Solutions

10.1

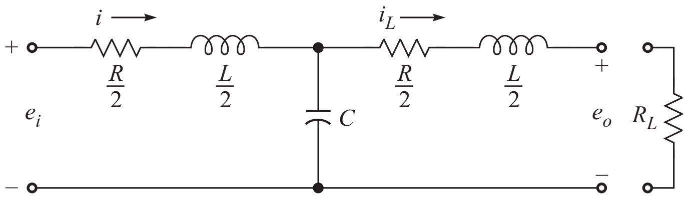

Fig. 1

Output of the cable is connected to a load R_L. Applying Kirchhoff's voltage law to both the loops, the first loop satisfies

$$\frac{R}{2}i + \frac{L}{2}Di + \frac{1}{CD}(i - i_L) = e_i \tag{1}$$

Multiplying by CD throughout, Eq. (1) becomes

$$\frac{RCD}{2}i + \frac{LCD^2}{2}i + i - i_L = e_i CD \tag{2}$$

The second loop satisfies the following equation

$$\left(\frac{R}{2} + R_L\right)i_L + \frac{L}{2}D\,i_L + \frac{1}{CD}(i_L - i) = 0 \tag{3}$$

Multiplying Eq. (3) throughout by CD

$$CD\left(\frac{R}{2} + R_L\right)i_L + \frac{LCD^2}{2}i_L + i_L - i = 0 \tag{4}$$

Assembling Eqs. (2) and (4) into a matrix

$$\begin{Bmatrix} i \\ i_L \end{Bmatrix} = \begin{bmatrix} \frac{RCD}{2} + \frac{LCD^2}{2} + 1 & -1 \\ -1 & \left(\frac{R}{2} + R_L\right) CD + \frac{LCD^2}{2} + 1 \end{bmatrix}^{-1} \begin{Bmatrix} e_i\, CD \\ 0 \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} i \\ i_L \end{Bmatrix} = \frac{1}{\det} \begin{bmatrix} \left(\frac{R}{2} + R_L\right) CD + \frac{LCD^2}{2} + 1 & 1 \\ 1 & \frac{RCD}{2} + \frac{LCD^2}{2} + 1 \end{bmatrix} \begin{Bmatrix} e_i\, CD \\ 0 \end{Bmatrix} \tag{6}$$

$$\det = CD \left[\frac{RCD}{2}\left(\frac{R}{2} + R_L\right) + \frac{RLCD^2}{4} + \frac{R}{2} + \frac{LCD^2}{2}\left(\frac{R}{2} + R_L\right) + \frac{L^2CD^3}{4} + \frac{L}{2}D + \left(\frac{R}{2} + R_L\right) + \frac{LD}{2} \right] \tag{7}$$

$$\det = CD\,(R + R_L) \left\{ \frac{D^3L^2C}{4(R+R_L)} + \frac{D^2LC}{2} + \frac{D}{(R+R_L)}\left[\frac{RC}{2}\left(\frac{R}{2} + R_L\right) + L\right] + 1 \right\} \tag{8}$$

$$i_L = \frac{e_i\, CD}{\det} \tag{9}$$

$$e_0 = i_L\, R_L \tag{10}$$

From Eqs. (8), (9) and (10)

$$\frac{e_0}{e_i}(D) = \frac{\dfrac{R_L}{(R+R_L)}}{\dfrac{D^3L^2C}{4(R+R_L)} + \dfrac{D^2LC}{2} + \dfrac{D}{(R+R_L)}\left[\dfrac{RC}{2}\left(\dfrac{R}{2} + R_L\right) + L\right] + 1} \tag{11}$$

$R = 20\ \Omega$, $C = 0.3\ \mu F$, $L = 0.2$ mH

Let $R_L = 1\ M\ \Omega$

using the above values in Eq. (11)

$$\frac{e_0}{e_i}(D) = \frac{0.999}{2.99 \times 10^{-21} D^3 + 3 \times 10^{-11} D^2 + 3 \times 10^{-6} D + 1} \tag{12}$$

For $R_L = \infty$, Eq. (12) becomes

$$\frac{e_0}{e_i}(D) = \frac{1}{3\times 10^{-11}D^2 + 3\times 10^{-6}D + 1} \tag{13}$$

The Bode plot of Eq. (13) is shown in Fig. 2.

