Chapter 2 • Pressure Distribution in a Fluid

2.1 For the two-dimensional stress field in Fig. P2.1, let

$$\sigma_{xx} = 3000 \text{ psf} \quad \sigma_{yy} = 2000 \text{ psf}$$
$$\sigma_{xy} = 500 \text{ psf}$$

Find the shear and normal stresses on plane AA cutting through at 30°.

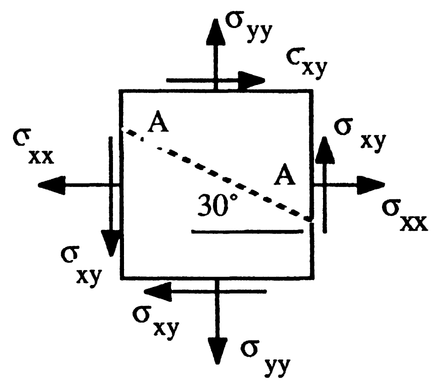
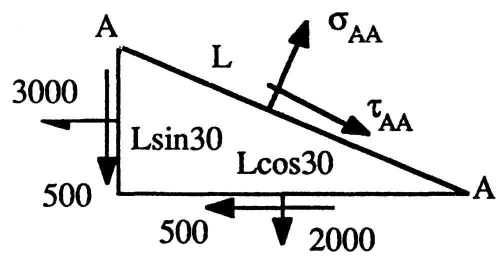


Fig. P2.1

Solution: Make cut "AA" so that it just hits the bottom right corner of the element. This gives the freebody shown at right. Now sum forces normal and tangential to side AA. Denote side length AA as "L."



$$\sum F_{n,AA} = 0 = \sigma_{AA}L - (3000\sin 30 + 500\cos 30)L\sin 30 - (2000\cos 30 + 500\sin 30)L\cos 30$$

Solve for $\sigma_{AA} \approx \mathbf{2683\ lbf/ft^2}$ *Ans.* (a)

$$\sum F_{t,AA} = 0 = \tau_{AA}L - (3000\cos 30 - 500\sin 30)L\sin 30 - (500\cos 30 - 2000\sin 30)L\cos 30$$

Solve for $\tau_{AA} \approx \mathbf{683\ lbf/ft^2}$ *Ans.* (b)

2.2 For the stress field of Fig. P2.1, change the known data to $\sigma_{xx} = 2000$ psf, $\sigma_{yy} = 3000$ psf, and $\sigma_n(AA) = 2500$ psf. Compute σ_{xy} and the shear stress on plane AA.

Solution: Sum forces normal to and tangential to AA in the element freebody above, with $\sigma_n(AA)$ known and σ_{xy} unknown:

$$\sum F_{n,AA} = 2500L - (\sigma_{xy}\cos 30° + 2000\sin 30°)L\sin 30° - (\sigma_{xy}\sin 30° + 3000\cos 30°)L\cos 30° = 0$$

Solve for $\sigma_{xy} = (2500 - 500 - 2250)/0.866 \approx \mathbf{-289\ lbf/ft^2}$ *Ans.* (a)

In like manner, solve for the shear stress on plane AA, using our result for σ_{xy}:

$$\sum F_{t,AA} = \tau_{AA}L - (2000\cos 30° + 289\sin 30°)L\sin 30° + (289\cos 30° + 3000\sin 30°)L\cos 30° = 0$$

Solve for $\tau_{AA} = 938 - 1515 \approx \mathbf{-577\ lbf/ft^2}$ *Ans.* (b)

This problem and Prob. 2.1 can also be solved using Mohr's circle.

2.3 A vertical clean glass piezometer tube has an inside diameter of 1 mm. When a pressure is applied, water at 20°C rises into the tube to a height of 25 cm. After correcting for surface tension, estimate the applied pressure in Pa.

Solution: For water, let Y = 0.073 N/m, contact angle θ = 0°, and γ = 9790 N/m^3. The capillary rise in the tube, from Example 1.9 of the text, is

$$h_{cap} = \frac{2Y\cos\theta}{\gamma R} = \frac{2(0.073\ N/m)\cos(0°)}{(9790\ N/m^3)(0.0005\ m)} = 0.030\ \text{m}$$

Then the rise due to applied pressure is less by that amount: h_{press} = 0.25 m − 0.03 m = 0.22 m. The applied pressure is estimated to be $p = \gamma h_{press}$ = (9790 N/m^3)(0.22 m) ≈ **2160 Pa** *Ans.*

2.4 Given a flow pattern with isobars $p_o - Bz + Cx^2$ = constant. Find an expression x = fcn(z) for the family of lines everywhere parallel to the local pressure gradient ∇p.

Solution: Find the slope (dx/dz) of the isobars and take the negative inverse and integrate:

$$\frac{d}{dz}(p_o - Bz + Cx^2) = -B + 2Cx\frac{dx}{dz} = 0, \quad \text{or:} \quad \frac{dx}{dz}\Big|_{p=const} = \frac{B}{2Cx} = \frac{-1}{(dx/dz)_{gradient}}$$

$$\text{Thus} \quad \frac{dx}{dz}\Big|_{gradient} = -\frac{2Cx}{B}, \quad \text{integrate} \int\frac{dx}{x} = \int\frac{-2C\,dz}{B}, \quad \mathbf{x = const\ e^{-2Cz/B}} \quad \textit{Ans.}$$

2.5 Atlanta, Georgia, has an average altitude of 1100 ft. On a U.S. standard day, pressure gage A reads 93 kPa and gage B reads 105 kPa. Express these readings in gage or vacuum pressure, whichever is appropriate.

Solution: We can find atmospheric pressure by either interpolating in Appendix Table A.6 or, more accurately, evaluate Eq. (2.27) at 1100 ft ≈ 335 m:

$$p_a = p_o\left(1-\frac{Bz}{T_o}\right)^{g/RB} = (101.35\text{ kPa})\left[1-\frac{(0.0065\text{ K/m})(335\text{ m})}{288.16\text{ K}}\right]^{5.26} \approx 97.4\text{ kPa}$$

Therefore:

Gage A = 93 kPa − 97.4 kPa = –4.4 kPa (gage) = **+4.4 kPa (vacuum)**

Gage B = 105 kPa − 97.4 kPa = **+7.6 kPa (gage)** *Ans.*

2.6 Express standard atmospheric pressure as a head, h = p/ρg, in (a) feet of ethylene glycol; (b) inches of mercury; (c) meters of water; and (d) mm of methanol.

Solution: Take the specific weights, $\gamma = \rho g$, from Table A.3, divide p_{atm} by γ:

(a) Ethylene glycol: $h = (2116\text{ lbf/ft}^2)/(69.7\text{ lbf/ft}^3) \approx$ **30.3 ft** *Ans.* (a)

(b) Mercury: $h = (2116\text{ lbf/ft}^2)/(846\text{ lbf/ft}^3) = 2.50\text{ ft} \approx$ **30.0 inches** *Ans.* (b)

(c) Water: $h = (101350\text{ N/m}^2)/(9790\text{ N/m}^3) \approx$ **10.35 m** *Ans.* (c)

(d) Methanol: $h = (101350\text{ N/m}^2)/(7760\text{ N/m}^3) = 13.1\text{ m} \approx$ **13100 mm** *Ans.* (d)

2.7 The deepest point in the ocean is 11034 m in the Mariana Tranch in the Pacific. At this depth $\gamma_{seawater} \approx 10520\text{ N/m}^3$. Estimate the absolute pressure at this depth.

Solution: Seawater specific weight at the surface (Table 2.1) is 10050 N/m^3. It seems quite reasonable to average the surface and bottom weights to predict the bottom pressure:

$$p_{bottom} \approx p_o + \gamma_{abg}h = 101350 + \left(\frac{10050+10520}{2}\right)(11034) = 1.136\text{E8 Pa} \approx \mathbf{1121\ atm} \quad \textit{Ans.}$$

2.8 A diamond mine is 2 miles below sea level. (a) Estimate the air pressure at this depth. (b) If a barometer, accurate to 1 mm of mercury, is carried into this mine, how accurately can it estimate the depth of the mine?

Solution: (a) Convert 2 miles = 3219 m and use a linear-pressure-variation estimate:

Then $p \approx p_a + \gamma h = 101{,}350\ \text{Pa} + (12\ \text{N/m}^3)(3219\ \text{m}) = 140{,}000\ \text{Pa} \approx \mathbf{140\ kPa}$ *Ans.* (a)

Alternately, the troposphere formula, Eq. (2.27), predicts a slightly higher pressure:

$$p \approx p_a(1 - Bz/T_o)^{5.26} = (101.3\ \text{kPa})[1-(0.0065\ \text{K/m})(-3219\ \text{m})/288.16\ \text{K}]^{5.26}$$
$$= \mathbf{147\ kPa} \quad \textit{Ans. (a)}$$

(b) The gage pressure at this depth is approximately 40,000/133,100 ≈ 0.3 m Hg or 300 mm Hg ±1 mm Hg or ±0.3% error. Thus the error in the actual depth is 0.3% of 3220 m or about **±10 m** if all other parameters are accurate. *Ans.* (b)

2.9 Integrate the hydrostatic relation by assuming that the isentropic bulk modulus, $B = \rho(\partial p/\partial\rho)_s$, is constant. Apply your result to the Mariana Trench, Prob. 2.7.

Solution: Begin with Eq. (2.18) written in terms of B:

$$dp = -\rho g\, dz = \frac{B}{\rho}d\rho, \quad \text{or:} \quad \int_{\rho_o}^{\rho}\frac{d\rho}{\rho^2} = -\frac{g}{B}\int_0^z dz = -\frac{1}{\rho}+\frac{1}{\rho_o} = -\frac{gz}{B}, \text{ also integrate:}$$

$$\int_{p_o}^{p} dp = B\int_{\rho_o}^{\rho}\frac{d\rho}{\rho} \quad \text{to obtain } p - p_o = B\ln(\rho/\rho_o)$$

Eliminate ρ between these two formulas to obtain the desired pressure-depth relation:

$$\mathbf{p = p_o - B\ln\left(1+\frac{g\rho_o z}{B}\right)} \quad \textit{Ans. (a)} \quad \text{With } B_{seawater} \approx 2.33E9 \text{ Pa from Table A.3,}$$

$$p_{Trench} = 101350 - (2.33E9)\ln\left[1+\frac{(9.81)(1025)(-11034)}{2.33E9}\right]$$
$$= 1.138E8\ \text{Pa} \approx \mathbf{1123\ atm} \quad \textit{Ans. (b)}$$

2.10 A closed tank contains 1.5 m of SAE 30 oil, 1 m of water, 20 cm of mercury, and an air space on top, all at 20°C. If p_{bottom} = 60 kPa, what is the pressure in the air space?

Solution: Apply the hydrostatic formula down through the three layers of fluid:

$$p_{bottom} = p_{air} + \gamma_{oil}h_{oil} + \gamma_{water}h_{water} + \gamma_{mercury}h_{mercury}$$

$$\text{or:} \quad 60000\ \text{Pa} = p_{air} + (8720\ \text{N/m}^3)(1.5\ \text{m}) + (9790)(1.0\ \text{m}) + (133100)(0.2\ \text{m})$$

Solve for the pressure in the air space: $p_{air} \approx \mathbf{10500\ Pa}$ *Ans.*

2.11 In Fig. P2.11, sensor A reads 1.5 kPa (gage). All fluids are at 20°C. Determine the elevations Z in meters of the liquid levels in the open piezometer tubes B and C.

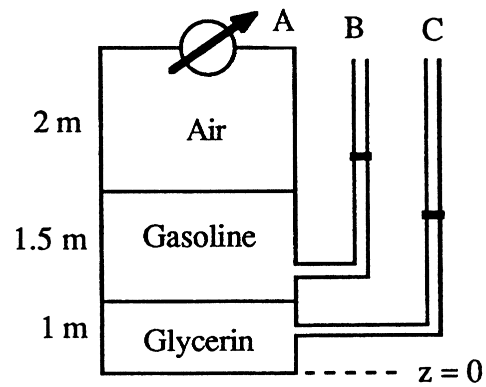


Fig. P2.11

Solution: (B) Let piezometer tube B be an arbitrary distance H above the gasoline-glycerin interface. The specific weights are $\gamma_{air} \approx 12.0\ N/m^3$, $\gamma_{gasoline} = 6670\ N/m^3$, and $\gamma_{glycerin} = 12360\ N/m^3$. Then apply the hydrostatic formula from point A to point B:

$$1500\ N/m^2 + (12.0\ N/m^3)(2.0\ m) + 6670(1.5 - H) - 6670(Z_B - H - 1.0) = p_B = 0\ (gage)$$

Solve for $Z_B = \mathbf{2.73\ m}$ (23 cm above the gasoline-air interface) *Ans.* (b)

Solution (C): Let piezometer tube C be an arbitrary distance Y above the bottom. Then

$$1500 + 12.0(2.0) + 6670(1.5) + 12360(1.0 - Y) - 12360(Z_C - Y) = p_C = 0\ (gage)$$

Solve for $Z_C = \mathbf{1.93\ m}$ (93 cm above the gasoline-glycerin interface) *Ans.* (c)

2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m^3?

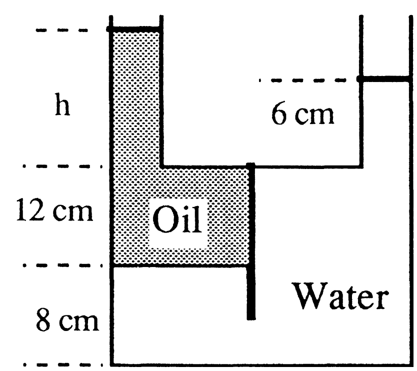


Fig. P2.12

Solution: For water take the density = 998 kg/m^3. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08\ m \approx \mathbf{8.0\ cm}$ *Ans.*

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

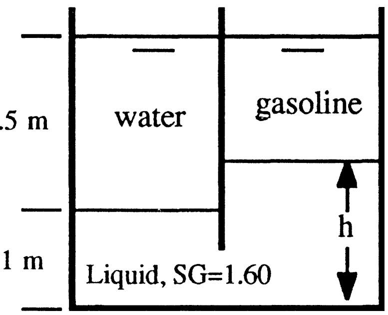


Fig. P2.13

Solution: Take water = 9790 N/m^3 and gasoline = 6670 N/m^3. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$p_{bottom} = (9790\ \mathrm{N/m^3})(1.5\ \mathrm{m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

$$\text{Solve for}\quad h = \mathbf{1.52\ m}\quad \textit{Ans.}$$

2.14 The closed tank in Fig. P2.14 is at 20°C. If the pressure at A is 95 kPa absolute, determine p at B (absolute). What percent error do you make by neglecting the specific weight of the air?

