{channel}} = 4R_h = 4(0.251\text{ m}) = \mathbf{1.00\ m}\quad \textit{Ans. (a)}$$

$$V = \frac{1}{n}R_h^{2/3}S_o^{1/2} = \frac{1}{0.022}(0.251\text{ m})^{2/3}\sqrt{0.0118} = 1.96\text{ m/s}$$

$$V_c = \sqrt{\frac{gA}{b_o}} = \sqrt{\frac{(9.81\text{ m/s}^2)[\pi(0.5\text{ m})^2/2]}{1.0\text{ m}}} = 1.96\text{ m/s},\quad \mathbf{Fr} = \frac{1.96\text{ m/s}}{1.96\text{ m/s}} = \mathbf{1.00}\quad \textit{Ans. (b)}$$

$$Q = VA = (1.96\text{ m/s})[\pi(0.5\text{ m})^2/2] = \mathbf{0.77\ m^3/s}\quad \textit{Ans. (c)}$$

10.59 Uniform water flow in a wide brick channel of slope 0.02° moves over a 10-cm bump as in Fig. P10.59. A slight depression in the water surface results. If the minimum depth over the bump is 50 cm, compute (a) the velocity over the bump; and (b) the flow rate per meter of width.

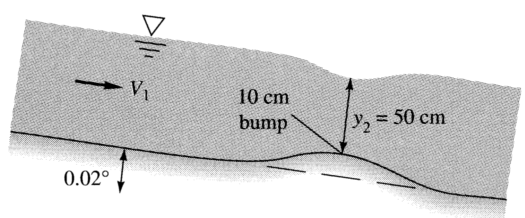


Fig. P10.59

Solution: For brickwork, take $n \approx 0.015$. Since the water level decreases over the bump, the upstream flow is *subcritical*. For a wide channel, $R_h = y/2$, and Eq. 10.39 holds:

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{2g} = 0,\quad q = V_1 y_1,\quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h,\quad \Delta h = 0.1\text{ m},\quad y_2 = 0.5\text{ m}$$

Meanwhile, for uniform flow, $q = \dfrac{1}{0.015} y_1 (y_1/2)^{2/3} \sqrt{\sin 0.02°} = 0.785 y_1^{5/3}$

Solve these two simultaneously for y_1 = 0.608 m, $\mathbf{V_1}$ **= 0.563 m/s** *Ans.* (a), and **q = 0.342 m³/s·m**. *Ans.* (b) [The upstream flow is subcritical, $Fr_1 \approx 0.23$.]

10.60 Modify Prob. 10.59 as follows. Again assuming uniform subcritical approach conditions (V_1, y_1), find (a) the flow rate and (b) y_2 for which the Froude number Fr_2 at the crest of the bump is exactly 0.7.

Solution: The basic analysis of Prob. 10.59 for uniform upstream flow plus a bump, still holds:

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{2g} = 0; \quad q = V_1 y_1; \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h; \quad \Delta h = 0.1 \text{ m}$$

$$\text{Uniform upstream flow:} \quad q = \frac{1}{0.015} y_1 \left(\frac{y_1}{2}\right)^{2/3} \sqrt{\sin(0.02°)} = 0.785 y_1^{5/3}$$

This time, however, y_2 is unknown, and we specify $Fr_2 = V_2/(g y_2)^{1/2} = \mathbf{0.7}$. Iteration or EES are necessary. The final results are that $Fr_2 = 0.7$ if:

$$y_1 = 0.203 \text{ m}; \quad V_1 = 0.271 \text{ m/s}; \quad V_2 = 0.642 \text{ m/s}$$

$$\boldsymbol{q = 0.055\ \mathbf{m^3/s/m}} \quad \textit{Ans. (a)}; \qquad \boldsymbol{y_2 = 0.0857\ \mathbf{m}} \quad \textit{Ans. (b)}$$

10.61 Modify Prob. 10.59 as follows: Again assuming uniform subcritical approach flow V_1, find (a) the flow rate q; and (b) the height y_2 for which the Froude number Fr_2 at the crest of the bump is exactly 1.0 (critical).

Solution: The basic analysis above, for uniform upstream flow plus a bump, still holds:

$$y_2^3 - E_2 y_2^2 + \frac{q^2}{2g} = 0, \quad q = V_1 y_1, \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h, \quad \Delta h = 0.1 \text{ m}$$

Meanwhile, for uniform flow, $q = \dfrac{1}{0.015} y_1 (y_1/2)^{2/3} \sqrt{\sin 0.02°} = 0.785 y_1^{5/3}$

This time, however, y_2 is unknown, and we need $Fr_2 = V_2/\sqrt{(gy_2)} = \mathbf{1.0}$. [At the crest in Prob. 10.60, $Fr_2 \approx 0.8$.] The iteration proceeds laboriously to the result:

$$Fr_2 = 1.0 \quad \text{if} \quad y_1 = 0.1916\ \text{m}; \quad V_1 = 0.261\ \frac{\text{m}}{\text{s}};$$

$$\mathbf{y_2 \approx 0.0635\ m} \quad \textit{Ans.}\ \text{(a)}; \quad \mathbf{q = 0.0500\ \frac{m^3}{m \cdot s}} \quad \textit{Ans.}\ \text{(b)}$$

[Finding the *critical* point is more difficult than finding a purely subcritical solution.]

10.62 Consider the flow in a wide channel over a bump, as in Fig. P10.62. One can estimate the water-depth change or *transition* with frictionless flow. Use continuity and the Bernoulli equation to show that

$$\frac{dy}{dx} = -\frac{dh/dx}{1 - V^2/(gy)}$$

Is the drawdown of the water surface realistic in Fig. P10.62? Explain under what conditions the surface might rise above its upstream position y_0.

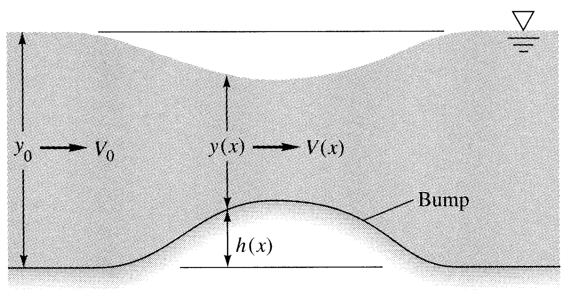


Fig. P10.62

Solution: This is a form of frictionless "gradually-varied" flow theory (Sect. 10.6). Use the frictionless energy equation from upstream to any point along the bump section:

$$\frac{p_{atm}}{\rho g} + \frac{V_o^2}{2g} + y_o = \frac{p_{atm}}{\rho g} + \frac{V^2}{2g} + h + y, \quad \text{differentiate:}\ 0 = \frac{VdV}{g} + dy + dh;$$

$$\text{Solve for:} \quad \mathbf{\frac{dy}{dx} = -\frac{dh/dx}{1 - V^2/(gy)}} \quad \textit{Ans.}$$

Assuming dh/dx > 0 in front (a 'bump'), dy/dx will be positive (a *rise* in water level) if the flow is *supercritical* (i.e., Fr > 1 or $V^2 > gy$).

10.63 In Fig. P10.62, let $V_o = 1$ m/s and $y_o = 1$ m. If the maximum bump height is 15 cm, estimate (a) the Froude number over the top of the bump; and (b) the maximum depression in the water surface.

Solution: Here we don't need to differentiate, just apply Eq. 10.39 directly:

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0, \quad \text{where } E_2 = \frac{V_1^2}{2g} + y_1 - h_{max} = \frac{(1)^2}{2(9.81)} + 1 - 0.15 = 0.901 \text{ m}$$

$$\text{Insert } \quad V_1 y_1 = 1.0 \quad \text{to get } \; y_2^3 - 0.901 y_2^2 + 0.051 = 0, \quad \text{solve for } \; y_2 \approx 0.826 \text{ m}$$

Thus the center depression is $\Delta\mathbf{z} = 1 - 0.15 - 0.826 \approx$ **0.024 m**. *Ans.* (b)
Also, $V_2 = 1.21$ m/s. The bump Froude number is $Fr_2 = 1.21/[9.81(.826)]^{1/2} \approx$ **0.425** *Ans.* (a).

10.64 In Fig. P10.62, let $V_o = 1$ m/s and $y_o = 1$ m. If the flow over the top of the bump is *exactly critical* (Fr = 1), determine the bump height h_{max}.

Solution: Here we guess bump heights until we find $Fr_2 = V_2/[gy_2]^{1/2} = 1.0$. [Clearly the bump must be higher than 15 cm, which only gave ≈ 0.425 above.] After considerable iteration (first guess h_{max}, then solve the resulting cubic for y_2) we find

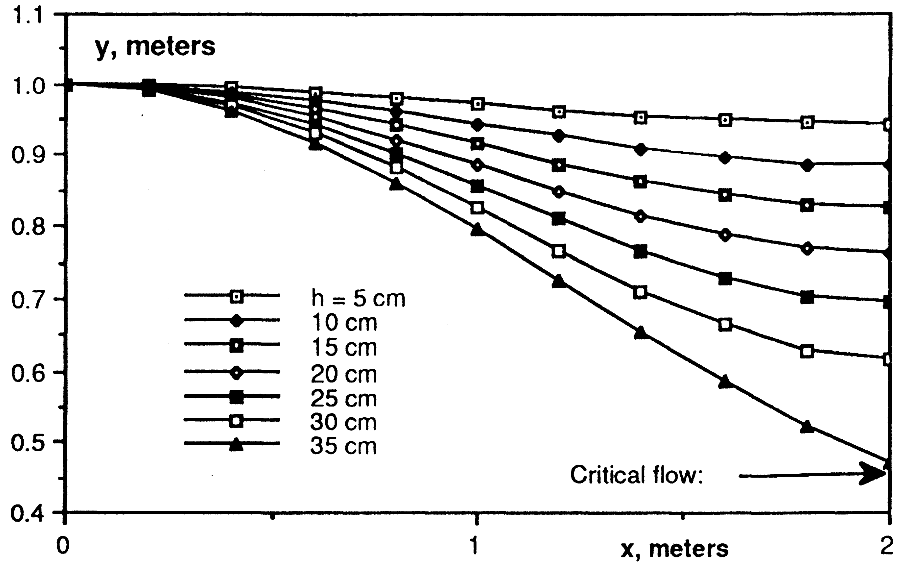
$$y_2^3 - E_2 y_2^2 + \frac{1}{2(9.81)} = 0, \quad E_2 = \frac{1}{2(9.81)} + 1 - h_{max}, \quad Fr_2 = 1.0 \quad \text{if } \; \mathbf{h_{max} \approx 0.35\ m} \quad \textit{Ans.}$$

[This corresponds to a water level $y_2 \approx 0.467$ m and $\Delta z = 0.1873$ m.]

10.65 Program and solve the differential equation of "frictionless flow over a bump," from Prob. 10.62, for entrance conditions $V_0 = 1$ m/s and $y_0 = 1$ m. Let the bump have the convenient shape $h = 0.5h_{max}[1 - \cos(2\pi x/L)]$, which simulates Fig. P10.62. Let $L = 3$ m, and generate a numerical solution for $y(x)$ in the bump region $0 < x < L$. If you have time for only one case, use h_{max} = 15 cm (Prob. 10.63), for which the maximum Froude number is 0.425. If more time is available, it is instructive to examine a complete family of surface profiles for $h_{max} \approx 1$ cm up to 35 cm (which is the solution of Prob. 10.64).

Solution: We solve the differential equation $dy/dx = -(dh/dx)/[1 - V^2/(gy)]$, with $h = 0.5h_{max}[1 - \cos(2\pi x/L)]$, plus continuity, $Vy = 1$ m²/s, subject to initial conditions V = 1.0

and y = 1.0 at x = 0. The plotted water profiles for various bump heights are as follows:



The Froude numbers at the point of maximum bump height are as follows:

h_{max}, cm:	0	5	10	15	20	25	30	35
Fr_{bump}:	0.319	0.348	0.383	0.425	0.479	0.550	0.659	1.000

10.66 In Fig. P10.62 let $V_o = 6$ m/s and $y_o = 1$ m. If the maximum bump height is 35 cm, estimate (a) the Froude number over the top of the bump; and (b) the maximum increase in the water-surface level.

Solution: This is a straightforward application of Eq. 10.39 for *supercritical* approach:

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{6}{\sqrt{9.81(1)}} = 1.92 > 1. \quad E_2 = \frac{V_1^2}{2g} + y_1 - \Delta h = \frac{(6)^2}{2(9.81)} + 1 - 0.35 = 2.48 \text{ m}$$

$$y_2^3 - E_2 y_2^2 + \frac{V_1^2 y_1^2}{2g} = y_2^3 - 2.48 y_2^2 + 1.835 = 0; \quad \text{solve } y_2 \approx \mathbf{1.19\ m} \quad \textit{Ans.} \text{ (b)}$$

$$\text{Then} \quad V_2 = 6(1)/1.19 = 5.03 \text{ m/s} \quad \text{and} \quad Fr_2 = V_2/\sqrt{(gy_2)} \approx \mathbf{1.47} \quad \textit{Ans.} \text{ (a)}$$

10.67 In Fig. P10.62 let $V_o = 5$ m/s and $y_o = 1$ m. If the flow over the top of the bump is exactly critical (Fr = 1), determine the bump height h_{max}.

Solution: The set-up is the same as Prob. 10.66, with a different V_o and with Fr_2 specified:

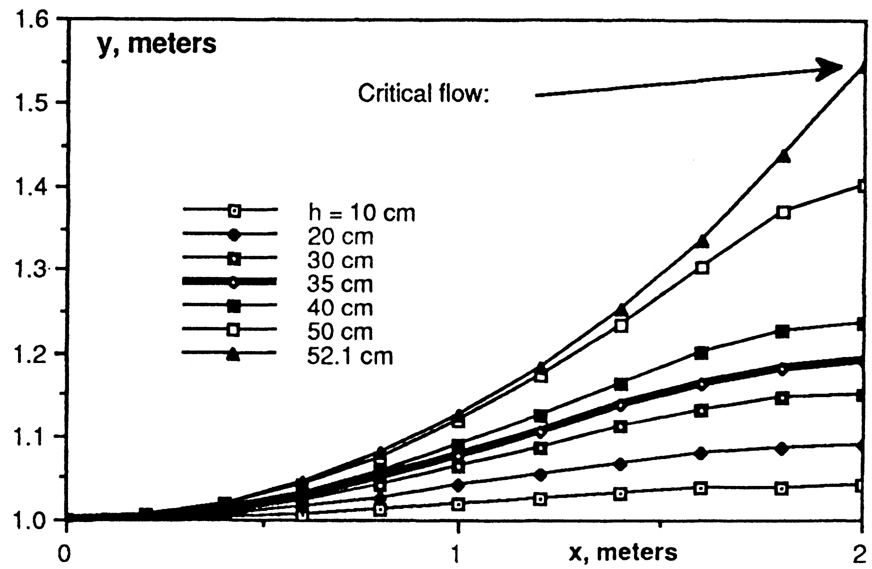
$$Fr_o = \frac{5}{\sqrt{9.81(1.0)}} = 1.596; \quad E_2 = \frac{5^2}{2(9.81)} + 1.0 - h_{\mathbf{max}} = 2.274 - h_{\mathbf{max}}; \quad V_o y_o = V_2 y_2$$

$$\text{Solve} \quad y_2^3 - E_2 y_2^2 + \frac{5^2}{2(9.81)} = 0 \quad \text{such that } Fr_2 = \frac{V_2}{\sqrt{9.81 y_2}} = 1.0$$

The solution is $y_2 = 1.366$ m, $V_2 = 3.66$ m/s, and $\boldsymbol{h_{\mathbf{max}} = 0.225}$ **m**. *Ans.*

10.68 Modify Prob. 10.65 to have a supercritical approach condition $V_0 = 6$ m/s and $y_0 = 1$ m. If you have time for only one case, use $h_{max} = 35$ cm (Prob. 10.66), for which the maximum Froude number is 1.47. If more time is available, it is instructive to examine a complete family of surface profiles for 1 cm < h_{max} < 52 cm (which is the solution to Prob. 10.67).

Solution: This is quite similar to the subcritical display in Prob. 10.65. The new family of supercritical-flow profiles is shown below:



The Froude numbers at the point of maximum bump height are as follows:

h_{max}, cm:	0	10	20	30	35	40	50	52.1
Fr_{bump}:	1.92	1.80	1.68	1.55	1.47	1.39	1.15	1.000

10.69 Given is the flow of a channel of large width b under a sluice gate, as in Fig. P10.69. Assuming frictionless steady flow with negligible upstream kinetic energy, derive a formula for the dimensionless flow ratio $Q^2/(y_1^3 b^2 g)$ as a function of the ratio y_2/y_1. Show by differentiation that the maximum flow rate occurs at $y_2 = 2y_1/3$.

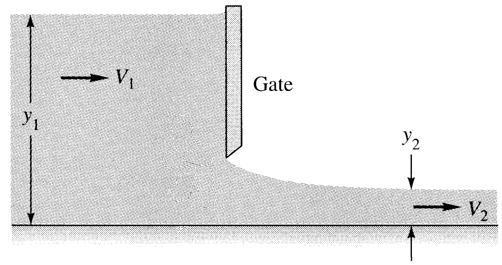


Fig. P10.69

Solution: With upstream kinetic energy neglected, the energy equation becomes

$$y_1 \approx y_2 + \frac{V_2^2}{2g} = y_2 + \frac{(Q/by_2)^2}{2g}; \quad \text{rearrange and multiply by } \left(y_2^2/y_1^3\right):$$

$$\mathbf{\frac{Q^2}{gb^2y_1^3} = 2(y_2/y_1)^2 - 2(y_2/y_1)^3} \quad \textit{Ans.}$$

Differentiate this with respect to (y_2/y_1) to find maximum Q at $\mathbf{y_2/y_1 = 2/3}$ *Ans.*

10.70 In Fig. P10.69 let $V_1 = 0.75$ m/s and $V_2 = 4.0$ m/s. Estimate (a) the flow rate per unit width; (b) y_2; and (c) Fr_2.

Solution: Equation (10.40) is not too useful because y_1 is unknown. Just use the basic equations:

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{(0.75\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{(4.0\text{ m/s})^2}{2(9.81\text{ m/s}^2)}$$

$$q = V_1 y_1 = (0.75\text{ m/s})y_1 = V_2 y_2 = (4.0\text{ m/s})y_2$$

(a, b, c) Iterate, or use EES, and the final solution is:

$q = \mathbf{0.726\ m^3/s/m}$ *Ans.* (a); $y_2 = \mathbf{0.182\ m}$ *Ans.* (b); $Fr_2 = V_2/(gy_2)^{1/2} = \mathbf{3.00}$ *Ans.* (c)

10.71 In Fig. P10.69 let $y_1 = 95$ cm and $y_2 = 50$ cm. Estimate the flow rate per unit width if the upstream kinetic energy is (a) neglected; and (b) included.

