champagne 6 inches above the bottom:

$$p_{AA} + (0.96 \times 62.4)\left(\frac{2}{12}\ \text{ft}\right) - (13.56 \times 62.4)\left(\frac{4}{12}\ \text{ft}\right) = p_{atmosphere} = 0\ \text{(gage)},$$

$$\text{or:}\quad P_{AA} = 272\ \text{lbf/ft}^2\ \text{(gage)}$$

Then the force on the bottom end cap is vertical only (due to symmetry) and equals the force at section AA plus the weight of the champagne below AA:

$$F = F_V = p_{AA}(\text{Area})_{AA} + W_{\text{6-in cylinder}} - W_{\text{2-in hemisphere}}$$

$$= (272)\frac{\pi}{4}(4/12)^2 + (0.96 \times 62.4)\pi(2/12)^2(6/12) - (0.96 \times 62.4)(2\pi/3)(2/12)^3$$

$$= 23.74 + 2.61 - 0.58 \approx \mathbf{25.8\ lbf} \quad \textit{Ans.}$$

2.88 Circular-arc *Tainter* gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action. Does the force pass through point O?

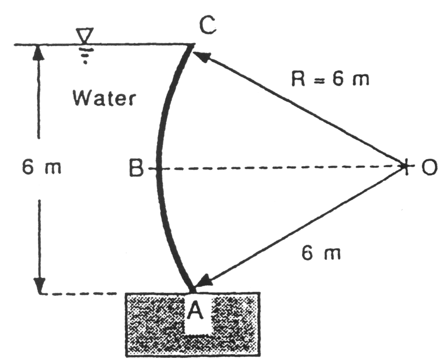
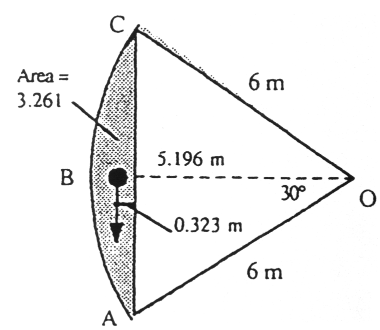


Fig. P2.88

Solution: The horizontal hydrostatic force is based on vertical projection:

$$F_H = \gamma h_{CG} A_{vert} = (9790)(3)(6 \times 1) = 176220\ \text{N} \quad \text{at 4 m below C}$$

The vertical force is *upward* and equal to the weight of the missing water in the segment ABC shown shaded below. Reference to a good handbook will give you the geometric properties of a circular segment, and you may compute that the segment area is 3.261 m^2 and its centroid is 5.5196 m from point O, or 0.3235 m from vertical line AC, as shown in the figure. The vertical (upward) hydrostatic force on gate ABC is thus



$$F_V = \gamma A_{ABC}(\text{unit width}) = (9790)(3.2611)$$

$$= 31926\ \text{N} \quad \text{at 0.4804 m from B}$$

The net force is thus $F = [F_H^2 + F_V^2]^{1/2} = \mathbf{179100\ N}$ per meter of width, acting upward to the right at an angle of **10.27°** and passing through a point 1.0 m below and 0.4804 m to the right of point B. This force passes, as expected, *right through point O*.

2.89 The tank in the figure contains benzene and is pressurized to 200 kPa (gage) in the air gap. Determine the vertical hydrostatic force on circular-arc section AB and its line of action.

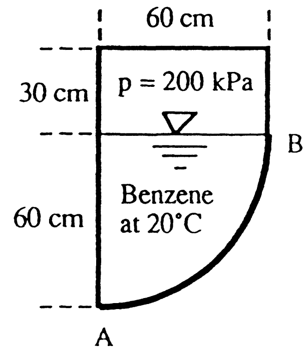


Fig. P2.89

Solution: Assume unit depth into the paper. The vertical force is the weight of benzene plus the force due to the air pressure:

$$F_V = \frac{\pi}{4}(0.6)^2(1.0)(881)(9.81) + (200{,}000)(0.6)(1.0) = \mathbf{122400\,\frac{N}{m}} \quad \textit{Ans.}$$

Most of this (120,000 N/m) is due to the air pressure, whose line of action is in the middle of the horizontal line through B. The vertical benzene force is 2400 N/m and has a line of action (see Fig. 2.13 of the text) at $4R/(3\pi) = 25.5$ cm to the right or A.

The moment of these two forces about A must equal to moment of the combined (122,400 N/m) force times a distance X to the right of A:

$$(120000)(30\text{ cm}) + (2400)(25.5\text{ cm}) = 122400(X), \quad \textit{solve for}\ \ \mathbf{X = 29.9\ cm} \quad \textit{Ans.}$$

The vertical force is **122400 N/m** (down), acting at **29.9 cm** to the right of A.

2.90 A 1-ft-diameter hole in the bottom of the tank in Fig. P2.90 is closed by a 45° conical plug. Neglecting plug weight, compute the force F required to keep the plug in the hole.

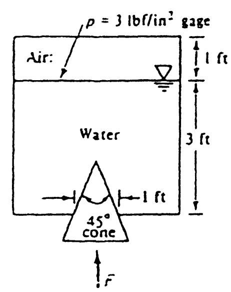


Fig. P2.90

Solution: The part of the cone that is inside the water is 0.5 ft in radius and h = 0.5/tan(22.5°) = 1.207 ft high. The force F equals the air gage pressure times the hole

area plus the weight of the water above the plug:

$$F = p_{gage}A_{hole} + W_{3\text{-ft-cylinder}} - W_{1.207\text{-ft-cone}}$$
$$= (3\times 144)\frac{\pi}{4}(1\text{ ft})^2 + (62.4)\frac{\pi}{4}(1)^2(3) - (62.4)\left[\left(\frac{1}{3}\right)\frac{\pi}{4}(1)^2(1.207)\right]$$
$$= 339.3 + 147.0 - 19.7 = \mathbf{467\ lbf} \quad \textit{Ans.}$$

2.91 The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally-spaced bolts. What is the force in each bolt required to hold the dome down?

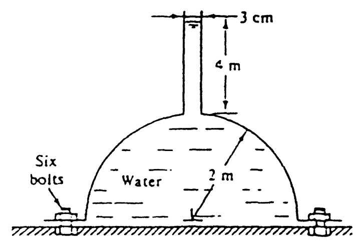


Fig. P2.91

Solution: Assuming no leakage, the hydrostatic force required equals the *weight of missing water*, that is, the water in a 4-m-diameter cylinder, 6 m high, minus the hemisphere and the small pipe:

$$F_{total} = W_{2\text{-m-cylinder}} - W_{2\text{-m-hemisphere}} - W_{3\text{-cm-pipe}}$$
$$= (9790)\pi(2)^2(6) - (9790)(2\pi/3)(2)^3 - (9790)(\pi/4)(0.03)^2(4)$$
$$= 738149 - 164033 - 28 = 574088\ \text{N}$$

The dome material helps with 30 kN of weight, thus the bolts must supply 574088–30000 or 544088 N. The force in each of 6 bolts is 544088/6 or $F_{bolt} \approx$ **90700 N** *Ans.*

2.92 A 4-m-diameter water tank consists of two half-cylinders, each weighing 4.5 kN/m, bolted together as in Fig. P2.92. If the end caps are neglected, compute the force in each bolt.

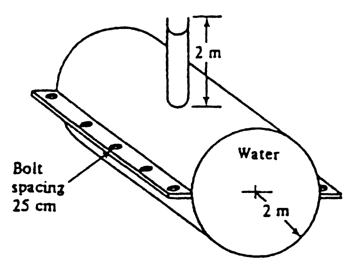


Fig. P2.92

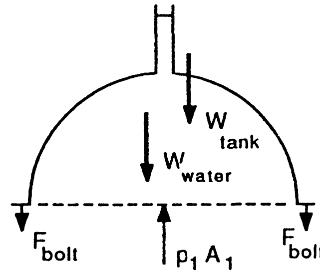
Solution: Consider a 25-cm width of upper cylinder, as at right. The water pressure in the bolt plane is

$$p_1 = \gamma h = (9790)(4) = 39160\ \text{Pa}$$

Then summation of vertical forces on this 25-cm-wide freebody gives

$$\sum F_z = 0 = p_1A_1 - W_{water} - W_{tank} - 2F_{bolt}$$

$$= (39160)(4\times 0.25) - (9790)(\pi/2)(2)^2(0.25)$$
$$-(4500)/4 - 2F_{bolt},$$



Solve for $F_{one\ bolt} = \mathbf{11300\ N}$ *Ans.*

2.93 In Fig. P2.93 a one-quadrant spherical shell of radius R is submerged in liquid of specific weight γ and depth h > R. Derive an analytic expression for the hydrodynamic force F on the shell and its line of action.

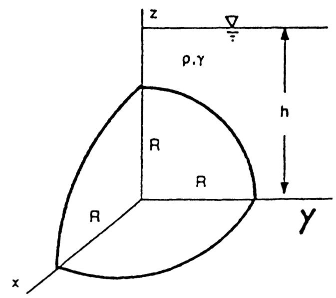


Fig. P2.93

Solution: The two horizontal components are identical in magnitude and equal to the force on the quarter-circle side panels, whose centroids are $(4R/3\pi)$ above the bottom:

$$\text{Horizontal components:} \quad F_x = F_y = \gamma h_{CG}A_{vert} = \gamma\left(h - \frac{4R}{3\pi}\right)\frac{\pi}{4}R^2$$

Similarly, the vertical component is the weight of the fluid above the spherical surface:

$$F_z = W_{cylinder} - W_{sphere} = \gamma\left(\frac{\pi}{4}R^2h\right) - \gamma\left(\frac{1}{8}\frac{4}{3}\pi R^3\right) = \gamma\frac{\pi}{4}R^2\left(h - \frac{2R}{3}\right)$$

There is no need to find the (complicated) centers of pressure for these three components, for we know that the resultant on a spherical surface *must pass through the center*. Thus

$$F = \left[F_x^2 + F_y^2 + F_z^2\right]^{1/2} = \boldsymbol{\gamma\frac{\pi}{4}R^2\left[(h - 2R/3)^2 + 2(h - 4R/3\pi)^2\right]^{1/2}} \quad \textit{Ans.}$$

2.94 The 4-ft-diameter log (SG = 0.80) in Fig. P2.94 is 8 ft long into the paper and dams water as shown. Compute the net vertical and horizontal reactions at point C.

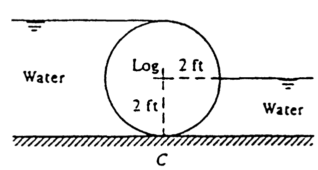
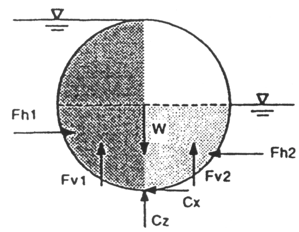


Fig. P2.94



Solution: With respect to the sketch at right, the horizontal components of hydrostatic force are given by

$$F_{h1} = (62.4)(2)(4\times 8) = 3994 \text{ lbf}$$

$$F_{h2} = (62.4)(1)(2\times 8) = 998 \text{ lbf}$$

The vertical components of hydrostatic force equal the weight of water in the shaded areas:

$$F_{v1} = (62.4)\frac{\pi}{2}(2)^2(8) = 3137 \text{ lbf}$$

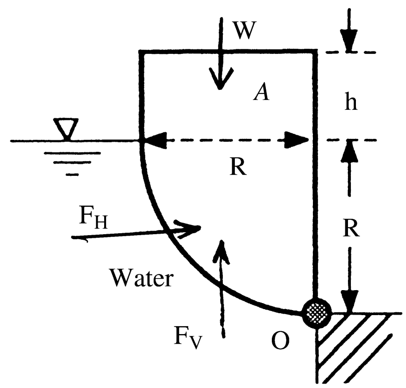
$$F_{v2} = (62.4)\frac{\pi}{4}(2)^2(8) = 1568 \text{ lbf}$$

The weight of the log is $W_{log} = (0.8 \times 62.4)\pi(2)^2(8) = 5018$ lbf. Then the reactions at C are found by summation of forces on the log freebody:

$$\sum F_x = 0 = 3994 - 998 - C_x, \quad \text{or} \quad C_x = \mathbf{2996\ lbf} \quad \textit{Ans.}$$

$$\sum F_z = 0 = C_z - 5018 + 3137 + 1568, \quad \text{or} \quad C_z = \mathbf{313\ lbf} \quad \textit{Ans.}$$

2.95 The uniform body A in the figure has width *b* into the paper and is in static equilibrium when pivoted about hinge O. What is the specific gravity of this body when (a) h = 0; and (b) h = R?



Solution: The water causes a horizontal and a vertical force on the body, as shown:

$$F_H = \gamma\frac{R}{2}Rb \quad at \quad \frac{R}{3} \ above\ O,$$

$$F_V = \gamma\frac{\pi}{4}R^2 b \quad at \quad \frac{4R}{3\pi} \ to\ the\ left\ of\ O$$

These must balance the moment of the body weight W about O:

$$\sum M_O = \frac{\gamma R^2 b}{2}\left(\frac{R}{3}\right) + \frac{\gamma\pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \frac{\gamma_s \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \gamma_s R h b\left(\frac{R}{2}\right) = 0$$

$$\textit{Solve for:}\quad SG_{body} = \frac{\gamma_s}{\gamma} = \left[\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{h}}{\mathbf{R}}\right]^{-1} \quad \textit{Ans.}$$

For h = 0, **SG = 3/2** *Ans.* (a). For h = R, **SG = 3/5** *Ans.* (b).

2.96 Curved panel BC is a 60° arc, perpendicular to the bottom at C. If the panel is 4 m wide into the paper, estimate the resultant hydrostatic force of the water on the panel.