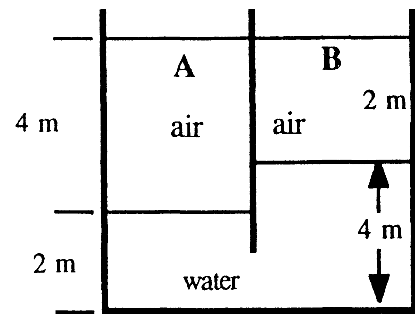


Fig. P2.14

Solution: First compute $\rho_A = p_A/RT = (95000)/[287(293)] \approx 1.13\ \mathrm{kg/m^3}$, hence $\gamma_A \approx (1.13)(9.81) \approx 11.1\ \mathrm{N/m^3}$. Then proceed around hydrostatically from point A to point B:

$$95000\ \mathrm{Pa} + (11.1\ \mathrm{N/m^3})(4.0\ \mathrm{m}) + 9790(2.0) - 9790(4.0) - \left(\frac{p_B}{RT}\right)(9.81)(2.0) = p_B$$

$$\text{Solve for}\quad p_B \approx \mathbf{75450\ Pa}\quad \textit{Accurate answer.}$$

If we neglect the air effects, we get a much simpler relation with comparable accuracy:

$$95000 + 9790(2.0) - 9790(4.0) \approx p_B \approx \mathbf{75420\ Pa}\quad \textit{Approximate answer.}$$

2.15 In Fig. P2.15 all fluids are at 20°C. Gage A reads 15 lbf/in^2 absolute and gage B reads 1.25 lbf/in^2 less than gage C. Compute (a) the specific weight of the oil; and (b) the actual reading of gage C in lbf/in^2 absolute.

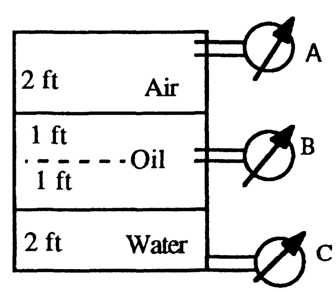


Fig. P2.15

Solution: First evaluate $\gamma_{air} = (p_A/RT)g = [15 \times 144/(1717 \times 528)](32.2) \approx 0.0767\ \text{lbf/ft}^3$. Take $\gamma_{water} = 62.4\ \text{lbf/ft}^3$. Then apply the hydrostatic formula from point B to point C:

$$p_B + \gamma_{oil}(1.0\ \text{ft}) + (62.4)(2.0\ \text{ft}) = p_C = p_B + (1.25)(144)\ \text{psf}$$

$$\text{Solve for}\quad \gamma_{oil} \approx \mathbf{55.2\ lbf/ft^3}\quad \textit{Ans.}\ \text{(a)}$$

With the oil weight known, we can now apply hydrostatics from point A to point C:

$$p_C = p_A + \sum \rho g h = (15)(144) + (0.0767)(2.0) + (55.2)(2.0) + (62.4)(2.0)$$

$$\text{or:}\quad p_C = 2395\ \text{lbf/ft}^2 = \mathbf{16.6\ psi}\quad \textit{Ans.}\ \text{(b)}$$

2.16 Suppose one wishes to construct a barometer using ethanol at 20°C (Table A-3) as the working fluid. Account for the equilibrium vapor pressure in your calculations and determine how high such a barometer should be. Compare this with the traditional mercury barometer.

Solution: From Table A.3 for ethanol at 20°C, $\rho = 789\ \text{kg/m}^3$ and $p_{vap} = 5700$ Pa. For a column of ethanol at 1 atm, the hydrostatic equation would be

$$p_{atm} - p_{vap} = \rho_{eth} g h_{eth},\quad \text{or:}\quad 101350\ \text{Pa} - 5700\ \text{Pa} = (789\ \text{kg/m}^3)(9.81\ \text{m/s}^2)h_{eth}$$

$$\text{Solve for}\quad h_{eth} \approx \mathbf{12.4\ m}\quad \textit{Ans.}$$

A mercury barometer would have $h_{merc} \approx 0.76$ m and would not have the high vapor pressure.

2.17 All fluids in Fig. P2.17 are at 20°C. If p = 1900 psf at point A, determine the pressures at B, C, and D in psf.

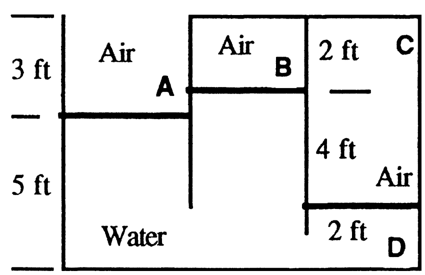


Fig. P2.17

Solution: Using a specific weight of 62.4 lbf/ft^3 for water, we first compute p_B and p_D:

$$p_B = p_A - \gamma_{water}(z_B - z_A) = 1900 - 62.4(1.0\ \text{ft}) = \mathbf{1838\ lbf/ft^2}\quad \textit{Ans.}\ \text{(pt. B)}$$

$$p_D = p_A + \gamma_{water}(z_A - z_D) = 1900 + 62.4(5.0\ \text{ft}) = \mathbf{2212\ lbf/ft^2}\quad \textit{Ans.}\ \text{(pt. D)}$$

Finally, moving up from D to C, we can neglect the air specific weight to good accuracy:

$$p_C = p_D - \gamma_{water}(z_C - z_D) = 2212 - 62.4(2.0\ \text{ft}) = \mathbf{2087\ lbf/ft^2}\quad \textit{Ans.}\ \text{(pt. C)}$$

The air near C has $\gamma \approx 0.074\ \text{lbf/ft}^3$ times 6 ft yields less than 0.5 psf correction at C.

2.18 All fluids in Fig. P2.18 are at 20°C. If atmospheric pressure = 101.33 kPa and the bottom pressure is 242 kPa absolute, what is the specific gravity of fluid X?

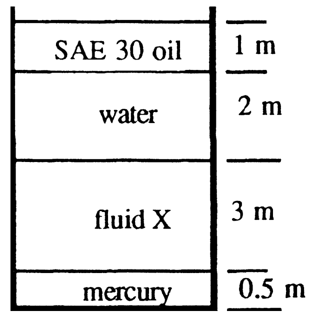


Fig. P2.18

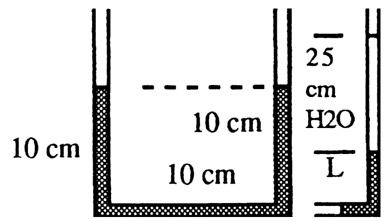
Solution: Simply apply the hydrostatic formula from top to bottom:

$$p_{bottom} = p_{top} + \Sigma \gamma h,$$

$$\text{or:}\quad 242000 = 101330 + (8720)(1.0) + (9790)(2.0) + \gamma_X(3.0) + (133100)(0.5)$$

$$\text{Solve for}\quad \gamma_X = 15273\ \text{N/m}^3,\quad \text{or:}\quad SG_X = \frac{15273}{9790} = \mathbf{1.56}\quad \textit{Ans.}$$

2.19 The U-tube at right has a 1-cm ID and contains mercury as shown. If 20 cm^3 of water is poured into the right-hand leg, what will be the free surface height in each leg after the sloshing has died down?



Solution: First figure the height of water added:

$$20\ \text{cm}^3 = \frac{\pi}{4}(1\ \text{cm})^2 h,\quad \text{or}\quad h = 25.46\ \text{cm}$$

Then, at equilibrium, the new system must have 25.46 cm of water on the right, and a 30-cm length of mercury is somewhat displaced so that "L" is on the right, 0.1 m on the bottom, and "0.2 – L" on the left side, as shown at right. The bottom pressure is constant:

$$p_{atm} + 133100(0.2 - L) = p_{atm} + 9790(0.2546) + 133100(L),\quad \text{or:}\quad L \approx 0.0906\ \text{m}$$

Thus right-leg-height = 9.06 + 25.46 = **34.52 cm** *Ans.*
left-leg-height = 20.0 – 9.06 = **10.94 cm** *Ans.*

2.20 The hydraulic jack in Fig. P2.20 is filled with oil at 56 lbf/ft^3. Neglecting piston weights, what force F on the handle is required to support the 2000-lbf weight shown?

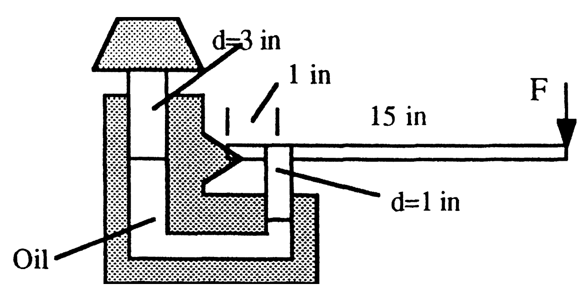


Fig. P2.20

Solution: First sum moments clockwise about the hinge A of the handle:

$$\sum M_A = 0 = F(15+1) - P(1),$$

or: $F = P/16$, where P is the force in the small (1 in) piston.

Meanwhile figure the pressure in the oil from the weight on the large piston:

$$p_{oil} = \frac{W}{A_{3\text{-in}}} = \frac{2000\text{ lbf}}{(\pi/4)(3/12\text{ ft})^2} = 40744\text{ psf},$$

$$\text{Hence}\quad P = p_{oil}A_{small} = (40744)\frac{\pi}{4}\left(\frac{1}{12}\right)^2 = 222\text{ lbf}$$

Therefore the handle force required is $F = P/16 = 222/16 \approx$ **14 lbf** *Ans.*

2.21 In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

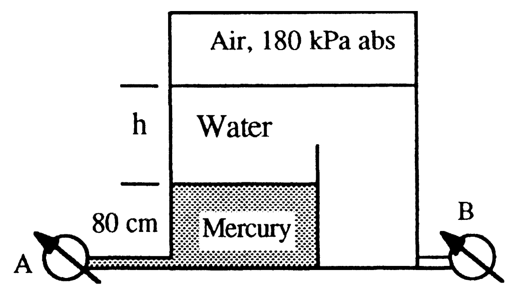


Fig. P2.21

Solution: Apply the hydrostatic formula from the air to gage A:

$$p_A = p_{air} + \sum \gamma h$$

$$= 180000 + (9790)h + 133100(0.8) = 350000\text{ Pa},$$

Solve for $h \approx$ **6.49 m** *Ans.* (a)

Then, with *h* known, we can evaluate the pressure at gage B:

$$p_B = 180000 + 9790(6.49 + 0.80) = 251000\text{ Pa} \approx \textbf{251 kPa}$$ *Ans.* (b)

2.22 The fuel gage for an auto gas tank reads proportional to the bottom gage pressure as in Fig. P2.22. If the tank accidentally contains 2 cm of water plus gasoline, how many centimeters "h" of air remain when the gage reads "full" in error?

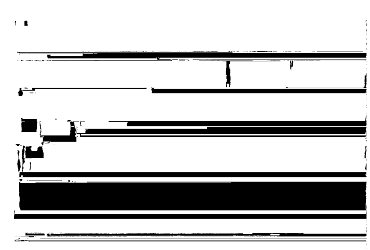


Fig. P2.22

Solution: Given $\gamma_{gasoline} = 0.68(9790) = 6657\ \text{N/m}^3$, compute the pressure when "full":

$$p_{full} = \gamma_{gasoline}(\text{full height}) = (6657\ \text{N/m}^3)(0.30\ \text{m}) = 1997\ \text{Pa}$$

Set this pressure equal to 2 cm of water plus "Y" centimeters of gasoline:

$$p_{full} = 1997 = 9790(0.02\ \text{m}) + 6657\text{Y}, \quad \text{or} \quad \text{Y} \approx 0.2706\ \text{m} = 27.06\ \text{cm}$$

Therefore the air gap h = 30 cm − 2 cm(water) − 27.06 cm(gasoline) ≈ **0.94 cm** *Ans.*

2.23 In Fig. P2.23 both fluids are at 20°C. If surface tension effects are negligible, what is the density of the oil, in kg/m^3?

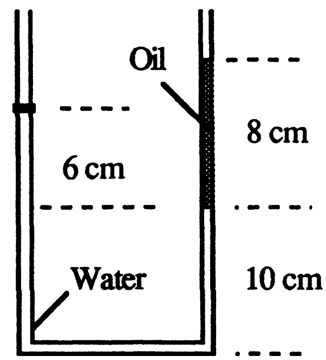


Fig. P2.23

Solution: Move around the U-tube from left atmosphere to right atmosphere:

$$p_a + (9790\ \text{N/m}^3)(0.06\ \text{m}) - \gamma_{oil}(0.08\ \text{m}) = p_a,$$

$$\text{solve for} \quad \gamma_{oil} \approx 7343\ \text{N/m}^3,$$

$$\text{or:} \quad \rho_{oil} = 7343/9.81 \approx \mathbf{748\ kg/m^3} \quad \textit{Ans.}$$

2.24 In Prob. 1.2 we made a crude integration of atmospheric density from Table A.6 and found that the atmospheric mass is approximately $m \approx 6.08\text{E}18$ kg. Can this result be used to estimate sea-level pressure? Can sea-level pressure be used to estimate m?

Solution: Yes, atmospheric pressure is essentially a result of the weight of the air above. Therefore the air weight divided by the surface area of the earth equals sea-level pressure:

$$p_{sea\text{-}level} = \frac{W_{air}}{A_{earth}} = \frac{m_{air}g}{4\pi R_{earth}^2} \approx \frac{(6.08\text{E}18\ \text{kg})(9.81\ \text{m/s}^2)}{4\pi(6.377\text{E}6\ \text{m})^2} \approx \mathbf{117000\ Pa} \quad \textit{Ans.}$$

This is a little off, thus our mass estimate must have been a little off. If global average sea-level pressure is actually 101350 Pa, then the mass of atmospheric air must be more nearly

$$m_{air} = \frac{A_{earth}p_{sea\text{-}level}}{g} \approx \frac{4\pi(6.377\text{E}6\ \text{m})^2(101350\ \text{Pa})}{9.81\ \text{m/s}^2} \approx \mathbf{5.28E18\ kg} \quad \textit{Ans.}$$

2.25 Venus has a mass of 4.90E24 kg and a radius of 6050 km. Assume that its atmosphere is 100% CO_2 (actually it is about 96%). Its surface temperature is 730 K, decreasing to 250 K at about z = 70 km. Average surface pressure is 9.1 MPa. Estimate the pressure on Venus at an altitude of 5 km.

Solution: The value of "g" on Venus is estimated from Newton's law of gravitation:

$$g_{Venus} = \frac{Gm_{Venus}}{R^2_{Venus}} = \frac{(6.67E\text{–}11)(4.90E24\text{ kg})}{(6.05E6\text{ m})^2} \approx 8.93\text{ m/s}^2$$

Now, from Table A.4, the gas constant for carbon dioxide is $R_{CO_2} \approx 189\text{ m}^2/(\text{s}^2\cdot\text{K})$. And we may estimate the Venus temperature lapse rate from the given information:

$$B_{Venus} \approx \frac{\Delta T}{\Delta z} \approx \frac{730 - 250\text{ K}}{70000\text{ m}} \approx 0.00686\text{ K/m}$$

Finally the exponent in the p(z) relation, Eq. (2.27), is "n" = g/RB = (8.93)/(189 × 0.00686) ≈ 6.89. Equation (2.27) may then be used to estimate p(z) at z = 10 km on Venus:

$$p_{5\text{ km}} \approx p_o(1 - Bz/T_o)^n \approx (9.1\text{ MPa})\left[1 - \frac{0.00686\text{ K/m}(5000\text{ m})}{730\text{ K}}\right]^{6.89} \approx \mathbf{6.5\ MPa} \quad \textit{Ans.}$$

2.26* A *polytropic atmosphere* is defined by the Power-law $p/p_o = (\rho/\rho_o)^m$, where m is an exponent of order 1.3 and p_o and ρ_o are sea-level values of pressure and density. (a) Integrate this expression in the static atmosphere and find a distribution $p(z)$. (b) Assuming an ideal gas, $p = \rho RT$, show that your result in (a) implies a linear temperature distribution as in Eq. (2.25). (c) Show that the standard B = 0.0065 K/m is equivalent to m = 1.235.