Solution: The result of Prob. 10.69 gives an excellent answer to part (a):

$$\text{Neglect } \frac{V_1^2}{2g}: \quad \frac{q^2}{2gy_1^3} = \left(\frac{y_2}{y_1}\right)^2 - \left(\frac{y_2}{y_1}\right)^3 = \left(\frac{50}{95}\right)^2 - \left(\frac{50}{95}\right)^3 = 0.1312 = \frac{q^2}{2(9.81)(0.95)^3}$$

$$\text{Solve for}\quad \mathbf{q \approx 1.49\ \frac{m^3}{s\cdot m}}\quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) Exact:}\quad y_2^3 - \left(\frac{V_1^2}{2g} + y_1\right)y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0,$$

$$\text{or:}\quad V_1^2 = \frac{2gy_2^2(y_1 - y_2)}{y_1^2 - y_2^2} = \frac{2(9.81)(0.5)^2(0.95-0.5)}{(0.95)^2 - (0.5)^2} = 3.383,$$

$$V_1 = 1.84\ \frac{m}{s},\quad \mathbf{q} = V_1 y_1 = \mathbf{1.75\ \frac{m^3}{s\cdot m}}\quad \textit{Ans.}\text{ (b)}$$

10.72 Water approaches the wide sluice gate in the figure, at V_1 = 0.2 m/s and y_1 = 1 m. Accounting for upstream kinetic energy, estimate, at outlet section 2, (a) depth; (b) velocity; and (c) Froude number.

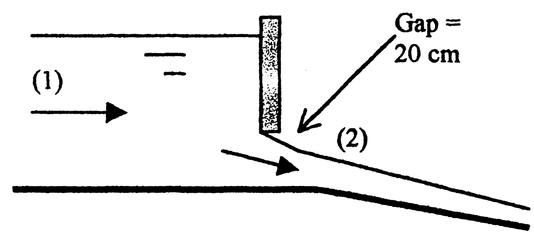


Fig. P10.72

Solution: (a) If we assume frictionless flow, the gap size is immaterial, and Eq. (10.40) applies:

$$y_2^3 - \left(y_1 + \frac{V_1^2}{2g}\right)y_2^2 + \frac{V_1^2 y_1^2}{2g} = 0 = y_2^3 - 1.00204y_2^2 + 0.00204$$

EES yields 3 solutions: $y_2 = 1.0$ m(trivial); -0.0442 m (impossible);

and the correct solution: $\mathbf{y_2 = 0.0462\ m}$ *Ans.* (a)

$$\text{(b)}\quad \text{From continuity,}\quad V_2 = \frac{V_1 y_1}{y_2} = \frac{(1.0)(0.2)}{0.0462} = \mathbf{4.33\ \frac{m}{s}}\quad \textit{Ans.}\text{ (b)}$$

$$\text{(c)}\quad \text{The Froude number is}\quad Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{4.33}{\sqrt{9.81(0.0462)}} = \mathbf{6.43}\quad \textit{Ans.}\text{ (c)}$$

10.73 In Fig. P10.69 suppose that y_1 = 1.4 m and the gate is raised so that its gap H is 15 cm. Estimate the resulting flow rate per unit width and the downstream depth.

Solution: The flow rate follows immediately from Eq. (10.41):

$$q = C_d H\sqrt{2gy_1}, \quad C_d = \frac{0.61}{\sqrt{1+0.61\,H/y_1}} = \frac{0.61}{\sqrt{1+0.61(0.15\text{ m})/(1.4\text{ m})}} = 0.591$$

$$q = (0.591)(0.15\text{ m})\sqrt{2(9.81\text{ m/s}^2)(1.4\text{ m})} = \mathbf{0.465\ m^3/s/m} \quad \textit{Ans.}$$

With q known, $V_1 = q/y_1 = 0.332$ m/s and $E_1 = 1.406\text{ m} = E_2$.

Solve for: $V_2 = 5.08$ m/s and $\mathbf{y_2 = 0.0915\ m}$ *Ans.* ($\text{Fr}_2 \approx 5.6$)

10.74 With respect to Fig. P10.69, show that, for frictionless flow, the upstream velocity may be related to the water levels by

$$V_1 = \sqrt{\frac{2g(y_1 - y_2)}{K^2 - 1}} \quad \text{where } K = y_1/y_2.$$

Solution: We have already shown this beautifully in Prob. 10.71*b*:

$$\text{Eq. 10.40:} \quad y_2^3 - \left(y_1 + V_1^2/2g\right)y_2^2 + (V_1y_1)^2/2g = 0;$$

$$\text{Solve for} \quad \mathbf{V_1} = \sqrt{\frac{\mathbf{2gy_2^2(y_1 - y_2)}}{\mathbf{y_1^2 - y_2^2}}} \quad \textit{Ans.}$$

10.75 A tank of water 1 m deep, 3 m long, and 4 m wide into the paper has a closed sluice gate on the right side, as in Fig. P10.75. At $t = 0$ the gate is opened to a gap of 10 cm. Assuming quasi-steady sluice-gate theory, estimate the time required for the water level to drop to 50 cm. Assume free outflow.

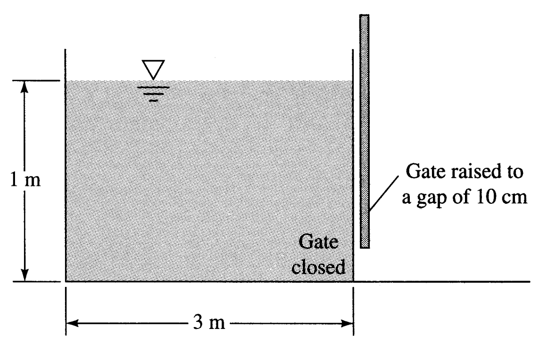


Fig. P10.75

Solution: Use a control volume surrounding the tank with Eq. 10.41 for the gate flow:

$$\frac{d}{dt}(\text{tank water}) = \frac{d}{dt}(bLy_1) = -Q_{out} = -C_dHb\sqrt{2gy_1}, \ C_d \approx 0.61/\sqrt{1+0.61H/y_1}$$

Because y_1 drops from 1.0 to 0.5 m, C_d also drops slowly from 0.592 to 0.576. Assume approximately constant $C_d \approx$ **0.584**, separate the variables and integrate:

$$\int_{y_o}^{y_1} \frac{dy_1}{\sqrt{y_1}} = -\frac{H}{L} C_d \sqrt{2g} \int_0^t dt, \quad \text{or:} \quad y_1 \approx \left(y_o^{1/2} - Kt\right)^2, \; y_o = 1\text{ m} \quad \text{and} \quad K = \frac{C_d H\sqrt{2g}}{2L}$$

$$K = \frac{0.584(0.1)\sqrt{2(9.81)}}{2(3.0)} = 0.0431. \quad \text{Set } y_1 = 0.5\text{ m}, \quad \text{solve for } \mathbf{t \approx 6.8\ s} \quad \textit{Ans.}$$

10.76 In Prob. 10.75 estimate what gap height *H* would cause the tank level to drop from 1 m to 30 cm in exactly 40 s. Assume free outflow.

Solution: We have the analytic solution to draining in Prob. 10.75. Just find *H*:

$$y_{1,final} = \left(y_o^{1/2} - Kt\right)^2 = 0.3\text{ m} = \left[(1.0\text{ m})^{1/2} - \frac{C_d H\sqrt{2(9.81\text{ m/s}^2)}}{2(3.0\text{ m})}(40\text{ s})\right]^2,$$

$$C_d = \frac{0.61}{\sqrt{1 + 0.61H/(1\text{ m})}}$$

We approximated $y_1 \approx 1$ m in the C_d formula, but the variation in $C_d \approx 0.6$ is negligible. A bit of iteration gives the final solution: $C_d \approx 0.605$, $\boldsymbol{H = 0.0253}$ **m**. *Ans.*

10.77 Equation 10.41 for the discharge coefficient is for *free* (nearly frictionless) outflow. If the outlet is *drowned*, as in Fig. 10.10*c*, there is dissipation and C_d drops sharply. Fig. P10.77 at right shows data from Ref. 3 on drowned vertical sluice gates. Use this chart to repeat Prob. 10.73, and plot the estimated flow rate versus y_2 in the range $0 < y_2 < 60$ cm.

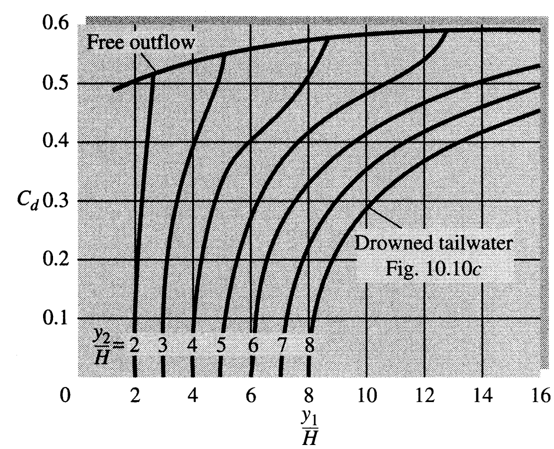
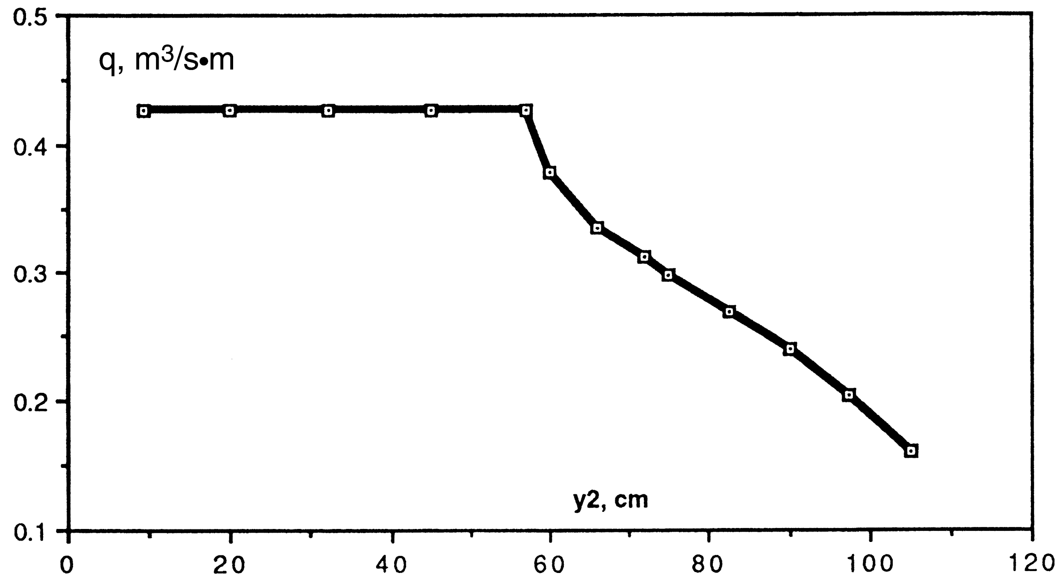


Fig. P10.77 [*from Ref. 2 of Chap. 10*]

Solution: Actually, for $y_1/H = 1.2/0.15 = 8$, there is no effect of drowning until $y_2/H > 3.8$, or $y_2 \approx 3.8(15) \approx 57$ cm. So let us plot out the flow rate a little further, up to $y_2 \approx 105$ cm, as shown on the next page. The effect of drowning is very sudden and sharp according to this correlation.



10.78 Repeat Prob. 10.75, to find the time to drain the tank from 1.0 m to 50 cm, if the gate is *drowned* downstream at $y_2 = 40$ cm. Again assume gap H = 10 cm.

Solution: With drowning, the discharge coefficient changes substantially during the drawdown, so our simple Prob. 10.75 solution, $y_1 \approx (1-Kt)^2$, is not valid. So we use the chart, Fig. P10.77, and calculate C_d and the flow rate as a function of tank depth:

y_1, m:	1.0	0.9	0.8	0.7	0.6	0.5
C_d:	0.58	0.58	0.52	0.45	0.40	0.33
q, $m^3/s{\cdot}m$:	0.257	0.244	0.206	0.167	0.137	0.103

Then we sum the approximate times to drop each 10 cm, as $\sum\Delta t = \sum(0.3\ m^3)/q_{avg}$. The approximate result is $\mathbf{t} = \sum \Delta t \approx \mathbf{8.6\ s}$, or 27% more than in Prob. 10.75. *Ans.*

10.79 Show that the Froude number downstream of a hydraulic jump will be given by $Fr_2 = 8^{1/2} Fr_1/[(1+8Fr_1^2)^{1/2} - 1]^{3/2}$. Does the formula remain correct if we reverse subscripts 1 and 2? Why?

Solution: Take the ratio of Froude numbers, use continuity, and eliminate y_2/y_1:

$$\frac{Fr_2}{Fr_1} = \frac{V_1}{\sqrt{gy_1}}\frac{\sqrt{gy_2}}{V_2} = \frac{V_1}{V_2}\sqrt{\frac{y_2}{y_1}}, \quad \text{but } \frac{V_1}{V_2} = \frac{y_2}{y_1} \text{ from continuity, so } \frac{Fr_2}{Fr_1} = \left(\frac{y_2}{y_1}\right)^{3/2}$$

$$\text{From Eq. 10.43,} \quad \frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2} - 1\right],$$

$$\text{so rearrange to} \quad \mathbf{Fr_2 = \frac{Fr_1\sqrt{8}}{\left[\sqrt{1+8Fr_1^2}-1\right]^{3/2}}} \quad \textit{Ans.}$$

The formula is indeed *symmetric*, you can reverse "1" and "2." *Ans.*

10.80 Water, flowing in a channel at 30-cm depth, undergoes a hydraulic jump of dissipation 71%. Estimate (a) the downstream depth; and (b) the volume flow.

Solution: We use the jump and dissipation relations, Eqs. (10.43) and (10.45):

$$\frac{2y_2}{y_1} = -1 + \sqrt{1+8Fr_1^2}, \quad Fr_1^2 = \frac{V_1^2}{gy_1}, \quad h_f = \frac{(y_2-y_1)^3}{4y_1y_2} = 0.71\left(y_1 + \frac{V_1^2}{2g}\right), \quad y_1 = 0.3 \text{ m}$$

Solve by EES: $V_1 = 16.1$ m/s; $Fr_1 = 9.38$; $y_2 = \mathbf{3.83\ m}$ *Ans.* (a);

$q = \mathbf{4.83\ m^3/s\cdot m}$ *Ans.* (b)

10.81 Water flows in a wide channel at $q = 25\ \text{ft}^3/\text{s}\cdot\text{ft}$ and $y_1 = 1$ ft and undergoes a hydraulic jump. Compute y_2, V_2, Fr_2, h_f, the percentage dissipation, and the horsepower dissipated per unit width. What is the critical depth?

Solution: This is a series of straightforward calculations:

$$V_1 = \frac{q}{y_1} = \frac{25}{1} = 25\ \frac{\text{ft}}{\text{s}}; \quad Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{25}{\sqrt{32.2(1)}} = 4.41; \quad E_1 = y_1 + \frac{V_1^2}{2g} \approx 10.7 \text{ ft}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(4.41)^2} - 1\right] = 5.75, \quad \text{or} \quad \mathbf{y_2 \approx 5.75\ ft} \quad \textit{Ans. (a)}$$

$$V_2 = q/y_2 = 25/5.75 \approx \mathbf{4.35\ \frac{ft}{s}}; \quad Fr_2 = \frac{4.35}{\sqrt{32.2(5.75)}} \approx \mathbf{0.32} \quad \textit{Ans. (b, c)}$$

$$h_f = (5.75-1)^3/[4(5.75)(1)] \approx 4.66 \text{ ft}, \quad \mathbf{\%\ dissipated} = 4.66/10.7 \approx \mathbf{44\%} \quad \textit{Ans. (d)}$$

$$\text{Power dissipated} = \rho g q h_f = (62.4)(25)(4.66) \div 550 \approx \mathbf{13.2\ hp/ft} \quad \textit{Ans.} \text{ (e)}$$

$$\text{Critical depth} \quad y_c = (q^2/g)^{1/3} = [(25)^2/32.2]^{1/3} \approx \mathbf{2.69\ ft} \quad \textit{Ans.} \text{ (f)}$$

10.82 Downstream of a wide hydraulic jump the flow is 4 ft deep and has a Froude number of 0.5. Estimate (a) y_1; (b) V_1; (c) Fr_1; (d) the percent dissipation; and (e) y_c.

Solution: As shown in Prob. 10.79, the hydraulic jump formulas are reversible. Thus, with Fr_1 known,

$$Fr_1 = \frac{Fr_2\sqrt{8}}{\left[\sqrt{1+8Fr_2^2}-1\right]^{3/2}} = \frac{(0.5)\sqrt{8}}{\left[\sqrt{1+8(0.5)^2}-1\right]^{3/2}} = \mathbf{2.26} \quad \textit{Ans.} \text{ (c)}$$

$$2\frac{y_1}{y_2} = 2\left(\frac{y_1}{4\text{ ft}}\right) = -1+\sqrt{1+8(0.5)^2} = 0.732, \quad \mathbf{y_1 = 1.46\ ft} \quad \textit{Ans.} \text{ (a)}$$

$$V_1 = Fr_1\sqrt{gy_1} = 2.26\sqrt{(32.2\text{ ft/s}^2)(1.46\text{ ft})} = \mathbf{15.5\ ft/s} \quad \textit{Ans.} \text{ (b)}$$

The percent dissipation follows from Eq. (10.45):

$$h_f = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(4.0\text{ ft}-1.46\text{ ft})^3}{4(4.0\text{ ft})(1.46\text{ ft})} = 0.696\text{ ft}; \quad E_1 = y_1 + \frac{V_1^2}{2g} = 1.46\text{ ft} + \frac{(15.5)^2}{2(32.2)} = 5.20\text{ ft}$$

$$\text{Percent dissipation} = \frac{h_f}{E_1} = \frac{0.696\text{ ft}}{5.20\text{ ft}} = 0.13 \quad \text{or} \quad \mathbf{13\%} \quad \textit{Ans.} \text{ (d)}$$

Finally, the critical depth for a wide channel is given by Eq. (10.30):

$$q = V_1y_1 = (15.5\text{ ft/s})(1.46\text{ ft}) = 22.7\text{ m}^2\text{/s}; \quad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(22.7)^2}{32.2}\right]^{1/3} = \mathbf{2.52\,ft} \quad \textit{Ans.} \text{ (e)}$$

10.83 A wide channel flow undergoes a hydraulic jump from 40 cm to 140 cm. Estimate (a) V_1; (b) V_2; (c) the critical depth; and (d) the percent dissipation.

