Chapter 9

Solutions

9.1 To derive the balanced-bridge relationship

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \tag{1}$$

Refer to figure 9.1. When the bridge is balanced, $e_{AC} = 0$, then current $i_m = 0$, and the currents through R_1 and R_4 are equal; similarly for R_2 and R_3.

$$i_1 R_1 = i_2 R_2 \tag{2}$$
$$i_1 R_4 = i_2 R_3 \tag{3}$$

Dividing Eqs. (2) and (3)

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \tag{4}$$

9.2 If $R_1 = R_2 = R_3 = R_4$ at balance, and if $\Delta R_2 = \Delta R_3 = 0$, $\Delta R_1 = -\Delta R_4$, to show that output voltage is a linear function of ΔR_1

$$e_{AC} = \left[\frac{R_1 + \Delta R_1}{(R_1 + \Delta R_1) + (R_4 + \Delta R_4)} - \frac{R_2 + \Delta R_2}{(R_2 + \Delta R_2) + (R_3 + \Delta R_3)}\right] \qquad \{(9.6)\} \quad (1)$$

For the given data, Eq. (1) becomes

$$\frac{e_{AC}}{E_{ex}} = \frac{R_1 + \Delta R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \tag{2}$$

$e_{AC} = 0$ for $\Delta R_1 = 0$ and is a linear function of ΔR_1

9.3 Objective: Qualitative discussion on the effect on bridge operation, for both the null method and the deflection method.

If the excitation source has an internal resistance R_s, this resistance is a series resistor in the lead wires going from the source to the bridge corners. Since a current i flows in these wires, the actual bridge excitation voltage will be reduced from E_{ex} by an amount iR_s. When any of the bridge resistances change, the current i will also change, giving a variable excitation to the bridge. For the deflection method, it would require new analysis to account for this.

For the null method, however, the relation $\frac{R_1}{R_4} = \frac{R_2}{R_3}$ would still be correct, since it is true for any excitation. However, the sensitivity of the null detection does depend on excitation voltage, so it would be affected to that extent.

9.4 We should have knowledge of the actual change in excitation voltage applied to the bridge corners. This change is equal to R_s times the current in its lead wires. We would calculate this current for nominal bridge conditions and then choose R_s accordingly.

9.5 In a Wheatstone bridge, $R_1 = 3000\ \Omega$, $R_2 = 6000\ \Omega$, $R_3 = 8000\ \Omega$, $R_4 = 4000\ \Omega$ open-circuit output voltage if $\Delta R_1 = 30$, $\Delta R_2 = -20$, $\Delta R_3 = 40$, $\Delta R_4 = -50\ \Omega$, $E_{ex} = 50$ V, $R_m = 20000\ \Omega$

Combining Eqs. (9.8) through (9.12)

$$\frac{e_{AC}}{E_{ex}} = \frac{R_4}{(R_1+R_4)^2}\Delta R_1 - \frac{R_3}{(R_2+R_3)^2}\Delta R_2 + \frac{R_2}{(R_2+R_3)^2}\Delta R_3 - \frac{R_1}{(R_1+R_4)^2}\Delta R_4 \qquad (1)$$

$$\frac{e_{AC}}{E_{ex}} = \frac{4000}{(3000+4000)^2}\times 30 - \frac{8000}{(6000+8000)^2}\times(-20) + \frac{6000}{(6000+8000)^2}\times 40 - \frac{3000}{(3000+4000)^2}\times -50 \qquad (2)$$

$$\frac{e_{AC}}{E_{ex}} = 0.0076 \qquad (3)$$

$$e_{AC} = 0.0076 \times 50 = 0.38 \text{ Volts} \qquad (4)$$

Equivalent resistance R_e is given by

$$R_e = \frac{R_2 R_3}{R_2+R_3} + \frac{R_1 R_4}{R_1+R_4} = \frac{6000\times 8000}{6000+8000} + \frac{3000\times 4000}{3000+4000} \qquad (5)$$

$$= 5143\,\Omega$$

$$\frac{e_{AC_L}}{e_{AC}} = \frac{1}{1+\frac{R_e}{R_m}} = \frac{1}{1+\frac{5143}{20000}} = 0.8 \qquad (6)$$