Solution: (a) In the hydrostatic Eq. (2.18) substitute for density in terms of pressure:

$$dp = -\rho g\,dz = -[\rho_o(p/p_o)^{1/m}]g\,dz, \quad \text{or:} \quad \int_{p_o}^{p} \frac{dp}{p^{1/m}} = -\frac{\rho_o g}{p_o^{1/m}}\int_0^z dz$$

Integrate and rearrange to get the result $\displaystyle \frac{\boldsymbol{p}}{\boldsymbol{p_o}} = \left[\mathbf{1} - \frac{\boldsymbol{(m-1)gz}}{\boldsymbol{m(p_o/\rho_o)}}\right]^{\boldsymbol{m/(m-1)}}$ *Ans.* (a)

(b) Use the ideal-gas relation to relate pressure ratio to temperature ratio for this process:

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^m = \left(\frac{p}{RT}\frac{RT_o}{p_o}\right)^m \quad \text{Solve for} \quad \frac{T}{T_o} = \left(\frac{p}{p_o}\right)^{(m-1)/m}$$

Using p/p_o from *Ans.* (a), we obtain $\dfrac{T}{T_o} = \left[1 - \dfrac{(m-1)gz}{mRT_o}\right]$ *Ans.* (b)

Note that, in using *Ans.* (a) to obtain *Ans.* (b), we have substituted $p_o/\rho_o = RT_o$.
(c) Comparing *Ans.* (b) with the text, Eq. (2.27), we find that lapse rate "B" in the text is equal to $(m-1)g/(mR)$. Solve for m if $B = 0.0065$ K/m:

$$m = \frac{g}{g - BR} = \frac{9.81\ m/s^2}{9.81\ m/s^2 - (0.0065\ K/m)(287\ m^2/s^2 - R)} = \mathbf{1.235} \quad \textit{Ans. (c)}$$

2.27 This is an *experimental* problem: Put a card or thick sheet over a glass of water, hold it tight, and turn it over without leaking (a glossy postcard works best). Let go of the card. Will the card stay attached when the glass is upside down? **Yes**: This is essentially a *water barometer* and, in principle, could hold a column of water up to 10 ft high!

2.28 What is the uncertainty in using pressure measurement as an altimeter? A gage on an airplane measures a local pressure of 54 kPa with an uncertainty of 3 kPa. The lapse rate is 0.006 K/m with an uncertainty of 0.001 K/m. Effective sea-level temperature is 10°C with an uncertainty of 5°C. Effective sea-level pressure is 100 kPa with an uncertainty of 2 kPa. Estimate the plane's altitude and its uncertainty.

Solution: Based on average values in Eq. (2.27), ($p = 54$ kPa, $p_o = 100$ kPa, $B = 0.006$ K/m, $T_o = 10$°C), $z_{avg} \approx$ **4835** m. Considering each variable separately (p, p_o, B, T_o), their predicted variations in altitude, from Eq. (2.27), are 8.5%, 3.1%, 0.9%, and 1.8%, respectively. Thus measured local pressure is the largest cause of altitude uncertainty. According to uncertainty theory, Eq. (1.43), the overall uncertainty is $\delta z = [(8.5)^2 + (3.1)^2 + (0.9)^2 + (1.8)^2]^{1/2} = 9.3\%$, or about 450 meters. Thus we can state the altitude as $z \approx \mathbf{4840 \pm 450\ m}$. *Ans.*

2.29 Show that, for an *adiabatic* atmosphere, $p = C(\rho)^k$, where C is constant, that

$$p/p_o = \left[1 - \frac{(k-1)gz}{kRT_o}\right]^{k/(k-1)}, \quad \text{where } k = c_p/c_v$$

Compare this formula for air at 5 km altitude with the U.S. standard atmosphere.

Solution: Introduce the adiabatic assumption into the basic hydrostatic relation (2.18):

$$\frac{dp}{dz} = -\rho g = \frac{d(C\rho^k)}{dz} = kC\rho^{k-1}\frac{d\rho}{dz}$$

Separate the variables and integrate:

$$\int C\rho^{k-2}\,d\rho = -\int \frac{g}{k}dz, \quad \text{or:} \quad \frac{C\rho^{k-1}}{k-1} = -\frac{gz}{k} + \text{constant}$$

The constant of integration is related to z = 0, that is, "constant" $= C\rho_o^{k-1}/(k-1)$. Divide this constant out and rewrite the relation above:

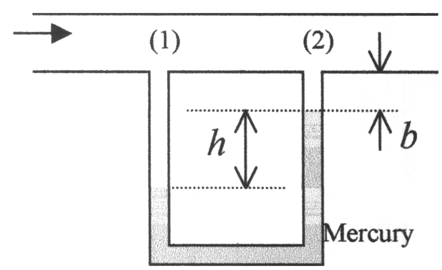
$$\left(\frac{\rho}{\rho_o}\right)^{k-1} = 1 - \frac{(k-1)gz}{kC\rho_o^{k-1}} = (p/p_o)^{(k-1)/k} \quad \text{since } p = C\rho^k$$

Finally, note that $C\rho_o^{k-1} = C\rho_o^k/\rho_o = p_o/\rho_o = RT_o$, where T_o is the surface temperature. Thus the final desired pressure relation for an adiabatic atmosphere is

$$\frac{p}{p_o} = \left[1 - \frac{(k-1)gz}{kRT_o}\right]^{k/(k-1)} \quad \textit{Ans.}$$

At z = 5,000 m, Table A.6 gives p = 54008 Pa, while the adiabatic formula, with k = 1.40, gives p = **52896 Pa**, or 2.1% lower.

2.30 A mercury manometer is connected at two points to a horizontal 20°C water-pipe flow. If the manometer reading is h = 35 cm, what is the pressure drop between the two points?



Solution: This is a classic manometer relation. The two legs of water of height b cancel out:

$$p_1 + 9790b + 9790h - 133100h - 9790b = p_2$$

$$p_1 - p_2 = (133{,}100 - 9790\ \text{N/m}^3)(0.35\ \text{m}) \approx \mathbf{43100\ Pa} \quad \textit{Ans.}$$

2.31 In Fig. P2.31 determine Δp between points A and B. All fluids are at 20°C.

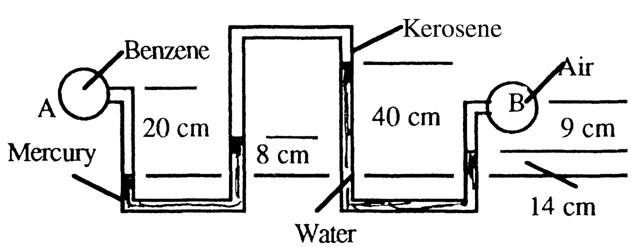


Fig. P2.31

Solution: Take the specific weights to be

Benzene:	8640 N/m^3	Mercury:	133100 N/m^3
Kerosene:	7885 N/m^3	Water:	9790 N/m^3

and γ_{air} will be small, probably around 12 N/m^3. Work your way around from A to B:

$$p_A + (8640)(0.20\ \text{m}) - (133100)(0.08) - (7885)(0.32) + (9790)(0.26) - (12)(0.09)$$
$$= p_B, \quad \text{or, after cleaning up,} \quad p_A - p_B \approx \mathbf{8900\ Pa} \quad \textit{Ans.}$$

2.32 For the manometer of Fig. P2.32, all fluids are at 20°C. If $p_B - p_A$ = 97 kPa, determine the height H in centimeters.

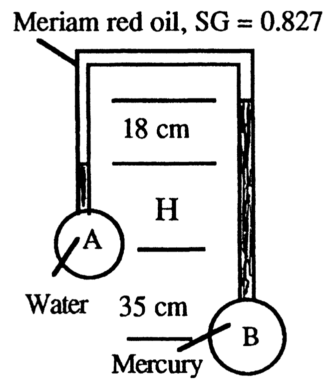


Fig. P2.32

Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\ \text{N/m}^3)(\text{H meters}) - 8096(0.18)$$
$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$
$$\text{Solve for} \quad H \approx 0.226\ \text{m} = \mathbf{22.6\ cm} \quad \textit{Ans.}$$

2.33 In Fig. P2.33 the pressure at point A is 25 psi. All fluids are at 20°C. What is the air pressure in the closed chamber B?

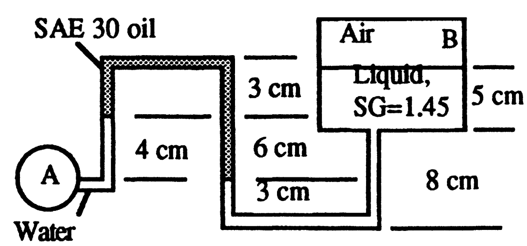


Fig. P2.33

Solution: Take γ = 9790 N/m^3 for water, 8720 N/m^3 for SAE 30 oil, and (1.45)(9790) = 14196 N/m^3 for the third fluid. Convert the pressure at A from 25 lbf/in^2 to 172400 Pa. Compute hydrostatically from point A to point B:

$$p_A + \sum \gamma h = 172400 - (9790\ \text{N/m}^3)(0.04\ \text{m}) + (8720)(0.06) - (14196)(0.10)$$
$$= p_B = 171100\ \text{Pa} \div 47.88 \div 144 = \mathbf{24.8\ psi} \quad \textit{Ans.}$$

2.34 To show the effect of manometer dimensions, consider Fig. P2.34. The containers (a) and (b) are cylindrical and are such that $p_a = p_b$ as shown. Suppose the oil-water interface on the right moves up a distance $\Delta h < h$. Derive a formula for the difference $p_a - p_b$ when (a) $d \ll D$; and (b) $d = 0.15D$. What is the % difference?

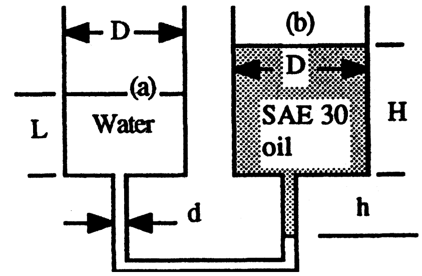


Fig. P2.34

Solution: Take $\gamma = 9790$ N/m^3 for water and 8720 N/m^3 for SAE 30 oil. Let "H" be the height of the oil in reservoir (b). For the condition shown, $p_a = p_b$, therefore

$$\gamma_{water}(L+h) = \gamma_{oil}(H+h), \quad \text{or:} \quad H = (\gamma_{water}/\gamma_{oil})(L+h) - h \tag{1}$$

Case (a), $d \ll D$: When the meniscus rises Δh, there will be no significant change in reservoir levels. Therefore we can write a simple hydrostatic relation from (a) to (b):

$$p_a + \gamma_{water}(L + h - \Delta h) - \gamma_{oil}(H + h - \Delta h) = p_b,$$

$$\text{or:} \quad \mathbf{p_a - p_b = \Delta h(\gamma_{water} - \gamma_{oil})} \quad \textit{Ans.} \text{ (a)}$$

where we have used Eq. (1) above to eliminate H and L. Putting in numbers to compare later with part (b), we have $\Delta p = \Delta h(9790 - 8720) = \mathit{1070}\ \Delta h$, with Δh in meters.

Case (b), $d = 0.15D$. Here we must account for reservoir volume changes. For a rise $\Delta h < h$, a volume $(\pi/4)d^2\Delta h$ of *water* leaves reservoir (a), decreasing "L" by $\Delta h(d/D)^2$, and an identical volume of oil enters reservoir (b), increasing "H" by the same amount $\Delta h(d/D)^2$. The hydrostatic relation between (a) and (b) becomes, for this case,

$$p_a + \gamma_{water}[L - \Delta h(d/D)^2 + h - \Delta h] - \gamma_{oil}[H + \Delta h(d/D)^2 + h - \Delta h] = p_b,$$

$$\text{or:} \quad p_a - p_b = \mathbf{\Delta h[\gamma_{water}(1 + d^2/D^2) - \gamma_{oil}(1 - d^2/D^2)]} \quad \textit{Ans.} \text{ (b)}$$

where again we have used Eq. (1) to eliminate H and L. If d is not small, this is a *considerable* difference, with surprisingly large error. For the case $d = 0.15$ D, with water and oil, we obtain $\Delta p = \Delta h[1.0225(9790) - 0.9775(8720)] \approx 1486\ \Delta h$ or **<u>39% more</u>** than (a).

2.35 Water flows upward in a pipe slanted at 30°, as in Fig. P2.35. The mercury manometer reads h = 12 cm. What is the pressure difference between points (1) and (2) in the pipe?

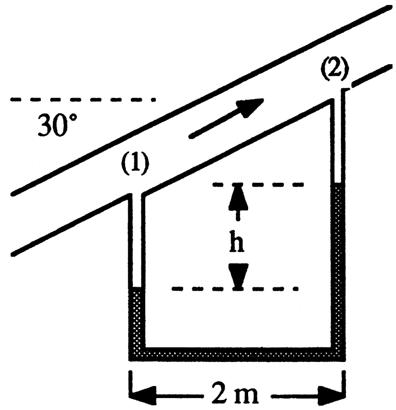


Fig. P2.35

Solution: The vertical distance between points 1 and 2 equals (2.0 m)tan 30° or 1.155 m. Go around the U-tube hydrostatically from point 1 to point 2:

$$p_1 + 9790h - 133100h - 9790(1.155\text{ m}) = p_2,$$

$$\text{or:}\quad p_1 - p_2 = (133100 - 9790)(0.12) + 11300 = \mathbf{26100\ Pa} \quad \textit{Ans.}$$

2.36 In Fig. P2.36 both the tank and the slanted tube are open to the atmosphere. If L = 2.13 m, what is the angle of tilt ϕ of the tube?

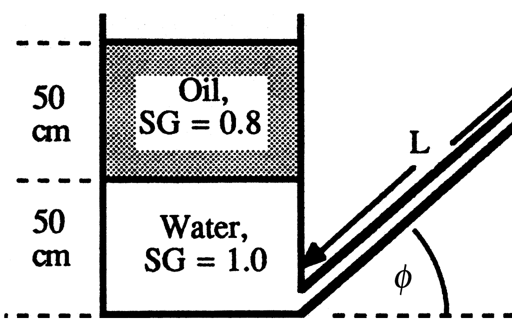


Fig. P2.36

Solution: Proceed hydrostatically from the oil surface to the slanted tube surface:

$$p_a + 0.8(9790)(0.5) + 9790(0.5) - 9790(2.13\sin\phi) = p_a,$$

$$\text{or:}\quad \sin\phi = \frac{8811}{20853} = 0.4225, \quad \text{solve}\quad \phi \approx \mathbf{25°} \quad \textit{Ans.}$$

2.37 The inclined manometer in Fig. P2.37 contains Meriam red oil, SG = 0.827. Assume the reservoir is very large. If the inclined arm has graduations 1 inch apart, what should θ be if each graduation represents 1 psf of the pressure p_A?

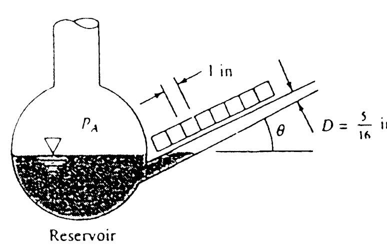


Fig. P2.37

Solution: The specific weight of the oil is (0.827)(62.4) = 51.6 lbf/ft^3. If the reservoir level does not change and ΔL = 1 inch is the scale marking, then

$$p_A(\text{gage}) = 1\,\frac{\text{lbf}}{\text{ft}^2} = \gamma_{oil}\Delta z = \gamma_{oil}\Delta L\ \sin\theta = \left(51.6\,\frac{\text{lbf}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ ft}\right)\sin\theta,$$

$$\text{or: } \sin\theta = 0.2325 \quad \text{or: } \theta = \mathbf{13.45°} \quad \textit{Ans.}$$

2.38 In the figure at right, new tubing contains gas whose density is greater than the outside air. For the dimensions shown, (a) find p_1(gage). (b) Find the error caused by assuming $\rho_{tube} = \rho_{air}$. (c) Evaluate the error if ρ_m = 860, ρ_a = 1.2, and ρ_t = 1.5 kg/m^3, H = 1.32 m, and h = 0.58 cm.