Solution: With the jump-height-ratio known, use Eq. 10.43:

$$\frac{2y_2}{y_1} = \frac{2(140)}{40} = 7 = \sqrt{1+8Fr_1^2}-1, \quad \text{solve for } Fr_1 \approx 2.81$$

$$V_1 = Fr_1\sqrt{gy_1} = 2.81\sqrt{9.81(0.4)} \approx \mathbf{5.56\ \frac{m}{s}} \quad \textit{Ans. (a)}; \quad V_2 = \frac{V_1 y_1}{y_2} \approx \mathbf{1.59\ \frac{m}{s}} \quad \textit{Ans. (b)}$$

$$q = V_1 y_1 = 5.56(0.4) = 2.22\ \frac{m^3}{s\cdot m}, \quad y_{crit} = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(2.22)^2}{9.81}\right]^{1/3} \approx \mathbf{0.80\ m} \quad \textit{Ans. (c)}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.4 + \frac{(5.56)^2}{2(9.81)} = 1.98\ m; \quad h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.4-0.4)^3}{4(0.4)(1.4)} = 0.45\ m$$

$$\%\ dissipation = h_f/E_1 = 0.45/1.98 \approx \mathbf{23\%} \quad \textit{Ans. (d)}$$

10.84 Consider the flow under the sluice gate of Fig. P10.84. If y_1 = 10 ft and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percentage of dissipation, and sketch the flow to scale with the EGL included. The channel is horizontal and wide.

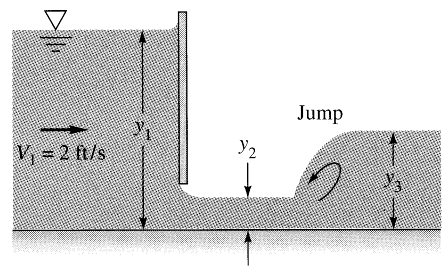


Fig. P10.84

Solution: First get the conditions at "2" by assuming a frictionless acceleration:

$$E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{(2)^2}{2(32.2)} = 10.062\ ft = E_2 = y_2 + \frac{V_2^2}{2g}, \quad \text{Also, } V_1y_1 = V_2y_2 = 20$$

$$\text{Solve for} \quad V_2 \approx 24.4\ ft/s; \ y_2 \approx \mathbf{0.820\ ft} \quad \textit{Ans. (a)} \quad Fr_2 = \frac{24.4}{\sqrt{32.2(0.820)}} \approx 4.75$$

$$\text{Jump:} \quad \frac{y_3}{y_2} = \frac{1}{2}\left[\sqrt{1+8Fr_2^2} - 1\right] \approx 6.23, \quad y_3 \approx \mathbf{5.11\ ft} \quad \textit{Ans. (b)}$$

$$E_2 = 10.062\ ft; \quad h_f = \frac{(y_3 - y_2)^3}{4y_2y_3} = \frac{(5.11-0.82)^3}{4(0.82)(5.11)} \approx 4.71\ ft,$$

$$\text{Dissipation} = \frac{4.71}{10.06} \approx \mathbf{47\%} \quad \textit{Ans. (c)}$$

10.85 In Prob. 10.72 the exit velocity from the sluice gate is 4.33 m/s. If there is a hydraulic jump at "3" just downstream of section 2, determine the downstream (a) velocity; (b) depth; (c) Froude number; and (d) percent dissipation.

Solution: If V_2 = 4.33 m/s, then $y_2 = V_1y_1/V_2 = (1.0)(0.2)/4.33 = 0.0462$ m, and the Froude number is $Fr_2 = V_2/[gy_2]^{1/2} = 6.43$. Now use hydraulic jump theory:

$$\frac{2y_3}{y_2} = -1+\sqrt{1+8(6.43)^2} = 17.2, \quad \text{or:} \quad \mathbf{y_3 = 0.398\ m} \quad \textit{Ans.}\ \text{(b)}$$

$$V_3 = \frac{q}{y_3} = \frac{(1.0)(0.2)}{0.398} = \mathbf{0.503\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$Fr_3 = \frac{V_3}{\sqrt{gy_3}} = \frac{0.503}{\sqrt{9.81(0.398)}} = \mathbf{0.255} \quad \textit{Ans.}\ \text{(c)}$$

$$h_f = \frac{(0.398-0.046)^3}{4(0.398)(0.046)} = 0.592\text{ m}; \quad \frac{h_f}{E_2} = \frac{0.592}{0.046+(4.33)^2/2g} = 0.59 \quad \textit{or} \quad \mathbf{59\%} \quad \textit{Ans.}\ \text{(d)}$$

10.86 A *bore* is a hydraulic jump which propagates upstream into a still or slower-moving fluid, as in Fig. 10.4*a*. Suppose that the still water is 2 m deep and the water behind the bore is 3 m deep. Estimate (a) the propagation speed of the bore and (b) the induced water velocity.

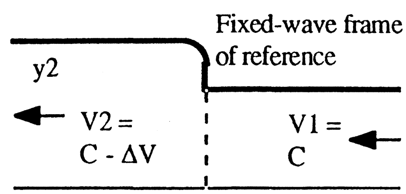


Fig. P10.86

Solution: The bore moves at speed C and induces a velocity ΔV behind it. If viewed in a *frame fixed to the wave*, as above, the approach velocity is $V_1 = C$ and, downstream, $V_2 = C - \Delta V$, as shown. We are given y_1 = 2 m and y_2 = 3 m so we can use Eq. 10.43:

$$\frac{y_2}{y_1} = \frac{3}{2} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2}-1\right], \quad \text{solve for } Fr_1 = 1.37$$

$$\text{Then} \quad V_1 = C = Fr_1\sqrt{gy_1} = 1.37\sqrt{9.81(2)} \approx \mathbf{6.07\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Meanwhile,} \quad V_2 = \frac{V_1y_1}{y_2} = \frac{6.07(2)}{3} = 4.04 = 6.07 - \Delta V, \quad \text{hence } \mathbf{\Delta V \approx 2.03\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(b)}$$

10.87 A *tidal bore* may occur when the ocean tide enters an estuary against an oncoming river discharge, such as on the Severn River in England. Suppose that the tidal bore is 10 ft deep and propagates at 13 mi/h upstream into a river which is 7 ft deep. Estimate the river current in kn.

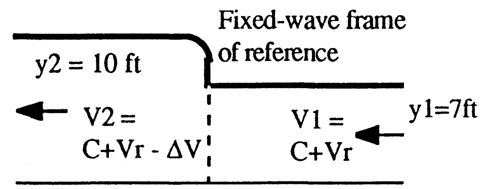


Fig. P10.87

Solution: Modify the analysis in 10.86 by superimposing a river velocity V_r onto the flow. Then, as shown, the approach velocity is $V_1 = C + V_r$, where C = 13 mi/h = 19.06 ft/s. We may again use Eq. 10.43 to find the Froude number:

$$\frac{y_2}{y_1} = \frac{10}{7} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2} - 1\right], \quad \text{solve for } Fr_1 = 1.32$$

$$\text{Then} \quad V_1 = C + V_r = Fr_1\sqrt{gy_1} = 1.32\sqrt{32.2(7)} = 19.77 \text{ ft/s} = 19.06 + V_r$$

$$\text{Thus} \quad \mathbf{V_r} = 0.71 \text{ ft/s} \approx \mathbf{0.42\ \ knots} \quad \textit{Ans.}$$

10.88 For the situation in Fig. P10.84, suppose that at section 3 the depth is 2 m and the Froude number is 0.25. Estimate (a) the flow rate per meter of width; (b) y_c; (c) y_1; (d) the percent dissipation in the jump; and (e) the gap height *H* of the gate.

Solution: We have enough information to immediately calculate the flow rate:

$$Fr_3 = 0.25 = \frac{V_3}{\sqrt{gy_3}} = \frac{V_3}{\sqrt{(9.81\text{ m/s}^2)(2\text{ m})}},$$

$$\text{Solve} \quad V_3 = 1.11\ \frac{\text{m}}{\text{s}}, \ q = V_3 y_3 = \mathbf{2.22}\ \frac{\mathbf{m^3}}{\mathbf{s{\cdot}m}} \quad \textit{Ans.}\text{ (a)}$$

The critical velocity is $y_c = (q^2/g)^{1/3} = [(2.22\text{ m}^2/\text{s})^2/(9.81\text{ m/s}^2)]^{1/3} = \mathbf{0.794\ m}$. *Ans.* (b)
We have to work our way back through the jump and sluice gate to find y_1:

$$\frac{2y_2}{y_3} = -1 + \sqrt{1+8Fr_3^2} = \frac{2y_2}{(2\text{ m})} = -1 + \sqrt{1+8(0.25)^2}, \quad \text{solve } y_2 = 0.225 \text{ m}$$

$$V_2 = q/y_2 = (2.22)/(0.225) = 9.86 \text{ m/s}, \quad E_2 = y_2 + \frac{V_2^2}{2g} = 0.225 + \frac{(9.86)^2}{2(9.81)} = 5.18 \text{ m}$$

$$E_2 = E_1 = 5.18\text{ m} = y_1 + \frac{V_1^2}{2g}, \quad q = V_1 y_1 = 2.22\frac{\text{m}^3}{\text{s{\cdot}m}}, \quad \text{solve } \mathbf{y_1 = 5.17\ m} \quad \textit{Ans.}\text{ (c)}$$

The head loss in the jump leads to percent dissipation:

$$h_f = \frac{(y_3 - y_2)^3}{4y_3y_2} = \frac{(2\text{ m} - 0.225\text{ m})^3}{4(2\text{ m})(0.225\text{ m})} = 3.11\text{ m},$$

$$\%\text{ dissipation} = \frac{h_f}{E_2} = \frac{3.11\text{ m}}{5.18\text{ m}} = \mathbf{60\%} \quad \textit{Ans.}\text{ (d)}$$

Finally, the gap height H follows from Eq. (10.41), assuming free discharge:

$$q = C_d H\sqrt{2gy_1} = 2.22\ \frac{\text{m}^3}{\text{s·m}} = \left[\frac{0.61}{\sqrt{1 + 0.61H/5.17\text{ m}}}\right] H\sqrt{2(9.81)(5.17\text{ m})},$$

$$\text{solve} \quad \mathbf{H = 0.37\ m} \quad \textit{Ans.}\text{ (e)}$$

10.89 Water 30 cm deep is in uniform flow down a 1° unfinished-concrete slope when a hydraulic jump occurs, as in Fig. P10.89. If the channel is very wide, estimate the water depth y_2 downstream of the jump.

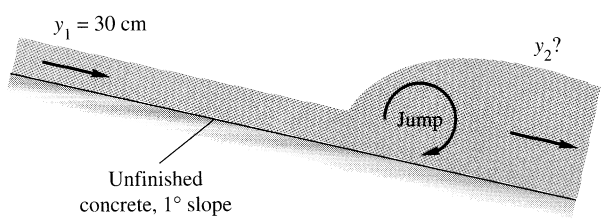


Fig. P10.89

Solution: For unfinished concrete, take $n \approx 0.014$. Compute the upstream velocity:

$$V_1 = \frac{1}{n}R_h^{2/3}S_o^{1/2} = \frac{1}{0.014}(0.3)^{2/3}(\sin 1°)^{1/2} \approx 4.23\ \frac{\text{m}}{\text{s}}; \quad Fr_1 = \frac{4.23}{\sqrt{9.81(0.3)}} \approx 2.465$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8(2.465)^2} - 1\right] \approx 3.02, \quad \mathbf{y_2} = 3.02(0.3) \approx \mathbf{0.91\ m} \quad \textit{Ans.}$$

10.90 Modify Prob. 10.89 as follows. Suppose that $y_2 = 1.5$ m and $y_1 = 30$ cm but the channel slope is not equal to 1 degree. Determine the proper slope for this condition.

Solution: For unfinished concrete take $n = 0.014$. The hydraulic jump formula gives the upstream Froude number and velocity:

$$\frac{2y_2}{y_1} = -1 + \sqrt{1 + 8Fr_1^2} = \frac{2(1.5\text{ m})}{0.3\text{ m}},$$

$$\text{Solve} \quad Fr_1 = 3.87, \; V_1 = Fr_1\sqrt{gy_1} = 3.87\sqrt{9.81(0.3)} = 6.64\text{ m/s}$$

$$V_{1,normal} = \frac{1}{0.014}(0.3\text{ m})^{2/3}\sqrt{S_o} = 6.64\text{ m/s}$$

$$\text{Solve} \quad S_o = 0.0431 \quad \textbf{\textit{or about 2.5°}} \quad Ans.$$

10.91 No doubt you used the horizontal-jump formula (10.43) to solve Probs. 10.89 and 10.90, which is reasonable since the slope is so small. However, Chow [ref. 3, p. 425] points out that hydraulic jumps are *higher* on sloped channels, due to "the weight of the fluid in the jump." Make a control-volume sketch of a sloping jump to show why this is so. The sloped-jump chart given in Chow's figure 15-20 may be approximated by the following curve fit:

$$\frac{2y_2}{y_1} \approx \left[\left(1 + 8\,\mathrm{Fr}_1^2\right)^{1/2} - 1\right]e^{3.5S_0}$$

where $0 < S_0 < 0.3$ are the channel slopes for which data are available. Use this correlation to modify your solution to Prob. 10.89. If time permits, make a graph of y_2/y_1 (≤ 20) versus Fr_1 (≤ 15) for various S_0 (≤ 0.3).

Solution: Include the water weight in a control volume around the jump:

$$\sum F_x = \frac{\rho g}{2}y_1^2 b - \frac{\rho g}{2}y_2^2 b + W\sin\theta$$

$$= \dot{m}(V_2 - V_1),$$

$$\dot{m} = \rho b y_1 V_1, \quad W \cong \rho g L b y_2$$

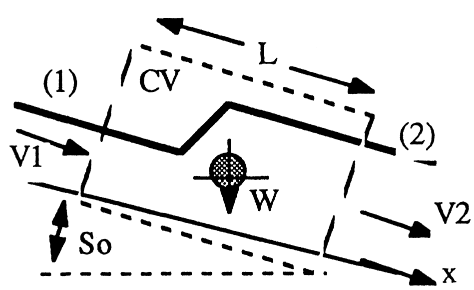


Fig. P10.91

Clean this up and rearrange:

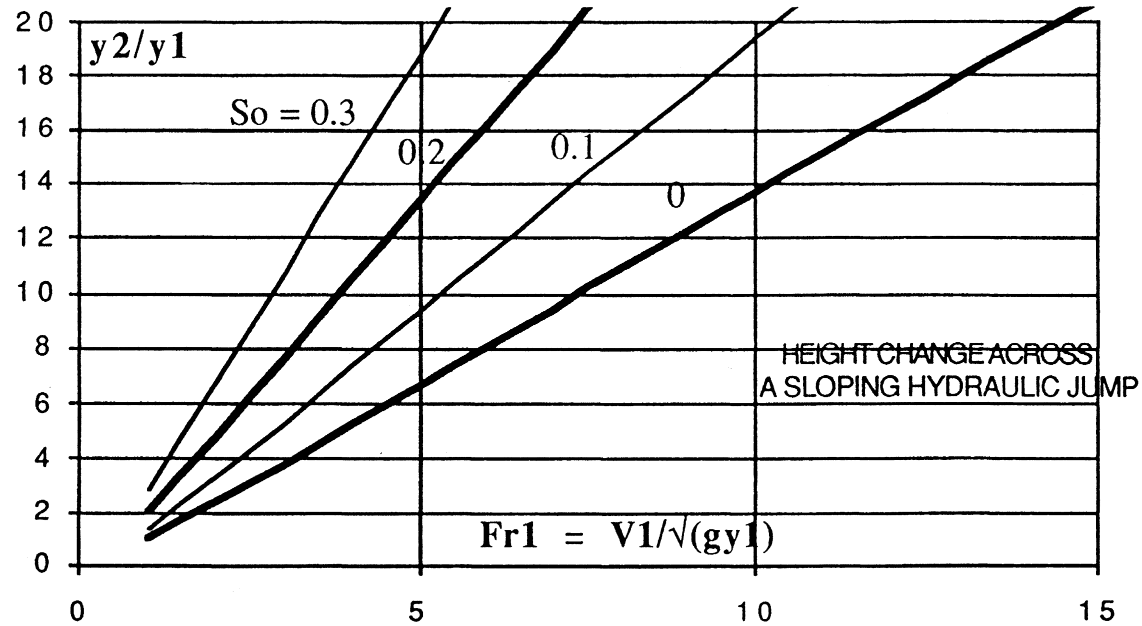
$$r^3(1 - KS_o) - r(\alpha + 1) + \alpha = 0, \quad \text{where } r = \frac{y_2}{y_1}, \quad \alpha = 2Fr_1^2, \quad K = \frac{L}{y_2} \approx 4 \quad \text{or so}$$

The solution to this cubic equation gives the jump height ratio r. If K = 0 (horizontal jump), the solution is Eq. 10.43. If K > 0 (sloping jump), the jump height increases

roughly as Chow's formula predicts. There is only a slight change to the result of Prob. 10.89:

$$y_{2,new} \approx y_{2,Prob.10.89}e^{3.5S_o} \approx 0.91e^{3.5\sin 1^\circ} \approx \mathbf{0.97\ m} \quad \textit{Ans.}$$

A plot of Chow's equation is shown below.



10.92 At the bottom of an 80-ft-wide spillway is a horizontal hydraulic jump with water depths 1 ft upstream and 10 ft downstream. Estimate (a) the flow rate; and (b) the horsepower dissipated.

Solution: With water depths known, Eq. 10.43 applies:

$$\frac{y_2}{y_1} = \frac{10}{1} = \frac{1}{2}\left[\sqrt{1+8Fr_1^2} - 1\right], \quad \text{solve for } Fr_1 = 7.42,\ V_1 = 7.42\sqrt{32.2(1)} \approx 42 \text{ ft/s}$$

$$\text{Then} \quad Q = V_1 y_1 b = (42)(1)(80) \approx \mathbf{3370\ ft^3/s} \quad \textit{Ans. (a)}$$

$$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(10-1)^3}{4(10)(1)} = 18.2 \text{ ft},$$

$$\text{Power dissipated} = \rho g Q h_f = (62.4)(3370)(18.2)$$

$$= 3.83E6\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \div 550 \approx \mathbf{7000\ hp} \quad \textit{Ans. (b)}$$

10.93 Water in a horizontal channel accelerates smoothly over a bump and then undergoes a hydraulic jump, as in Fig. P10.93. If $y_1 = 1$ m and $y_3 = 40$ cm, estimate (a) V_1; (b) V_3; (c) y_4; and (d) the bump height h.