9.6

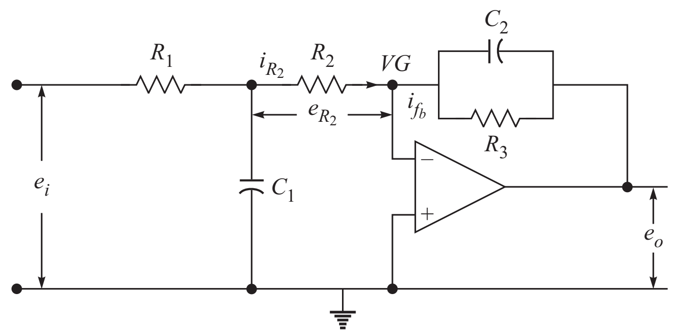


Fig. 1 [Fig. 9.21(a)]

The op-amp draws no input current and 'VG' is a virtual ground

The feedback current is given by

$$i_{fb} = \frac{e_0 - 0}{\dfrac{R_2}{R_3\, C_2\, D + 1}} \tag{1}$$

The input part of the circuit shown in Fig. 1 can be redrawn as

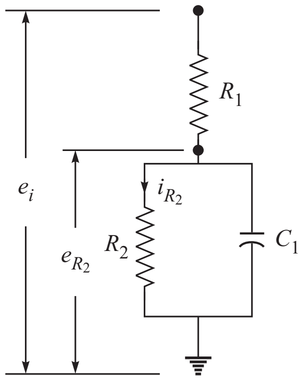


Fig. 2: Combining virtual and real ground of Fig. 1 for the input circuit.

$$e_{R_2} = \left\{ \frac{\dfrac{R_2}{R_2\, C_1\, D + 1}}{R_1 + \dfrac{R_2}{R_2\, C_1\, D + 1}} \right\} e_i \tag{2}$$

$$e_{R_2} = \frac{\dfrac{e_i\, R_2}{R_1 + R_2}}{\left(\dfrac{R_1\, R_2}{R_1 + R_2}\right) C_1\, D + 1} \tag{3}$$

$$i_{R_2} = \frac{e_{R_2}}{R_2} = \frac{\dfrac{e_i}{R_1 + R_2}}{\left(\dfrac{R_1\, R_2}{R_1 + R_2}\right) C_1\, D + 1} \tag{4}$$

$$i_{fb} = i_{R_2} \tag{5}$$

From Eqs. (1), (4) and (5)

$$\frac{\dfrac{e_0}{R_2}}{R_3\, C_2\, D + 1} = \frac{\dfrac{e_i}{R_1 + R_2}}{\left(\dfrac{R_1\, R_2}{R_1 + R_2}\right) C_1\, D + 1} \tag{6}$$

$$\frac{e_0}{e_i} = \frac{\dfrac{R_2}{R_1 + R_2}}{(R_3 C_2 D + 1)\left[\left(\dfrac{R_1 R_2}{R_1 + R_2}\right) C_1 D + 1\right]} \tag{7}$$

Equation (7) is a second-order low pass filter

9.7 Objective: To derive Eq. (9.45)

Refer to Fig. 9.15 (a) and (b)

Here the current produced by the crystal must be equal to the op-amp feedback current

$$i_{fb} = \frac{e_0}{\dfrac{R_f}{R_f C_f D + 1}} \tag{1}$$

Crystal current is given by

$$i_c = K_q D x_i \tag{2}$$

where $$K = \frac{K_q}{C_f} \tag{3}$$

From Eqs. (2) and (3)

$$i_c = K C_f D x_i \tag{4}$$

From Eqs. (1) and (4)

$$\frac{e_0 (R_f C_f D + 1)}{R_f} = K C_f D x_i \tag{5}$$

$$\frac{e_0}{x_i}(D) = \frac{K R_f C_f D}{R_f C_f D + 1} \tag{6}$$

Let $$\tau = R_f C_f$$

$$\frac{e_0}{x_i}(D) = \frac{K \tau D}{\tau D + 1} \tag{7}$$

9.8 The top end of the capacitor is at the summing junction, which is a virtual ground. The bottom end is at the actual ground. So the two ends are at the same potential resulting in the capacitor being effectively shorted out.

