Chapter 9 • Compressible Flow

9.1 An ideal gas flows adiabatically through a duct. At section 1, p_1 = 140 kPa, T_1 = 260°C, and V_1 = 75 m/s. Farther downstream, p_2 = 30 kPa and T_2 = 207°C. Calculate V_2 in m/s and $s_2 - s_1$ in J/(kg·K) if the gas is (a) air, k = 1.4, and (b) argon, k = 1.67.

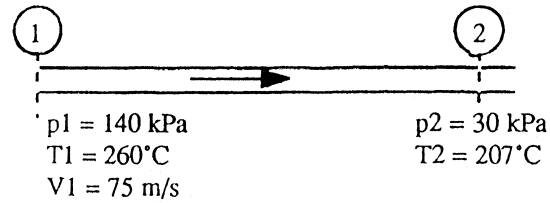


Fig. P9.1

Solution: (a) For air, take k = 1.40, R = 287 J/kg·K, and c_p = 1005 J/kg·K. The adiabatic steady-flow energy equation (9.23) is used to compute the downstream velocity:

$$c_pT + \frac{1}{2}V^2 = \text{constant} = 1005(260) + \frac{1}{2}(75)^2 = 1005(207) + \frac{1}{2}V_2^2 \quad \text{or} \quad \mathbf{V_2 \approx 335\ \frac{m}{s}} \quad \textit{Ans.}$$

$$\text{Meanwhile, } s_2 - s_1 = c_p\ln(T_2/T_1) - R\ln(p_2/p_1) = 1005\ln\left(\frac{207+273}{260+273}\right) - 287\ln\left(\frac{30}{140}\right),$$

$$\text{or} \quad s_2 - s_1 = -105 + 442 \approx \mathbf{337\ J/kg\cdot K} \quad \textit{Ans.}\text{ (a)}$$

(b) For argon, take k = 1.67, R = 208 J/kg·K, and c_p = 518 J/kg·K. Repeat part (a):

$$c_pT + \frac{1}{2}V^2 = 518(260) + \frac{1}{2}(75)^2 = 518(207) + \frac{1}{2}V_2^2, \quad \text{solve } \mathbf{V_2 = 246\ \frac{m}{s}} \quad \textit{Ans.}$$

$$s_2 - s_1 = 518\ln\left(\frac{207+273}{260+273}\right) - 208\ln\left(\frac{30}{140}\right) = -54 + 320 \approx \mathbf{266\ J/kg\cdot K} \quad \textit{Ans.}\text{ (b)}$$

9.2 Solve Prob. 9.1 if the gas is steam. Use two approaches: (a) an ideal gas from Table A.4; and (b) real steam from the steam tables [15].

Solution: For steam, take k = 1.33, R = 461 J/kg·K, and c_p = 1858 J/kg·K. Then

$$c_pT + \frac{1}{2}V^2 = 1858(260) + \frac{1}{2}(75)^2 = 1858(207) + \frac{1}{2}V_2^2, \quad \text{solve } \mathbf{V_2 \approx 450\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$s_2 - s_1 = 1858\ln\left(\frac{207+273}{260+273}\right) - 461\ln\left(\frac{30}{140}\right) = -195 + 710 \approx \mathbf{515\ J/kg\cdot K} \quad \textit{Ans.}\text{ (a)}$$

(b) For real steam, we look up each enthalpy and entropy in the Steam Tables:

$$\text{at 140 kPa and 260°C,} \quad \text{read } h_1 = 2.993E6\ \frac{J}{kg};$$
$$\text{at 30 kPa and 207°C,} \quad h_2 = 2.893E6\ \frac{J}{kg}$$

$$\text{Then} \quad h + \frac{1}{2}V^2 = 2.993E6 + \frac{1}{2}(75)^2 = 2.893E6 + \frac{1}{2}V_2^2, \quad \text{solve } \mathbf{V_2 \approx 453\ \frac{m}{s}} \quad \textit{Ans. (b)}$$

$$\text{at 140 kPa and 260°C,} \quad \text{read } s_1 = 7915\frac{J}{kg\cdot K}, \quad \text{at 30 kPa and 207°C,} \quad s_2 = 8427\ \frac{J}{kg\cdot K}$$

$$\text{Thus} \quad s_2 - s_1 = 8427 - 7915 \approx \mathbf{512\ J/kg\cdot K} \quad \textit{Ans. (b)}$$

These are within ±1% of the ideal gas estimates (a). Steam is nearly ideal in this range.

9.3 If 8 kg of oxygen in a *closed tank* at 200°C and 300 kPa is heated until the pressure rises to 400 kPa, calculate (a) the new temperature; (b) the total heat transfer; and (c) the change in entropy.

Solution: For oxygen, take $k = 1.40$, $R = 260$ J/kg·K, and $c_v = 650$ J/kg·K. Then

$$\rho_1 = \rho_2, \quad \therefore\ T_2 = T_1(p_2/p_1) = (200+273)\left(\frac{400}{300}\right) = 631\text{ K} \approx \mathbf{358°C} \quad \textit{Ans. (a)}$$

$$Q = mc_v\Delta T = (8)(650)(358-200) \approx \mathbf{8.2E5\ J} \quad \textit{Ans. (b)}$$

$$s_2 - s_1 = mc_v\ln(T_2/T_1) = (8)(650)\ln\left(\frac{358+273}{200+273}\right) \approx \mathbf{1500\ \frac{J}{K}} \quad \textit{Ans. (c)}$$

9.4 Compressibility becomes important when the Mach number > 0.3. How fast can a two-dimensional cylinder travel in sea-level standard air before compressibility becomes important *somewhere* in its vicinity?

Solution: For sea-level air, $T = 288$ K, $a = [1.4(287)(288)]^{1/2} = \mathbf{340}$ m/s. Recall from Chap. 8 that incompressible theory predicts $V_{max} = 2U_\infty$ on a cylinder. Thus

$$Ma_{max} = \frac{V_{max}}{a} = \frac{2U_\infty}{340} = 0.3 \quad \text{when } U_\infty = \frac{0.3(340)}{2} \approx \mathbf{51\ \frac{m}{s} = 167\ \frac{ft}{s}} \quad \textit{Ans.}$$

9.5 Steam enters a nozzle at 377°C, 1.6 MPa, and a steady speed of 200 m/s and accelerates isentropically until it exits at saturation conditions. Estimate the exit velocity and temperature.

Solution: At saturation conditions, steam is *not ideal*. Use the Steam Tables:

$$\text{At } 377°\text{C and } 1.6\text{ MPa, read} \quad h_1 = 3.205\text{E}6\text{ J/kg} \quad \text{and} \quad s_1 = 7153\text{ J/kg}\cdot\text{K}$$

$$\text{At } \textit{saturation} \quad \text{for } s_1 = s_2 = 7153, \quad \text{read } p_2 = 185\text{ kPa},$$

$$\mathbf{T_2 = 118°C}, \quad \text{and } h_2 = 2.527\text{E}6\text{ J/kg}$$

$$\text{Then} \quad h + \frac{1}{2}V^2 = 3.205\text{E}6 + \frac{1}{2}(200)^2 = 2.527\text{E}6 + \frac{1}{2}V_2^2, \quad \text{solve } \mathbf{V_2 \approx 1180\ \frac{m}{s}} \quad \textit{Ans.}$$

This exit flow is **<u>supersonic</u>**, with a Mach number exceeding 2.0. We are assuming with this calculation that a (supersonic) shock wave does not form.

9.6 Helium at 300°C and 200 kPa, in a closed container, is cooled to a pressure of 100 kPa. Estimate (a) the new temperature, in °C; and (b) the change in entropy, in J/(kg·K).

Solution: From Table A.4 for helium, $k = 1.66$ and $R = 2077\ m^2/s^2\cdot K$. Convert 300°C to 573 K.
(a) The density is unchanged because the container is constant volume. Thus

$$\frac{p_2}{p_1} = \frac{100\text{ kPa}}{200\text{ kPa}} = \frac{\rho_2 RT_2}{\rho_1 RT_1} = \frac{T_2}{T_1} = \frac{T_2}{573\ K}, \quad \text{solve for } T_2 = 287\ K = \mathbf{14°C} \quad \textit{Ans. (a)}$$

(b) Evaluate $c_p = kR/(k-1) = 1.66(2077)/(1.66-1) = 5224\ m^2/s^2\cdot K$. From Eq. (9.8),

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = 5244\ln\left(\frac{287\text{ K}}{573\text{ K}}\right) - 2077\ln\left(\frac{100\text{ kPa}}{200\text{ kPa}}\right)$$

$$= \mathbf{-2180\ \frac{J}{kg\cdot K}} \quad \textit{Ans. (b)}$$

9.7 Carbon dioxide ($k = 1.28$) enters a constant-area duct at 400°F, 100 lbf/in^2 absolute, and 500 ft/s. Farther downstream the properties are $V_2 = 1000$ ft/s and $T_2 = 900$°F. Compute (a) p_2, (b) the heat added between sections, (c) the entropy change between sections, and (d) the mass flow per unit area. *Hint:* This problem requires the continuity equation.

Solution: For carbon dioxide, take k = 1.28, R = 1130 ft·lbf/slug·°R, and c_p = 5167 ft·lbf/slug·°R. (a) The downstream pressure is computed from one-dimensional continuity:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2, \text{ cancel } A, \quad \frac{p_1}{RT_1}V_1 = \frac{p_2}{RT_2}V_2, \text{ cancel } R,$$

$$\text{or:} \quad p_2 = p_1(T_2/T_1)(V_1/V_2) = 100\left(\frac{900+460}{400+460}\right)\left(\frac{500}{1000}\right) = \mathbf{79\ psia} \quad \textit{Ans.}\text{ (a)}$$

(b) The steady-flow energy equation, with no shaft work, yields the heat transfer per mass:

$$q = c_p(T_2 - T_1) + \frac{1}{2}\left(V_2^2 - V_1^2\right) = 5167(900-400) + \frac{1}{2}[(1000)^2 - (500)^2]$$

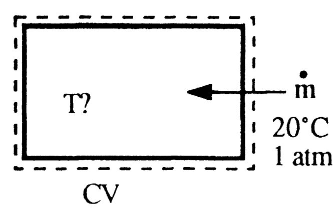
$$\text{or:} \quad q = 2.96E6\ \frac{\text{ft·lbf}}{\text{slug}} \div 32.2 \div 778.2 \approx \mathbf{118\ \frac{Btu}{lbm}} \quad \textit{Ans.}\text{ (b)}$$

(c, d) Finally, the entropy change and mass flow follow from the properties known above:

$$s_2 - s_1 = 5167\ln\left(\frac{900+460}{400+460}\right) - 1130\ln\left(\frac{79}{100}\right) = 2368 + 266 \approx \mathbf{2630\ \frac{ft·lbf}{slug\text{-}°R}} \quad \textit{Ans.}\text{ (c)}$$

$$\dot{m}/A = \rho_1 V_1 = \left[\frac{100\times 144}{1130(400+460)}\right](500) \approx \mathbf{7.4\ \frac{slug}{s·ft^2}} \quad \textit{Ans.}\text{ (d)}$$

9.8 Atmospheric air at 20°C enters and fills an insulated tank which is initially evacuated. Using a control-volume analysis from Eq. (3.63), compute the tank air temperature when it is full.



Solution: The energy equation during filling of the adiabatic tank is

$$\frac{dQ}{dt} + \frac{dW_{shaft}}{dt} = 0 + 0 = \frac{dE_{CV}}{dt} - h_{atm}\dot{m}_{entering}, \quad \text{or, after filling,}$$

$$E_{CV,final} - E_{CV,initial} = h_{atm}m_{entered}, \quad \text{or:} \quad mc_v T_{tank} = mc_p T_{atm}$$

$$\text{Thus} \quad \mathbf{T_{tank}} = (c_p/c_v)T_{atm} = (1.4)(20+273) \approx \mathbf{410\ K = 137°C} \quad \textit{Ans.}$$

9.9 Liquid hydrogen and oxygen are burned in a combustion chamber and fed through a rocket nozzle which exhausts at exit pressure equal to ambient pressure of 54 kPa. The nozzle exit diameter is 45 cm, and the jet exit density is 0.15 kg/m^3. If the exhaust gas has

a molecular weight of 18, estimate (a) the exit gas temperature; (b) the mass flow; and (c) the thrust generated by the rocket.

NOTE: *Sorry, we forgot to give the exit velocity, which is 1600 m/s.*

Solution: (a) From Eq. (9.3), estimate R_{gas} and hence the gas exit temperature:

$$R_{gas} = \frac{\Lambda}{M} = \frac{8314}{18} = 462\ \frac{J}{kg\cdot K},\quad \text{hence } T_{exit} = \frac{p}{R\rho} = \frac{54000}{462(0.15)} \approx \mathbf{779\ K} \quad \textit{Ans.}\ \text{(a)}$$

(b) The mass flow follows from the velocity which we forgot to give:

$$\dot{m} = \rho A V = \left(0.15\ \frac{kg}{m^3}\right)\frac{\pi}{4}(0.45)^2(1600) \approx \mathbf{38\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(b)}$$

(c) The thrust was derived in Problem 3.68. When $p_{exit} = p_{ambient}$, we obtain

$$Thrust = \rho_e A_e V_e^2 = \dot{m} V_e = 38(1600) \approx \mathbf{61{,}100\ N} \quad \textit{Ans.}\ \text{(c)}$$

9.10 A certain aircraft flies at the same Mach number regardless of its altitude. Compared to its speed at 12000-m Standard Altitude, it flies 127 km/h faster at sea level. Determine its Mach number.

Solution: At sea level, $T_1 = 288.16$ K. At 12000 m standard, $T_2 = 216.66$ K. Then

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340.3\ \frac{m}{s};\quad a_2 = \sqrt{kRT_2} = 295.0\ \frac{m}{s}$$

$$\text{Then}\quad \Delta V_{plane} = Ma(a_2 - a_1) = Ma(340.3 - 295.0) = [127\ km/h] = 35.27\ m/s$$

$$\text{Solve for}\quad \mathbf{Ma} = \frac{35.27}{45.22} \approx \mathbf{0.78} \quad \textit{Ans.}$$

9.11 At 300°C and 1 atm, estimate the speed of sound of (a) nitrogen; (b) hydrogen; (c) helium; (d) steam; and (e) uranium hexafluoride $^{238}UF_6$ ($k \neq 1.06$).

Solution: The gas constants are listed in Appendix Table A.4 for all but uranium gas (e):

(a) nitrogen: k = 1.40, R = 297, T = 300 + 273 = 573 K:

$$a = \sqrt{kRT} = \sqrt{1.40(297)(573)} \approx \mathbf{488\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

(b) hydrogen: k = 1.41, R = 4124, $a = \sqrt{1.41(4124)(573)} \approx \mathbf{1825\ m/s}$ *Ans.* (b)

(c) helium: k = 1.66, R = 2077: $a = \sqrt{1.66(2077)(573)} \approx \mathbf{1406\ m/s}$ *Ans.* (c)

(d) steam: $k = 1.33,\ R = 461:\quad a = \sqrt{1.33(461)(573)} \approx$ **593 m/s** *Ans.* (d)

(e) For uranium hexafluoride, we need only to compute *R* from the molecular weight:

$$\text{(e) } {}^{238}UF_6:\quad M = 238 + 6(19) = 352,\quad \therefore\ R = \frac{8314}{352} \approx 23.62\ m^2/s^2\cdot K$$

then $a = \sqrt{1.06(23.62)(573)} \approx$ **120 m/s** *Ans.* (e)

9.12 Assume that water follows Eq. (1.19) with $n \approx 7$ and $B \approx 3000$. Compute the bulk modulus (in kPa) and the speed of sound (in m/s) at (a) 1 atm; and (b) 1100 atm (the deepest part of the ocean). (c) Compute the speed of sound at 20°C and 9000 atm and compare with the measured value of 2650 m/s (A. H. Smith and A. W. Lawson, *J. Chem. Phys.*, vol. 22, 1954, p. 351).

Solution: We may compute these values by differentiating Eq. (1.19) with $k \approx 1.0$:

$$\frac{p}{p_a} = (B+1)(\rho/\rho_a)^n - B;\quad \text{Bulk modulus } K = \rho\frac{dp}{d\rho} = n(B+1)p_a(\rho/\rho_a)^n,\quad a = \sqrt{K/\rho}$$

We may then substitute numbers for water, with $p_a = 101350$ Pa and $\rho_a = 998\ kg/m^3$:

(a) at 1 atm: $K_{water} = 7(3001)(101350)(1)^7 \approx$ **2.129E9 Pa** (21007 atm) *Ans.* (a)

speed of sound $a_{water} = \sqrt{K/\rho} = \sqrt{2.129E9/998} \approx$ **1460 m/s** *Ans.* (a)

$$\text{(b) at 1100 atm:}\quad \rho = 998\left(\frac{1100+3000}{3001}\right)^{1/7} = 998(1.0456) \approx 1044\ kg/m^3$$

$K = K_{atm}(1.0456)^7 = (2.129E9)(1.3665) =$ **2.91E9 Pa** (28700 atm) *Ans.* (b)

$a = \sqrt{K/\rho} = \sqrt{2.91E9/1044} \approx$ **1670 m/s** *Ans.* (b)

$$\text{(c) at 9000 atm:}\quad \rho = 998\left(\frac{9000+3000}{3001}\right)^{1/7} = 1217\ \frac{kg}{m^3};\quad K = K_a\left(\frac{1217}{998}\right)^7,$$

or: $K = 8.51E9$ Pa, $a = \sqrt{K/\rho} = \sqrt{8.51E9/1217} \approx$ **2645 m/s** (within 0.2%) *Ans.* (c)

9.13 Assume that the airfoil of Prob. 8.84 is flying at the same angle of attack at 6000 m standard altitude. Estimate the forward velocity, in mi/h, at which supersonic flow (and possible shock waves) will appear on the airfoil surface.

Solution: At 6000 m, from Table A.6, $a = 316.5$ m/s. From the data of Prob. 8.84, the highest surface velocity is about $1.29U_\infty$ and occurs at about the quarter-chord point.

When that velocity reaches the speed of sound, shock waves may begin to form:

$$a = 316.5 \text{ m/s} = 1.29U_\infty, \quad \text{hence } U_\infty \approx 245 \text{ m/s} = \mathbf{549\ mi/h} \quad Ans.$$

9.14 Assume steady adiabatic flow of a perfect gas. Show that the energy Eq. (9.21), when plotted as *a* versus *V*, forms an ellipse. Sketch this ellipse; label the intercepts and the regions of subsonic, sonic, and supersonic flow; and determine the ratio of the major and minor axes.

Solution: In Eq. (9.21), simply replace enthalpy by its equivalent in speed of sound:

$$h + \frac{1}{2}V^2 = \text{constant} = c_pT + \frac{1}{2}V^2 = \frac{kR}{k-1}T + \frac{1}{2}V^2 = \frac{a^2}{k-1} + \frac{1}{2}V^2,$$

$$\text{or:} \quad \mathbf{a^2 + \frac{k-1}{2}V^2 = constant = a_o^2 = \frac{k-1}{2}V_{max}^2} \text{ (ellipse)} \quad Ans.$$

This ellipse is shown below. The axis ratio is $\mathbf{V_{max}/a_o = [2/(k-1)]^{1/2}}$. *Ans.*

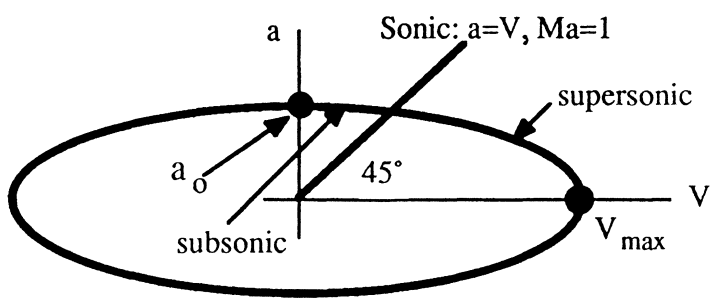


Fig. P9.14

9.15 A weak pressure wave (sound wave), with a pressure change Δp ≈ 40 Pa, propagates through still air at 20°C and 1 atm. Estimate (a) the density change; (b) the temperature change; and (c) the velocity change across the wave.

Solution: For air at 20°C, speed of sound $a \approx 343$ m/s, and $\rho = 1.2$ kg/m^3. Then

$$\Delta p \approx \rho C\Delta V,\ C \approx a, \quad \text{thus } 40 = (1.2)(343)\Delta V, \quad \text{solve for } \Delta V \approx \mathbf{0.097\ \frac{m}{s}} \quad Ans. \text{ (a)}$$

$$\Delta\rho = (\rho + \Delta\rho)\frac{\Delta V}{C} = (1.2 + \Delta\rho)\frac{0.097}{343}, \quad \text{solve for } \Delta\rho \approx \mathbf{0.00034\ kg/m^3} \quad Ans. \text{ (b)}$$

$$\frac{T + \Delta T}{T} \approx \left(\frac{p + \Delta p}{p}\right)^{(k-1)/k}, \text{ or: } \frac{293 + \Delta T}{293} \approx \left(\frac{101350 + 40}{101350}\right)^{\frac{0.4}{1.4}}, \quad \Delta T \approx \mathbf{0.033\ K} \quad Ans. \text{ (c)}$$

9.16 A weak pressure wave (sound wave) Δp propagates through still air. Discuss the type of reflected pulse which occurs, and the boundary conditions which must be satisfied, when the wave strikes normal to, and is reflected from, (a) a solid wall; and (b) a free liquid surface.

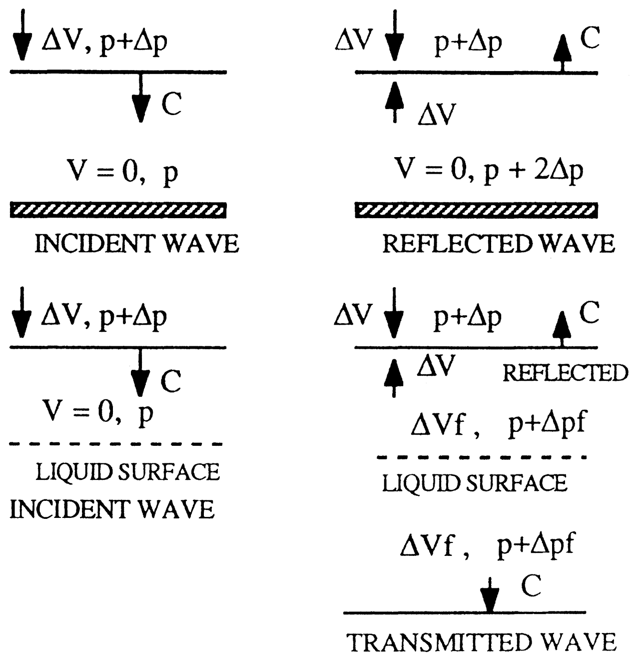


Fig. P9.16

Solution: (a) When reflecting from a solid wall, the velocity to the wall must be zero, so the wall pressure rises to p + 2Δp to create a compression wave which cancels out the oncoming particle motion ΔV.

(b) When a compression wave strikes a liquid surface, it reflects and transmits to keep the particle velocity ΔVf and the pressure p + Δpf the same across the liquid interface:

$$\Delta V_f = \frac{2\rho C\,\Delta V}{\rho C + \rho_{liq}C_{liq}}; \qquad \Delta p_f = \frac{2\rho_{liq}C_{liq}\,\Delta p}{\rho C + \rho_{liq}C_{liq}} \quad \textit{Ans.}\text{ (b)}$$

If $\rho_{liq}C_{liq} \geq\geq \rho C$ of air, then $\Delta V_f \approx 0$ and $\Delta p_f \approx 2\Delta p$, which is case (a) above.

9.17 A submarine at a depth of 800 m sends a sonar signal and receives the reflected wave back from a similar submerged object in 15 s. Using Prob. 9.12 as a guide, estimate the distance to the other object.

Solution: It probably makes little difference, but estimate *a* at a depth of 800 m:

$$\text{at 800 m,} \quad p = 101350 + 1025(9.81)(800) = 8.15E6\ \text{Pa} = 80.4\ \text{atm}$$

$$p/p_a = 80.4 = 3001(\rho/1025)^7 - 3000, \quad \text{solve } \rho \approx 1029 \text{ kg/m}^3$$

$$a = \sqrt{n(B+1)p_a(\rho/\rho_a)^7/\rho} = \sqrt{7(3001)(101350)(1029/1025)^7/1029} \approx 1457 \text{ m/s}$$

Hardly worth the trouble: One-way distance $\approx a\Delta t/2 = 1457(15/2) \approx$ **10900 m**. *Ans.*

9.18 Race cars at the Indianapolis Speedway average speeds of 185 mi/h. After determining the altitude of Indianapolis, find the Mach number of these cars and estimate whether compressibility might affect their aerodynamics.

Solution: Rush to the Almanac and find that Indianapolis is at 220 m altitude, for which Table A.6 predicts that the standard speed of sound is 339.4 m/s = 759 mi/h. Thus the Mach number is

$$Ma_{racer} = V/a = 185 \text{ mph}/759 \text{ mph} = \mathbf{0.24} \quad \textit{Ans.}$$

This is less than 0.3, so the Indianapolis Speedway need not worry about compressibility.

9.19 The Concorde aircraft flies at Ma ≈ 2.3 at 11-km standard altitude. Estimate the temperature in °C at the front stagnation point. At what Mach number would it have a front stagnation point temperature of 450°C?

Solution: At 11-km standard altitude, T ≈ 216.66 K, a = √(kRT) = 295 m/s. Then

$$T_{nose} = T_o = T\left(1 + \frac{k-1}{2}Ma^2\right) = 216.66[1 + 0.2(2.3)^2] = 446 \text{ K} \approx \mathbf{173°C} \quad \textit{Ans.}$$

If, instead, $T_o = 450°C = 723 \text{ K} = 216.66(1 + 0.2\, Ma^2)$, solve Ma ≈ **3.42** *Ans.*

9.20 A gas flows at V = 200 m/s, p = 125 kPa, and T = 200°C. For (a) air and (b) helium, compute the maximum pressure and the maximum velocity attainable by expansion or compression.

Solution: Given (V, p, T), we can compute Ma, T_o and p_o and then $V_{max} = \sqrt{(2c_pT_o)}$:

$$\text{(a) air:} \quad Ma = \frac{V}{\sqrt{kRT}} = \frac{200}{\sqrt{1.4(287)(200+273)}} = \frac{200}{436} = 0.459$$

$$\text{Then} \quad p_{max} = p_o = p\left(1 + \frac{k-1}{2}Ma^2\right)^{k/(k-1)} = 125[1 + 0.2(0.459)^2]^{3.5} \approx \mathbf{144\ kPa} \quad \textit{Ans. (a)}$$

$$T_o = (200+273)[1 + 0.2(0.459)^2] = 493 \text{ K}, \quad V_{max} = \sqrt{2(1005)(493)} \approx \mathbf{995\ m/s} \quad \textit{Ans. (a)}$$

(b) For helium, k = 1.66, R = 2077 $m^2/s^2\cdot K$, $c_p = kR/(k-1) = 5224\ m^2/s^2\cdot K$. Then

$$Ma = 200/\sqrt{1.66(2077)(473)} \approx 0.157, \quad p_o = 125[1+0.33(0.157)^2]^{\frac{1.66}{0.66}} \approx \mathbf{128\ kPa}$$

$$T_o = 473[1+0.33(0.157)^2] = 477\ K, \quad V_{max} = \sqrt{2(5224)(477)} \approx \mathbf{2230\ m/s} \quad \textit{Ans. } (b)$$

9.21 CO_2 expands isentropically through a duct from p_1 = 125 kPa and T_1 = 100°C to p_2 = 80 kPa and V_2 = 325 m/s. Compute (a) T_2; (b) Ma_2; (c) T_o; (d) p_o; (e) V_1; and (f) Ma_1.

Solution: For CO_2, from Table A.4, take k = 1.30 and R = 189 J/kg·K. Compute the specific heat: $c_p = kR/(k-1) = 1.3(189)/(1.3-1) = 819$ J/kg·K. The results follow in sequence:

(a) $T_2 = T_1(p_2/p_1)^{(k-1)/k} = (373\ K)(80/125)^{(1.3-1)/1.3} = \mathbf{336\ K}$ *Ans.* (a)

(b) $a_2 = \sqrt{kRT_2} = \sqrt{(1.3)(189)(336)} = 288$ m/s, $Ma_2 = V_2/a_2 = 325/288 = \mathbf{1.13}$ *Ans.* (b)

(c) $T_{o1} = T_{o2} = T_2\left(1+\dfrac{k-1}{2}Ma_2^2\right) = (336)\left[1+\dfrac{0.3}{2}(1.13)^2\right] = \mathbf{401\ K}$ *Ans.* (c)

(d) $p_{o1} = p_{o2} = p_2\left(1+\dfrac{k-1}{2}Ma_2^2\right)^{1.3/(1.3-1)} = (80)\left[1+\dfrac{0.3}{2}(1.13)^2\right]^{1.3/0.3} = \mathbf{171\ kPa}$ *Ans.* (d)

(e) $T_{o1} = 401\ K = T_1 + \dfrac{V_1^2}{2c_p} = 373 + \dfrac{V_1^2}{2(819)}$, Solve for $\mathbf{V_1 = 214\ m/s}$ *Ans.* (e)

(f) $a_1 = \sqrt{kRT_1} = \sqrt{(1.3)(189)(373)} = 303$ m/s, $Ma_1 = V_1/a_1 = 214/303 = \mathbf{0.71}$ *Ans.* (f)

9.22 Given the pitot stagnation temperature and pressure and the static-pressure measurements in Fig. P9.22, estimate the air velocity V, assuming (a) incompressible flow and (b) compressible flow.

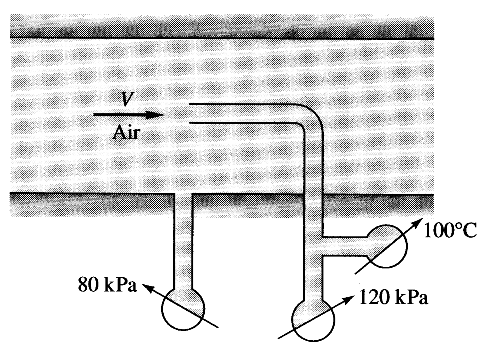


Fig. P9.22

Solution: Given p = 80 kPa, p_o = 120 kPa, and T = 100°C = 373 K. Then

$$\rho_o = \frac{p_o}{RT_o} = \frac{120000}{287(373)} = 1.12\ kg/m^3$$

(a) 'Incompressible':

$$\rho = \rho_o,\ V \approx \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2(120000-80000)}{1.12}} \approx \mathbf{267\ \frac{m}{s}}\ (7\%\ \text{low}) \quad \textit{Ans.}\ \text{(a)}$$

(b) Compressible: $T = T_o(p/p_o)^{(k-1)/k} = 373(80/120)^{0.4/1.4} = 332$ K. Then $T_o = 373$ K $= T + V^2/2c_p = 332 + V^2/[2(1005)]$, solve for **V = 286 m/s**. *Ans.* (b)

9.23 A large rocket engine delivers hydrogen at 1500°C and 3 MPa, $k = 1.41$, $R = 4124$ J/kg·K, to a nozzle which exits with gas pressure equal to the ambient pressure of 54 kPa. Assuming isentropic flow, if the rocket thrust is 2 MN, estimate (a) the exit velocity; and (b) the mass flow of hydrogen.

Solution: Compute $c_p = kR/(k-1) = 14180$ J/kg·K. For isentropic flow, compute

$$\rho_o = \frac{p_o}{RT_o} = \frac{3E6}{4124(1773)} = 0.410\ \frac{\text{kg}}{\text{m}^3},\ \therefore\ \rho_e = \rho_o\left(\frac{p_e}{p_o}\right)^{\frac{1}{k}} = 0.410\left(\frac{54E3}{3E6}\right)^{\frac{1}{1.41}} = 0.0238\ \frac{\text{kg}}{\text{m}^3}$$

$$T_e = \frac{54000}{4124(0.0238)} = 551\text{ K},\quad T_o = 1773 = 551 + \frac{V_e^2}{2(14180)},$$

$$\text{Solve}\quad \mathbf{V_{exit} \approx 5890\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{From Prob. 3.68,}\quad \textit{Thrust} = 2E6\ N = \dot{m}V_e = \dot{m}(5890),\quad \text{solve}\ \mathbf{\dot{m} \approx 340\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(b)}$$

9.24 For low-speed (nearly incompressible) gas flow, the stagnation pressure can be computed from Bernoulli's equation

$$p_0 = p + \frac{1}{2}\rho V^2$$

(a) For higher subsonic speeds, show that the isentropic relation (9.28*a*) can be expanded in a power series as follows:

$$p_0 \approx p + \frac{1}{2}\rho V^2\left(1 + \frac{1}{4}\text{Ma}^2 + \frac{2-k}{24}\text{Ma}^4 + \cdots\right)$$

(b) Suppose that a pitot-static tube in air measures the pressure difference $p_0 - p$ and uses the Bernoulli relation, with stagnation density, to estimate the gas velocity. At what Mach number will the error be 4 percent?

Solution: Expand the isentropic formula into a binomial series:

$$\frac{p_o}{p} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{k/(k-1)} = 1+\frac{k}{k-1}\frac{k-1}{2}\mathrm{Ma}^2 + \frac{k}{k-1}\frac{1}{2}\left(\frac{k}{k-1}-1\right)\left(\frac{k-1}{2}\mathrm{Ma}^2\right)^2 + \cdots$$

$$= 1+\frac{k}{2}\mathrm{Ma}^2 + \frac{k}{8}\mathrm{Ma}^4 + \frac{k(2-k)}{48}\mathrm{Ma}^6 + \cdots$$

Use the ideal gas identity $(1/2)\rho V^2 \equiv (1/2)kp(\mathrm{Ma}^2)$ to obtain

$$\frac{p_o - p}{(1/2)\rho V^2} = \mathbf{1+\frac{1}{4}Ma^2 + \frac{2-k}{24}Ma^4} + \cdots \quad \textit{Ans.}$$

The error in the incompressible formula, $2\Delta p/\rho_o V^2$, is 4% when

$$\frac{V}{\sqrt{2(p_o - p)/\rho}} = \sqrt{\frac{\rho_o/\rho}{1+(1/4)\mathrm{Ma}^2 + [(2-k)/24]\mathrm{Ma}^4}} = 1.04,$$

$$\text{where} \quad \frac{\rho_o}{\rho} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{1/(k-1)}$$

For $k = 1.4$, solve this for 4% error at **Ma ≈ 0.576** *Ans.*

9.25 If it is known that the air velocity in the duct is 750 ft/s, use that mercury manometer measurement in Fig. P9.25 to estimate the static pressure in the duct, in psia.

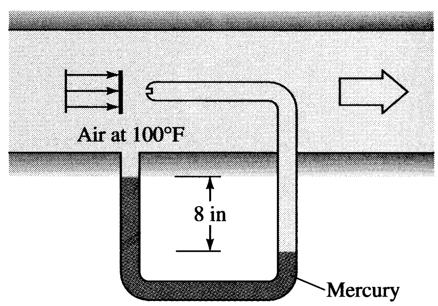


Fig. P9.25

Solution: Estimate the air specific weight in the manometer to be, say, 0.07 lbf/ft^3. Then

$$p_o - p\big|_{\text{measured}} = (\rho g_{\text{mercury}} - \rho g_{\text{air}})h = (846-0.07)\left(\frac{8}{12}\ \text{ft}\right) \approx 564\ \text{lbf/ft}^2$$

$$\text{Given} \quad T = 100°F = 560°R, \quad a = \sqrt{kRT} = \sqrt{1.4(1717)(560)} \approx 1160\ \text{ft/s}$$

$$\text{Then} \quad \mathrm{Ma} = V/a = 750/1160 \approx 0.646$$

$$\text{Finally,} \quad \frac{p_o - p}{p} = [1+0.2(0.646)^2]^{3.5} - 1 = 1.324 - 1 = 0.324 = \frac{564}{p}$$

Solve for $p_{\text{static}} \approx 1739$ psf ≈ **12.1 lbf/in²** (abs) *Ans.*

9.26 Show that for isentropic flow of a perfect gas if a pitot-static probe measures p_0, p, and T_0, the gas velocity can be calculated from

$$V^2 = 2c_pT_0\left[1-\left(\frac{p}{p_0}\right)^{(k-1)/k}\right]$$

What would be a source of error if a shock wave were formed in front of the probe?

Solution: Assuming isentropic flow past the probe,

$$T = T_o(p/p_o)^{(k-1)/k} = T_o - \frac{V^2}{2c_p}, \quad \text{solve } \mathbf{V^2 = 2c_pT_o\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} \quad \textit{Ans.}$$

If there is a *shock wave* formed in front of the probe, this formula will yield the air velocity inside the shock wave, because the probe measures p_{o2} *inside* the shock. The stagnation pressure in the outer stream is *greater,* as is the velocity outside the shock.

9.27 In many problems the sonic (*) properties are more useful reference values than the stagnation properties. For isentropic flow of a perfect gas, derive relations for p/p^*, T/T^*, and ρ/ρ^* as functions of the Mach number. Let us help by giving the density-ratio formula:

$$\rho/\rho^* = \left[\frac{k+1}{2+(k-1)\,\mathrm{Ma}^2}\right]^{1/(k-1)}$$

Solution: Simply introduce (and then cancel out) the stagnation properties:

$$\frac{\rho}{\rho^*} = \frac{\rho/\rho_o}{\rho^*/\rho_o} = \frac{\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}}{\left(1+\frac{k-1}{2}\right)^{-1/(k-1)}} \equiv \left[\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right]^{1/(k-1)} \quad \textit{Ans.}$$

$$\text{similarly,}\quad \frac{p}{p^*} = \frac{p/p_o}{p^*/p_o} \equiv \left[\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right]^{k/(k-1)} \quad \text{and} \quad \frac{T}{T^*} = \frac{T/T_o}{T^*/T_o} = \frac{k+1}{2+(k-1)\mathrm{Ma}^2} \quad \textit{Ans.}$$

9.28 A large vacuum tank, held at 60 kPa absolute, sucks sea-level standard air through a converging nozzle of throat diameter 3 cm. Estimate (a) the mass flow rate; and (b) the Mach number at the throat.