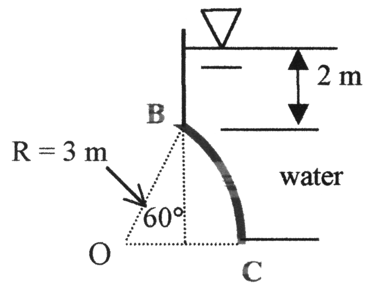
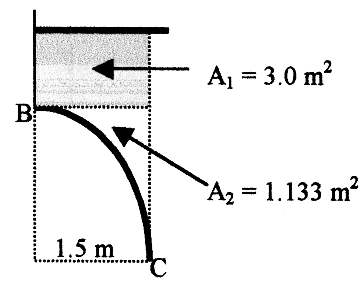


Fig. P2.96

Solution: The horizontal force is,

$$\begin{aligned} F_H &= \gamma h_{CG} A_h \\ &= (9790\ \text{N/m}^3)[2 + 0.5(3\sin 60°)\ \text{m}] \\ &\quad \times[(3\sin 60°)\text{m}(4\ \text{m})] \\ &= 335{,}650\ \text{N} \end{aligned}$$



The vertical component equals the weight of water above the gate, which is the sum of the rectangular piece above BC, and the curvy triangular piece of water just above arc BC—see figure at right. (The curvy-triangle calculation is messy and is not shown here.)

$$F_V = \gamma(\text{Vol})_{\text{above BC}} = (9790\ \text{N/m}^3)[(3.0 + 1.133\ \text{m}^2)(4\ \text{m})] = 161{,}860\ \text{N}$$

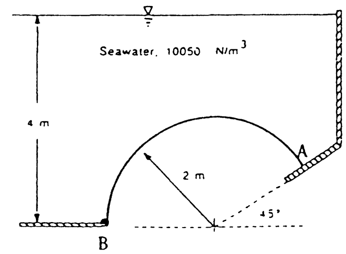
The resultant force is thus,

$$F_R = [(335{,}650)^2 + (161{,}860)^2]^{1/2} = 372{,}635\ \text{N} = \mathbf{373\ kN} \quad \textit{Ans.}$$

This resultant force acts along a line which passes through point O at

$$\theta = \tan^{-1}(161{,}860/335{,}650) = \mathbf{25.7°}$$

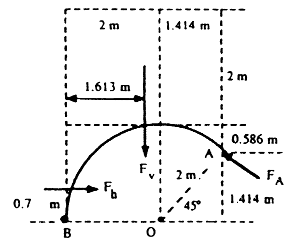
2.97 Gate AB is a 3/8th circle, 3 m wide into the paper, hinged at B and resting on a smooth wall at A. Compute the reaction forces at A and B.



Solution: The two hydrostatic forces are

$$F_h = \gamma h_{CG} A_h$$
$$= (10050)(4 - 0.707)(1.414 \times 3)$$
$$= 140 \text{ kN}$$

$$F_v = \text{weight above AB} = 240 \text{ kN}$$



To find the reactions, we need the lines of action of these two forces—a laborious task which is summarized in the figure at right. Then summation of moments on the gate, about B, gives

$$\sum M_{B,\text{clockwise}} = 0 = (140)(0.70) + (240)(1.613) - F_A(3.414), \quad \text{or} \quad F_A = \mathbf{142\ kN} \quad \textit{Ans.}$$

Finally, summation of vertical and horizontal forces gives

$$\sum F_z = B_z + 142\sin 45° - 240 = 0, \quad \text{or} \quad B_z = \mathbf{139\ kN}$$
$$\sum F_x = B_x - 142\cos 45° = 0, \quad \text{or} \quad B_x = \mathbf{99\ kN} \quad \textit{Ans.}$$

2.98 Gate *ABC* in Fig. P2.98 is a quarter circle 8 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the gate and the line of action of the resultant force.

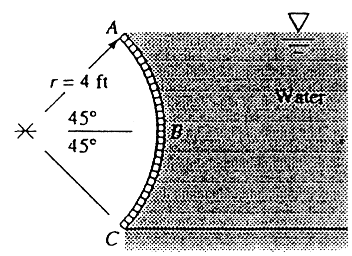
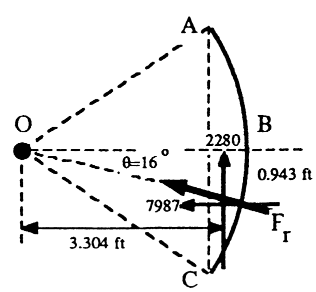


Fig. P2.98

Solution: The horizontal force is

$$F_h = \gamma h_{CG} A_h = (62.4)(2.828)(5.657 \times 8)$$
$$= \mathbf{7987\ lbf} \leftarrow$$

located at



$$y_{cp} = -\frac{(1/12)(8)(5.657)^3}{(2.828)(5.657 \times 8)} = -0.943 \text{ ft}$$

$$\text{Area ABC} = (\pi/4)(4)^2 - (4\sin 45°)^2$$
$$= 4.566 \text{ ft}^2$$

$$\text{Thus} \quad F_v = \gamma \text{Vol}_{ABC} = (62.4)(8)(4.566) = \mathbf{2280\ lbf} \uparrow$$

The resultant is found to be

$$F_R = [(7987)^2 + (2280)^2]^{1/2} = \mathbf{8300\ lbf} \quad \text{acting at } \theta = 15.9° \text{ through the center O.} \quad \textit{Ans.}$$

2.99 A 2-ft-diam sphere weighing 400 kbf closes the 1-ft-diam hole in the tank bottom. Find the force F to dislodge the sphere from the hole.

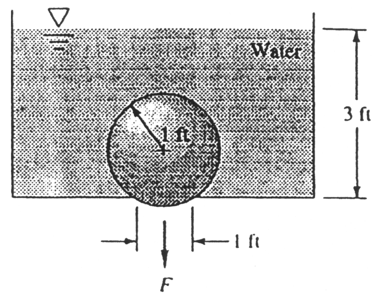
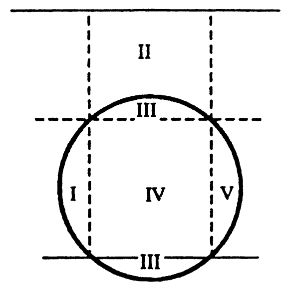


Fig. P2.99



Solution: <u>NOTE: This problem is laborious</u>! Break up the system into regions I,II,III,IV, & V. The respective volumes are:

$$\upsilon_{III} = 0.0539\ \text{ft}^3; \quad \upsilon_{II} = 0.9419\ \text{ft}^3$$

$$\upsilon_{IV} = \upsilon_I = \upsilon_V = 1.3603\ \text{ft}^3$$

Then the hydrostatic forces are:

$$F_{down} = \gamma\upsilon_{II} = (62.4)(0.9419) = 58.8\ \text{lbf}$$

$$F_{up} = \gamma(\upsilon_I + \upsilon_V) = (62.4)(2.7206) = 169.8\ \text{lbf}$$

Then the required force is $F = W + F_{down} - F_{up} = 400 + 59 - 170 = \mathbf{289\ lbf}\uparrow$ *Ans.*

2.100 Pressurized water fills the tank in Fig. P2.100. Compute the hydrostatic force on the conical surface ABC.

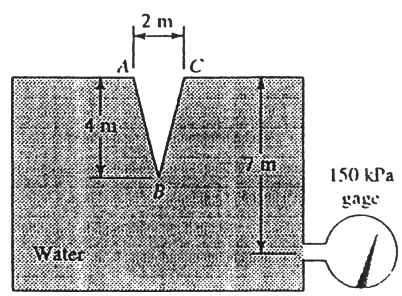
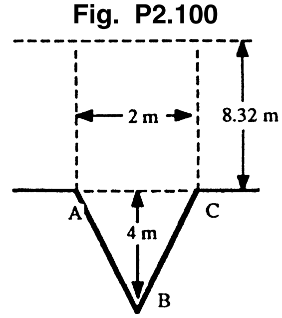


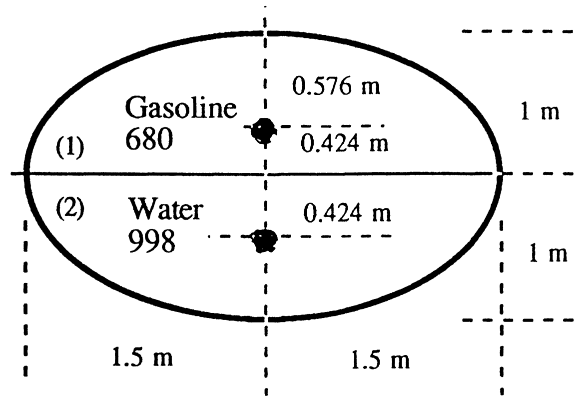
Fig. P2.100



Solution: The gage pressure is equivalent to a fictitious water level $h = p/\gamma = 150000/9790 = 15.32$ m above the gage or 8.32 m above AC. Then the vertical force on the cone equals the weight of fictitious water above ABC:

$$F_V = \gamma \text{Vol}_{above} = (9790)\left[\frac{\pi}{4}(2)^2(8.32) + \frac{1}{3}\frac{\pi}{4}(2)^2(4)\right] = \mathbf{297{,}000\ N} \quad \textit{Ans.}$$

2.101 A fuel tank has an elliptical cross-section as shown, with gasoline in the (vented) top and water in the bottom half. Estimate the total hydrostatic force on the flat end panel of the tank. The major axis is 3 m wide. The minor axis is 2 m high.



Solution: The centroids of the top and bottom halves are 4(1 m)/(3π) = 0.424 m from the center, as shown. The area of each half ellipse is (π/2)(1 m)(1.5 m) = 2.356 m^2. The forces on panel #1 in the gasoline and on panel #2 in the water are:

$$F_1 = \rho_1 g h_{CG1} A_1 = (680)(9.81)(0.576)(2.356) = 9050 \text{ N}$$

$$F_2 = p_{CG2} A_2 = [680(1.0) + 998(0.424)](9.81)(2.356) = 25500 \text{ N}$$

Then the total hydrostatic force on the end plate is 9050 + 25500 ≈ **34600 N** *Ans.*

2.102 A cubical tank is 3 × 3 × 3 m and is layered with 1 meter of fluid of specific gravity 1.0, 1 meter of fluid with SG = 0.9, and 1 meter of fluid with SG = 0.8. Neglect atmospheric pressure. Find (a) the hydrostatic force on the bottom; and (b) the force on a side panel.

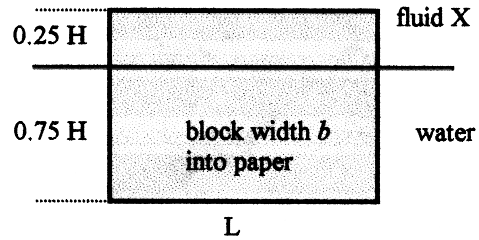
Solution: (a) The force on the bottom is the bottom pressure times the bottom area:

$$F_{bot} = p_{bot} A_{bot} = (9790 \text{ N/m}^3)[(0.8\times 1 \text{ m}) + (0.9\times 1 \text{ m}) + (1.0\times 1 \text{ m})](3 \text{ m})^2$$
$$= \mathbf{238{,}000\ N} \quad \textit{Ans.} \text{ (a)}$$

(b) The hydrostatic force on the side panel is the sum of the forces due to each layer:

$$F_{side} = \sum \gamma h_{CG} A_{side} = (0.8\times 9790 \text{ N/m}^3)(0.5 \text{ m})(3 \text{ m}^2) + (0.9\times 9790 \text{ N/m}^3)(1.5 \text{ m})(3 \text{ m}^2)$$
$$+ (9790 \text{ N/m}^3)(2.5 \text{ m})(3 \text{ m}^2) = \mathbf{125{,}000\ kN} \quad \textit{Ans.} \text{ (b)}$$

2.103 A solid block, of specific gravity 0.9, floats such that 75% of its volume is in water and 25% of its volume is in fluid X, which is layered above the water. What is the specific gravity of fluid X?



Solution: The block is sketched at right. A force balance is

$$0.9\gamma(\text{HbL}) = \gamma(0.75\text{HbL}) + \text{SG}_X\gamma(0.25\text{HbL})$$
$$0.9 - 0.75 = 0.25\text{SG}_X, \quad \boldsymbol{SG_X = 0.6} \quad \textit{Ans.}$$

2.104 The can in Fig. P2.104 floats in the position shown. What is its weight in newtons?

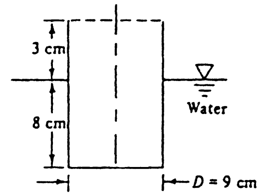


Fig. P2.104

Solution: The can weight simply equals the weight of the displaced water:

$$W = \gamma\upsilon_{\text{displaced}} = (9790)\frac{\pi}{4}(0.09\text{ m})^2(0.08\text{ m}) = \mathbf{5.0\ N} \quad \textit{Ans.}$$

2.105 Archimedes, when asked by King Hiero if the new crown was pure gold (SG = 19.3), found the crown weight in air to be 11.8 N and in water to be 10.9 N. Was it gold?

Solution: The buoyancy is the difference between air weight and underwater weight:

$$B = W_{\text{air}} - W_{\text{water}} = 11.8 - 10.9 = 0.9\text{ N} = \gamma_{\text{water}}\upsilon_{\text{crown}}$$

$$\text{But also} \quad W_{\text{air}} = (\text{SG})\gamma_{\text{water}}\upsilon_{\text{crown}}, \quad \text{so } W_{\text{in water}} = B(\text{SG} - 1)$$

$$\text{Solve for} \quad \text{SG}_{\text{crown}} = 1 + W_{\text{in water}}/B = 1 + 10.9/0.9 = \mathbf{13.1\ (not\ pure\ gold)} \quad \textit{Ans.}$$

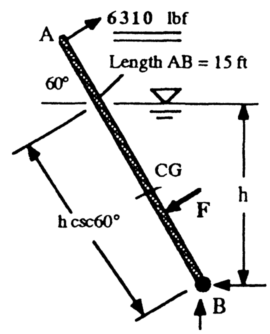
2.106 A spherical helium balloon is 2.5 m in diameter and has a total mass of 6.7 kg. When released into the U. S. Standard Atmosphere, at what altitude will it settle?

Solution: The altitude can be determined by calculating the air density to provide the proper buoyancy and then using Table A.3 to find the altitude associated with this density:

$$\rho_{air} = m_{balloon}/Vol_{sphere} = (6.7\text{ kg})/[\pi(2.5\text{ m}^3)/6] = 0.819\text{ kg/m}^3$$

From Table A.3, atmospheric air has $\rho = 0.819\text{ kg/m}^3$ at an altitude of about **4000 m**. *Ans.*

2.107 Repeat Prob. 2.62 assuming that the 10,000 lbf weight is aluminum (SG = 2.71) and is hanging submerged in the water.