9.9

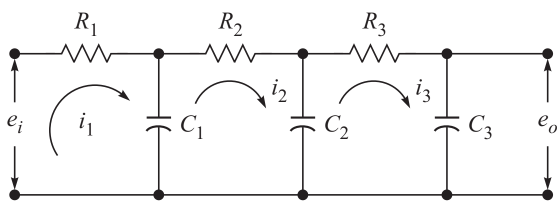


Fig. 3 Fig. 9.20a

Applying Kirchhoff's voltage law for the three loops of Fig. 1

$$i_1 R_1 + \frac{1}{C_1 D}(i_1 - i_2) = e_i \tag{1}$$

$$i_2 R_2 + \frac{1}{C_2 D}(i_2 - i_3) + \frac{1}{C_1 D}(i_2 - i_1) = 0 \tag{2}$$

$$i_3 R_3 + \frac{1}{C_3 D} \times i_3 + \frac{1}{C_2 D}(i_3 - i_2) = 0 \tag{3}$$

From Eqs. (1), (2) and (3)

$$\begin{bmatrix} R_1 + \frac{1}{C_1 D} & -\frac{1}{C_1 D} & 0 \\ -\frac{1}{C_1 D} & R_2 + \frac{1}{C_2 D} + \frac{1}{C_1 D} & -\frac{1}{C_2 D} \\ 0 & -\frac{1}{C_2 D} & R_3 + \frac{1}{C_2 D} + \frac{1}{C_3 D} \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} e_i \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

only 'i_3' needs to be solved from Eq. (4)

$$e_0 = \frac{i_3}{C_3 D} \tag{5}$$

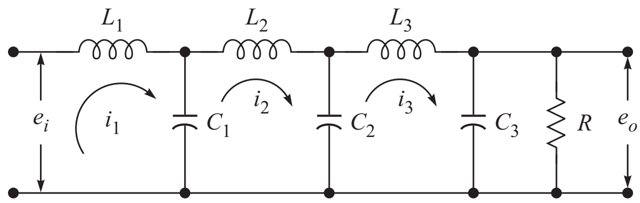


Fig. 4 Fig. 9.20b

Applying Kirchhoff's voltage law for the three loops of Fig. 2.

$$L_1 Di_1 + \frac{1}{C_1 D}(i_1 - i_2) = e_i \tag{6}$$

$$L_2 Di_2 + \frac{1}{C_1 D}(i_2 - i_1) + \frac{1}{C_2 D}(i_2 - i_3) = 0 \tag{7}$$

$$L_3 Di_3 + \frac{1}{C_2 D}(i_3 - i_2) + \frac{R}{RC_3 D + 1} i_3 = 0 \tag{8}$$

$$\begin{bmatrix} L_1 D + \dfrac{1}{C_1 D} & -\dfrac{1}{C_1 D} & 0 \\ -\dfrac{1}{C_1 D} & L_2 D + \dfrac{1}{C_1 D} + \dfrac{1}{C_2 D} & -\dfrac{1}{C_2 D} \\ 0 & -\dfrac{1}{C_2 D} & L_3 D + \dfrac{1}{C_2 D} + \dfrac{R}{R C_3 D + 1} \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} e_i \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

Equation (9) can be solved for i_3

$$e_0 = \frac{i_3 R}{R C_3 D + 1} \tag{10}$$

9.10 To derive the transfer function of the hydromechanical filter

The upper bellows provide an instantaneous response to p_i, which is then cancelled by the delayed response of the lower bellows. The delayed response 'p' is found from conservation of volume for lower bellows. All moving parts are assumed to be massless.

Volume flow through the tube = Volume storage in the bellows.