Solution: For sea-level air take $T_o = 288$ K, $\rho_o = 1.225$ kg/m^3, and $p_o = 101350$ Pa. The pressure ratio is given, and we can assume isentropic flow with $k = 1.4$:

$$\frac{p_e}{p_o} = \frac{60000}{101350} = \left(1 + 0.2Ma_e^2\right)^{-3.5}, \quad \text{solve } \mathbf{Ma_e \approx 0.899} \quad \textit{Ans.} \text{ (b)}$$

We can then solve for exit temperature, density, and velocity, finally mass flow:

$$\rho_e = \rho_o[1 + 0.2(0.899)^2]^{-2.5} \approx 0.842\ \frac{\text{kg}}{\text{m}^3}, \quad T_e = \frac{p_e}{R\rho_e} = \frac{60000}{287(0.842)} \approx 248\text{ K}$$

$$V_e = Ma_e a_e = 0.899[1.4(287)(248)]^{1/2} \approx 284\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,} \quad \dot{m} = \rho_e A_e V_e = (0.842)\frac{\pi}{4}(0.03)^2(284) \approx \mathbf{0.169\ \frac{kg}{s}} \quad \textit{Ans.} \text{ (a)}$$

9.29 Steam from a large tank, where T = 400°C and p = 1 MPa, expands isentropically through a small nozzle until, at a section of 2-cm diameter, the pressure is 500 kPa. Using the Steam Tables, estimate (a) the temperature; (b) the velocity; and (c) the mass flow at this section. Is the flow subsonic?

Solution: "Large tank" is code for stagnation values, thus $T_o = 400°C$ and $p_o = 1$ MPa. This problem involves dogwork in the tables and well illustrates why we use the ideal-gas law so readily. Using $k \approx 1.33$ for steam, we find the flow is slightly supersonic:

$$\text{Ideal-gas simplification:} \quad \frac{p_o}{p} = \frac{1000}{500} = 2.0 \approx \left[1 + \left(\frac{1.33-1}{2}\right)\text{Ma}^2\right]^{\frac{1.33}{0.33}},$$

$$\text{Solve} \quad \mathbf{Ma \approx 1.08}$$

That was quick. Instead, plow about in the S.I. Steam Tables, assuming constant entropy:

$$\text{At } T_o = 400°\text{C} \quad \text{and } p_o = 1\text{ MPa}, \quad \text{read } s_o \approx 7481\ \frac{\text{J}}{\text{kg·K}} \quad \text{and} \quad h_o \approx 3.264\text{E}6\ \frac{\text{J}}{\text{kg}}$$

$$\text{Then, at p} = 0.5\text{ MPa, assuming s} = s_o, \quad \text{read T} \approx 304°\text{C} \approx \mathbf{577\ K} \quad \textit{Ans.} \text{ (a)}$$

$$\text{Also read} \quad h \approx 3.074\text{E}6\text{ J/kg} \quad \text{and} \quad \rho \approx 1.896\text{ kg/m}^3.$$

With h and h_o known, the velocity follows from the adiabatic energy equation:

$$h + V^2/2 = h_o, \quad \text{or} \quad 3.074E6 + V^2/2 = 3.264E6\ \frac{J}{kg}\left(\text{or } \frac{m^2}{s^2}\right),$$

$$\text{Solve} \quad V \approx \mathbf{618\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

The speed of sound is not in *my* Steam Tables, however, the "isentropic exponent" is:

$$\gamma_{isen} \approx 1.298 \text{ at } p = 500 \text{ kPa and } T = 304°C. \text{ Then} \quad a \approx \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{1.298(5E5)}{1.896}} \approx 585\ \frac{m}{s}$$

$$\text{Then} \quad Ma = V/a = \frac{618}{585} \approx \mathbf{1.06} \quad \textit{Ans.}\text{ (c)} \quad \text{(slightly supersonic)}$$

We could have done nearly as well (±2%) by simply assuming an ideal gas with $k \approx 1.33$.

9.30 Oxygen flows in a duct of diameter 5 cm. At one section, T_o = 300°C, p = 120 kPa, and the mass flow is 0.4 kg/s. Estimate, at this section, (a) V; (b) Ma; and (c) ρ_o.

Solution: For oxygen, from Table A.4, take k = 1.40 and R = 260 J/kg·K. Compute the specific heat: $c_p = kR/(k-1) = 1.4(260)/(1.4-1) = 910$ J/kg·K. Use energy and mass together:

$$T + \frac{V^2}{2c_p} = T_o, \quad \text{or:} \quad T + \frac{V^2}{2(910\ m^2/s^2K)} = 573\ K$$

$$\dot{m} = \rho A V = \frac{p}{RT} A V = \left[\frac{120000\ Pa}{(260\ m^2/s^2K)T}\right]\left(\frac{\pi}{4}\right)(0.05\ m)^2 V = 0.4\ kg/s$$

$$\text{Solve for} \quad T = 542\ K \quad \text{and} \quad \mathbf{V = 239\ m/s} \quad \textit{Ans.}\text{ (a)}$$

With T and V known, we can easily find the Mach number and stagnation density:

$$Ma = \frac{V}{\sqrt{kRT}} = \frac{239}{\sqrt{1.4(260)(542)}} = \frac{239\ m/s}{444\ m/s} = \mathbf{0.538} \quad \textit{Ans.}\text{ (b)}$$

$$\rho = \frac{p}{RT} = \frac{120000}{260(542)} = 0.852\ \frac{kg}{m^3}$$

$$\rho_o = \rho\left(1 + \frac{k-1}{2}Ma^2\right)^{1/(k-1)} = 0.852\left[1 + \frac{0.4}{2}(0.538)^2\right]^{1/0.4} = \mathbf{0.98\ \frac{kg}{m^3}} \quad \textit{Ans.}\text{ (c)}$$

9.31 Air flows adiabatically through a duct. At one section, V_1 = 400 ft/s, T_1 = 200°F, and p_1 = 35 psia, while farther downstream V_2 = 1100 ft/s and p_2 = 18 psia. Compute (a) Ma_2; (b) U_{max}; and (c) p_{o2}/p_{o1}.

Solution: (a) Begin by computing the stagnation temperature, which is constant (adiabatic):

$$T_{o1} = T_{o2} = T_1 + \frac{V^2}{2c_p} = (200+460) + \frac{(400)^2}{2(6010)} = 673°R = T_2 + \frac{V^2}{2c_p}$$

$$\text{Then}\quad T_2 = 673 - \frac{(1100)^2}{2(6010)} = 573°R,$$

$$Ma_2 = \frac{V_2}{a_2} = \frac{1100}{\sqrt{1.4(1717)(573)}} = \frac{1100}{1173} \approx \mathbf{0.938} \quad \textit{Ans.}\text{ (a)}$$

(b) $U_{max} = \sqrt{2c_pT_o} = \sqrt{2(6010)(673)} \approx \mathbf{2840\ ft/s}$ *Ans.* (b)

(c) We need $Ma_1 = V_1/a_1 = 400/\sqrt{1.4(1717)(200+460)} = 400/1260 \approx 0.318$

$$\text{then}\quad p_{o1} = p_1\left(1+0.2Ma_1^2\right)^{3.5} = 1.072p_1 = 37.53 \text{ psia}$$

and $p_{o2} = p_2\left(1+0.2Ma_2^2\right)^{3.5} = 1.763p_2 = 31.74$ psia, $\therefore \dfrac{p_{o2}}{p_{o1}} = \dfrac{31.74}{37.53} \approx \mathbf{0.846}$ *Ans.* (c)

9.32 The large compressed-air tank in Fig. P9.32 exhausts from a nozzle at an exit velocity of 235 m/s. The mercury manometer reads h = 30 cm. Assuming isentropic flow, compute the pressure (a) in the tank and (b) in the atmosphere. (c) What is the exit Mach number?

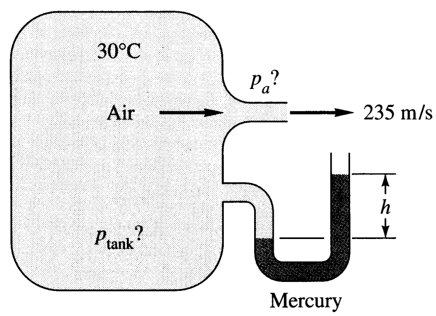


Fig. P9.32

Solution: The tank temperature = T_o = 30°C = 303 K. Then the exit jet temperature is

$$T_e = T_o - \frac{V_e^2}{2c_p} = 303 - \frac{(235)^2}{2(1005)} = 276\text{ K}, \quad \therefore Ma_e = \frac{235}{\sqrt{1.4(287)(276)}} \approx \mathbf{0.706} \quad \textit{Ans.}\text{ (c)}$$

$$\text{Then}\quad \frac{p_{tank}}{p_e} = \left(1+0.2Ma_e^2\right)^{3.5} = \mathbf{1.395} \quad \text{and} \quad p_{tank} - p_e = (\rho_{mercury} - \rho_{tank})gh$$

$$\text{Guess}\quad \rho_{tank} \approx 1.6\text{ kg/m}^3, \quad \therefore p_o - p_e \approx (13550-1.6)(9.81)(0.30) \approx \mathbf{39900\ Pa}$$

Solve the above two simultaneously for $p_e \approx \mathbf{101\ kPa}$ and $p_{tank} \approx \mathbf{140.8\ kPa}$ *Ans.* (a, b)

9.33 Air flows isentropically from a reservoir, where p = 300 kPa and T = 500 K, to section 1 in a duct, where $A_1 = 0.2\ m^2$ and $V_1 = 550$ m/s. Compute (a) Ma_1; (b) T_1; (c) p_1; (d) $\dot{m}$; and (e) A*. Is the flow choked?

Solution: Use the energy equation to calculate T_1 and then get the Mach number:

$$T_1 = T_o - \frac{V_1^2}{2c_p} = 500 - \frac{(550)^2}{2(1005)} = \mathbf{350\ K} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Then}\quad a_1 = \sqrt{1.4(287)(350)} = 375\text{ m/s},\quad Ma_1 = V_1/a_1 = \frac{550}{375} \approx \mathbf{1.47} \quad \textit{Ans.}\text{ (a)}$$

The flow **<u>must be choked</u>** in order to produce supersonic flow in the duct. *Answer.*

$$p_1 = p_o\Big/\left(1 + 0.2\ Ma_1^2\right)^{3.5} = 300/[1 + 0.2(1.47)^2]^{3.5} \approx \mathbf{86\ kPa} \quad \textit{Ans.}\text{ (c)}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{86000}{287(350)} \approx 0.854\ \frac{kg}{m^3},\quad \therefore\ \dot{m} = \rho AV = (0.854)(0.2)(550) \approx \mathbf{94}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\text{ (d)}$$

$$\text{Finally,}\quad \frac{A}{A^*} = \frac{1}{Ma}\frac{(1 + 0.2Ma^2)^3}{1.728} = 1.155 \quad \text{if}\ \ Ma = 1.47,$$

$$\therefore\ A^* = \frac{0.2}{1.155} \approx \mathbf{0.173\ m^2} \quad \textit{Ans.}\text{ (e)}$$

9.34 Steam in a tank at 450°F and 100 psia exhausts through a converging nozzle of throat area 0.1-in^2 to a 1-atm environment. Compute the initial mass flow rate (a) for an ideal gas; and (b) from the Steam Tables.

Solution: For steam, from Table A.4, let R = 461 J/kg·K and k = 1.33. Then the critical pressure ratio is

$$\frac{p_o}{p^*} = \left[1 + \frac{0.33}{2}\right]^{1.33/0.33} = 1.85,\quad \text{hence}\ \ p_{exit} = p^* = \frac{100}{1.85} = 54.04\text{ psia}$$

The nozzle is **<u>choked</u>** and exits at a pressure higher than 1 atm. Use Eq. 9.46 for k = 1.33:

$$\dot{m}_{max} = k^{1/2}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}\frac{p_oA^*}{\sqrt{RT_o}} = 0.6726\frac{(100\times 144)(0.1/144)}{\sqrt{2759(450+460)}} \approx \mathbf{0.00424}\ \frac{\mathbf{slug}}{\mathbf{s}}$$

Ans. (a) *Ideal*

(b) For non-ideal (Steam Table) calculations, we first establish the stagnation entropy: at $T_o = 450°F$ and $p_o = 100$ psia, read $s_o = 1.6814$ Btu/lbm°F and $h_o = 1253.7$ Btu/lbm. We need to guess the exit pressure **p* ≈ 54 psia** from part (a), otherwise we will be here all month iterating in the Steam Tables, especially for the speed of sound. Then

$$\text{at 54 psia and } s = 1.6814\ \frac{\text{Btu}}{\text{lbm·°F}},\ \text{read } T \approx 328°\text{F},\ h \approx 1197.8\ \frac{\text{Btu}}{\text{lbm}},\ \text{and } v = 8.443\ \frac{\text{ft}^3}{\text{lbm}}$$

$$\text{Energy equation:}\quad h_o = h + V^2/2,\quad \text{or}\quad V = \sqrt{2(1253.7 - 1197.8)(778)(32.2)} \approx \mathbf{1673\ ft/s}$$

$$\text{We also need}\quad \rho = 1/v = \frac{1}{8.443(32.2)} \approx \mathbf{0.00368\ slug/ft^3}$$

To determine if the exit flow is *sonic*, evaluate the "isentropic exponent" in the Tables:

$$\text{at 328°F and 54 psia, read } \gamma_{\text{isen}} \approx 1.31,\ \text{then } a = \sqrt{\gamma p/\rho} = \sqrt{1.31(54 \times 144)/(0.00368)}$$

or $a \approx$ **1664 ft/s**. This is close enough to V = 1673 ft/s, don't iterate any more!

$$\text{Then } \dot{m}_{\text{tables}} = \rho AV = (0.00368)(0.1/144)(1670) \approx \mathbf{0.00427\ \frac{slug}{s}}\quad \textit{Ans.}\ \text{(b)}\ \textit{non-ideal}$$

Even though we expand to *very near the saturation line*, the ideal-gas theory predicts the mass flow and exit pressure to within 1% and the exit temperature to within 2%.

9.35 Helium, at $T_o = 400$ K, enters a nozzle isentropically. At section 1, where $A_1 = 0.1\ m^2$, a pitot-static arrangement (see Fig. P9.25) measures stagnation pressure of 150 kPa and static pressure of 123 kPa. Estimate (a) Ma_1; (b) mass flow; (c) T_1; and (d) A*.

Solution: For helium, from Table A.4, take $k = 1.66$ and $R = 2077$ J/kg·K. (a) The local pressure ratio is given, hence we can estimate the Mach number:

$$\frac{p_o}{p_1} = \frac{150}{123} = \left[1 + \frac{1.66-1}{2}Ma_1^2\right]^{1.66/(1.66-1)},\quad \text{solve for}\ \mathbf{Ma_1 \approx 0.50}\quad \textit{Ans.}\ \text{(a)}$$

Use this Mach number to estimate local temperature, density, velocity, and mass flow:

$$T_1 = \frac{T_o}{1 + (k-1)Ma_1^2/2} = \frac{400}{1 + 0.33(0.50)^2} \approx \mathbf{370\ K}\quad \textit{Ans.}\ \text{(c)}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{123000}{2077(370)} \approx 0.160\ \frac{\text{kg}}{\text{m}^3}$$

$$V_1 = Ma_1 a_1 = 0.50[1.66(2077)(370)]^{1/2} \approx \mathbf{565}\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,}\quad \dot{m} = \rho_1 A_1 V_1 = (0.160)(0.1)(565) \approx \mathbf{9.03\ \frac{kg}{s}} \quad \textit{Ans.}\text{(b)}$$

Finally, A* can be computed from Eq. (9.44), using $k = 1.66$:

$$\frac{A_1}{A^*} = \frac{1}{Ma_1}\left[\frac{1+0.33\,Ma_1^2}{(1.66+1)/2}\right]^{(1/2)(2.66)/(0.66)} \approx 1.323,\quad \mathbf{A^* \approx 0.0756\ m^2} \quad \textit{Ans.}\text{(d)}$$

9.36 An air tank of volume 1.5 m^3 is at 800 kPa and 20°C when it begins exhausting through a converging nozzle to sea-level conditions. The throat area is 0.75 cm^2. Estimate (a) the initial mass flow; (b) the time to blow down to 500 kPa; and (c) the time when the nozzle ceases being choked.

Solution: For sea level, p_{ambient} = 101.35 kPa < $0.528p_{\text{tank}}$, hence the flow is choked until the tank pressure drops to $p_{\text{ambient}}/0.528$ = 192 kPa. (a) We obtain

$$\dot{m}_{initial} = \dot{m}_{\max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{800000(0.75E\text{–}4\ \text{m}^2)}{\sqrt{287(293)}} = \mathbf{0.142}\ \frac{\text{kg}}{\text{s}} \quad \textit{Ans.}\text{(a)}$$

(b) For a control volume surrounding the tank, a mass balance gives

$$\frac{d}{dt}(\rho_o \upsilon) = \frac{\upsilon}{RT_o}\frac{dp_o}{dt} = -\dot{m} = -0.6847\frac{p_o A^*}{\sqrt{RT_o}},\quad \text{separate the variables:}$$

$$\frac{p(t)}{p(0)} = \exp\left[-0.6847\frac{A^*\sqrt{RT_o}}{\upsilon}t\right] = \mathbf{e}^{-0.00993\text{t}} \quad \text{until p(t) drops to 192 kPa}$$

At 500 kPa, we obtain 500/800 = exp(–0.00993t), or **t ≈ 47 s** *Ans.* (b)

At choking (192 kPa), 192/800 = exp(–0.00993t), or **t ≈ 144 s** *Ans.* (c)

9.37 Make an exact control volume analysis of the blowdown process in Fig. P9.37, assuming an insulated tank with negligible kinetic and potential energy. Assume critical flow at the exit and show that both p_o and T_o *decrease* during blowdown. Set up first-order differential equations for $p_o(t)$ and $T_o(t)$ and reduce and solve as far as you can.

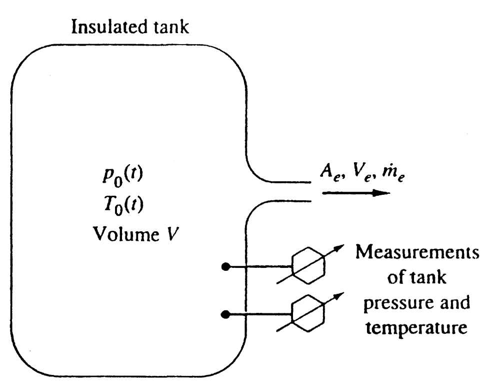


Fig. P9.37

Solution: For a CV around the tank, write the mass and the energy equations:

$$\text{mass:}\quad \frac{d}{dt}(\rho_o \upsilon) = -\dot{m}, \quad \text{or} \quad \frac{d}{dt}\left(\frac{p_o}{RT_o}\upsilon\right) = -B\frac{p_o}{\sqrt{T_o}}, \quad \text{where } B = \frac{0.6847A^*}{\sqrt{R}}$$

$$\text{energy:}\quad \frac{dQ}{dt} + \frac{dW}{dt} = 0 = \frac{d}{dt}\left(\frac{p_o}{RT_o}\upsilon c_v T_o\right) + \dot{m}c_p T_o$$

We may rearrange and combine these to give a single differential equation for T_o:

$$\frac{dT_o}{dt} = -CT_o^{3/2}, \quad \text{where } C = \frac{0.6847}{\upsilon}(k-1)A^*\sqrt{R}, \quad \text{or} \quad \int\frac{dT_o}{T_o^{3/2}} = -C\int dt$$

$$\text{Integrate:}\quad \mathbf{T_o(t)} = \left[\frac{\mathbf{1}}{\sqrt{\mathbf{T_o(0)}}} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{Ct}\right]^{\mathbf{-2}} \quad \textit{Ans.}$$

With $T_o(t)$ known, we could go back and solve the mass relation for $p_o(t)$, but in fact that is not necessary. We simply use the isentropic-flow assumption:

$$\frac{p_o(t)}{p_o(0)} = \left[\frac{T_o(t)}{T_o(0)}\right]^{k/(k-1)} = \left[\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{CT_o^{1/2}(0)t}\right]^{\mathbf{-2k/(k-1)}} \quad \left(C = \frac{0.6847(k-1)A^*\sqrt{R}}{\upsilon}\right) \quad \textit{Ans.}$$

Clearly, tank pressure also decreases with time as the tank blows down.

9.38 Prob. 9.37 makes an ideal senior project or combined laboratory and computer problem, as described in Ref. 30, sec. 8.6. In Bober and Kenyon's lab experiment, the tank had a volume of 0.0352 ft^3 and was initially filled with air at 50 lb/in^2 gage and 72°F. Atmospheric pressure was 14.5 lb/in^2 absolute, and the nozzle exit diameter was 0.05 in. After 2 s of blowdown, the measured tank pressure was 20 lb/in^2 gage and the tank temperature was –5°F. Compare these values with the theoretical analysis of Prob. 9.37.

Solution: Use the formulas derived in Prob. 9.37 above, with the given data:

$$T_o(0) = 72 + 460 = 532°R,$$

$$\text{“C”} = \frac{0.6847(\pi/4)(0.05/12)^2(1.4-1)\sqrt{1717}}{0.0352\ ft^3} \approx 0.0044\ \frac{1}{s\cdot°R^{1/2}}$$

$$\text{Then}\quad T_o(t) \approx T_o(0)\left[1 + \frac{1}{2}(0.0044)\sqrt{532}t\right]^{-2} = 532/[1 + 0.0507t]^2$$

$$\text{Similarly,}\quad p_o = p_o(0)[T_o/T_o(0)]^{k/(k-1)} = (50 + 14.5\ \text{psia})/[1 + 0.0507t]^7$$

Some numerical predictions from these two formulas are as follows:

t, sec:	0	0.5	1.0	1.5	2.0
T_o, °R:	532.0	506.0	481.9	459.5	**438.6°R**
p_o, psia:	64.5	54.1	45.6	38.6	**32.8 psia**

At t = 2 sec, the tank temperature is 438.6°R = **–21.4°F**, compared to –5°F measured. At t = 2 sec, the tank pressure is 32.8 psia = **18.3 psig,** compared to 20 psig measured. The discrepancy is probably due to heat transfer through the tank walls warming the air.

9.39 Consider isentropic flow in a channel of varying area, between sections 1 and 2. Given $Ma_1 = 2.0$, we desire that V_2/V_1 equal 1.2. Estimate (a) Ma_2 and (b) A_2/A_1. (c) Sketch what this channel looks like, for example, does it converge or diverge? Is there a throat?

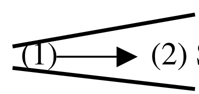
Solution: This is a problem in iteration, ideally suited for EES. Algebraically,

$$\frac{V_2}{V_1} = \frac{Ma_2 a_2}{Ma_1 a_1} = \frac{Ma_2}{Ma_1}\frac{\cancel{a_o}\left[1+0.2Ma_2^2\right]^{-1/2}}{\cancel{a_o}\left[1+0.2Ma_1^2\right]^{-1/2}} = 1.2, \quad \text{given that } Ma_1 = 2.0$$

For adiabatic flow, a_o is constant and cancels. Introducing $Ma_1 = 2.0$, we have to solve $Ma_2/[1+0.2Ma_2^2]^{1/2} \approx 1.789$. By iteration, the solution is: $\mathbf{Ma_2 = 2.98}$ *Ans.* (a)

$$\text{Then} \quad \frac{\boldsymbol{A_2}}{\boldsymbol{A_1}} = \frac{A_2/A^*}{A_1/A^*} = \frac{4.1547}{1.6875}(\text{Table B.1}) \approx \mathbf{2.46} \quad \textit{Ans.}\text{ (b)}$$

There is no throat, it is a **supersonic expansion**. *Ans.* (c)



9.40 Air, with stagnation conditions of 800 kPa and 100°C, expands isentropically to a section of a duct where A_1 = 20 cm^2 and p_1 = 47 kPa. Compute (a) Ma_1; (b) the throat area; and (c) $\dot{m}$. At section **2**, between the throat and section 1, the area is 9 cm^2. (d) Estimate the Mach number at section 2.

Solution: Use the downstream pressure to compute the Mach number:

$$\frac{p_o}{p_1} = \frac{800}{47} = \left(1+0.2Ma_1^2\right)^{3.5}, \quad \text{solve } \mathbf{Ma_1 \approx 2.50} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Flow is } \textbf{choked}\text{:} \quad \frac{A_1}{A^*} = \frac{20}{A^*} = \frac{1}{Ma_1}\frac{\left(1+0.2Ma_1^2\right)^3}{1.728} = 2.63,$$

$$\therefore \quad A^* = \frac{20}{2.63} \approx \mathbf{7.6\ cm^2} \quad \textit{Ans.}\text{ (b)}$$

$$\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{800000(7.6E\text{–}4)}{\sqrt{287(373)}} \approx \mathbf{1.27\ kg/s} \quad \textit{Ans.}\ (c)$$

$$\text{Finally, at } A_2 = 9\,cm^2, \quad \frac{A_2}{A^*} = \frac{9.0}{7.6}\Big|_{supersonic} = \frac{1}{Ma_2}\frac{\left(1+0.2\,Ma_2^2\right)^3}{1.728},$$

$$\text{solve} \quad \mathbf{Ma_2 \approx 1.50} \quad \textit{Ans.}\ (d)$$

9.41 Air, with a stagnation pressure of 100 kPa, flows through the nozzle in Fig. P9.41, which is 2 m long and has an area variation approximated by

$$A \approx 20 - 20x + 10x^2$$

with A in cm^2 and x in m. It is desired to plot the complete family of isentropic pressures $p(x)$ in this nozzle, for the range of inlet pressures $1 < p(0) < 100$ kPa. Indicate those inlet pressures which are not physically possible and discuss briefly. If your computer has an online graphics routine, plot at least 15 pressure profiles; otherwise just hit the highlights and explain.

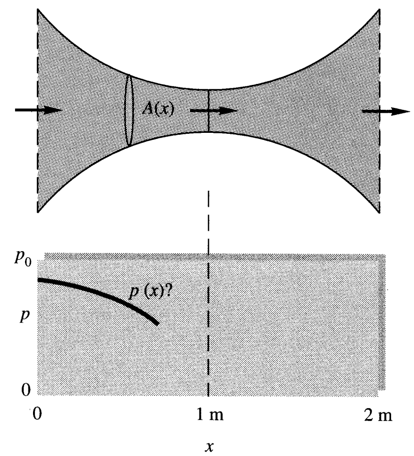


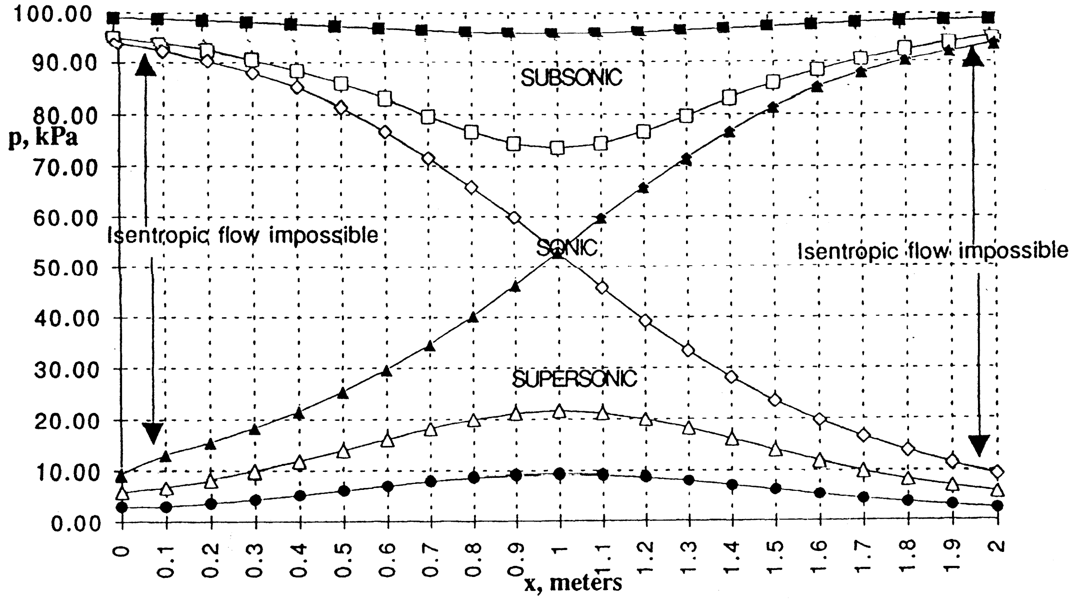
Fig. P9.41

Solution: There is a subsonic entrance region of high pressure and a supersonic entrance region of low pressure, both of which are bounded by a sonic (critical) throat, and both of which have a ratio $A_{x=0}/A^* = 2.0$. From Table B.1 or Eq. (9.44), we find these two conditions to be bounded by

a) subsonic entrance: $A/A^* = 2.0$, $Ma_e \approx 0.306$, $p_e \approx 0.9371p_o \approx$ **93.71 kPa**

b) supersonic entrance: $A/A^* = 2.0$, $Ma_e \approx 2.197$, $p_e \approx 0.09396p_o \approx$ **9.396 kPa**

Thus *no isentropic flow can exist* between entrance pressures $9.396 < p_e < 93.71$ kPa. The complete family of isentropic pressure curves is shown in the graph on the following page. They are **not** easy to find, because we have to convert implicitly from area ratio to Mach number.



9.42 A bicycle tire is filled with air at 169.12 kPa (abs) and 30°C. The valve breaks, and air exhausts into the atmosphere of 100 kPa (abs) and 20°C. The valve exit is 2-mm-diameter and is the smallest area in the system. Assuming one-dimensional isentropic flow, (a) find the initial Mach number, velocity, and temperature at the exit plane. (b) Find the initial mass flow rate. (c) Estimate the exit velocity using the *incompressible Bernoulli equation*. How well does this estimate agree with part (a)?

Solution: (a) Flow is *not* choked, because the pressure ratio is less than 1.89:

$$\frac{p_o}{p} = \frac{169.12}{100} = \left(1+0.2Ma_e^2\right)^{3.5}, \quad \text{solve } \mathbf{Ma_e = 0.90}; \quad \text{Read } T_e = 0.8606T_o = \mathbf{261\ K}$$

$$V_e = Ma_e a_e = (0.90)\sqrt{1.4(287)(261)} = 0.90(324) = \mathbf{291\ \frac{m}{s}} \quad \textit{Ans.} \text{ (a)}$$

(b) Evaluate the exit density at Ma = 0.90 and thence the mass flow:

$$\rho_e = \frac{p_e}{RT_e} = \frac{100000}{287(261)} = 1.335\ \frac{\text{kg}}{\text{m}^3},$$

$$\textit{Then} \quad \dot{m} = \rho_e A_e V_e = (1.335)\frac{\pi}{4}(0.002)^2(291) = \mathbf{0.00122\ \frac{kg}{s}} \quad \textit{Ans.} \text{ (b)}$$

(c) Assume $\rho = \rho_o = \rho_{tire}$, for how would we know ρ_{exit} if we didn't use compressible-flow theory? Then the incompressible Bernoulli relation predicts

$$\rho_o = \frac{p_o}{RT_o} = \frac{169120}{287(303)} = 1.945\ \frac{kg}{m^3}$$

$$V_{e,inc} \approx \sqrt{\frac{2\Delta p}{\rho_o}} = \sqrt{\frac{2(169120-100000)}{1.945}} \approx \mathbf{267\ \frac{m}{s}} \quad \textit{Ans.}\ (c)$$

This is **8% lower** than the "exact" estimate in part (a).

9.43 Air flows isentropically through a duct with T_o = 300°C. At two sections with identical areas of 25 cm^2, the pressures are p_1 = 120 kPa and p_2 = 60 kPa. Determine (a) the mass flow; (b) the throat area, and (c) Ma_2.

Solution: If the areas are the same and the pressures *different*, than section (1) must be subsonic and section (2) supersonic. In other words, we need to find where

$$\frac{p_1/p_o}{p_2/p_o} = \frac{120}{60} = 2.0 \quad \text{for the same } A_1/A^* = A_2/A^*\text{—search Table B.1 (isentropic)}$$

$$\text{After laborious but straightforward iteration, } Ma_1 = 0.729, \quad \mathbf{Ma_2 \approx 1.32} \quad \textit{Ans.}\ (c)$$

$$A/A^* = 1.075 \text{ for both sections, } A^* = 25/1.075 = \mathbf{23.3\ cm^2} \quad \textit{Ans.}\ (b)$$

With critical area and stagnation conditions known, we may compute the mass flow:

$$p_o = 120[1+0.2(0.729)^2]^{3.5} \approx 171 \text{ kPa} \quad \text{and} \quad T_o = 300+273 = 573 \text{ K}$$

$$\dot{m} = 0.6847 p_o A^*/[RT_o]^{1/2} = 0.6847(171000)(0.00233)/[287(573)]^{1/2}$$

$$\dot{m} \approx \mathbf{0.671\ \frac{kg}{s}} \quad \textit{Ans.}\ (a)$$

9.44 In Prob. 3.34 we knew nothing about compressible flow at the time so merely assumed exit conditions p_2 and T_2 and computed V_2 as an application of the continuity equation. Suppose

that the throat diameter is 3 in. For the given stagnation conditions in the rocket chamber in Fig. P3.34 and assuming $k = 1.4$ and a molecular weight of 26, compute the actual exit velocity, pressure, and temperature according to one-dimensional theory. If $p_a = 14.7$ lbf/in^2 absolute, compute the thrust from the analysis of Prob. 3.68. This thrust is entirely independent of the stagnation temperature (check this by changing T_o to 2000°R if you like). Why?

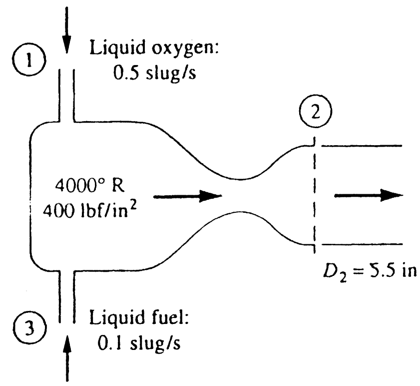


Fig. P3.34

Solution: If **M** = 26, then $R_{gas} = 49720/26 = 1912$ ft·lbf/slug°R. Assuming choked flow in the throat (to produce a supersonic exit), the exit area ratio yields the exit Mach number:

$$\frac{A_e}{A^*} = \left(\frac{D_e}{D^*}\right)^2 = \left(\frac{5.5}{3.0}\right)^2 = 3.361, \text{ whence Eq. 9.45 (for } k = 1.4\text{) predicts } Ma_e \approx \mathbf{2.757}$$

$$\text{Then isentropic } p_e = 400/[1+0.2(2.757)^2]^{3.5} \approx \mathbf{15.7\ psia} \quad \textit{Ans.}$$

$$T_e = 4000°R/[1+0.2(2.757)^2] \approx \mathbf{1587°R} \quad \textit{Ans.}$$

$$\text{Then} \quad V_e = Ma_e\sqrt{kRT_e} = 2.757\sqrt{1.4(1912)(1587)} \approx \mathbf{5680\ ft/s} \quad \textit{Ans.}$$

$$\text{We also need} \quad \rho_e = p_e/RT_e = (15.7\times 144)/[1912(1587)] \approx 0.000747\ \text{slug/ft}^3$$

$$\text{From Prob. 3.68, Thrust } F = A_e\left[\rho_e V_e^2 + (p_e - p_a)\right],$$

$$\text{or:} \quad F = \frac{\pi}{4}\left(\frac{5.5}{12}\right)^2[0.000747(5680)^2 + (15.7-14.7)\times 144] \approx \mathbf{4000\ lbf} \quad \textit{Ans.}$$

Thrust is independent of T_o because $\rho_e \propto 1/T_o$ and $V_e \propto \sqrt{T_o}$, so T_o cancels out.

9.45 At a point upstream of the throat of a converging-diverging nozzle, the properties are $V_1 = 200$ m/s, $T_1 = 300$ K, and $p_1 = 125$ kPa. If the exit flow is supersonic, compute, from isentropic theory, (a) $\dot{m}$; and (b) A_1. The throat area is 35 cm^2.

Solution: We begin by computing the Mach number at section 1 for air:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(300)} = 347\ \text{m/s}, \quad \therefore\ Ma_1 = 200/347 \approx \mathbf{0.576}$$

Given that the exit flow is supersonic, we know that A* is the *throat*. Then we find

$$\text{At } M_1 = 0.576, \quad A_1/A^* \approx 1.218, \quad \text{thus } A_1 = 1.218(35) \approx \mathbf{42.6\ cm^2} \quad \textit{Ans. (b)}$$

$$\rho_1 = p_1/RT_1 = 125000/[287(300)] \approx 1.45\ \text{kg/m}^3$$

$$\text{Finally,} \quad \dot{m} = \rho_1 A_1 V_1 = (1.45)(42.6\text{E–}4)(200) \approx \mathbf{1.24\ kg/s} \quad \textit{Ans. (a)}$$

9.46 If the writer did not falter, the results of Prob. 9.43 are (a) 0.671 kg/s, (b) 23.3 cm^2, and (c) 1.32. Do not tell your friends who are still working on Prob. 9.43. Consider a control volume which encloses the nozzle between these two 25-cm^2 sections. If the pressure outside the duct is 1 atm, determine the total force acting on this section of nozzle.