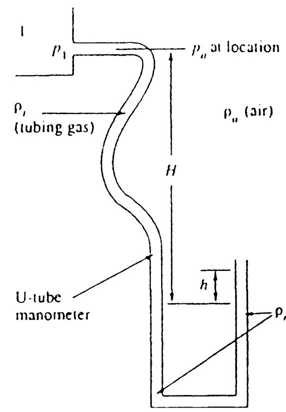


Fig. P2.38

Solution: (a) Work hydrostatically around the manometer:

$$p_1 + \rho_t gH = p_a + \rho_m gh + \rho_a g(H-h),$$

$$\text{or: } \mathbf{p_{1\,gage} = (\rho_m - \rho_a)gh - (\rho_t - \rho_a)gH} \quad \textit{Ans.}\text{ (a)}$$

(b) From (a), the error is the last term: **Error = $-(\rho_t - \rho_a)$gH** *Ans.* (b)

(c) For the given data, the normal reading is (860 – 1.2)(9.81)(0.0058) = 48.9 Pa, and

$$\textit{Error} = -(1.50-1.20)(9.81)(1.32) = \mathbf{-3.88\ Pa}\ (\textit{about } 8\%) \quad \textit{Ans.}\text{ (c)}$$

2.39 In Fig. P2.39 the right leg of the manometer is open to the atmosphere. Find the gage pressure, in Pa, in the air gap in the tank. Neglect surface tension.

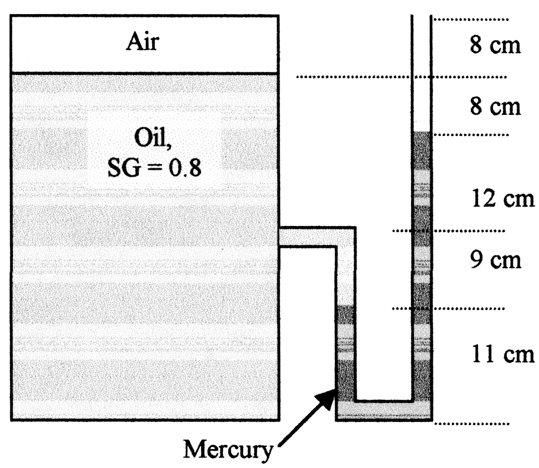


Fig. P2.39

Solution: The two 8-cm legs of air are negligible (only 2 Pa). Begin at the right mercury interface and go to the air gap:

$$0\text{ Pa-gage} + (133100\text{ N/m}^3)(0.12+0.09\text{ m}) - (0.8\times 9790\text{ N/m}^3)(0.09-0.12-0.08\text{ m}) = p_{airgap}$$

$$\text{or: } p_{airgap} = 27951\text{ Pa} - 2271\text{ Pa} \approx \mathbf{25700\ Pa\text{-}gage} \quad \textit{Ans.}$$

2.40 In Fig. P2.40 the pressures at A and B are the same, 100 kPa. If water is introduced at A to increase p_A to 130 kPa, find and sketch the new positions of the mercury menisci. The connecting tube is a uniform 1-cm in diameter. Assume no change in the liquid densities.

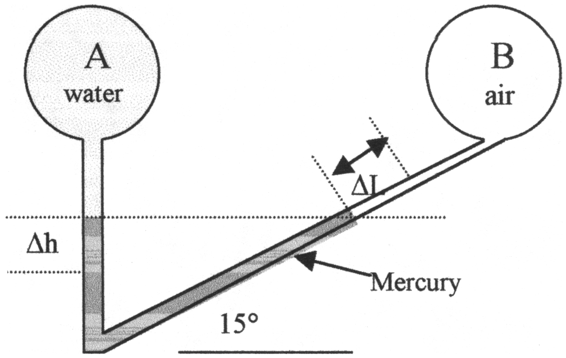


Fig. P2.40

Solution: Since the tube diameter is constant, the volume of mercury will displace a distance Δh down the left side, equal to the volume increase on the right side; Δh = ΔL. Apply the hydrostatic relation to the pressure change, beginning at the right (air/mercury) interface:

$$p_B + \gamma_{Hg}(\Delta L \sin\theta + \Delta h) - \gamma_W(\Delta h + \Delta L \sin\theta) = p_A \quad \text{with } \Delta h = \Delta L$$

$$\text{or: } 100{,}000 + 133100(\Delta h)(1+\sin 15°) - 9790(\Delta h)(1+\sin 15°) = p_A = 130{,}000 \text{ Pa}$$

$$\text{Solve for } \quad \Delta h = (30{,}000 \text{ Pa})/[(133100 - 9790 \text{ N/m}^2)(1+\sin 15°)] = \mathbf{0.193\ m} \quad \textit{Ans.}$$

The mercury in the left (vertical) leg will drop 19.3 cm, the mercury in the right (slanted) leg will rise 19.3 cm along the slant and 0.05 cm in vertical elevation.

2.41 The system in Fig. P2.41 is at 20°C. Determine the pressure at point A in pounds per square foot.

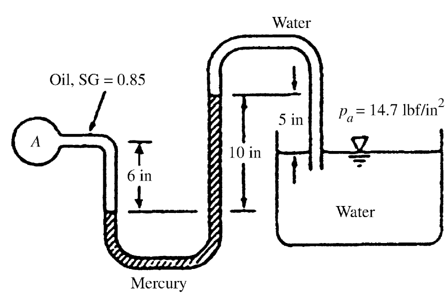


Fig. P2.41

Solution: Take the specific weights of water and mercury from Table 2.1. Write the hydrostatic formula from point A to the water surface:

$$p_A + (0.85)(62.4 \text{ lbf/ft}^3)\left(\frac{6}{12}\text{ ft}\right) - (846)\left(\frac{10}{12}\right) + (62.4)\left(\frac{5}{12}\right) = p_{atm} = (14.7)(144)\frac{\text{lbf}}{\text{ft}^2}$$

$$\text{Solve for } \quad p_A = \mathbf{2770\ lbf/ft^2} \quad \textit{Ans.}$$

2.42 Small pressure differences can be measured by the two-fluid manometer in Fig. P2.42, where ρ_2 is only slightly larger than ρ_1. Derive a formula for $p_A - p_B$ if the reservoirs are very large.

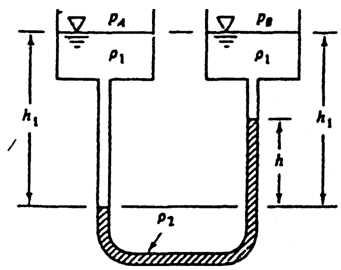


Fig. P2.42

Solution: Apply the hydrostatic formula from A to B:

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$$

$$\text{Solve for} \quad \mathbf{p_A - p_B = (\rho_2 - \rho_1)gh} \quad \textit{Ans.}$$

If $(\rho_2 - \rho_1)$ is very small, h will be very large for a given Δp (a sensitive manometer).

2.43 The traditional method of measuring blood pressure uses a *sphygmomanometer*, first recording the highest (*systolic*) and then the lowest (*diastolic*) pressure from which flowing "Korotkoff" sounds can be heard. Patients with dangerous hypertension can exhibit systolic pressures as high as 5 lbf/in^2. Normal levels, however, are 2.7 and 1.7 lbf/in^2, respectively, for systolic and diastolic pressures. The manometer uses mercury and air as fluids. (a) How high should the manometer tube be? (b) Express normal systolic and diastolic blood pressure in millimeters of mercury.

Solution: (a) The manometer height must be at least large enough to accommodate the largest systolic pressure expected. Thus apply the hydrostatic relation using 5 lbf/in^2 as the pressure,

$$h = p_B/\rho g = (5\ \text{lbf/in}^2)(6895\ \text{Pa/lbf/in}^2)/(133100\ \text{N/m}^3) = 0.26\ \text{m}$$

$$\text{So make the height about} \quad \mathbf{30\ cm}. \quad \textit{Ans.}\ \text{(a)}$$

(b) Convert the systolic and diastolic pressures by dividing them by mercury's specific weight.

$$h_{\text{systolic}} = (2.7\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)/(846\ \text{lbf/ft}^3) = 0.46\ \text{ft Hg} = 140\ \text{mm Hg}$$

$$h_{\text{diastolic}} = (1.7\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)/(846\ \text{lbf/ft}^3) = 0.289\ \text{ft Hg} = 88\ \text{mm Hg}$$

The systolic/diastolic pressures are thus **140/88 mm Hg**. *Ans.* (b)

2.44 Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The mercury manometer reads a 6-in height. The pressure drop $p_2 - p_1$ is partly due to friction and partly due to gravity. Determine the total pressure drop and also the part due to friction only. Which part does the manometer read? Why?

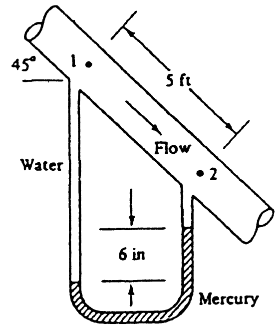


Fig. P2.44

Solution: Let "h" be the distance down from point 2 to the mercury-water interface in the right leg. Write the hydrostatic formula from 1 to 2:

$$p_1 + 62.4\left(5\sin 45° + h + \frac{6}{12}\right) - 846\left(\frac{6}{12}\right) - 62.4h = p_2,$$

$$p_1 - p_2 = \underbrace{(846 - 62.4)(6/12)}_{\text{....friction loss...}} - \underbrace{62.4(5\sin 45°)}_{\text{..gravity head..}} = 392 - 221$$

$$= \mathbf{171\,\frac{lbf}{ft^2}} \quad \textit{Ans.}$$

The manometer reads only the *friction loss* of 392 lbf/ft^2, not the gravity head of 221 psf.

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than $P_{atmosphere}$?

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$p_{atm} + (0.85)(9790)(0.4\text{ m})$$
$$-(133100)(0.15\text{ m}) - (12)(0.30\text{ m})$$
$$+(9790)(0.45\text{ m}) = p_A,$$

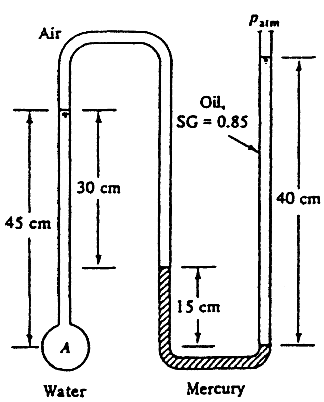


Fig. P2.45

$$\text{or:}\quad p_A = p_{atm} - 12200\text{ Pa} = \mathbf{12200\ Pa\ (vacuum)} \quad \textit{Ans.}$$

2.46 In Fig. P2.46 both ends of the manometer are open to the atmosphere. Estimate the specific gravity of fluid X.

Solution: The pressure at the bottom of the manometer must be the same regardless of which leg we approach through, left or right:

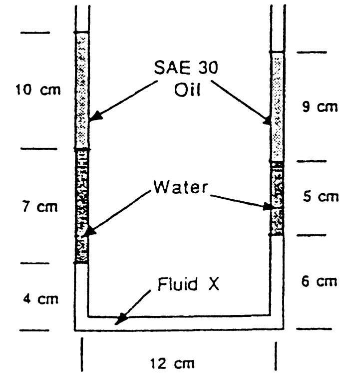


Fig. P2.46

$$p_{atm} + (8720)(0.1) + (9790)(0.07) + \gamma_X(0.04) \quad \text{(left leg)}$$

$$= p_{atm} + (8720)(0.09) + (9790)(0.05) + \gamma_X(0.06) \quad \text{(right leg)}$$

$$\text{or:} \quad \gamma_X = 14150 \text{ N/m}^3, \quad SG_X = \frac{14150}{9790} \approx \mathbf{1.45} \quad \textit{Ans.}$$

2.47 The cylindrical tank in Fig. P2.47 is being filled with 20°C water by a pump developing an exit pressure of 175 kPa. At the instant shown, the air pressure is 110 kPa and H = 35 cm. The pump stops when it can no longer raise the water pressure. Estimate "H" at that time.

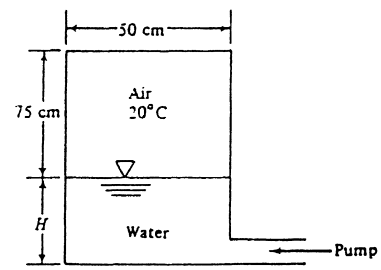


Fig. P2.47

Solution: At the end of pumping, the bottom water pressure must be 175 kPa:

$$p_{air} + 9790H = 175000$$

Meanwhile, assuming isothermal air compression, the final air pressure is such that

$$\frac{p_{air}}{110000} = \frac{Vol_{old}}{Vol_{new}} = \frac{\pi R^2(0.75 \text{ m})}{\pi R^2(1.1 \text{ m} - H)} = \frac{0.75}{1.1 - H}$$

where R is the tank radius. Combining these two gives a quadratic equation for H:

$$\frac{0.75(110000)}{1.1 - H} + 9790H = 175000, \quad \text{or} \quad H^2 - 18.98H + 11.24 = 0$$

The two roots are H = 18.37 m (ridiculous) or, properly, H = **0.614 m** *Ans.*

2.48 Conduct an experiment: Place a thin wooden ruler on a table with a 40% overhang, as shown. Cover it with 2 full-size sheets of newspaper. (a) Estimate the total force on top

of the newspaper due to air pressure. (b) With everyone out of the way, perform a karate chop on the outer end of the ruler. (c) Explain the results in b.

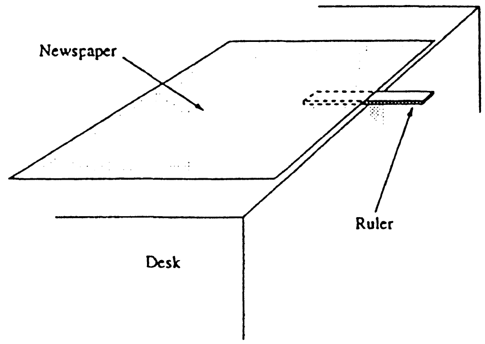


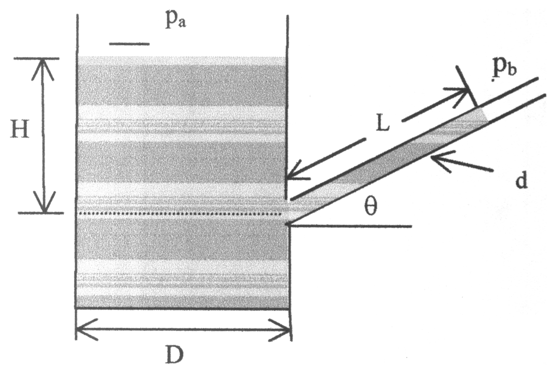
Fig. P2.48

Results: (a) Newsprint is about 27 in (0.686 m) by 22.5 in (0.572 m). Thus the force is:

$$F = pA = (101325\ Pa)(0.686\ m)(0.572\ m)$$
$$= \mathbf{39700\ N!} \quad \textit{Ans.}$$

(b) The newspaper will hold the ruler, which will probably *break* due to the chop. *Ans.*
(c) Chop is fast, air does not have time to rush in, partial vacuum under newspaper. *Ans.*

2.49 An inclined manometer, similar in concept to Fig. P2.37, has a vertical cylinder reservoir whose cross-sectional area is 35 times that of the tube. The fluid is ethylene glycol at 20°C. If $\theta = 20°$ and the fluid rises 25 cm above its zero-difference level, measured along the slanted tube, what is the actual pressure difference being measured?