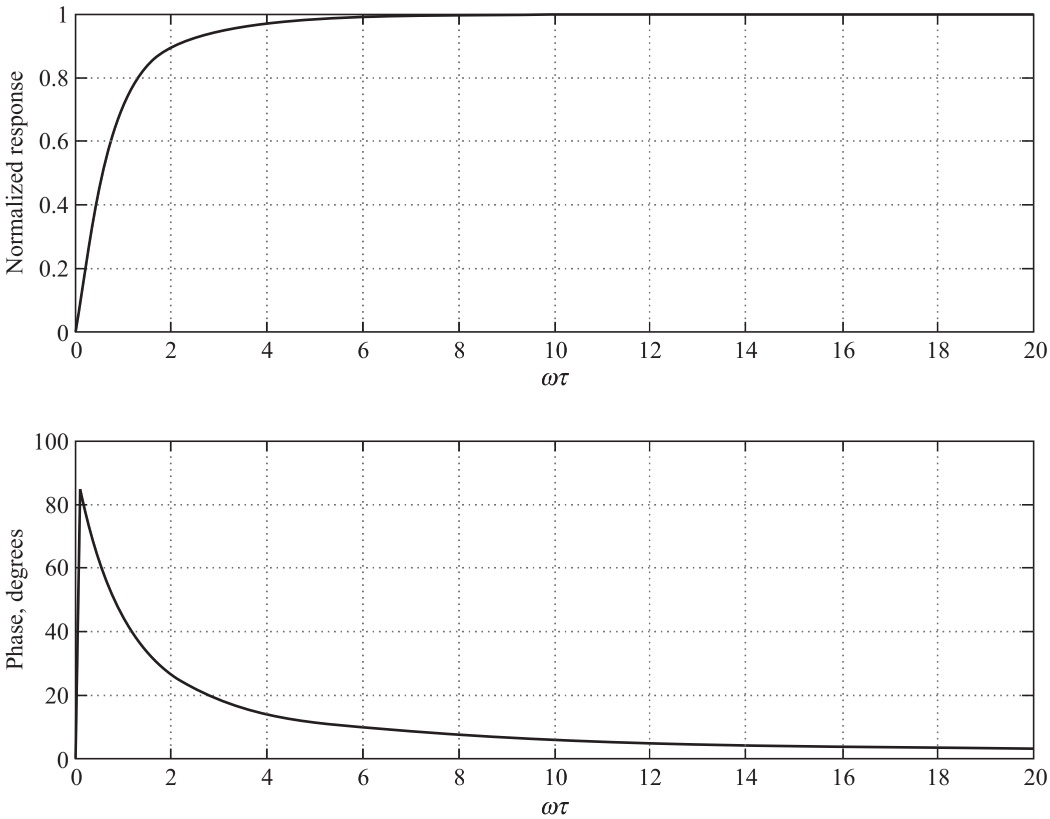


Fig. 1 Magnitude and phase of the hydromechanical filter.

$$K_{p_x} = m/Pa \quad K_b = Pa/m^3, \quad K_t = m^3/Pa$$

$$(p_i - p)\ K_t dt = \frac{dp}{K_b} \tag{1}$$

$$\frac{dp}{dt} + K_b\ K_t\, p = K_b\ K_t\ p_i \tag{2}$$

$$\tau = K_b\ K_t$$

$$(\tau D + 1)p = p_i \tag{3}$$

$$x_0 = K_{p_x}\left(p_i - \frac{p_i}{\tau D + 1}\right) \tag{4}$$

$$\frac{x_0}{p_i}(D) = \frac{K_{p_x}\tau D}{\tau D + 1} \tag{5}$$

The normalized response of Eq. 5 is shown in Fig. 1 therefore, the above system is a high-pass filter.

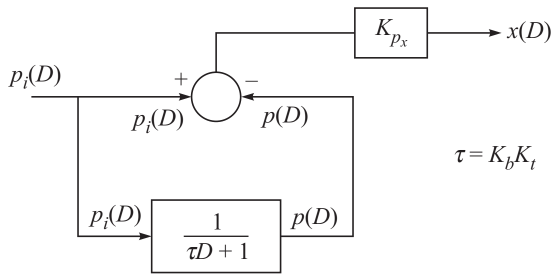


Fig. 2 Block diagram of the hydromechanical filter.