Solution: Refer back to Prob. 2.62 for details. The only difference is that the force applied to gate AB by the weight is less due to buoyancy:

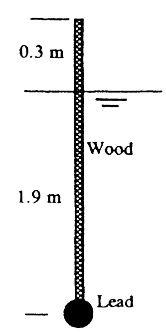
$$F_{net} = \frac{(SG-1)}{SG}\gamma \upsilon_{body} = \frac{2.71-1}{2.71}(10000) = 6310\text{ lbf}$$

This force replaces "10000" in the gate moment relation (see Prob. 2.62):

$$\sum M_B = 0 = 6310(15) - (288.2h^2)\left(\frac{h}{2}\csc 60° - \frac{h}{6}\csc 60°\right) - 4898(7.5\cos 60°)$$

$$\text{or: } h^3 = 76280/110.9 = 688, \quad \text{or: } h = \mathbf{8.83\text{ ft}} \quad \textit{Ans.}$$

2.108 A yellow pine rod (SG = 0.65) is 5 cm by 5 cm by 2.2 m long. How much lead (SG = 11.4) is needed at one end so that the rod will float vertically with 30 cm out of the water?



Solution: The weight of wood and lead must equal the buoyancy of immersed wood and lead:

$$W_{wood} + W_{lead} = B_{wood} + B_{lead},$$

$$\text{or: } (0.65)(9790)(0.05)^2(2.2) + 11.4(9790)\upsilon_{lead} = (9790)(0.05)^2(1.9) + 9790\upsilon_{lead}$$

$$\text{Solve for } \upsilon_{lead} = 0.000113\text{ m}^3 \quad \text{whence } W_{lead} = 11.4(9790)\upsilon_{lead} = \mathbf{12.6\text{ N}} \quad \textit{Ans.}$$

2.109 The float level h of a <u>hydrometer</u> is a measure of the specific gravity of the liquid. For stem diameter D and total weight W, if h = 0 represents SG = 1.0, derive a formula for h as a function of W, D, SG, and γ_o for water.

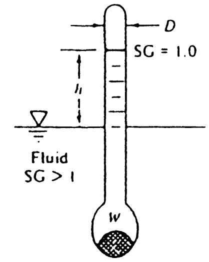
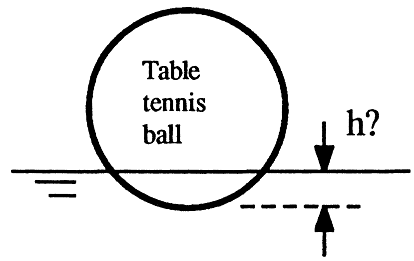


Fig. P2.109

Solution: Let submerged volume be υ_o when SG = 1. Let $A = \pi D^2/4$ be the area of the stem. Then

$$W = \gamma_o \upsilon_o = (SG)\gamma_o(\upsilon_o - Ah), \quad \text{or:} \quad \mathbf{h} = \frac{\mathbf{W(SG-1)}}{\mathbf{SG}\gamma_o(\pi \mathbf{D}^2/4)} \quad \textit{Ans.}$$

2.110 An average table tennis ball has a diameter of 3.81 cm and a mass of 2.6 gm. Estimate the (small) depth h at which the ball will float in water at 20°C and sea-level standard air if air buoyancy is (a) neglected; or (b) included.



Solution: For both parts we need the volume of the submerged spherical segment:

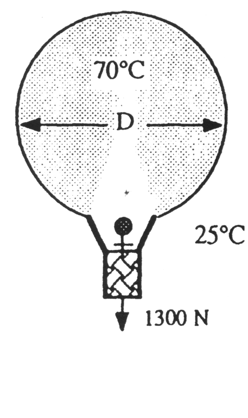
$$W = 0.0026(9.81) = 0.0255\ N = \rho_{water} g \frac{\pi h^2}{3}(3R - h), \quad R = 0.01905 \text{ m}, \quad \rho = 998\ \frac{kg}{\text{m}^3}$$

(a) Air buoyancy is neglected. Solve for h ≈ 0.00705 m = **7.05 mm** *Ans.* (a)
(b) Also include air buoyancy on the exposed sphere volume in the air:

$$0.0255\ N = \rho_w g \upsilon_{seg} + \rho_{air} g\left[\frac{4}{3}\pi R^3 - \upsilon_{seg}\right], \quad \rho_{air} = 1.225\ \frac{kg}{\text{m}^3}$$

The air buoyancy is only one-80th of the water. Solve ***h* = 7.00 mm** *Ans.* (b)

2.111 A hot-air balloon must support its own weight plus a person for a total weight of 1300 N. The balloon material has a mass of 60 g/m^2. Ambient air is at 25°C and 1 atm. The hot air inside the balloon is at 70°C and 1 atm. What diameter spherical balloon will just support the weight? Neglect the size of the hot-air inlet vent.



Solution: The buoyancy is due to the difference between hot and cold air density:

$$\rho_{cold} = \frac{p}{RT_{cold}} = \frac{101350}{(287)(273+25)} = 1.185\,\frac{kg}{m^3};\quad \rho_{hot} = \frac{101350}{287(273+70)} = 1.030\,\frac{kg}{m^3}$$

The buoyant force must balance the known payload of 1300 N:

$$W = 1300\text{ N} = \Delta\rho g\text{ Vol} = (1.185 - 1.030)(9.81)\frac{\pi}{6}D^3,$$

$$\text{Solve for}\quad D^3 = 1628 \quad\text{or}\quad D_{balloon} \approx \mathbf{11.8\ m} \quad \textit{Ans.}$$

Check to make sure the balloon material is not excessively heavy:

$$W(\text{balloon}) = (0.06\text{ kg/m}^2)(9.81\text{ m/s}^2)(\pi)(11.8\text{ m})^2 \approx 256\text{ N} \quad \text{OK, only 20\% of } W_{total}.$$

2.112 The uniform 5-m-long wooden rod in the figure is tied to the bottom by a string. Determine (a) the string tension; and (b) the specific gravity of the wood. Is it also possible to determine the inclination angle θ?

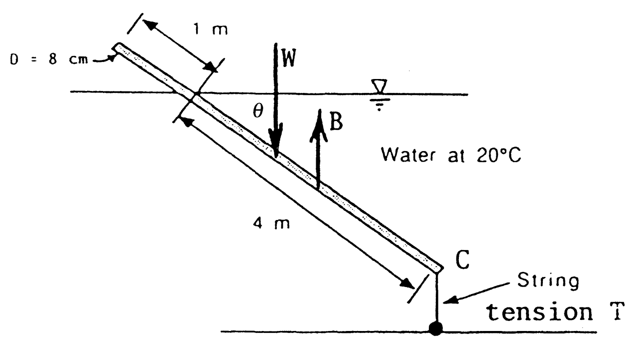


Fig. P2.112

Solution: The rod weight acts at the middle, 2.5 m from point C, while the buoyancy is 2 m from C. Summing moments about C gives

$$\sum M_C = 0 = W(2.5\sin\theta) - B(2.0\sin\theta), \quad\text{or}\quad W = 0.8B$$

$$\text{But}\quad B = (9790)(\pi/4)(0.08\text{ m})^2(4\text{ m}) = 196.8\text{ N}.$$

$$\text{Thus}\quad W = 0.8B = 157.5\text{ N} = SG(9790)(\pi/4)(0.08)^2(5\text{ m}),\quad\text{or:}\quad SG \approx \mathbf{0.64} \quad \textit{Ans. (b)}$$

Summation of vertical forces yields

$$\text{String tension}\quad T = B - W = 196.8 - 157.5 \approx \mathbf{39\ N} \quad \textit{Ans. (a)}$$

These results are independent of the angle θ, which cancels out of the moment balance.

2.113 A *spar buoy* is a rod weighted to float vertically, as in Fig. P2.113. Let the buoy be maple wood (SG = 0.6), 2 in by 2 in by 10 ft, floating in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added at the bottom so that $h = 18$ in?

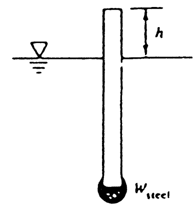


Fig. P2.113

Solution: The relevant volumes needed are

$$\text{Spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(10) = 0.278\ \text{ft}^3; \quad \text{Steel volume} = \frac{W_{steel}}{7.85(62.4)}$$

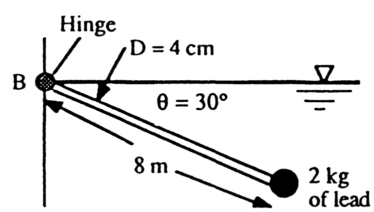
$$\text{Immersed spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(8.5) = 0.236\ \text{ft}^3$$

The vertical force balance is: buoyancy $B = W_{wood} + W_{steel}$,

$$\text{or:}\quad 1.025(62.4)\left[0.236 + \frac{W_{steel}}{7.85(62.4)}\right] = 0.6(62.4)(0.278) + W_{steel}$$

$$\text{or:}\quad 15.09 + 0.1306W_{steel} = 10.40 + W_{steel}, \quad \text{solve for}\quad W_{steel} \approx \mathbf{5.4\ lbf} \quad \textit{Ans.}$$

2.114 The uniform rod in the figure is hinged at B and in static equilibrium when 2 kg of lead (SG = 11.4) are attached at its end. What is the specific gravity of the rod material? What is peculiar about the rest angle $\theta = 30°$?



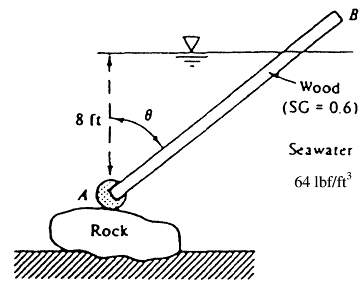
Solution: First compute buoyancies: $B_{rod} = 9790(\pi/4)(0.04)^2(8) = 98.42$ N, and $W_{lead} = 2(9.81) = 19.62$ N, $B_{lead} = 19.62/11.4 = 1.72$ N. Sum moments about B:

$$\sum M_B = 0 = (SG-1)(98.42)(4\cos 30°) + (19.62 - 1.72)(8\cos 30°) = 0$$

$$\text{Solve for}\quad \mathbf{SG_{rod} = 0.636} \quad \textit{Ans.}\text{ (a)}$$

The angle θ drops out! The rod is neutrally stable for **any tilt angle!** *Ans.* (b)

2.115 The 2 inch by 2 inch by 12 ft spar buoy from Fig. P2.113 has 5 lbm of steel attached and has gone aground on a rock. If the rock exerts no moments on the spar, compute the angle of inclination *θ*.



Solution: Let ζ be the submerged length of spar. The relevant forces are:

$$W_{wood} = (0.6)(64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(12) = 12.8 \text{ lbf} \quad \textit{at distance } 6\,\mathbf{sin}\,\theta \textit{ to the right of } A \downarrow$$

$$Buoyancy = (64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)\zeta = 1.778\zeta \quad \textit{at distance } \frac{\zeta}{2}\mathbf{sin}\,\theta \textit{ to the right of } A \uparrow$$

The steel force acts right through A. Take moments about A:

$$\sum M_A = 0 = 12.8(6\,\mathbf{sin}\,\theta) - 1.778\zeta\left(\frac{\zeta}{2}\mathbf{sin}\,\theta\right)$$

$$\textit{Solve for} \quad \zeta^2 = 86.4, \quad \textit{or} \quad \zeta = 9.295\,\textit{ft} \quad (\textit{submerged length})$$

Thus the angle of inclination $\theta = \cos^{-1}(8.0/9.295) = \mathbf{30.6°}$ *Ans.*

2.116 When the 12-cm cube in the figure is immersed in 20°C ethanol, it is balanced on the beam scale by a 2-kg mass. What is the specific gravity of the cube?

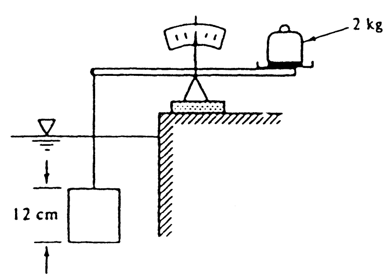


Fig. P2.116

Solution: The scale force is 2(9.81) = 19.62 N. The specific weight of ethanol is 7733 N/m^3. Then

$$F = 19.62 = (W - B)_{cube} = (\gamma_{cube} - 7733)(0.12 \text{ m})^3$$

$$\text{Solve for } \gamma_{cube} = 7733 + 19.62/(0.12)^3 \approx \mathbf{19100\ N/m^3} \quad \textit{Ans.}$$

2.117 The balloon in the figure is filled with helium and pressurized to 135 kPa and 20°C. The balloon material has a mass of 85 g/m^2. Estimate (a) the tension in the mooring line, and (b) the height in the standard atmosphere to which the balloon will rise if the mooring line is cut.

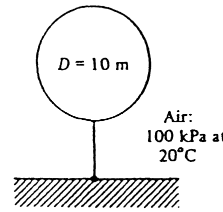


Fig. P2.117

Solution: (a) For helium, from Table A-4, R = 2077 m^2/s^2/K, hence its weight is

$$W_{helium} = \rho_{He} g \upsilon_{balloon} = \left[\frac{135000}{2077(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 1139\ N$$

Meanwhile, the total weight of the balloon material is

$$W_{balloon} = \left(0.085\ \frac{kg}{\text{m}^2}\right)\left(9.81\ \frac{\text{m}}{s^2}\right)[\pi(10\ \text{m})^2] = 262\ N$$

Finally, the balloon buoyancy is the weight of displaced air:

$$B_{air} = \rho_{air} g \upsilon_{balloon} = \left[\frac{100000}{287(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 6108\ N$$

The difference between these is the tension in the mooring line:

$$T_{line} = B_{air} - W_{helium} - W_{balloon} = 6108 - 1139 - 262 \approx \mathbf{4700\ N} \quad \textit{Ans.}\ \text{(a)}$$

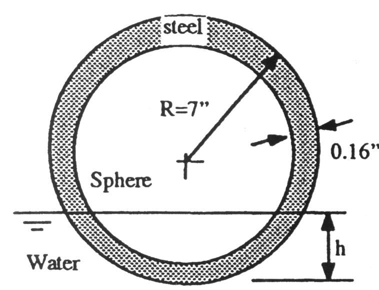
(b) If released, and the balloon remains at 135 kPa and 20°C, equilibrium occurs when the balloon air buoyancy exactly equals the total weight of 1139 + 262 = 1401 N:

$$B_{air} = 1401\ N = \rho_{air}(9.81)\frac{\pi}{6}(10)^3, \quad or: \quad \rho_{air} \approx 0.273\ \frac{kg}{\text{m}^3}$$

From Table A-6, this standard density occurs at approximately

$$\mathbf{Z \approx 12{,}800\ m} \quad \textit{Ans.}\ \text{(b)}$$

2.118 A 14-in-diameter hollow sphere of steel (SG = 7.85) has 0.16 in wall thickness. How high will this sphere float in 20°C water? How much weight must be added inside to make the sphere neutrally buoyant?