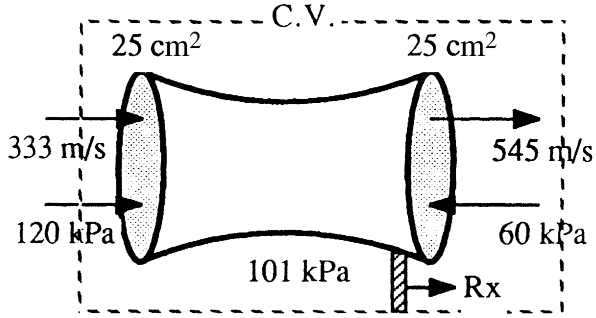


Fig. P9.46

Solution: The control volume encloses this portion of duct as in the figure above. To complete the analysis, we need the velocities at sections 1 and 2:

$$Ma_1 = 0.729,\ T_o = 573\text{ K},\ T_1 = T_o\big/\left(1+.2Ma_1^2\right) = 518\text{ K},\ a_1 = 456\ \frac{m}{s},$$

$$V_1 = Ma_1 a_1 = 0.729(456) = \mathbf{333}\text{ m/s}; \quad \text{similarly, } Ma_2 = 1.32 \text{ leads to } V_2 = \mathbf{545}\text{ m/s}$$

Then the control-volume x-momentum relation yields, for steady flow,

$$\sum F_x = R_x + (p_1 - p_2)_{gage} A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad R_x = (0.671)(545-333) + (-120000+60000)(0.0025) = 142 - 150 = \mathbf{-8\ N} \quad \textit{Ans.}$$

Things are pretty well balanced, and there is a small 8-N support force R_x to the *left*.

9.47 In wind-tunnel testing near Mach 1, a small area decrease caused by model blockage can be important. Let the test section area be 1 sq.m. and unblocked conditions are Ma = 1.1 and T = 20°C. What model area will first cause the test section to choke? If the model cross-section is 0.004 sq.m., what % change in test-section velocity results?

Solution: First evaluate the unblocked test conditions:

$$T = 293°K,\ a = \sqrt{kRT} = \sqrt{1.4(287)(293)} = 343\ \frac{m}{s},\quad \therefore\ V = (1.1)(343) = \mathbf{377\ \frac{m}{s}}$$

$$\text{Also,}\quad \frac{A}{A^*} = \frac{[1+0.2(1.1)^2]^3}{1.728(1.1)} = \underset{\text{(unblocked)}}{1.007925},\quad \text{or}\quad A^* = 1.0/1.007925 \approx \mathbf{0.99214\ m^2}$$

If A is blocked by 0.004 m^2, then $A_{new} = 1.0 - 0.004 = 0.996\ m^2$, and now

$$\frac{A_{new}}{A^*} = \frac{0.996}{0.99214} = 1.00389,\quad \text{solve Eq. (9.45) for Ma(blocked)} \approx \mathbf{1.0696}$$

$$\text{Same } T_o = 364\text{ K, new } T = 296\text{ K, new } a = 345\text{ m/s, new } \mathbf{V} = \text{Ma}(a) \approx \mathbf{369\ \frac{m}{s}}\quad \textit{Ans.}$$

Thus a 0.4% decrease in test section area has caused a 2.1% decrease in test velocity.

9.48 A force F = 1100 N pushes a piston of diameter 12 cm through an insulated cylinder containing air at 20°C, as in Fig. P9.48. The exit diameter is 3 mm, and p_a = 1 atm. Estimate (a) V_e, (b) V_p, and (c) $\dot{m}_e$.

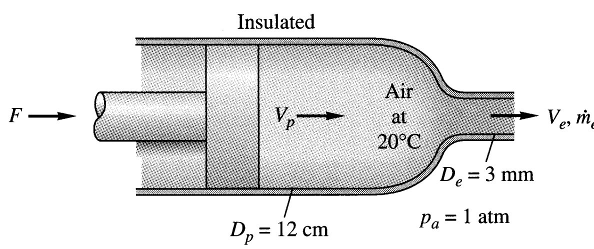


Fig. P9.48

Solution: First find the pressure inside the large cylinder:

$$p_p = \frac{F}{A} + 1\text{ atm} = \frac{1100}{(\pi/4)(0.12)^2} + 101350 \approx 198600 = 1.96\text{ atm}$$

Since this is greater than (1/0.5283) atm, the small cylinder is ***choked***, and thus

$$V_{exit} = \sqrt{\frac{2k}{k+1}RT_o} = \sqrt{\frac{2(1.4)}{1.4+1}(287)(293)} \approx \mathbf{313\ m/s}\quad \textit{Ans. (a)}$$

$$V_{piston} = (\rho_e/\rho_p)(A_e/A_p)V_e = (0.6339)(0.003/0.12)^2(313) = \mathbf{0.124\ m/s}\quad \textit{Ans. (b)}$$

$$\text{Finally,}\quad \dot{m} = \dot{m}_{max} = 0.6847\frac{(198600)(\pi/4)(0.003)^2}{\sqrt{287(293)}} \approx \mathbf{0.00331\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (c)}$$

The mass flow increases with F, but the piston velocity and exit velocity are independent of F if the exit flow is choked.

9.49 Consider the venturi nozzle of Fig. 6.40*c*, with D = 5 cm and d = 3 cm. Air stagnation temperature is 300 K, and the upstream velocity V_1 = 72 m/s. If the throat pressure is 124 kPa, estimate, with isentropic flow theory, (a) p_1; (b) Ma_2; and (c) the mass flow.

Solution: Given one-dimensional isentropic flow of air. The problem looks sticky—sparse, scattered information, implying laborious iteration. But the energy equation yields V_1 and Ma_1:

$$T_o = T_1 + \frac{V_1^2}{2c_p} = 300\,\text{K} = T_1 + \frac{(72\text{ m/s})^2}{2(1005\,\text{J/kg·K})},\quad \text{solve for } T_1 = 297.4\,\text{K}$$

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{72}{\sqrt{1.4(287)(297.4)}} = \frac{72\,\text{m/s}}{346\,\text{m/s}} = 0.208$$

Area-ratio calculations will then yield A^* and Ma_2 and then p_o and p_1:

$$\frac{A_1}{A^*} = \frac{(\pi/4)(0.05\,\text{m})^2}{A^*} = \frac{\left(1+0.2\,Ma_1^2\right)^3}{1.728\,Ma_1} = \frac{[1+0.2(0.208)^2]^3}{1.728(0.208)} = 2.85,$$

$$\text{Solve}\quad A^* = 0.0006886\text{ m}^2$$

$$\frac{A_2}{A^*} = \frac{(\pi/4)(0.03\text{ m})^2}{0.0006886\text{ m}^2} = \frac{\left(1+0.2Ma_2^2\right)^3}{1.728Ma_2} = 1.027,\quad \text{Solve}\ \boldsymbol{Ma_2 = 0.831}\quad \textit{Ans.}\text{ (b)}$$

$$p_o = p_2\left(1+0.2\,Ma_2^2\right)^{3.5} = (124\,\text{kPa})[1+0.2(0.831)^2]^{3.5} = 195\,\text{kPa}$$

$$p_1 = p_o\Big/\left(1+0.2\,Ma_1^2\right)^{3.5} = (195\,\text{kPa})/[1+0.2(0.208)^2]^{3.5} = \mathbf{189\ kPa}\quad \textit{Ans.}\text{ (a)}$$

The mass flow follows from any of several formulas. For example:

$$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_1}{RT_1}\right)A_1V_1 = \left[\frac{189000}{287(297.4)}\right]\left(\frac{\pi}{4}\right)(0.05)^2(72) = \mathbf{0.313\ \frac{kg}{s}}\quad \textit{Ans.}\text{ (c)}$$

9.50 Argon expands isentropically at 1 kg/s in a converging nozzle with D_1 = 10 cm, p_1 = 150 kPa, and T_1 = 100°C. The flow discharges to a pressure of 101 kPa. (a) What is the nozzle exit diameter? (b) How much further can the ambient pressure be reduced before it affects the inlet mass flow?

Solution: For argon, from Table A.4, R = 208 J/kg·K and k = 1.67.

$$\rho_1 = \frac{150000}{208(373)} = 1.93\ \frac{\text{kg}}{\text{m}^3},\quad \dot{m} = 1\ \frac{\text{kg}}{\text{s}} = 1.93\frac{\pi}{4}(0.1)^2 V_1,\quad \therefore\quad V_1 = 66\ \frac{\text{m}}{\text{s}}$$

$$Ma_1 = \frac{66}{\sqrt{1.67(208)(373)}} = 0.183,\quad \frac{A_1}{A^*} = \frac{1}{0.183}\left[\frac{1+0.335(0.183)^2}{(1+1.67)/2}\right]^{\frac{1.67+1}{2(1.67-1)}} = 3.14$$

Thus $A^* = A_1/3.14 = 0.00250\ \text{m}^2 = (\pi/4)D_e^2$, solve $\mathbf{D_{exit} = 0.0564\ m}$ *Ans.* (a)

$$p_o = 150[1+0.335(0.183)^2]^{\frac{1.67}{0.67}} = 154\ \text{kPa},$$

$$\frac{p_e}{p_o} = \frac{101}{154} = \left(1+0.335\,Ma_e^2\right)^{\frac{-1.67}{0.67}},\quad \mathbf{Ma_e = 0.743}$$

Thus the exit flow is *not* choked. We could decrease the ambient pressure to **75 kPa** before the flow would choke. The maximum mass flow is about 1.01 kg/s.

9.51 Air, at stagnation conditions of 500 K and 200 kPa, flow through a nozzle. At section 1, where A = 12 cm^2, the density is 0.32 kg/m^3. Assuming isentropic flow, (a) find the mass flow. (b) Is the flow choked? Is so, estimate A*. Also estimate (c) p_1; and (d) Ma_1.

Solution: Evaluate stagnation density, density ratio, and Mach number:

$$\rho_o = \frac{p_o}{RT_o} = \frac{200000}{287(500)} = 1.39\ \frac{\text{kg}}{\text{m}^3};$$

$$\frac{\rho_o}{\rho} = \frac{1.39}{0.32} = \left(1+0.2\,Ma_1^2\right)^{2.5},\quad \text{solve}\ \mathbf{Ma_1 = 2.00}\quad \textit{Ans.}\ \text{(d)}$$

$$T_1 = 500/[1+0.2(2.00)^2] = 278\ \text{K},\quad V_1 = Ma_1 a_1 = 2.00[1.4(287)(278)]^{1/2} = 668\ \frac{\text{m}}{\text{s}}$$

$$\text{Finally,}\quad \dot{m} = \rho_1 A_1 V_1 = 0.32(12E\text{–}4)(668) = \mathbf{0.257\ \frac{kg}{s}}\quad \textit{Ans.}\ \text{(a)}$$

The **flow is clearly choked**, because Ma_1 is supersonic. A throat exists:

$$\dot{m} = 0.257 = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{200000A^*}{\sqrt{287(500)}},$$

$$\text{solve} \quad \mathbf{A^* = 0.000710\ m^2} \quad \textit{Ans.}\text{ (b)}$$

(c) Also calculate

$$p_1 = \frac{p_o}{\left(1+0.2\,Ma_1^2\right)^{3.5}} = \frac{200000}{[1+0.2(2.00)^2]^{3.5}} = 25500\,\text{Pa} \quad \textit{Ans.}\text{ (c)}$$

9.52 A converging-diverging nozzle exits smoothly to sea-level standard atmosphere. It is supplied by a 40-m^3 tank initially at 800 kPa and 100°C. Assuming isentropic flow, estimate(a) the throat area; and (b) the tank pressure after 10 sec of operation. NOTE: The exit area is **10 cm^2** (this was omitted in the first printing).

Solution: The phrase "exits smoothly" means that exit pressure = atmospheric pressure, which is 101 kPa. Then the pressure ratio specifies the exit Mach number:

$$p_o/p_{exit} = \frac{800}{101} = \left[1+0.2\,Ma_e^2\right]^{3.5}, \quad \text{solve for } \mathbf{Ma_{exit} \approx 2.01}$$

$$\text{Thus} \quad A_e/A^* = 1.695 \quad \text{and} \quad A^* = (10\text{ cm}^2)/1.695 \approx \mathbf{5.9\ cm^2} \quad \textit{Ans.}\text{(a)}$$

$$\text{Further,} \quad \dot{m} = \dot{m}_{max} = 0.6847(800000)(0.00059)\big/\sqrt{287(373)} \approx 0.99\text{ kg/s}$$

The initial mass in the tank is quite large because of large volume and high pressure:

$$\rho_o = \frac{p_o}{RT_o} = \frac{800000}{287(373)} \approx 7.47\ \frac{kg}{m^3}, \quad \text{thus } m_{tank,t=0} = \rho\upsilon = (7.47)(40) \approx \mathbf{299\ kg}$$

After 10 sec, blowing down at 0.99 kg/s, we have about 299 − 10 ≈ 289 kg left in the tank. The pressure will drop to about 800(289/299) ≈ **773 kPa**. *Ans.* (b).

9.53 Air flows steadily from a reservoir at 20°C through a nozzle of exit area 20 cm^2 and strikes a vertical plate as in Fig. P9.53. The flow is subsonic throughout. A force of 135 N is required to hold the plate stationary. Compute (a) V_e, (b) Ma_e, and (c) p_0 if p_a = 101 kPa.

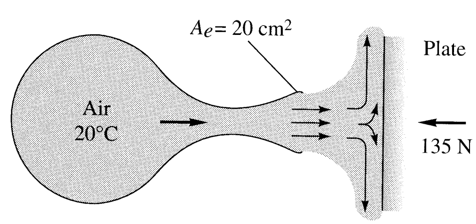


Fig. P9.53

Solution: Assume p_e = 1 atm. For a control volume surrounding the plate, we deduce that

$$F = 135\,\text{N} = \rho_e V_e^2 A_e = k p_e Ma_e^2 A_e = 1.4(101350)(0.002\ \text{m}^2)Ma_e^2,$$

$$\text{or}\quad \mathbf{Ma_e \approx 0.69}\quad \textit{Ans. (b)}$$

$$T_e = 293/[1+0.2(0.69)^2] \approx 268\ \text{K},\quad a_e = \sqrt{1.4(287)(268)} = 328\ \frac{\text{m}}{\text{s}},\ \text{thus}$$

$$V_e = a_e Ma_e = (328)(0.69) \approx \mathbf{226\ m/s}\quad \textit{Ans. (a)}$$

$$\text{Finally,}\quad p_{tank} = p_o = 101350[1+0.2(0.69)^2]^{3.5} \approx \mathbf{139000\ Pa}\quad \textit{Ans. (c)}$$

9.54 For flow of air through a normal shock, the upstream conditions are V_1 = 600 m/s, T_{o1} = 500 K, and p_{o1} = 700 kPa. Compute the downstream conditions Ma_2, V_2, T_2, p_2, and p_{o2}.

Solution: First compute the upstream Mach number:

$$T_1 = T_o - \frac{V_1^2}{2c_p} = 500 - \frac{(600)^2}{2(1005)} = 321\ \text{K},$$

$$a_1 = \sqrt{1.4(287)(321)} = 359\ \frac{\text{m}}{\text{s}},\quad Ma_1 = \frac{600}{359} = \mathbf{1.67}$$

$$\text{so}\quad Ma_2 = \sqrt{\frac{0.4(1.67)^2+2}{2.8(1.67)^2-0.4}} \approx \mathbf{0.648}\quad \textit{Ans.}\qquad \frac{V_2}{V_1} = \frac{2+0.4(1.67)^2}{2.4(1.67)^2} = 0.465$$

$$\text{or}\quad V_2 = 0.465(600) \approx \mathbf{279\ m/s}\quad \textit{Ans.}$$

Continue with the temperature and pressure ratios across the shock:

$$\frac{T_2}{T_1} = [2+0.4(1.67)^2]\left[\frac{2.8(1.67)^2-0.4}{(2.4(1.67))^2}\right] = 1.44,\ T_2 = 1.44(321) \approx \mathbf{461\ K}\quad \textit{Ans.,}$$

$$p_1 = \frac{700}{[1+0.2(1.67)^2]^{3.5}} = 148\ \text{kPa},\quad \frac{p_2}{p_1} = \frac{2.8(1.67)^2-0.4}{2.4} = 3.09,$$

$$\text{or}\quad p_2 \approx \mathbf{458\ kPa}\quad \textit{Ans.}$$

Finally, we can compute the downstream stagnation pressure in two ways:

$$p_{o2} = p_2\left(1+0.2Ma_2^2\right)^{3.5} = 458[1+0.2(0.648)^2]^{3.5} \approx \mathbf{607\ kPa}\quad \textit{Ans.}$$

Check Table B.2, at $Ma_1 = 1.67$, $p_{o2}/p_{o1} \approx 0.868$, $p_{o2} = 0.868(700) \approx 607$ kPa (*check*)

9.55 Air, supplied by a reservoir at 450 kPa, flows through a converging-diverging nozzle whose throat area is 12 cm^2. A normal shock stands where $A_1 = 20\ cm^2$. (a) Compute the pressure just downstream of this shock. Still farther downstream, where $A_3 = 30\ cm^2$, estimate (b) p_3; (c) A_3*; and (d) Ma_3.

Solution: If a shock forms, the throat must be **choked** (sonic). Use the area ratio at (1):

$$\frac{A_1}{A*} = \frac{20}{12} = 1.67, \quad \text{or} \quad Ma_1 \approx 1.985, \quad \text{whence } p_1 = \frac{450}{[1+0.2(1.985)^2]^{3.5}} \approx 59 \text{ kPa}$$

$$\text{Then, across the shock,} \quad \frac{p_2}{p_1} = \frac{2.8(1.985)^2 - 0.4}{2.4} = 4.43,$$

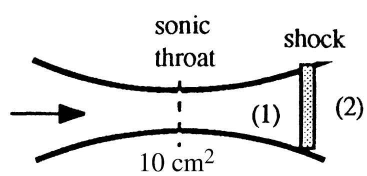
$$p_2 = 4.43(59) \approx \mathbf{261\ kPa} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Across the shock, at } Ma_1 = 1.985, \quad \frac{A_2^*}{A_1^*} = 1.374, \quad A_2^* = 1.374(12) \approx \mathbf{16.5\ cm^2} \quad \textit{Ans.}\ \text{(c)}$$

$$\text{At } A_3 = 30\ cm^2, \quad \frac{A_3}{A_2^*} = \frac{30}{16.5} = 1.82, \quad \text{whence subsonic } Ma_3 \approx \mathbf{0.34} \quad \textit{Ans.}\ \text{(d)}$$

$$\text{Finally,} \quad p_{o2} = \frac{p_{o1}}{1.374} = 328 \text{ kPa}, \quad p_3 = \frac{328}{[1+0.2(0.34)^2]^{3.5}} \approx \mathbf{303\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

9.56 Air from a reservoir at 20°C and 500 kPa flows through a duct and forms a normal shock downstream of a throat of area 10 cm^2. By an odd coincidence it is found that the stagnation pressure downstream of this shock exactly equals the throat pressure. What is the area where the shock wave stands?



Solution: If a shock forms, the throat is **sonic**, $A* = 10\ cm^2$. Now

$$p_1^* = 0.5283 p_{o1} = 0.5283(500) \approx \mathbf{264\ kPa} = p_{o2} \quad \text{also}$$

$$\text{Then} \quad \frac{p_{o2}}{p_{o1}} = \frac{264}{500} = 0.5283: \quad \text{Table B.2, read } Ma_1 \approx 2.43$$

$$\text{So} \quad A_1/A_1^* = \frac{[1+0.2(2.43)^2]^{3.0}}{1.728(2.43)} \approx 2.47, \quad \text{or} \quad A_1(\text{at shock}) = 2.47(10) \approx \mathbf{24.7\ cm^2} \quad \textit{Ans.}$$

9.57 Air flows from a tank through a nozzle into the standard atmosphere, as in Fig. P9.57. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the tank pressure; and (b) the mass flow.

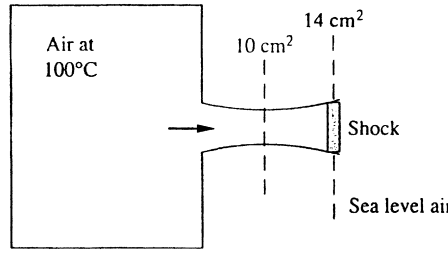


Fig. P9.57

Solution: The throat <u>must</u> be *sonic*, and the area ratio at the shock gives the Mach number:

$$A_1/A^* = \frac{14}{10} = 1.4 = \frac{\left[1+0.2Ma_1^2\right]^3}{1.728Ma_1}, \quad \text{solve } Ma_1 \approx 1.76 \text{ upstream of the shock}$$

$$\text{Then}\quad p_2/p_1|_{shock} = \frac{2.8(1.76)^2 - 0.4}{2.4} \approx 3.46, \quad p_2 = 1\text{ atm}, \quad p_1 = \frac{101350}{3.46} \approx 29289\text{ Pa}$$

$$\text{Thus}\quad p_{tank} = p_{o1} = 29289[1+0.2(1.76)^2]^{3.5} \approx \mathbf{159100\ Pa} \quad \textit{Ans.}\text{ (a)}$$

Given that T_o = 100°C = 373 K and a critical throat area of 10 cm^2, we obtain

$$\dot{m} = \dot{m}_{max} = 0.6847 p_o A^*\Big/\sqrt{RT_o} = 0.6847(159100)(0.001)\Big/\sqrt{287(373)}$$

$$\approx \mathbf{0.333\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (b)}$$

9.58 <u>Argon</u> (Table A.4) approaches a normal shock with V_1 = 700 m/s, p_1 = 125 kPa, and T_1 = 350 K. Estimate (a) V_2, and (b) p_2. (c) What pressure p_2 would result if the same velocity change V_1 to V_2 were accomplished *isentropically*?

Solution: For argon, take k = 1.67 and R = 208 J/kg·K. Determine the Mach number upstream of the shock:

$$a_1 = \sqrt{kRT_1} = \sqrt{1.67(208)(350)} \approx 349\ \frac{m}{s}; \quad Ma_1 = V_1/a_1 = \frac{700}{349} \approx \mathbf{2.01}$$

$$\text{Then}\quad \frac{p_2}{p_1}\Big|_{shock} = \frac{2(1.67)(2.01)^2 - 0.67}{1.67+1} \approx 4.79, \quad \text{or} \quad p_2 = 4.79(125) \approx \mathbf{599\ kPa} \quad \textit{Ans.}\text{ (b)}$$

$$\text{and}\quad V_2/V_1 = \frac{0.67(2.01)^2 + 2}{2.67(2.01)^2} \approx 0.437, \quad \text{or} \quad V_2 = 0.437(700) = \mathbf{306\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

For an "isentropic" calculation, assume the same **density ratio** across the shock:

$$\rho_2/\rho_1 = V_1/V_2 = \frac{1}{0.437} = 2.29; \quad \text{Isentropic: } p_2/p_1 \approx (\rho_2/\rho_1)^k,$$

$$\text{or:} \quad p_{2,\text{isentropic}} \approx 125(2.29)^{1.67} \approx \mathbf{498\ kPa} \quad \textit{Ans.}\ \text{(c)}$$

9.59 Air, at stagnation conditions of 450 K and 250 kPa, flows through a nozzle. At section 1, where the area = 15 cm^2, there is a normal shock wave. If the mass flow is 0.4 kg/s, estimate (a) the Mach number; and (b) the stagnation pressure just downstream of the shock.

Solution: If there is a shock wave, then the mass flow is maximum:

$$\dot{m}_{max} = 0.4\ \frac{\text{kg}}{\text{s}} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{250000A^*}{\sqrt{287(450)}}, \quad \text{solve } A^* = 0.000840\ \text{m}^2$$

$$\text{Then} \quad \frac{A_1}{A^*} = \frac{0.0015}{0.00084} = 1.786 \quad \text{Table B.1: Read } Ma_{1,upstream} \approx 2.067$$

$$\text{Finally, from Table B.2, read } \mathbf{Ma_{1,\,downstream}} \approx \mathbf{0.566} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{Also, Table B.2: } \frac{p_{o2}}{p_{o1}} = 0.690, \quad \mathbf{p_{0,downstream}} = 0.690(250) \approx \mathbf{172\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

9.60 When a pitot tube such as Fig. (6.30) is placed in a supersonic flow, a normal shock will stand in front of the probe. Suppose the probe reads p_0 = 190 kPa and p = 150 kPa. If the stagnation temperature is 400 K, estimate the (supersonic) Mach number and velocity upstream of the shock.

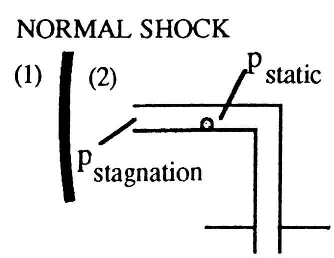


Fig. P9.60

Solution: We can immediately find Ma *inside* the shock:

$$p_{o2}/p_2 = \frac{190}{150} = 1.267 = \left(1 + 0.2\text{Ma}_2^2\right)^{3.5}, \quad \text{solve Ma}_2 \approx 0.591$$

$$\text{Then, across the shock,} \quad \text{Ma}_1^2 = \frac{0.4(0.591)^2 + 2}{2.8(0.591)^2 - 0.4}, \quad \text{solve } \mathbf{Ma_1} \approx \mathbf{1.92} \quad \textit{Ans.}$$

$$T_1 = \frac{400}{[1+0.2(1.92)^2]} = 230\text{ K},\quad a_1 = \sqrt{1.4(287)(230)} \approx 304\text{ m/s},$$

$$V_1 = Ma_1 a_1 = (1.92)(304) \approx \mathbf{585\ m/s}\quad \textit{Ans.}$$

9.61 Repeat Prob. 9.56 except this time let the odd coincidence be that the *static* pressure downstream of the shock exactly equals the throat pressure. What is the area where the shock wave stands?

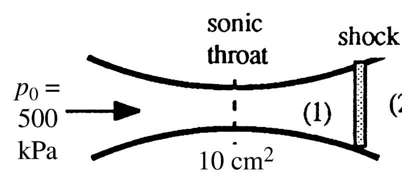


Fig. P9.61

Solution: If a shock forms, the throat is **sonic**, A* = 10 cm^2. Now

$$p_1^* = 0.5283p_{o1} = 0.5283(500) = 264\text{ kPa} = p_2 \quad \text{downstream of the shock}$$

$$\text{Given}\ \ p_1 = 500/\left(1+0.2Ma_1^2\right)^{3.5}\quad \text{and}\quad p_2/p_1 = \left(2.8Ma_1^2 - 0.4\right)\big/(2.4)\quad \text{and}\quad p_2 = 264$$

Solve iteratively for $Ma_1 \approx 2.15$ ($p_1 = 51$ kPa), $A_1/A^* = 1.92$, $\therefore$ $\mathbf{A_1 \approx 19.2\ cm^2}$ *Ans.*

9.62 An atomic explosion propagates into still air at 14.7 psia and 520°R. The pressure just inside the shock is 5000 psia. Assuming *k* = 1.4, what are the speed C of the shock and the velocity V just inside the shock?

Solution: The pressure ratio tells us the Mach number of the shock motion:

$$p_2/p_1 = \frac{5000}{14.7} = 340 = \frac{2.8Ma_1^2 - 0.4}{2.4},\quad \text{solve for}\ \ Ma_1 \approx \mathbf{17.1}$$

$$a_1 = \sqrt{1.4(1717)(520)} = 1118\text{ ft/s},\quad \therefore\ V_1 = C = 17.1(1118) \approx \mathbf{19100\ \frac{ft}{s}}\quad \textit{Ans.}\text{ (a)}$$

We then compute the velocity ratio across the shock and thence the relative motion inside:

$$V_2/V_1 = \frac{0.4(17.1)^2 + 2}{2.4(17.1)^2} = 0.1695,\quad \therefore\quad V_2 = 0.1695(19100) = 3240\text{ ft/s}$$

$$\text{Then}\quad V_{inside} = C - V_2 = 19100 - 3240 \approx \mathbf{15900\ ft/s}\quad \textit{Ans.}\text{ (b)}$$

9.63 Sea-level standard air is sucked into a vacuum tank through a nozzle, as in Fig. P9.63. A normal shock stands where the nozzle area is 2 cm^2, as shown. Estimate (a) the pressure in the tank; and (b) the mass flow.

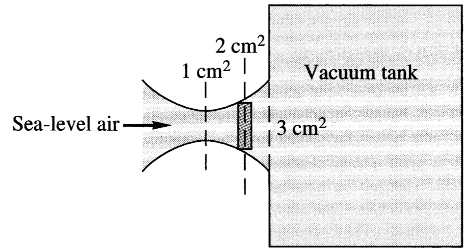


Fig. P9.63

Solution: The flow at the exit section ("3") is *subsonic* (after a shock) therefore must equal the tank pressure. Work our way to 1 and 2 at the shock and thence to 3 in the exit:

$$p_{01} = 101350 \text{ Pa}, \quad A_1/A^* = 2.0, \quad \text{thus } Ma_1 \approx 2.1972, \quad p_1 = \frac{101350}{[1+0.2(2.2)^2]^{3.5}} \approx 9520 \text{ Pa}$$

$$\frac{p_2}{p_1} = \frac{2.8(2.2)^2 - 0.4}{2.4} = 5.47, \quad \therefore\ p_2 = 5.47(9520) \approx 52030 \text{ Pa}$$

$$\text{Also compute} \quad A_2^*/A_1^* \approx 1.59, \quad \text{or} \quad A_2^* = \mathbf{1.59\ cm^2}$$

Also compute p_{o2} = 101350/1.59 = 63800 Pa. Finally compute $A_3/A_2^* = 3/1.59 = 1.89$, read Ma_3 = 0.327, whence $p_3 = 63800/[1 + 0.2(0.327)^2]^{3.5} \approx$ **59200 Pa**. *Ans.* (a).
With T_o = 288 K, the (critical) mass flow = $0.6847p_oA^*/\sqrt{(RT_o)}$ = **0.0241 kg/s**. *Ans.* (b)

9.64 Air in a large tank at 100°C and 150 kPa exhausts to the atmosphere through a converging nozzle with a 5-cm^2 throat area. Compute the exit mass flow if the atmospheric pressure is (a) 100 kPa; (b) 60 kPa; and (c) 30 kPa.

Solution: Choking occurs when $p_{atmos} < 0.5283p_{tank}$ = 79 kPa. Therefore the first case is *not* choked, the second two cases *are*. For the first case, with T_o = 100°C = 373 K,

$$\text{(a)} \ \frac{p_o}{p_e} = \frac{150}{100} = 1.5 = \left(1+0.2Ma_e^2\right)^{3.5}, \quad \text{solve } Ma_e = 0.784,\ T_e = \frac{373}{1+0.2(0.784)^2} = 332 \text{ K}$$

$$\text{and} \quad a_e = \sqrt{1.4(287)(332)} \approx 365\ \frac{m}{s}, \quad V_e = 0.784(365) = 286 \text{ m/s},$$

$$\text{and} \quad \rho_e = p_e/RT_e = 1.05 \text{ kg/m}^3, \text{ finally: } \dot{m} = 1.05(0.0005)(286) = \mathbf{0.150\ kg/s} \quad \textit{Ans. (a)}$$

Both cases (b) and (c) are *choked*, with $p_{atm} \le 79$ kPa, and the mass flow is maximum and driven by tank conditions T_o and p_o:

$$\text{(b, c)} \ \dot{m} = \dot{m}_{max} = \frac{0.6847p_oA^*}{\sqrt{(RT_o)}} = \frac{0.6847(150000)(0.0005)}{\sqrt{287(373)}} \approx \mathbf{0.157\ \frac{kg}{s}} \quad \textit{Ans. (b, c)}$$

9.65 Air flows through a converging-diverging nozzle between two large reservoirs, as in Fig. P9.65. A mercury manometer reads h = 15 cm. Estimate the downstream reservoir pressure. Is there a shock wave in the flow? If so, does it stand in the exit plane or farther upstream?

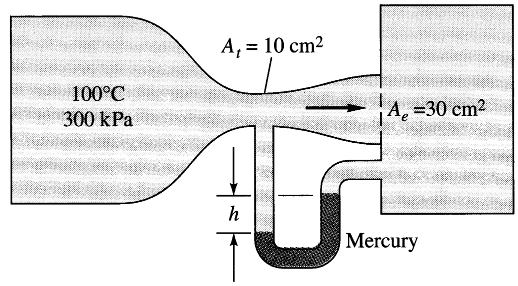


Fig. P9.65

Solution: The manometer reads the pressure drop between throat and exit tank:

$$p_{throat} - p_{tank\#2} = (\rho_{mercury} - \rho_{air})gh \approx (13550 - 0)(9.81)(0.15) \approx 19940 \text{ Pa}$$

The lowest possible $p_{throat} = p^* = 0.5283(300) = 158.5$ kPa, for which $p_e \approx 138.5$ kPa

But **this** p_e is much lower than would occur in the duct for isentropic subsonic flow.

We can check also to see if isentropic *supersonic* flow is a possibility: With $A_e/A^* = 3.0$, the exit Mach number would be 2.64, corresponding to $p_e = 0.047p_o \approx 14$ kPa (?). This is much too low, so that case fails also.

Suppose we had supersonic flow with a normal shock wave in the exit plane:

$$A_e/A^* = 3.0, \;\; Ma_e \approx 2.64, \;\; p_e = 14 \text{ kPa}, \;\; \frac{p_{tank\#2}}{p_e} = \frac{2.8(2.64)^2 - 0.4}{2.4} = 7.95,$$

or: $p_{tank\#2} = 7.95(14) \approx 113$ kPa, compared to p_{tank} (manometer reading) ≈ 138.5 kPa

This doesn't match either, the flow <u>expanded too much</u> before the shock wave. Therefore the correct answer is: a **normal shock wave upstream of the exit plane**. *Ans.*

9.66 In Prob. 9.65 what would be the mercury manometer reading if the nozzle were operating exactly at supersonic "design" conditions?

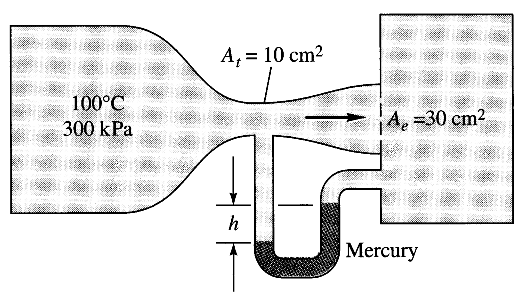


Fig. P9.65

Solution: We worked out this idealized isentropic-flow condition in Prob. 9.65:

$$\text{Design flow:} \quad \frac{A_e}{A^*} = 3.0, \quad Ma_e = 2.64, \quad p_e = 14 \text{ kPa} = p_{tank\,\#2}; \quad p^* = p_{throat} = 158.5 \text{ kPa}$$

Then $p_t - p_e = 158500 - 14200 = 144300 \approx (13550 - 0)(9.81)h,$

solve $\mathbf{h \approx 1.09\ m}$ *Ans.*

9.67 In Prob. 9.65 estimate the complete range of manometer readings h for which the flow through the nozzle is isentropic, except possibly in the exit plane.

Solution: First analyze *subsonic* flow throughout the duct: from Ma ≈ 0 to A/A* = 3.0:

Subsonic choked flow: A/A* = 3.0, $Ma_e \approx 0.1975$, $p_e = 292$ kPa, $p^* = p_t = 158.5$ kPa

Then $(p_t - p_e) = (158500 - 292000) = (13550 - 0)(9.81)h,$

or $h \approx \mathbf{-1.00\ m}$ (high side on the left)

So, for subsonic isentropic flow, we measure $\mathbf{-1.00\ m < h < 0}$ *Ans.*

Now analyze *supersonic* isentropic flow, possibly with a normal shock at the exit. One case if a fully overexpanded exit, $p_e = 0$, $\Delta p = 158500 = 13550(9.81)h$, $h = +1.19$ m. At the other extreme (see Prob. 9.65), a normal shock in the exit causes $p_e \approx 113$ kPa, $\Delta p = (158500 - 113000) = 13550(9.81)h$, or $h = +0.34$ m. The complete range is:

$\mathbf{-1.00\ m < h = 0}$ (subsonic flow) and $\mathbf{+0.34 < h < 1.19\ m}$ (supersonic flow) *Ans.*

9.68 Air in a tank at 120 kPa and 300 K exhausts to the atmosphere through a 5-cm^2-throat converging nozzle at a rate of 0.12 kg/s. What is the atmospheric pressure? What is the maximum mass flow possible at low atmospheric pressure?

Solution: Let us answer the second question first, to see where 0.12 kg/s stands:

$$\dot{m}_{max} = \frac{0.6847 p_o A^*}{\sqrt{RT_o}} = \frac{0.6847(120000)(0.0005)}{\sqrt{287(300)}}$$

$$\approx \mathbf{0.140\ kg/s} \quad \textit{Ans. (b)} \quad (\text{if } p_{atm} < 63 \text{ kPa})$$

So the given mass flow is about 86% of maximum and $p_{atm} > 63$ kPa. We could just go at it, guess the exit pressure and iterating, or we could express it more elegantly:

$$\dot{m} = \rho AV = \frac{\rho_o}{(1+0.2Ma^2)^{2.5}} A\, Ma \sqrt{kR}\sqrt{\frac{T_o}{1+0.2Ma^2}} = \frac{\text{Const Ma}}{(1+0.2Ma^2)^3},$$

where Const ≈ 0.2419 in SI units. If $\dot{m} = 0.12$ kg/s, we thus solve for Ma:

$$Ma \approx 0.496(1+0.2Ma^2)^3 \quad \text{to obtain } Ma \approx 0.62,\ p_{atm} \approx \mathbf{92.6\ kPa} \quad \textit{Ans.}$$

9.69 With reference to Prob. 3.68, show that the thrust of a rocket engine exhausting into a vacuum is given by

$$F = \frac{p_0 A_e\left(1 + k\,\mathrm{Ma}_e^2\right)}{\left(1 + \dfrac{k-1}{2}\mathrm{Ma}_e^2\right)^{k/(k-1)}}$$

where A_e = exit area
Ma_e = exit Mach number
p_0 = stagnation pressure in combustion chamber

Note that stagnation temperature does not enter into the thrust.