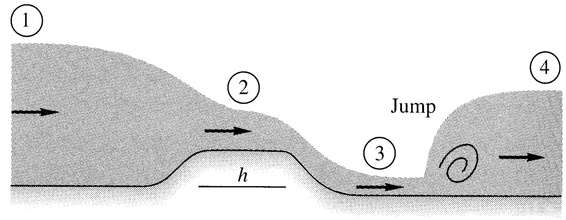


Fig. P10.93

Solution: Assume frictionless flow except in the jump. From point 1 to point 3:

$$\text{Energy: } E_1 = 1 + \frac{V_1^2}{2(9.81)} = E_3 = 0.4 + \frac{V_3^2}{2(9.81)}; \quad \text{Continuity: } V_1(1.0) = V_3(0.4)$$

Solve simultaneously by iteration for $V_1 \approx \mathbf{1.50\ \frac{m}{s}}$ *Ans.* (a) $V_3 = \mathbf{3.74\ \frac{m}{s}}$ *Ans.* (b)

The flow after the bump is supercritical: $Fr_3 = 3.74/\sqrt{[9.81(0.4)]} \approx \mathbf{1.89}$. For the jump,

$$\frac{y_4}{y_3} = \frac{1}{2}\left[\sqrt{1+8(1.89)^2} - 1\right] = 2.22, \quad y_4 = 2.22(0.4) \approx \mathbf{0.89\ m} \quad \textit{Ans. (c)}$$

Finally, use energy from “1” to “2” and note that the bump flow must be *critical*:

$$E_1 = 1.115 = E_2 = y_2 + \frac{V_2^2}{2g} + h; \quad V_1 y_1 = 1.50 = V_2 y_2; \quad \text{and:} \quad V_2 = \sqrt{9.81 y_2}$$

Solve simultaneously for $y_2 \approx 0.61$ m; $V_2 \approx 2.45$ m/s; $\mathbf{h \approx 0.20\ m}$ *Ans.* (d)

10.94 For the flow pattern of Fig. P10.93, consider the following different, and barely sufficient, data. The upstream velocity $V_1 = 1.5$ m/s, and the bump height h is 27 cm. Find (a) y_1; (b) y_2; (c) y_3; and (d) y_4.

Solution: Begin by writing out the various useful equations:

$$E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g} + h_{bump} = E_3 = y_3 + \frac{V_3^2}{2g} \neq E_4$$

$$q = V_1 y_1 = V_2 y_2 = V_3 y_3 = V_4 y_4$$

$$\frac{2y_4}{y_3} = -1 + \sqrt{1 + 8Fr_3^2} \quad \text{where} \quad Fr_3 = \frac{V_3}{\sqrt{g y_3}}$$

Critical flow over the bump: $Fr_2 = 1.0, \quad V_2 = \sqrt{g y_2}$

Our only data are $V_1 = 1.5$ m/s and $h = 0.27$ m. Our best hope is to type all these relations out in EES and limit all variables to be positive numbers. The final results are:

$$y_1 = \mathbf{1.18\ m} \quad \textit{Ans. (a)}; \quad y_2 = \mathbf{0.684\ m} \quad \textit{Ans. (b)};$$
$$y_3 = \mathbf{0.430\ m} \quad \textit{Ans. (c)}; \quad y_4 = \mathbf{1.024\ m} \quad \textit{Ans. (d)}$$

10.95 A 10-cm-high bump in a wide horizontal channel creates a hydraulic jump just upstream and the flow pattern in Fig. P10.95. Neglect losses except in the jump. If $y_3 = 30$ cm, estimate (a) V_4; (b) y_4; (c) V_1; and (d) y_1.

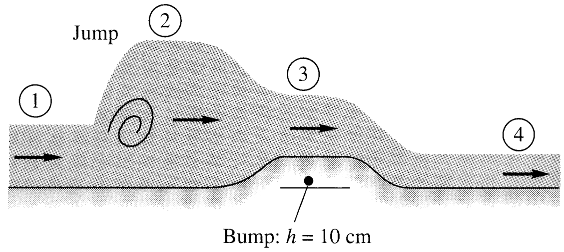


Fig. P10.95

Solution: Since section "2" is subcritical and "4" is supercritical, assume "3" is *critical*:

$$V_3 = \sqrt{gy_3} = \sqrt{9.81(0.3)} = 1.72\ \frac{m}{s}, \quad \text{thus } q = V_j y_j|_{2,3,4} = 1.72(0.3) = 0.515\ m^3/s\cdot m$$

$$y_3 + \frac{V_3^2}{2g} + h = 0.3 + \frac{(1.72)^2}{2(9.81)} + 0.1 = 0.55\ m = E_4 = y_4 + \frac{V_4^2}{2(9.81)}; \quad \text{and} \quad V_4 y_4 = 0.515$$

$$\text{Solve for} \quad y_4 \approx \mathbf{0.195\ m} \quad \textit{Ans. (b)};\ V_4 \approx \mathbf{2.64\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

$$\text{Also, } E_2 = 0.55 = y_2 + \frac{V_2^2}{2g} \quad \text{and} \quad V_2 y_2 = 0.515, \quad \text{solve } y_2 \approx 0.495\ m,\ V_2 \approx 1.04\ m/s$$

$$Fr_2 = \frac{1.04}{\sqrt{9.81(0.495)}} \approx 0.472, \quad \text{Jump: } \frac{y_1}{y_2} = \frac{1}{2}\left[\sqrt{1+8(0.472)^2} - 1\right] = 0.334 = \frac{y_1}{0.495}$$

$$\text{Thus} \quad y_1 \approx \mathbf{0.165\ m} \quad \textit{Ans. (d)}; \quad V_1 = \frac{0.515}{0.165} \approx \mathbf{3.11\ \frac{m}{s}} \quad \textit{Ans. (c)}$$

10.96 Show that the Froude numbers on either side of a hydraulic jump are related by the simple formula $Fr_2 = Fr_1(y_1/y_2)^{3/2}$.

Solution: This relation follows immediately from continuity, q = Vy = constant:

$$\frac{Fr_2}{Fr_1} = \frac{V_2\sqrt{gy_1}}{\sqrt{gy_2}V_1} = \frac{V_2}{V_1}\sqrt{\frac{y_1}{y_2}} = \frac{qy_1}{y_2 q}\sqrt{\frac{y_1}{y_2}} = \left(\frac{y_1}{y_2}\right)^{3/2} \quad \textit{Ans.}$$

10.97 A brickwork rectangular channel 4 m wide is flowing at 8.0 m^3/s on a slope of 0.1°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 1.5 m; (c) 2 m?

Solution: For brickwork, take $n \approx 0.015$. Then, with $q = Q/b = 8/4 = 2.0\ m^3/s\cdot m$,

$$Q = 8\ \frac{m^3}{s} = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.015}(4y_n)\left(\frac{4y_n}{4+2y_n}\right)^{2/3}(\sin 0.1°)^{1/2},$$

$$\text{solve for} \quad y_n \approx 0.960\ m$$

whereas $y_c = (q^2/g)^{1/3} = [(2)^2/9.81]^{1/3} \approx 0.742$ m. Since $y_n > y_c$, **slope is mild** *Ans.*

All three of the given depths—1.0, 1.5, and 2.0 meters—are above y_n on Fig. 10.14*c*, hence **all three are on M-1 curves**. *Ans.*

10.98 A gravelly-earth wide channel is flowing at 10.0 m^3/s per meter on a slope of 0.75°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 2 m; (c) 3 m?

Solution: For gravelly earth, take $n \approx 0.025$. Then, with $R_h = y$ itself,

$$q = 10\ \frac{m^3}{s\cdot m} = \frac{1}{n}\frac{A}{b} R_h^{2/3} S_o^{1/2} = \frac{1}{0.025}(y_n) y_n^{2/3}(\sin 0.75°)^{1/2}, \quad \text{solve for } y_n \approx 1.60\ m$$

whereas $y_c = (q^2/g)^{1/3} = [(10)^2/9.81]^{1/3} \approx 2.17$ m. Since $y_c > y_n$, **slope is steep** *Ans.*

The three given depths fits nicely into the spaces between y_n and y_c in Fig. 10.14*a*:

$y = 1\ m < y_n < y_c$: **S-3 curve** *Ans.* (a) $y_n < y = 2\ m < y_c$: **S-2 curve** *Ans.* (b)

$y_n < y_c < y = 3\ m$: **S-1 curve** *Ans.* (c)

10.99 A clay tile V-shaped channel, of included angle 60°, is flowing at 1.98 m^3/s on a slope of 0.33°. Is this a mild, critical, or steep slope? What type of gradually-varied-solution curve are we on if the local water depth is (a) 1 m; (b) 2 m; (c) 3 m?

Solution: For clay tile, take $n \approx 0.014$. For a 60° Vee-channel, from Example 10.5 of the text, $A = y^2\cot 60°$ and $R_h = (y/2)\cos 60°$. For uniform flow,

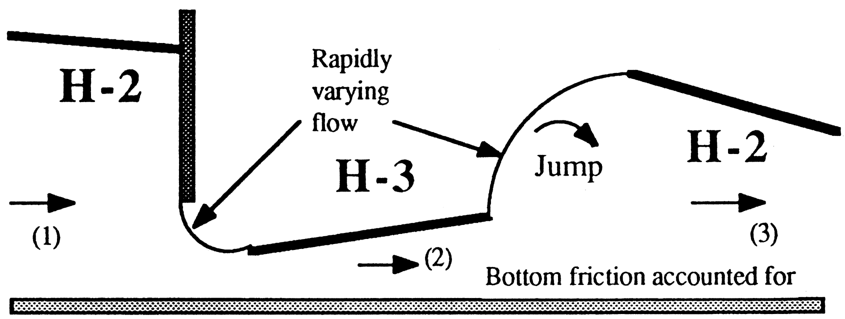
$$Q = 1.98 = \frac{1}{n}AR_h^{2/3}S_o^{1/2} = \frac{1}{0.014}(y^2\cot 60°)\left(\frac{y}{2}\cos 60°\right)^{2/3}(\sin 0.33°)^{1/2},$$

solve for $y_n \approx 1.19$ m; whereas $y_c = \left(\dfrac{2Q^2}{g\cot^2 60°}\right)^{1/5} \approx 1.19$ also. Slope is **critical**. *Ans.*

Fig. 10.14b: $y = 1\text{ m} < y_c$: **C-3 curve** *Ans.* (a); $y = 2$ or $3\text{ m} > y_c$: **C-1 curve** *Ans.* (b, c)

10.100 If bottom friction is included in the sluice-gate flow of Prob. 10.84, the depths (y_1, y_2, y_3) will vary with *x*. Sketch the type and shape of gradually-varied solution curve in each region (1,2,3) and show the regions of rapidly-varying flow.

Solution: The expected curves are all of the "H" (horizontal) type and are shown below:



Ans.

Fig. P10.100

10.101 Consider the gradual change from the profile beginning at point *a* in Fig. P10.101 on a mild slope S_{o1} to a *mild* but steeper slope S_{o2} downstream. Sketch and label the gradually-varied solution curve(s) *y*(x) expected.

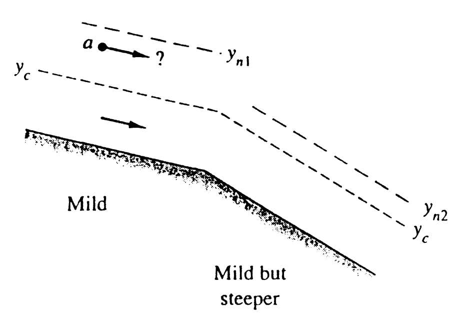
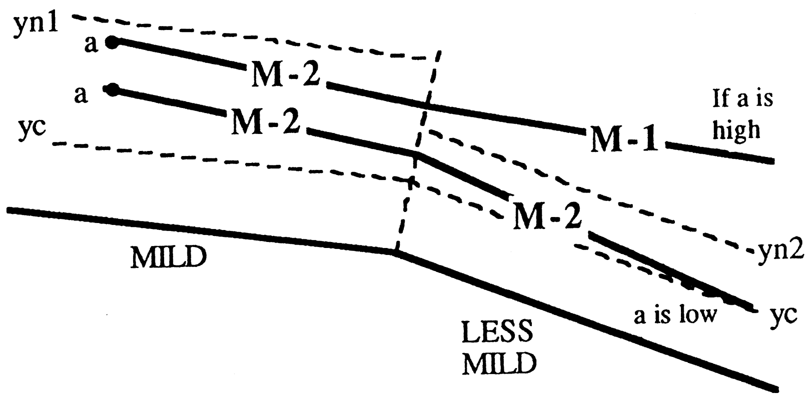


Fig. P10.101

Solution: There are two possible profiles, depending upon whether or not the initial M-2 profile slips below the new normal depth y_{n2}. These are shown on the next page:



10.102 The wide channel flow in Fig. P10.102 changes from a steep slope to one even steeper. Beginning at points a and b, sketch and label the water surface profiles which are expected for gradually-varied flow.

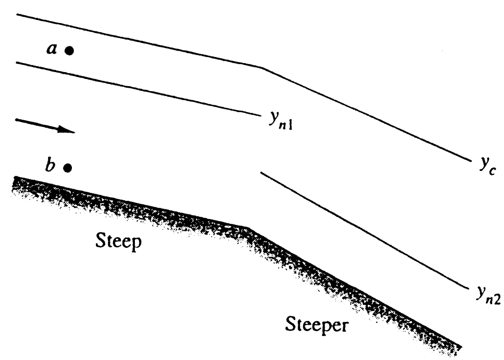
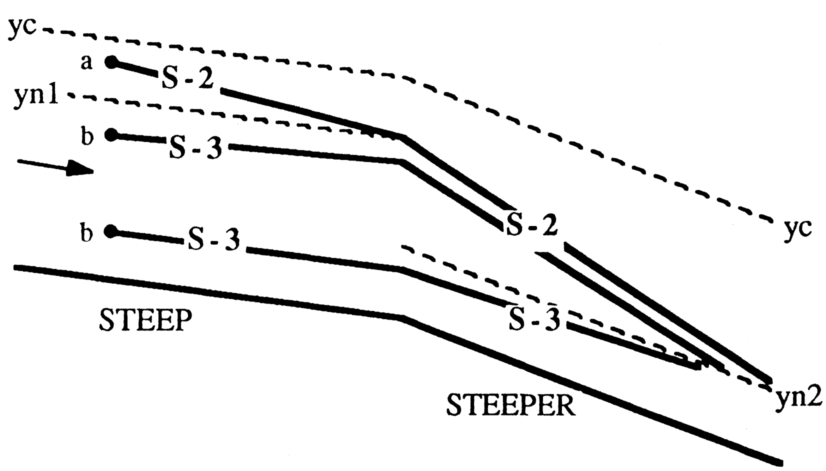


Fig. P10.102

Solution: The point-*a* curve will approach each normal depth in turn. Point-*b* curves, depending upon initial position, may approach y_{n2} either from above or below, as shown in the sketch below.



10.103 A circular painted-steel channel, of radius 50 cm, is running half-full at 1.2 m^3/s on a slope of 5 m/km. Determine (a) whether the slope is mild or steep; and (b) what type

of gradually-varied solution applies at this point. (c) Use the approximate method of Eq. (10.52), and a single depth increment $\Delta y = 5$ cm, to calculate the estimated Δx for this new y.

Solution: (a) To classify the slope, we need to compute y_n and y_c. Take $n = 0.014$. The geometric properties of the partly-full circular duct are taken from the discussion of Eq. (10.20):

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right); \quad P = 2R\theta; \quad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right); \quad y = R[1 + \sin(90° - \theta)]$$

where θ is measured from the bottom of the circle (see Fig. 10.6*a*). For normal flow,

$$Q = 1.2\ \frac{\text{m}^3}{\text{s}} = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{0.014}\left[(0.5)^2\left(\theta - \frac{\sin 2\theta}{2}\right)\right]\left[\frac{0.5}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)\right]^{2/3}\sqrt{0.005}$$

EES seems indicated, and the solution is $\theta_n = 107.9°$ and $\boldsymbol{y_n} = \mathbf{0.654\ m}$. Next, for this non-rectangular channel, critical flow occurs when

$$A_c = \left(\frac{b_o Q^2}{g}\right)^{1/3} \quad \text{where } b_o = 2R\cos(90° - \theta) \quad \text{and} \quad Q = 1.2\ \frac{\text{m}^3}{\text{s}}$$

Again, EES is handy, and the solution is $\theta_c = 104.8°$ and $\boldsymbol{y_c} = \mathbf{0.628\ m}$.
(a) Thus $y_c < y_n$, and the channel slope is therefore **mild**. *Ans.* (a)
(b) From Fig. 10.14c, since we are starting at $y = 0.5$ m, which is less than y_c, we will proceed for Fr > 1 along an **M-3 curve**. *Ans.* (b)
(c) We are to find Δx required to move from $y = 0.5$ m to $y = 0.55$ m in one step ($\Delta y = 0.05$ m), using the numerical method of Eq. (10.52). At the initial depth,

$$y_1 = 0.5\text{ m};\ V_1 = 3.06\text{ m/s};\ E_1 = 0.976\text{ m};\ R_{h1} = 0.25\text{ m};\ S_1 = n^2V^2/R_h^{4/3} = 0.0116$$

Similarly, at the final depth,

$$y_2 = 0.55\text{ m};\ V_2 = 2.71\text{ m/s};\ E_2 = 0.925\text{ m};\ R_{h2} = 0.265\text{ m};\ S_2 = n^2V^2/R_h^{4/3} = 0.00847$$

The numerical approximation, Eq. (10.52), then predicts:

$$\Delta x = \frac{E_2 - E_1}{S_o - S_{avg}} = \frac{0.925 - 0.976\text{ m}}{0.005 - (0.0116 + 0.00847)/2} = \frac{-.051}{-0.00504} \approx \mathbf{10.1\ m} \quad \textit{Ans.}$$

10.104 The rectangular channel flow in Fig. P10.104 expands to a cross-section 50% wider. Beginning at points *a* and *b*, sketch and label the water-surface profiles which are expected for gradually-varied flow.

Solution: Three types of dual curves are possible: S2/S2, S3/S2, and S3/S3, as shown:

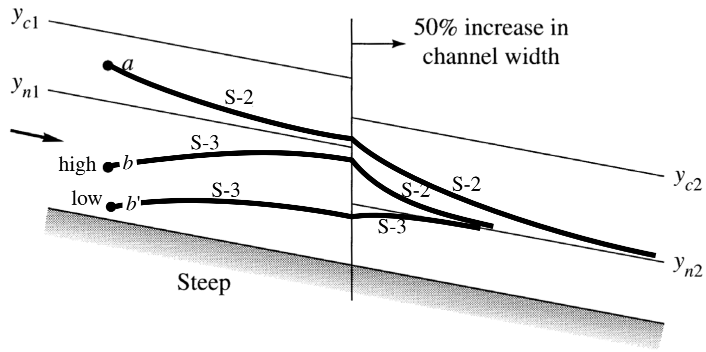


Fig. P10.104

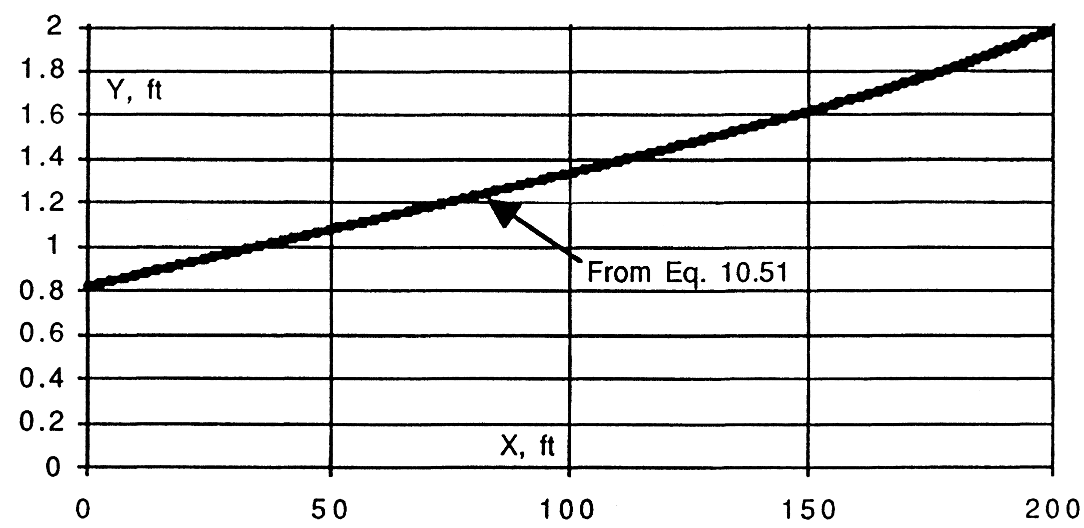
10.105 In Prob. 10.84 the frictionless solution is $y_2 = 0.82$ ft, which we denote as $x = 0$ just downstream of the gate. If the channel is horizontal with $n = 0.018$ and there is no hydraulic jump, compute from gradually-varied theory the downstream distance where $y = 2.0$ ft.