Solution: The weight of the steel is

$$W_{steel} = \gamma \text{Vol} = (7.85)(62.4)\frac{\pi}{6}\left[\left(\frac{14}{12}\right)^3 - \left(\frac{13.68}{12}\right)^3\right]$$
$$= 27.3 \text{ lbf}$$

This is equivalent to 27.3/62.4 = 0.437 ft^3 of displaced water, whereas $\upsilon_{sphere} = 0.831$ ft^3.

Therefore the sphere floats *slightly above* its midline, such that the sphere segment volume, of height *h* in the figure, equals the displaced volume:

$$\upsilon_{segment} = 0.437 \text{ ft}^3 = \frac{\pi}{3}h^2(3R - h) = \frac{\pi}{3}h^2[3(7/12) - h]$$

Solve for h = 0.604 ft ≈ **7.24 in** *Ans.*

In order for the sphere to be *neutrally* buoyant, we need another (0.831 − 0.437) = 0.394 ft^3 of displaced water, so we need additional weight ΔW = 62.4(0.394) ≈ **25 lbf**. *Ans.*

2.119 With a 5-lbf-weight placed at one end, the uniform wooden beam in the figure floats at an angle θ with its upper right corner at the surface. Determine (a) θ; (b) γ_{wood}.

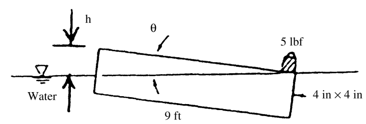


Fig. P2.119

Solution: The total wood volume is (4/12)^2(9) = 1 ft^3. The exposed distance $h = 9\tan\theta$. The vertical forces are

$$\sum F_z = 0 = (62.4)(1.0) - (62.4)(h/2)(9)(4/12) - (SG)(62.4)(1.0) - 5 \text{ lbf}$$

The moments of these forces about point C at the right corner are:

$$\sum M_C = 0 = \gamma(1)(4.5) - \gamma(1.5h)(6 \text{ ft}) - (SG)(\gamma)(1)(4.5 \text{ ft}) + (5 \text{ lbf})(0 \text{ ft})$$

where $\gamma = 62.4$ lbf/ft^3 is the specific weight of water. Clean these two equations up:

$$1.5h = 1 - SG - 5/\gamma \quad \text{(forces)} \qquad 2.0h = 1 - SG \quad \text{(moments)}$$

Solve simultaneously for SG ≈ **0.68** *Ans.* (b); h = 0.16 ft; $\theta \approx$ **1.02°** *Ans.* (a)

2.120 A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?

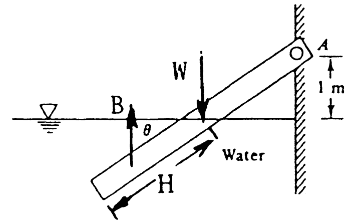


Fig. P2.120

Solution: The total beam volume is $3(.1)^2 = 0.03\ \text{m}^3$, and therefore its weight is $W = (0.65)(9790)(0.03) = 190.9$ N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is $B = (9790)(0.1)^2H = 97.9H$ newtons, acting at H/2 from the lower end. Sum moments about point A:

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:}\quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225\ \text{m}$$

Geometry: $3 - H = 1.775$ m is out of the water, or: $\sin\theta = 1.0/1.775$, or $\theta \approx \mathbf{34.3°}$ *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b,h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) $\gamma_b = \gamma/3$; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

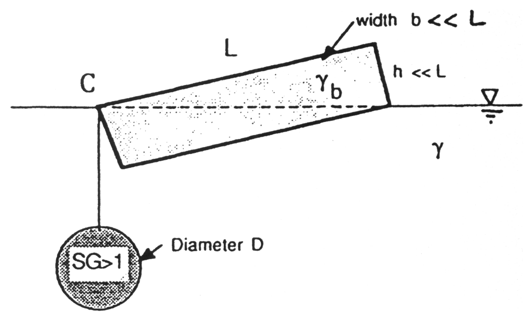


Fig. P2.121

Solution: The beam weight $W = \gamma_b Lhb$ and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals $B = \gamma Lhb/2$ and acts at L/3 from the left corner. Sum moments about the left corner, point C:

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{\text{sphere}} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that} \quad D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG - 1)}}\right]^{1/3} \quad \textit{Ans. (b)}$$

2.122 A uniform block of steel (SG = 7.85) will "float" at a mercury-water interface as in the figure. What is the ratio of the distances *a* and *b* for this condition?

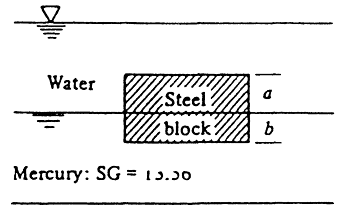


Fig. P2.122

Solution: Let *w* be the block width into the paper and let γ be the water specific weight. Then the vertical force balance on the block is

$$7.85\gamma(a+b)Lw = 1.0\gamma aLw + 13.56\gamma bLw,$$

$$\text{or: } 7.85a + 7.85b = a + 13.56b, \quad \text{solve for } \frac{a}{b} = \frac{13.56 - 7.85}{7.85 - 1} = \mathbf{0.834} \quad \textit{Ans.}$$

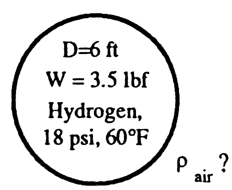
2.123 A spherical balloon is filled with helium at sea level. Helium and balloon material together weigh 500 N. If the net upward lift force on the balloon is also 500 N, what is the diameter of the balloon?

Solution: Since the net upward force is 500 N, the buoyancy force is 500 N plus the weight of the balloon and helium, or B = 1000 N. From Table A.3, the density of air at sea level is 1.2255 kg/m^3.

$$B = 1000\text{ N} = \rho_{air} g V_{balloon} = (1.2255)(9.81)(\pi/6)D^3$$

$$\mathbf{D = 5.42\ m} \quad \textit{Ans.}$$

2.124 A balloon weighing 3.5 lbf is 6 ft in diameter. If filled with hydrogen at 18 psia and 60°F and released, at what U.S. standard altitude will it be neutral?



Solution: Assume that it remains at 18 psia and 60°F. For hydrogen, from Table A-4, $R \approx 24650$ ft^2/(s^2·°R). The density of the hydrogen in the balloon is thus

$$\rho_{H_2} = \frac{p}{RT} = \frac{18(144)}{(24650)(460+60)} \approx 0.000202 \text{ slug/ft}^3$$

In the vertical force balance for neutral buoyancy, only the outside air density is unknown:

$$\sum F_z = B_{air} - W_{H_2} - W_{balloon} = \rho_{air}(32.2)\frac{\pi}{6}(6)^3 - (0.000202)(32.2)\frac{\pi}{6}(6)^3 - 3.5\text{ lbf}$$

$$\text{Solve for} \quad \rho_{air} \approx 0.00116 \text{ slug/ft}^3 \approx 0.599 \text{ kg/m}^3$$

From Table A-6, this density occurs at a standard altitude of **6850 m ≈ 22500 ft**. *Ans.*

2.125 Suppose the balloon in Prob. 2.111 is constructed with a diameter of 14 m, is filled at sea level with hot air at 70°C and 1 atm, and released. If the hot air remains at 70°C, at what U.S. standard altitude will the balloon become neutrally buoyant?

Solution: Recall from Prob. 2.111 that the hot air density is $p/RT_{hot} \approx 1.030 \text{ kg/m}^3$. Assume that the entire weight of the balloon consists of its material, which from Prob. 2.111 had a density of 60 grams per square meter of surface area. Neglect the vent hole. Then the vertical force balance for neutral buoyancy yields the air density:

$$\sum F_z = B_{air} - W_{hot} - W_{balloon}$$

$$= \rho_{air}(9.81)\frac{\pi}{6}(14)^3 - (1.030)(9.81)\frac{\pi}{6}(14)^3 - (0.06)(9.81)\pi(14)^2$$

$$\text{Solve for} \quad \rho_{air} \approx 1.0557 \text{ kg/m}^3.$$

From Table A-6, this air density occurs at a standard altitude of **1500 m**. *Ans.*

2.126 A block of wood (SG = 0.6) floats in fluid X in Fig. P2.126 such that 75% of its volume is submerged in fluid X. Estimate the gage pressure of the air in the tank.

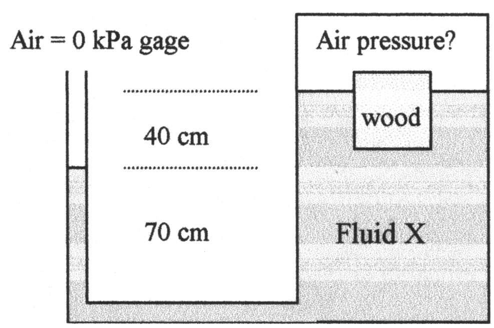


Fig. P2.126

Solution: In order to apply the hydrostatic relation for the air pressure calculation, the density of Fluid X must be found. The buoyancy principle is thus first applied. Let the block have volume V. Neglect the buoyancy of the air on the upper part of the block. Then

$$0.6\gamma_{water}V = \gamma_X(0.75V) + \cancel{\gamma_{air}(0.25V)}; \quad \gamma_X \approx 0.8\gamma_{water} = 7832 \ N/\text{m}^3$$

The air gage pressure may then be calculated by jumping from the left interface into fluid X:

$$0 \text{ Pa-gage} - (7832 \text{ N/m}^3)(0.4 \text{ m}) = p_{air} = -3130 \text{ Pa-gage} = \textbf{3130 Pa-vacuum} \quad \textit{Ans.}$$

2.127* Consider a cylinder of specific gravity $S < 1$ floating vertically in water ($S = 1$), as in Fig. P2.127. Derive a formula for the stable values of D/L as a function of S and apply it to the case $D/L = 1.2$.

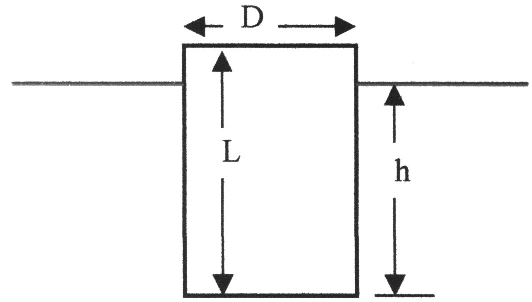


Fig. P2.127

Solution: A vertical force balance provides a relation for h as a function of S and L,

$$\gamma\pi D^2 h/4 = S\gamma\pi D^2 L/4, \quad \text{thus} \quad h = SL$$

To compute stability, we turn Eq. (2.52), centroid G, metacenter M, center of buoyancy B:

$$MB = I_o/v_{sub} = \frac{\frac{\pi}{4}(D/2)^4}{\frac{\pi}{4}Dh} = MG + GB \quad \textit{and substituting } h = SL, \quad \frac{D^2}{16SL} = MG + GB$$

where GB = L/2 – h/2 = L/2 – SL/2 = L(1 – S)/2. For neutral stability, MG = 0. Substituting,

$$\frac{D^2}{16SL} = 0 + \frac{L}{2}(1 - S) \quad \textit{solving for } D/L, \quad \frac{\mathbf{D}}{\mathbf{L}} = \sqrt{\mathbf{8S(1-S)}} \quad \textit{Ans.}$$

For D/L = 1.2, $S^2 - S - 0.18 = 0$ giving $\mathbf{0 \le S \le 0.235}$ and $\mathbf{0.765 \le S \le 1}$ *Ans.*

2.128 The iceberg of Fig. 2.20 can be idealized as a cube of side length L as shown. If seawater is denoted as S = 1, the iceberg has S = 0.88. Is it stable?

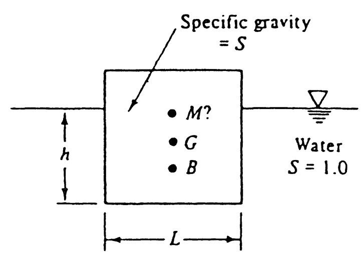


Fig. P2.128

Solution: The distance *h* is determined by

$$\gamma_w h L^2 = S\gamma_w L^3, \quad \text{or:} \quad h = SL$$

The center of gravity is at L/2 above the bottom, and B is at h/2 above the bottom. The metacenter position is determined by Eq. (2.52):

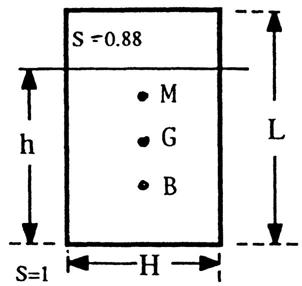
$$MB = I_o/\upsilon_{sub} = \frac{L^4/12}{L^2 h} = \frac{L^2}{12h} = \frac{L}{12S} = MG + GB$$

Noting that GB = L/2 − h/2 = L(1 − S)/2, we may solve for the metacentric height:

$$MG = \frac{L}{12S} - \frac{L}{2}(1-S) = 0 \quad \text{if } S^2 - S + \frac{1}{6} = 0, \text{ or: } S = 0.211 \quad \text{or} \quad 0.789$$

Instability: 0.211 < S < 0.789. Since the iceberg has S = 0.88 > 0.789, **it is stable**. *Ans.*

2.129 The iceberg of Prob. 2.128 may become unstable if its width decreases. Suppose that the height is L and the depth into the paper is L but the width decreases to H < L. Again with S = 0.88 for the iceberg, determine the ratio H/L for which the iceberg becomes unstable.



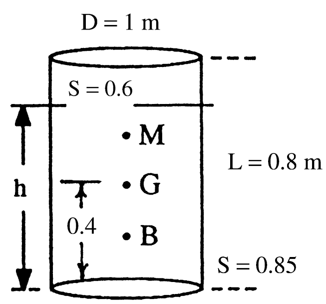
Solution: As in Prob. 2.128, the submerged distance h = SL = 0.88L, with G at L/2 above the bottom and B at h/2 above the bottom. From Eq. (2.52), the distance MB is

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{LH^3/12}{HL(SL)} = \frac{H^2}{12SL} = MG + GB = MG + \left(\frac{L}{2} - \frac{SL}{2}\right)$$

Then neutral stability occurs when MG = 0, or

$$\frac{H^2}{12SL} = \frac{L}{2}(1-S), \quad \text{or} \quad \frac{H}{L} = [6S(1-S)]^{1/2} = [6(0.88)(1-0.88)]^{1/2} = \mathbf{0.796} \quad \textit{Ans.}$$

2.130 Consider a wooden cylinder (SG = 0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (SG = 0.85)?



Solution: A vertical force balance gives

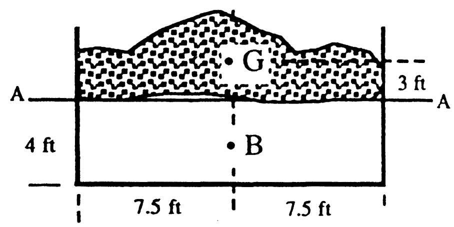
$$0.85\pi R^2 h = 0.6\pi R^2(0.8 \text{ m}),$$
$$\text{or:} \quad h = 0.565 \text{ m}$$

The point B is at h/2 = 0.282 m above the bottom. Use Eq. (2.52) to predict the metacenter location:

$$MB = I_o/\upsilon_{sub} = [\pi(0.5)^4/4]/[\pi(0.5)^2(0.565)] = 0.111 \text{ m} = MG + GB$$

Now GB = 0.4 m − 0.282 m = 0.118 m, hence **MG** = 0.111 − 0.118 = **−0.007 m**. This float position is thus **slightly unstable**. The cylinder would turn over. *Ans.*

2.131 A barge is 15 ft wide and floats with a draft of 4 ft. It is piled so high with gravel that its center of gravity is 3 ft above the waterline, as shown. Is it stable?