Solution: In a vacuum, $p_{atm} = 0$, the solution to Prob. 3.68 is

$$F = \rho_e A_e V_e^2 + A_e(p_e - 0) = A_e\left(p_e + \rho_e V_e^2\right),$$

$$\text{but } \rho_e V_e^2 \equiv k p_e \mathrm{Ma}_e^2, \quad \text{hence } F = p_e A_e\left(1 + k\mathrm{Ma}_e^2\right)$$

For isentropic flow, $p_e = p_o \Big/ \left(1 + \dfrac{k-1}{2}\mathrm{Ma}_e^2\right)^{k/(k-1)}$, $\therefore$ $\mathbf{F} = \dfrac{\mathbf{p_o A_e\left(1 + kMa_e^2\right)}}{\left(\mathbf{1} + \dfrac{\mathbf{k-1}}{\mathbf{2}}\mathbf{Ma_e^2}\right)^{\mathbf{k/(k-1)}}}$ *Ans.*

9.70 Air, at stagnation temperature 100°C, expands isentropically through a nozzle of 6-cm^2 throat area and 18-cm^2 exit area. The mass flow is at its maximum value of 0.5 kg/s. Estimate the exit pressure for (a) subsonic; and (b) supersonic exit flow.

Solutions: These are conditions "C" and "H" in Fig. 9.12b. The mass flow yields p_o:

$$\dot{m} = \dot{m}_{max} = 0.5\ \frac{kg}{s} = \frac{0.6847 p_o A^*}{\sqrt{(RT_o)}} = \frac{0.6847 p_o (0.0006)}{\sqrt{287(373)}}, \quad \text{solve } p_o \approx \mathbf{398\ kPa}$$

(a) Subsonic: $A_e/A^* = 18/6 = 3.0$: $\mathrm{Ma}_e \approx 0.1975$,

$$p_e = \frac{398}{\left(1 + 0.2\mathrm{Ma}_e^2\right)^{3.5}} \approx \mathbf{388\ kPa} \quad \textit{Ans. (a)}$$

(b) Supersonic: $A_e/A^* = 3.0$, $\mathrm{Ma}_e \approx 2.64$, $p_e = \dfrac{398}{[1 + 0.2(2.64)^2]^{3.5}} \approx \mathbf{19\ kPa}$ *Ans.* (b)

9.71 For the nozzle of problem 9.70, allowing for non-isentropic flow, what is the range of exit tank pressures p_b for which (a) the diverging nozzle flow is fully supersonic; (b) the

exit flow is subsonic; (c) the mass flow is independent of p_b; (d) the exit plane pressure p_e is independent of p_b; and (e) $p_e < p_b$?

Solution: In Prob. 9.70 we computed p_o = 398 kPa and the two 'design' Mach numbers.

(a) If a normal shock at the exit, Ma_1 = 2.64 and p_1 = 19 kPa, then across the shock p_2/p_1 = 7.95, p_b = 7.95(19) ≈ 150 kPa. Conclusion: $\mathbf{0 < p_b < 150\ kPa}$ *Ans.* (a)

Above this, *subsonic* exit flow occurs, for $\mathbf{150 < p_b < 398\ kPa}$ *Ans.* (b)

(c) The throat is *choked*, $\dot{m} = \dot{m}_{max}$ if $\mathbf{0 < p_b < 388\ kPa}$ (see Prob. 9.70a) *Ans.* (c)

(d) The exit-plane pressure is independent of p_b for $\mathbf{0 < p_b < 150\ kPa}$ *Ans.* (d)

(e) Exit plane pressure $p_e < p_b$ for $\mathbf{19 < p_b < 150\ kPa}$ *Ans.* (e)

9.72 A large tank at 500 K and 165 kPa feeds air to a converging nozzle. The back pressure outside the nozzle exit is sea-level standard. What is the appropriate exit diameter if the desired mass flow is 72 kg/h?

Solution: Given T_o = 500 K and p_o = 165 kPa. The pressure ratio across the nozzle is (101.35 kPa)/(165 kPa) = 0.614 > 0.528. Therefore the flow is not choked but instead exits at a high subsonic Mach number, with $p_{throat} = p_{atm}$ = 101.35 kPa. Equation (9.47) is handy:

$$\frac{\dot{m}}{A}\frac{\sqrt{RT_o}}{p_o} = \sqrt{\frac{2k}{k-1}\left(\frac{p}{p_o}\right)^{2/k}\left[1-\left(\frac{p}{p_o}\right)^{(k-1)/k}\right]} = \frac{(72/3600)}{A}\frac{\sqrt{287(500)}}{165000} = \frac{4.59E\text{–}5\ \text{m}^2}{A}$$

$$= \sqrt{\frac{2(1.4)}{0.4}\left(\frac{101}{165}\right)^{2/1.4}\left[1-\left(\frac{101}{165}\right)^{0.4/1.4}\right]} = 0.673$$

Solve for $A_{exit} = 6.82E\text{–}5\ \text{m}^2 = \frac{\pi}{4}D_{exit}^2$, Solve $\mathbf{D_{exit} = 0.0093\ m}$ *Ans.*

9.73 Air flows isentropically in a converging-diverging nozzle with a throat area of 3 cm^2. At section 1, the pressure is 101 kPa, T_1 = 300 K, and V_1 = 868 m/s. (a) Is the nozzle choked? Determine (b) A_1; and (c) the mass flow. Suppose, without changing stagnation conditions of A_1, the flexible throat is reduced to 2 cm^2. Assuming shock-free flow, will there be any changes in the gas properties at section 1? If so, calculate the new p_1, V_1, and T_1 and explain.

Solution: Check the Mach number. If choked, calculate the mass flow:

$$Ma_1 = \frac{V_1}{A_1} = \frac{868}{\sqrt{1.4(287)(300)}} = 2.50 \quad \text{Supersonic: the nozzle is } \textbf{choked.} \quad \textit{Ans.}\text{(a)}$$

$$p_o = 101[1 + 0.2(2.50)^2]^{3.5} = 1726 \text{ kPa}; \quad T_o = 300[1 + 0.2(2.50)^2] = 675 \text{ K}$$

$$\text{At } Ma_1 = 2.50,\ \frac{A_1}{A^*} = 2.64 \quad \therefore\ A_1 = 2.64(3) = \mathbf{7.91\ cm^2} \quad \textit{Ans.}\text{ (b)}$$

$$\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{1726000(0.0003)}{\sqrt{287(675)}} = \mathbf{0.805\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (c)}$$

If p_o and T_o are unchanged and the throat (A*) is reduced from 3.0 to 2.0 cm^2, the mass flow is cut by one-third and, if A_1 remains the same (7.91 cm^2), the area ratio changes and the Mach number will change at section 1:

$$\text{New} \quad \frac{A_1}{A^*_{new}} = \frac{7.91}{2.0} = 3.955; \quad \text{Table B.1:} \quad \text{read } Ma_{1,new} \approx \mathbf{2.928}$$

Since the Mach number changes, all properties at section 1 change:

$$p_{1,new} = \frac{1726000}{[1 + 0.2(2.928)^2]^{3.5}} = \mathbf{52300\ Pa}$$

$$T_{1,new} = \frac{675}{[1 + 0.2(2.928)^2]} = \mathbf{249\ K}$$

$$V_{1,new} = 2.928\sqrt{1.4(287)(249)} = \mathbf{926\ \frac{m}{s}}$$

A practical question might be: Does the new, reduced throat shape avoid flow separation and shock waves?

9.74 The perfect-gas assumption leads smoothly to Mach-number relations which are very convenient (and tabulated). This is not so for a real gas such as steam. To illustrate, let steam at T_0 = 500°C and p_0 = 2 MPa expand isentropically through a converging nozzle whose exit area is 10 cm^2. Using the steam tables, find (a) the exit pressure and (b) the mass flow when the flow is sonic, or choked. What complicates the analysis?

Solution: Never mind looking up Steam Tables, the big complication is finding the Mach number—even the software in modern thermo books does not contain the speed of sound. We can make a preliminary estimate with "ideal" steam, $k \approx 1.33$, $R \approx 461$ J/kg·K:

$$\text{Approximation:} \quad p_e \approx \frac{p_o}{[1+(k-1)/2]^{k/(k-1)}} = \frac{2.0\text{ MW}}{[1+0.33/2]^{1.33/0.33}} \approx \mathbf{1.08\ Mpa}$$

$$\dot{m}_{max} \approx 0.6726 p_o A^* \big/ \sqrt{RT_o} = 0.6726(2{,}000{,}000)(0.001) \big/ \sqrt{461(773)} \approx \mathbf{2.25\ kg/s}$$

This gives us an idea of where to look for the exit flow: around p ≈ 1.1 MPa. We can try 1.10 MPa, which is too high, $Ma_e < 1$, and 1.05 MPa, which is too low, $Ma_e < 1$, and finally converge to about **1.096 MPa** for the sonic-flow, isentropic exit pressure:

$$T_o = 500°\text{C}, \; p_o = 2.0\text{ MPa}, \quad \text{read } s_o \approx 7432\text{ J/kg·K and } h_o = 3.467\text{E6 J/kg}$$

$$\text{Then, at } p_e = \mathbf{1.096\ MPa} \quad \textit{Ans. (a)} \quad \text{read } T_e \approx 676\text{ K}, \; \rho_e \approx 3.563\text{ kg/m}^3,$$

$$h_e \approx 3.269\text{E6 J·kg}, \quad \therefore \; V = \sqrt{2(h_o - h)} = 629\text{ m/s, also read } a_e \approx 629\text{ m/s (OK, sonic)}$$

$$\text{Finally,} \quad \dot{m} = \rho_e A_e V_e = (3.563)(0.001)(629) \approx \mathbf{2.24\ kg/s} \quad \textit{Ans. (b)}$$

All that effort, and we ended up only 0.5% lower than our ideal-gas estimate!

9.75 A double-tank system in Fig. P9.75 has two identical converging nozzles of 1-in^2 throat area. Tank 1 is very large, and tank 2 is small enough to be in steady-flow equilibrium with the jet from tank 1. Nozzle flow is isentropic, but entropy changes between 1 and 3 due to jet dissipation in tank 2. Compute the mass flow. (If you give up, Ref. 14, pp. 288–290, has a good discussion.)

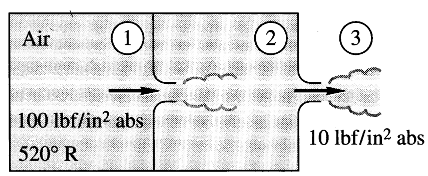


Fig. P9.75

Solution: We know that $\rho_1 V_1 = \rho_2 V_2$ from continuity. Since p_{atm} is so low, we may assume that the second nozzle is choked, but the first nozzle is probably not choked. We may guess values of p_2 and compare the computed values of flow through each nozzle:

$$\text{Assume 2nd nozzle choked:} \quad \rho_3 V_3 = \frac{0.6847 p_2}{\sqrt{(RT_o)}} \quad \text{with } T_o = 520°\text{R} = \text{constant}$$

$$\text{Guess} \quad p_2 \approx 80\text{ psia:} \; \frac{100}{80} = \left(1 + 0.2 Ma_1^2\right)^{3.5} \quad \text{or } Ma_1 \approx 0.574, \quad \text{then } \rho_1 \approx 0.0138 \; \frac{\text{slug}}{\text{ft}^3}$$

and $V_1 \approx 621$ ft/s, or $\rho_1 V_1 \approx 8.54$ slug/s·ft^2, whereas $\rho_3 V_3 \approx 8.35$ slug/s·ft^2

Guess $p_2 \approx 81$ psia: $Ma_1 \approx 0.557$, $\rho_1 V_1 = 8.38\ \dfrac{\text{slug}}{\text{s·ft}^2}$ and $\rho_3 V_3 \approx 8.45\ \dfrac{\text{slug}}{\text{s·ft}^2}$

Interpolate to: $p_2 \approx 80.8$ psia, $Ma_1 \approx 0.561$, $\mathbf{\dot{m} = \rho AV = 0.0585\ \dfrac{slug}{s}}$ *Ans.*

9.76 A large reservoir at 20°C and 800 kPa is used to fill a small insulated tank through a converging-diverging nozzle with 1-cm^2 throat area and 1.66-cm^2 exit area. The small tank has a volume of 1 m^3 and is initially at 20°C and 100 kPa. Estimate the elapsed time when (a) shock waves begin to appear inside the nozzle; and (b) the mass flow begins to drop below its maximum value.

Solution: During this entire time the nozzle is choked, so let's compute the mass flow:

$$\dot{m}_e = \dot{m}_{max} = \frac{0.6847\, p_o A^*}{\sqrt{(RT_o)}} = \frac{0.6847(800000)(0.0001)}{\sqrt{287(293)}} \approx \mathbf{0.189\ kg/s}$$

Meanwhile, a control volume around the small tank reveals a linear pressure rise with time:

$$\dot{m}_e = \frac{d}{dt}(m_{tank}) = \frac{d}{dt}(\rho \upsilon_{tank}) = \frac{d}{dt}\left(\frac{p_{tank}}{RT_o}\upsilon\right), \quad \text{or:} \quad \mathbf{\frac{dp_{tank}}{dt} \approx \frac{RT_o\dot{m}}{\upsilon}}$$

Carrying out the numbers gives $\dfrac{dp}{dt} \approx \dfrac{287(293)(0.189)}{1.0} \approx 15900\ \dfrac{\text{Pa}}{\text{s}} \approx \text{constant}$

We are assuming, for simplicity, that the tank stagnation temperature remains at 293 K. Shock waves move into the nozzle when the tank pressure rises above what would occur if the nozzle exit plane were to have a normal shock:

$$\text{If}\quad \frac{A_e}{A^*} = \frac{1.66}{1.0}, \quad \text{then}\ Ma_e \approx 1.98, \quad p_1 = \frac{800000}{[1+0.2(1.98)^2]^{3.5}} \approx 105400\ \text{Pa.}$$

$$\text{After the shock,}\quad \frac{p_2}{p_1} = \frac{2.8(1.98)^2 - 0.4}{2.4} \approx 4.41, \quad \therefore\ p_2 = p_{tank} = 4.41(105.4) \approx 465\ \text{kPa}$$

Above this tank pressure, the <u>shock wave moves into the nozzle</u>. The time lapse is

$$\Delta t_{\text{shocks in nozzle}} = \frac{\Delta p_{tank}}{dp/dt} = \frac{465000 - 100000\ \text{Pa}}{15900\ \text{Pa/s}} \approx \mathbf{23\ sec} \quad \textit{Ans.}\ \text{(a)}$$

Assuming the tank pressure rises smoothly and the shocks do not cause any instability or anything, the nozzle ceases to be choked when p_{tank} rises above a subsonic isentropic exit:

$$\frac{A_e}{A^*} = 1.66, \quad \text{than } \ Ma_e(\text{subsonic}) \approx 0.380, \quad p_e = \frac{800000}{[1+0.2(0.38)^2]^{3.5}} \approx 724300 \text{ Pa}$$

$$\text{Then} \quad \Delta t_{\text{choking stops}} \approx \frac{\Delta p}{dp/dt} = \frac{724300-100000}{15900} \approx \mathbf{39\ sec} \quad \textit{Ans.}\text{ (b)}$$

9.77 A perfect gas (not air) expands isentropically through a supersonic nozzle with an exit area 5 times its throat area. The exit Mach number is 3.8. What is the specific heat ratio of the gas? What might this gas be? If p_o = 300 kPa, what is the exit pressure of the gas?

Solution: We must iterate the area-ratio formula, Eq. (9.44), for *k*:

$$Ma_e = 3.8, \quad \frac{A_e}{A^*} = 5 = \frac{1}{3.8}\left[\frac{1+\frac{k-1}{2}(3.8)^2}{(k+1)/2}\right]^{\frac{k+1}{2(k-1)}}, \quad \text{Solve for } \ \mathbf{k \approx 1.667} \quad \textit{Ans.}\text{ (a)}$$

Monatomic gas: could be *helium* or *argon*. *Ans.* (b)

$$\text{With } k \text{ known,} \quad p_e = \frac{300 \text{ kPa}}{\left[1+\left(\frac{1.667-1}{2}\right)(3.8)^2\right]^{1.667/0.667}} \approx \mathbf{3.7\ kPa} \quad \textit{Ans.}\text{ (c)}$$

9.78 The orientation of a hole can make a difference. Consider holes A and B in Fig. P9.78, which are identical but reversed. For the given air properties on either side, compute the mass flow through each hole and explain the difference.

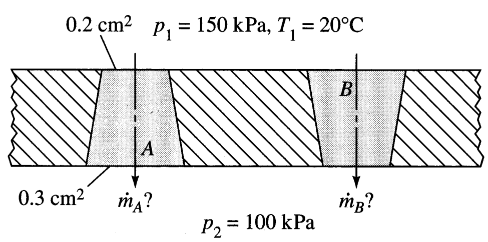


Fig. P9.78

Solution: Case B is a converging nozzle with $p_2/p_1 = 100/150 = 0.667 > 0.528$, therefore case B is **not choked**. Case A is **choked at the entrance** and expands to a (subsonic) pressure of 100 kPa, which we may check from a subsonic calculation. The results are:

$$\text{Nozzle } \mathbf{B}\text{:} \quad \frac{p_2}{p_o} = 0.667, \quad \text{read } \ Ma_2 \approx 0.784, \quad T_e = \frac{293}{[1+0.2(0.784)^2]} \approx 261 \text{ K,}$$

$$\rho_e = \frac{100000}{287(261)} = 1.34\ \frac{\text{kg}}{\text{m}^3},\quad a_e = 324\ \frac{\text{m}}{\text{s}},\quad V_e = 254\ \frac{\text{m}}{\text{s}},$$

$$\dot{m} = \rho_e A_e V_e \approx \mathbf{0.0068\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(B)}$$

$$\text{Nozzle } \mathbf{A}\text{:}\quad \dot{m} = \dot{m}_{max} = \frac{0.6847(150000)(0.0002)}{\sqrt{287(293)}} \approx \mathbf{0.071\ \frac{kg}{s}}\ \text{(5\% more)}\quad \textit{Ans.}\ \text{(A)}$$

9.79 A large reservoir at 600 K supplies air flow through a converging-diverging nozzle with a throat area of 2 cm^2. A normal shock wave forms at a section of area 6 cm^2. Just downstream of this shock, the pressure is 150 kPa. Calculate (a) the pressure in the throat; (b) the mass flow; and (c) the pressure in the reservoir.

Solution: The throat is choked, and just upstream of the shock is a supersonic flow at an area ratio $A/A^* = (6\ cm^2)/(2\ cm^2) = 3.0$. From Table B.1 estimate $Ma_1 = 2.64$. That is,

$$\frac{A_1}{A^*} = 3.0 = \frac{\left(1+0.2Ma_1^2\right)^3}{1.728Ma_1},\quad \text{Solve } Ma_1 = 2.637$$

(a, c) The pressure ratio across the shock is given by Eq. (9.55) or Table B.2:

$$\frac{p_2}{p_1} = \frac{150\,\text{kPa}}{p_1} = \frac{1}{k+1}\left(2kMa_1^2 - k + 1\right)$$

$$= \frac{1}{2.4}[2(1.4)(2.637)^2 - 0.4] = 7.95,\quad \text{or}\quad p_1 = 18.9\ \text{kPa}$$

$$p_{\boldsymbol{tank}} = p_o = p_1\left(1+0.2Ma_1^2\right)^{3.5} = (18.9)[1+0.2(2.637)^2]^{3.5} = \mathbf{399\ kPa}\quad \textit{Ans.}\ \text{(c)}$$

$$\text{At the throat,}\quad p = p^* = 0.5283p_o = (0.5283)(399\ \text{kPa}) = \mathbf{211\ kPa}\quad \textit{Ans.}\ \text{(a)}$$

(b) To avoid bothering with density and velocity, Eq. (9.46b) is handy for choked flow.

$$\dot{m}_{max} = \frac{0.6847p_oA^*}{\sqrt{RT_o}} = \frac{0.6847(399000\,\text{Pa})(0.0002\ \text{m}^2)}{\sqrt{(287\,\text{m}^2/\text{s}^2\text{K})(600\,\text{K})}} = \mathbf{0.132\ kg/s}\quad \textit{Ans.}\ \text{(b)}$$

9.80 A sea-level automobile tire is initially at 32 lbf/in^2 gage pressure and 75°F. When it is punctured with a hole which resembles a converging nozzle, its pressure drops to 15 lbf/in^2 gage in 12 min. Estimate the size of the hole, in thousandths of an inch.

Solution: The volume of the tire is **2.5 ft²**. With $p_{atm} \approx 14.7$ psi, the absolute pressure drops from 46.7 psia to 29.7 psia, both of which are sufficient to cause a **choked** exit. A theory for isothermal blowdown of a choked tank was given in Prob. 9.36:

$$p_{tank} = p(0)\exp\left[-0.6847\frac{A^*\sqrt{RT_o}}{\upsilon}t\right],$$

$$\text{or:}\quad 29.7 = 46.7\exp\left[-0.6847\frac{A^*\sqrt{1717(535)}}{2.5}12\times 60\right],$$

$$\text{solve}\quad \mathbf{A^* = 2.4E{-}6\ ft^2}\ \ (d = 0.021\text{ in})\quad \textit{Ans.}$$

9.81 Helium, in a large tank at 100°C and 400 kPa, discharges to a receiver through a converging-diverging nozzle designed to exit at Ma = 2.5 with exit area 1.2 cm². Compute (a) the receiver pressure and (b) the mass flow at design conditions. (c) Also estimate the range of receiver pressures for which mass flow will be a maximum.

Solution: For helium (Table A.4), take k = 1.66 and R = 2077 J/kg·°K. At the exit,

$$p_e = p_o\Big/\left[1+\left(\frac{k-1}{2}\right)Ma_e^2\right]^{k/(k-1)} = 400/[1+0.33(2.5)^2]^{1.66/0.66} \approx \mathbf{24\ kPa}\quad \textit{Ans. (a)}$$

$$\text{Also,}\ T_e = \frac{373\text{ K}}{[1+0.33(2.5)^2]} = 122\text{ K},\quad a_e = \sqrt{1.66(2077)(122)} = 648\ \frac{m}{s},$$

$$V_e = Ma_e a_e = 1620\ \frac{m}{s},\quad \rho_e = \frac{p_e}{RT_e} = 0.0947\ \frac{kg}{m^3},\quad \dot{m} = \rho_e A_e V_e \approx \mathbf{0.0184\ \frac{kg}{s}}\quad \textit{Ans. (b)}$$

We could also compute A/A* ≈ **2.15**, whence A* ≈ 0.56 cm², and use the "maximum mass flow formula," Eq. (9.46) to compute the throat mass flow, also = 0.0184 kg/s. This choked flow persists up to the "isentropic subsonic exit" condition:

$$\frac{A}{A^*} = 2.15 = \frac{1}{Ma}\left[\frac{2+(k-1)Ma^2}{k+1}\right]^{\frac{k+1}{2(k-1)}}\ \text{for } k = 1.66.\quad \text{Solve for } Ma_{subsonic} \approx 0.275$$

$$p_e = \frac{400}{[1+0.33(0.275)^2]^{1.66/0.66}} \approx 376\text{ kPa}.\quad \text{Choked flow for } \mathbf{0 < p_e < 376\ kPa}\quad \textit{Ans. (c)}$$

9.82 Air at 500 K flows through a converging-diverging nozzle with throat area of 1 cm² and exit area of 2.7 cm². When the mass flow is 182.2 kg/h, a pitot-static probe

placed in the exit plane reads p_0 = 250.6 kPa and p = 240.1 kPa. Estimate the exit velocity. Is there a normal shock wave in the duct? If so, compute the Mach number just downstream of this shock.

Solution: These numbers **just don't add up** to a purely isentropic flow. For example, p_o/p = 250.6/240.1 yields Ma ≈ **0.248**, whereas A/A* = 2.7 gives Ma ≈ **0.221**. If the mass flow is *maximum*, we can estimate the upstream stagnation pressure:

$$\dot{m} \overset{?}{=} \dot{m}_{max} = \frac{182.2}{3600} \overset{?}{=} 0.6847\frac{p_{o1}(0.0001)}{\sqrt{287(500)}} \quad \text{if} \;\; p_{o1} \approx \mathbf{280\,kPa}$$

This doesn't check with the measured value of 250.6 kPa, nor does an isentropic choked subsonic expansion lead to p_{exit} = 240.1—it gives **271** kPa instead. We conclude that **there is a normal shock wave in the duct** before the exit plane, reducing p_o:

$$\text{Normal shock:} \quad \frac{p_{o2}}{p_{o1}} = \frac{250.6}{280.0} = 0.895 \quad \text{if} \;\; Ma_1 \approx 1.60 \quad \text{and} \quad \mathbf{Ma_2 \approx 0.67} \quad \textit{Ans.}\text{ (b)}$$

$$\text{This checks with} \quad A_2^* \approx 1.12\ \text{cm}^2, \;\; \frac{A_e}{A_2^*} = 2.42, \;\; Ma_e \approx 0.248 \text{ (as above)}$$

$$T_e = \frac{500}{[1+0.2(0.248)^2]} = 494\ \text{K}, \quad a_e = \sqrt{kRT_e} = 445\ \frac{m}{s}, \quad \mathbf{V_e} = Ma_e a_e \approx \mathbf{110\ \frac{m}{s}}$$

9.83 When operating at design conditions (smooth exit to sea-level pressure), a rocket engine has a thrust of 1 million lbf. The chamber pressure and temperature are 600 lbf/in^2 absolute and 4000°R, respectively. The exhaust gases approximate k = 1.38 with a molecular weight of 26. Estimate (a) the exit Mach number and (b) the throat diameter.

Solution: "Design conditions" mean isentropic expansion to p_e = 14.7 psia = 2116 lbf/ft^2:

$$\frac{p_o}{p_e} = \frac{600}{14.7} = 40.8 = \left[1+\left(\frac{1.38-1}{2}\right)Ma_e^2\right]^{1.38/0.38}, \quad \text{solve for} \;\; \mathbf{Ma_e \approx 3.06} \quad \textit{Ans.}\text{ (a)}$$

From Prob. 3.68, if $p_e = p_a$,

$$F = \rho_e A_e V_e^2 = kp_eA_eMa_e^2 = 1.38(2116)A_e(3.06)^2 = 1E6\ \text{lbf}, \quad \text{solve} \;\; A_e \approx 36.6\ \text{ft}^2$$

Assuming an isentropic expansion to $\mathrm{Ma_e} \approx 3.06$, we can compute the throat area:

$$\frac{A_e}{A^*} = \frac{36.6}{A^*} = \frac{1}{3.06}\left[\frac{2+0.38(3.06)^2}{1.38+1}\right]^{\frac{2.38}{2(0.38)}} = 4.65, \quad \text{or} \quad A^* = \frac{36.6}{4.65} = 7.87\ \mathrm{ft}^2 = \frac{\pi}{4}D^{*2}$$

Solve for throat diameter $\mathbf{D^* \approx 3.2\ ft}$ *Ans.* (b)

9.84 Air flows through a duct as in Fig. P9.84, where $A_1 = 24\ \mathrm{cm}^2$, $A_2 = 18\ \mathrm{cm}^2$, and $A_3 = 32\ \mathrm{cm}^2$. A normal shock stands at section 2. Compute (a) the mass flow, (b) the Mach number, and (c) the stagnation pressure at section 3.

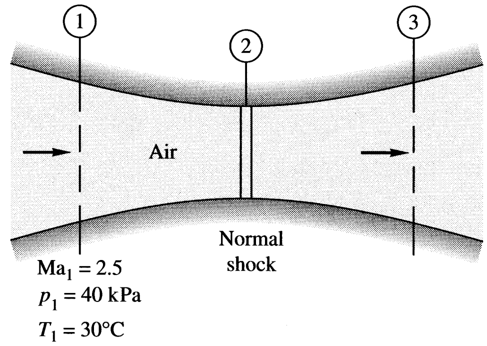


Fig. P9.84

Solution: We have enough information at section 1 to compute the mass flow:

$$a_1 = \sqrt{1.4(287)(30+273)} \approx 349\ \mathrm{m/s}, \quad V_1 = 2.5(349) = 872\ \frac{\mathrm{m}}{\mathrm{s}}, \quad \rho_1 = \frac{p_1}{RT_1} = 0.46\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$\text{Then} \quad \dot{m} = \rho_e A_e V_e = 0.46(0.0024)(872) \approx \mathbf{0.96\ kg/s} \quad \textit{Ans.}\ (a)$$

Now move isentropically from 1 to 2 upstream of the shock and thence across to 3:

$$\mathrm{Ma}_1 = 2.5, \quad \therefore \quad \frac{A_1}{A_1^*} = 2.64, \quad A_1^* = \frac{24}{2.64} = 9.1\ \mathrm{cm}^2, \quad \text{and} \quad \frac{A_2}{A_1^*} = \frac{18}{9.1} = 1.98$$

Read $\mathrm{Ma}_{2,\mathrm{upstream}} \approx \mathbf{2.18}$, $p_{o1} = p_{o2} = 40[1+0.2(2.5)^2]^{3.5} \approx 683\ \mathrm{kPa}$, across the shock, $\frac{A_3^*}{A_2^*} = 1.57$, $A_3^* = 14.3\ \mathrm{cm}^2$, $\frac{A_3}{A_3^*} = 2.24\big|_{\mathrm{sub}}$, $\mathbf{Ma_3 \approx 0.27}$ *Ans.* (b)

Finally, go back and get the stagnation pressure ratio across the shock:

$$\text{at } \mathrm{Ma}_2 \approx 2.18, \quad \frac{p_{o3}}{p_{o2}} \approx 0.637, \quad \therefore \quad p_{o3} = 0.637(683) \approx \mathbf{435\ kPa} \quad \textit{Ans.}\ (c)$$

9.85 A large tank at 300 kPa delivers air through a nozzle of 1-cm^2 throat area and 2.2-cm^2 exit area. A normal shock wave stands in the exit plane. The temperature just downstream

of this shock is 473 K. Calculate (a) the temperature in the large tank; (b) the receiver pressure; and (c) the mass flow.

Solution: First find the Mach number just upstream of the shock and the temperature ratio:

$$\frac{A_{exit}}{A^*} = \frac{2.2\ \text{cm}^2}{1.0\ \text{cm}^2} = 2.2 = \frac{\left(1+0.2Ma_1^2\right)^3}{1.728Ma_1}, \quad \text{solve for } Ma_1 = 2.303$$

Across the shock: $$\frac{T_2}{T_1} = \frac{473\,\text{K}}{T_1} = \frac{[2+0.4(2.303)^2][2.8(2.303)^2-0.4]}{(2.4)^2(2.303)^2} = 1.95,\ T_1 = 243\,\text{K}$$

$$T_{tank} = T_o = T_1\left(1+0.2Ma_1^2\right) = (243\ \text{K})[1+0.2(2.303)^2] = \mathbf{500\ K} \quad \textit{Ans.}\ \text{(a)}$$

Finally, compute the pressures just upstream and downstream of the shock:

$$p_1 = p_o\Big/\left(1+0.2Ma_1^2\right)^{3.5} = (300\ \text{kPa})/[1+0.2(2.303)^2]^{3.5} = 23.9\ \text{kPa}$$

$$p_2 = p_{receiver} = \frac{p_1}{k+1}\left(2kMa_1^2 - k + 1\right) = \frac{23.9\ \text{kPa}}{(1.4+1)}\ [2(1.4)(2.303)^2 - 0.4]$$

$$= \mathbf{144\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

9.86 Air enters a 3-cm diameter pipe 15 m long at V_1 = 73 m/s, p_1 = 550 kPa, and T_1 = 60°C. The friction factor is 0.018. Compute V_2, p_2, T_2, and p_{02} at the end of the pipe. How much additional pipe length would cause the exit flow to be sonic?

Solution: First compute the inlet Mach number and then get $(fL/D)_1$:

$$a_1 = \sqrt{1.4(287)(60+273)} = 366\ \frac{m}{s}, \quad Ma_1 = \frac{73}{366} \approx 0.20, \quad \text{read } \left(\frac{fL}{D}\right) = 14.53,$$

for which p/p* = 5.4554, T/T* = 1.1905, V/V* = 0.2182, and $p_o/p_o^* = 2.9635$

Then $(fL/D)_2 = 14.53 - (0.018)(15)/(0.03) \approx \mathit{5.53}$, read $Ma_2 \approx \mathbf{0.295}$

At this new Ma_2, read p/p* ≈ 3.682, T/T* ≈ 1.179, V/V* ≈ 0.320, $p_o/p_o^* \approx 2.067$. Then

$$V_2 = V_1\frac{V_2/V^*}{V_1/V^*} = 73\left(\frac{0.320}{0.218}\right) \approx \mathbf{107\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$p_2 = 550\left(\frac{3.682}{5.455}\right) \approx \mathbf{371\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

$$T_2 = 333\left(\frac{1.179}{1.190}\right) \approx \mathbf{330\ K} \quad \textit{Ans.}\ \text{(c)}$$

Now we need p_{o1} to get p_{o2}:

$$p_{o1} = 550[1+0.2(0.2)^2]^{3.5} \approx 566\text{ kPa},\quad \text{so}\ \ p_{o2} = 566\left(\frac{2.067}{2.964}\right) \approx \mathbf{394\ kPa}$$

The extra distance we need to choke the exit to sonic speed is $(fL/D)_2 = 5.53$. That is,

$$\Delta L = 5.53\frac{D}{f} = 5.53\left(\frac{0.03}{0.018}\right) \approx \mathbf{9.2\ m}\quad \textit{Ans.}$$

9.87 Air enters an adiabatic duct of L/D = 40 at V_1 = 170 m/s and T_1 = 300 K. The flow at the exit is choked. What is the average friction factor in the duct?

Solution: Noting that Ma_{exit} = 1.0, compute Ma_1, find fL/D and hence *f*:

$$Ma_1 = \frac{V_1}{a_1} = \frac{170}{\sqrt{1.4(287)(300)}} = \frac{170}{347} = 0.49,$$

$$\text{Table B.3:}\quad \text{read}\ \ f\frac{L}{D} \approx 1.15 \quad \text{Then}\ \ f = \frac{1.15}{40} \approx \mathbf{0.029}\quad \textit{Ans.}$$

9.88 Air enters a 5- by 5-cm square duct at V_1 = 900 m/s and T_1 = 300 K. The friction factor is 0.02. For what length duct will the flow exactly decelerate to Ma = 1.0? If the duct length is 2 m, will there be a normal shock in the duct? If so, at what Mach number will it occur?

Solution: First compute the inlet Mach number, which is decidedly supersonic:

$$Ma_1 = \frac{V_1}{a_1} = \frac{900}{\sqrt{1.4(287)(300)}} \approx 2.59,$$

$$\text{read}\ \ (fL/D)_1 \approx 0.451,\quad \text{whence}\ \ L^*\big|_{Ma=1} = 0.451\left(\frac{0.05}{0.02}\right) \approx \mathbf{1.13\ m}\quad \textit{Ans.}$$

[We are taking the "hydraulic diameter" of the square duct to be 5 cm.] If the actual duct length = 2 m > L*, then there **must be a normal shock** in the duct. By trial and error, we need a total dimensionless length (fL/D) = 0.02(2)/0.05 ≈ **0.8**. The result is:

$$Ma_1 = 2.59,\ \ \frac{fL}{D}\Big|_1 = 0.451,\quad Ma_2 = \mathbf{2.14},\ \ \frac{fL}{D}\Big|_2 = 0.345,$$

$$\text{shock: } Ma_3 = 0.555,\ \ \frac{fL}{D}\Big|_3 = 0.695$$

$$\text{Total}\quad fL/D = 0.451 - 0.345 + 0.695 = 0.801\ \text{(close enough)}\quad \therefore\ \mathbf{Ma_2 = 2.14}\quad \textit{Ans.}$$

9.89 Carbon dioxide flows through an insulated pipe 25 m long and 8 cm in diameter. The friction factor is 0.025. At the entrance, $p = 300$ kPa and $T = 400$ K. The mass flow is 1.5 kg/s. Estimate the pressure drop by (a) compressible; and (b) incompressible (Sect. 6.6) flow theory. (c) For what pipe length will the exit flow be choked?

Solution: For CO_2, from Table A.4, take $k = 1.30$ and $R = 189$ J/kg·K. Tough calculation, no appendix tables for CO_2, should probably use EES. Find inlet density, velocity, Mach number:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{300000\,\text{Pa}}{(189\,\text{J/kg·K})(400\,\text{K})} = 3.97\ \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = 1.5\ \frac{\text{kg}}{\text{s}} = \rho_1 A_1 V_1 = \left(3.97\ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(0.08\ \text{m})^2 V_1, \quad \text{Solve for } V_1 = 75.2\ \text{m/s}$$

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{75.2\,\text{m/s}}{\sqrt{(1.3)(189\,\text{m}^2/\text{s}^2\text{·K})(400\,\text{K})}} = \frac{75.2\,\text{m/s}}{313.5\,\text{m/s}} = \mathbf{0.240}$$

Between section 1 (inlet) and section 2 (exit), the **change** in (fL/D) equals (0.025)(25 m)/(0.08 m) = 7.813. We have to find the correct exit Mach number from this change:

$$fL^*/D = \frac{1-Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right]$$

For $k = 1.3$ and $Ma_1 = 0.240$ compute $(fL^*/D)_1 = 10.190$

Then $(fL^*/D)_2 = 10.190 - 7.813 = 2.377$ for what Mach number?