Solution: The volume of the fluid rising into the tube, $\pi d^2\Delta h/4$, must equal the volume decrease in the reservoir. Thus H decreases by $(d/D)^2\ \Delta h$ where,

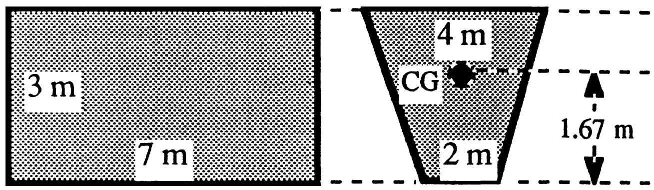
$$\Delta h = L\sin\theta = (0.25\text{ m})\sin 20° = 0.0855\text{ m}$$
$$\Delta H = (d/D)^2\Delta h = \Delta h/35 = 0.0024\text{ m}$$

Applying the hydrostatic relation,

$$p_a + \gamma(-\Delta H) - \gamma\Delta h = p_b$$

$$p_a - p_b = \gamma(\Delta H + \Delta h) = (1117\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.0855\text{ m} + 0.0024\text{ m}) = 963\text{ Pa}$$
$$\mathbf{\Delta p \approx 963\ Pa} \quad \textit{Ans.}$$

2.50 A vat filled with oil (SG = 0.85) is 7 m long and 3 m deep and has a trapezoidal cross-section 2 m wide at the bottom and 4 m wide at the top, as shown in Fig. P2.50. Compute (a) the weight of oil in the vat; (b) the force on the vat bottom; and (c) the force on the trapezoidal end panel.



Solution: (a) The total volume of oil in the vat is (3 m)(7 m)(4 m + 2 m)/2 = 63 m^3. Therefore the weight of oil in the vat is

$$W = \gamma_{oil}(\text{Vol}) = (0.85)(9790\ \text{N/m}^3)(63\ \text{m}^3) = \mathbf{524{,}000\ N} \quad \textit{Ans.}\ \text{(a)}$$

(b) The force on the horizontal bottom surface of the vat is

$$F_{bottom} = \gamma_{oil} h_{CG} A_{bottom} = (0.85)(9790)(3\ \text{m})(2\ \text{m})(7\ \text{m}) = \mathbf{350{,}000\ N} \quad \textit{Ans.}\ \text{(b)}$$

Note that F is <u>less</u> than the total weight of oil—the student might explain why they differ? (c) I found in my statics book that the centroid of this trapezoid is 1.33 m below the surface, or 1.67 m above the bottom, as shown. Therefore the side-panel force is

$$F_{side} = \gamma_{oil} h_{CG} A_{side} = (0.85)(9790)(1.33\ \text{m})(9\ \text{m}^2) = \mathbf{100{,}000\ N} \quad \textit{Ans.}\ \text{(c)}$$

These are large forces. Big vats have to be strong!

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

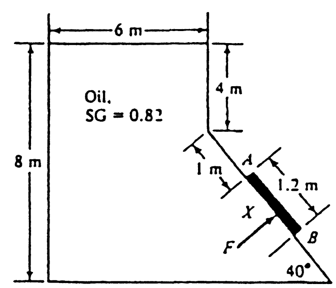


Fig. P2.51

Solution: The centroidal depth of the gate is

$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028\ \text{m},$$

$$\text{hence} \quad F_{AB} = \gamma_{oil} h_{CG} A_{gate} = (0.82 \times 9790)(5.028)(1.2 \times 0.8) = \mathbf{38750\ N} \quad \textit{Ans.}$$

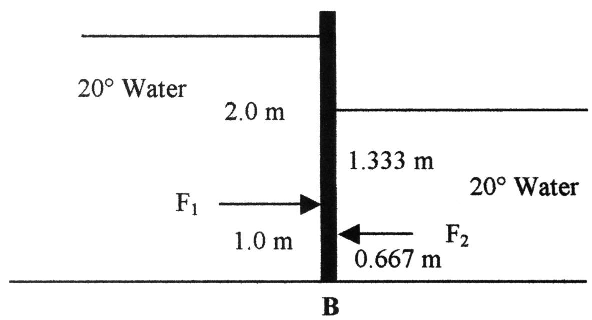
The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3 \sin 40°}{(5.028)(1.2 \times 0.8)} = -0.0153\ \text{m}$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx \mathbf{0.615\ m}$ *Ans.*

2.52 A vertical lock gate is 4 m wide and separates 20°C water levels of 2 m and 3 m, respectively. Find the moment about the bottom required to keep the gate stationary.

Solution: On the side of the gate where the water measures 3 m, F_1 acts and has an h_{CG} of 1.5 m; on the opposite side, F_2 acts with an h_{CG} of 1 m.



$$F_1 = \gamma h_{CG1} A_1 = (9790)(1.5)(3)(4) = 176,220\ \text{N}$$

$$F_2 = \gamma h_{CG2} A_2 = (9790)(1.0)(2)(4) = 78,320\ \text{N}$$

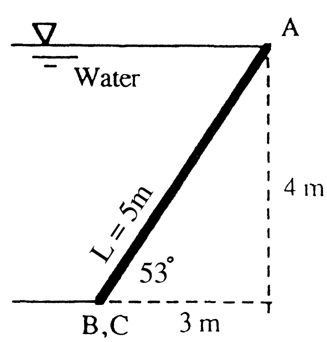
$$y_{CP1} = [-(1/12)(4)(3)^3 \sin 90°]/[(1.5)(4)(3)] = -0.5\ \text{m; so } F_1 \text{ acts at } 1.5 - 0.5$$
$$= 1.0\ \text{m above B}$$

$$y_{CP2} = [-(1/12)(4)(2)^3 \sin 90°]/[(1)(4)(2)] = -0.333\ \text{m; } F_2 \text{ acts at } 1.0 - 0.33$$
$$= 0.67\ \text{m above B}$$

Taking moments about points B (see the figure),

$$\sum M_B = (176{,}220\ \text{N})(1.0\ \text{m}) - (78{,}320\ \text{N})(0.667\ \text{m})$$
$$= 124{,}000\ \text{N}\cdot\text{m}; \quad \boldsymbol{M_{\text{bottom}} = 124\ \text{kN}\cdot\text{m.}}$$

2.53 Panel ABC in the slanted side of a water tank (shown at right) is an isoceles triangle with vertex at A and base BC = 2 m. Find the water force on the panel and its line of action.



Solution: (a) The centroid of ABC is 2/3 of the depth down, or 8/3 m from the surface. The panel area is (1/2)(2 m)(5 m) = 5 m^2. The water force is

$$F_{ABC} = \gamma h_{CG} A_{panel} = (9790)(2.67\ \text{m})(5\ \text{m}^2) = \mathbf{131{,}000\ N} \quad \textit{Ans.}\ \text{(a)}$$

(b) The moment of inertia of ABC is $(1/36)(2\text{ m})(5\text{ m})^3 = 6.94\text{ m}^4$. From Eq. (2.44),

$$y_{CP} = -I_{xx}\sin\theta/(h_{CG}A_{panel}) = -6.94\sin(53°)/[2.67(5)] = \mathbf{-0.417\ m} \quad \textit{Ans.}\text{ (b)}$$

The center of pressure is 3.75 m down from A, or 1.25 m up from BC.

2.54 In Fig. P2.54, the hydrostatic force F is the same on the bottom of all three containers, even though the weights of liquid above are quite different. The three bottom shapes and the fluids are the same. This is called the *hydrostatic paradox*. Explain why it is true and sketch a freebody of each of the liquid columns.

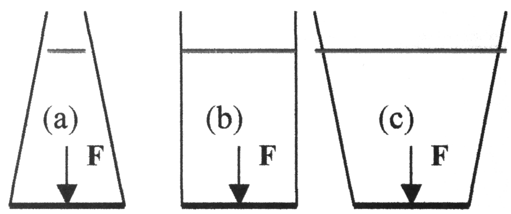
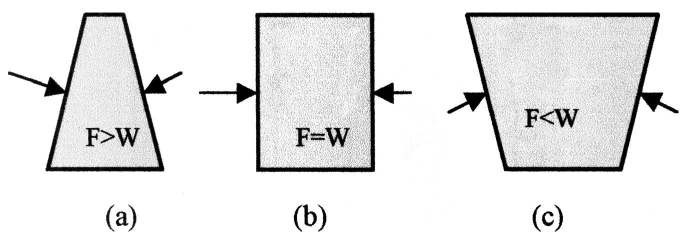


Fig. P2.54

Solution: The three freebodies are shown below. Pressure on the side-walls balances the forces. In (a), downward side-pressure components help add to a light W. In (b) side pressures are horizontal. In (c) upward side pressure helps reduce a heavy W.



2.55 Gate AB in Fig. P2.55 is 5 ft wide into the paper, hinged at A, and restrained by a stop at B. Compute (a) the force on stop B; and (b) the reactions at A if $h = 9.5$ ft.

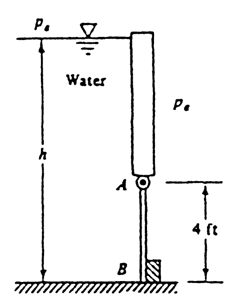


Fig. P2.55

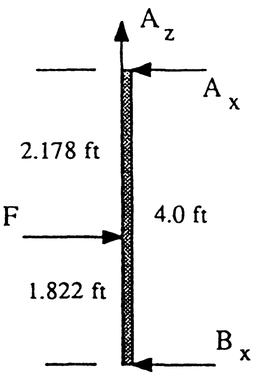
Solution: The centroid of AB is 2.0 ft below A, hence the centroidal depth is $h + 2 - 4 = 7.5$ ft. Then the total hydrostatic force on the gate is

$$F = \gamma h_{CG}A_{gate} = (62.4\text{ lbf/ft}^3)(7.5\text{ ft})(20\text{ ft}^2) = 9360\text{ lbf}$$

The C.P. is below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(1/12)(5)(4)^3\sin 90°}{(7.5)(20)} = -0.178 \text{ ft}$$

This is shown on the freebody of the gate at right. We find force B_x with moments about A:



$$\sum M_A = B_x(4.0) - (9360)(2.178) = 0,$$

$$\text{or: } B_x = \mathbf{5100\ lbf} \quad \text{(to left)} \quad \textit{Ans.}\text{ (a)}$$

The reaction forces at A then follow from equilibrium of forces (with *zero* gate weight):

$$\sum F_x = 0 = 9360 - 5100 - A_x, \quad \text{or:} \quad A_x = \mathbf{4260\ lbf} \quad \text{(to left)}$$

$$\sum F_z = 0 = A_z + W_{gate} \approx A_z, \quad \text{or:} \quad A_z = \mathbf{0\ lbf} \quad \textit{Ans.}\text{ (b)}$$

2.56 For the gate of Prob. 2.55 above, stop "B" breaks if the force on it equals 9200 lbf. For what water depth *h* is this condition reached?

Solution: The formulas must be written in terms of the unknown centroidal depth:

$$h_{CG} = h - 2 \quad F = \gamma h_{CG}A = (62.4)h_{CG}(20) = 1248h_{CG}$$

$$y_{CP} = -\frac{I_{XX}\sin\theta}{h_{CG}A} = -\frac{(1/12)(5)(4)^3\sin 90°}{h_{CG}(20)} = -\frac{1.333}{h_{CG}}$$

Then moments about A for the freebody in Prob. 2.155 above will yield the answer:

$$\sum M_A = 0 = 9200(4) - (1248h_{CG})\left(2 + \frac{1.333}{h_{CG}}\right), \quad \text{or} \quad h_{CG} = 14.08 \text{ ft}, \ h = \mathbf{16.08\ ft} \quad \textit{Ans.}$$

2.57 The tank in Fig. P2.57 is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on panel BC, (a) from a single formula; (b) by computing horizontal and vertical forces separately, in the spirit of curved surfaces.

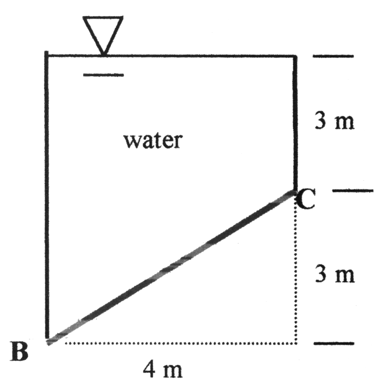


Fig. P2.57

Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790\ \text{N/m}^3)(3+1.5\ \text{m})(5\ \text{m}\times 2\ \text{m}) = 440{,}550\ \text{N} = \mathbf{441\ kN} \quad \textit{Ans. (a)}$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

$$F_H = (9790)(4.5)(3\times 2) = 264{,}330\ \text{N} = \mathbf{264\ kN} \quad \textit{Ans. (b)}$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790)[(3)(4)+(1/2)(4)(3)](2) = 352{,}440 = \mathbf{352\ kN} \quad \textit{Ans. (b)}$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2}$ = **441 kN.**

2.58 In Fig. P2.58, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?

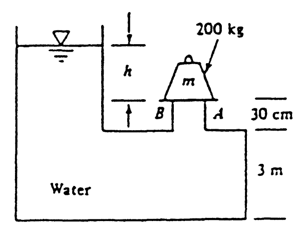


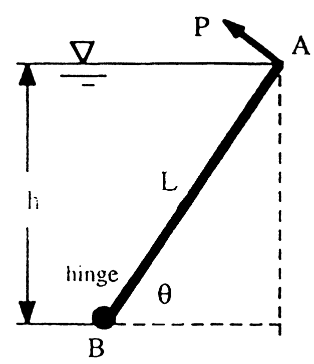
Fig. P2.58

Solution: The centroidal depth is exactly equal to *h* and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG} A_{gate} = (9790\ \text{N/m}^3)\ h\frac{\pi}{4}(0.8\ \text{m})^2 = W = (200)(9.81)\ \text{N}$$

$$\text{Solve for} \quad h = \mathbf{0.40\ m} \quad \textit{Ans.}$$

2.59 Gate AB has length L, width b into the paper, is hinged at B, and has negligible weight. The liquid level h remains at the top of the gate for any angle θ. Find an analytic expression for the force P, perpendicular to AB, required to keep the gate in equilibrium.



Solution: The centroid of the gate remains at distance L/2 from A and depth h/2 below

the surface. For any θ, then, the hydrostatic force is F = γ(h/2)Lb. The moment of inertia of the gate is $(1/12)bL^3$, hence $y_{CP} = -(1/12)bL^3\sin\theta/[(h/2)Lb]$, and the center of pressure is $(L/2 - y_{CP})$ from point B. Summing moments about hinge B yields

$$PL = F(L/2 - y_{CP}), \quad \text{or:} \quad \mathbf{P = (\gamma hb/4)(L - L^2\sin\theta/3h)} \quad \textit{Ans.}$$

2.60 The pressure in the air gap is 8000 Pa gage. The tank is cylindrical. Calculate the net hydrostatic force (a) on the bottom of the tank; (b) on the cylindrical sidewall CC; and (c) on the annular plane panel BB.