Solution: Example 2.10 applies to this case, with L = 7.5 ft and H = 4 ft:

$$MA = \frac{L^2}{3H} - \frac{H}{2} = \frac{(7.5\text{ ft})^2}{3(4\text{ ft})} - \frac{4\text{ ft}}{2} = 2.69\text{ ft}, \quad \text{where “A” is the waterline}$$

Since G is **<u>3</u>** ft *above* the waterline, MG = 2.69 − 3.0 = **−0.31 ft, unstable**. *Ans.*

2.132 A solid right circular cone has SG = 0.99 and floats vertically as shown. Is this a stable position?

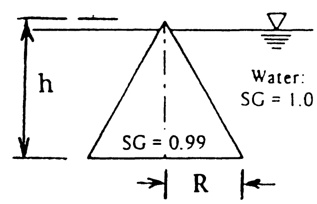


Fig. P2.132

Solution: Let *r* be the radius at the surface and let *z* be the exposed height. Then

$$\sum F_z = 0 = \gamma_w \frac{\pi}{3}(R^2h - r^2z) - 0.99\gamma_w \frac{\pi}{3}R^2h, \quad \text{with } \frac{z}{h} = \frac{r}{R}.$$

$$\text{Thus} \quad \frac{z}{h} = (0.01)^{1/3} = 0.2154$$

The cone floats at a draft $\zeta = h - z = 0.7846h$. The centroid G is at 0.25h above the bottom. The center of buoyancy B is at the centroid of a frustrum of a (submerged) cone:

$$\zeta = \frac{0.7846h}{4}\left(\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right) = 0.2441h \quad \text{above the bottom}$$

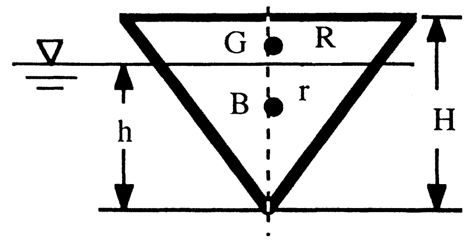
Then Eq. (2.52) predicts the position of the metacenter:

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{\pi(0.2154R)^4/4}{0.99\pi R^2h} = 0.000544\frac{R^2}{h} = MG + GB$$

$$= MG + (0.25h - 0.2441h) = MG + 0.0594h$$

Thus MG > 0 (**stability**) if $(R/h)^2 \geq 10.93$ or $\mathbf{R/h \geq 3.31}$ *Ans.*

2.133 Consider a uniform right circular cone of specific gravity S < 1, floating with its vertex down in water, S = 1.0. The base radius is R and the cone height is H, as shown. Calculate and plot the stability parameter MG of this cone, in dimensionless form, versus H/R for a range of cone specific gravities S < 1.



Solution: The cone floats at height *h* and radius *r* such that B = W, or:

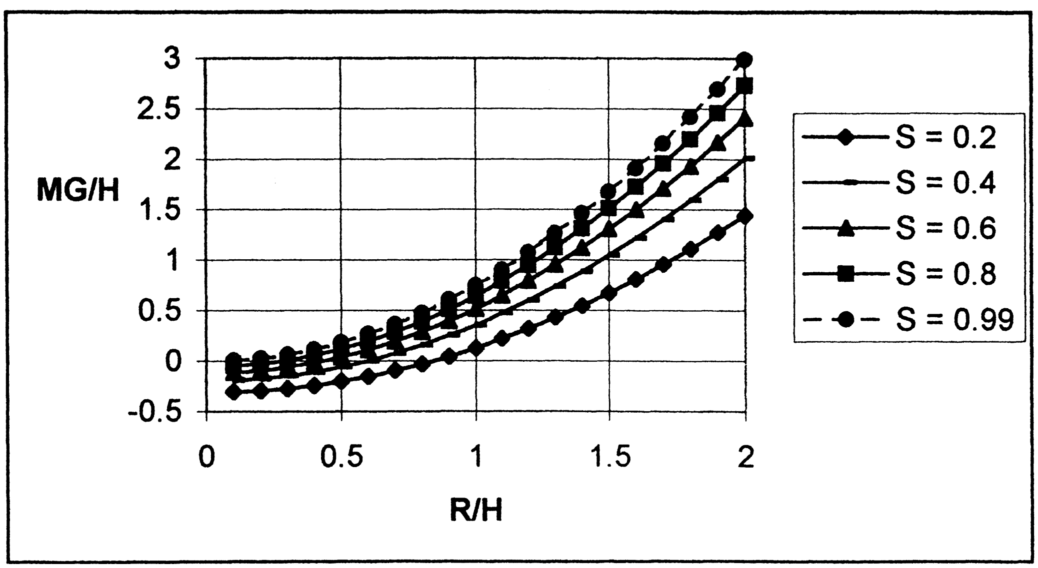
$$\frac{\pi}{3}r^2h(1.0) = \frac{\pi}{3}R^2H(S), \quad or: \quad \frac{h^3}{H^3} = \frac{r^3}{R^3} = S < 1$$

Thus r/R = h/H = $S^{1/3}$ = ζ for short. Now use the stability relation:

$$MG + GB = MG + \left(\frac{3H}{4} - \frac{3h}{4}\right) = \frac{I_o}{\upsilon_{sub}} = \frac{\pi r^4/4}{\pi r^2 h/3} = \frac{3\zeta R^2}{4H}$$

Non-dimensionalize in the final form: $\mathbf{\frac{MG}{H} = \frac{3}{4}\left(\zeta\frac{R^2}{H^2} - 1 + \zeta\right)}$, $\zeta = S^{1/3}$ *Ans.*

This is plotted below. Floating cones pointing *down* are stable unless slender, R ≪ H.



2.134 When floating in water (SG = 1), an equilateral triangular body (SG = 0.9) might take *two* positions, as shown at right. Which position is more stable? Assume large body width into the paper.

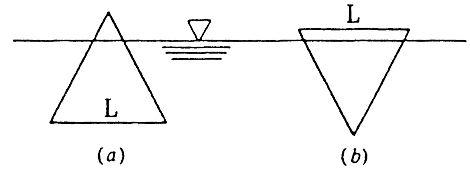


Fig. P2.134

Solution: The calculations are similar to the floating cone of Prob. 2.132. Let the triangle be L by L by L. List the basic results.

(a) Floating with point *up*: Centroid G is 0.289L above the bottom line, center of buoyancy B is 0.245L above the bottom, hence GB = (0.289 – 0.245)L ≈ 0.044L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.0068L = MG + GB = MG + 0.044L$$

$$\text{Hence} \quad MG = \mathbf{-0.037L} \quad \textbf{Unstable} \quad \textit{Ans. (a)}$$

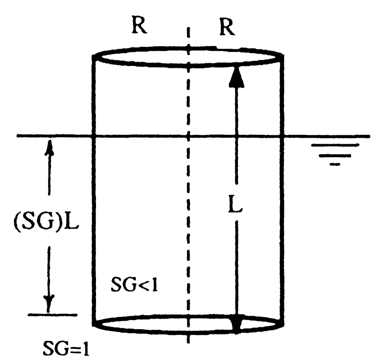
(b) Floating with point *down*: Centroid G is 0.577L above the bottom point, center of buoyancy B is 0.548L above the bottom point, hence GB = (0.577 – 0.548)L ≈ 0.0296L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.1826L = MG + GB = MG + 0.0296L$$

$$\text{Hence} \quad MG = \mathbf{+0.153L} \quad \textbf{Stable} \quad \textit{Ans. (b)}$$

2.135 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG, floating in water (SG = 1) with its axis *vertical*. Show that the body is stable if

$$R/L > [2SG(1 - SG)]^{1/2}$$



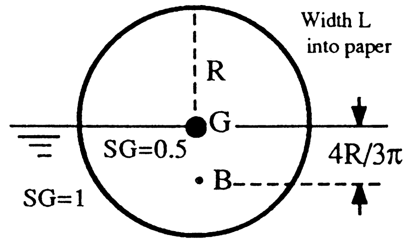
Solution: For a given SG, the body floats with a draft equal to (SG)L, as shown. Its center of gravity G is at L/2 above the bottom. Its center of buoyancy B is at (SG)L/2 above the bottom. Then Eq. (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{\pi R^4/4}{\pi R^2(SG)L} = \frac{R^2}{4(SG)L} = MG + GB = MG + \frac{L}{2} - SG\frac{L}{2}$$

$$\text{Thus} \quad MG > 0 \text{ (stability)} \quad \text{if} \quad \mathbf{R^2/L^2 > 2SG(1 - SG)} \quad \textit{Ans.}$$

For example, if SG = 0.8, stability requires that R/L > 0.566.

2.136 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG = 0.5, floating in water (SG = 1) with its axis *horizontal*. Show that the body is stable if L/R > 2.0.

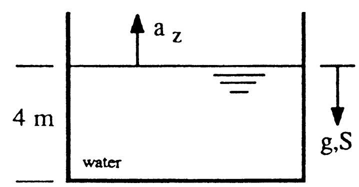


Solution: For the given SG = 0.5, the body floats centrally with a draft equal to R, as shown. Its center of gravity G is exactly at the surface. Its center of buoyancy B is at the centroid of the immersed semicircle: 4R/(3π) below the surface. Equation (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{(1/12)(2R)L^3}{\pi(R^2/2)L} = \frac{L^2}{3\pi R} = MG + GB = MG + \frac{4R}{3\pi}$$

$$\text{or:}\quad MG = \frac{L^2}{3\pi R} - \frac{4R}{3\pi} > 0 \text{ (stability)} \quad \text{if } \mathbf{L/R > 2} \quad \textit{Ans.}$$

2.137 A tank of water 4 m deep receives a constant upward acceleration a_z. Determine (a) the gage pressure at the tank bottom if a_z = 5 m^2/s; and (b) the value of a_z which causes the gage pressure at the tank bottom to be 1 atm.



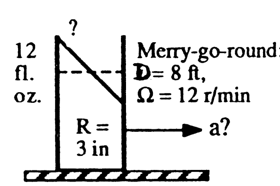
Solution: Equation (2.53) states that $\nabla p = \rho(\mathbf{g} - \mathbf{a}) = \rho(-\mathbf{k}g - \mathbf{k}a_z)$ for this case. Then, for part (a),

$$\Delta p = \rho(g + a_z)\Delta S = (998 \text{ kg/m}^3)(9.81 + 5 \text{ m}^2\text{/s})(4 \text{ m}) = \mathbf{59100 \text{ Pa (gage)}} \quad \textit{Ans.}\text{ (a)}$$

For part (b), we know Δp = 1 atm but we don't know the acceleration:

$$\Delta p = \rho(g + a_z)\Delta S = (998)(9.81 + a_z)(4.0) = 101350 \text{ Pa} \quad \text{if } \mathbf{a_z = 15.6 \frac{m}{s^2}} \quad \textit{Ans.}\text{ (b)}$$

2.138 A 12 fluid ounce glass, 3 inches in diameter, sits on the edge of a merry-go-round 8 ft in diameter, rotating at 12 r/min. How full can the glass be before it spills?



Solution: First, how high is the container? Well, 1 fluid oz. = 1.805 in^3, hence 12 fl. oz. = 21.66 in^3 = $\pi(1.5\text{ in})^2h$, or $h \approx 3.06$ *in*—It is a fat, nearly square little glass. Second, determine the acceleration toward the center of the merry-go-round, noting that the angular velocity is Ω = (12 rev/min)(1 min/60 s)(2π rad/rev) = 1.26 rad/s. Then, for r = 4 ft,

$$a_x = \Omega^2 r = (1.26\text{ rad/s})^2(4\text{ ft}) = 6.32\text{ ft/s}^2$$

Then, for steady rotation, the water surface in the glass will slope at the angle

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{6.32}{32.2 + 0} = 0.196, \quad \text{or:} \quad \Delta h_{\text{left to center}} = (0.196)(1.5\text{ in}) = 0.294\text{ in}$$

Thus the glass should be filled to no more than 3.06 – 0.294 ≈ 2.77 inches
This amount of liquid is $\upsilon = \pi(1.5\text{ in})^2(2.77\text{ in})$ = 19.6 in^3 ≈ **10.8 fluid oz.** *Ans.*

2.139 The tank of liquid in the figure P2.139 accelerates to the right with the fluid in rigid-body motion. (a) Compute a_x in m/s^2. (b) Why doesn't the solution to part (a) depend upon fluid density? (c) Compute gage pressure at point A if the fluid is glycerin at 20°C.