Solution: Given $q = Vy = 20$ ft^3/s·ft, the critical depth is $y_c = (q^2/32.3)^{1/3} = 2.32$ ft, hence we are on an **H-2** curve (see Fig. 10.14d) which will approach y_c from below. We solve the basic differential equation 10.51 for horizontal wide-channel flow ($S_o = 0$):

$$\frac{dy}{dx} = \frac{-n^2q^2/(\alpha^2y^{10/3})}{1-q^2/(gy^3)} = -\frac{(0.018)^2(20)^2/(2.208y^{10/3})}{1-(20)^2/(32.2y^3)} \quad \text{for } y_o = 0.82 \text{ ft at } x = 0 \text{ ft.}$$

The water level increases until **y = 2.0 ft at x ≈ 200 ft**. *Ans.*

The complete solution y(x) is shown below.



10.106 A rectangular channel with $n = 0.018$ and a constant slope of 0.0025 increases its width linearly from b to $2b$ over a distance L, as in Fig. P10.106. (a) Determine the variation y(x) along the channel if b = 4 m, L = 250 m, y(0) = 1.05 m, and Q = 7 m^3/s. (b) Then, if your computer program is working well, determine y(0) for which the exit flow will be exactly critical.

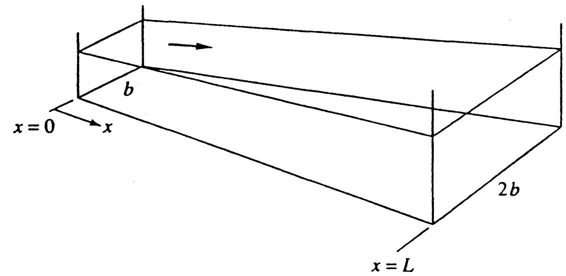


Fig. P10.106

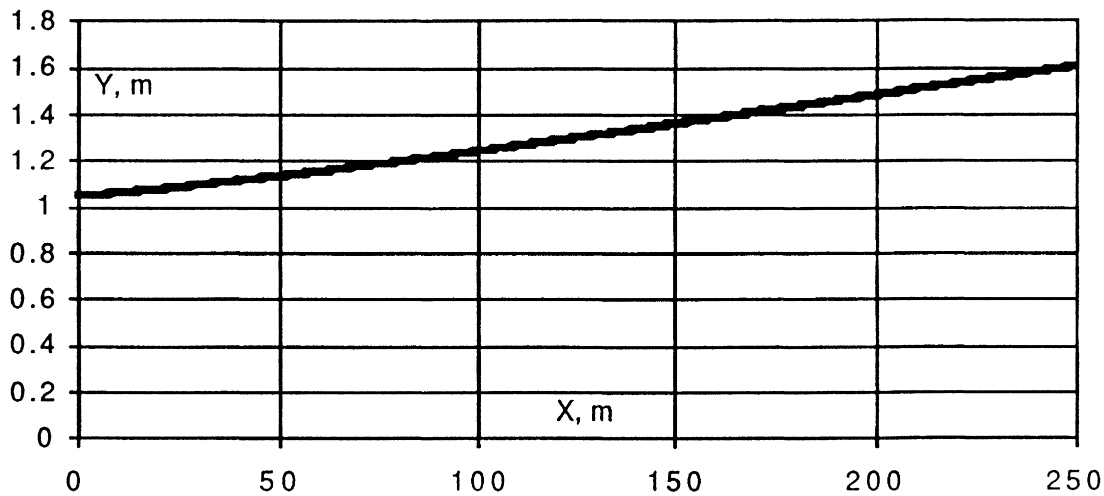
Solution: We are to solve the gradually-varied-flow relation, Eq. 10.51:

$$\frac{dy}{dx} = \frac{S_o - S}{1 - V^2/(9.81y)}, \quad \text{where } S = \frac{n^2V^2}{R_h^{4/3}}, \quad V = \frac{Q}{by}, \quad R_h = \frac{by}{b+2y}, \quad b = 4\left(1 + \frac{x}{250}\right)$$

For reference purposes, compute $y_c = 0.68$ m and $y_n = 0.88$ m at x = 0, compared to $y_c = 0.43$ m and $y_n = 0.53$ m at x = 250 m. For initial depth y(0) = 1.05, we are on an M-2 curve (see Fig. 10.14*c*) and we compute **y(L) ≈ 1.61 m**. *Ans.* (a)

The curve y(x) is shown below.

The writer **cannot find any y(0) for which the exit flow is critical**. *Ans.* (b)



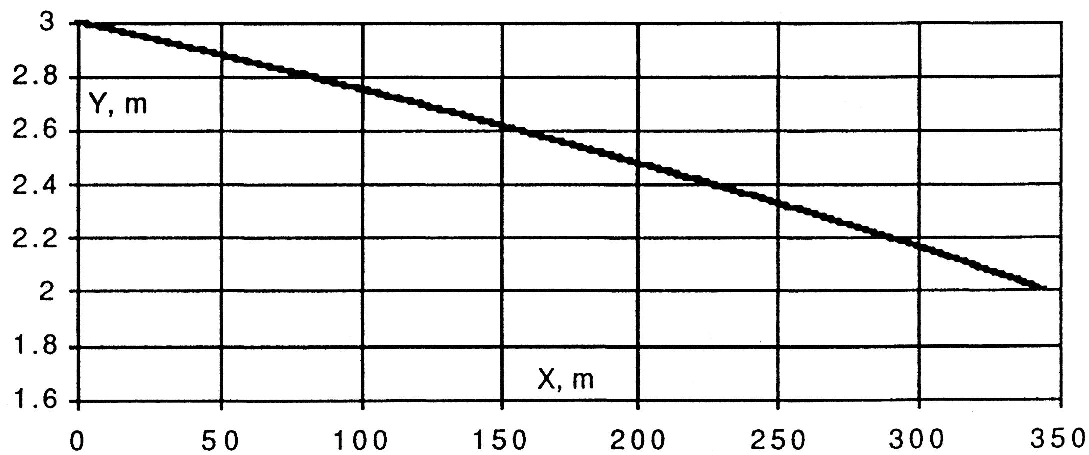
This figure shows y(x) for Prob. 10.106 when y(0) = 1.05 m.

10.107 A clean-earth wide-channel flow is flowing up an *adverse* slope with $S_o = -0.002$. If the flow rate is $q = 4.5\ m^3/s\cdot m$, use gradually-varied theory to compute the distance for the depth to drop from 3.0 to 2.0 meters.

Solution: For clean earth, take $n \approx 0.022$. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/(\alpha^2 y^{10/3})}{1 - q^2/(gy^3)},\quad S_o = -0.002,\ \alpha = 1.0,\ q = 4.5,\ n = 0.022,\ y(0) = 3.0\text{ m}$$

The complete graph y(x) is shown below. The depth = 2.0 m when **x = 345 m**. *Ans.*

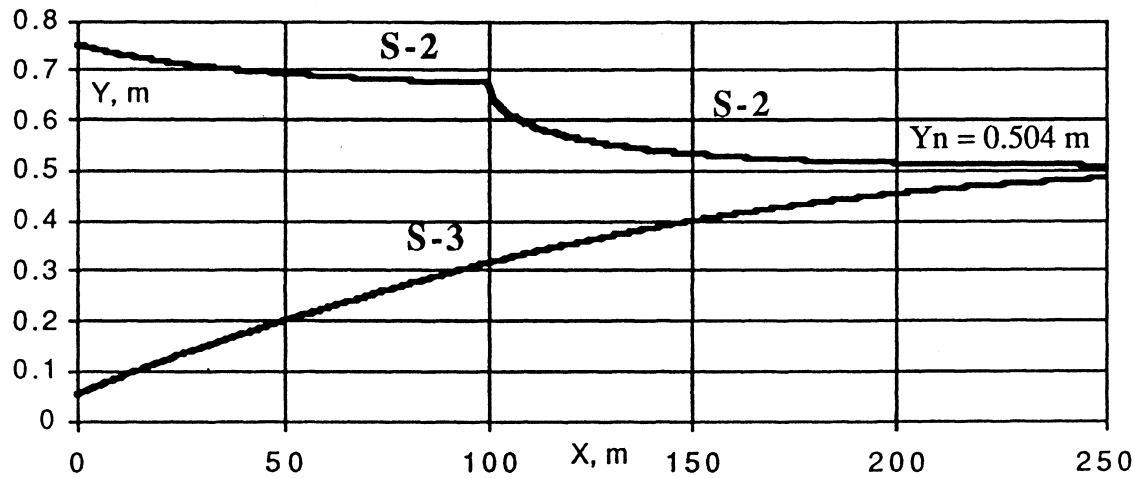


10.108 Illustrate Prob. 10.104 with a numerical example. Let the channel be rectangular with a width $b_1 = 10$ m for $0 < x < 100$ m, expanding to $b_2 = 15$ m for $100 < x < 250$ m. The flow rate is 27 m^3/s, and $n = 0.012$. Compute the water depth at $x = 250$ m for initial depth $y(0)$ equal to (a) 75 cm and (b) 5 cm. Compare your results with the discussion in Prob. 10.104.

Solution: The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2V^2/R_h^{4/3}}{1 - Q^2/(gb^2y^3)},\quad \text{where } V = \frac{Q}{A},\ R_h = \frac{by}{b+2y},\ y(0) = 75\text{ cm and 5 cm}$$

The two graphs are shown on the next page. The upper is S-2/S-2, the lower curve is S-3/S-3, both approach the downstream normal depth $y_n \approx$ **0.504 m**. *Ans.*



10.109 Figure P10.109 illustrates a free overfall or *dropdown* flow pattern, where a channel flow accelerates down a slope and falls freely over an abrupt edge. As shown, the flow reaches critical just before the overfall. Between y_c and the edge the flow is rapidly varied and does not satisfy gradually varied theory. Suppose that the flow rate is q = 1.3 $m^3/(s\cdot m)$ and the surface is unfinished cement. Use Eq. (10.51) to estimate the water depth 300 m upstream as shown.

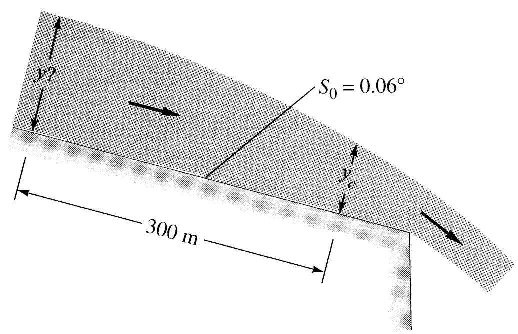


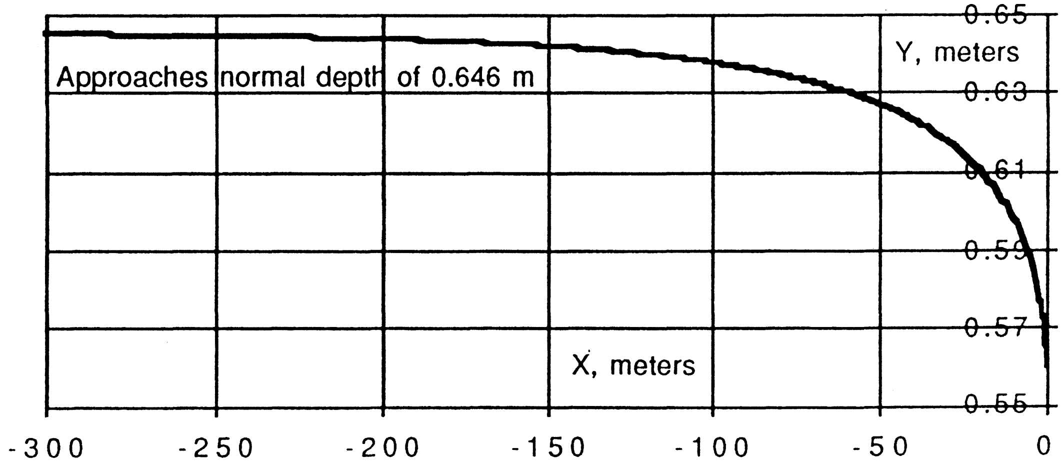
Fig. P10.109

Solution: For unfinished cement, take $n \approx 0.012$. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/y^{10/3}}{1 - q^2/(gy^3)}, \quad S_o = \sin(0.06°) = 0.00105, \quad q = 1.3\ \frac{m^3}{s\cdot m}, \quad n = 0.012,$$

$$y(0) = y_{critical} = (q^2/g)^{1/3} = [(1.3)^2/9.81]^{1/3} = 0.556\text{ m}, \quad \text{integrate for } \Delta x < 0.$$

The solution grows rapidly at first and then approaches, at about 150 m upstream, the normal depth of **0.646 m** for this flow rate and roughness. The profile is shown on the next page.



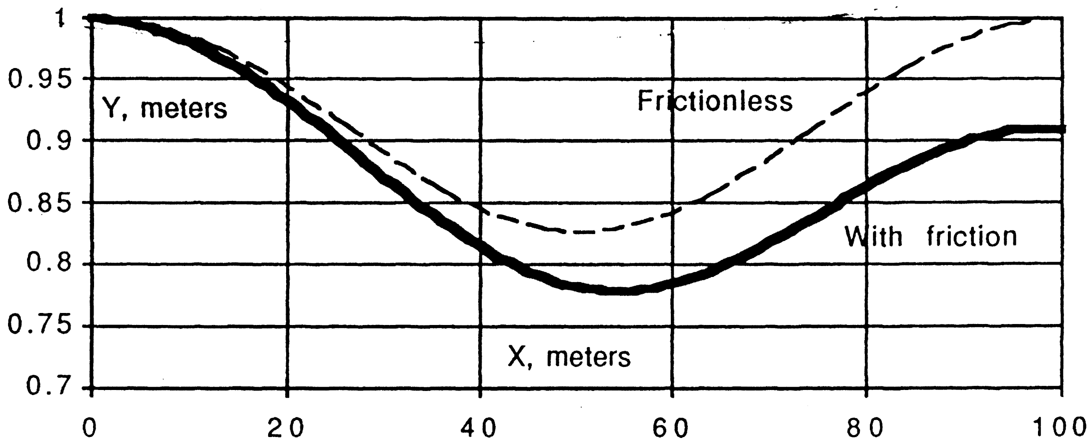
10.110 We assumed frictionless flow in solving the bump case, Prob. 10.65, for which $V_2 = 1.21$ m/s and $y_2 = 0.826$ m over the crest when $h_{max} = 15$ cm, $V_1 = 1$ m/s, and $y_1 = 1$ m. However, if the bump is long and rough, friction may be important. Repeat Prob. 10.65 for the same bump shape, $h = 0.5h_{max}[1 - \cos(2\pi x/L)]$, to compute conditions (a) at the crest and (b) at the end of the bump, $x = L$. Let $h_{max} = 15$ cm and $L = 100$ m, and assume a clean-earth surface.

Solution: For clean earth, take $n = 0.022$. The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2 q^2 y^{-10/3}}{1 - q^2/(gy^3)}, \quad q = 1.0\ \frac{\text{m}^3}{\text{s}\cdot\text{m}},$$

$$S_o = \frac{dh}{dx}\Big|_{bump} = -\frac{\pi h_{max}}{2L}\sin\left(\frac{2\pi x}{L}\right), \quad y(0) = 1\text{ m}$$

We integrate this for clean earth ($n = 0.022$) and also for *frictionless* flow (Prob. 10.65), $n = 0$. The results are shown on the next page. The frictionless profile drops to $y = 0.826$ m at the crest and returns to $y = 1.0$ m at the end, x = L = 100 m. The *frictional* flow drops lower, to **y = 0.782 m at the crest** [*Ans.* (a)] and even lower, to y = 0.778 m at x = 54 m, and then does not recover fully, ending up at **y = 0.909 m at x = L**. [*Ans.* (b)]



10.111 Solve Prob. 10.105 (a horizontal variation along an H-3 curve) by the approximate method of Eq. (10.52), beginning at (x, y) = (0, 0.82 ft) and using a depth increment Δy = 0.2 ft. (The final increment should be Δy = 0.18 ft to bring us exactly to y = 2.0 ft.)

Solution: The procedure is explained in Example 10.10 of the text. Recall that n = 0.018 and the flow rate is q = 20 ft^3/s/ft. The numerical method uses Eq. (10.52) to compute Δx for a given Δy. The "friction slope" $S = n^2V^2/(2.208y^{4/3})$. The bed slope S_o = 0 (horizontal). The tabulated results below indicate that a depth of 2.0 ft is reached at a distance $\boldsymbol{x \approx}$ **195 ft** downstream.

[NOTE: Prob. 10.105 had a more accurate numerical solution $x \approx 200$ ft.]

y, ft	$V = 20/y$	$E = y + V^2/2g$	S	S-avg	Δx, ft	$x = \sum\Delta x$, ft
0.82	24.390	10.057	0.1137	n/a	n/a	0.000
1.02	19.608	6.990	0.0549	0.084	36.37	36.37
1.22	16.393	5.393	0.0303	0.043	37.49	73.86
1.42	14.085	4.500	0.0182	0.024	36.82	110.68
1.62	12.346	3.987	0.0118	0.015	34.25	144.93
1.82	10.989	3.695	0.0080	0.010	29.56	174.49
2.00	10.000	3.553	0.0058	0.007	20.63	**195.12**

10.112 The clean-earth channel in Fig. P10.112 is 6 m wide and slopes at 0.3°. Water flows at 30 m^3/s in the channel and enters a reservoir so that the channel depth is 3 m just before the entry. Assuming gradually-varied flow, how far is L to a point upstream where y = 2m? What type of curve is the water surface?