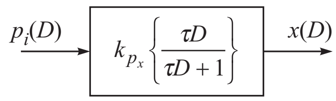
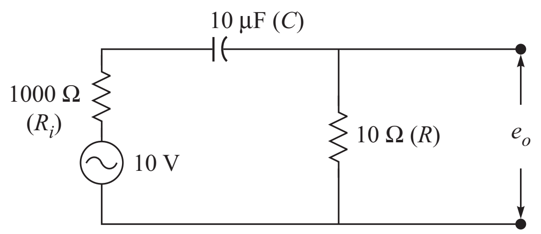


Fig. 3 Overall transfer function of Fig. 2.

9.11



$$e_i = 10 \text{ V (peak to peak)}$$

$$e_0 \quad \text{for} \quad f = 0,\ 100,\ 1000,\ 10000, \text{ and } 100{,}000 \text{ Hz}$$

$$R_{e_q} = R_i + R \tag{1}$$

$$Z(D) = R_{e_q} + \frac{1}{CD} \tag{2}$$

$$i = \frac{e_i}{Z(D)} = \frac{e_i}{R_{e_q} + \dfrac{1}{CD}} = \frac{CD\,e_i}{R_{e_q}CD + 1} \tag{3}$$

Let $D = j\omega$

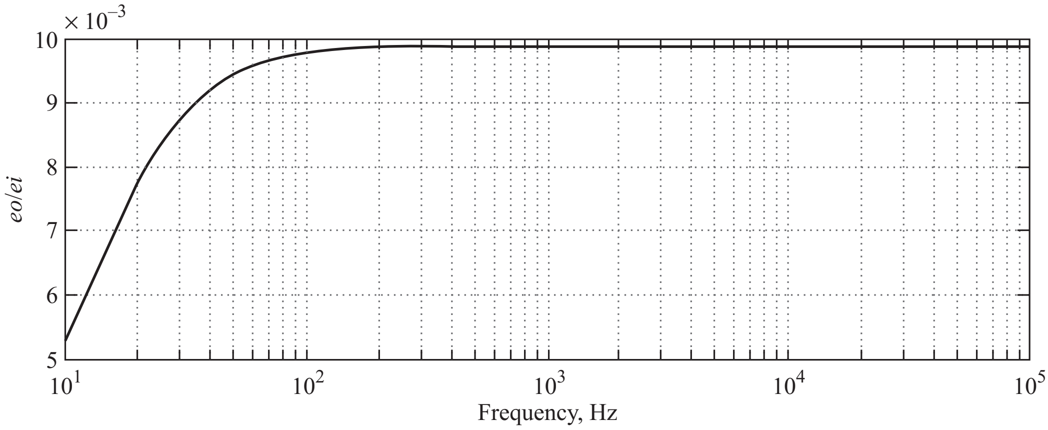
$$i(j\omega) = \frac{j\omega C e_i}{j\omega C R_{e_q} + 1} \tag{4}$$

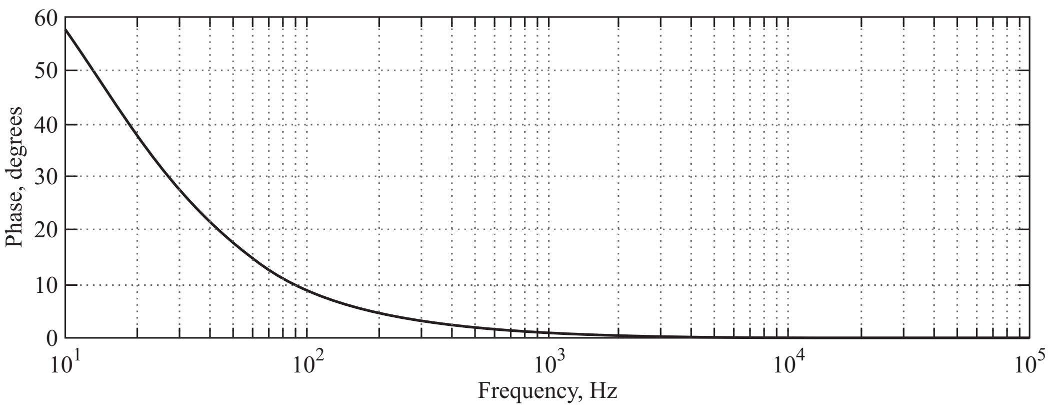
$$e_0(j\omega) = Ri(j\omega) = \frac{j\omega RC e_i}{j\omega C R_{e_q} + 1} \tag{5}$$