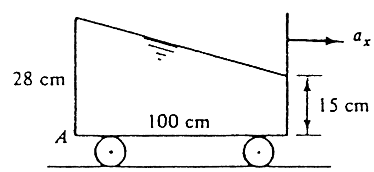


Fig. P2.139

Solution: (a) The slope of the liquid gives us the acceleration:

$$\mathbf{\tan\theta} = \frac{a_x}{g} = \frac{28 - 15\text{ cm}}{100\text{ cm}} = 0.13, \quad \text{or:} \quad \theta = 7.4°$$

$$\textit{thus} \quad a_x = 0.13g = 0.13(9.81) = \mathbf{1.28\ m/s^2} \quad \textit{Ans.}\text{ (a)}$$

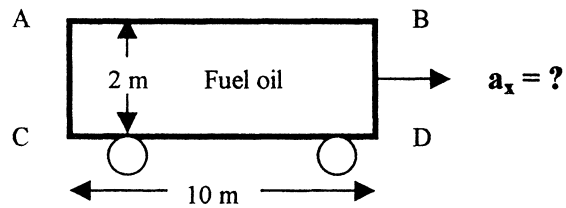
(b) Clearly, the solution to (a) is purely geometric and does not involve fluid density. *Ans.* (b)
(c) From Table A-3 for glycerin, ρ = 1260 kg/m^3. There are many ways to compute p_A. For example, we can go straight down on the left side, using only gravity:

$$p_A = \rho g\Delta z = (1260\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.28\text{ m}) = \mathbf{3460\ Pa\ (gage)} \quad \textit{Ans.}\text{ (c)}$$

Or we can start on the right side, go down 15 cm with g and across 100 cm with a_x:

$$p_A = \rho g\Delta z + \rho a_x\Delta x = (1260)(9.81)(0.15) + (1260)(1.28)(1.00)$$
$$= 1854 + 1607 = \mathbf{3460\ Pa} \quad \textit{Ans.}\text{ (c)}$$

2.140 Suppose that the elliptical-end fuel tank in Prob. 2.101 is 10 m long and filled completely with fuel oil (ρ = 890 kg/m^3). Let the tank be pulled along a horizontal road in rigid-body motion. Find the acceleration and direction for which (a) a constant-pressure surface extends from the top of the front end to the bottom of the back end; and (b) the top of the back end is at a pressure 0.5 atm lower than the top of the front end.



Solution: (a) We are given that the isobar or constant-pressure line reaches from point C to point B in the figure above, θ is *negative*, hence the tank is *decelerating*. The elliptical shape is immaterial, only the 2-m height. The isobar slope gives the acceleration:

$$\tan\theta_{C-B} = -\frac{2\text{ m}}{10\text{ m}} = -0.2 = \frac{a_x}{g}, \quad hence\ \ a_x = -0.2(9.81) = \mathbf{-1.96\ m/s^2} \quad Ans.\ \text{(a)}$$

(b) We are now given that p_A (back end top) is lower than p_B (front end top)—see the figure above. Thus, again, the isobar must slope upward through B but not necessarily pass through point C. The pressure difference along line AB gives the correct *deceleration*:

$$\Delta p_{A-B} = -0.5(101325\ Pa) = \rho_{oil} a_x \Delta x_{A-B} = \left(890\ \frac{kg}{\text{m}^3}\right) a_x (10\text{ m})$$

$$solve\ for \quad a_x = \mathbf{-5.69\ m/s^2} \quad Ans.\ \text{(b)}$$

This is more than part (a), so the isobar angle must be steeper:

$$\tan\theta = \frac{-5.69}{9.81} = -0.580, \quad hence\ \ \theta_{isobar} = -30.1°$$

The isobar in part (a), line CB, has the angle $\theta_{(a)} = \tan^{-1}(-0.2) = -11.3°$.

2.141 The same tank from Prob. 2.139 is now accelerating while rolling *up* a 30° inclined plane, as shown. Assuming rigid-body motion, compute (a) the acceleration **a**, (b) whether the acceleration is up or down, and (c) the pressure at point A if the fluid is mercury at 20°C.

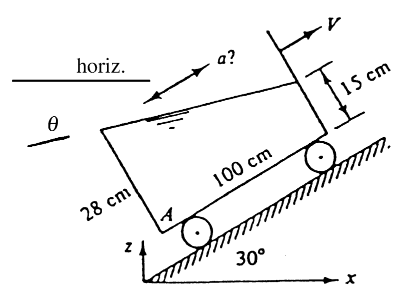


Fig. P2.141

Solution: The free surface is tilted at the angle $\theta = -30° + 7.41° = -22.59°$. This angle must satisfy Eq. (2.55):

$$\tan\theta = \tan(-22.59°) = -0.416 = a_x/(g + a_z)$$

But the 30° incline constrains the acceleration such that $a_x = 0.866a$, $a_z = 0.5a$. Thus

$$\tan\theta = -0.416 = \frac{0.866a}{9.81 + 0.5a}, \quad \text{solve for} \quad \mathbf{a \approx -3.80\,\frac{m}{s^2}\ (down)} \quad \textit{Ans.}\ (a, b)$$

The cartesian components are $a_X = -3.29\ m/s^2$ and $a_Z = -1.90\ m/s^2$.
(c) The distance ΔS normal from the surface down to point A is (28 cosθ) cm. Thus

$$p_A = \rho[a_x^2 + (g + a_z)^2]^{1/2} = (13550)[(-3.29)^2 + (9.81 - 1.90)^2]^{1/2}(0.28\cos 7.41°)$$
$$\approx \mathbf{32200\ Pa\ (gage)} \quad \textit{Ans.}\ (c)$$

2.142 The tank of water in Fig. P2.142 is 12 cm wide into the paper. If the tank is accelerated to the right in rigid-body motion at 6 m/s^2, compute (a) the water depth at AB, and (b) the water force on panel AB.

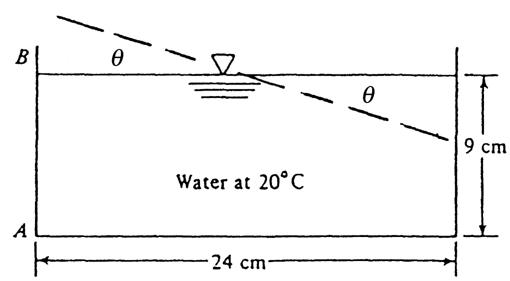


Fig. P2.142

Solution: From Eq. (2.55),

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta \approx 31.45°$$

Then surface point B on the left rises an additional $\Delta z = 12\tan\theta \approx 7.34$ cm,

$$\text{or:} \quad \text{water depth AB} = 9 + 7.34 \approx \mathbf{16.3\ cm} \quad \textit{Ans.}\ (a)$$

The water pressure on AB varies linearly due to gravity only, thus the water force is

$$F_{AB} = p_{CG}A_{AB} = (9790)\left(\frac{0.163}{2}\ m\right)(0.163\ m)(0.12\ m) \approx \mathbf{15.7\ N} \quad \textit{Ans.}\ (b)$$

2.143 The tank of water in Fig. P2.143 is full and open to the atmosphere (p_{atm} = 15 psi = 2160 psf) at point A, as shown. For what acceleration a_x, in ft/s^2, will the pressure at point B in the figure be (a) atmospheric; and (b) zero absolute (neglecting cavitation)?

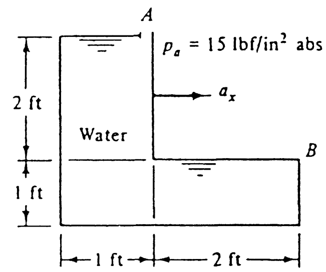


Fig. P2.143

Solution: (a) For $p_A = p_B$, the imaginary 'free surface isobar' should join points A and B:

$$\mathbf{tan}\,\theta_{AB} = \mathbf{tan}\,45° = 1.0 = a_x/g, \quad \text{hence } a_x = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}\ \text{(a)}$$

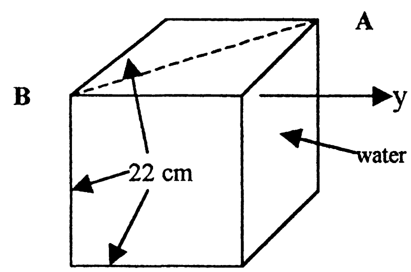
(b) For $p_B = 0$, the free-surface isobar must tilt even more than 45°, so that

$$p_B = 0 = p_A + \rho g\Delta z - \rho a_x \Delta x = 2160 + 1.94(32.2)(2) - 1.94a_x(2),$$

$$\textit{solve} \quad a_x = \mathbf{589\ ft/s^2} \quad \textit{Ans.}\ \text{(b)}$$

This is a very high acceleration (18 g's) and a very steep angle, $\theta = \tan^{-1}(589/32.2) = 87°$.

2.144 Consider a hollow cube of side length 22 cm, full of water at 20°C, and open to p_{atm} = 1 atm at top corner A. The top surface is horizontal. Determine the rigid-body accelerations for which the water at opposite top corner B will *cavitate*, for (a) horizontal, and (b) vertical motion.



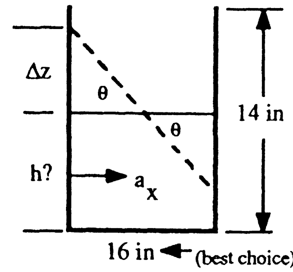
Solution: From Table A-5 the vapor pressure of the water is 2337 Pa. (a) Thus cavitation occurs first when accelerating horizontally along the diagonal AB:

$$p_A - p_B = 101325 - 2337 = \rho a_{x,AB}\Delta L_{AB} = (998)a_{x,\,AB}(0.22\sqrt{2}),$$

$$\textit{solve} \quad a_{x,AB} = \mathbf{319\ m/s^2} \quad \textit{Ans.}\ \text{(a)}$$

If we moved along the *y* axis shown in the figure, we would need $a_y = 319\sqrt{2} = 451$ m/s^2. (b) For *vertical* acceleration, ***nothing would happen***, both points A and B would continue to be atmospheric, although the pressure at deeper points would change. *Ans.*

2.145 A fish tank 16-in by 27-in by 14-inch deep is carried in a car which may experience accelerations as high as 6 m/s^2. Assuming rigid-body motion, estimate the maximum water depth to avoid spilling. Which is the best way to align the tank?



Solution: The best way is to *align the 16-inch width with the car's direction of motion*, to minimize the vertical surface change Δz. From Eq. (2.55) the free surface angle will be

$$\tan\theta_{max} = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{thus } \Delta z = \frac{16''}{2}\tan\theta = 4.9 \text{ inches } (\theta = 31.5°)$$

Thus the tank should contain no more than 14 – 4.9 ≈ **9.1 inches of water**. *Ans.*

2.146 The tank in Fig. P2.146 is filled with water and has a vent hole at point A. It is 1 m wide into the paper. Inside is a 10-cm balloon filled with helium at 130 kPa. If the tank accelerates to the right at 5 m/s/s, at what angle will the balloon lean? Will it lean to the left or to the right?

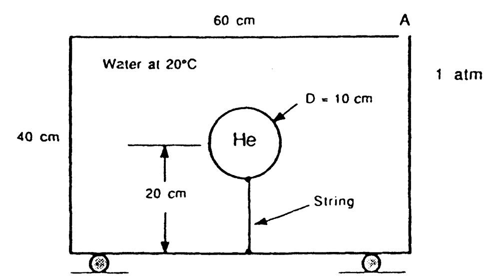
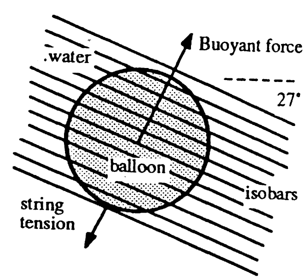


Fig. P2.146

Solution: The acceleration sets up pressure isobars which slant down and to the right, in both the water *and* in the helium. This means there will be a buoyancy force on the balloon up and to the right, as shown at right. It must be balanced by a string tension down and to the left. If we neglect balloon material weight, the balloon leans ***up and to the right*** at angle



$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{5.0}{9.81}\right) \approx \mathbf{27°} \quad \textit{Ans.}$$

measured from the vertical. This acceleration-buoyancy effect may seem counter-intuitive.

2.147 The tank of water in Fig. P2.147 accelerates uniformly by rolling without friction down the 30° inclined plane. What is the angle θ of the free surface? Can you explain this interesting result?

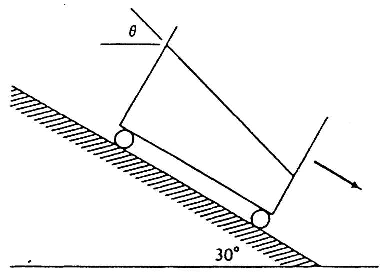


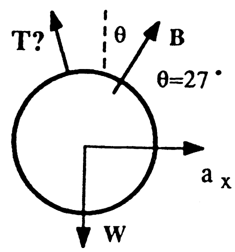
Fig. P2.147

Solution: If frictionless, $\Sigma F = W \sin\theta = ma$ along the incline and thus $a = g \sin 30° = 0.5g$.

$$\text{Thus } \tan\theta = \frac{a_x}{g + a_z} = \frac{0.5g\cos 30°}{g - 0.5g\sin 30°}; \quad \text{solve for } \theta = \mathbf{30°\,!} \quad \textit{Ans.}$$

The free surface aligns itself exactly parallel with the 30° incline.

2.148 Modify Prob. 2.146 as follows: Let the 10-cm-diameter sphere be concrete (SG = 2.4) hanging by a string from the *top*. If the tank accelerates to the right at 5 m/s/s, at what angle will the balloon lean? Will it lean to the left or to the right?



Solution: This problem differs from 2.146 only in the heavy weight of the solid sphere, which still reacts to the acceleration but not due to an internal "pressure gradient." The x-directed forces are not in balance. The equations of motion are

$$\sum F_x = m_{sphere}a_x = B_x + T_X,$$

$$\text{or:} \quad T_x = a_x(2.4 - 1.0)(998)\frac{\pi}{6}(0.1)^3 = 3.66 \text{ N}$$

$$\sum F_z = 0 = B_z + T_z - W,$$

$$\text{or:} \quad T_z = g(2.4 - 1.0)(998)\frac{\pi}{6}(0.1)^3 = 7.18 \text{ N}$$

$$\text{Thus} \quad T = (T_x^2 + T_z^2)^{1/2} = \mathbf{8.06\ N} \quad \text{acting at } \theta = \text{atan}\left(\frac{3.66}{7.18}\right) = \mathbf{27°}$$

The concrete sphere hangs down and to the left at an angle of 27°. *Ans.*

2.149 The waterwheel in Fig. P2.149 lifts water with 1-ft-diameter half-cylinder blades. The wheel rotates at 10 r/min. What is the water surface angle θ at pt. A?

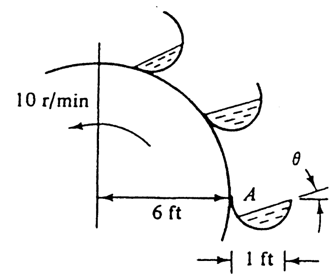


Fig. P2.149

Solution: Convert Ω = 10 r/min = 1.05 rad/s. Use an average radius R = 6.5 ft. Then

$$a_x = \Omega^2 R = (1.05)^2(6.5) \approx 7.13 \text{ ft/s}^2 \quad \text{toward the center}$$

$$\text{Thus} \quad \tan\theta = a_x/g = 7.13/32.2, \quad \text{or:} \quad \theta = \mathbf{12.5°} \quad \textit{Ans.}$$

2.150 A cheap accelerometer can be made from the U-tube at right. If L = 18 cm and D = 5 mm, what will *h* be if $a_x = 6 \text{ m/s}^2$?