Then iterate (or use EES) to the exit value $Ma_2 = \mathbf{0.408}$

Now compute $p_1/p^* = 4.452$ and $p_2/p^* = 2.600$

Then $p_2 = p_1(p_2/p^*)/(p_1/p^*) = (300\,\text{kPa})(2.600)/(4.452) = 175$ kPa

The desired compressible pressure drop $= 300 - 175 = \mathbf{125\,kPa}$ *Ans.* (a)

(b) The incompressible flow theory (Chap. 6) simply predicts that

$$\Delta p_{inc} = \frac{fL}{D}\frac{\rho_1}{2}V_1^2 = (7.813)\left(\frac{3.97\,\text{kg/m}^3}{2}\right)\left(75.2\ \frac{\text{m}}{\text{s}}\right)^2 = 88000\,\text{Pa} = \mathbf{88\ kPa} \quad \textit{Ans. (b)}$$

The incompressible estimate is 30% low. Finally, the inlet value of (fL/D) tells us the maximum possible pipe length for choking at the exit:

$$L_{\max} = \left.\frac{fL^*}{D}\right|_1\left(\frac{D}{f}\right) = (10.19)\left(\frac{0.08\ \text{m}}{0.025}\right) = \mathbf{32.6\ m} \quad \textit{Ans. (c)}$$

9.90 Air, supplied at p_0 = 700 kPa and T_0 = 330 K, flows through a converging nozzle into a pipe of 2.5-cm diameter which exits to a near vacuum. If $\bar{f} = 0.022$, what will be the mass flow through the pipe if its length is (a) 0 m, (b) 1 m, and (c) 10 m?

Solution: (a) With no pipe (L = 0), the mass-flow is simply the isentropic maximum:

$$\dot{m} = \dot{m}_{max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{700000(\pi/4)(0.025)^2}{\sqrt{287(330)}} \approx \mathbf{0.764}\ \frac{\mathbf{kg}}{\mathbf{s}} \quad \textit{Ans.}\text{ (a)}$$

(b) With a finite length L = 1 m, the flow will choke in the exit plane instead:

$$Ma_e = 1.0,\ \frac{fL}{D} = \frac{0.022(1.0)}{0.025} = 0.88, \quad \text{read } Ma_1(\text{entrance}) \approx 0.525$$

$$\text{Then} \quad T_1 = 330/[1+0.2(0.525)^2] = 313\text{ K}, \quad a_1 = \sqrt{1.4(287)(313)} \approx 354\text{ m/s},$$

$$V_1 = Ma_1 a_1 = 186\text{ m/s}, \quad p_1 = 700/[1+0.2(0.525)^2]^{3.5} = 580\text{ kPa},$$

$$\rho_1 = p_1/(RT_1) = 6.46\text{ kg/m}^3$$

Finally, then, $\dot{m} = \rho_1 A_1 V_1 = (6.46)(\pi/4)(0.025)^2(186) \approx \mathbf{0.590}\ \frac{\mathbf{kg}}{\mathbf{s}}$ (23% less) *Ans.* (b)

(c) Repeat part (b) for a much longer length, L = 10 m:

$$\frac{fL}{D} = \frac{0.022(10)}{0.025} = 8.8, \quad Ma_1 = 0.246, \quad T_1 = 326\text{ K}, \quad a_1 = 362\ \frac{m}{s}, \quad V_1 = 89\ \frac{m}{s},$$

$$\text{also,} \quad p_1 = 671\text{ kPa}, \quad \rho_1 = 7.17\ \frac{kg}{m^3}, \quad \dot{m} = \rho_1 A_1 V_1 \approx \mathbf{0.314}\ \frac{\mathbf{kg}}{\mathbf{s}}\ (59\%\text{ less}) \quad \textit{Ans.}\text{ (c)}$$

9.91 Air flows steadily from a tank through the pipe in Fig. P9.91. There is a converging nozzle on the end. If the mass flow is 3 kg/s and the flow is choked, estimate (a) the Mach number at section 1; and (b) the pressure in the tank.

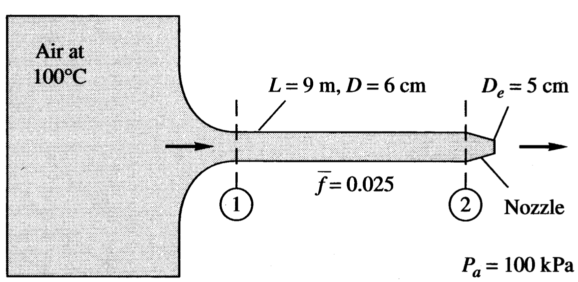


Fig. P9.91

Solution: For adiabatic flow, T* = constant = $T_o/1.2 = 373/1.2 = 311$ K. The flow chokes in the small exit nozzle, D = 5 cm. Then we estimate Ma_2 from isentropic theory:

$$\frac{A_2}{A^*} = \left(\frac{6\text{ cm}}{5\text{ cm}}\right)^2 = 1.44, \quad \text{read } Ma_2(\text{subsonic}) \approx 0.45, \text{ for which } fL/D\big|_2 \approx 1.52,$$

$$p_2/p^* \approx 2.388, \quad p_{o2}/p_o^* \approx 1.449, \quad \rho_2/\rho^* \approx 2.070, \quad T_2/T^* = 1.153 \quad \text{or} \quad T_2 \approx 359\text{ K}$$

$$\text{Given} \quad \dot{m} = 3\ \frac{kg}{s} = \rho_2 A_2 V_2 = \frac{p_2}{287(359)}\left(\frac{\pi}{4}\right)(0.06)^2(0.45)\sqrt{1.4(287)(359)},$$

$$\text{Solve for} \quad p_2 \approx 640\text{ kPa}. \quad \text{Then} \quad p^* = 640/2.388 \approx 268\text{ kPa}$$

$$\text{At section 1,} \quad \frac{fL}{D} = \frac{fL}{D}\bigg|_2 + \frac{f\Delta L}{D} = 1.52 + \frac{0.025(9)}{0.06} \approx 5.27, \quad \text{read } Ma_1 \approx \mathbf{0.30} \quad \textit{Ans. (a)}$$

$$\text{for which} \quad p_1/p^* \approx 3.6, \quad \text{or} \quad p_1 \approx 3.6(268) \approx 965\text{ kPa}.$$

Assuming isentropic flow in the inlet nozzle,

$$p_{tank} \approx 965[1 + 0.2(0.30)^2]^{3.5} \approx \mathbf{1030\ kPa} \quad \textit{Ans. (b)}$$

9.92 Modify Prob. 9.91 as follows: Let the tank pressure be 700 kPa, and let the nozzle be *choked*. Determine (a) Ma_2; and (b) the mass flow. Keep T_o = 100°C.

Solution: This is the reverse of Prob. 9.91 and is easier. The Mach numbers are the same, since they depend only upon fL/D (which is the same) and the two nozzle area ratios. If we didn't know the solution to Prob. 9.91, we would guess Ma_1, work out Ma_2 and see if the flow then expands exactly to a sonic exit at the second nozzle. Repeat, if necessary, until the progression through the pipe and the second nozzle is choked. The results are:

$$Ma_1 = 0.30, \quad \text{compute } p_1 = 700/[1 + 0.2(0.30)^2]^{3.5} \approx 658\text{ kPa}. \quad \text{In Table B.3, read}$$

$$p_1/p^* \approx 3.6, \quad \text{or} \quad p^* = \frac{658}{3.6} \approx 183\text{ kPa}. \quad \text{Also read } fL/D\big|_1 \approx 5.27, \text{ subtract } f\Delta L/D \text{ of } 3.75$$

$$\text{to find } fL/D\big|_2 \approx 1.52, \text{ read } Ma_2 \approx \mathbf{0.45} \quad \textit{Ans. (a)} \quad \text{Table B.1:} \quad A_2/A^* \approx 1.44$$

$$\text{Then} \quad A_{exit}/A^* = \frac{1.44}{(6/5)^2} \approx 1.0 \text{ (exactly what we want, } \textit{sonic flow exit}\text{).}$$

$$\text{Go back to sections 1 or 2 to compute} \quad \dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \approx \mathbf{2.04\ kg/s} \quad \textit{Ans. (b)}$$

9.93 Air flows adiabatically in a 3-cm-diameter duct with f = 0.015. At the entrance, V = 950 m/s and T = 250 K. How far down the tube will (a) the Mach number be 1.8; and (b) the flow be choked?

Solution: (a) Find the entrance Mach number and its value of fL/d:

$$Ma_1 = \frac{950}{\sqrt{1.4(287)(250)}} = 3.00; \quad \text{Table B.3:} \quad \text{read} \quad f\frac{L_1}{D} = 0.5222; \quad \text{at} \quad Ma_2 = 1.8,$$

$$\text{read} \quad f\frac{L_2}{D} = 0.2419; \quad \Delta\left(f\frac{L}{D}\right) = 0.5222 - 0.2419 \approx 0.28,$$

$$\Delta L = \frac{0.28(0.03)}{0.015} = \mathbf{0.56\ m} \quad \textit{Ans. (a)}$$

(b) To go all the way to choking requires the full change

$$f\Delta L_1/D = 0.5222, \quad \text{or:} \quad \Delta L_{choke} = (0.5222)(0.03)/(0.015) = \mathbf{1.04\ m} \quad \textit{Ans. (b)}$$

9.94 Compressible pipe flow with friction, Sec. 9.7, assumes constant stagnation enthalpy and mass flow but variable momentum. Such a flow is often called *Fanno flow*, and a line representing all possible property changes on a temperature-entropy chart is called a Fanno line. Assuming an ideal gas with $k = 1.4$ and the data of Prob. 9.86, draw a Fanno line for a range of velocities from very low ($Ma \ll 1$) to very high ($MA \gg 1$). Comment on the meaning of the maximum-entropy point on this curve.

Solution: Recall from Prob. 9.86 that, at Section 1 of the pipe, $V_1 = 73$ m/s, $p_1 = 550$ kPa, and $T_1 = 60°C = 333$ K, with $f \approx 0.018$. We can then easily compute $Ma_1 \approx 0.20$, $\rho_1 = 5.76$ kg/m^3, $V_{max} = 822$ m/s, and $T_o = 336$ K. Our basic algebraic equations are:

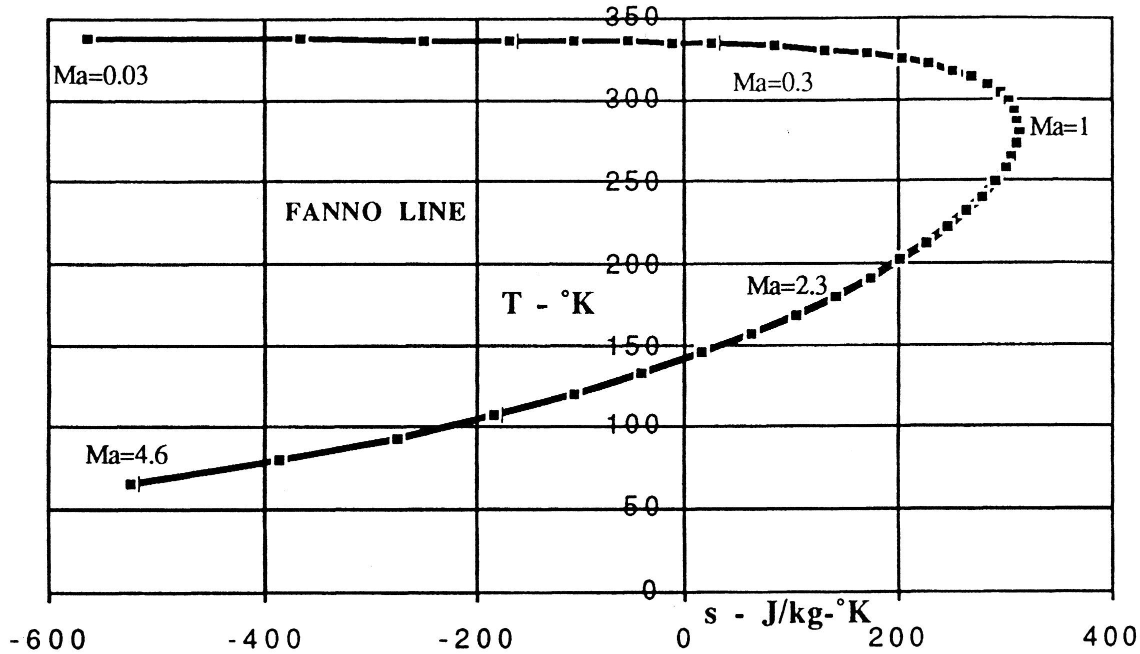
$$\text{Energy:} \quad T = T_o - \frac{V^2}{2c_p}, \quad \text{or:} \quad \mathbf{T = 336\ K - \frac{V^2}{2(1005)}} \qquad \textbf{(a)}$$

$$\text{Continuity:} \quad \rho V = \rho_1 V_1 = 5.76(73), \quad \text{or:} \quad \boldsymbol{\rho = 420/V} \qquad \textbf{(b)}$$

$$\text{Entropy:} \quad \mathbf{s = 718\ ln(T/333) - 287\ ln(\rho/5.76)} \qquad \textbf{(c)}$$

We simply let *V* vary from, say, 10 m/s to 800 m/s, compute ρ from (b) and *T* from (a) and *s* from (c), then plot *T* versus *s*. [We have arbitrarily set s = 0 at state 1.]

The result of this exercise forms the **Fanno Line** for this flow, shown on the next page. Some Mach numbers are listed, subsonic on the top, supersonic on the bottom, and exactly **sonic** at the right-hand (maximum-entropy) side. *Ans.*



9.95 Helium (Table A.4) enters a 5-cm-diameter pipe at $p_1 = 550$ kPa, $V_1 = 312$ m/s, and $T_1 = 40°C$. The friction factor is 0.025. If the flow is choked, determine (a) the length of the duct and (b) the exit pressure.

Solution: For helium, take k = 1.66 and R = 2077 J/kg·K. We have no tables for k = 1.66, have to do our best anyway. Compute the Mach number at section 1:

$$a_1 = \sqrt{(1.66)(2077)(40+273)} \approx 1039 \text{ m/s}, \quad Ma_1 = V_1/a_1 = \frac{312}{1039} \approx \mathbf{0.300}$$

$$\text{Eq. 9.66:} \quad \frac{fL}{D} = \frac{1-(0.3)^2}{1.66(0.3)^2} + \frac{2.66}{2(1.66)}\ln\left[\frac{(2.66)(0.3)^2}{2+(0.66)(0.3)^2}\right] \approx 4.37 \quad \text{at section 1}$$

$$\text{Choked:} \quad fL/D\big|_2 = 0, \quad \therefore\ L = 4.37D/f = 4.37(0.05)/(0.025) \approx \mathbf{8.7\ m} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Also,} \quad p_1/p^* = \frac{1}{0.3}\left[\frac{2.66}{2+0.66(0.3)^2}\right]^{1/2} \approx 3.79, \quad \therefore\ p_{exit} = p^* = \frac{550}{3.79} \approx \mathbf{145\ kPa} \quad \textit{Ans.}\text{(b)}$$

9.96 Methane (CH_4) flows through an insulated 15-cm-diameter pipe with f = 0.023. Entrance conditions are 600 kPa, 100°C, and a mass flow of 5 kg/s. What lengths of pipe will (a) choke the flow; (b) raise the velocity by 50%; (c) decrease the pressure by 50%?

Solution: For methane (CH_4), from Table A.4, take $k = 1.32$ and $R = 518$ J/kg·K. Tough calculation, no appendix tables for methane, should probably use EES. Find inlet density, velocity, Mach number:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{600000\,\text{Pa}}{(518\,\text{J/kg·K})(373\,\text{K})} = 3.11\ \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = 5\ \text{kg/s} = \rho_1 A_1 V_1 = (3.11\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.15\ \text{m})^2 V_1, \quad \text{solve for } V_1 = 91.1\ \text{m/s}$$

$$a_1 = \sqrt{kRT_1} = \sqrt{1.32(518)(373)} = 505\ \text{m/s}, \quad Ma_1 = \frac{V_1}{a_1} = \frac{91.1\ \text{m/s}}{505\ \text{m/s}} = 0.180$$

Now we have to work out the pipe-friction relations, Eqs. (9.66) and (9.68), for $k = 1.32$. We need *fL*/D, V/V**, and *p/p** at the inlet, Ma = 0.18:

$$\frac{fL^*}{D} = \frac{1-Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right] = 19.63 \quad \text{at } Ma_1 = 0.18 \quad \text{and} \quad k = 1.32$$

$$\text{Solve} \quad L^*_{choking} = 19.63\frac{D}{f} = 19.63\left(\frac{0.15\ m}{0.023}\right) = \mathbf{128\,m} \quad \textit{Ans. (a)}$$

$$\frac{fL^*}{D} = \frac{1-Ma^2}{kMa^2} + \frac{k+1}{2k}\ln\left[\frac{(k+1)Ma^2}{2+(k-1)Ma^2}\right] = 19.63 \quad \text{at } Ma_1 = 0.18 \quad \text{and} \quad k = 1.32$$

$$\text{Solve} \quad L^*_{choking} = 19.63\frac{D}{f} = 19.63\left(\frac{0.15\,m}{0.023}\right) = \mathbf{128\,m} \quad \textit{Ans. (a)}$$

$$\frac{p}{p^*} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]^{1/2} = 5.954 \quad \text{at } Ma_1 = 0.18 \quad \text{and} \quad k = 1.32$$

$$\text{Decrease 50\% to:} \quad p/p^* = 2.977 \quad \text{Solve for: } Ma_2 = 0.358,\ fL^*/D = 3.46$$

$$\text{Solve:} \quad \Delta L^* = (19.63 - 3.46)\frac{D}{f} = 16.17\left(\frac{0.15\,\text{m}}{0.023}\right) = \mathbf{105\,m} \quad \textit{Ans. (c)}$$

9.97 By making a few algebraic substitutions, show that Eq. (9.74), or the relation in Prob. 9.96, may be written in the density form

$$\rho_1^2 = \rho_2^2 + \rho^{*2}\left(\frac{2k}{k+1}\frac{\bar{f}L}{D} + 2\ln\frac{\rho_1}{\rho_2}\right)$$

Why is this formula awkward if one is trying to solve for the mass flow when the pressures are given at sections 1 and 2?

Solution: This much less laborious algebraic derivation is left as a student exercise. There are two awkward bits: (1) we don't know ρ_1 and ρ_2; and (2) we don't know ρ^* either.

9.98 Compressible *laminar* flow, $f \approx 64/\text{Re}$, may occur in capillary tubes. Consider air, at stagnation conditions of 100°C and 200 kPa, entering a tube 3 cm long and 0.1 mm in diameter. If the receiver pressure is near vacuum, estimate (a) the average Reynolds number, (b) the Mach number at the entrance, and (c) the mass flow in kg/h.

Solution: The pipe is choked, "receiver pressure near vacuum," so L = L* and we need only to correctly guess the inlet Mach number and iterate until the Table B.3 value of (fL/D) matches the actual value, with f ≈ 64/Re from laminar pipe theory. Since Re = ρVD/μ and ρV is constant due to mass conservation, Re varies only due to the change in μ with temperature (from about 2.1E–5 in the entrance to 1.9E–5 kg/m·s at the exit). We assume $\mu_{avg} \approx$ 2.0E–5 kg/m·s. Try Ma_1 from 0.1 to 0.2 and find **0.12** to be the best estimate:

$$\mathbf{Ma_1 \approx 0.12} \quad \textit{Ans.}\text{(b)} \quad \text{Table B.3:} \quad (\text{fL/D})_1 \approx 45.4, \text{ also compute}$$

$$T_1 = 372\text{ K},\ V_1 = 46\text{ m/s},\ \rho_1 = 1.85\ \frac{\text{kg}}{\text{m}^3},\ \mathbf{Re_{avg}} = \frac{1.85(46)(0.0001)}{2.0\text{E–}5} = \mathbf{430} \quad \textit{Ans.}\text{(a)}$$

$$\text{Then} \quad f_{\text{laminar}} \approx 64/430 = 0.15,\ f(L/D) = 0.15(300) \approx 45.0 \text{ (close enough for me!)}$$

$$\text{The mass flow is } \dot{m} = \rho_1 A_1 V_1 = 1.85(\pi/4)(0.0001)^2(46) \approx \mathbf{6.74E\text{–}7\ \frac{kg}{s}} \quad \textit{Ans.}\text{(c)}$$

(0.00243 kg/h)

9.99 A compressor forces air through a smooth pipe 20 m long and 4 cm in diameter, as in Fig. P9.99. The air leaves at 101 kPa and 200°C. The compressor data for pressure rise versus mass flow are shown in the figure. Using the Moody chart to estimate $\bar{f}$, compute the resulting mass flow.

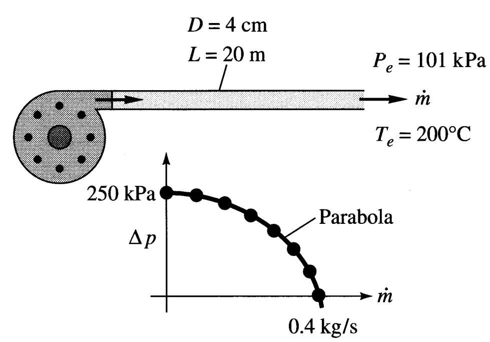


Fig. P9.99

Solution: The compressor performance is approximate by the parabolic relation

$$\Delta p_{\text{compressor}} \approx 250 - 1563\ \dot{m}^2, \text{ with } \Delta p \text{ in kPa and } \dot{m} \text{ in kg/s}$$

We must match this to the pressure drop due to friction in the pipe. For preliminaries, compute $\rho_e = p_e/RT_e = \mathbf{0.744}$ kg/m^3, and $a_e = \sqrt{(kRT_e)} = \mathbf{436}$ m/s. Guess the mass flow:

$$\dot{m} \stackrel{?}{=} 0.2 \text{ kg/s}, \quad \text{then } Re = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.2)}{\pi(0.04)(2.5E\text{–}5)} \approx 255000,\ f_{Moody} \approx 0.015$$

$$V_e = \frac{Re\mu}{\rho_e D} = \frac{255000(2.5E\text{–}5)}{0.744(0.04)} \approx 214\ \frac{m}{s}, \quad Ma_e = \frac{214}{436} = 0.491, \quad \text{read } \left.\frac{fL}{D}\right|_e \approx 1.15,$$

$$\text{also read} \quad p_e/p^* \approx 2.18,\ p^* = \frac{101000}{2.18} \approx 46300\,\text{Pa}$$

Then, at the pipe entrance (Sect. 1), we may compute fL/D and find the pressure there:

$$\left.\frac{fL}{D}\right|_1 = 1.15 + \frac{0.015(20)}{0.04} = 8.65, \quad \text{read } Ma_1 \approx 0.248,\ \frac{p_1}{p^*} \approx 4.39,$$

$$\therefore \quad p_2 = 4.39(46300) \quad \text{or} \quad p_2 \approx 203 \text{ kPa},$$

$$\text{or} \quad \Delta p_{pipe} = 203 - 101 \approx 102 \text{ kPa} \quad \text{whereas } \Delta p_{comp} \approx 187\,\text{kPa } (\dot{m} \text{ too small})$$

We increase the mass flow until $\Delta p_{pipe} \approx \Delta p_{compressor}$. The final converged result is:

$$\dot{m} = \mathbf{0.256}\ \frac{kg}{s},\ Re \approx 326000,\ f \approx 0.0142,\ V_e \approx 274\ \frac{m}{s},\ Ma_e \approx 0.628,\ \text{read}$$

$$fL/D|_e \approx 0.39,\ p_e/p^* \approx 1.68,\ p^* \approx 60.1 \text{ kPa, add } f\Delta L/D = 7.1 \text{ to get} \quad fL/D|_1 \approx \mathbf{7.49},$$

$$\text{read } Ma_1 \approx 0.263,\ p_1/p^* \approx 4.14,\ p_2 \approx 249 \text{ kPa},\ \Delta p_{pipe} \approx 148 \text{ kPa} = \Delta p_{compressor} (\text{OK})$$

For these operating conditions, the approximate flow rate is **0.256 kg/s**. *Ans.*

9.100 Modify Prob. 9.99 as follows: Find the length of 4-cm-diameter pipe for which the pump pressure rise will be exactly 200 kPa.

Solution: With Δp known, we can immediately compute the mass flow and Re, f, etc.:

$$\Delta p = 200 = 250 - 1563\dot{m}^2, \quad \text{solve } \dot{m} \approx 0.179 \text{ kg/s}, \quad Re = \frac{4\dot{m}}{\pi\mu D} = 228000$$

$$V_e = \frac{\mu Re}{\rho D} = 191\ \frac{m}{s},\ Ma_e = \frac{191}{436} \approx 0.44, \quad \text{read } fL/D|_e \approx 1.69,\ p_e/p^* = 2.44,$$

$$p^* = \frac{101}{2.44} = 41.4 \text{ kPa},\ f_{Moody} \approx 0.0152, \quad \text{Now we want } p_1 - p_e = 200 \text{ kPa}$$

Thus $p_1 = 200 + 101 = 301\text{ kPa},\ \dfrac{p_1}{p^*} = \dfrac{301}{41.4} = 7.27,\quad \text{read Ma}_1 \approx 0.151,\ \dfrac{fL}{D}\Big|_1 \approx 27.9$

Then $\dfrac{f\Delta L}{D} = \dfrac{fL}{D}\Big|_1 - \dfrac{fL}{D}\Big|_e = 27.9 - 1.69 \approx 26.2 = \dfrac{0.0152\,L}{0.04},\quad \text{solve } \mathbf{L_{pipe} \approx 69\ m}\quad$ *Ans.*

9.101 How do the compressible-pipe-flow formulas behave for small pressure drops? Let air at 20°C enter a tube of diameter 1 cm and length 3 m. If $\bar{f} = 0.028$ with p_1 = 102 kPa and p_2 = 100 kPa, estimate the mass flow in kg/h for (a) isothermal flow, (b) adiabatic flow, and (c) incompressible flow (Chap. 6) at the entrance density.

Solution: For a pressure change of only 2%, all three estimates are nearly the same. Begin by noting that fL/D = 0.028(3.0/0.01) = **8.4**, and ρ_1 = 102000/[287(293)] ≈ **1.213** kg/m^3. Take these estimates in order:

(a) Isothermal: $\left(\dfrac{\dot{m}}{A}\right)^2 = \dfrac{p_1^2 - p_2^2}{RT[fL/D + 2\ln(p_1/p_2)]} = \dfrac{(102000)^2 - (100000)^2}{287(293)[8.4 + 2\ln(102/100)]} = 569$

Then $\dot{m}/A \approx 23.9,\ \dot{m}_{isothermal} = 23.9(\pi/4)(0.01)^2 \approx \mathbf{0.00187\ \dfrac{kg}{s}}$ *Ans.* (a)

(b) Adiabatic: Given $T_o \approx 293$ K, $a_o = \sqrt{kRT_o} = 343$ m/s, use Eqs. 9.74 and 9.75:

Converges to $\dfrac{V_1}{V_2} = 0.9803,\ V_1^2 = \dfrac{(343)^2[1-(0.9803)^2]}{1.4(8.4) + 2.4\ln(1.02)} = 388,\ \text{or } V_1 \approx 19.7\text{ m/s}$

Then $\dot{m} = \rho_1 A V_1 = 1.213(\pi/4)(0.01)^2(19.7) = \mathbf{0.00188\ kg/s}$ *Ans.* (b)

(c) Incompressible: $\Delta p = (fL/D)(\rho/2)V^2$, or $2000 = (8.4)(1.213/2)V^2$, or $V \approx 19.8$ m/s.

Then $\dot{m}_{incompressible} = \rho AV = 1.213(\pi/4)(0.01)^2(19.8) \approx \mathbf{0.00189\ kg/s}$. *Ans.* (c)

9.102 Air at 550 kPa and 100°C enters a smooth 1-m-long pipe and then passes through a second smooth pipe to a 30-kPa reservoir, as in Fig. P9.102. Using the Moody chart to compute *f*, estimate the mass flow through this system. Is the flow choked?

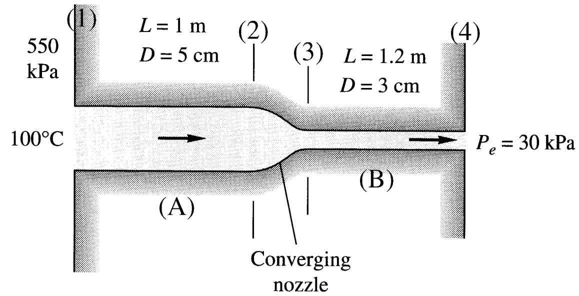


Fig. P9.102

Solution: Label the pipes "A" and "B" as shown. Given $(L/D)_A = 20$ and $(L/D)_B = 40$. Label the relevant sections 1, 2, 3, 4 as shown. With $p_{o1}/p_e = 550/30 = 18.3$, these short pipes are *sure* to be choked, with an exit pressure p_4 much larger than 30 kPa. One way is to guess Ma_1 and work your way through to section 4 to require $Ma_4 = 1.0$ (choked). Take a constant average viscosity $\mu = 2.2E{-}5$ kg/m·s. Assume isentropic expansion to section 1 from the reservoir, frictional flow through pipe A, isentropic expansion from 2 to 3, and a second frictional flow through pipe B to section 4. The correct solution is $Ma_1 \approx 0.18$:

$$Ma_1 = 0.18,\ T_1 = \frac{373\text{ K}}{1+0.2(0.18)^2} = 371\text{ K},\quad p_1 = \frac{450000\text{ Pa}}{[1+0.2(0.18)^2]^{3.5}} = 440000\,\text{Pa},$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{440000}{287(371)} = 4.14\ \frac{\text{kg}}{\text{m}^3},\quad V_1 = Ma_1\sqrt{kRT_1} = 69.5\ \frac{\text{m}}{\text{s}},\quad \text{Re}_1 = \frac{\rho_1 V_1 D_A}{\mu} = 653{,}000$$

$$\text{Moody chart: } f_A \approx 0.0125;\ \text{Table B.3: } \frac{fL}{D}\Big|_1 = 18.54,\ \frac{fL}{D}\Big|_2 = 18.54 - 0.0125(20) = 18.29$$

Pipe A is so short that the Mach number hardly changes. At $(fL/D)_2 = 18.29$, read $Ma_2 \approx 0.181$. Now, at $Ma_2 = 0.181$, determine $A_2/A^* = 3.26$, hence $A_3/A^* = (3.26)(3/5)^2 = 1.17$, read $Ma_3 = 0.613$ and $(fL/D)_3 = 0.442$. Stop to calculate $\rho_3 = 3.49$ kg/m^3, $V_3 = 229$ m/s, $Re_B = 1.09E6$, from the Moody chart, $f_B = 0.0115$. Then $(fL/D)_4 = 0.442 - 0.0115(40) =$ **–0.018**. (?) This last value should have been exactly $(fL/D)_4 = 0$ if the exit Mach number is 1.0. But we were close. The mass flow follows from the conditions at section 1:

$$\dot{m} = \rho_1 A_1 V_1 = \left(4.14\ \frac{\text{kg}}{\text{m}^3}\right)\left[\frac{\pi}{4}(0.05\text{ m})^2\right]\left(69.5\ \frac{\text{m}}{\text{s}}\right) \approx \mathbf{0.565\ \frac{kg}{s}}\quad \textit{Ans.}$$

EES can barely improve upon this: $Ma_1 =$ **0.1792**, yielding a mass flow of **0.5616 kg/s**. The exit pressure is $p_4 = 201$ kPa, far larger than the receiving reservoir pressure of 30 kPa.

9.103 Natural gas, with $k \approx 1.3$ and a molecular weight of 16, is to be pumped through 100 km of 81-cm-diameter pipeline. The downstream pressure is 150 kPa. If the gas enters at 60°C, the mass flow is 20 kg/s, and $\bar{f} = 0.024$, estimate the required entrance pressure for (a) isothermal flow and (b) adiabatic flow.

Solution: The gas constant is $R_{gas} = 8314/16 \approx 520$ J/kg·K. First use Eq. 9.73:

$$\text{(a) Isothermal:}\quad \left(\frac{\dot{m}}{A}\right)^2 = \left[\frac{20}{(\pi/4)(0.81)^2}\right]^2 = \frac{p_1^2 - p_2^3}{RT[fL/D + 2\ln(p_1/p_2)]}$$

$$= \frac{p_1^2 - (150000)^2}{520(333)[2963 + 2\ln(p_1/150000)]},$$

$$\text{solve for } \mathbf{p_1 \approx 892\ kPa} \quad \textit{Ans. (a)}$$

Part (a) indicates a low inlet Mach number, ≈ 0.02, so $T_e \approx T_o$, $a_e \approx a_o \approx 475$ m/s. Then use Eqs. (9.74) and (9.75)—the latter simply indicates that the bracket [] $\approx$ 1.000. Then

$$\frac{V_1}{V_2} \approx \frac{p_2}{p_1} \quad \text{and} \quad V_1^2 = \frac{a_o^2[1-(V_1/V_2)^2]}{k(fL/D)+(k+1)\ln(V_2/V_1)} = \frac{(475)^2[1-(V_1/V_2)^2]}{1.3(2963)+2.3\ln(V_2/V_1)},$$

$$\text{plus } \rho_1 V_1 = \rho_2 V_2 = \dot{m}/A = 38.81\ \text{kg/s·m}^2.\ \text{Solve for } V_1 = 7.54\ \frac{m}{s},\ \rho_1 = 5.15\ \frac{kg}{m^3},$$

$$\mathbf{p_1} = \rho_1 R T_1 = 5.15(520)(333) \approx \mathbf{892\ kPa}$$

9.104 A tank of oxygen (Table A.4) at 20°C is to supply an astronaut through an umbilical tube 12 m long and 1.5 cm in diameter. The exit pressure in the tube is 40 kPa. If the desired mass flow is 90 kg/h and f = 0.025, what should be the air pressure in the tank?

Solution: For oxygen, from Table A.4, take $k = 1.40$ and $R = 260$ J/kg·K. Given T_o = 293 K and $f\Delta L/D = (0.025)(12\ \text{m})/(0.015\ \text{m}) = 20$. Use isothermal flow, Eq. (9.73), as a first estimate:

$$\left(\frac{\dot{m}}{A}\right)^2 = \left[\frac{90/3600\ \text{kg/s}}{(\pi/4)(0.015\ \text{m})^2}\right]^2 = \frac{p_1^2 - p_2^2}{RT[fL/D + 2\ln(p_1/p_2)]}$$

$$= \frac{p_1^2 - (40000)^2}{(260)(293)[20 + 2\ln(p_1/40000)]} \quad \text{Solve for } p_1 \approx 192\ \text{kPa}$$

This is a very good estimate of p_1, but we really need *adiabatic* flow, Eqs. (9.66) and (9.68*a*). First estimate the entrance Mach number from p_1 and $T_1 \approx T_o$:

$$\rho_1 \approx \frac{p_1}{RT_o} = \frac{192000}{260(293)} \approx 2.52\ \frac{\text{kg}}{\text{m}^3},\quad \dot{m} = \frac{90\text{ kg}}{3600\text{ s}} = \rho_1 A V_1 \approx (2.52)\frac{\pi}{4}(0.015\,\text{m})^2 V_1$$

$$\text{solve}\quad V_1 \approx 56\text{ m/s},\quad a_1 \approx \sqrt{kRT_o} = \sqrt{1.4(260)(293)} \approx 327\text{ m/s},\ Ma_1 \approx \frac{56}{327} \approx 0.17$$

We can guess Ma_1 around 0.17, find $(fL^*/D)_1$, subtract $(f\Delta L/D) = 20$, find the new Mach number and p^*, thence back up to obtain p_1. Iterate to convergence. For example:

$\text{Ma}_1 = 0.17$, $(fL^*/D)_1 = 21.12$, $p_1/p^* = 6.43$, $(fL^*/D)_2 = 21.12 - 20 = 1.12$, compute

$\text{Ma}_2 = 0.49$, $p_2/p^* = 2.16$, $p^* = 40000/2.16 = 18500$ Pa, $p_1 = 6.43(18500) = 119000$ Pa,

$$T_1 = T_o/[1 + 0.2(0.17)^2] = 291\text{ K},\ \rho_1 = 1.57\text{ kg/m}^3,\ \text{mass flow} = \rho_1 A V_1 \approx 55\text{ kg/h}$$

The mass flow is too low, so try Ma_1 a little higher. The iteration is remarkably sensitive to Mach number because the correct exit flow is close to sonic. The final converged solution is

$$\text{Ma}_1 = 0.1738,\ \text{Ma}_2 = 0.7792,\quad \boldsymbol{p_1 = 189.4\ \textbf{kPa}}\quad \textit{Ans.}$$

This problem is clearly well suited to EES, which converges rapidly to the final pressure.

9.105 Air enters a 5-cm-diameter pipe at $p_1 = 200$ kPa and $T_1 = 350$ kPa. The downstream receiver pressure is 74 kPa and the friction factor is 0.02. If the exit is choked, what is (a) the length of the pipe, and (b) the mass flow? (c) If p_1, T_1 and $p_{receiver}$ stay the same, what pipe length will cause the mass flow to increase by 50% over (b)? *Hint*: In (c) the exit pressure does not equal receiver pressure.