$$|e_0(j\omega)| = \frac{\omega RC e_i}{\sqrt{1 + (\omega C R_{e_q})^2}} \tag{6}$$

$$e_0(j\omega) = \frac{\pi}{2} - \tan^{-1} R_{e_q}\ \omega c \tag{7}$$

Equations (6) and (7) are plotted on below.





9.12 The two analog multipliers used here are scaled for a 10-volt full scale system, that is, the output of the multiplier is the product of the two input voltages divided by 10. The easiest way to get the system Eqs. (9.47), (9.48) and (9.49) is to "work backward" from the output voltage e_{l_p} through the second integrator. That is, the input to the right most multiplier must be-10$RCD\ e_{l_p}/V_c$. This signal is then multiplied by 2ζ and passed through the left most summer. The output of this summer can again be found by going backwards through the first integrator

$$\omega_n = \frac{V_c}{10RC} \tag{1}$$

$$-e_i - e_{l_p} - \frac{20\zeta RCDe_{l_p}}{V_c} = \frac{100R^2C^2D^2}{V_c^2} e_{l_p} \tag{2}$$

$$\frac{e_{l_p}}{-e_i}(D) = \frac{1}{\dfrac{100R^2C^2D^2}{V_c^2} + \dfrac{20\zeta RCD}{V_c} + 1} \tag{3}$$

From Eqs. (1) and (3)

$$\frac{e_{l_p}}{-e_i}(D) = \frac{1}{\dfrac{D^2}{\omega_n^2} + \dfrac{2\zeta D}{\omega_n} + 1} \qquad \{\text{Eq. } 9.47\} \tag{4}$$

$$\frac{e_{h_p}}{e_{l_p}} = \frac{100R^2C^2D^2}{V_c^2} \tag{5}$$

From Eqs. (1) and (5)

$$\frac{e_{h_p}}{e_{l_p}} = \frac{D^2}{\omega_n^2} \tag{6}$$

From Eqs. (4) and (6)

$$\frac{e_{h_p}}{-e_i}(D) = \frac{\dfrac{D^2}{\omega_n^2}}{\dfrac{D^2}{\omega_n^2} + \dfrac{2\zeta D}{\omega_n} + 1} \qquad \{\text{Eq. } 9.48\} \tag{7}$$

$$\frac{e_{b_p}}{-e_i}(D) = \frac{2\zeta 10RCD}{V_c} \tag{7}$$

From Eqs. (4) and (7)

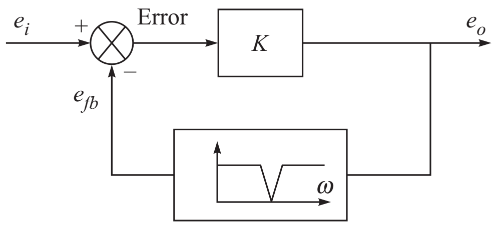
$$\frac{e_{b_p}}{-e_i}(D) = \frac{\dfrac{20\zeta RCD}{V_c}}{\dfrac{D^2}{\omega_n^2} + \dfrac{2\zeta D}{\omega_n} + 1} \qquad \{\text{Eq. } 9.49\} \tag{8}$$

using Eq. (1), Eq. (8) can be simplified as

$$\frac{e_{b_p}}{-e_i}(D) = \frac{\dfrac{2\zeta D}{\omega_n}}{\dfrac{D^2}{\omega_n^2} + \dfrac{2\zeta D}{\omega_n} + 1} \tag{9}$$

Note: Equations (2), (5) and (7) can be obtained from D.H. Sheingold (Ref. 23) page 582

9.13 Notch filter in the feedback path of a high-gain feedback system to construct a bandpass filter



Notch filter

If required, it can be mathematically proved that a notch filter in the feedback path can become a bandpass filter in the closed loop response: by assuming $G(s) = K$ as the forward transfer function and $H(s)$ for the notch filter and obtaining the closed-loop response function.