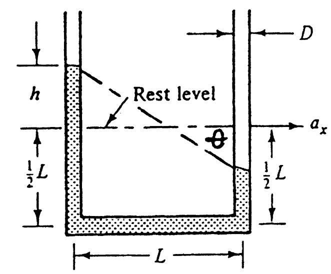


Fig. P2.150

Solution: We assume that the diameter is so small, $D \ll L$, that the free surface is a "point." Then Eq. (2.55) applies, and

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta = 31.5°$$

$$\text{Then} \quad h = (L/2)\tan\theta = (9 \text{ cm})(0.612) = \mathbf{5.5\ cm} \quad \textit{Ans.}$$

Since $h = (9 \text{ cm})a_x/g$, the scale readings are indeed linear in a_x, but I don't recommend it as an actual accelerometer, there are too many inaccuracies and disadvantages.

2.151 The U-tube in Fig. P2.151 is open at A and closed at D. What uniform acceleration a_x will cause the pressure at point C to be atmospheric? The fluid is water.

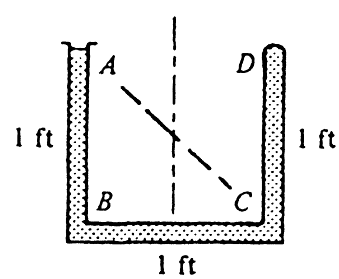
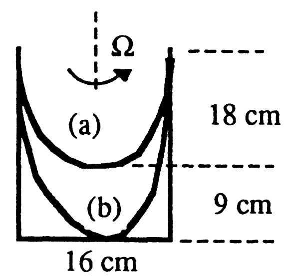


Fig. P2.151

Solution: If pressures at A and C are the same, the "free surface" must join these points:

$$\theta = 45°, \; a_x = g\tan\theta = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}$$

2.152 A 16-cm-diameter open cylinder 27 cm high is full of water. Find the central rigid-body rotation rate for which (a) one-third of the water will spill out; and (b) the bottom center of the can will be exposed.



Solution: (a) One-third will spill out if the resulting paraboloid surface is 18 cm deep:

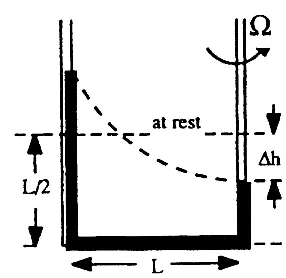
$$h = 0.18\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)},\quad \text{solve for}\quad \Omega^2 = 552,$$

$$\Omega = 23.5\text{ rad/s} = \mathbf{224\ r/min}\quad \textit{Ans.}\ \text{(a)}$$

(b) The bottom is barely exposed if the paraboloid surface is 27 cm deep:

$$h = 0.27\text{ m} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)},\quad \text{solve for}\ \ \Omega = 28.8\text{ rad/s} = \mathbf{275\ r/min}\quad \textit{Ans.}\ \text{(b)}$$

2.153 Suppose the U-tube in Prob. 2.150 is not translated but instead is *rotated about the right leg* at 95 r/min. Find the level *h* in the left leg if L = 18 cm and D = 5 mm.

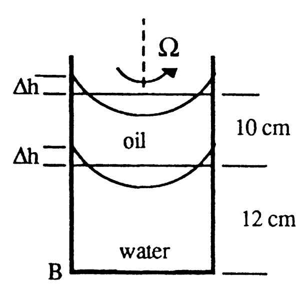


Solution: Convert Ω = 95 r/min = 9.95 rad/s. Then "R" = L = 18 cm, and, since $D \ll L$,

$$\Delta h = \frac{\Omega^2 R^2}{4g} = \frac{(9.95)^2 (0.18)^2}{4(9.81)} = 0.082\text{ m},$$

$$\text{thus}\quad h_{\text{left leg}} = 9 + 8.2 = \mathbf{17.2\ cm}\quad \textit{Ans.}$$

2.154 A very deep 18-cm-diameter can has 12 cm of water, overlaid with 10 cm of SAE 30 oil. It is rotated about the center in rigid-body motion at 150 r/min. (a) What will be the shapes of the interfaces? (b) What and where will be the maximum fluid pressure?



Solution: Convert Ω = 150 r/min = 15.7 rad/s. (a) The parabolic surfaces which result are entirely independent of the fluid density, hence both interfaces will curl up into the same-shape paraboloid, with a deflection Δh up at the wall and down in the center:

$$\Delta h = \frac{\Omega^2 R^2}{4g} = \frac{(15.7)^2(0.09)^2}{4(9.81)} = 0.051 \text{ m} = \mathbf{5.1\ cm} \quad \textit{Ans.} \text{ (a)}$$

(b) The fluid pressure will be highest at point B in the bottom corner. We can compute this by moving straight down through the oil and water at the wall, with gravity only:

$$p_B = \rho_{oil} g \Delta z_{oil} + \rho_{water} g \Delta z_{water}$$
$$= (891)(9.81)(0.1 \text{ m}) + (998)(9.81)(0.051 + 0.12 \text{ m}) = \mathbf{2550\ Pa\ (gage)} \quad \textit{Ans.} \text{ (b)}$$

2.155 For what uniform rotation rate in r/min about axis C will the U-tube fluid in Fig. P2.155 take the position shown? The fluid is mercury at 20°C.

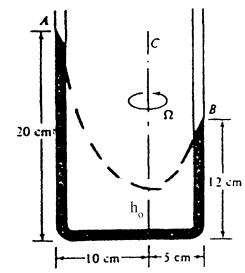


Fig. P2.155

Solution: Let h_o be the height of the free surface at the centerline. Then, from Eq. (2.64),

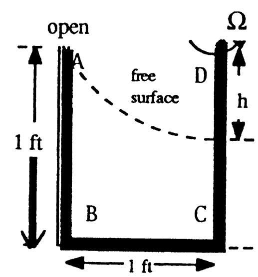
$$z_B = h_o + \frac{\Omega^2 R_B^2}{2g}; \quad z_A = h_o + \frac{\Omega^2 R_A^2}{2g}; \quad R_B = 0.05 \text{ m} \quad \text{and} \quad R_A = 0.1 \text{ m}$$

$$\text{Subtract:} \quad z_A - z_B = 0.08 \text{ m} = \frac{\Omega^2}{2(9.81)}[(0.1)^2 - (0.05)^2],$$

$$\text{solve} \quad \Omega = 14.5 \ \frac{\text{rad}}{\text{s}} = \mathbf{138\ \frac{r}{min}} \quad \textit{Ans.}$$

The fact that the fluid is mercury does not enter into this "kinematic" calculation.

2.156 Suppose the U-tube of Prob. 2.151 is rotated about axis DC. If the fluid is water at 122°F and atmospheric pressure is 2116 psfa, at what rotation rate will the fluid begin to vaporize? At what point in the tube will this happen?



Solution: At 122°F = 50°C, from Tables A-1 and A-5, for water, ρ = 988 kg/m^3 (or 1.917 slug/ft^3) and p_v = 12.34 kPa (or 258 psf). When spinning around DC, the free surface comes down from point A to a position *below* point D, as shown. Therefore the fluid pressure is lowest at point D (*Ans.*). With *h* as shown in the figure,

$$p_D = p_{vap} = 258 = p_{atm} - \rho gh = 2116 - 1.917(32.2)h, \quad h = \Omega^2R^2/(2g)$$

Solve for h ≈ 30.1 ft (!) Thus the drawing is wildly distorted and the dashed line falls **far below** point C! (The solution is correct, however.)

$$\text{Solve for} \quad \Omega^2 = 2(32.2)(30.1)/(1\text{ ft})^2 \quad \text{or:} \quad \Omega = 44\text{ rad/s} = \mathbf{420\ rev/min}. \quad \textit{Ans.}$$

2.157 The 45° V-tube in Fig. P2.157 contains water and is open at A and closed at C. (a) For what rigid-body rotation rate will the pressure be equal at points B and C? (b) For the condition of part (a), at what point in leg BC will the pressure be a minimum?

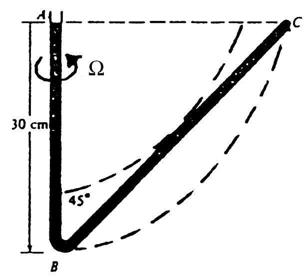


Fig. P2.157

Solution: (a) If pressures are equal at B and C, they must lie on a constant-pressure paraboloid surface as sketched in the figure. Taking $z_B = 0$, we may use Eq. (2.64):

$$z_C = 0.3\text{ m} = \frac{\Omega^2R^2}{2g} = \frac{\Omega^2(0.3)^2}{2(9.81)}, \quad \text{solve for} \quad \Omega = 8.09\ \frac{\text{rad}}{\text{s}} = \mathbf{77}\ \frac{\mathbf{rev}}{\mathbf{min}} \quad \textit{Ans. (a)}$$

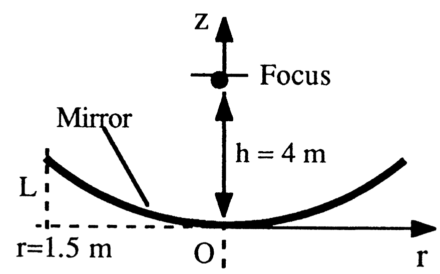
(b) The minimum pressure in leg BC occurs where the highest paraboloid pressure contour is tangent to leg BC, as sketched in the figure. This family of paraboloids has the formula

$$z = z_o + \frac{\Omega^2r^2}{2g} = r\tan 45°, \quad \text{or:} \quad z_o + 3.333r^2 - r = 0 \quad \text{for a pressure contour}$$

$$\text{The minimum occurs when} \quad dz/dr = 0, \quad \text{or} \quad r \approx \mathbf{0.15\ m} \quad \textit{Ans. (b)}$$

The minimum pressure occurs *halfway between points B and C.*

2.158* It is desired to make a 3-m-diameter parabolic telescope mirror by rotating molten glass in rigid-body motion until the desired shape is achieved and then cooling the glass to a solid. The focus of the mirror is to be 4 m from the mirror, measured along the centerline. What is the proper mirror rotation rate, in rev/min?



Solution: We have to review our math book, or Mark's Manual, to recall that the *focus* F of a parabola is the point for which all points on the parabola are equidistant from both the focus and a so-called "directrix" line (which is one focal length below the mirror). For the focal length *h* and the *z-r* axes shown in the figure, the equation of the parabola is given by $\mathbf{r^2 = 4hz}$, with $h = 4$ m for our example.
Meanwhile the equation of the free-surface of the liquid is given by $\mathbf{z = r^2\Omega^2/(2g)}$.
Set these two equal to find the proper rotation rate:

$$z = \frac{r^2\Omega^2}{2g} = \frac{r^2}{4h}, \quad \text{or:} \quad \Omega^2 = \frac{g}{2h} = \frac{9.81}{2(4)} = 1.226$$

$$\textit{Thus} \quad \Omega = 1.107\ \frac{\text{rad}}{\text{s}}\left(\frac{60}{2\pi}\right) = \mathbf{10.6\ rev/min} \quad \textit{Ans.}$$

The focal point F is far above the mirror itself. If we put in $r = 1.5$ m and calculate the mirror depth "L" shown in the figure, we get L ≈ 14 centimeters.

2.159 The three-legged manometer in Fig. P2.159 is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

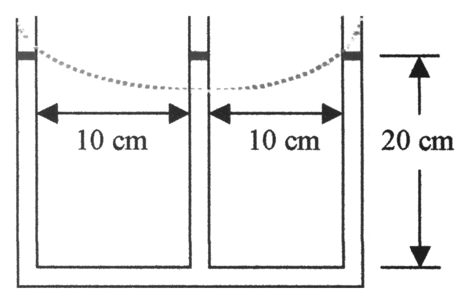


Fig. P2.159

Solution: (a) The free-surface during rotation is visualized as the **red** dashed line in Fig. P2.159. The outer right and left legs experience an increase which is one-half that of the central leg, or $\Delta h_O = \Delta h_C/2$. The total displacement between outer and center menisci is, from Eq. (2.64) and Fig. 2.23, equal to $\Omega^2R^2/(2g)$. The center meniscus

falls two-thirds of this amount and feeds the outer tubes, which each rise one-third of this amount above the rest position:

$$\boldsymbol{\Delta h_{outer} = \frac{1}{3}\Delta h_{total} = \frac{\Omega^2 R^2}{6g} \qquad \Delta h_{center} = -\frac{2}{3}\Delta h_{total} = -\frac{\Omega^2 R^2}{3g}} \quad \textit{Ans. (a)}$$

For the particular case $R = 10$ cm and $\Omega = 120$ r/min $= (120)(2\pi/60) = 12.57$ rad/s, we obtain

$$\frac{\Omega^2 R^2}{2g} = \frac{(12.57\text{ rad/s})^2(0.1\text{ m})^2}{2(9.81\text{ m/s}^2)} = 0.0805\text{ m};$$

$$\Delta h_O \approx \mathbf{0.027\ m}\ (up) \quad \Delta h_C \approx \mathbf{-0.054\ m}\ (down) \quad \textit{Ans. (b)}$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE-2.1 A gage attached to a pressurized nitrogen tank reads a gage pressure of 28 inches of mercury. If atmospheric pressure is 14.4 psia, what is the absolute pressure in the tank?

(a) 95 kPa (b) 99 kPa (c) 101 kPa **(d) 194 kPa** (e) 203 kPa

FE-2.2 On a sea-level standard day, a pressure gage, moored below the surface of the ocean (SG = 1.025), reads an absolute pressure of 1.4 MPa. How deep is the instrument?

(a) 4 m **(b) 129 m** (c) 133 m (d) 140 m (e) 2080 m

FE-2.3 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point A is 1 atmosphere, what is the absolute pressure at point B?

(a) 5.6 kPa (b) 10.9 kPa **(c) 106.9 kPa** (d) 112.2 kPa (e) 157.0 kPa

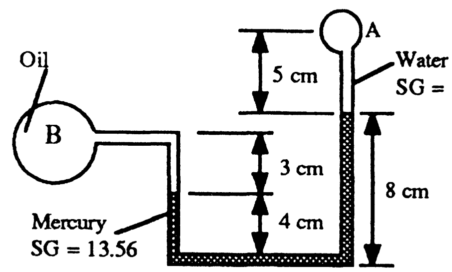


Fig. FE-2.3

FE-2.4 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point B is 14 psia, what is the absolute pressure at point B?