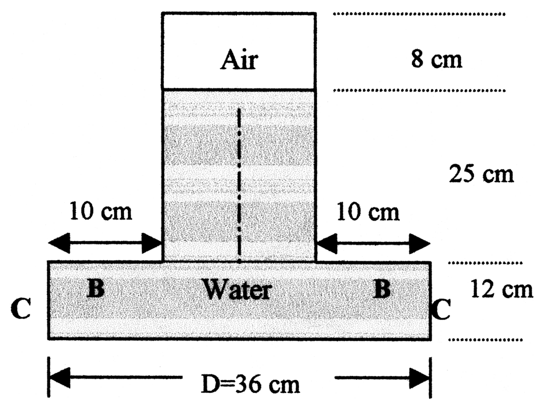


Fig. P2.60

Solution: (a) The bottom force is simply equal to bottom pressure times bottom area:

$$p_{bottom} = p_{air} + \rho_{water} g|\Delta z| = 8000 \text{ Pa} + (9790 \text{ N/m}^3)(0.25 + 0.12 \text{ m}) = 11622 \text{ Pa-gage}$$

$$F_{bottom} = p_{bottom}A_{bottom} = (11622 \text{ Pa})(\pi/4)(0.36 \text{ m})^2 = \mathbf{1180\ N} \quad \textit{Ans.}\text{ (a)}$$

(b) The net force on the cylindrical sidewall CC is **zero** due to symmetry. *Ans.* (b)

(c) The force on annular region CC is, like part (a), the pressure at CC times the area of CC:

$$p_{CC} = p_{air} + \rho_{water} g|\Delta z|_{CC} = 8000 \text{ Pa} + (9790 \text{ N/m}^3)(0.25 \text{ m}) = 10448 \text{ Pa-gage}$$

$$F_{CC} = p_{CC}A_{CC} = (10448 \text{ Pa})(\pi/4)[(0.36 \text{ m})^2 - (0.16 \text{ m})^2] = \mathbf{853\ N} \quad \textit{Ans.}\text{ (c)}$$

2.61 Gate AB in Fig. P2.61 is a homogeneous mass of 180 kg, 1.2 m wide into the paper, resting on smooth bottom B. All fluids are at 20°C. For what water depth *h* will the force at point B be zero?

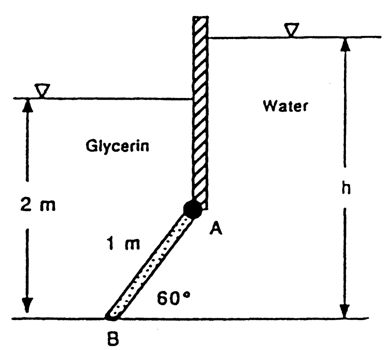


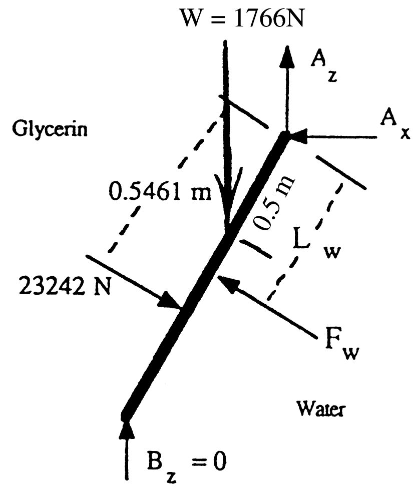
Fig. P2.61

Solution: Let γ= 12360 N/m^3 for glycerin and 9790 N/m^3 for water. The centroid of

AB is 0.433 m vertically below A, so $h_{CG} = 2.0 - 0.433 = 1.567$ m, and we may compute the glycerin force and its line of action:

$$F_g = \gamma \bar{h} A = (12360)(1.567)(1.2) = 23242 \text{ N}$$

$$y_{CP,g} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{(1.567)(1.2)} = -0.0461 \text{ m}$$



These are shown on the freebody at right. The water force and its line of action are shown without numbers, because they depend upon the centroidal depth on the water side:

$$F_w = (9790)h_{CG}(1.2)$$

$$y_{CP} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{h_{CG}(1.2)} = -\frac{0.0722}{h_{CG}}$$

The weight of the gate, W = 180(9.81) = 1766 N, acts at the centroid, as shown above. Since the force at B equals zero, we may sum moments counterclockwise about A to find the water depth:

$$\sum M_A = 0 = (23242)(0.5461) + (1766)(0.5\cos 60°) - (9790)h_{CG}(1.2)(0.5 + 0.0722/h_{CG})$$

Solve for $h_{CG,water} = 2.09$ m, or: $h = h_{CG} + 0.433 = \mathbf{2.52\ m}$ *Ans.*

2.62 Gate AB in Fig. P2.62 is 15 ft long and 8 ft wide into the paper, hinged at B with a stop at A. The gate is 1-in-thick steel, SG = 7.85. Compute the 20°C water level h for which the gate will start to fall.

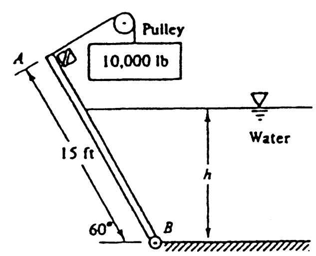
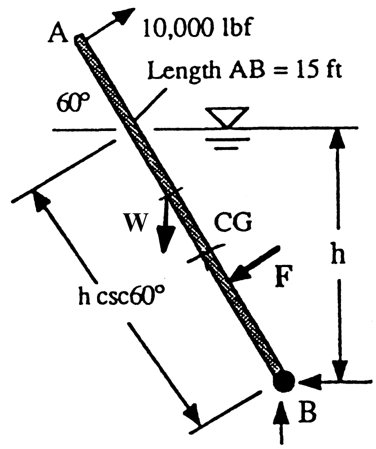


Fig. P2.62

Solution: Only the length (h csc 60°) of the gate lies below the water. Only this part

contributes to the hydrostatic force shown in the freebody at right:



$$F = \gamma h_{CG} A = (62.4)\left(\frac{h}{2}\right)(8h\csc 60^\circ)$$

$$= 288.2h^2 \text{ (lbf)}$$

$$y_{CP} = -\frac{(1/12)(8)(h\csc 60^\circ)^3 \sin 60^\circ}{(h/2)(8h\csc 60^\circ)}$$

$$= -\frac{h}{6}\csc 60^\circ$$

The weight of the gate is (7.85)(62.4 lbf/ft^3)(15 ft)(1/12 ft)(8 ft) = 4898 lbf. This weight acts downward at the CG of the *full gate* as shown (not the CG of the submerged portion). Thus, W is 7.5 ft above point B and has moment arm (7.5 cos 60° ft) about B.

We are now in a position to find *h* by summing moments about the hinge line B:

$$\sum M_B = (10000)(15) - (288.2h^2)[(h/2)\csc 60^\circ - (h/6)\csc 60^\circ] - 4898(7.5\cos 60^\circ) = 0,$$

$$\text{or:}\quad 110.9h^3 = 150000 - 18369,\quad h = (131631/110.9)^{1/3} = \mathbf{10.6\ ft}\quad \textit{Ans.}$$

2.63 The tank in Fig. P2.63 has a 4-cm-diameter plug which will pop out if the hydrostatic force on it reaches 25 N. For 20°C fluids, what will be the reading *h* on the manometer when this happens?

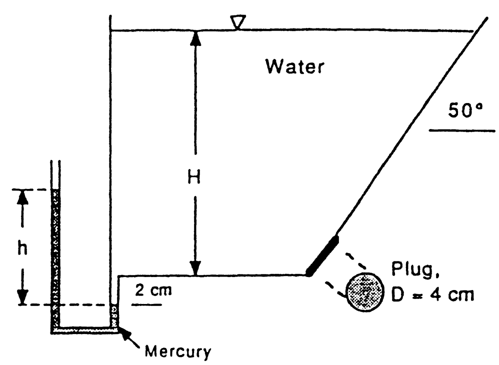


Fig. P2.63

Solution: The water depth when the plug pops out is

$$F = 25\text{ N} = \gamma h_{CG} A = (9790)h_{CG}\frac{\pi(0.04)^2}{4}$$

$$\text{or}\quad h_{CG} = 2.032\text{ m}$$

It makes little numerical difference, but the mercury-water interface is a little deeper than this, by the amount (0.02 sin 50°) of plug-depth, plus 2 cm of tube length. Thus

$$p_{atm} + (9790)(2.032 + 0.02\sin 50^\circ + 0.02) - (133100)h = p_{atm},$$

$$\text{or:}\quad h \approx \mathbf{0.152\ m}\quad \textit{Ans.}$$

2.64 Gate ABC in Fig. P2.64 has a fixed hinge at B and is 2 m wide into the paper. If the water level is high enough, the gate will open. Compute the depth *h* for which this happens.

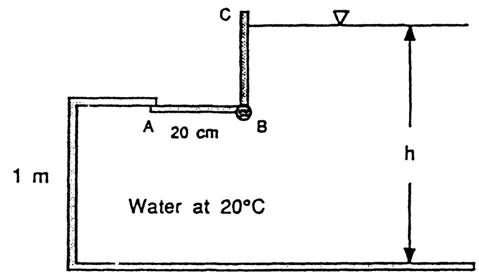
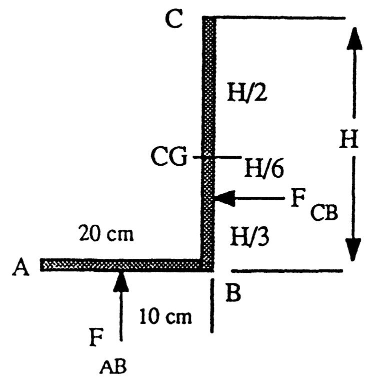


Fig. P2.64

Solution: Let *H* = (h − 1 meter) be the depth down to the level AB. The forces on AB and BC are shown in the freebody at right. The moments of these forces about B are equal when the gate opens:



$$\sum M_B = 0 = \gamma H(0.2)b(0.1) = \gamma\left(\frac{H}{2}\right)(Hb)\left(\frac{H}{3}\right)$$

or: $H = 0.346$ m,

$$h = H + 1 = \mathbf{1.346\ m} \quad \textit{Ans.}$$

This solution is independent of both the water density and the gate width *b* into the paper.

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

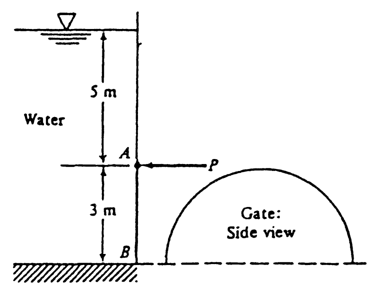
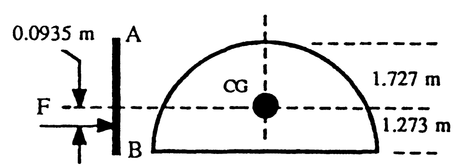


Fig. P2.65

Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is



$$F = \gamma h_{CG} A = (9790)(5.0 + 1.727)\frac{\pi}{2}(3)^2 = 931000 \text{ N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(0.10976)(3)^4 \sin 90°}{(5+1.727)(\pi/2)(3)^2} = -0.0935 \text{ m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P, \quad \text{or:} \quad P = \mathbf{366000\ N} \quad \textit{Ans.}$$

2.66 Dam ABC in Fig. P2.66 is 30 m wide into the paper and is concrete (SG ≈ 2.40). Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over? Would fluid seepage under the dam change your argument?

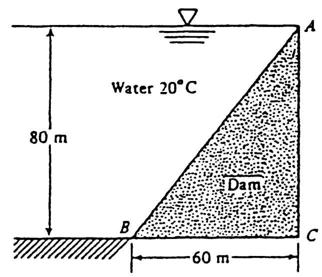
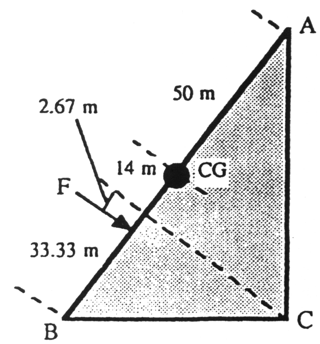


Fig. P2.66

Solution: The centroid of surface AB is 40 m deep, and the total force on AB is

$$F = \gamma h_{CG} A = (9790)(40)(100 \times 30)$$
$$= 1.175E9\ \text{N}$$



The line of action of this force is two-thirds of the way down along AB, or 66.67 m from A. This is seen either by inspection (A is at the surface) or by the usual formula:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(30)(100)^3\sin(53.13°)}{(40)(30 \times 100)} = -16.67\ \text{m}$$

to be added to the 50-m distance from A to the centroid, or 50 + 16.67 = 66.67 m. As shown in the figure, the line of action of F is 2.67 m to the left of a line up from C normal to AB. The moment of F about C is thus

$$M_C = FL = (1.175E9)(66.67 - 64.0) \approx \mathbf{3.13E9\ N \cdot m} \quad \textit{Ans.}$$

This moment is counterclockwise, hence it cannot tip over the dam. If there were *seepage* under the dam, the main support force at the bottom of the dam would shift to the left of point C and might indeed cause the dam to tip over.

2.67 Generalize Prob. 2.66 with length AB as “H”, length BC as “L”, and angle ABC as “*q*”, with width “b” into the paper. If the dam material has specific gravity “SG”, with no seepage, find the critical angle θ_c for which the dam will just tip over to the right. Evaluate this expression for SG = 2.40.

Solution: By geometry, L = H$\cos\theta$ and the vertical height of the dam is Hsinq. The

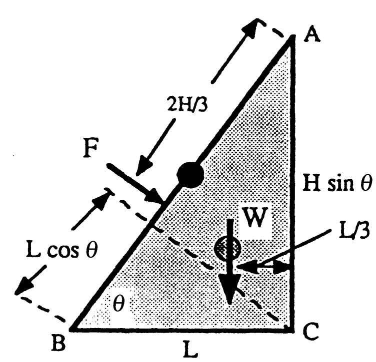


Fig. P2.67

force F on surface AB is γ(H/2)(sinθ)Hb, and its position is at 2H/3 down from point A, as shown in the figure. Its moment arm about C is thus (H/3 − Lcosθ). Meanwhile the weight of the dam is W = (SG)γ(L/2)H(sinθ)b, with a moment arm L/3 as shown. Then summation of clockwise moments about C gives, for critical "tip-over" conditions,

$$\sum M_C = 0 = \left(\gamma \frac{H}{2}\sin\theta\ Hb\right)\left[\frac{H}{3} - L\cos\theta\right] - \left[SG(\gamma)\frac{L}{2}H\sin\theta\ b\right]\left(\frac{L}{3}\right) \quad \text{with } L = H\cos\theta.$$

$$\text{Solve for} \quad \cos^2\theta_c = \frac{1}{\mathbf{3+SG}} \quad \textit{Ans.}$$

Any angle greater than θ_c will cause tip-over to the right. For the particular case of concrete, SG ≈ 2.40, cosθ_c ≈ 0.430, or θ_c ≈ **64.5°**, which is greater than the given angle θ = 53.13° in Prob. 2.66, hence there was no tipping in that problem.