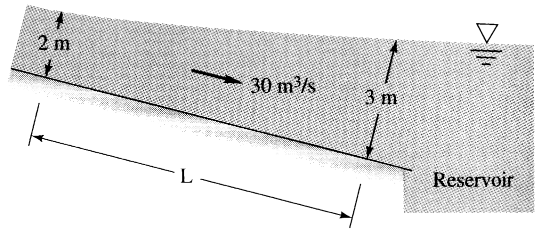
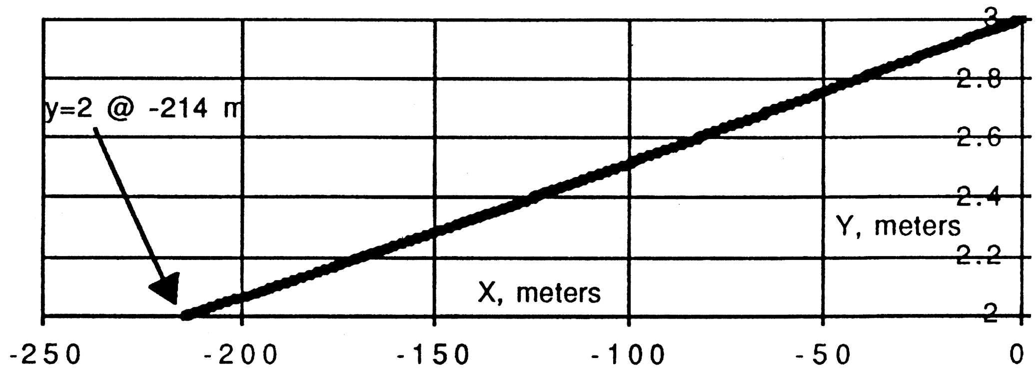


Fig. P10.112

Solution: For clean earth, take $n \approx 0.022$. The differential equation is Eq. 10.51:

$$\frac{dy}{dx} = \frac{S_o - n^2Q^2/(A^2R_h^{4/3})}{1 - Q^2b/(gA^3)}, \quad \text{where } S_o = 0.3°,\ Q = 30\ \frac{m^3}{s},\ b = 6\ m,\ A = by,\ y(0) = 3.0$$

To begin, compute $y_n \approx 1.51$ m and $y_c \approx 1.37$ m, hence $y_c < y_n < y$: we are on a *mild* slope above the normal depth, hence we are on an **M-1** curve. *Ans.*
Begin at y(0) = 3 m and integrate backwards ($\Delta x < 0$) until y = 2 m at **L = 214 m**. *Ans.*



10.113 Figure P10.113 shows a channel contraction section often called a *venturi flume* [from Ref. 23 of Chap. 10], because measurements of y_1 and y_2 can be used to meter the flow rate. Show that if losses are neglected and the flow is one-dimensional and subcritical, the flow rate is given by

$$Q = \left[\frac{2g(y_1 - y_2)}{1/\left(b_2^2y_2^2\right) - 1/\left(b_1^2y_1^2\right)}\right]^{1/2}$$

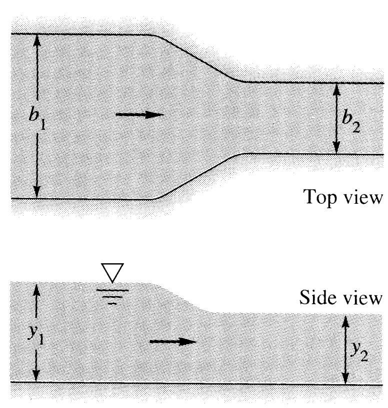


Fig. P10.113

Apply this to the special case $b_1 = 3$ m, $b_2 = 2$ m, and $y_1 = 1.9$ m. Find the flow rate (a) if $y_2 = 1.5$ m; and (b) find the depth y_2 for which the flow becomes critical in the throat.

Solution: Given the water depths, continuity and energy allow us to eliminate one velocity:

$$\text{Continuity: } Q = V_1y_1b_1 = V_2y_2b_2; \quad \text{Energy: } y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$\text{Eliminate } V_1 \text{ to obtain } \quad V_2 = [2g(y_1 - y_2)/(1-\alpha^2)]^{1/2} \quad \text{where } \alpha = (y_2b_2)/(y_1b_1)$$

$$\text{or:} \quad \mathbf{Q} = V_2y_2b_2 = \left[\mathbf{2g(y_1 - y_2)}\Big/\left\{\mathbf{b_2^{-2}y_2^{-2} - b_1^{-2}y_1^{-2}}\right\}\right]^{\mathbf{1/2}} \quad \textit{Ans.}$$

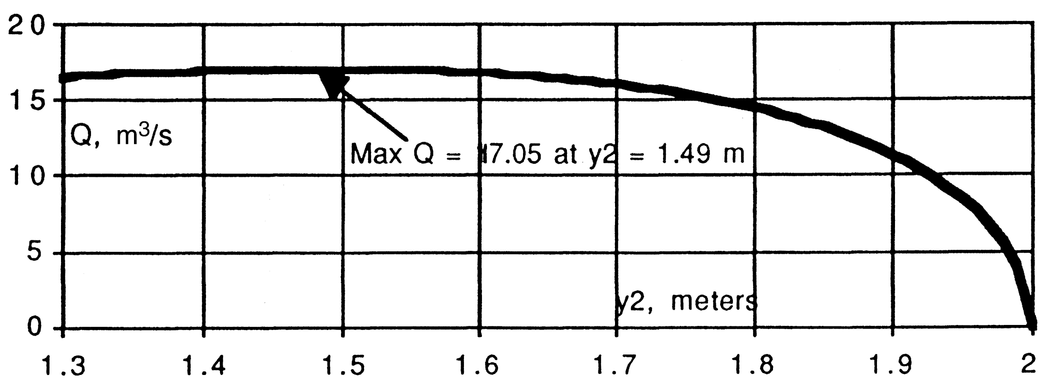
Evaluate the solution we just found:

$$Q = \left[\frac{2(9.81)(1.9-1.5)}{(2)^{-2}(1.5)^{-2} - (3)^{-2}(1.9)^{-2}}\right]^{1/2} \approx \mathbf{9.88}\ \frac{\mathbf{m^3}}{\mathbf{s}} \quad \textit{Ans. (a)}$$

For this part (a), $Fr_2 = V_2/\sqrt{(gy_2)} \approx \mathbf{0.86}$.
(b) To find critical flow, keep reducing y_2 until $Fr_2 = 1.0$. This converges to $\mathbf{y_2 \approx 1.372\ m}$. [for which Q = **10.1** m^3/s] *Ans.* (b)

10.114 Investigate the possibility of *choking* in the venturi flume of Fig. P10.113. Let b_1 = 4 ft, b_2 = 3 ft, and y_1 = 2 ft. Compute the values of y_2 and V_1 for a flow rate of (a) 30 ft^3/s and (b) 35 ft^3/s. Explain your vexation.

Solution: You can't get anywhere near either Q = 30 or Q = 35 m^3/s, the flume **chokes** (becomes critical in the throat) at about Q = 17.05 m^3/s, when $\mathbf{y_2 \approx 1.49\ m}$, as shown in the graph below. **Ans**.



10.115 Gradually varied theory, Eq. (10.49), neglects the effect of *width* changes, db/dx, assuming that they are small. But they are not small for a short, sharp contraction such as the venturi flume in Fig. P10.113. Show that, for a rectangular section with $b = b(x)$, Eq. (10.49) should be modified as follows:

$$\frac{dy}{dx} \approx \frac{S_o - S + [V^2/(gb)](db/dx)}{1 - \mathrm{Fr}^2}$$

Investigate a criterion for reducing this relation to Eq. (10.49).

Solution: We use the same energy equation, 10.47, but modify continuity, 10.47:

$$\text{Energy: } \frac{dy}{dx} + \frac{V}{g}\frac{dV}{dx} = S_o - S; \quad \text{continuity: } V = \frac{Q}{by}, \quad \therefore \frac{dV}{dx} = -\frac{Q}{by^2}\frac{dy}{dx} - \frac{Q}{yb^2}\frac{db}{dx}$$

$$\text{or: } \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx} - \frac{V}{b}\frac{db}{dx}; \quad \text{combine: } \mathbf{\frac{dy}{dx}\left(1 - \frac{V^2}{gy}\right) \approx S_o - S + \frac{V^2}{gb}\frac{db}{dx}} \quad \textit{Ans.}$$

Obviously, we can neglect the last term (width expansion) and obtain Eq. 10.49 if

$$\frac{V^2}{gb}\frac{db}{dx} \ll S_o - S = \mathcal{O}\left(\frac{f}{4R_h}\frac{V^2}{2g}\right), \quad \text{or: } \mathbf{\frac{db}{dx} \approx \frac{\Delta b}{L} \ll \frac{f}{8}\frac{b}{R_h}} \quad \textit{Ans.} \text{ (approximate)}$$

Since $(f/8) = \mathcal{O}(0.01)$ and $(b/R_h) = \mathcal{O}(1)$, we are OK unless $\Delta b \approx L$ (large expansion).

10.116 Investigate the possibility of *frictional effects* in the venturi flume of Prob. 10.113, part (a), for which the frictionless solution is $Q = 9.88$ m³/s. Let the contraction be 3 m long and the measurements of y_1 and y_2 be at positions 3 m upstream and 3 m downstream of the contraction, respectively. Use the modified gradually varied theory of Prob. 10.115, with $n = 0.018$ to estimate the flow rate.

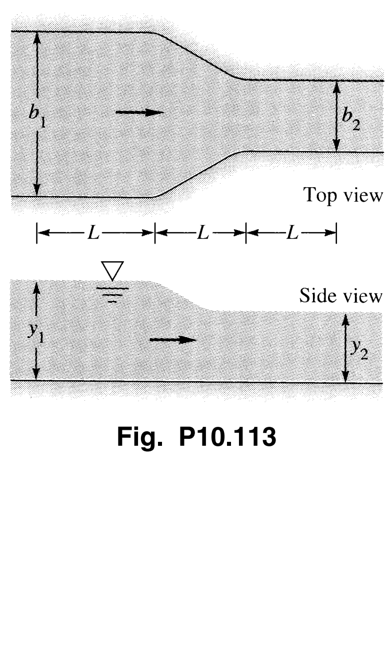


Fig. P10.113

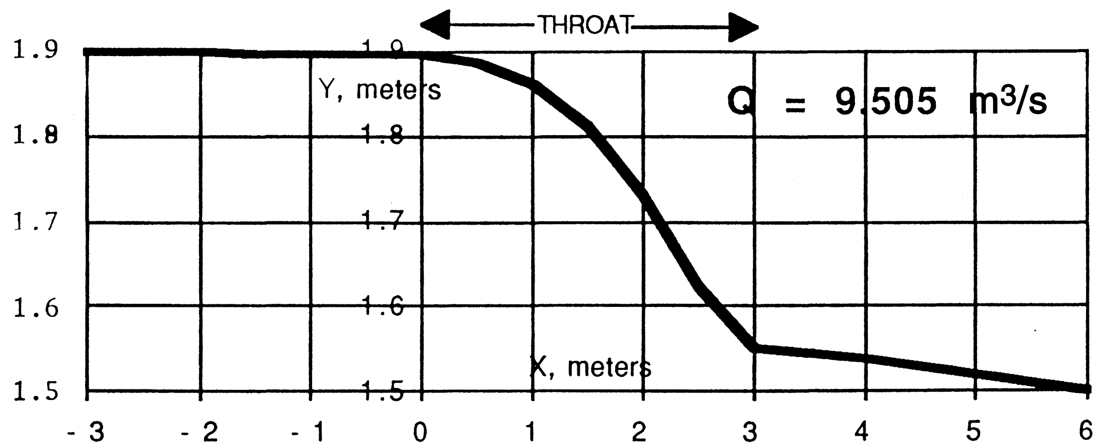
Solution: We use the differential equation and assume a smooth contraction:

$$\frac{dy}{dx} = \frac{S_o - S + (V^2/gb)(db/dx)}{1 - V^2/(gy)}, \quad S = \frac{n^2V^2}{R_h^{4/3}}$$

$$\text{and} \quad b \approx b_1 - \frac{b_1 - b_2}{2}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right] \text{ in the throat}$$

Assume horizontal ($S_o = 0$) and integrate from $x = -L$ to $x = +2L$ where $L = 3$ m

Note that $V = Q/(by)$ with $b = b_1$ for $-L < x < 0$ and $b = b_2$ for $L < x < 2L$. The given numerical values are $b_1 = 3$ m, $b_2 = 2$ m, and $y(0) = y_1 = 1.9$ m, find Q if $y_2 = 1.5$ m. The solution is shown in the graph below. The flow rate is $\mathbf{Q = 9.505}$ m^3/s. *Ans.*
Here friction causes about a 4% change in the predicted flow rate.



10.117 A full-width weir in a horizontal channel is 5 m wide and 80 cm high. The upstream depth is 1.5 m. Estimate the flow rate for (a) a sharp-crested weir; and (b) a round-nosed broad-crested weir.

Solution: We are given b = 5 m, Y = 0.8 m, and H = 1.5 − 0.8 = 0.7 m. Then

$$C_{d,sharp} = 0.564 + 0.0846\left(\frac{0.7}{0.8}\right) \approx 0.638, \quad Q = C_d b\sqrt{g}\, H^{3/2}$$

$$\text{or:} \quad Q_{sharp} = 0.638(5.0)\sqrt{9.81}(0.7)^{3/2} \approx \mathbf{5.85\ m^3/s} \quad \textit{Ans. (a)}$$

For the round-nosed broad-crested weir, we don't know the length or the roughness, so assume it is fairly short and smooth:

$$C_d \approx 0.544; \quad Q_{round,broad} \approx 0.544(5.0)\sqrt{9.81}(0.7)^{1.5} \approx \mathbf{5.0\ m^3/s} \quad \textit{Ans. (b)}$$

10.118 Using a Bernoulli-type analysis similar to Fig. 10.16*a*, show that the theoretical discharge of the V-shaped weir in Fig. P10.118 is given by

$$Q = 0.7542 g^{1/2} \tan\alpha H^{5/2}$$

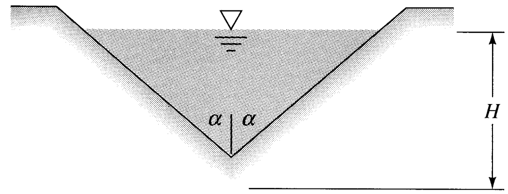


Fig. P10. 118

Solution: As in Eq. 10.52, assume that velocity V in any strip of height *dz* and width *b*, where *z* is measured down from the top, is V ≈ √(2gz) and integrate for the flow rate:

$$Q = \int_{weir} V\,dA = \int_0^H \sqrt{2gz}\; b\,dz, \quad \text{where } b = b_o(1 - z/H) \quad \text{and} \quad b_o = \text{top width.}$$

$$\text{Thus} \quad Q = \int_0^H \sqrt{2gz}\; b_o\left(1 - \frac{z}{H}\right)dz = \frac{4}{15}\sqrt{2g}\,\frac{b_o}{H}H^{5/2},$$

$$\text{but from Fig. P10.118} \quad \frac{b_o}{H} = 2\tan\alpha$$

$$\text{Finally, then,} \quad Q_{V\text{-notch}} = \frac{8}{15}\sqrt{2g}\tan\alpha H^{5/2} \approx \mathbf{0.7542\sqrt{g}\tan\alpha\,H^{5/2}} \quad \textit{Ans.}$$

10.119 Data by A. T. Lenz for water at 20°C (reported in Ref. 19) show a significant increase of discharge coefficient of V-notch weirs (Fig. P10.118) at low heads. For $\alpha = 20°$, some measured values are as follows:

H, ft:	0.2	0.4	0.6	0.8	1.0
C_d:	0.499	0.470	0.461	0.456	0.452

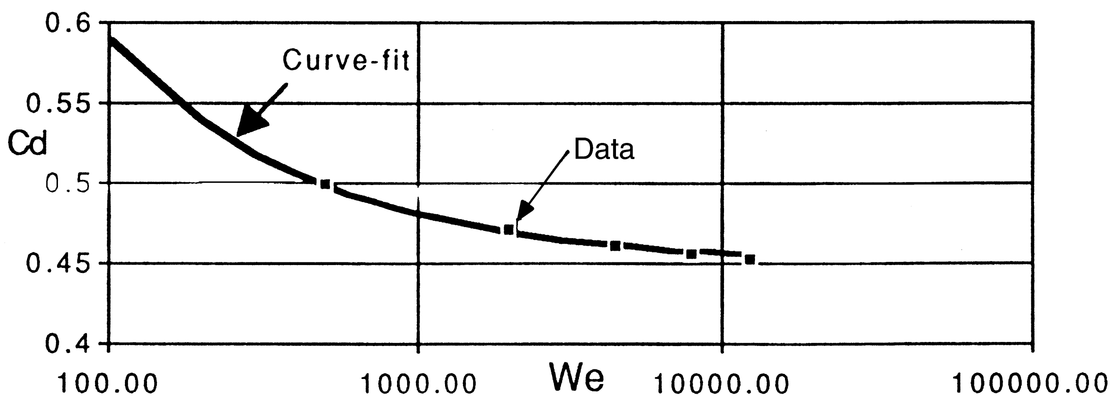
Solution: There is little or no Reynolds number effect. We can ascribe the entire effect to surface tension Y, or Weber number $We = \rho g H^2/Y$.

A Power-law curve-fits the raw data and hence also fits the Weber-number form:

$$C_d = 0.449 + \frac{0.0060}{[\text{H in ft}]^{1.3}} \pm 0.5\%$$

Convert to dimensionless form: $\mathbf{C_d \approx 0.449 + \dfrac{2.8}{We^{0.65}}}$, where $\mathrm{We} = \dfrac{\rho g H^2}{Y}$ *Ans.*

The data and this curve-fit are shown on the graph below.



10.120 The rectangular channel in Fig. P10.120 contains a V-notch weir as shown. The intent is to meter flow rates between 2.0 and 6.0 m^3/s with an upstream hook gage set to measure water depths between 2.0 and 2.75 m. What are the most appropriate values for the notch height *Y* and the notch half-angle α?

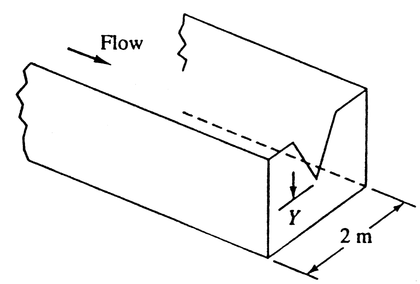


Fig. P10.120

Solution: There is an *exact* solution to this problem which uses the full range of water depth to measure the full range of flow rates. Of course, there are also a wide variety of combinations of (α, Y) which do a good job but have somewhat less range and accuracy. Anyway, for the "solution" to this problem, match each flow to each depth, with H = y – Y and using Table 10.2(*c*) for the flow-rate correlation:

$$y = 2\text{ m:}\quad Q = 2\ \frac{m^3}{s} = 0.44\tan\alpha\sqrt{g}(2.0 - Y)^{5/2};$$

$$\text{also:}\quad Q = 6 = 0.44\tan\alpha\sqrt{g}(2.75 - Y)^{5/2}$$

Divide these two to get $\mathbf{Y \approx 0.64\ m}$ *Ans.* Back substitute for $\alpha \approx \mathbf{34°}$ *Ans.*

10.121 Water flow in a rectangular channel is to be metered by a thin-plate weir with side contractions, as in Table 10.2b, with L = 6 ft and Y = 1 ft. It is desired to measure flow rates between 1500 and 3000 gal/min with only a 6-in change in upstream water depth. What is the most appropriate length for the weir width b?