But here, we shall only give a conceptual explanation. The system configuration is as shown in Fig. 1. For a high gain K, the feedback system strives to keep the error near zero. When frequency ω is near the notch, it takes a large e_0 to cancel e_i, where as it takes a smaller e_0 to cancel e_i when the frequency is away from the notch. This behavior in the closed loop response resembles a bandpass filter in the band ω, same as that of the notch.

9.14 To derive Eq. 9.60.

$$\frac{e_0}{e_i}(D) = -\frac{Z_{fb}}{Z_i} = -\frac{R}{\dfrac{1}{CD}} = -RCD$$

The above equation is obtained from Fig. 9.28(a) Which is an inverting amplifier (op amp)

9.15 For Fig. 9.28(a)

$$e_i = e^i_{\text{signal}} + e^i_{\text{noise}}$$

$$e^i_{\text{signal}} = 10 \sin 20t \tag{1}$$

$$e^i_{\text{noise}} = 0.1 \sin 377t \tag{2}$$

$$\left|\frac{e^i_{\text{signal}}}{e^i_{\text{noise}}}\right| = 100 \text{ before differentiation}$$

$$e^0_{\text{signal}} = -RCD\,\{10 \sin 20t\}$$

$$= -RC\,\{200 \cos 20t\} \tag{3}$$

$$e^0_{\text{noise}} = -RCD\,\{0.1 \sin 377t\}$$
$$= -RC\,\{37.7 \sin 377t\} \tag{4}$$

$$\left|\frac{e^0_{\text{signal}}}{e^0_{\text{noise}}}\right| = \frac{200}{37.7} = 5.31 \tag{5}$$

Therefore, signal to noise ratio has decreased after differentiating the signal e_i

9.16

$$S_x(f) \text{ Spectral density in Hz} = 0.001\ \frac{V^2}{\text{Hz}}$$

$$S_x(\omega) \text{ spectral density in rad/s} = \frac{0.001}{2\pi}\ \frac{V^2}{\text{rad/s}}$$

$$S_x(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow S_y(\omega)$$

m.s. of
$$y(t) = \int_{-\infty}^{\infty} |H(\omega)|^2 S_x(\omega)\ d\omega \tag{1}$$

Where, $H(\omega)$ is the system transfer function for the present problem

$$|H(\omega)| = |D| = \omega \tag{2}$$

From Eqs. (1) and (2)

$$\text{m.s. of } y(t) = \int_0^{2\pi\times 10000} \frac{\omega^2 \times 0.001 d\omega}{2\pi} \tag{3}$$

$$= \frac{0.001}{2\pi}\,\frac{\omega^3}{3}\Bigg|_0^{2\pi\times 10000}$$

$$= 1.3159 \times 10^{10}\ (114715 \text{ rms})$$

$$\text{m.s. of } x(t) = \frac{0.001}{2\pi} \times 2\pi \times 10000 = 10\ (3.16 \text{ rms})$$

9.17 To derive Eq. (9.62)

As in Problem 9.14 and reference to Fig. 9.28C

$$\frac{e_0}{e_i}(D) = \frac{Z_{fb}}{Z_i} = \frac{\dfrac{R_2}{R_2 C_2 D + 1}}{R_1 + \dfrac{1}{C_1 D}} \tag{1}$$

$$\frac{e_0}{e_i}(D) = \frac{R_2 C_1 D}{(R_2 C_2 D + 1)(R_1 C_1 D + 1)} \qquad \text{\{Eq. (9.62)\}} \quad (2)$$