2.68 Isosceles triangle gate AB in Fig. P2.68 is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?

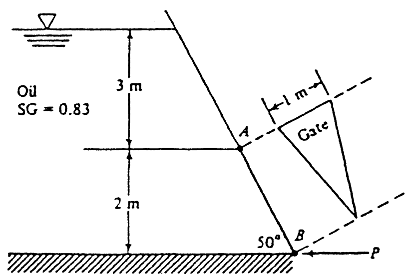
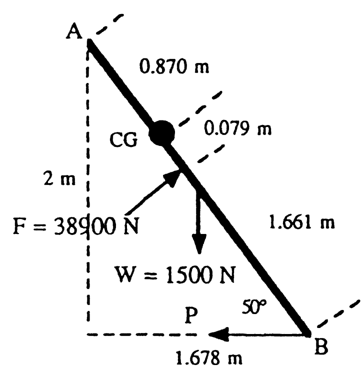


Fig. P2.68

Solution: The gate is 2.0/sin 50° = 2.611 m long from A to B and its area is 1.3054 m^2. Its centroid is 1/3 of the way down from A, so the centroidal depth is 3.0 + 0.667 m. The force on the gate is

$$F = \gamma h_{CG} A = (0.83)(9790)(3.667)(1.3054)$$
$$= 38894 \text{ N}$$



The position of this force is below the centroid:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$= -\frac{(1/36)(1.0)(2.611)^3\sin 50°}{(3.667)(1.3054)} = -0.0791 \text{ m}$$

The force and its position are shown in the freebody at upper right. The gate weight of 1500 N is assumed at the centroid of the plate, with moment arm 0.559 meters about point A. Summing moments about point A gives the required force P:

$$\sum M_A = 0 = P(2.0) + 1500(0.559) - 38894(0.870 + 0.0791),$$

$$\text{Solve for} \quad P = \mathbf{18040\ N} \quad \textit{Ans.}$$

2.69 Panel BCD is semicircular and line BC is 8 cm below the surface. Determine (a) the hydrostatic force on the panel; and (b) the moment of this force about D.

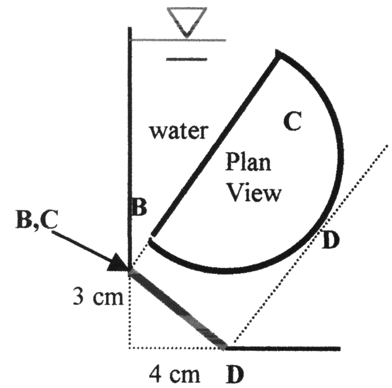


Fig. P2.69

Solution: (a) The radius of BCD is 5 cm. Its centroid is at $4R/3\pi$ or $4(5\text{ cm})/3\pi$ = 2.12 cm down along the slant from BC to D. Then the vertical distance down to the centroid is $h_{CG} = 8\text{ cm} + (2.12\text{ cm})\cos(53.13°) = 9.27\text{ cm}$.

The force is the centroidal pressure times the panel area:

$$F = \gamma h_{CG} A = (9790\text{ N/m}^3)(0.0927\text{ m})(\pi/2)(0.05\text{ m})^2 = \mathbf{3.57\text{ N}} \quad \textit{Ans.}\text{ (a)}$$

(b) Point D is (0.05 − 0.0212) = 0.288 cm from the centroid. The moment of F about D is thus

$$M_D = (3.57\text{ N})(0.05\text{ m} - 0.0212\text{ m}) = \mathbf{0.103\text{ N}\cdot\text{m}} \quad \textit{Ans.}\text{ (b)}$$

2.70 The cylindrical tank in Fig. P2.70 has a 35-cm-high cylindrical insert in the bottom. The pressure at point B is 156 kPa. Find (a) the pressure in the air space; and (b) the force on the top of the insert. Neglect air pressure outside the tank.

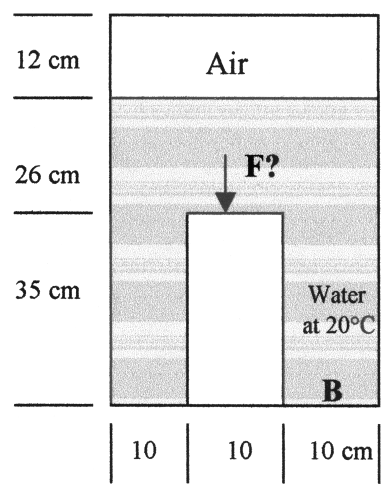


Fig. P2.70

Solution: (a) The pressure in the air space can be found by working upwards hydrostatically from point B:

$$156{,}000\text{ Pa} - (9790\text{ N/m}^3)(0.35 + 0.26\text{ m}) = p_{air} \approx 150{,}000\text{ Pa} = \mathbf{150\text{ kPa}} \quad \textit{Ans.}\text{ (a)}$$

(b) The force on top of the insert is simply the pressure on the insert times the insert area:

$$p_{\text{insert top}} = 156{,}000\text{ Pa} - (9790\text{ N/m}^3)(0.35\text{ m}) = 152{,}600\text{ Pa}$$

$$F_{insert} = p_{insert} A_{insert} = (152600\text{ Pa})(\pi/4)(0.1\text{ m})^2 = \mathbf{1200\text{ N}} \quad \textit{Ans.}\text{ (b)}$$

2.71 In Fig. P2.71 gate AB is 3 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.40). What sphere diameter is just right to close the gate?

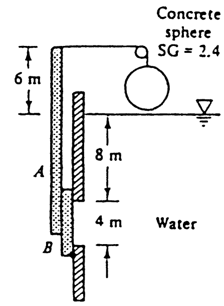


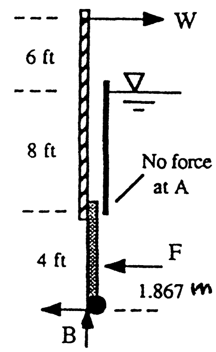
Fig. P2.71

Solution: The centroid of AB is 10 m down from the surface, hence the hydrostatic force is

$$F = \gamma h_{CG} A = (9790)(10)(4\times 3) = 1.175E6 \text{ N}$$

The line of action is slightly below the centroid:

$$y_{CP} = -\frac{(1/12)(3)(4)^3 \sin 90°}{(10)(12)} = -0.133 \text{ m}$$



Sum moments about B in the freebody at right to find the pulley force or weight W:

$$\sum M_B = 0 = W(6+8+4 \text{ m}) - (1.175E6)(2.0 - 0.133 \text{ m}), \quad \text{or} \quad W = 121800 \text{ N}$$

Set this value equal to the weight of a solid concrete sphere:

$$W = 121800 \text{ N} = \gamma_{\text{concrete}} \frac{\pi}{6} D^3 = (2.4)(9790)\frac{\pi}{6} D^3, \quad \text{or:} \quad D_{\text{sphere}} = \mathbf{2.15 \text{ m}} \quad \textit{Ans.}$$

2.72 Gate B is 30 cm high and 60 cm wide into the paper and hinged at the top. What is the water depth *h* which will first cause the gate to open?

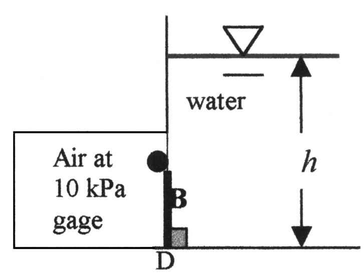


Fig. P2.72

Solution: The minimum height needed to open the gate can be assessed by calculating the hydrostatic force on each side of the gate and equating moments about the hinge. The air pressure causes a force, F_{air}, which acts on the gate at 0.15 m above point D.

$$F_{air} = (10{,}000 \text{ Pa})(0.3 \text{ m})(0.6 \text{ m}) = 1800 \text{ N}$$

Since the air pressure is uniform, F_{air} acts at the centroid of the gate, or 15 cm below the hinge. The force imparted by the water is simply the hydrostatic force,

$$F_w = (\gamma h_{CG}A)_w = (9790\ \text{N/m}^3)(h - 0.15\ \text{m})(0.3\ \text{m})(0.6\ \text{m}) = 1762.2h - 264.3$$

This force has a center of pressure at,

$$y_{CP} = \frac{\frac{1}{12}(0.6)(0.3)^3(\mathbf{sin}90)}{(h-0.15)(0.3)(0.6)} = \frac{0.0075}{h-0.15} \quad \text{with } h \text{ in meters}$$

Sum moments about the hinge and set equal to zero to find the minimum height:

$$\sum M_{hinge} = 0 = (1762.2h - 264.3)[0.15 + (0.0075/(h-0.15))] - (1800)(0.15)$$

This is quadratic in *h*, but let's simply solve by iteration: h = **1.12 m** *Ans.*

2.73 Weightless gate AB is 5 ft wide into the paper and opens to let fresh water out when the ocean tide is falling. The hinge at A is 2 ft above the freshwater level. Find *h* when the gate opens.

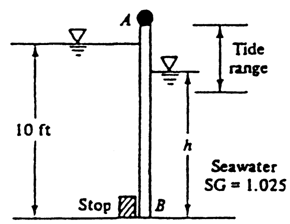


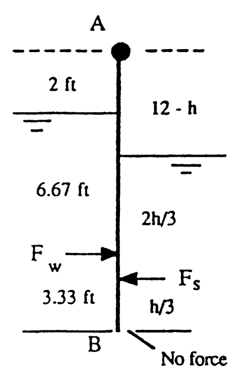
Fig. P2.73

Solution: There are two different hydrostatic forces and two different lines of action. On the water side,

$$F_w = \gamma h_{CG}A = (62.4)(5)(10 \times 5) = 15600\ \text{lbf}$$

positioned at 3.33 ft above point B. In the seawater,

$$F_s = (1.025 \times 62.4)\left(\frac{h}{2}\right)(5h) = 159.9h^2\ \ (\text{lbf})$$



positioned at h/3 above point B. Summing moments about hinge point A gives the desired seawater depth h:

$$\sum M_A = 0 = (159.9h^2)(12 - h/3) - (15600)(12 - 3.33),$$

$$\text{or} \quad 53.3h^3 - 1918.8h^2 + 135200 = 0, \quad \text{solve for} \quad h = \mathbf{9.85\ ft} \quad \textit{Ans.}$$

2.74 Find the height H in Fig. P2.74 for which the hydrostatic force on the rectangular panel is the same as the force on the semicircular panel below. Find the force on each panel and set them equal:

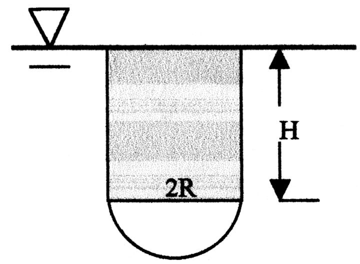


Fig. P2.74

$$F_{rect} = \gamma h_{CG} A_{rect} = \gamma(H/2)[(2R)(H)] = \gamma RH^2$$

$$F_{semi} = \gamma h_{CG} A_{semi} = \gamma(H + 4R/3\pi)[(\pi/2)R^2]$$

Set them equal, cancel γ: $RH^2 = (\pi/2)R^2H + 2R^3/3$, or: $H^2 - (\pi/2)RH - 2R^2/3 = 0$

Solution: $H = \boldsymbol{R[\pi/4 + \{(\pi/4)^2 + 2/3\}^{1/2}] \approx 1.92R}$ *Ans.*

2.75 Gate AB in the figure is hinged at A, has width b into the paper, and makes smooth contact at B. The gate has density ρ_S and uniform thickness t. For what gate density, expressed as a function of (h, t, ρ, θ), will the gate just begin to lift off the bottom? Why is your answer independent of L and b?

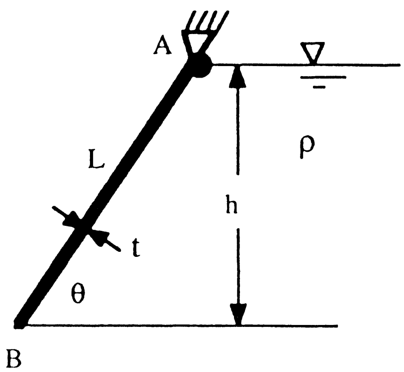


Fig. P2.75

Solution: Gate weight acts down at the center between A and B. The hydrostatic force acts at two-thirds of the way down the gate from A. When "beginning to lift off," there is no force at B. Summing moments about A yields

$$W\frac{L}{2}cos\theta = F\frac{2L}{3}, \quad F = \rho g\frac{h}{2}bL, \quad W = \rho_s gbLt$$

Combine and solve for the density of the gate. *L* and *b* and *g* drop out!

$$\rho_s = \frac{\mathbf{2h}}{\mathbf{3t\,cos\theta}}\rho \quad \textit{Ans.}$$

2.76 Panel BC in Fig. P2.76 is circular. Compute (a) the hydrostatic force of the water on the panel; (b) its center of pressure; and (c) the moment of this force about point B.

Solution: (a) The hydrostatic force on the gate is:

$$F = \gamma h_{CG} A = (9790\ \text{N/m}^3)(4.5\ \text{m})\sin 50°(\pi)(1.5\ \text{m})^2 = \mathbf{239\ kN} \quad \textit{Ans. (a)}$$

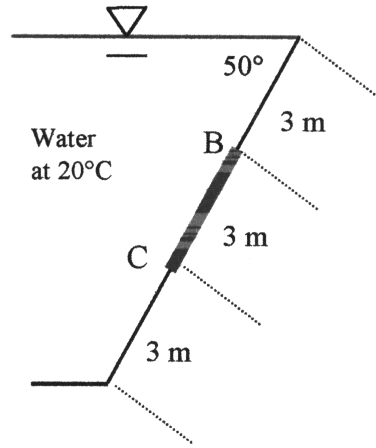


Fig. P2.76

(b) The center of pressure of the force is:

$$y_{CP} = \frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{\frac{\pi}{4}r^4\sin\theta}{h_{CG}A}$$

$$= \frac{\frac{\pi}{4}(1.5)^4\sin 50°}{(4.5\sin 50°)(\pi)(1.5^2)} = \mathbf{0.125\ m} \quad \textit{Ans. (b)}$$

Thus y is **1.625 m** down along the panel from B (or 0.125 m down from the center of the circle).
(c) The moment about B due to the hydrostatic force is,

$$M_B = (238550\ \text{N})(1.625\ \text{m}) = 387{,}600\ \text{N}\cdot\text{m} = \mathbf{388\ kN\cdot m} \quad \textit{Ans. (c)}$$

2.77 Circular gate ABC is hinged at B. Compute the force just sufficient to keep the gate from opening when h = 8 m. Neglect atmospheric pressure.

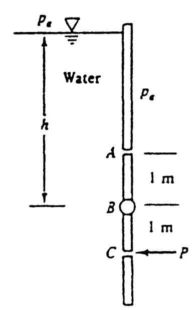


Fig. P2.77

Solution: The hydrostatic force on the gate is

$$F = \gamma h_{CG} A = (9790)(8\ \text{m})(\pi\ \text{m}^2) = 246050\ \text{N}$$

This force acts below point B by the distance

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)(1)^4\sin 90°}{(8)(\pi)} = -0.03125\ \text{m}$$

Summing moments about B gives $P(1\ \text{m}) = (246050)(0.03125\ \text{m})$, or $P \approx \mathbf{7690\ N}$ *Ans.*

2.78 Analyze Prob. 2.77 for arbitrary depth *h* and gate radius *R* and derive a formula for the opening force P. Is there anything unusual about your solution?