Solution: We are given Y = 1 ft, so water level y_1 and weir width b are the unknowns. Apply Table 10.2(b) to each flow rate, noting that $y_2 = y_1 + 0.5$ ft:

$$Q_1 = 1500\text{ gpm} = 3.342\ \frac{ft^3}{s} = 0.581[b - 0.1(y_1 - 1)]\sqrt{32.2}(y_1 - 1)^{3/2}$$

$$Q_2 = 3000\text{ gpm} = 6.684\ \frac{ft^3}{s} = 0.581[b - 0.1(y_1 - 0.5)]\sqrt{32.2}(y_1 - 0.5)^{3/2}$$

Solve simultaneously by iteration for $y_1 \approx 1.80$ ft and $\mathbf{b \approx 1.50\ ft}$ *Ans.*

10.122 In 1952 E. S Crump developed the triangular weir shape shown in Fig. P10.122 [Ref. 19, chap. 4]. The front slope is 1:2 to avoid sediment deposition, and the rear slope is 1:5 to maintain a stable tailwater flow. The beauty of the design is that it has a unique discharge correlation up to near-drowning conditions, $H_2/H_1 \le 0.75$:

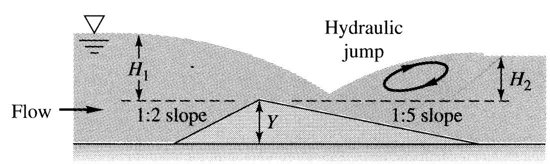


Fig. P10.122 The Crump weir [19, chap. 4]

$$Q = C_d b g^{1/2}\left(H_1 + \frac{V_1^2}{2g} - k_h\right)^{3/2} \quad \text{where } C_d \approx 0.63 \quad \text{and} \quad k_h \approx 0.3\text{ mm}$$

The term k_h is a low-head loss factor. Suppose that the weir is 3 m wide and has a crest height Y = 50 cm. If the water depth upstream is 65 cm, estimate the flow rate in gal/min.

Solution: We are given weir height Y = 50 cm and upstream depth y_1 = 65 cm, hence $H_1 = 65 - 50 = 15$ cm. Apply the formula, which has an unknown (but low) velocity:

$$Q = 0.63(3.0)\sqrt{9.81}\left(0.15 + \frac{V_1^2}{2(9.81)} - 0.0003\right)^{3/2}, \quad \text{where } V_1 = \frac{Q}{by_1} = \frac{Q}{3(0.65)}$$

Very slight iteration is needed to find $Q \approx 0.349\ m^3/s \approx \mathbf{5500\ \frac{gal}{min}}$ *Ans.*

10.123 The Crump weir in Prob. 10.122 is for *modular* flow, that is, when the flow rate is independent of downstream tailwater. When the weir becomes drowned, the flow rate decreases by the following factor:

$$Q = Q_{modular} f \qquad \text{where } f \approx 1.035\left[0.817 - \left(\frac{H_2^*}{H_1^*}\right)^4\right]^{0.0647}$$

for $0.70 \le H_2^*/H_1^* \le 0.93$, where H^* denotes $H_1 + V_1^2/(2g) - k_h$ for brevity. The weir is then *double-gaged* to measure both H_1 and H_2. Suppose that the weir crest is 1 m high and 2 m wide. If the measured upstream and downstream water depths are 2.0 and 1.9 m, respectively, estimate the flow rate in gal/min. Comment on the possible uncertainty of your estimate.

Solution: Again, as in Prob. 10.123, we do not know the velocities (which are fairly low) so we have to iterate the formula slightly:

$$Q = 0.63(2.0)\sqrt{9.81}\left(H_2^*\right)^{3/2} f, \quad \text{where } H^* = H + \frac{V^2}{2g} - k_h,\ V = \frac{Q}{by} \text{ for both 1 and 2}$$

$$\text{Given} \quad H_1 = 2 - 1 = 1 \text{ m and } H_2 = 1.9 - 1 = 0.9 \text{ m, iterate slightly to}$$

$$\mathbf{Q} \approx 3.84 \text{ m}^3/\text{s} \approx \mathbf{61000\ gal/min} \quad \textit{Ans.}$$

This estimate is uncertain to at least ±5%. The formula itself is a curve-fit and therefore probably uncertain. In addition, the formula is very sensitive to the measured values of y_1 and y_2. For example, a **1%** error in these measurements causes a **10%** change in Q. *Ans.*

10.124 Water flows at 600 ft^3/s in a rectangular channel 22 ft wide with $n \approx 0.024$ and a slope of 0.1°. A dam increases the depth to 15 ft, as in Fig. P10.124. Using gradually varied theory, estimate the distance L upstream at which the water depth will be 10 ft. What type of solution curve are we on? What should be the water depth asymptotically far upstream?

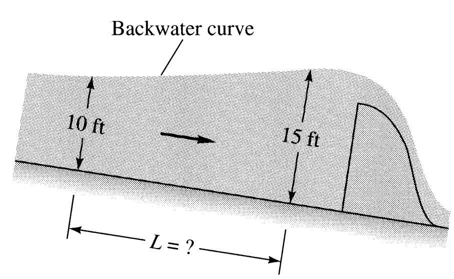


Fig. P10.124

Solution: With depth given just upstream of the dam, we do not have to make a "weir" calculation, but instead go directly to the gradually-varied-flow calculation. Note first that

$$Q = 600 = \frac{1.486}{0.024}(22y_n)\left(\frac{22y_n}{22 + 2y_n}\right)^{2/3}(\sin 0.1°)^{1/2}, \quad \text{solve for normal depth} \quad y_n \approx 4.74$$

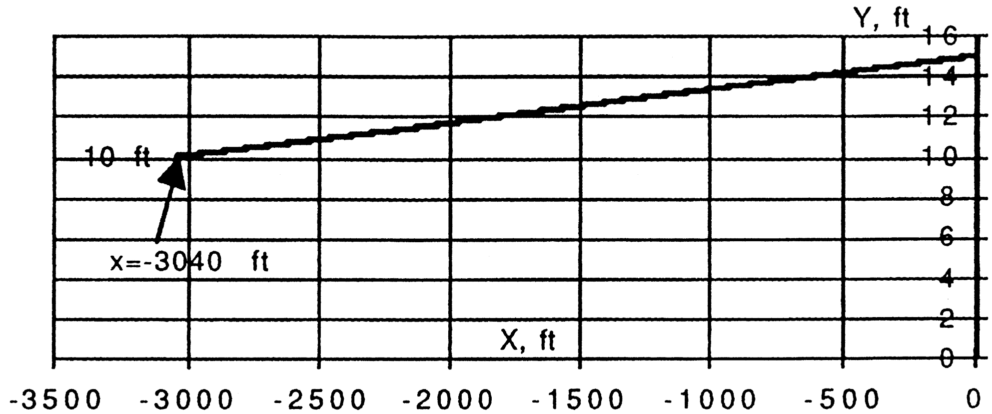
$$\text{Also,}\quad y_c = (q^2/g)^{1/3} = \left[\frac{(600/22)^2}{32.2}\right]^{1/3} \quad \text{or} \quad \text{critical depth} \quad y_c \approx 2.85$$

Therefore, since $y_c < y_n < y$, we are on a mild-slope **M-1 curve**. *Ans.* (Fig. 10.4c) The basic differential equation is:

$$\frac{dy}{dx} = \frac{S_o - n^2q^2\Big/\left(2.208y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)},$$

$$\text{where} \quad q = \frac{600}{22}, \quad R_h = \frac{22y}{22+2y}, \quad n = 0.024, \quad S_o = \sin 0.1°$$

The graph of y(x) is shown below. We reach y = 10 ft at **x = –3040 ft**. *Ans.*
If we keep integrating backward, we reach $v = v_n \approx$ **4.74 ft**.



10.125 The Tupperware dam on the Blackstone River is 12 ft high, 100 ft wide, and sharp-edged. It creates a backwater similar to Fig. P10.124. Assume that the river is a weedy-earth rectangular channel 100 ft wide with a flow rate of 800 ft^3/s. Estimate the water-depth 2 mi upstream of the dam if $S_o = 0.001$.

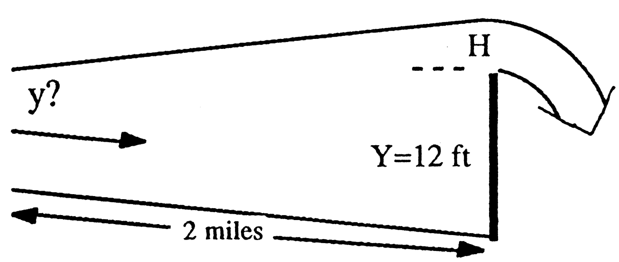


Fig. P10.125

Solution: First use the weir formula to establish the water depth just upstream of the dam:

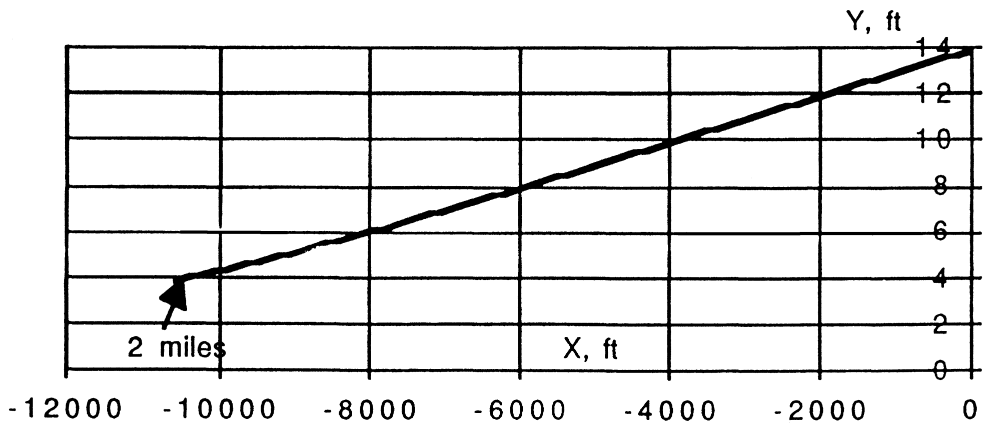
$$Q = 800\ \frac{ft^3}{s} = C_d b\sqrt{g}\,H^{3/2} = \left(0.564 + 0.0846\frac{H}{Y}\right)(100)\sqrt{32.2}\,H^{3/2},$$

$$\text{Solve for } H \approx 1.81 \text{ ft}$$

Therefore the initial water depth is y(0) = Y + H = 12 + 1.81 = 13.81 ft. For weedy earth, take $n \approx 0.030$. We are on an **M-1 curve,** with a basic differential equation

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/\left(2.208y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)}, \quad \text{where } q = \frac{800}{100},\ R_h = \frac{100y}{100+2y},\ n = 0.030,\ S_o = 0.001$$

to be integrated backwards ($\Delta x < 0$) for 2 miles (10560 ft). The result is shown in the graph below.



At x = −2 miles = −10560 ft, the water depth is **y ≈ 3.8 ft**. *Ans.*
[Another mile upstream and we would asymptotically reach the normal depth of 2.71 ft.]

10.126 Suppose that the rectangular channel of Fig. P10.120 is made of riveted steel with a flow of 8 m^3/s. If α = 30° and Y = 50 cm, estimate, from gradually-varied theory, the water depth 100 meters upstream.

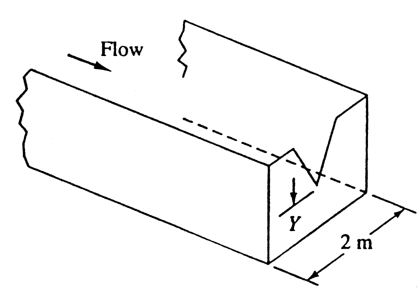


Fig. P10.120

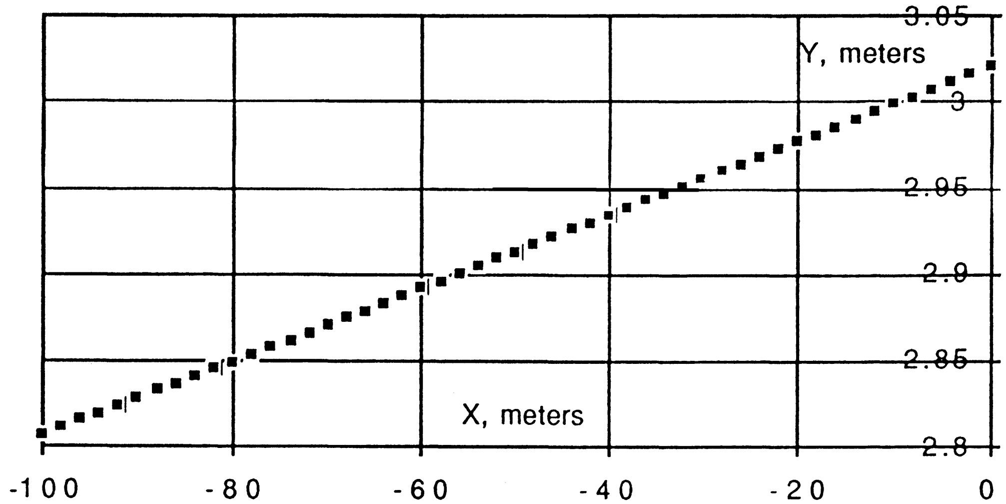
Solution: For riveted steel, take $n \approx 0.015$. First use the weir formula to get y(0):

$$Q = 8\ \frac{m^3}{s} = 0.44\tan 30^\circ\sqrt{9.81}H^{5/2}, \quad \text{solve } H \approx 2.52 \text{ m},\ y(0) = 2.52 + 0.5 = \mathbf{3.02\ m}$$

The basic differential equation is

$$\frac{dy}{dx} = \frac{S_o - n^2q^2/\left(y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)}, \quad \text{where } q = \frac{8}{2},\ R_h = \frac{2y}{2+2y},\ n = 0.015,\ S_o = \sin 0.15°$$

Integrate backwards ($\Delta x < 0$) for 100 m. The result is shown in the graph below:



At $x = -100$ m, the water depth is $\mathbf{y \approx 2.81\ m}$. *Ans.*
[Another 1000 m upstream and we asymptotically reach the normal depth of 1.62 m.]

10.127 A horizontal gravelly earth channel 2 m wide contains a full-width Crump weir (Fig. P10.122) 1 m high. If the weir is not drowned, estimate, from gradually varied theory, the flow rate for which the water depth 100 m upstream will be 2 m.

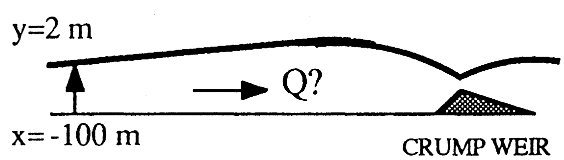


Fig. P10.127

Solution: With Q unknown, we need to *combine* weir and gradually-varied theories:

$$Q_{Crump} = 0.63(2.0)\sqrt{9.81}\left(H + \frac{V^2}{2g} - 0.0003\text{ m}\right)^{3/2}, \quad V = \frac{Q}{(2.0)y_o}$$

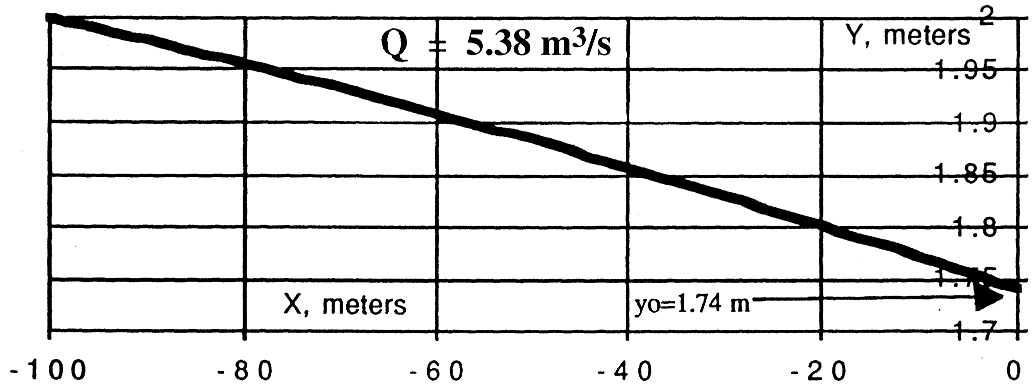
$$\frac{dy}{dx} = \frac{S_o - n^2q^2/\left(y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)}, \quad \text{where } q = \frac{Q}{2.0},\ R_h = \frac{2y}{2+2y},\ S_o = 0,\ n_{gravelly} \approx 0.025$$

With Q and H unknown, guess Q, find H, y(0) = Y + H = 0.5 m + H, then integrate backwards along the "backwater" curve to x = –100 m, see if water depth there is 2 m.

A list of such guesses is given as follows, after *much* digital computation:

Q, m^3/s:	1.0	2.0	3.0	4.0	5.0	**5.38**
H, m:	0.230	0.451	0.678	0.912	1.150	1.242
y(at –100 m), m:	0.814	1.111	1.385	1.648	1.904	**2.000**

After iteration, the proper flow rate is **$Q \approx 5.38\ m^3/s$**. *Ans.*
[This gives H ≈ 1.24 m and y(0) ≈ 1.74 m.]



10.128 A rectangular channel 4 m wide is blocked by a broadcrested weir 2 m high, as in Fig. P10.128. The channel is horizontal for 200 m upstream and then slopes at 0.7° as shown. The flow rate is 12 m^3/s, and n = 0.03. Compute the water depth y at 300 m upstream from gradually varied theory.

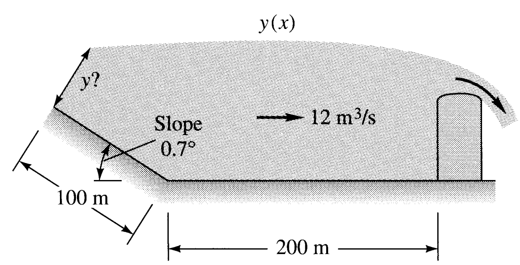


Fig. P10.128

Solution: First use (smooth) weir theory to establish the depth just upstream of the dam:

$$Q = 12\ \frac{m^3}{s} \approx 0.544(4.0)\sqrt{9.81}H^{3/2}, \quad \text{solve for } H \approx 1.46\text{ m}, \quad \therefore\ y(0) = H + Y = 3.46\text{ m}$$

$$\frac{dy}{dx} = \frac{S_o - n^2q^2\Big/\left(y^2R_h^{4/3}\right)}{1 - q^2/(gy^3)}, \quad \text{where } q = \frac{12}{4},\ R_h = \frac{4y}{4+2y},\ n = 0.03, \text{ two values of } S_o$$

Integrate backwards ($\Delta x < 0$) for 200 m with $S_o = 0$, then for 100 m with $S_o = \sin 0.7°$. After 200 m, the depth is y = 3.56 m. Then at x = –300 m, **y ≈ 2.37 m**. *Ans.*

Both water profiles are nearly linear:

x, m:	0	–50	–100	–150	–200	–225	–250	–275	–300
y, m:	3.46	3.49	3.51	3.53	3.56	3.26	2.96	2.66	**2.37 m**

[At x ≈ –400 m, the sloped flow will approach its normal depth of 1.05 m.]