Solution: (a) Here the exit pressure *does* equal the receiver pressure:

$$\frac{p_1}{p^*} = \frac{200}{74} = 2.70;\quad \text{Table B.3:}\quad \text{read}\ \ Ma_1 = 0.399,\ f\frac{L}{D} = 2.327,$$

$$\therefore\quad L = \frac{2.327(0.05)}{0.02} = \mathbf{5.82\ m}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)}\ V = Ma_1 a_1 = 0.399\sqrt{1.4(287)(350)} = 150\ \frac{\text{m}}{\text{s}},\quad \rho_1 = \frac{p_1}{RT_1} = \frac{200000}{287(350)} = 1.99\ \frac{\text{kg}}{\text{m}^3}$$

$$\dot{m} = \rho_1 A_1 V_1 = (1.99)\frac{\pi}{4}(0.05)^2(150) = \mathbf{0.585\ \frac{kg}{s}}\quad \textit{Ans.}\ \text{(b)}$$

(c) If mass flow increases 50%, and ρ_1 and A_1 are the same, then V_1 and Ma_1 must increase 50%, hence we can immediately calculate the new Mach number:

$$Ma_{1,new} = 1.5(0.399) = 0.599;$$

$$\text{Table B.3:} \quad f\frac{L_{new}}{D} = 0.497, \quad L_{new} = 0.497\frac{0.05}{0.02} = \mathbf{1.24\,m} \quad \textit{Ans.}\text{ (c)}$$

Check in Table B.3 that the exit pressure is $p^*_{new} = 113\text{ kPa} > 74\text{ kPa} = p_{receiver}$.

9.106 Air at 300 K flows through a duct 50 m long with $\bar{f} = 0.019$. What is the minimum duct diameter which can carry the flow without choking if the entrance velocity is (a) 50 m/s, (b) 150 m/s, and (c) 420 m/s?

Solution: With velocities and speed of sound known, compute Ma and get fL*/D:

$$\text{If } T_1 = 300\text{ K},\ a_1 = \sqrt{1.4(287)(300)} \approx 347\text{ m/s}$$

$$\text{(a) } V_1 = 50\text{ m/s},\ Ma_1 = \frac{50}{347} = 0.144,\ \text{read } \frac{fL^*}{D} \approx 30.6 = \frac{0.019(50)}{D},\ D < \mathbf{0.031\,m} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) } V_1 = 150\text{ m/s},\ Ma_1 = 0.432,\ \text{read } fL^*/D \approx 1.80 = 0.019(50)/D,\ D < \mathbf{0.53\,m} \quad \textit{Ans.}\text{ (b)}$$

$$\text{(c) } V_1 = 420\text{ m/s},\ Ma_1 = 1.21,\ \text{read } fL^*/D \approx 0.036 = 0.019(50)/D,\ D < \mathbf{26\,m} \quad \textit{Ans.}\text{ (c)}$$

9.107 A fuel-air mixture, assumed equivalent to air, enters a duct combustion chamber at V_1 = 104 m/s and T_1 = 300 K. What amount of heat addition in kJ/kg will cause the exit flow to be choked? What will be the exit Mach number and temperature if 504 kJ/kg is added during combustion?

Solution: Evaluate stagnation temperature and initial Mach number:

$$T_o = T_1 + \frac{V_1^2}{2c_p} = 300 + \frac{(104)^2}{2(1005)} = 305\text{ K}; \quad Ma_1 = \frac{104}{\sqrt{1.4(287)(300)}} \approx 0.30$$

$$\text{Table B.4:} \quad T_o/T_o^* = 0.3469, \quad \text{hence } T_o^* \approx \left(\frac{305}{0.3469}\right) \approx 880\text{ K}$$

$$\text{Thus} \quad q_{choke} = c_p\Delta T_{o,max} = 1005(880 - 305) \approx 5.78\text{E5 J/kg} = \mathbf{578\ kJ/kg} \quad \textit{Ans.}\text{ (a)}$$

A heat addition of 504 kJ/kg is (just barely) less than maximum, should nearly choke:

$$T_{o2} = T_{o1} + \frac{q}{c_p} = 305 + \frac{540000}{1005} \approx 842\text{ K},\quad \frac{T_{o2}}{T_o^*} = \frac{842}{880} = 0.957,\quad \therefore\ Ma_2 \approx \mathbf{0.78}\quad \textit{Ans.}\text{ (b)}$$

Finally, <u>without using Table B.4</u>, $T_2 = 842/[1+0.2(0.78)^2] \approx \mathbf{751\ K}$ *Ans.* (c)

9.108 What happens to the inlet flow of Prob. 9.107 if the combustion yields 1500 kJ/kg heat addition and p_{o1} and T_{o1} remain the same? How much is the mass flow reduced?

Solution: The flow will choke down to a lower mass flow such that $T_{o2} = T_o^*$:

$$T_{o2} = T_o^* = 305 + \frac{1500000}{1005} = 1798\text{ K},\quad \text{thus } \frac{T_{o1}}{T_o^*} = \frac{305}{1798} = 0.17,\quad Ma_{1,new} \approx \mathbf{0.198}$$

$$(\dot{m}/A)_{new} = \rho_1 V_1 = \rho_1 a_1 Ma_1 = \rho_o a_o Ma_1/[1+0.2Ma_1^2]^3 \quad \text{if } p_{o1},\ T_{o1},\ \rho_{o1} \text{ are the same.}$$

$$\text{Then}\quad \frac{\dot{m}_{new}}{\dot{m}_{old}} = \frac{0.198}{0.30}\left[\frac{1+0.2(0.30)^2}{1+0.2(0.198)^2}\right] \approx \mathbf{0.68}\ \text{(about 32\% less flow)}\quad \textit{Ans.}$$

9.109 A jet engine at 7000-m altitude takes in 45 kg/s of air and adds 550 kJ/kg in the combustion chamber. The chamber cross section is 0.5 m^2, and the air enters the chamber at 80 kPa and 5°C. After combustion the air expands through an isentropic converging nozzle to exit at atmospheric pressure. Estimate (a) the nozzle throat diameter, (b) the nozzle exit velocity, and (c) the thrust produced by the engine.

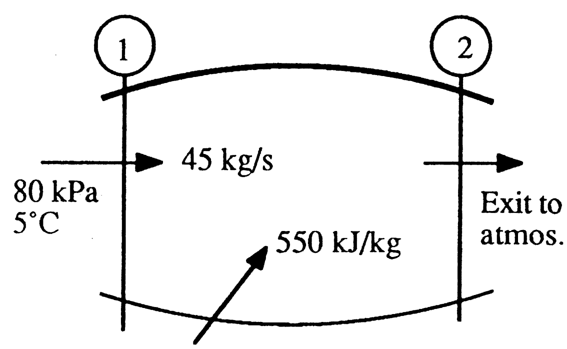


Fig. P9.109

Solution: At 700-m altitude, $p_a = 41043$ Pa, $T_a = 242.66$ K to use as exit conditions.

$$\rho_1 = \frac{p_1}{RT_1} = \frac{80000}{287(278)} = 1.00\ \frac{kg}{m^3},\quad \dot{m} = 45\ \frac{kg}{s} = \rho AV = 1.00(0.5)V_1,\quad V_1 = 90\,m/s$$

$$\text{Ma}_1 = \frac{90}{\sqrt{1.4(287)(278)}} = \mathbf{0.27}, \quad \text{Table B.4: } T_{o1}/T_o^* \approx 0.29,$$

$$T_{o1} = 278 + (90)^2/[2(1005)] \approx 282\text{ K}, \quad \therefore\ T_o^* = 282/0.29 \approx \mathbf{973\ K}$$

$$\text{Add heat: } T_{o2} = 282 + \frac{550000}{1005} \approx 829\text{ K}, \quad \text{thus } \frac{T_{o2}}{T_o^*} = \frac{829}{973} \approx 0.85, \quad \text{read } \text{Ma}_2 \approx \mathbf{0.63}$$

$$\text{also read } p_1/p^* \approx 2.18,\ p_2/p^* \approx 1.54, \quad \therefore\ p_2 = 80(1.54/2.18) \approx 57\text{ kPa},$$

$$p_{o2} = p_2\left[1 + 0.2\,\text{Ma}_2^2\right]^{3.5} = 57[1 + 0.2(0.63)^2]^{3.5} \approx 74\,\text{kPa}$$

With data now known at section 2, expand isentropically to the atmosphere:

$$\frac{p_e}{p_{o2}} = \frac{41043}{57000} = 0.72 = \left[1 + 0.2\text{Ma}_e^2\right]^{-3.5}, \quad \text{solve } \text{Ma}_e = 0.70, \ \frac{T_e}{T_{o2}} = \frac{T_e}{829} = 0.910\ ,$$

$$\text{Solve } T_e \approx 755\text{ K}, \quad \rho_e = p_e/RT_e \approx 0.189\text{ kg/m}^3, \quad a_e = \sqrt{kRT_e} = 551\text{ m/s}\ ,$$

$$V_e = \text{Ma}_e a_e \approx \mathbf{385\ m/s} \quad \textit{Ans. (b)}$$

$$\dot{m} = 45 = 0.189(385)\frac{\pi}{4}D_e^2, \quad \text{solve } D_e \approx \mathbf{0.89\ m} \quad \textit{Ans. (a)}$$

$$\text{Finally, if } p_e = p_{atm}, \text{ from Prob. 3.68, } F_{thrust} = \dot{m}V_e = 45(385) \approx \mathbf{17300\ N} \quad \textit{Ans. (c)}$$

9.110 Compressible pipe flow with heat addition, Sec. 9.8, assumes constant momentum ($p + \rho V^2$) and constant mass flow but variable stagnation enthalpy. Such a flow is often called *Rayleigh flow*, and a line representing all possible property changes on an temperature-entropy chart is called a *Rayleigh line*. Assuming air passing through the flow state $p_1 = 548$ kPa, $T_1 = 588$ K, $V_1 = 266$ m/s, and $A = 1$ m^2, draw a Rayleigh curve of the flow for a range of velocities from very low ($\text{Ma} \ll 1$) to very high ($\text{Ma} \gg 1$). Comment on the meaning of the maximum-entropy point on this curve.

Solution: First evaluate the Mach number and density at the reference state:

$$\rho = \frac{p}{RT} = \frac{548000}{287(588)} \approx 3.25\ \frac{\text{kg}}{\text{m}^3}; \quad \text{Ma} = \frac{V}{\sqrt{kRT}} = \frac{266}{\sqrt{1.4(287)(588)}} = 0.55$$

Our basic algebraic equations are then:

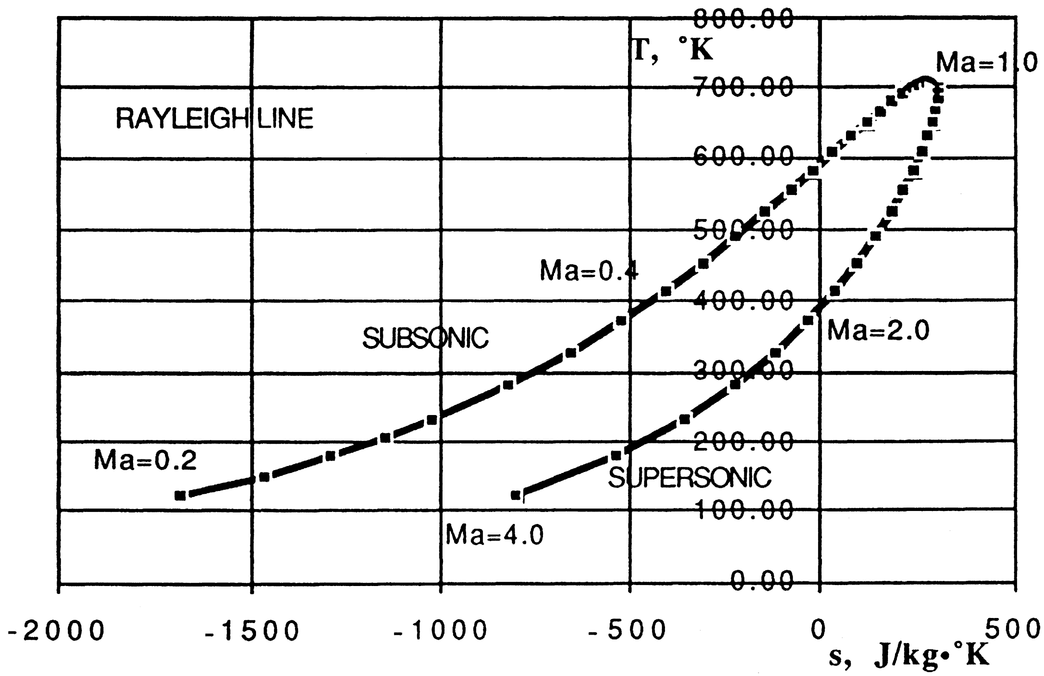
$$\text{Momentum: } \mathbf{p + \rho V^2} = 548000 + 3.25(266)^2 = \mathbf{778000} \quad \text{(a)}$$

$$\text{Continuity: } \rho V = 3.25(266), \quad \text{or: } \rho = \mathbf{864/V} \quad \text{(b)}$$

$$\text{Entropy: } \mathbf{s = 718\ln(T/588) - 287\ln(\rho/3.25)} \quad \text{(c)}$$

We simply let V vary from, say, 10 m/s to 800 m/s, compute ρ from (b), p from (a), $T = p/\rho T$, and s from (c), then plot T versus s. [We have arbitrarily set s = 0 at state 1.]

The result of this exercise forms the **Rayleigh Line** for this flow, shown below. Some Mach numbers are listed, subsonic on the top, supersonic on the bottom, and exactly **sonic** at the right-hand (maximum-entropy) side. *Ans.*



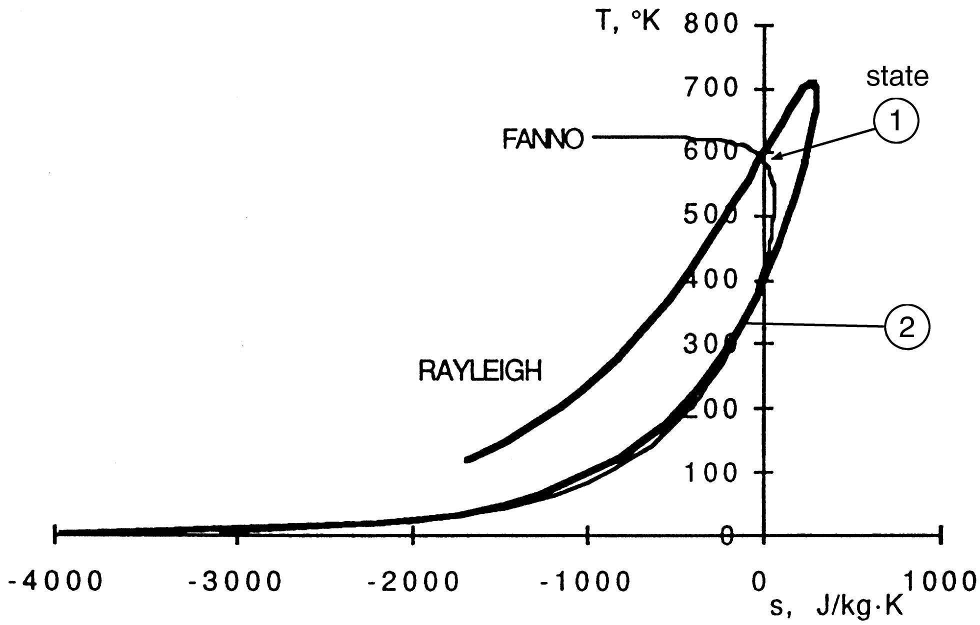
9.111 Add to your Rayleigh line of Prob. 9.110 a Fanno line (see Prob. 9.94) for stagnation enthalpy equal to the value associated with state 1 in Prob. 9.110. The two curves will intersect at state 1, which is subsonic, and also at a certain state 2, which is supersonic. Interpret these two cases vis-a-vis Table B.2.

Solution: For $T_1 = 588$ K and $V_1 = 266$ m/s, the stagnation temperature is

$$T_o = T_1 + \frac{V^2}{2c_p} = 588 + \frac{(266)^2}{2(1005)} \approx \mathbf{623\ K}; \quad \text{Elsewhere,}\ T = 623 - \frac{V^2}{2(1005)} \qquad \textbf{(d)}$$

Also, from Prob. 9.110, $\rho = 864/V$ (**b**) and $s = 728\ln\left(\frac{T}{588}\right) - 287\ln\left(\frac{\rho}{3.25}\right)$ **(c)**

By varying V and computing (T, ρ, s) from (b, c, d), we plot the Fanno line and add it to the previous Rayleigh line. The composite graph is as follows:



The subsonic intersection is state 1, $Ma_1 \approx$ **0.55**, $T_2 \approx 588$ K, and the supersonic intersection is at $Ma_2 \approx$ **2.20**, where, for example, $T_2 \approx 316$ K. Also, $s_1 > s_2$. These two points thus correspond to the two sides of a **normal shock wave**, where "2" is the supersonic upstream and "1" the subsonic downstream condition. We may check these results in Table B-2, where, at Ma ≈ **2.20**, the temperature ratio across the shock is 1.857—for *our* calculations, this ratio is 588 K/316 K ≈ 1.86 (agreement would be perfect if we kept more significant figures). Shock flow satisfies **all** the four equations of Rayleigh and Fanno flow combined—continuity, momentum, energy, and the equation of state.

9.112 Air enters a duct subsonically at section 1 at 1.2 kg/s. When 650 kW of heat is added, the flow chokes at the exit at $p_2 = 95$ kPa and $T_2 = 700$ K. Assuming frictionless heat addition, estimate (a) the velocity; and (b) the stagnation pressure at section 1.

Solution: Since the exit is choked, $p_2 = p^*$ and $T_2 = T^*$ and, of course, $Ma_2 = 1.0$. Then

$$V_2 = V^* = \sqrt{kRT^*} = \sqrt{1.4(287)(700)} \approx 530 \text{ m/s}, \quad \text{and} \quad q = \dot{Q}/\dot{m} = \frac{650}{1.2} = 542\ \frac{\text{kJ}}{\text{kg}}$$

$$\text{Also,} \quad T_o^* = 1.2T_2 = 1.2(700) = 840 \text{ K}; \quad \text{hence } T_{o1} = 840 - \frac{542000}{1005} \approx 301 \text{ K}$$

$$\text{Then}\quad T_{o1}/T_o^* = \frac{301}{840} = 0.358;\quad \text{Table B.4: read } Ma_1 \approx \mathbf{0.306},\ \text{read } V_1/V^* \approx 0.199$$

$$\text{So}\quad V_1 = 530(0.199) \approx \mathbf{105\ m/s}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{Also read}\quad p_{o1}/p_o^* \approx 1.196,\quad \text{where } p_o^* = p_2/0.5283 \approx 180\ \text{kPa},$$

$$\text{Hence}\quad p_{o1} \approx 1.196(180) \approx \mathbf{215\ kPa}\quad \textit{Ans.}\ \text{(b)}$$

9.113 Air enters a constant-area duct at $p_1 = 90$ kPa, $V_1 = 520$ m/s, and $T_1 = 558°C$. It is then *cooled* with negligible friction until it exists at $p_2 = 160$ kPa. Estimate (a) V_2; (b) T_2; and (c) the total amount of cooling in kJ/kg.

Solution: We have enough information to estimate the inlet Ma_1 and go from there:

$$a_1 = \sqrt{1.4(287)(558+273)} = 578\ \frac{m}{s},\quad \therefore\ Ma_1 = \frac{520}{578} \approx \mathbf{0.90},\quad \text{read } \frac{p_1}{p^*} = 1.1246,$$

$$\text{or}\quad p^* = \frac{90}{1.1246} = 80.0\ \text{kPa},\quad \text{whence } \frac{p_2}{p^*} = \frac{160}{80} \approx 2.00,\quad \text{read } Ma_2 \approx \mathbf{0.38},$$

$$\text{read}\quad T_2/T^* = 0.575,\ V_2/V^* = 0.287,\ T_{o2}/T_o^* \approx 0.493$$

We have to back off to section 1 to determine the critical (*) values of T, V, T_o:

$$Ma_1 = 0.9,\ T_1/T^* = 1.0245,\ T^* = \frac{558+273}{1.0245} = 811\ \text{K},\ T_2 = 0.575(811) \approx \mathbf{466\ K}\quad \textit{Ans.}\ \text{(b)}$$

$$\text{also,}\quad V_1/V^* = 0.911,\ V^* = \frac{520}{0.911} = 571\ \text{m/s},\quad \text{so } V_2 = 0.287(571) \approx \mathbf{164\ m/s}\quad \textit{Ans.}\ \text{(a)}$$

$$T_{o1}/T_o^* = 0.9921,\ \text{where } T_{o1} = T_1 + V_1^2/2c_p = 966\ \text{K},\ T_o^* = \frac{966}{0.9921} = 973\ \text{K}$$

$$\text{Finally,}\quad T_{o2} = 0.493(973) = 480\ \text{K},$$

$$\mathbf{q_{cooling}} = c_p\Delta T_o = 1.005(966-480) \approx \mathbf{489\ \frac{kJ}{kg}}\quad \textit{Ans.}\ \text{(c)}$$

9.114 We have simplified things here by separating friction (Sec. 9.7) from heat addition (Sec. 9.8). Actually, they often occur together, and their effects must be evaluated simultaneously. Show that, for flow with friction *and* heat transfer in a constant-diameter pipe, the continuity, momentum, and energy equations may be combined into the following differential equation for Mach-number changes:

$$\frac{d\mathrm{Ma}^2}{\mathrm{Ma}^2} = \frac{1+k\mathrm{Ma}^2}{1-\mathrm{Ma}^2}\frac{dQ}{c_pT} + \frac{k\mathrm{Ma}^2[2+(k-1)\mathrm{Ma}^2]}{2(1-\mathrm{Ma}^2)}\frac{f\,dx}{D}$$

where *dQ* is the heat added. A complete derivation, including many additional combined effects such as area change and mass addition, is given in chap. 8 of Ref. 8.

Solution: This derivation is algebraically complicated and is left as an exercise for the student. One can set good problems using this equation for studies in combined friction and heat transfer, using the Reynolds analogy between friction and heat transfer [Ref. 8].

9.115 Air flows subsonically in a duct with negligible friction. When heat is added in the amount of 948 kJ/kg, the pressure drops from p_1 = 200 kPa to p_2 = 106 kPa. Using one-dimensional theory, estimate (a) Ma_1; (b) T_1; and (c) V_1.

Solution: We need one missing piece of information: $\mathbf{T_{o1} = 305\ K}$. Then guess Ma_1:

$$Ma_1 \overset{?}{=} 0.2: \quad \text{read } T_{o1}/T_o^* = 0.1736,\ p_1/p^* = 2.2727, \text{ then } p_2/p^* = 2.2727\left(\frac{106}{200}\right) \approx 1.205$$

$$\text{then read} \quad Ma_2 \approx 0.84,\ T_{o2}/T_o^* = 0.978,\ T_{o2} = 305\left(\frac{0.978}{0.1736}\right) \approx 1719\text{ K, compute}$$

$$q = c_p\Delta T_o = 1.005(1719 - 305) \approx 1420\text{ kJ/kg (too much, } >948 \text{ as given)}$$

$$Ma_1 \overset{?}{=} 0.3: \quad \text{gives } p_2/p^* = 1.129,\ Ma_2 \approx 0.90,\ T_{o2} \approx 887\text{ K},\ q = c_p\Delta T_o = 585\ \frac{kJ}{kg}(<948)$$

$$\text{Converges to} \quad Ma_1 = \mathbf{0.24} \quad \textit{Ans. (a)} \quad T_1 = \mathbf{302\ K} \quad \textit{Ans. (b)}$$

$$T_{o2} \approx 1247\text{ K}, \quad q = 947\ \frac{kJ}{kg}\text{(OK)}$$

$$\text{Also,} \quad V_1 = Ma_1\sqrt{kRT_1} = 0.24\sqrt{1.4(287)(302)} \approx \mathbf{84\ m/s} \quad \textit{Ans. (b)}$$

9.116 An observer at sea level does not hear an aircraft flying at 12000 ft standard altitude until it is 5 statute miles past her. Estimate the aircraft speed in ft/sec.

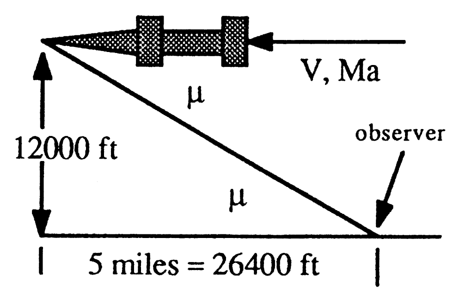


Fig. P9.116

Solution: The average temperature over this range is 498°R, hence

$$\bar{a} = \sqrt{kRT} = \sqrt{1.4(1717)(498)} = 1094 \text{ ft/s}, \quad \text{and} \quad \tan\mu = \frac{12000}{26400} = 0.455,$$

$$\text{or:} \quad \mu \approx 24.4°, \; Ma_{plane} = \csc\mu \approx 2.42, \; \mathbf{V_{plane}} = 2.42(1094) \approx \mathbf{2648\ \frac{ft}{s}} \quad \textit{Ans.}$$

9.117 An observer at sea level does not hear an aircraft flying at 6000-m standard altitude until 15 seconds after it has passed overhead. Estimate the aircraft speed in m/s.

Solution: This is a messier version of Prob. 9.116 above. The average temperature over this range of altitudes is 268 K. Then the appropriate Mach-wave geometry is

$$\mu = \tan^{-1}\left(\frac{6000}{15V}\right) = \sin^{-1}\left(\frac{1}{Ma}\right) = \sin^{-1}\left(\frac{a}{V}\right), \quad \text{where } a = \sqrt{1.4(287)(268)} \approx 328 \text{ m/s}$$

$$\text{Solve iteratively for} \quad \mu \approx 35°, \quad Ma \approx 1.74, \quad \mathbf{V_{plane}} = 1.74(328) \approx \mathbf{572\ \frac{m}{s}} \quad \textit{Ans.}$$

9.118 A particle moving at uniform velocity in sea-level standard air creates the two disturbance spheres shown in Fig. P9.118. Compute the particle velocity and Mach number.

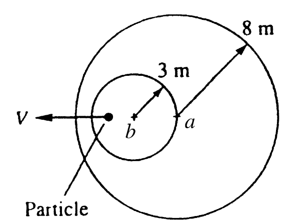


Fig. P9.118

Solution: If point "a" represents t = 0 units, the particle reaches point "b" in 8 – 3 = 5 units. But the distance from *a* to *b* is only 3 units. Therefore the (subsonic) Mach number is

$$Ma = \frac{V\Delta t}{a\Delta t} = \frac{3 \text{ units}}{5 \text{ units}} \approx \mathbf{0.6} \quad \textit{Ans.}\text{ (a)}$$

$$V_{particle} = Ma(a) = 0.6\sqrt{1.4(287)(288\text{ K})} \approx \mathbf{204\ m/s} \quad \textit{Ans.}\text{ (b)}$$

9.119 The particle in Fig. P9.119 is moving supersonically in sea-level standard air. From the two disturbance spheres shown, compute the particle (a) Mach number; (b) velocity; and (c) Mach angle.

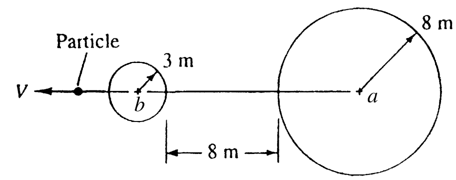


Fig. P9.119

Solution: If point "a" represents t = 0 units, the particle reaches point "b" in 8 – 3 = 5 units. But the distance from *a* to *b* is 8 + 8 + 3 = **19** units. Therefore the Mach number is

$$\mathrm{Ma} = \frac{\mathrm{V}\,\Delta t}{\mathrm{a}\,\Delta t} = \frac{19\text{ units}}{5\text{ units}} \approx \mathbf{3.8} \quad \textit{Ans.}\text{ (a)} \qquad \mu_{\text{wave}} = \sin^{-1}\left(\frac{1}{3.8}\right) \approx \mathbf{15.3°} \quad \textit{Ans.}\text{ (c)}$$

$$\mathrm{V}_{\text{particle}} = \mathrm{Ma(a)} = 3.8\sqrt{1.4(287)(288\text{ K})} \approx \mathbf{1290\ m/s} \quad \textit{Ans.}\text{ (b)}$$

9.120 The particle in Fig. P9.120 is moving in sea-level standard air. From the two disturbance spheres shown, estimate (a) the position of the particle at this instant; and (b) the temperature in °C at the front stagnation point of the particle.

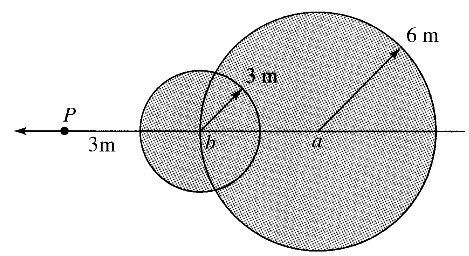


Fig. P9.120

Solution: Given sea-level temperature = 288 K. If point "a" represents t = 0 units, the particle reaches point "b" in 6 – 3 = 3 units. But the distance from *a* to *b* is **6** units. Therefore the particle Mach number is

$$\mathrm{Ma} = \frac{6}{3} = \mathbf{2.0}, \quad \therefore\ \mathrm{T}_{\text{stagnation}} = \mathrm{T_o} = 288[1 + 0.2(2.0)^2] \approx \mathbf{518\ K} \quad \textit{Ans.}\text{ (b)}$$

$\mu = \sin^{-1}\left(\frac{1}{2}\right) = 30°$, and the particle is at point "P" at **6 meters ahead of "b."** *Ans.* (a)

9.121 A thermistor probe, in the shape of a needle parallel to the flow, reads a static temperature of –25°C when inserted in the stream. A conical disturbance of half-angle 17° is formed. Estimate (a) the Mach number; (b) the velocity; and (c) the stagnation temperature of the stream.

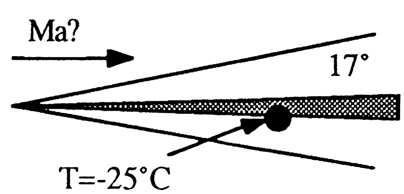


Fig. P9.121

Solution: If the needle is "very thin," it reads the *stream* static temperature, $T_\infty \approx -25°C$ = 248 K. We are given the Mach angle, $\mu = 17°$, so everything else follows readily:

$$\mathrm{Ma}_\infty = \csc\mu = \csc 17° = \mathbf{3.42} \quad \textit{Ans.}\ \text{(a)}$$

$$T_o = 248[1+0.2(3.42)^2] = 828\ \text{K} = \mathbf{555°C} \quad \textit{Ans.}\ \text{(c)}$$

$$U_\infty = \mathrm{Ma}_\infty a_\infty = 3.42\sqrt{1.4(287)(248)} = 3.42(316) \approx \mathbf{1080\ m/s} \quad \textit{Ans.}\ \text{(b)}$$

9.122 Supersonic air takes a 5° compression turn, as in Fig. P9.122. Compute the downstream pressure and Mach number and wave angle, and compare with small-disturbance theory.

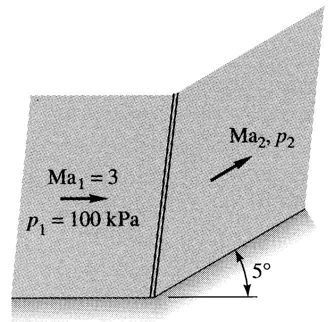


Fig. P9.122

Solution: From Fig. 9.23, $\beta \approx 25°$, and we can iterate Eq. (9.86) to a closer estimate:

$$\mathrm{Ma}_1 = 3.0,\ \theta = 5°,\ \text{compute } \beta = \mathbf{23.133°},$$

$$p_2/p_1 = 1.454, \quad \mathbf{p_2 = 145.4\ Pa}$$

From Eq. 9.83f, compute $\mathbf{Ma_2 = 2.750}$ *Ans.* (a, b, c) Exact oblique shock theory.

This is a small deflection. The linear theory of Eqs. 9.88 and 9.89 is reasonably accurate:

$$\mu = \sin^{-1}\left(\frac{1}{3}\right) = 19.47°, \quad \sin\beta = \sin\mu + \frac{(k+1)\tan\theta}{4\cos\mu} + \cdots = \frac{1}{3} + 0.0557 + \cdots \approx 0.389$$

$$\text{or:} \quad \beta_{\text{linear}} \approx \mathbf{22.9°}\ (1\%\ \text{low}) \quad \frac{\Delta p}{p_1} \approx \frac{1.4(3)^2}{\sqrt{3^2-1}}\tan 5° = 0.39,$$

$$p_2 \approx 100(1.39) \approx \mathbf{139\ kPa}\ (4\%\ \text{low})$$

9.123 Modify Prob. 9.122 as follows: Let the 5° turn be in the form of five separate compression turns of 1° each. Compute the final Mach number and pressure, and compare the pressure with an isentropic expansion to the same final Mach number.

Solution: Even the above 5° turn in 9.122 is nearly isentropic, but let's do it again:

Turn angle:	0°–1°	1°–2°	2°–3°	3°–4°	4°–5°
Wave angle β:	20.158°	20.514°	20.876°	21.245°	21.620°
Ma-downstream:	2.949	2.898	2.849	2.800	**2.753** *Ans.*
Pressure ratio:	1.0802	1.0790	1.0778	1.0766	1.0754

The total pressure ratio across the shock is

$$p_{final}/p_\infty = (1.0802)(1.0790)(1.0778)(1.0766)(1.0754) \approx \mathbf{1.4544} \quad \textit{Ans.}\ \text{(exact)}$$

$$p_{isentropic}/p_\infty = \frac{(p/p_o)_{final}}{(p/p_o)_{stream}} = \left[\frac{1+0.2(2.753)^2}{1+0.2(3.0)^2}\right]^{-3.5} \approx \mathbf{1.4547} \quad \textit{Ans.}\ \text{(nearly the same)}$$

9.124 When a sea-level air flow approaches a ramp of angle 20°, an oblique shock wave forms as in Figure P9.124. Calculate (a) Ma_1; (b) p_2; (c) T_2 ; and (d) V_2.

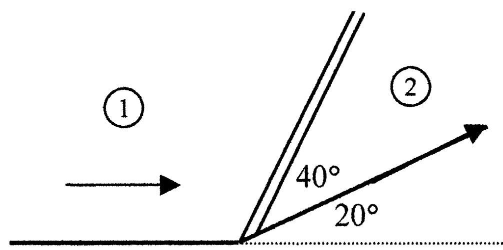


Fig. P9.124

Solution: For sea-level air, take p_1 = 101.35 kPa, T_1 = 288.16 K, and ρ_1 = 1.2255 kg/m^3. (a) The approach Mach number is determined by the specified angles, β = 60° and θ = 20°. From Fig. 9.23 we read that Ma_1 is slightly less than 2.0. More accurately, use Eq. (9.86):

$$\tan\theta = \frac{2\cot\beta\left(Ma_1^2\sin^2\beta - 1\right)}{Ma_1^2(k+\cos 2\beta)+2} \quad \text{for } \theta = 20° \quad \text{and} \quad \beta = 60°$$

Iterate, or use EES, to find that $Ma_1 = \mathbf{1.87}$. *Ans.* (a)

(b, c) With Ma_1 known, use Eqs. (9.83*a, c*) to find p_2 and T_2:

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left(2kMa_1^2\sin^2\beta - k + 1\right) = 2.893,\ p_2 = 2.893(101.35\text{ kPa}) = \mathbf{293\ kPa} \quad \textit{Ans.}\ \text{(b)}$$

$$\frac{T_2}{T_1} = \left[2+(k-1)Ma_1^2\sin^2\beta\right]\frac{2kMa_1^2\sin^2\beta - k+1}{(k+1)^2Ma_1^2\sin^2\beta} = 1.401,$$

$$T_2 = 1.401(288.16\text{ K}) = \mathbf{404\ K} \quad \textit{Ans.}\ \text{(c)}$$

(d) Finally, to find V_2, first find V_1 from the approach Mach number, then use Eq. (9.83b):

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340\ \frac{\text{m}}{\text{s}},\ V_1 = a_1 Ma_1 = (340)(1.87) = 636\ \frac{\text{m}}{\text{s}}$$

$$\frac{V_2}{V_1} = \frac{\cos\beta}{\cos(\beta-\theta)} = \frac{\cos 60°}{\cos 40°} = 0.653,\ V_2 = 0.653(636\text{ m/s}) = \mathbf{415\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(d)}$$

9.125 Show that, as the upstream Mach number approaches infinity, the Mach number downstream of an attached oblique-shock wave will have the value

$$\mathrm{Ma}_2 \approx \sqrt{\frac{k-1}{2k\sin^2(\beta-\theta)}}$$

Solution: This is a limiting result of Eqs. (9.82) and (9.83f) as $\mathrm{Ma}_1 \to \infty$:

$$\text{Eq. (9.83f):} \quad \lim_{\mathrm{Ma}_1\to\infty}\left|\frac{(k-1)\mathrm{Ma}_{1n}^2+2}{2k\mathrm{Ma}_{n1}^2-(k-1)}\right| = \frac{k-1}{2k} = \mathrm{Ma}_{n2}^2 = \mathrm{Ma}_2^2\sin^2(\beta-\theta)$$

$$\text{Solve for} \quad \mathbf{Ma_2} = \sqrt{\frac{\mathbf{k-1}}{\mathbf{2k\sin^2}(\beta-\theta)}} \quad \textit{Ans.}$$

9.126 Consider airflow at $\mathrm{Ma}_1 = 2.2$. Calculate, to two decimal places, (a) the deflection angle for which the downstream flow is sonic; and (b) the maximum deflection angle.