Solution: Referring to Fig. P2.77, the force F and its line of action are given by

$$F = \gamma h_{CG} A = \gamma h(\pi R^2)$$

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)R^4 \sin 90^\circ}{h(\pi R^2)} = -\frac{R^2}{4h}$$

Summing moments about the hinge line B then gives

$$\sum M_B = 0 = (\gamma h \pi R^2)\left(\frac{R^2}{4h}\right) - P(R), \quad \text{or:} \quad P = \frac{\boldsymbol{\pi}}{\mathbf{4}}\boldsymbol{\gamma}\mathbf{R}^3 \quad \textit{Ans.}$$

What is unusual, at least to non-geniuses, is that the result is independent of depth *h*.

2.79 Gate ABC in Fig. P2.79 is 1-m-square and hinged at B. It opens automatically when the water level is high enough. Neglecting atmospheric pressure, determine the lowest level *h* for which the gate will open. Is your result independent of the liquid density?

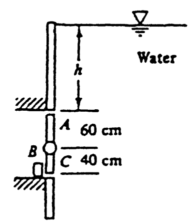
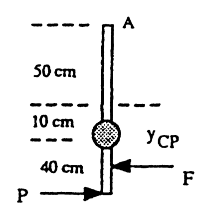


Fig. P2.79

Solution: The gate will open when the hydrostatic force F on the gate is *above B*, that is, when



$$|y_{CP}| = \frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$= \frac{(1/12)(1\text{ m})(1\text{ m})^3 \sin 90^\circ}{(h + 0.5\text{ m})(1\text{ m}^2)} < 0.1\text{ m},$$

$$\text{or:} \quad h + 0.5 > 0.833\text{ m}, \quad \text{or:} \quad \mathbf{h > 0.333\text{ m}} \quad \textit{Ans.}$$

Indeed, this result is independent of the liquid density.

2.80 For the closed tank of Fig. P2.80, all fluids are at 20°C and the air space is pressurized. If the outward net hydrostatic force on the 40-cm by 30-cm panel at the bottom is 8450 N, estimate (a) the pressure in the air space; and (b) the reading *h* on the manometer.

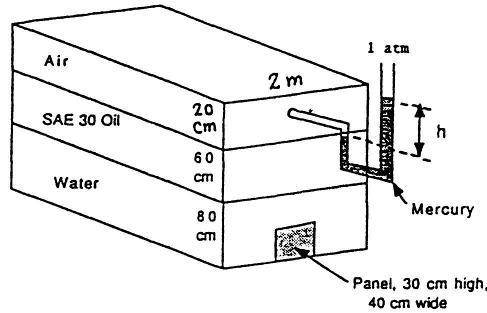


Fig. P2.80

Solution: The force on the panel yields water (gage) pressure at the centroid of the panel:

$$F = 8450\text{ N} = p_{CG}A = p_{CG}(0.3 \times 0.4\text{ m}^2), \quad \text{or} \quad p_{CG} = 70417\text{ Pa (gage)}$$

This is the water pressure 15 cm above the bottom. Now work your way back through the two liquids to the air space:

$$p_{\text{air space}} = 70417\text{ Pa} - (9790)(0.80 - 0.15) - 8720(0.60) = \mathbf{58800\ Pa} \quad \textit{Ans. } \text{(a)}$$

Neglecting the specific weight of air, we move out through the mercury to the atmosphere:

$$58800\text{ Pa} - (133100\text{ N/m}^3)h = p_{atm} = 0\text{ (gage)}, \quad \text{or:} \quad h = \mathbf{0.44\ m} \quad \textit{Ans. } \text{(b)}$$

2.81 Gate AB is 7 ft into the paper and weighs 3000 lbf when submerged. It is hinged at B and rests against a smooth wall at A. Find the water level *h* which will just cause the gate to open.

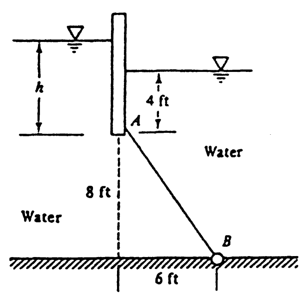
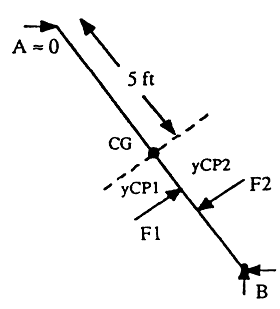


Fig. P2.81



Solution: On the right side, $h_{CG} = 8$ ft, and

$$F_2 = \gamma h_{CG2} A_2$$
$$= (62.4)(8)(70) = 34944\text{ lbf}$$

$$y_{CP2} = -\frac{(1/12)(7)(10)^3 \sin(53.13°)}{(8)(70)}$$
$$= -0.833\text{ ft}$$

On the right side, we have to write everything in terms of the centroidal depth $h_{CG1} = h + 4$ ft:

$$F_1 = (62.4)(h_{CG1})(70) = 4368h_{CG1}$$

$$y_{CP1} = -\frac{(1/12)(7)(10)^3 \sin(53.13°)}{h_{CG1}(70)} = -\frac{6.67}{h_{CG1}}$$

Then we sum moments about B in the freebody above, taking $F_A = 0$ (gate opening):

$$\sum M_B = 0 = 4368h_{CG1}\left(5 - \frac{6.67}{h_{CG1}}\right) - 34944(5 - 0.833) - 3000(5\cos 53.13°),$$

$$\text{or:} \quad h_{CG1} = \frac{183720}{21840} = 8.412 \text{ ft}, \quad \text{or:} \quad h = h_{CG1} - 4 = \mathbf{4.41\ ft} \quad \textit{Ans.}$$

2.82 The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of hydrostatic force against the dam and the point CP where the resultant strikes the dam.

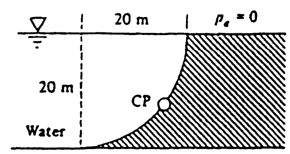
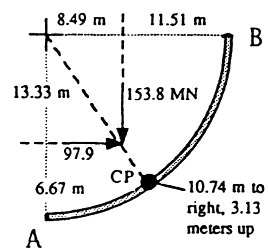


Fig. P2.82

Solution: The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert}$$
$$= (9790 \text{ N/m}^3)(10 \text{ m})(20 \times 50 \text{ m}^2)$$
$$= \mathbf{97.9\ MN} \quad \textit{Ans.}$$



This force acts 2/3 of the way down or 13.33 m from the surface, as in the figure at right. The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma(\text{Vol})_{dam} = (9790 \text{ N/m}^3)\frac{\pi}{4}(20 \text{ m})^2(50 \text{ m}) = \mathbf{153.8\ MN} \quad \textit{Ans.}$$

This vertical component acts through the centroid of the water above the dam, or $4R/3\pi = 4(20 \text{ m})/3\pi = 8.49$ m to the right of point A, as shown in the figure. The resultant hydrostatic force is $F = [(97.9 \text{ MN})^2 + (153.8 \text{ MN})^2]^{1/2}$ = **182.3 MN** acting down at an angle of **32.5°** from the vertical. The line of action of F strikes the circular-arc dam AB at the center of pressure CP, which is **10.74 m to the right and 3.13 m up from point A**, as shown in the figure. *Ans.*

2.83 Gate AB is a quarter-circle 10 ft wide and hinged at B. Find the force F just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf.

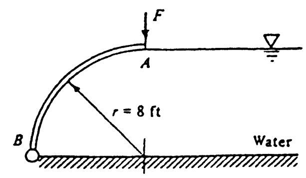
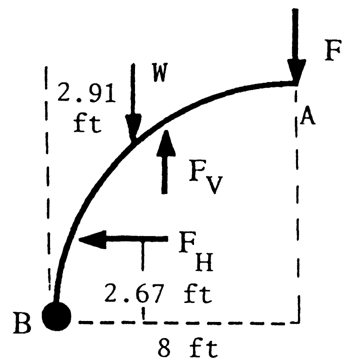


Fig. P2.83

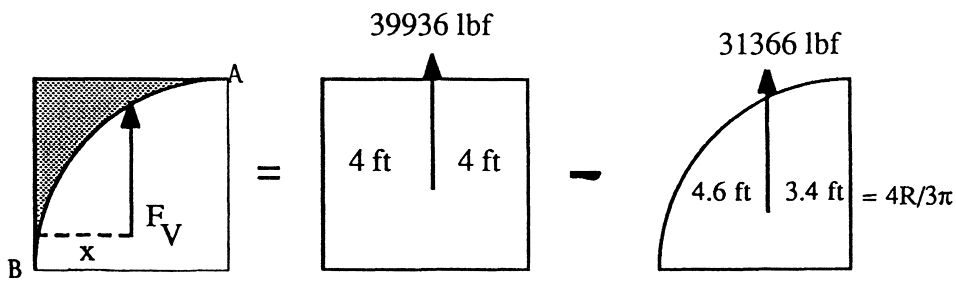
Solution: The horizontal force is computed as if AB were vertical:

$$F_H = \gamma h_{CG} A_{vert} = (62.4)(4\ \text{ft})(8\times 10\ \text{ft}^2)$$
$$= 19968\ \text{lbf} \quad \text{acting 5.33 ft below A}$$



The vertical force equals the weight of the missing piece of water above the gate, as shown below.

$$F_V = (62.4)(8)(8\times 10) - (62.4)(\pi/4)(8)^2(10)$$
$$= 39936 - 31366 = 8570\ \text{lbf}$$



The line of action *x* for this 8570-lbf force is found by summing moments from above:

$$\sum M_B(\text{of } F_V) = 8570x = 39936(4.0) - 31366(4.605), \quad \text{or} \quad x = 1.787\ \text{ft}$$

Finally, there is the 3000-lbf gate weight W, whose centroid is $2R/\pi = 5.093$ ft from force F, or 8.0 − 5.093 = 2.907 ft from point B. Then we may sum moments about hinge B to find the force F, using the freebody of the gate as sketched at the top-right of this page:

$$\sum M_B(\text{clockwise}) = 0 = F(8.0) + (3000)(2.907) - (8570)(1.787) - (19968)(2.667),$$

$$\text{or} \quad F = \frac{59840}{8.0} = \mathbf{7480\ lbf} \quad \textit{Ans.}$$

2.84 Determine (a) the total hydrostatic force on curved surface AB in Fig. P2.84 and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.

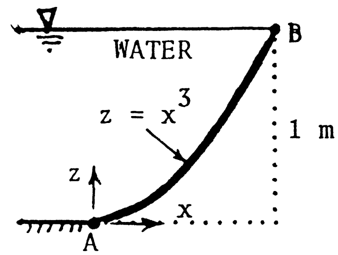
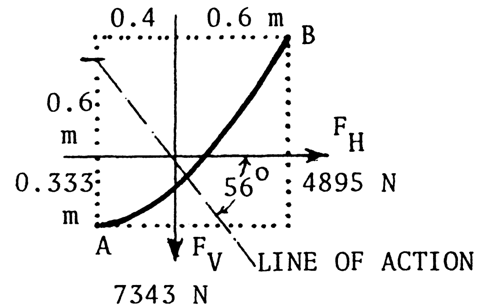


Fig. P2.84

Solution: The horizontal force is

$$F_H = \gamma h_{CG} A_{vert} = (9790\ \text{N/m}^3)(0.5\ \text{m})(1\times 1\ \text{m}^2) = 4895\ \text{N} \quad \text{at } 0.667\ \text{m below B.}$$

For the cubic-shaped surface AB, the weight of water above is computed by integration:



$$F_V = \gamma b \int_0^1 (1 - x^3)\,dx = \frac{3}{4}\gamma b$$

$$= (3/4)(9790)(1.0) = 7343\ \text{N}$$

The line of action (water centroid) of the vertical force also has to be found by integration:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^1 x(1-x^3)\,dx}{\int_0^1 (1-x^3)\,dx} = \frac{3/10}{3/4} = 0.4\ \text{m}$$

The vertical force of 7343 N thus acts at 0.4 m to the right of point A, or 0.6 m to the left of B, as shown in the sketch above. The resultant hydrostatic force then is

$$F_{total} = [(4895)^2 + (7343)^2]^{1/2} = \mathbf{8825\ N} \quad \text{acting at } \mathbf{56.31°} \text{ down and to the right.} \quad \textit{Ans.}$$

This result is shown in the sketch at above right. The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.

2.85 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

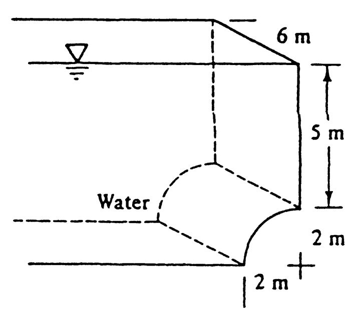


Fig. P2.85

Solution: The horizontal component is

$$F_H = \gamma h_{CG} A_{vert} = (9790)(6)(2\times 6)$$
$$= \mathbf{705000\ N} \quad \textit{Ans. (a)}$$

The vertical component is the weight of the fluid above the quarter-circle panel:

$$F_V = W(2 \text{ by } 7 \text{ rectangle}) - W(\text{quarter-circle})$$
$$= (9790)(2\times7\times6) - (9790)(\pi/4)(2)^2(6)$$
$$= 822360 - 184537 = \mathbf{638000\ N} \quad \textit{Ans.} \text{ (b)}$$

2.86 The quarter circle gate BC in Fig. P2.86 is hinged at C. Find the horizontal force *P* required to hold the gate stationary. The width *b* into the paper is 3 m.

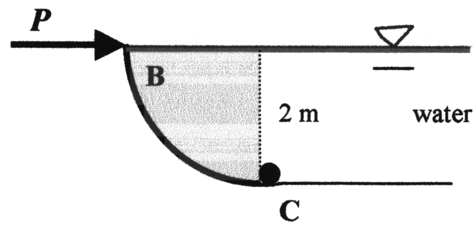


Fig. P2.86

Solution: The horizontal component of water force is

$$F_H = \gamma h_{CG} A = (9790 \text{ N/m}^3)(1 \text{ m})[(2 \text{ m})(3 \text{ m})] = 58{,}740 \text{ N}$$

This force acts 2/3 of the way down or 1.333 m down from the surface (0.667 m up from C). The vertical force is the weight of the quarter-circle of water above gate BC:

$$F_V = \gamma(\text{Vol})_{\text{water}} = (9790 \text{ N/m}^3)[(\pi/4)(2 \text{ m})^2(3 \text{ m})] = 92{,}270 \text{ N}$$

F_V acts down at $(4R/3\pi) = 0.849$ m to the left of C. Sum moments clockwise about point C:

$$\sum M_C = 0 = (2 \text{ m})P - (58740 \text{ N})(0.667 \text{ m}) - (92270 \text{ N})(0.849 \text{ m}) = 2P - 117480$$

Solve for $P = 58{,}700 \text{ N} = \mathbf{58.7\ kN}$ *Ans.*

2.87 The bottle of champagne (SG = 0.96) in Fig. P2.87 is under pressure as shown by the mercury manometer reading. Compute the net force on the 2-in-radius hemispherical end cap at the bottom of the bottle.

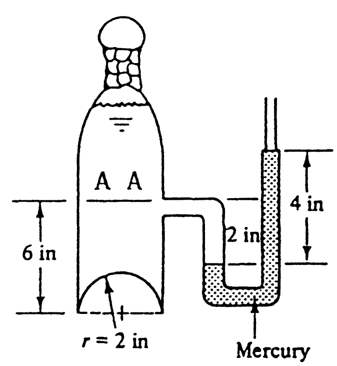


Fig. P2.87

Solution: First, from the manometer, compute the gage pressure at section AA in the