(a) 11 kPa (b) 41 kPa (c) 86 kPa **(d) 91 kPa** (e) 101 kPa

FE-2.5 A tank of water (SG = 1.0) has a gate in its vertical wall 5 m high and 3 m wide. The top edge of the gate is 2 m below the surface. What is the hydrostatic force on the gate?

(a) 147 kN (b) 367 kN (c) 490 kN **(d) 661 kN** (e) 1028 kN

FE-2.6 In Prob. FE-2.5 above, how far below the surface is the center of pressure of the hydrostatic force?

(a) 4.50 m (b) 5.46 m (c) 6.35 m (d) 5.33 m **(e) 4.96 m**

FE-2.7 A solid 1-m-diameter sphere floats at the interface between water (SG = 1.0) and mercury (SG = 13.56) such that 40% is in the water. What is the specific gravity of the sphere?

(a) 6.02 (b) 7.28 (c) 7.78 **(d) 8.54** (e) 12.56

FE-2.8 A 5-m-diameter balloon contains helium at 125 kPa absolute and 15°C, moored in sea-level standard air. If the gas constant of helium is 2077 $m^2/(s^2 \cdot K)$ and balloon material weight is neglected, what is the net lifting force of the balloon?

(a) 67 N (b) 134 N (c) 522 N **(d) 653 N** (e) 787 N

FE-2.9 A square wooden (SG = 0.6) rod, 5 cm by 5 cm by 10 m long, floats vertically in water at 20°C when 6 kg of steel (SG = 7.84) are attached to the lower end. How high above the water surface does the wooden end of the rod protrude?

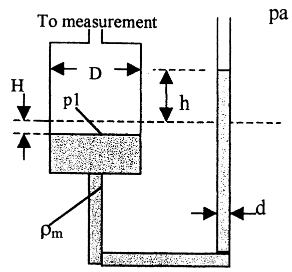
(a) 0.6 m (b) 1.6 m **(c) 1.9 m** (d) 2.4 m (e) 4.0 m

FE-2.10 A floating body will always be stable when its

(a) CG is above the center of buoyancy (b) center of buoyancy is below the waterline (c) center of buoyancy is above its metacenter (d) metacenter is above the center of buoyancy **(e) metacenter is above the CG**

COMPREHENSIVE PROBLEMS

C2.1 Some manometers are constructed as in the figure at right, with one large reservoir and one small tube open to the atmosphere. We can then neglect movement of the reservoir level. If the reservoir is not large, its level will move, as in the figure. Tube height h is measured from the zero-pressure level, as shown.



(a) Let the reservoir pressure be high, as in the Figure, so its level goes down. Write an exact Expression for p_{1gage} as a function of h, d, D, and gravity g. (b) Write an approximate expression for p_{1gage}, neglecting the movement of the reservoir. (c) Suppose h = 26 cm, p_a = 101 kPa, and ρ_m = 820 kg/m^3. Estimate the ratio (D/d) required to keep the error in (b) less than 1.0% and also < 0.1%. Neglect surface tension.

Solution: Let H be the downward movement of the reservoir. If we neglect air density, the pressure difference is $p_1 - p_a = \rho_m g(h + H)$. But volumes of liquid must balance:

$$\frac{\pi}{4}D^2H = \frac{\pi}{4}d^2h, \quad \text{or:} \quad H = (d/D)^2 h$$

Then the pressure difference (exact except for air density) becomes

$$\boldsymbol{p_1 - p_a = p_{1gage} = \rho_m gh(1 + d^2/D^2)} \quad \textit{Ans.} \text{ (a)}$$

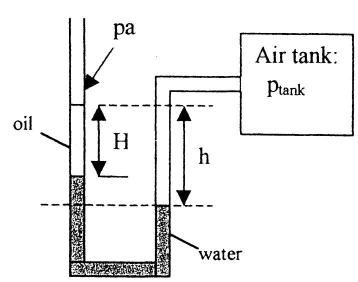
If we ignore the displacement H, then $\boldsymbol{p_{1gage} \approx \rho_m gh}$ *Ans.* (b)

(c) For the given numerical values, h = 26 cm and ρ_m = 820 kg/m^3 are irrelevant, all that matters is the ratio d/D. That is,

$$Error\ E = \frac{\Delta p_{exact} - \Delta p_{approx}}{\Delta p_{exact}} = \frac{(d/D)^2}{1+(d/D)^2}, \quad or: \quad D/d = \sqrt{(1-E)/E}$$

For E = 1% or 0.01, D/d = $[(1 - 0.01)/0.01]^{1/2} \geq$ **9.95** *Ans.* (c-1%)
For E = 0.1% or 0.001, D/d = $[(1 - 0.001)/0.001]^{1/2} \geq$ **31.6** *Ans.* (c-0.1%)

C2.2 A prankster has added oil, of specific gravity SG_o, to the left leg of the manometer at right. Nevertheless, the U-tube is still to be used to measure the pressure in the air tank. (a) Find an expression for h as a function of H and other parameters in the problem.
(b) Find the special case of your result when $p_{tank} = p_a$.
(c) Suppose H = 5 cm, p_a = 101.2 kPa, SG_o = 0.85, and p_{tank} is 1.82 kPa higher than p_a. Calculate h in cm, ignoring surface tension and air density effects.



Solution: Equate pressures at level *i* in the tube:

$$p_i = p_a + \rho g H + \rho_w g(h - H) = p_{tank},$$

$$\rho = SG_o\rho_w \quad \text{(ignore the column of air in the right leg)}$$

$$\text{Solve for:} \quad \boldsymbol{h = \frac{p_{tk} - p_a}{\rho_w g} + H(1 - SG_o)} \quad \textit{Ans.} \text{ (a)}$$

If $p_{tank} = p_a$, then

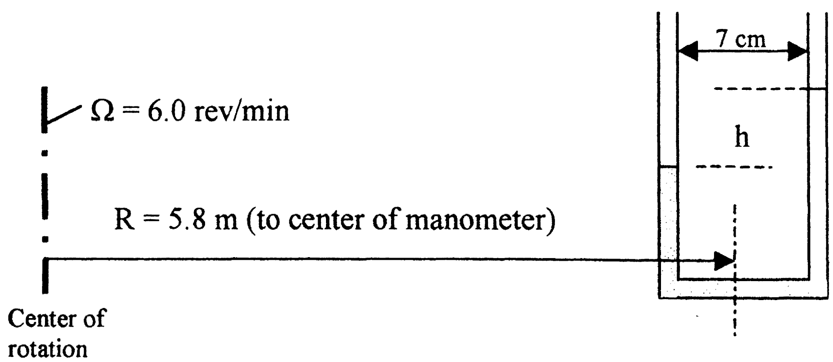
$$\boldsymbol{h = H(1 - SG_o)} \quad \textit{Ans.} \text{ (b)}$$

(c) For the particular numerical values given above, the answer to (a) becomes

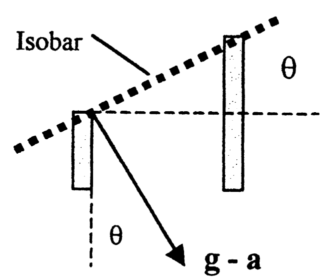
$$h = \frac{1820\ Pa}{998(9.81)} + 0.05(1 - 0.85) = 0.186 + 0.0075 = 0.193\ m = \mathbf{19.3\ cm} \quad \textit{Ans.} \text{ (c)}$$

Note that this result is not affected by the actual value of atmospheric pressure.

C2.3 Professor F. Dynamics, riding the merry-go-round with his son, has brought along his U-tube manometer. (You never know when a manometer might come in handy.) As shown in Fig. C2.2, the merry-go-round spins at constant angular velocity and the manometer legs are 7 cm apart. The manometer center is 5.8 m from the axis of rotation. Determine the height difference *h* in two ways: (a) approximately, by assuming rigid body translation with **a** equal to the average manometer acceleration; and (b) exactly, using rigid-body rotation theory. How good is the approximation?



Solution: (a) Approximate: The average acceleration of the manometer is $R_{avg}\Omega^2 = 5.8[6(2\pi/60)]^2 = 2.29$ rad/s toward the center of rotation, as shown. Then



$$\tan(\theta) = a/g = 2.29/9.81 = h/(7\text{ cm}) = 0.233$$

$$\text{Solve for} \quad h = \mathbf{1.63\ cm} \quad \textit{Ans.} \text{ (a)}$$

(b) Exact: The isobar in the figure at right would be on the parabola $z = C + r^2\Omega^2/(2g)$, where C is a constant. Apply this to the left leg (z_1) and right leg (z_2). As above, the rotation rate is $\Omega = 6.0*(2\pi/60) = 0.6283$ rad/s. Then

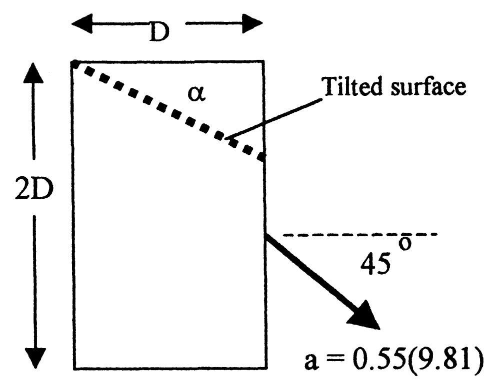
$$h = z_2 - z_1 = \frac{\Omega^2}{2g}(r_2^2 - r_1^2) = \frac{(0.6283)^2}{2(9.81)}[(5.8+0.035)^2 - (5.8-0.035)^2]$$
$$= \mathbf{0.0163\ m} \quad \textit{Ans.}\ \text{(b)}$$

This is nearly identical to the approximate answer (a), because $R \gg \Delta r$.

C2.4 A student sneaks a glass of cola onto a roller coaster ride. The glass is cylindrical, twice as tall as it is wide, and filled to the brim. He wants to know what percent of the cola he should drink before the ride begins, so that none of it spills during the big drop, in which the roller coaster achieves 0.55-g acceleration at a 45° angle below the horizontal. Make the calculation for him, neglecting sloshing and assuming that the glass is vertical at all times.

Solution: We have both horizontal and vertical acceleration. Thus the angle of tilt α is

$$\tan\alpha = \frac{a_x}{g + a_z} = \frac{0.55g\cos 45°}{g - 0.55g\sin 45°} = 0.6364$$



Thus $\alpha = 32.47°$. The tilted surface strikes the centerline at $R\tan\alpha = 0.6364R$ below the top. So the student should drink the cola until its rest position is 0.6364R below the top. The percentage drop in liquid level (and therefore liquid volume) is

$$\%\ removed = \frac{0.6364R}{4R} = 0.159 \quad or\!: \quad \mathbf{15.9\%} \quad \textit{Ans.}$$

C2.5 Dry adiabatic lapse rate is defined as DALR = –dT/dz when T and p vary isentropically. Assuming $T = Cp^a$, where $a = (\gamma - 1)/\gamma$, $\gamma = c_p/c_v$, (a) show that DALR = $g(\gamma - 1)/(\gamma R)$, R = gas constant; and (b) calculate DALR for air in units of °C/km.

Solution: Write T(p) in the form $T/T_o = (p/p_o)^a$ and differentiate:

$$\frac{dT}{dz} = T_o a\left(\frac{p}{p_o}\right)^{a-1}\frac{1}{p_o}\frac{dp}{dz}, \quad \text{But for the hydrostatic condition: } \frac{dp}{dz} = -\rho g$$

Substitute $\rho = p/RT$ for an ideal gas, combine above, and rewrite:

$$\frac{dT}{dz} = -\frac{T_o}{p_o} a\left(\frac{p}{p_o}\right)^{a-1} \frac{p}{RT} g = -\frac{ag}{R}\left(\frac{T_o}{T}\right)\left(\frac{p}{p_o}\right)^{a}. \quad \text{But: } \frac{T_o}{T}\left(\frac{p}{p_o}\right)^{a} = 1 \ (\textit{isentropic})$$

Therefore, finally,

$$-\frac{dT}{dz} = DALR = \frac{ag}{R} = \mathbf{\frac{(\gamma-1)g}{\gamma R}} \quad \textit{Ans. } \text{(a)}$$

(b) Regardless of the actual air temperature and pressure, the DALR for **air** equals

$$DALR = -\frac{dT}{dz}\Big|_s = \frac{(1.4-1)(9.81\ m/s^2)}{1.4(287\ m^2/s^2/°C)} = 0.00977\ \frac{°C}{m} = \mathbf{9.77\ \frac{°C}{km}} \quad \textit{Ans. } \text{(b)}$$

C2.6 Use the approximate pressure-density relation for a “soft” liquid,

$$dp = a^2 d\rho, \quad \text{or} \quad p = p_o + a^2(\rho - \rho_o)$$

to derive a formula for the density distribution $\rho(z)$ and pressure distribution $p(z)$ in a column of soft liquid. Then find the force F on a vertical wall of width b, extending from $z = 0$ down to $z = -h$, and compare with the incompressible result $F = \rho_o g h^2 b/2$.

Solution: Introduce this $p(\rho)$ relation into the hydrostatic relation (2.18) and integrate:

$$dp = a^2 d\rho = -\gamma\, dz = -\rho g\, dz, \quad \text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\int_0^z \frac{g\, dz}{a^2}, \quad \text{or:} \quad \boldsymbol{\rho = \rho_o e^{-gz/a^2}} \quad \textit{Ans.}$$

assuming constant a^2. Substitute into the $p(\rho)$ relation to obtain the pressure distribution:

$$p \approx p_o + a^2 \rho_o [e^{-gz/a^2} - 1] \tag{1}$$

Since p(z) increases with z at a greater than linear rate, the center of pressure will always be <u>*a little lower than predicted by linear theory*</u> (Eq. 2.44). Integrate Eq. (1) above, neglecting p_o, into the pressure force on a vertical plate extending from $z = 0$ to $z = -h$:

$$F = -\int_0^{-h} pb\, dz = \int_{-h}^{0} a^2 \rho_o (e^{-gz/a^2} - 1) b\, dz = \mathbf{ba^2 \rho_o \left[\frac{a^2}{g}(e^{gh/a^2} - 1) - h\right]} \quad \textit{Ans.}$$

In the limit of small depth change relative to the “softness” of the liquid, $h \ll a^2/g$, this reduces to the linear formula $F = \rho_o g h^2 b/2$ by expanding the exponential into the first three terms of its series. For “hard” liquids, the difference in the two formulas is negligible. For example, for water ($a \approx 1490$ m/s) with h = 10 m and b = 1 m, the linear formula predicts F = 48950<u>**0**</u> N while the exponential formula predicts F = 48950<u>**7**</u> N.