Solution: We are near the peak of the (invisible) curve for $\mathrm{Ma}_1 = 2.2$ in Fig. 9.23. The wave angles are ≈ 65°, which we guess for finding the sonic downstream condition:

$$\mathrm{Ma}_1 = 2.2, \text{ guess } \beta \approx 65°, \text{ and using Eq. 9.86,}$$

$$\text{compute } \theta \approx 26.1° \text{ and } \mathrm{Ma}_2 = 0.92 \text{ (not quite sonic)}$$

$$\text{Converges to} \quad \mathrm{Ma}_2 = 1.000 \text{ when } \beta \approx 61.9° \text{ and } \theta \approx \mathbf{25.9°} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Maximum deflection occurs at} \quad \beta \approx 64.6° \text{ and } \theta_{max} \approx \mathbf{26.1°} \quad \textit{Ans.}\text{ (b)}$$

9.127 Do the Mach waves upstream of an oblique-shock wave intersect with the shock? Assuming supersonic downstream flow, do the downstream Mach waves intersect the shock? Show that for small deflections the shock-wave angle β lies halfway between μ_1 and $\mu_2 + \theta$ for any Mach number.

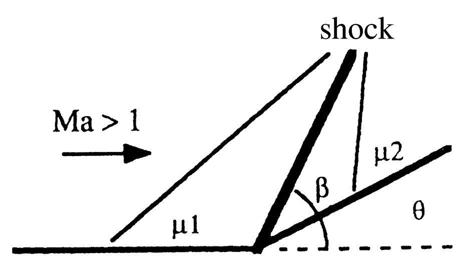


Fig. P9.127

Solution: Yes, Mach waves both upstream and downstream will intersect the shock:

$$\text{Linear theory:} \quad \beta \approx \mu_1 + \frac{(k+1)\mathrm{Ma}_1^2}{4\sqrt{(\mathrm{Ma}_1^2-1)}}\theta \quad \text{and} \quad \beta-\theta \approx \mu_2 - \frac{(k+1)\mathrm{Ma}_1^2}{4\sqrt{(\mathrm{Ma}_1^2-1)}}\theta$$

Thus, to first order (small deflection), the shock wave angle β will lie **halfway between** μ_1 and ($\mu_2 + \theta$), as sketched in the figure above.

9.128 Air flows past a two-dimensional wedge-nosed body as in Fig. P9.128. Determine the wedge half-angle δ for which the horizontal component of the total pressure force on the nose is 35 kN/m of depth into the paper.

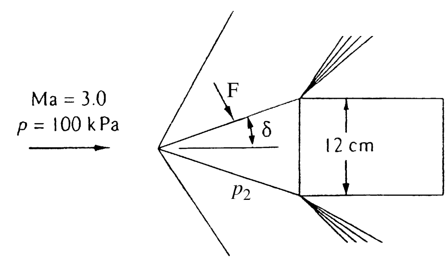


Fig. P9.128

Solution: *Regardless* of the wedge angle δ, the horizontal force equals the pressure inside the shock times the projected vertical area of the nose:

$$F_{horiz} = p_2 A_{vert\ proj} = p_2(0.12\,m)(1.0\,m) = 35000\,N, \quad \text{or} \quad p_2 = 291700\ Pa$$

$$\text{Eq. (9.83a):}\quad \frac{p_2}{p_1} = \frac{291700}{100000} = 2.917 = \frac{1}{2.4}\left[2.8Ma_{1n}^2 - 0.4\right], \quad \text{solve } Ma_1 \sin\beta = 1.63$$

$$\text{or}\quad \beta = \sin^{-1}\left(\frac{1.63}{3.0}\right) = \mathbf{32.81°} \quad \text{Use Eq. (9.86) to compute } \delta_{\mathbf{wedge}} \approx \mathbf{15.5°} \quad \textit{Ans.}$$

9.129 Air flows at supersonic speed toward a compression ramp, as in Fig. P9.129. A scratch on the wall at *a* creates a wave of 30° angle, while the oblique shock has a 50° angle. What is (a) the ramp angle θ; and (b) the wave angle ϕ caused by a scratch at *b*?

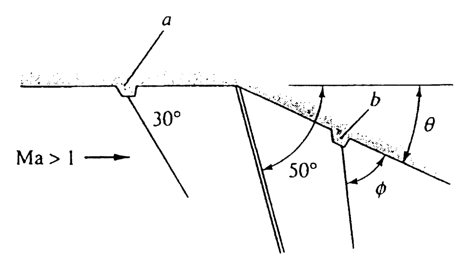


Fig. P9.129

Solution: The two "scratches" cause *Mach waves* which are directly related to Mach No.:

$$\mu_1 = 30°, \quad Ma_1 = \csc 30° = \mathbf{2.0}, \quad \beta = 50°, \quad \text{Eq. 9.86 yields } \theta \approx \mathbf{18.13°} \quad \textit{Ans. (a)}$$

$$\text{Then}\quad Ma_{2n} = 0.690 = Ma_2 \sin(50 - 18.13°),$$

$$Ma_2 = 1.307, \quad \phi = \sin^{-1}\left(\frac{1}{1.307}\right) \approx \mathbf{49.9°} \quad \textit{Ans. (b)}$$

9.130 Modify Prob. P9.129 as follows: If the wave angle ϕ is 42°, determine (a) the shock wave angle (it is *not* 50°) and (b) the deflection angle θ.

Solution: Referring to Fig. P9.129, we already know that $Ma_1 = 2.0$ because the Mach wave angle at point *a* is 30°. Now we know the Mach wave angle inside the shock:

$$\sin(\phi) = \sin(42°) = \frac{1}{Ma_2} \quad \text{or:} \quad Ma_2 = 1.494$$

With $Ma_1 = 2.0$ and $Ma_2 = 1.494$, we can determine β and θ from Eqs. (9.83*f*) and (9.86):

$$Ma_2^2 \sin^2(\beta-\theta) = \frac{(k-1)Ma_1^2\sin^2\beta + 2}{2kMa_1^2\sin^2\beta - k + 1}; \quad \tan\theta = \frac{2\cot\beta\left(Ma_1^2\sin^2\beta - 1\right)}{Ma_1^2(k+\cos 2\beta)+2}$$

This is an excellent job for EES, with the results $\beta = \mathbf{43.78°}$ *Ans.* (a) and $\theta = \mathbf{13.80°}$ *Ans.* (b)

9.131 The following formula has been suggested as an alternate to Eq. (9.86) to relate upstream Mach number to the oblique shock wave angle β and turning angle θ:

$$\sin^2\beta = \frac{1}{Ma_1^2} + \frac{(k+1)\sin\beta\sin\theta}{2\cos(\beta-\theta)}$$

Can you prove or disprove this relation? If not, try a few numerical values and compare with the results from Eq. (9.86).

Solution: The formula is quite correct and serves as an interesting alternative to Eq. (9.86). Notice that one can immediately solve for Ma_1 if β and θ are known, which would have been a great help in Prob. 9.124. For details of the proof, see page 371 of R. M. Olson, *Essentials of Engineering Fluid Mechanics*, 4th ed., Harper and Row, New York, 1980.

9.132 Air flows at Ma = 3 and p = 10 psia toward a wedge of 16° angle at zero incidence, as in Fig. P9.132. (a) If the pointed edge is forward, what is the pressure at point A? If the blunt edge is forward, what is the pressure at point B?

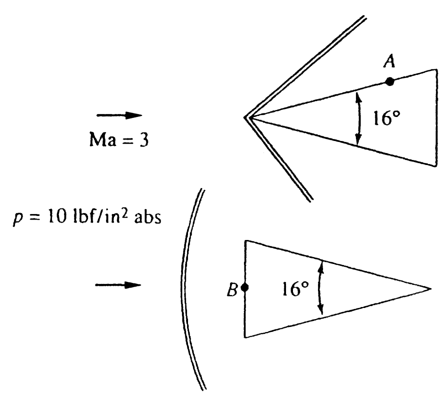


Fig. P9.132

Solution: For Ma = 3, $\theta = 8°$, Eq. 9.86:

$$\beta = 25.61°,$$

$$p_A/p_1 = \frac{2.8(3\sin 25.61°)^2 - 0.4}{2.4} = 1.80,$$

$$\therefore \quad p_A \approx \mathbf{18.0\ psia} \quad \textit{Ans. (a)}$$

(b) A <u>normal shock</u> forms, and $p_B = p_{o2}$ inside the shock. Given $p_{o1} = p_1/0.0272 = 367$ psia, Table B.2, Ma = 3: $p_{o2}/p_{o1} = 0.3283$, hence $p_{o2} = 0.3283(367) =$ **121 psia**. *Ans.* (b)

9.133 Air flows supersonically toward the double-wedge system in the figure. The (x,y) coordinates of the tips are given. Both wedges have 15° deflection angles. The shock wave of the forward wedge strikes the tip of the aft wedge. What is the freestream Mach number?

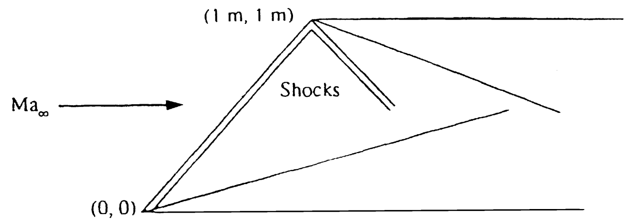


Fig. P9.133

Solution: However tricky the shock reflection might be at the upper (aft) wedge, the problem is solved by knowing the shock angle at the lower (forward) wedge:

$$\beta = \mathbf{tan}^{-1}(1.0) = 45°, \quad \theta = 15°, \quad \textit{Eq.}(9.86)\,\textit{yields} \quad \mathbf{Ma_\infty = 2.01} \quad \textit{Ans.}$$

9.134 When an oblique shock strikes a solid wall, it reflects as a shock of sufficient strength to cause the exit flow Ma_3 to be parallel to the wall, as in Fig. P9.134. For airflow with $Ma_1 = 2.5$ and $p_1 = 100$ kPa, compute Ma_3, p_3, and the angle ϕ.

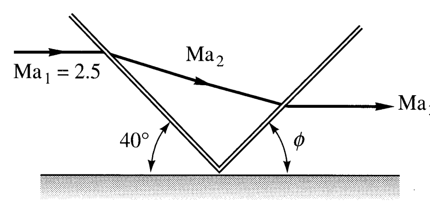


Fig. P9.134

Solution: With $\beta_1 = 40°$, we can compute the first shock deflection, which then must turn back the same amount through the second shock:

$$\mathrm{Ma_{1n}} = 2.5\sin 40° = 1.607; \quad \mathrm{Eq.}(9.86){:}\ \theta_1 = 17.68°,\ \mathrm{Ma_{2n}} = 0.666,\ \mathrm{Ma_2} = 1.754,$$

$$\text{Also } \theta_2 = 17.68°, \quad \text{solve } \beta_2 = 60.45°,\ \mathrm{Ma_{2n}} = 1.526,\ \mathrm{Ma_{3n}} = 0.692 = \mathrm{Ma_2}\sin(\beta_2 - \theta_2),$$

$$\text{Finally} \quad \mathbf{Ma_3 \approx 1.02} \quad \textit{Ans.}\text{ (a)} \quad p_2/p_1 = 2.85,\ p_2 = 285\text{ kPa},\ p_3/p_2 = 2.55.$$

$$\text{Keep going:} \quad p_3 = 2.55(285) \approx \mathbf{727\ kPa} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Finally,} \quad \phi = \beta_2 - \theta_2 = 60.56 - 17.68 \approx \mathbf{42.8°} \quad \textit{Ans.}\text{ (c)}$$

9.135 A bend in the bottom of a supersonic duct flow induces a shock wave which reflects from the upper wall, as in Fig. P9.135. Compute the Mach number and pressure in region 3.

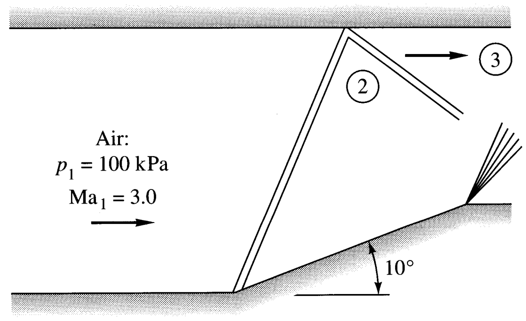


Fig. P9.135

Solution: Given $\theta = 10°$, find state 2:

$$Ma_1 = 3.0, \quad \theta_1 = 10°,$$
$$\text{Eq. 9.86 predicts } \beta_1 \approx 27.38°,$$
$$Ma_{1n} = 1.380, \quad Ma_{2n} = 0.748,$$

$$\therefore\ Ma_2 = 2.505, \quad \theta_2 = \theta_1 = 10°, \quad \beta_2 = 31.80°, \quad Ma_{2n} = 1.32, \quad Ma_{3n} = 0.776,$$

$$\therefore\ Ma_3 = \mathbf{2.09} \quad \textit{Ans.}$$

$$\text{Meanwhile,} \quad p_2/p_1 = 2.054, \quad \text{or} \quad p_2 = 205.4\text{ kPa},$$

$$\text{and} \quad p_3/p_2 = 1.866, \quad p_3 = 1.866(205.4) \approx \mathbf{383\ kPa} \quad \textit{Ans.}$$

9.136 Figure P9.136 is a special application of Prob. 9.135. With careful design, one can orient the bend on the lower wall so that the reflected wave is exactly canceled by the return bend, as shown. This is a method of reducing the Mach number in a channel (a supersonic diffuser). If the bend angle is $\phi = 10°$, find (a) the downstream width h and (b) the downstream Mach number. Assume a weak shock wave.

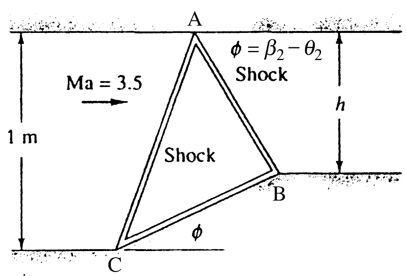


Fig. P9.136

Solution: The important thing is to find the angle ϕ between the *second* shock and the upper wall, as shown in the figure. With initial deflection = 10°, proceed forward to "3":

$$Ma_1 = 3.5, \theta_1 = 10°, \quad \text{compute } \beta_1 = 24.384°, \ Ma_2 = 2.904, \ \theta_2 = \theta_1 = 10°, \ \beta_2 = 28.096°,$$

$$\phi_{\text{upper wall}} = \beta_2 - \theta_2 = 28.096 - 10 = 18.096°, \ Ma_3 = 2.427 \quad \textit{Ans.}\text{ (b)}$$

Length CB in the figure = (1 m)/sin(24.384°) = 2.422 m, angle ACB = 14.384°, angle CBA = β_2 = 28.096°, by the law of sines, AB/sin(14.384°) = 2.422/sin(28.096°) or the length AB = 1.278 m. Finally, duct width h = 1.278sin(18.096°) ≈ **0.40 m**. *Ans.* (a)

The horizontal distance from one lower corner to the next is 3.42 m. The length of CB is 3.47 m. Thus the shocks are not drawn to scale in the figure.

9.137 A 6° half-angle wedge creates the reflected shock system in Fig. P9.137. If $Ma_3 = 2.5$, find (a) Ma_1; and (b) the angle α.

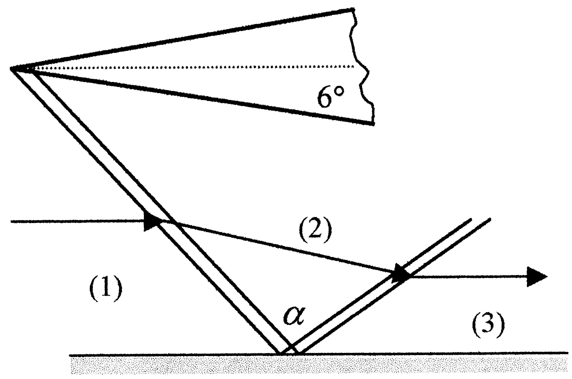


Fig. P9.137

Solution: (a) We have to go backward from region 3 to regions 2 and 1, using Eq. (9.86). In both cases the turning angle is $\theta = 6°$. EES is of course excellent for this task, otherwise the iteration will be laborious. The results are:

$$Ma_2 = 2.775, \quad \beta_2 = 25.66°, \quad \mathbf{Ma_1 = 3.084}, \quad \beta_1 = 23.36° \quad \textit{Ans.}\text{ (a)}$$

Since the wall is horizontal, it is clear from the geometry of Fig. P9.137 that

$$\alpha = 180° - 25.66° - 23.36° = \mathbf{130.98°} \quad \textit{Ans.}\text{ (b)}$$

9.138 The supersonic nozzle of Fig. P9.138 is *overexpanded* (case G of Fig. 9.12) with $A_e/A_t = 3.0$ and a stagnation pressure of 350 kPa. If the jet edge makes a 4° angle with the nozzle centerline, what is the back pressure p_r in kPa?

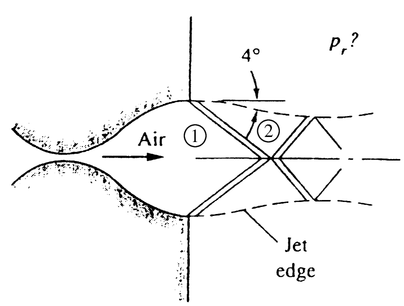


Fig. P9.138

Solution: The nozzle is clearly choked because there are shock waves downstream. Thus

$$\frac{A_e}{A^*} = 3.0, \quad \text{read } Ma_e = Ma_1 = 2.64, p_e = 350/[1+0.2(2.64)^2]^{3.5} = 16.5 \text{ kPa}$$

$$\theta = 4°, \quad \text{Eq. 9.86 gives } \beta_2 = 25.3°, Ma_{1n} = 2.64 \sin 25.3° = 1.125, p_2/p_1 = 1.311$$

$$\text{Thus} \quad p_2 = p_{receiver} = 1.311(16.5) \approx \mathbf{21.7\ kPa} \quad \textit{Ans.}$$

9.139 Airflow at Ma = 2.2 takes a compression turn of 12° and then another turn of angle θ in Fig. P9.139. What is the maximum value of θ for the second shock to be *attached*? Will the two shocks intersect for any θ less than θ_{max}?

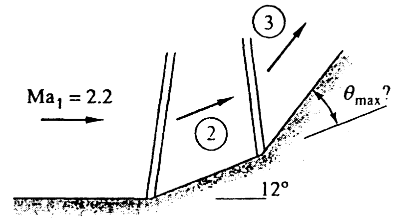


Fig. P9.139

Solution: First get the conditions in section (2) and then iterate for θ_{max}:

$$Ma_1 = 2.2,\ \theta = 12°, \text{Eq. 9.86: } \beta_1 = 37.87°, Ma_{1n} = 1.351,\ Ma_{2n} = 0.762 = Ma_2 \sin(\beta - \theta)$$

$$\text{or: } Ma_2 \approx \mathbf{1.745}. \text{ For this Mach number, estimate } \theta_{max} \approx 18° \text{ from Fig. 9.23}$$

$$\text{Iterate, by trial and error, find } \quad \theta_{\mathbf{max}} \approx \mathbf{18.02°} \quad \textit{Ans.}$$

NOTE: The two shocks β_1 and β_2 **always** intersect for any $\theta_2 < \theta_{max}$. *Ans.*

9.140 The solution to Prob. 9.122 is Ma_2 = 2.750 and p_2 = 145.5 kPa. Compare these results with an isentropic compression turn of 5°, using Prandtl-Meyer theory.

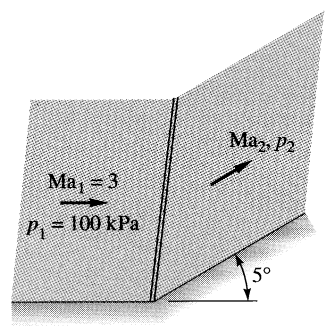


Fig. P9.122

Solution: Find ω for Ma = 3 and subtract 5°:

$$Ma_1 = 3.0, \quad \text{Table B.5: } \omega_1 = 49.76°,$$

$$\omega_2 = 49.76 - 5 = 44.76°, \text{ read } Ma_2 \approx \mathbf{2.753}$$

$$\text{Then} \quad p_2 = p_1\left(\frac{p_2/p_o}{p_1/p_o}\right) = 100\left[\frac{1+0.2(2.753)^2}{1+0.2(3.0)^2}\right]^{-3.5} \approx \mathbf{145.4\ kPa} \quad \textit{Ans.}$$

This is almost identical to the shock wave result, because a 5° turn is nearly isentropic.

9.141 Supersonic airflow takes a 5° expansion turn, as in Fig. P9.141. Compute the downstream Mach number and pressure and compare with small-disturbance theory.

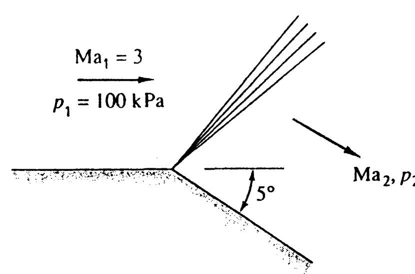


Fig. P9.141

Solution: Find ω for Ma = 3 and *add* 5°:

$$Ma_1 = 3.0, \quad \text{Table B.5: } \omega_1 = 49.76°,$$

$$\omega_2 = 49.76 + 5 = 54.76°,$$

$$\text{Read} \quad Ma_2 \approx \mathbf{3.274} \quad \textit{Ans.}$$

$$\text{Then}\quad p_2 = p_1\left(\frac{p_2/p_o}{p_1/p_o}\right) = 100\left[\frac{1+0.2(3.274)^2}{1+0.2(3.0)^2}\right]^{-3.5} \approx \mathbf{66.7\ kPa}\quad \textit{Ans.}$$

The linear theory is not especially accurate because even a 5° turn is slightly nonlinear:

$$\text{Eq. 9.89:}\quad \frac{\Delta p}{p} \approx \frac{k\,Ma^2}{\sqrt{Ma^2-1}}\tan\theta_2 = \frac{1.4(3)^2}{\sqrt{3^2-1}}\tan(-5°) = -0.390,$$

$$p = 100(1-0.390) \approx \mathbf{61\ kPa}\quad \textit{Ans.}\ (9\%\ \text{low})$$

9.142 A supersonic airflow at Ma_1 = 3.2 and p_1 = 50 kPa undergoes a compression shock followed by an isentropic expansion turn. The flow deflection is 30° for each turn. Compute Ma_2 and p_2 if (a) the shock is followed by the expansion and (b) the expansion is followed by the shock.

Solution: The solution is given in the form of the two sketches below. A shock wave with a 30° turn is a hugely non-isentropic flow, so the final conditions are nowhere near the original and they do not agree with each other either.

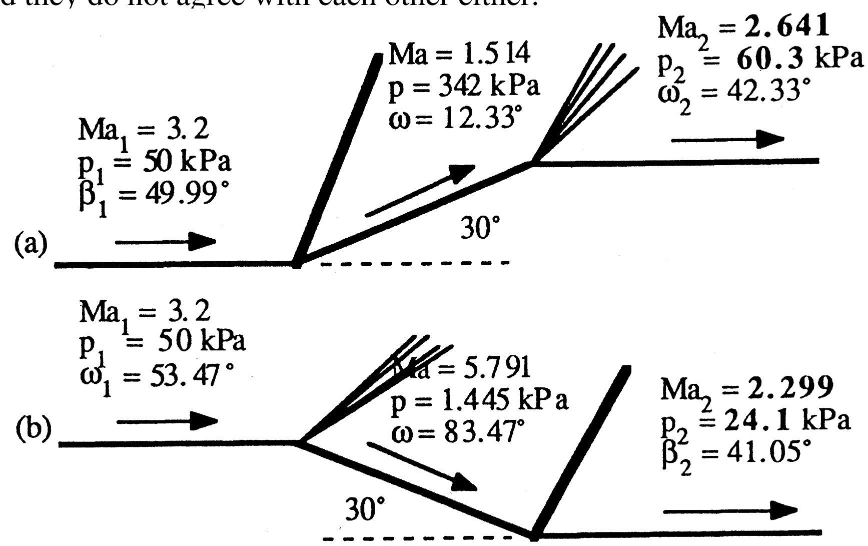


Fig. P9.142

9.143 Airflow at Ma_1 = 3.2 passes through a 25° oblique-shock deflection. What isentropic expansion turn is required to bring the flow back to (a) Ma_1 and (b) p_1?

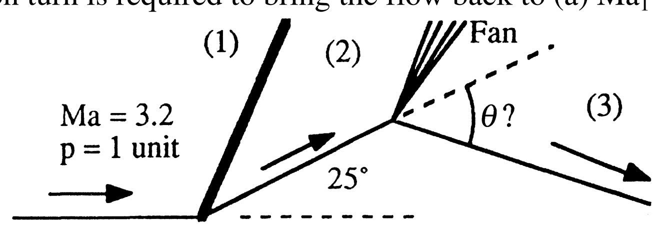


Fig. P9.143

Solution: First work out state (2):

$$Ma_1 = 3.2,\ \beta_1 = 42.56°,\ Ma_2 = 1.83,\ p_2 = 5.30 \text{ units},\quad \omega_2 = 21.59°$$

(a) $Ma_3 = 3.2$ means $\omega_2 = 53.47°$, or $\theta = 53.47 - 21.59 \approx \mathbf{31.9°}$ *Ans.* (a)

(b) $p_3 = 1$ unit means $Ma_3 = 2.906$, $\omega_3 = 47.90°$, $\theta = 47.90 - 21.59 = \mathbf{26.3°}$ *Ans.* (b)

9.144 Consider a smooth isentropic compression turn of 20°, as in Fig. P9.144. The Mach waves thus generated will form a converging fan. Sketch this fan as accurately as possible, using five equally-spaced waves, and demonstrate how the fan indicates the probable formation of an oblique shock wave.

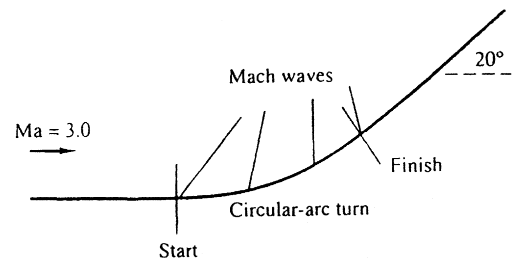
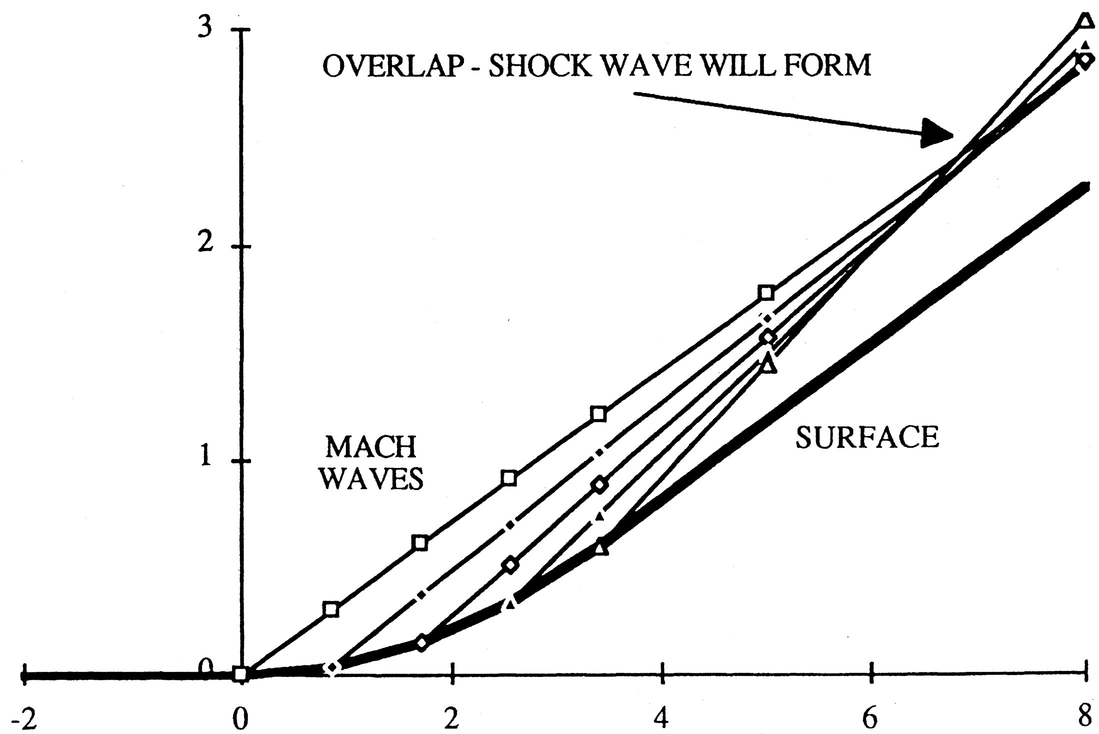


Fig. P9.144

Solution: The desired sketch is shown below. The final state after the 20° isentropic turn is Ma = 2.125. The Mach waves cross each other, so an oblique shock will form. In the figure below, the *y*-axis is exaggerated for clarity of the Mach waves.



9.145 Air at $Ma_1 = 2.0$ and $p_1 = 100$ kPa undergoes an isentropic expansion to a downstream pressure of 50 kPa. What is the desired turn angle in degrees?

Solution: This is a real 'quickie' compared to what we have been doing for the past few problems. Isentropic expansion to a new pressure specifies the downstream Mach number:

$$p_o = p_1\left[1+0.2\,Ma_1^2\right]^{3.5} = 100[1+0.2(2)^2]^{3.5} = 782 \text{ kPa}$$

$$p_2/p_o = \frac{50}{782} = 0.0639, \quad \text{read } Ma_2 \approx 2.44, \quad \text{read } \omega_2 \approx 37.79°,$$

$$\text{while} \quad \omega_1 \approx 26.38°, \quad \therefore\ \Delta\theta = 37.79 - 26.38 \approx \mathbf{11.4°} \quad \textit{Ans.}$$

9.146 Air flows supersonically over a surface which changes direction twice, as in Fig. P9.146. Calculate (a) Ma_2; and (b) p_3.

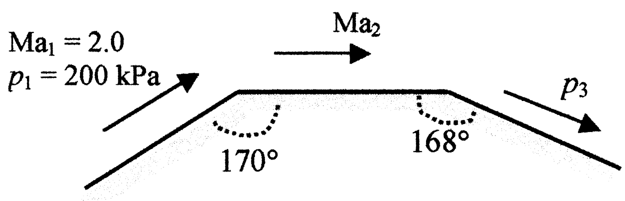


Fig. P9.146

Solution: (a) At the initial condition $Ma_1 = 2.0$, from Table B.5 read $\omega_1 = 26.38°$. The first turn is 10°, so $\omega_2 = 26.38 + 10 = 36.38°$. From Table B.5 read $Ma_2 = 2.38$. For more accuracy, use Eq. (9.99) to obtain $\mathbf{Ma_2 = 2.385}$. *Ans.* (a)

(b) The second turn is 12°, so $\omega_3 = 36.38 + 12 = 48.38°$. From Table B.5 read $Ma_3 = 2.93$. For more accuracy, use Eq. (9.99) to obtain $Ma_2 = 2.9296$. (Not worth the extra effort.) To find pressures, we need the stagnation pressure, which is constant:

$$p_o = p_1\left(1+0.2\,Ma_1^2\right)^{3.5} = (200 \text{ kPa})[1+0.2(2.0)^2]^{3.5} = 1565 \text{ kPa}$$

$$\text{Then} \quad p_3 = p_o\Big/\left(1+0.2\,Ma_3^2\right)^{3.5} = (1565)/[1+0.2(2.9296)^2]^{3.5} = \mathbf{47.4\ kPa} \quad \textit{Ans.}\text{ (b)}$$

9.147 A converging-diverging nozzle with a 4:1 exit-area ratio and $p_0 = 500$ kPa operates in an underexpanded condition (case 1 of Fig. 9.12) as in Fig. P9.147. The receiver pressure is $p_a = 10$ kPa, which is less than the exit pressure, so that expansion waves

form outside the exit. For the given conditions, what will the Mach number Ma_2 and the angle ϕ of the edge of the jet be? Assume $k = 1.4$ as usual.

Solution: Get the Mach number in the exit and then execute a Prandtl-Meyer expansion:

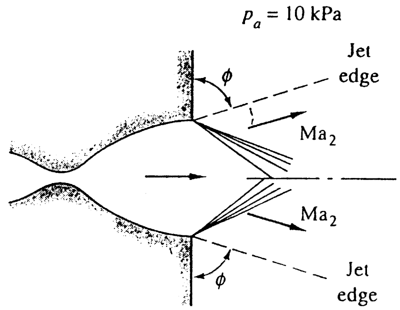


Fig. P9.147

$$\frac{A_e}{A^*} = 4.0, \quad \text{read } Ma_e \approx 2.94, \quad \text{Table B.5: } \omega_1 = 48.59°, \quad p_{o1} = p_{o2} = 500\text{ kPa}$$

$$p_o/p_2 = \frac{500}{10} = 50, \quad \text{read } Ma_2 \approx \mathbf{3.21} \quad \textit{Ans.}\text{ (a)} \quad \text{Read } \omega_2 = 53.61°,$$

$$\therefore \quad \Delta\theta = 53.61 - 48.59 = 5.02°, \quad \text{or} \quad \phi_{\text{see figure above}} = 90 - \Delta\theta \approx \mathbf{85.0°} \quad \textit{Ans.}\text{ (b)}$$

9.148 Air flows supersonically over a circular-arc surface as in Fig. P9.148. Estimate (a) the Mach number Ma_2 and (b) the pressure p_2 as the flow leaves the circular surface.

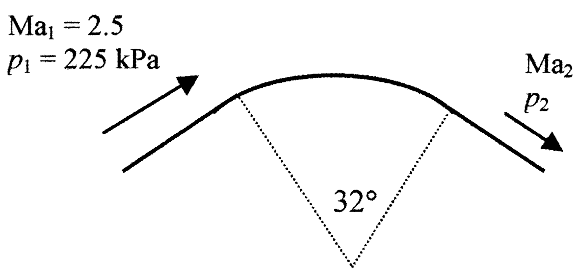


Fig. P9.148

Solution: (a) At the initial condition Ma_1 = 2.5, from Table B.5 read ω_1 = 39.12°. (a) Circular arc or not, the turn angle is 32°, so ω_2 = 39.12 + 32 = 71.12°. From Table B.5 read Ma_2 = 4.44. For more accuracy, use Eq. (9.99) to obtain $\mathbf{Ma_2 = 4.437}$. *Ans.* (a)

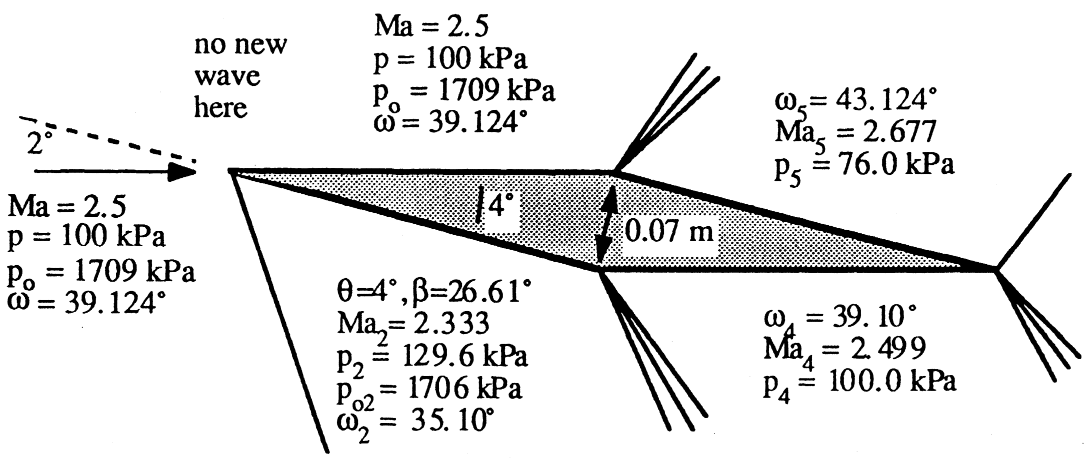
(b) To find p_2, first find the stagnation pressure:

$$p_o = p_1\left(1 + 0.2\,Ma_1^2\right)^{3.5} = (150\text{ kPa})[1 + 0.2(2.5)^2]^{3.5} = 2563\text{ kPa}$$

$$\text{Then} \quad p_2 = p_o \Big/ \left(1 + 0.2\,Ma_2^2\right)^{3.5} = (2563)/[1 + 0.2(4.437)^2]^{3.5} = \mathbf{9.6\text{ kPa}} \quad \textit{Ans.}\text{ (b)}$$

9.149 Repeat Example 9.21 for an angle of attack of **2°**. Is the lift coefficient *linear* with α in this range of $0° < \alpha < 8°$? Why does the drag coefficient <u>not</u> have the simple parabolic form $C_D \approx K\alpha^2$ in this range?

Solution: The various calculations for surface pressure are listed in the sketch below.



The forces are calculated as in Example 9.21:

$$F = \frac{1}{2}(29.6 + 24.0)(2\,\text{m}^2) \approx 53.6\,\text{kN}, \quad P = \frac{1}{2}(24.0 + 29.6)(0.07)(1) \approx 1.87\,\text{kN}$$

$$L = F\cos 2° - P\sin 2° = 53.5\,\text{kN}, \quad C_L = 53.5\Big/\frac{k}{2}(100)(2.5)^2(2) = \frac{53.5}{875} = \mathbf{0.0611} \quad \textit{Ans.}\ \text{(a)}$$

$$D = F\sin 2° + P\cos 2° \approx 3.74\,\text{kN}, \quad C_D = 3.74/875 \approx \mathbf{0.00427} \quad \textit{Ans.}\ \text{(b)}$$

The lift is quite linear, $C_L \approx \mathbf{1.75}\boldsymbol{\alpha}_{\text{radians}}$ (compared with 1.75α also in linearized theory), but the drag is not purely parabolic through the origin, $C_D \approx \underline{\mathbf{0.00209}} + \mathbf{1.79}\boldsymbol{\alpha}^2$ (compared with $\underline{0.00212} + 1.75\alpha^2$ according to Ackeret theory, Eq. 9.107). The reason is that there is **thickness-drag** for this diamond-shaped airfoil. *Ans.*

9.150 A flat plate airfoil with chord C = 1.2 m is to have a lift of 30 kN/m when flying at 5000-m standard altitude with U_∞ = 641 m/s. Using Ackeret theory, estimate (a) the angle of attack; and (b) the drag force in N/m.