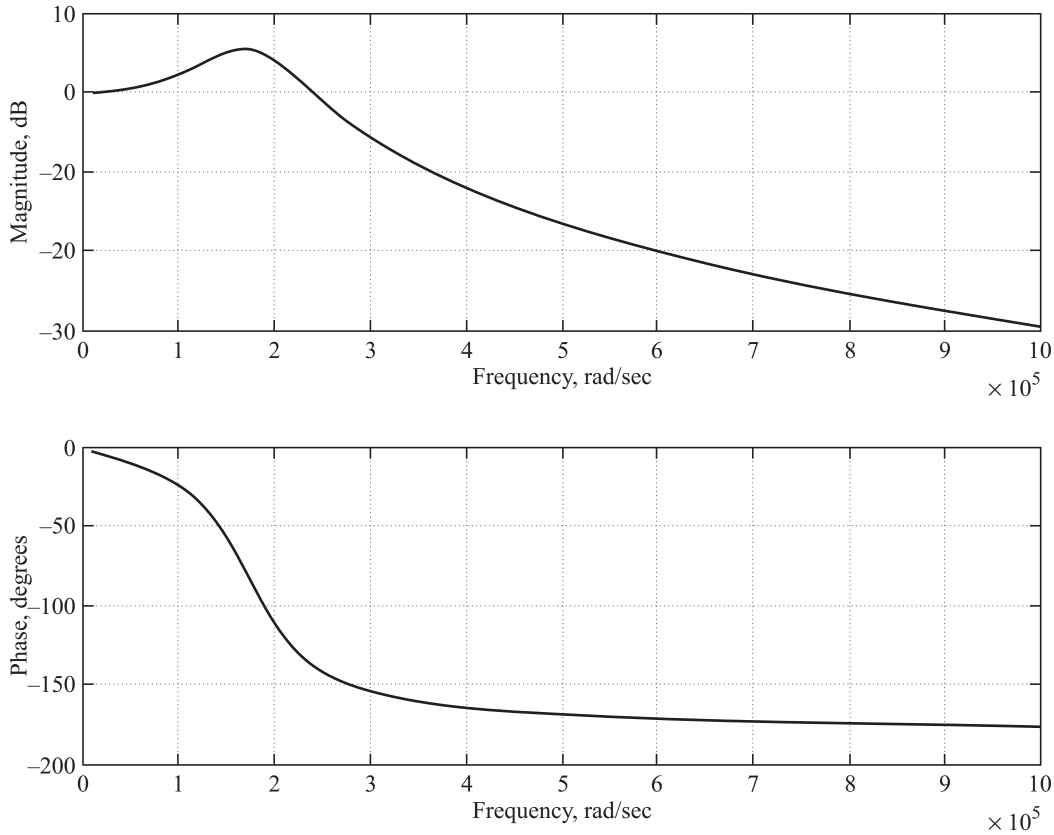

Fig. 2

10.2 Synchro repeater system

Number of transmitters	1
Number of receivers	5
Torque gradient of a single pair, for short cable	0.0035 N-m/deg
Percentage of torque gradient lost due to cable resistance	10% per ohm
Friction of each receiver	0.00035 N-m/deg
Cable resistance	0.16667 Ω/m

L_{max} = ?

Toque gradient, when all the devices are conncted

$$= \frac{0.0035 \times 2}{5+1}$$

$$= 1.16667 \times 10^{-3} \text{ N-m/deg}$$

Total torque gradient lost due to cable resistance

$$= 2 \times 0.0035 \times 0.1$$

$$= 7 \times 10^{-4} \text{ N-m/deg/}\Omega$$

$$= 7 \times 10^{-4} \times 0.16667\, L_{max}\, \frac{\text{N-m}}{\text{deg}}$$

$$= 1.16667 \times 10^{-4}\, L_{max}$$

Total torque gradient between devices

– torque gradient lost due to cable resistance = friction

$$1.16667 \times 10^{-3} - 1.16667 \times 10^{-4}\, L_{max} = 0.00035$$

$$L_{max} = 7 \text{ m (between each pair)}$$

10.3

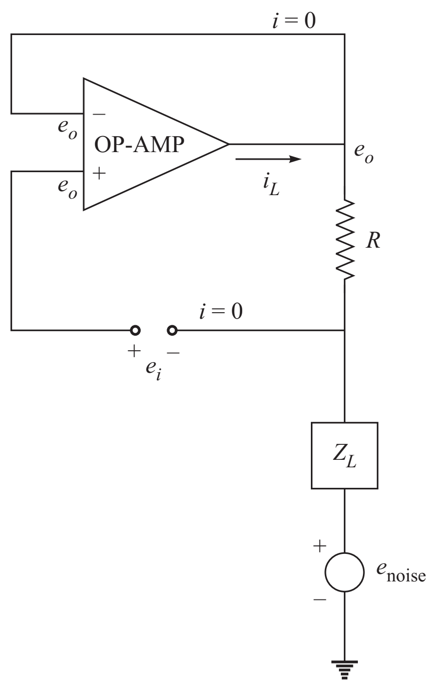

Fig. 1 Basic op-amp current source (Fig. 10.3)

The op-amp strives to maintain the positive and negative amplifier inputs equal, by adjusting e_0, the negative input is directly connected to e_0, so the positive input will also be at the same value because of the very high op-amp gain. Also, the current into each of the amplifier inputs is negligibly small. Since

the positive input terminal and the output terminal are at the same voltage, a loop through them, R and e_i results in $i_L = e_i/R$. This will hold even if there is a "noise voltage" present that changes the potential at the "bottom" of the load impedance Z_L. If there is any change in the noise voltage of load, the op-amp would change e_0 to bring the load current back to the value commanded by e_i.