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: <u>Answers</u>

FE10.1 Consider a rectangular channel 3 m wide laid on a 1° slope. If the water depth is 2 m, the hydraulic radius is

(a) 0.43 m (b) 0.6 m (**c**) **<u>0.86 m</u>** (d) 1.0 m (e) 1.2 m

FE10.2 For the channel of Prob. FE10.1, the most efficient water depth (best flow for a given slope and resistance) is

(a) 1 m (**b**) **<u>1.5 m</u>** (c) 2 m (d) 2.5 m (e) 3 m

FE10.3 If the channel of Prob. FE10.1 is built of rubble cement (Manning's $n \approx 0.020$), what is the uniform flow rate when the water depth is 2 m?

(a) 6 m^3/s (b) 18 m^3/s (**c**) **<u>36 m^3/s</u>** (d) 40 m^3/s (e) 53 m^3/s

FE10.4 For the channel of Prob. FE10.1, if the water depth is 2 m and the uniform flow rate is 24 m^3/s, what is the approximate value of Manning's roughness factor n?

(a) 0.015 (b) 0.020 (c) 0.025 (**d**) **<u>0.030</u>** (e) 0.035

FE10.5 For the channel of Prob. FE10.1, if Manning's roughness factor $n \approx 0.020$ and $Q \approx 24\ m^3/s$, what is the normal depth y_n?

(a) 1 m (**b**) **<u>1.5 m</u>** (c) 2 m (d) 2.5 m (e) 3 m

FE10.6 For the channel of Prob. FE10.1, if $Q \approx 24\ m^3/s$, what is the critical depth y_c?

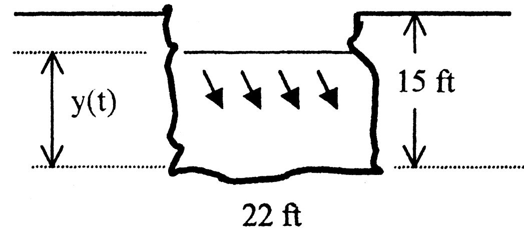
(a) 1.0 m (b) 1.26 m (c) 1.5 m (**d**) **<u>1.87 m</u>** (e) 2.0 m

FE10.7 For the channel of Prob. FE10.1, if $Q \approx 24\ m^3/s$ and the depth is 2 m, what is the Froude number of the flow?

(a) 0.50 (b) 0.77 (**c**) **<u>0.90</u>** (d) 1.00 (e) 1.11

COMPREHENSIVE APPLIED PROBLEMS

C10.1 February 1998 saw the failure of the earthen dam impounding California Jim's Pond in southern Rhode Island. The resulting flood raised temporary havoc in the nearby village of Peace Dale. The pond is 17 acres in area and 15 ft deep and was full from heavy rains. The breach in the dam was 22 ft wide and 15 ft deep. Estimate the time required to drain the pond to a depth of 2 ft.



Solution: Unfortunately, Table 10.2, item *b*, does not really apply, because the breach is so *deep*:, H = y > 0.5Y = 0. Nevertheless, it's all we have, and ponds don't rupture every day, so let's use it! A control volume around the pond yields

$$\frac{d}{dt}\left(\int d\upsilon_{pond}\right) + Q_{out} = 0,$$

$$or:\quad A_{pond}\frac{dy}{dt} = -Q_{out} = -0.581(b-0.1y)g^{1/2}y^{3/2},$$

$$b = 22 \text{ ft},\quad A_{pond} = 17 \text{ acres} = 740{,}520 \text{ ft}^2$$

If we neglect the "edge contraction" term "–0.1y" compared to b = 22 ft, this first-order differential equation has the solvable form

$$\frac{dy}{dt} \approx -Cy^{3/2},\quad \text{where } C = \frac{0.581(22\text{ ft})(32.2)^{1/2}}{740520} \approx 9.8\text{E–5 ft}^{-1/2}\text{sec}^{-1}$$

$$\text{Separate and integrate:}\quad \int_{15ft}^{2ft}\frac{dy}{y^{3/2}} = -C\int_0^t dt,\quad or:\quad \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{15}} = Ct$$

$$\text{Finally, solve}\quad t_{drain\text{-}to\text{-}2ft} \approx \frac{1.414-0.516}{9.8\text{E–5}} = 9160 \text{ s} = \mathbf{2.55\ h}\quad \textit{Ans.}$$

If we used a spreadsheet and kept the term "–0.1y", we would predict a *time-to-drain-to-2 ft* or about **2.61** hours. The theory is too crude to distinguish between these estimates.

C10.2 A circular, unfinished concrete drainpipe is laid on a slope of 0.0025 and is planned to carry from 50 to 300 ft^3/s of run-off water. Design constraints are that (1) the water depth should be no more than 3/4 of the diameter; and (2) the flow should always be subcritical. What is the appropriate pipe diameter to satisfy these requirements? If no commercial pipe is exactly this calculated size, should you buy the next smallest or the next largest pipe?

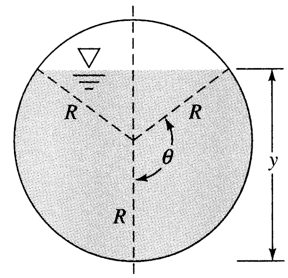


Fig. C10.2

Solution: For unfinished concrete $n \approx 0.014$. From the geometry of Fig. 10.6, 3/4-full corresponds to an angle $\theta = 120°$. This level should be able to carry the maximum 300 ft^3/s flow:

$$\theta = 120°: \quad A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 2.53R^2; \quad R_h = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2}\right) = 0.603R$$

$$\text{Try} \quad Q = 300\ \frac{\text{ft}^3}{\text{s}} = \frac{1.49}{n} A R_h^{2/3} S_o^{1/2} = \frac{1.49}{0.014}(2.53R^2)(0.603R)^{2/3}\sqrt{0.0025}$$

$$\text{Solve for} \quad R_{3/4\text{-}full} \approx \mathbf{3.64\ ft} \quad \textit{Ans.}$$

Now check to see if this flow is subcritical:

$$A_c = \left(\frac{b_0 Q^2}{g}\right)^{1/3} = \left[\frac{(1.732)(3.64)(Q_c)^2}{32.2}\right]^{1/3} = 2.53(3.64)^2$$

$$\text{Solve for} \quad Q_c = 438\ \frac{\text{ft}^3}{\text{s}} > 300, \quad \therefore \text{ flow is subcritical}$$

Even at max flow, three-quarters full, the Froude number is only 0.68. Scanning the other flow rates, down to 50 ft^3/s, yields *smaller* drainpipes. Therefore we conclude that a pipe of diameter **D ≈ 7.3 ft** will do the job. Pick the nearest ***larger*** available size. *Ans.*

C10.3 Extend Prob. 10.72, whose solution was V_2 = 4.33 m/s. Use gradually-varied theory to estimate the water depth 10 m down at section 3 for (a) the 5° unfinished concrete slope shown in the figure; and (b) for an *upward* (–5°) adverse slope. (c) When you find out that (b) is *impossible*, explain why and repeat for an adverse slope of (–1°).

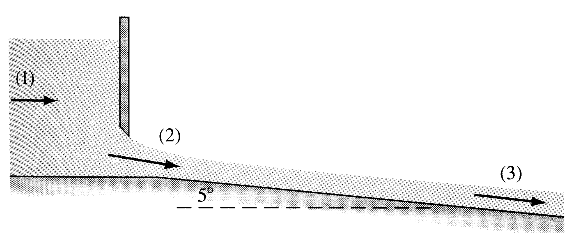


Fig. P10.72

Solution: For unfinished concrete take $n \approx 0.014$. Note that y(0) = 0.0462 m. For the given flow rate $q = 0.2$ m^3/s·m, first calculate reference depths:

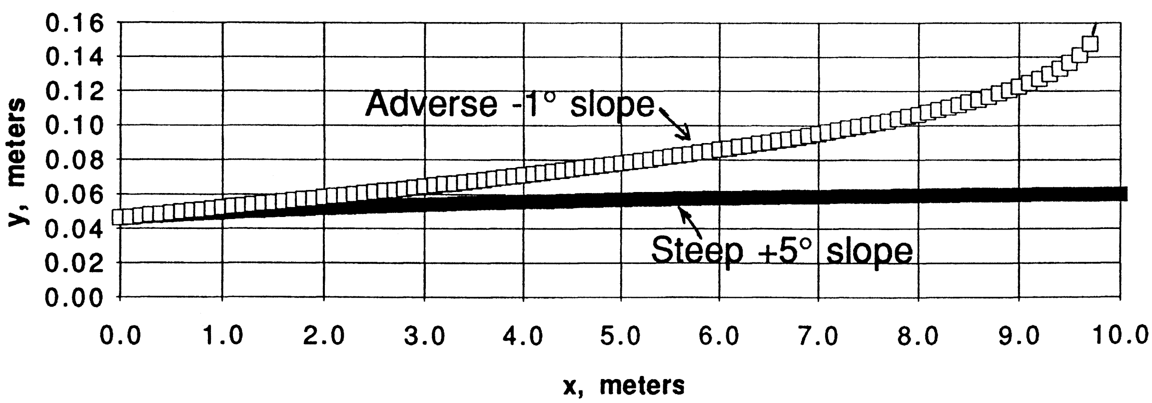
$$q = 0.2 = \frac{1}{0.014}(y)(y^{2/3})\sqrt{\sin 5°}, \quad \text{solve normal depth } y_n \approx 0.0611 \text{ m}$$

$$\text{Critical depth:} \quad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.2^2}{9.81}\right)^{1/3} = 0.160 \text{ m} \quad \therefore \text{ Steep S-3 curve.}$$

The channel is "wide," so the formulation of Example 10.8 is appropriate:

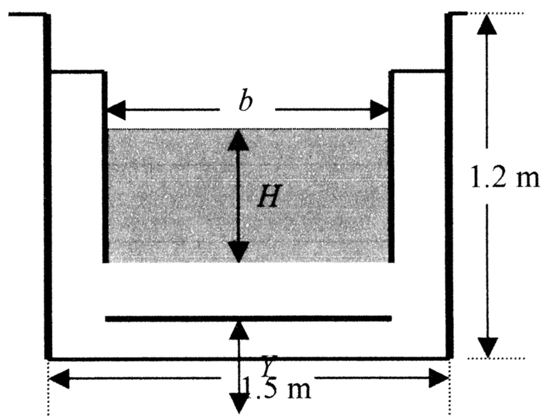
$$\frac{dy}{dx} = \frac{S_0 - n^2q^2/y^{10/3}}{1 - q^2/(gy^3)}, \quad S_o = \sin \pm 5°, \ n = 0.014, \ q = 0.2, \ y(0) = 0.0462 \text{ m}, \ y(10 \text{ m}) = ?$$

Numerical integration, by Excel or MATLAB or whatever, for the *steep* (**S-3**) slope yields ***y* = 0.060 m** at x = 10 m, nearly critical, as shown below. *Ans.* (a) Integration for the adverse (–5°) slope goes to *critical* at about x = 4.5 m—**the theory fails for x > 4.5 m**. *Ans.* (b) If we go back and take a *smaller* adverse slope of (–1°), we obtain **y ≈ 0.16 m** at x = 9.8 m—the flow goes critical near x = L, as shown in the graph below. *Ans.* (c)



C10.4 It is desired to meter an asphalt rectangular channel of width 1.5 m which is designed for uniform flow at a depth of 70 cm and a slope of 0.0036. The vertical sides of the channel are 1.2 m high. Consider using a thin-plate rectangular weir, either full or partial width (Table 10.2*a*, *b*) for this purpose. Sturm [7, p. 51] recommends, for accurate correlation, that such a weir have $Y \geq 9$ cm and $H/Y \leq 2.0$. Determine the feasibility of installing such a weir which will be accurate and yet not cause the water to overflow the sides of the channel.

Solution: For asphalt take $n = 0.016$. We have only one partial-width formula, and that is from Table 10.2b. We are slightly outside the limits of applicability, but we will use it anyway:



$$Q_{weir} = 0.581(b - 0.1H)g^{1/2}H^{3/2}$$

where b and H are shown in the figure.

Meanwhile, calculate the total flow rate in the expected normal flow:

$$Q = \frac{1}{n}AR_h^{2/3}\sqrt{S_o} = \frac{1}{0.016}[1.5\ m(0.7\ m)]\left[\frac{1.5\ m(0.7\ m)}{1.5\ m + 2(0.7\ m)}\right]^{2/3}\sqrt{0.0036} = \mathbf{2.00}\ \frac{\mathbf{m^3}}{\mathbf{s}}$$

The weir discharge must equal this flow rate. Let us begin by making b equal to the full available width of 1.5 m and holding Y to the minimum height of 9 cm. The weir formula is:

$$Q = 2.0\ \frac{m^3}{s} = 0.581(1.5\ m - 0.1H)(9.81\ m/s^2)^{1/2}H^{3/2}, \quad \text{solve for}\ \ H = 0.845\ m$$

Note that H is independent of Y, but the ratio $H/Y = 0.845/0.09 = 9.4$, which far exceeds: Sturm's recommendation $H/Y \le 2.0$. If we raise Y to $H/2 = 0.423$ m, the total upstream water depth is $H + Y = 1.27$ m, which overflows the channel walls. If we back down to Y = 0.35 m, the upstream depth is only 1.195 m, so **a wide-weir design is possible with *H/Y* = 2.4, not bad**.

Similarly, we can try shorter values of b, but either (1) the upstream depth will exceed 1.2 m, or (2) the ratio H/Y will exceed 2.0. Here are some possible scenarios:

$\boldsymbol{b = 1.4}$ **m**: $H = 0.89$ m; $H + Y \approx 1.2$ m if $\boldsymbol{Y = 0.31}$ **m** and $H/Y = 2.9$

$\boldsymbol{b = 1.25}$ **m**: $H = 0.97$ m; $H + Y \approx 1.2$ m if $\boldsymbol{Y = 0.23}$ **m** and $H/Y = 4.2$

$b = 1.0$ m: $H = 1.16$ m; $H + Y \approx 1.2$ m if $Y = 0.04$ m and $H/Y = 29.0$

The first two of these are plausible, although *H/Y* is larger than 2.0. The third result is not recommended because *Y* is too far below 9 cm. **We conclude that reasonable designs are possible, but they slightly violate the constraints on the formulas we are using**.

C10.5 Figure C10.5 shows a hydraulic model of a *compound weir*, that is, one which combines two different shapes. (a) Other than measurement, for which it might be useful, what could be the engineering reason for such a weir? (b) For the prototype river, assume that both sections have sides at a 70° angle to the vertical, with the bottom section having a base width of 2 m and the upper section having a base width of 4.5 m, including the cut-out portion. The heights of lower and upper horizontal sections are 1 m and 2 m, respectively. Use engineering estimates and make a plot of upstream water depth versus Petaluma River flow rate in the range 0 to 4 m^3/s. (c) For what river flow rate will the water overflow the top of the dam?

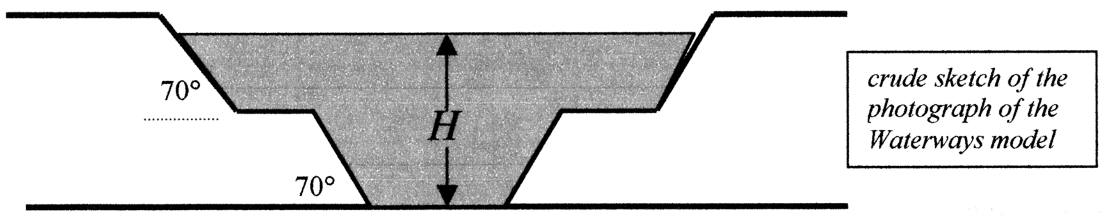


Fig. C10.5

Solution: We have no formulas in the text for a compound weir shape, but we can still use the concept of weir flow and estimate the discharge for various water depths.
(a) A good reason for using a narrow bottom portion of the weir is to maintain a reasonable upstream depth at low flow, then widen to maintain a lower depth at high flow. It also allows a more accurate flow measurement during low flow. *Ans.* (a)
(b) Rather than derive a whole new theory for compound weirs, we will assume that the bottom portion is more or less rectangular, based on average width *b*, with the top portion also assumed rectangular with its flow rate added onto the lower portion. For example, if $H = 1$ m (the top of the lower portion), the flow rate is estimated by Table 10.2*b*:

$$b_{avg} = \frac{2.728 + 2.0\ m}{2} = 2.364\ m,\ Q \approx 0.58(b_{avg} - 0.1H)g^{1/2}H^{3/2}$$

$$\text{or: } \quad Q_{lower} = 0.58[2.364\ m - 0.1(1\ m)](9.81\ m/s^2)^{1/2}(1\ m)^{3/2} \approx 4.1\ m^3/s$$

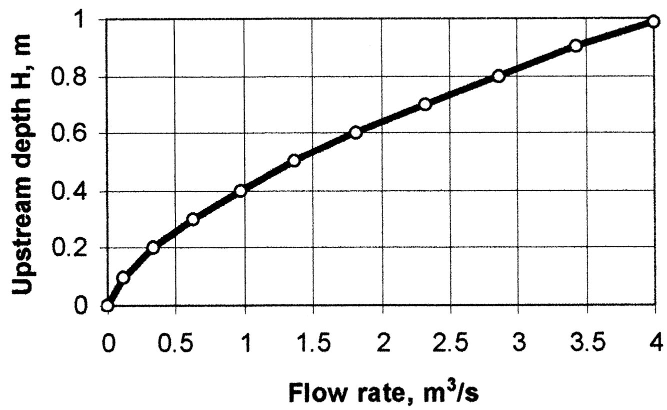
Then if $H = 2$ m > 1 m, we figure Q_{upper} the same way and add on the lower portion flow. Again take $H = 1$ m, that is, the height of the flow above the lower part of the weir:

$$b_{avg} = \frac{4.5 + 5.23\ m}{2} = 4.87\ m, \quad Q \approx 0.58(b_{avg} - 0.1H)g^{1/2}H^{3/2}$$

$$\textbf{\textit{or}}: \quad Q_{upper} = 0.58[4.87\ m - 0.1(1\ m)](9.81\ m/s^2)^{1/2}(1\ m)^{3/2} \approx 8.7\ m^3/s$$

$$\textbf{\textit{Thus}} \quad Q_{total} = Q_{lower} + Q_{upper} = 4.1 + 8.7 \approx \mathbf{12.8\ m^3/s}$$

Flow rates greater than this value of 12.8 m^3/s will **overflow the top of the weir**. *Ans.* (b)
A plot of Q versus H for the range $0 < Q < 4$ m^3/s is shown below.



Problem C10.5