Solution: At 5000 m, $\rho = 0.7361\ \text{kg/m}^3$, T = 256 K, and p = 54008 Pa. Compute Ma:

$$\text{Ma} = \frac{641}{\sqrt{1.4(287)(256)}} = 2.00, \quad C_L = \frac{30000}{(1/2)(0.7361)(641)^2(1.2)(1)} = 0.1653 \approx \frac{4\alpha}{\sqrt{(2)^2 - 1}}$$

$$\text{Solve for} \quad \alpha = 0.0716\ \text{rad} \approx \mathbf{4.10°} \quad \textit{Ans.}$$

With angle of attack known, Ackeret theory simply predicts that

$$D = L\tan\alpha = 30000\tan(4.10°) \approx \mathbf{2150\ N/m} \quad \textit{Ans.}\text{ (b)}$$

9.151 Air flows at Ma = 2.5 past a half-wedge airfoil whose angles are 4°, as in Fig. P9.151. Compute the lift and drag coefficients at α equal to (a) 0°; and (b) 6°.

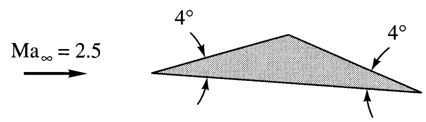


Fig. P9.151

Solution: Let's use Ackeret theory here:

$$\text{(a) } \alpha = 0°\text{:}\quad C_L \approx \mathbf{0};\quad C_D \approx \frac{4}{\sqrt{(2.5)^2 - 1}}\left[0^2 + \frac{1}{2}(\tan^2 4° + 0)\right] \approx \mathbf{0.00427} \quad \textit{Ans.}\text{(a)}$$

[A complete shock-expansion calculation gives $C_L \approx -0.0065$, $C_D \approx 0.00428$.]

$$\text{(b) } \alpha = 6° = 0.105\text{ rad:}\quad C_L = \frac{4(0.105)}{\sqrt{5.25}} \approx \mathbf{0.183},$$

$$C_D = \frac{4}{\sqrt{5.25}}[(0.105)^2 + (1/2)(\tan^2 4° + 0)] \approx \mathbf{0.0234} \quad \textit{Ans.}\text{(b)}$$

[A complete shock-expansion calculation gives $C_L \approx 0.179$, $C_D \approx 0.0219$.]

9.152 A supersonic airfoil has a parabolic symmetric shape for upper and lower surfaces

$$y_{u,l} = \pm 2t\left(\frac{x}{C} - \frac{x^2}{C^2}\right)$$

such that the maximum thickness is t at $x = \frac{1}{2}C$. Compute the drag coefficient at zero incidence by Ackeret theory, and compare with a symmetric double wedge of the same thickness.

Solution: Evaluate the mean-square surface slope and then use Eq. (9.107):

$$y'|_{\text{upper}} = \frac{2t}{C}\left(1 - \frac{2x}{C}\right),$$

$$\overline{y'^2} = \frac{1}{C}\int_0^C \left[\frac{2t}{C}\left(1 - \frac{2x}{C}\right)\right]^2 dx = \frac{4}{3}\frac{t^2}{C^2} \quad \text{(for both upper and lower surfaces)}$$

$$\text{At } \alpha = 0,\quad C_D = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left[0^2 + \frac{1}{2}\left\{2\left(\frac{4}{3}\frac{t^2}{C^2}\right)\right\}\right],\quad \text{or:}\quad C_D = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left[\frac{4}{3}\frac{t^2}{C^2}\right] \quad \textit{Ans.}$$

For a *double-wedge* foil of the same thickness, y′ = t/C on both surfaces:

$$\overline{y'^2}\big|_{avg} = t^2/C^2, \quad \text{Eq. 9.107 yields } C_{D,\text{double-wedge}} \approx \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left[\frac{t^2}{C^2}\right] \quad \textbf{(25\% less)} \quad \textit{Ans.}$$

9.153 A supersonic transport has a mass of 65 Mg and cruises at 11-km standard altitude at a Mach number of 2.25. If the angle of attack is 2° and its wings can be approximated by flat plates, estimate (a) the required wing area; and (b) the thrust.

Solution: At 11 km (Table B.6), take $p_\infty = 22612$ Pa. (a) Use linearized theory:

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(2\pi/180)}{\sqrt{2.25^2 - 1}} = 0.0693 = \frac{W}{\frac{k}{2}pMa^2 A} = \frac{65000(9.81)}{0.7(22612)(2.25)^2 A},$$

$$\mathbf{A = 115\ m^2} \quad \textit{Ans. (a)}$$

(b) According to linearized (Ackeret) theory, if there is no thickness drag, then

$$\text{Drag} = \text{Lift} \times \alpha, \quad \text{or} \quad \text{Drag} = \text{Thrust} = 65000(9.81)(2\pi/180) = \mathbf{22{,}300\ N} \quad \textit{Ans. (b)}$$

9.154 A symmetric supersonic airfoil has its upper and lower surfaces defined by a sine waveshape:

$$y = \frac{t}{2}\sin\frac{\pi x}{C}$$

where t is the maximum thickness, which occurs at $x = C/2$. Use Ackeret theory to derive an expression for the drag coefficient at zero angle of attack. Compare your result with Ackeret theory for a symmetric double-wedge airfoil of the same thickness.

Solution: Evaluate the mean-square surface slope and then use Eq. (9.107):

$$y' = \frac{\pi t}{2C}\cos\left(\frac{\pi x}{C}\right), \quad \overline{y'^2} = \frac{1}{C}\int_0^C \left(\frac{\pi t}{2C}\right)^2 \cos^2\left(\frac{\pi x}{C}\right)dx = \frac{\pi^2}{8}\left(\frac{t}{C}\right)^2$$

$$\text{At } \alpha = 0, \quad C_D = \frac{4}{\sqrt{(Ma_\infty^2 - 1)}}\left[0^2 + \frac{1}{2}\left\{2\left(\frac{\pi^2}{8}\frac{t^2}{C^2}\right)\right\}\right],$$

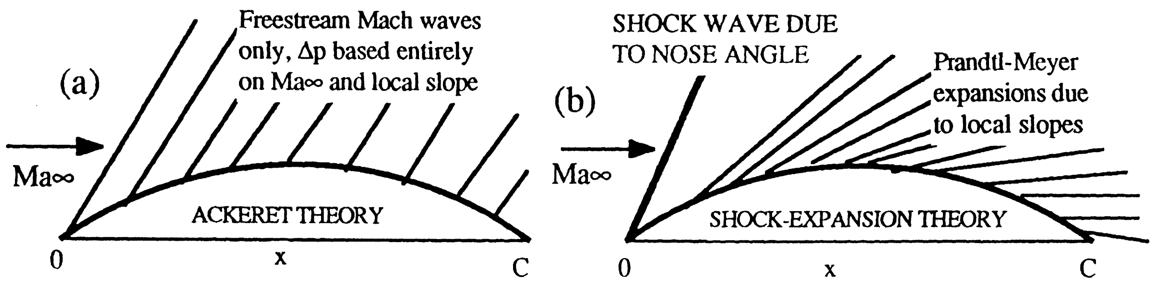
$$\text{or:} \quad \mathbf{C_D = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left(\frac{\pi^2}{8}\right)\left(\frac{t}{C}\right)^2} \quad \textit{Ans.}$$

Meanwhile, for a double-wedge of the same thickness *t*, from Prob. 9.152,

$$C_{D,\text{double-wedge}} = \frac{4}{\sqrt{Ma_\infty^2 - 1}}\left(\frac{t}{C}\right)^2 \quad \text{(19\% less)} \quad \textit{Ans.}$$

9.155 For the sine-wave airfoil shape of Prob. 9.154, with $Ma_\infty = 2.5$, $k = 1.4$, $t/C = 0.1$, and $\alpha = 0°$, plot (without computing the overall forces) the pressure distribution $p(x)/p_\infty$ along the upper surface for (a) Ackeret theory and (b) an oblique shock plus a continuous Prandtl-Meyer expansion.

Solution: A sketch of the physical differences between the two theories is shown below:



For Ackeret theory, simply evaluate the local slopes and apply Eq. (9.89):

$$y = \frac{t}{2}\sin\left(\frac{\pi x}{C}\right), \quad \frac{dy}{dx} = \frac{\pi t}{2C}\cos\left(\frac{\pi x}{C}\right), \quad \frac{p}{p_\infty} \approx 1 + \frac{k\,Ma_\infty^2}{\sqrt{Ma_\infty^2 - 1}}\frac{dy}{dx}, \quad \text{where } \frac{t}{C} = \mathbf{0.1}$$

For shock-expansion theory, evaluate the nose slope and solve the leading edge shock:

$$\frac{dy}{dx}\Big|_{nose} = \frac{\pi t}{2C} = \frac{\pi}{2}(0.1) = \tan\theta_o, \quad \theta_o \approx 8.93°, \quad Ma_1 = 2.5, \quad \text{solve } \beta \approx 30.85°,$$

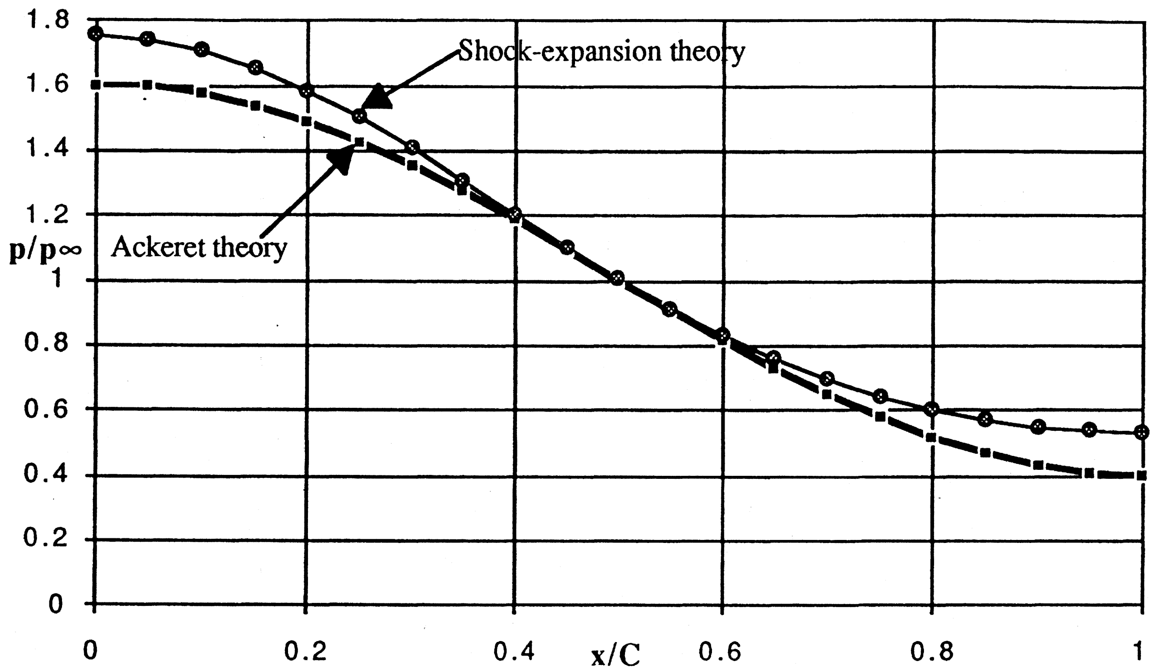
$$Ma_2 = 2.130, \quad \omega_2 = 29.905°, \quad p_2/p_\infty = 1.750 \text{ just inside the shock.}$$

Continue Ma(x) based on $\omega(x) = \omega_2 + \tan^{-1}(dy/dx)$ plus $p_{o2} = \text{constant (isentropic)}$

The results may be tabulated and plotted as shown below:

x/C:	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Ackeret theory, p/p_∞:	1.60	1.57	1.49	1.35	1.19	1.00	0.81	0.65	0.51	0.43	0.40
Shock-expansion theory: p/p_∞:	1.75	1.71	1.58	1.40	1.20	1.00	0.83	0.70	0.60	0.55	0.53

The agreement is pretty good, and the **integrated drag** is in even better agreement.



9.156 A thin circular-arc airfoil is shown in Fig. P9.156. The leading edge is parallel to the free stream. Using linearized (small-turning-angle) supersonic-flow theory, derive a formula for the lift and drag coefficient for this orientation, and compare with Ackeret-theory results for an angle of attack $\alpha = \tan^{-1}(h/L)$.

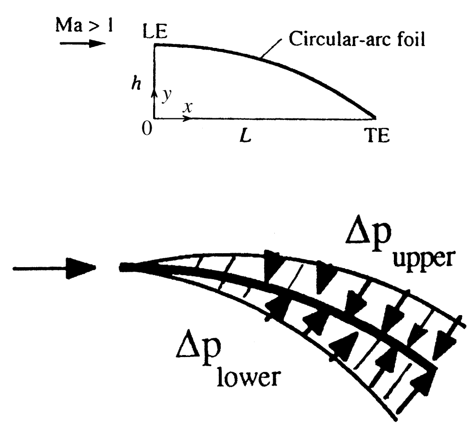


Fig. P9.156

Solution: For the (x,y) coordinate system shown, the formula for the plate surface is

$$y_{foil} = \sqrt{R^2 - x^2} - R + h,$$

$$\text{where } R = \frac{L^2 + h^2}{2h}, \quad \text{and} \quad \frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}}$$

If $h \ll L$ (small disturbances), $R \gg L, h$. Since the flow approaches parallel to the leading edge, the pressure distribution on the airfoil looks like the bottom figure on the previous page. We compute this pressure distribution from the linearized expression, Eq. (9.89):

$$\frac{\Delta p}{p_\infty} \approx B\frac{dy}{dx}, \quad \text{where } B = \frac{kMa_\infty^2}{\sqrt{Ma_\infty^2 - 1}}; \quad \text{Thus } \Delta p_{lower} - \Delta p_{upper} = \frac{2Bx}{\sqrt{R^2 - x^2}} p_\infty$$

$$\text{Lift} = \int_{foil} \Delta p_{total}\, dA_{foil} = 2Bp_\infty \int_0^L x(R^2 - x^2)^{-1/2} \cos\theta\, b\, dx,$$

$$\text{where} \quad \cos\theta = \left[1 + (dy/dx)^2\right]^{-1/2}$$

Carry out this integration, assuming that $h \ll L$, $R \gg L$, chord length $C \approx L$, to obtain

$$\text{Lift} \approx \frac{2kMa_\infty^2}{\sqrt{Ma_\infty^2 - 1}} p_\infty bh, \quad \text{or} \quad \mathbf{C_L} = \frac{\text{Lift}}{(k/2)Ma_\infty^2 p_\infty bL} \approx \frac{\mathbf{4}}{\sqrt{\mathbf{Ma_\infty^2 - 1}}}\frac{\mathbf{h}}{\mathbf{L}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{But} \quad h/L \approx \alpha_{radians}, \quad \text{therefore } C_L \approx \left[4\Big/\sqrt{Ma_\infty^2 - 1}\right]\alpha, \quad \text{which is exactly Ackeret theory.}$$

$$\text{Similarly,} \quad \text{Drag} = 2Bp_\infty \int_0^L x(R^2 - x^2)^{-1/2} \sin\theta\, b\, dx \approx \left[2kMa_\infty^2\left(Ma_\infty^2 - 1\right)^{-1/2}\right] p_\infty bh\left(\frac{4h}{3L}\right)$$

$$\text{or:} \quad \mathbf{C_D} \approx \left[4/\sqrt{Ma_\infty^2 - 1}\right](h/L)^2\left(1 + \frac{1}{3}\right) \approx \mathbf{C_L}\boldsymbol{\alpha}^\mathbf{2}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{3}}\right) \quad \textit{Ans.}\text{ (b)}$$

This is <u>exactly the same</u> as Ackeret theory. The extra term "1/3" is the "thickness-drag" contribution (actually the <u>**camber**</u> slope contribution) from Eqs. (9.106, 107) of the text.

9.157 Prove from Ackeret theory that for a given supersonic airfoil shape with sharp leading and trailing edges and a given thickness, the minimum-thickness drag occurs for a symmetric double-wedge shape.

Solution: This (final) problem is merely to <u>alert the reader</u> to such a theorem. The proof itself is very laborious and is not intended to be assigned to students.

The proof involves first showing that, for any foil surface shape between two points, a *straight line* gives the lowest drag. Then you show that, for any max thickness, the lowest straight-line drag shape is the symmetrical double-wedge airfoil of, e.g., Fig. E9.21 of the text. A complete proof is given in the text by F. Cheers (Ref. 3 of Chapter 9).

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

In the following problems, assume one-dimensional flow of ideal air, R = 287 J/(kg·K) and k = 1.4.

FE9.1 For steady isentropic flow, if the absolute temperature increases 50%, by what ratio does the static pressure increase?

(a) 1.12 (b) 1.22 (c) 2.25 (d) 2.76 **(e) 4.13**

FE9.2 For steady isentropic flow, if the density doubles, by what ratio does the static pressure increase?

(a) 1.22 (b) 1.32 (c) 1.44 **(d) 2.64** (e) 5.66

FE9.3 A large tank, at 500 K and 200 kPa, supplies isentropic air flow to a nozzle. At section 1, the pressure is only 120 kPa. What is the Mach number at this section?

(a) 0.63 (b) 0.78 **(c) 0.89** (d) 1.00 (e) 1.83

FE9.4 In Prob. FE9.3 what is the temperature at section 1?

(a) 300 K (b) 408 K (c) 417 K **(d) 432 K** (e) 500 K

FE9.5 In Prob. FE9.3, if the area at section 1 is 0.15 m^2, what is the mass flow?

(a) 38.1 kg/s **(b) 53.6 kg/s** (c) 57.8 kg/s (d) 67.8 kg/s (e) 77.2 kg/s

FE9.6 For steady isentropic flow, what is the maximum possible mass flow through the duct in Fig. FE9.6? (T_o = 400 K, p_o = 300 kPa)

(a) 9.5 kg/s (b) 15.1 kg/s (c) 26.2 kg/s **(d) 30.3 kg/s** (e) 52.4 kg/s

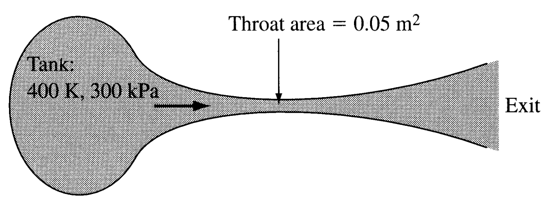


Fig. FE9.6

FE9.7 If the exit Mach number in Fig. FE9.6 is 2.2, what is the exit area?

(a) 0.10 m^2 (b) 0.12 m^2 (c) 0.15 m^2 (d) 0.18 m^2 (e) 0.22 m^2

FE9.8 If there are no shock waves and the pressure at one duct section in Fig. FE9.6 is 55.5 kPa, what is the velocity at that section?

(a) 166 m/s (b) 232 m/s **(c) 554 m/s** (d) 706 m/s (e) 774 m/s

FE9.9 If, in Fig. FE9.6, there is a normal shock wave at a section where the area is 0.07 m^2, what is the air density just upstream of that shock?

(a) 0.48 kg/m^3 **(b) 0.78 kg/m^3** (c) 1.35 kg/m^3 (d) 1.61 kg/m^3 (e) 2.61 kg/m^3

FE9.10 In Prob. FE9.9, what is the Mach number just downstream of the shock wave?

(a) 0.42 (b) 0.55 **(c) 0.63** (d) 1.00 (e) 1.76

COMPREHENSIVE PROBLEMS

C9.1 The converging-diverging nozzle in the figure has a design Mach number of 2.0 at the exit plane for isentropic flow from tank *a* to *b*. (a) Find the exit area A_e and back pressure p_b which will allow design conditions. (b) The back pressure grows as tank *b* fills with air, until a normal shock wave appear in the exit plane. At what back pressure does this occur? (c) If tank *b* remains at constant T = 20°C, how long will it take for the flow to go from condition (a) to condition (b)?

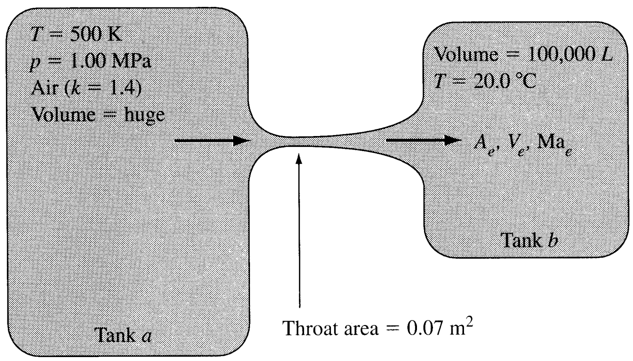


Fig. C9.1

Solution: (a) Compute the isentropic pressure ratio and area ratio for Ma = 2.0:

$$Ma_e = 2.0\text{: Table B.1: } \frac{p_e}{p_o} = 0.1278, \quad p_e = 0.1278(1E6) = \mathbf{128{,}000\,Pa} \quad \textit{Ans.}\text{(a)}$$

$$\frac{A_e}{A^*} = 1.6875, \quad A_e = 1.6875(0.07) = \mathbf{0.118\,m^2} \quad \textit{Ans.}\text{(a)}$$

(b) Compute the pressure ratio across a shock for Ma = 2.0:

$$Ma_e = 2.0\text{: Table B.2: } \frac{p_b}{p_e} = 4.5, \quad p_b = 4.5(128000) = \mathbf{575{,}000\,Pa} \quad \textit{Ans.}\text{(b)}$$

(c) Compute the (constant) mass flow and the mass needed to fill the tank:

$$\dot{m} = \dot{m}_{\max} = 0.6847\frac{p_o A^*}{\sqrt{RT_o}} = 0.6847\frac{(1E6)(0.07)}{\sqrt{287(500)}} = 126.5\ \frac{\text{kg}}{\text{s}} = \textit{constant}$$

$$\Delta m_{tank} = (\rho_{final} - \rho_{initial})\upsilon_{tank} = \left[\frac{575000}{287(293)} - \frac{128000}{287(293)}\right](100\,\text{m}^2) = 531.9\,\text{kg}$$

Since mass flow is constant, the time required is simply the ratio of these two:

$$\Delta t_{required} = \frac{\Delta m}{\dot{m}} = \frac{531.9\,\text{kg}}{126.5\,\text{kg/s}} = \mathbf{4.2\,sec} \quad \textit{Ans.}\text{(c)}$$

C9.2 Two large air tanks, one at 400 K and 300 kPa and the other at 300 K and 100 kPa, are connected by a straight tube 6 m long and 5 cm in diameter. The average friction factor is 0.0225. Assuming adiabatic flow, estimate the mass flow through the tube.

Solution: The higher-pressure tank denotes the inlet *stagnation* conditions, p_o = 300 kPa and T_o = 400 K. The flow will be subsonic, but we have no idea whether it is choked. Assume that the tube exit pressure equals the receiver pressure, 100 kPa. We must iterate—an ideal job for EES! We *do* know (fΔL/D):

$$f\frac{\Delta L}{D} = 0.0225\left(\frac{6.0}{0.05}\right) = 2.70$$

If p_2 = 100 kPa, we must ensure that the inlet Mach number is just sufficient that the inlet stagnation pressure p_{o1} = 300 kPa:

Guess Ma_2, back off (fΔL/D) = 2.70, find Ma_1, check p*, p_1, and p_{o1}. Example:

$$Ma_2 = 1.0, \quad p^* = p_2 = 100\,\text{kPa}, \quad f\frac{L_2^*}{D} = 0, \quad f\frac{L_1^*}{D} = 2.70,$$

$$\text{Read} \quad Ma_1 = \mathbf{0.380,} \quad \frac{p_1}{p^*} = 2.84 \quad \text{Then} \quad p_1 = 2.84(100) = 284\,\text{kPa},$$

$$\frac{p_1}{p_{o1}} = [1 + 0.2(0.380)^2]^{-3.5}, \quad \text{solve} \quad p_{o1} = 314\,\text{kPa} \neq 300$$

So we back off and try values of Ma_2 < 1.0 and proceed until the inlet matches. The solution (performed by the author using EES) is

$$Ma_2 = 0.962, \quad Ma_1 = 0.380, \quad p_2 = p^* = 100\,\text{kPa}, \quad p_1 = 271.5\,\text{kPa}, \quad T_1 = 389\,\text{K},$$

$$\rho_1 = 2.434\,\frac{\text{kg}}{\text{m}^3}, \quad V_1 = 150\,\frac{\text{m}}{\text{s}}, \quad \dot{m} = \rho_1\frac{\pi}{4}D^2V_1 = \mathbf{0.718}\,\frac{\textbf{kg}}{\textbf{s}} \quad \textit{Ans.}$$

So the tube is *nearly*, but not quite, choked.

C9.3 Fig. C9.3 shows the exit of a converging-diverging nozzle, where an oblique shock pattern is formed. In the exit plane, which has an area of 15 cm^2, the air pressure is 16 kPa and the temperature is 250 K. Just outside the exit shock, which makes an angle of 50° with the exit plane, the temperature is 430 K. Estimate (a) the mass flow; (b) the throat area; (c) the turning angle of the exit flow; and, in the tank supplying the air, (d) the pressure and (e) the temperature.

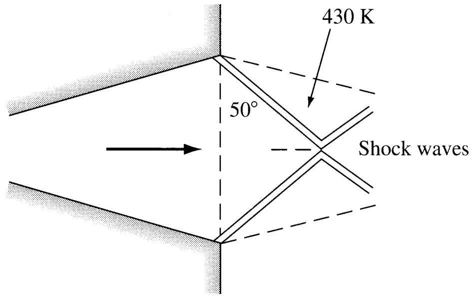


Fig. C9.3

Solution: We know the temperature ratio and the shock wave angle, so we can muddle through oblique-shock-wave theory to find the shock conditions:

$$\frac{T_2}{T_1} = \frac{430}{250} = 1.72; \quad \beta = 40° \quad \textit{Iterate Eqns.}(9.83)\ \textit{or use EES}!$$

$$\textit{Solution}: \quad Ma_{exit} = 3.17; \quad \rho_1 = 0.223\ \frac{\text{kg}}{\text{m}^3}; \quad \frac{A_1}{A^*} = 4.99 = \frac{15}{A_{throat}},$$

$$\mathbf{A_{throat} = 3.00\ cm^2} \quad \textit{Ans.}\ \text{(b)}$$

$$V_1 = Ma_1 a_1 = 1006\ \frac{\text{m}}{\text{s}}, \quad \dot{m} = \rho_1 A_1 V_1 = \mathbf{0.336\ \frac{kg}{s}} \quad \textit{Ans.}\ \text{(a)} \quad \theta_{shock} = \mathbf{22.88°} \quad \textit{Ans.}\ \text{(c)}$$

With the exit Mach number known, it is easy to compute stagnation conditions:

$$p_{o,tank} = 16[1 + 0.2(3.17)^2]^{3.5} = \mathbf{760\ kPa} \quad \textit{Ans.}\ \text{(d)}$$

$$T_{o,tank} = 250[1 + 0.2(3.17)^2] = \mathbf{753\ K} \quad \textit{Ans.}\ \text{(e)}$$

C9.5 Consider one-dimensional steady flow of a non-ideal gas, steam, in a converging nozzle. Stagnation conditions are p_o = 100 kPa and T_o = 200°C. The nozzle exit diameter is 2 cm. If the nozzle exit pressure is 70 kPa, (a) calculate the mass flow and the exit temperature for real steam, from the Steam Tables or using EES. (As a first estimate, assume steam to be an ideal gas from Table A.4.) Is the flow choked? Why is EES unable to estimate the exit Mach number? (b) Find the nozzle exit pressure and mass flow for which the steam flow *is* choked, using EES or the Steam Tables.

Solution: (a) For steam as an ideal gas, from Table A.4, k = 1.33 and R = 461 J/kg·K. First use this approximation to find the exit Mach number:

$$\frac{p}{p_o} = \frac{70\ \text{kPa}}{100\ \text{kPa}} \approx \left(1 + \frac{0.33}{2} Ma_e^2\right)^{-1.33/0.33}, \quad \text{solve for } Ma_{exit} \approx 0.75 \quad \text{Flow is not choked}$$

For real steam, we use EES. The nice Power-law ideal-gas formulas, Eqs. (9.26–9.28), are invalid, but the energy equation (9.22) is valid, and the nozzle flow is isentropic. First evaluate

$$h_o = \text{ENTHALPY(steam, p = 100, T = 473)} = 2875 \text{ kJ/kg·K}$$

$$s_o = \text{ENTROPY(steam, p = 100, T = 473)} = 7.833 \text{ kJ/kg·K}$$

Then use the energy equation with the known pressure at the exit:

$$h_o = 2875 = h_e + \frac{V_e^2}{2(1000)} \quad \text{with } h_e = \text{ENTHALPY}(steam, p = 70, s = 7.833)$$

EES returns the result $h_e = 2800\,\text{kJ/kg}$ and $V_e = 385 \text{ m/s}$

The specific request was for the exit temperature and the mass flow:

$$T_e = \text{TEMPERATURE(steam, p = 70, s = 7.833)} = \mathbf{434\ K} \quad \textit{Ans.}\text{ (a)}$$

$$\rho_e = \text{DENSITY(steam, p = 70, s = 7.833)} = 0.351 \text{ kg/m}^3$$

$$\text{mass flow} = \rho_e A_e V_e = (0.351 \text{ kg/m}^3)(\pi/4)(0.02 \text{ m})^2(385 \text{ m/s}) = \mathbf{0.0425\ kg/s} \quad \textit{Ans.}\text{ (a)}$$

EES cannot calculate Mach numbers directly because it does not contain the speed of sound. However, we can estimate $a_e \approx \sqrt{(\delta p/\delta\rho)_{s=so}}$ by evaluating density slightly above and below p_e, with the result $a_e \approx 513$ m/s. Hence $Ma_e \approx 385/513 \approx$ **0.75** (the same as the ideal gas!).

(b) We are asked to determine p_e for which the flow ***is*** choked, using EES. Ideal gas theory for $k \approx 1.33$ predicts from Eq. (9.32) that $p^*/p_o \approx 0.54$. For EES, real steam, we use the same procedure as in part (a) above and reduce p_{exit} gradually until the mass flow is a maximum. The final result is

$$p_e/p_o = 0.542, \quad Ma_e \approx 1.00, \quad \mathbf{\dot{m}_{max} = 0.04517\ kg/s} \quad \textit{Ans.}\text{ (b)}$$

C9.6 Extend Prob. C9.5 as follows. Let the nozzle be converging-diverging, with an exit diameter of 3 cm. Assume isentropic flow. (a) Find the exit Mach number, pressure, and temperature for an ideal gas, from Table A.4. Does the mass flow agree the value of 0.0452 kg/s in Prob. C9.5? (b) Investigate, briefly, the use of EES for this problem and explain why part (a) is unrealistic and poor convergence of EES is obtained. [HINT: Study the pressure and temperature state predicted by part (a).]

Solution: (a) For steam as an ideal gas, from Table A.4, $k = 1.33$ and $R = 461$ J/kg·K.

$$\frac{A_{exit}}{A*} = \frac{(\pi/4)(0.03)^2}{(\pi/4)(0.02)^2} = 2.25 = \frac{1}{Ma_e}\left\{\frac{\left[1+0.5(k-1)Ma_e^2\right]}{0.5(k+1)}\right\}^{0.5(k+1)/(k-1)} \quad \text{for } k = 1.33$$

Solve for $\mathbf{Ma_e = 2.27}$ *Ans.* (a)

For the exit pressure, temperature, and mass flow, use the ideal Power-law relations:

$$T_e = T_o\Big/\left[1+0.5(k-1)Ma_e^2\right] = 473/[1+0.165(2.27)^2] = \mathbf{256\ K} \quad \textit{Ans. (a)}$$

$$p_e = p_o\Big/\left[1+0.5(k-1)Ma_e^2\right]^{k/(k-1)} = 100/[1+0.165(2.27)^2]^{4.03} = \mathbf{8.4\ kPa} \quad \textit{Ans. (a)}$$

$$\rho_e = \frac{p_e}{RT_e} = 0.0713\ \frac{\text{kg}}{\text{m}^3}; \quad V_e = Ma_e a_e = 898\ \frac{\text{m}}{\text{s}}; \quad \dot{\mathbf{m}} = \rho_e A_e V_e = \mathbf{0.0452\ kg/s} \quad \textit{Ans. (a)}$$

The mass flow does equal the choked-flow value from Prob. C9.5, as expected.

(b) Recall from Prob. C9.5 that $p_o = 100$ kPa and $T_o = 200°C$, which corresponds for real steam (EES) to $h_o = 2875$ kJ/kg·K and $s_o = 7.833$ kJ/kg. We proceed through the nozzle, using the energy equation, $h_o = h + V^2/2$, plus the condition of constant entropy. We also know that the flow is choked at 0.0452 kg/s with a throat diameter of 2 cm. The results are, for real steam,

$\mathrm{Ma_e} = \mathbf{2.22}$; $p_e = \mathbf{10.2\ kPa}$; $T_e = \mathbf{319\ K}$; $\rho_e = \mathbf{0.0725\ kg/m^3}$; Quality = **96%**

The ideal-gas theory is still reasonably accurate at this point, but it is unrealistic, since the real steam has entered the *two-phase (wet) region*.

C9.7 Professor Gordon Holloway and his student, Jason Bettle, of the University of New Brunswick, obtained the following tabulated data for blow-down air flow through a converging-diverging nozzle similar in shape to Fig. P3.22. The supply tank pressure and temperature were 29 psig and 74°F, respectively. Atmospheric pressure was 14.7 psia. Wall pressures and centerline stagnation pressures were measured in the expansion section, which was a frustrum of a cone. The nozzle throat is at $x = 0$.

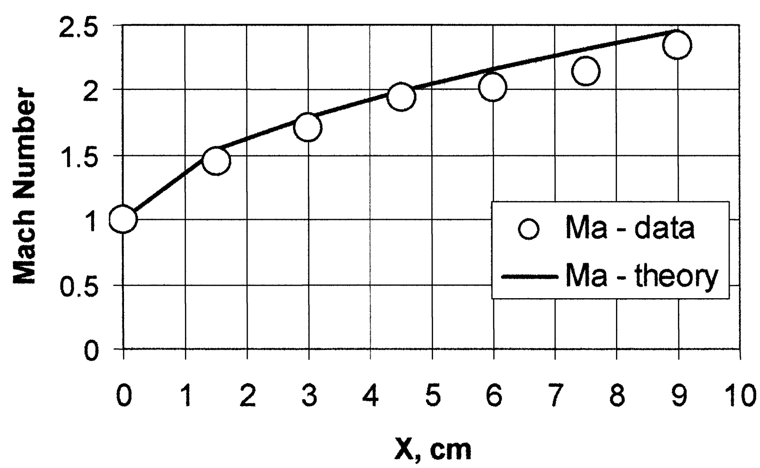
x (cm):	0	1.5	3	4.5	6	7.5	9
Diameter (cm):	1.00	1.098	1.195	1.293	1.390	1.488	1.585
p_{wall} (psig):	7.7	–2.6	–4.9	–7.3	–6.5	–10.4	–7.4
$p_{stagnation}$ (psig):	29	26.5	22.5	18	16.5	14	10

Use the stagnation pressure data to estimate the local Mach number. Compare the measured Mach numbers and wall pressures with the predictions of one-dimensional theory. For $x > 9$ cm, the stagnation pressure data was not thought by Holloway and Bettle to be a valid measure of Mach number. What is the probable reason?

Solution: From the cone's diameters we can determine A/A* and compute theoretical Mach numbers and pressures from Table B.1. From the measured stagnation pressures we can compute measured (supersonic) Mach numbers, because a normal shock forms in front of the probe. The ratio p_{o2}/p_{o1} from Eq. (9.58) or Table B.2 is used to estimate the Mach number.

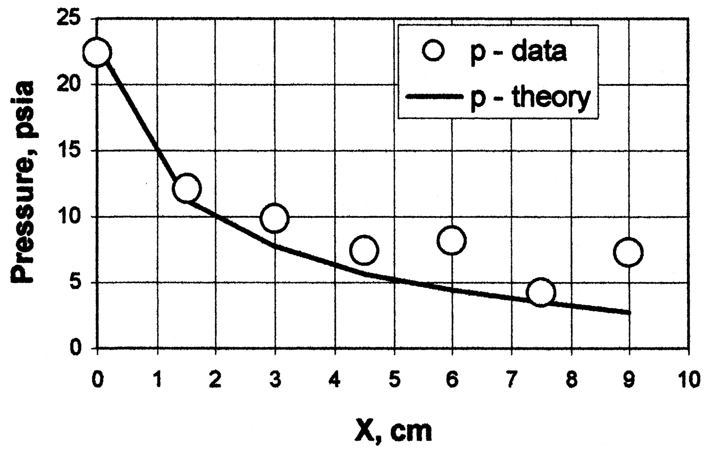
x (cm):	0	1.5	3	4.5	6	7.5	9
Ma-theory:	1.00	1.54	1.79	1.99	2.16	2.31	2.45
p_w-theory (psia):	23.1	11.2	7.72	5.68	4.36	3.44	2.77

The comparison of measured and theoretical Mach number is shown in the graph below.



Problem C9.7

The comparison of measured and theoretical static pressure is shown in the graph below.



Holloway and Bettle discounted the data for x > 9 cm, which gave Mach numbers and pressures widely divergent from theory. It is probably that a normal shock formed in the duct.