Equation (2) is a second-order type of low-pass filter

9.18 To derive Eq. (9.63)

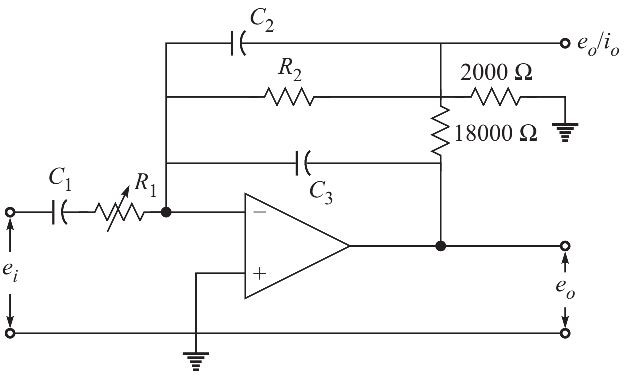


Fig. 1 Electronic differentiators {Fig. 9.28}

The above circuit shown in Fig. 1 is used for measuring the rate of charging or discharging of batteries. The output is read on a meter which may be either connected to e_0 or $\frac{e_0}{10}$, depending on the magnitude of output.

First we have to prove how the $\frac{e_0}{10}$ terminal is accomplished.

Figure 2 shows part of the circuit of Fig. 1. In this figure, due to high impedance of the feedback circuit elements, most of the current flows through the 18000Ω and 2000Ω resistors.

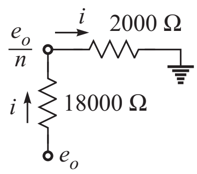


Fig. 2

Let the voltage at the junction of 18000 Ω and 2000 Ω resistors be

$$= \frac{e_0}{n} \qquad (1)$$

Current 'i' is given by

$$\frac{e_0}{18000 + 2000} = i \qquad (2)$$

Voltage drop across 2000Ω is given by

$$\frac{e_0}{n} = i2000 = \frac{e_0}{20000} \times 2000 \tag{3}$$

Therefore, $n = 10$

Feedback current is given by

$$\frac{\frac{e_0}{10} \times (R_2 C_2 D + 1)}{R_2} + e_0 C_3 D = i_{fb} \tag{4}$$

Also,

$$i_{fb} = \frac{e_i - 0}{R_1 + \frac{1}{C_1 D}} = \frac{e_i C_1 D}{R_1 C_1 D + 1} \tag{5}$$

From Eqs. (4) and (5)

$$e_0 \left\{ \frac{(R_2 C_2 D + 1)}{10 R_2} + C_3 D \right\} = e_i \left\{ \frac{C_1 D}{R_1 C_1 D + 1} \right\} \tag{6}$$

$$\frac{e_0}{e_i}(D) = \frac{10 R_2 C_1 D}{(R_1 C_1 D + 1)\,[10 R_2 C_3 D + R_2 C_2 D + 1]} \tag{7}$$

$$\frac{e_0}{e_i}(D) = \frac{10 R_2 C_1 D}{(R_1 C_1 D + 1)\,[1 + R_2\,(10 C_3 + C_2)D]} \tag{8}$$

9.19 Objective: using the system of Eq. (9.64) and e_i of Prob. 9.15, to compute the signal to noise ratio at both the input and output. From Prob. 9.15, the signal to noise ratio at the input is 100.

$$\frac{e_0}{e_i}(D) = \frac{24000D}{(0.44D + 1)\,(1.764D + 1)} \qquad \{\text{Eq. 9.64}\} \tag{1}$$

$$e^0_{\text{signal}} = \frac{10 \sin 20t \times 24000(j20)}{[0.44(j20) + 1]\,[1.764(j20) + 1]} \tag{2}$$

$$|e^0_{\text{signal}}| = 15350 \tag{3}$$

$$e^0_{\text{noise}} = \frac{0.1 \times \sin 377t \times 24000(j377)}{[0.44(j377) + 1]\,[1.764(j377) + 1]} \tag{4}$$

$$|e^0_{\text{noise}}| = 8.2$$

$$\left| \frac{e^0_{\text{signal}}}{e^0_{\text{noise}}} \right| = \frac{15350}{8.2} = 1872 \tag{5}$$

Although the signal to noise ratio is very high, the differentiator of Eq. (1) is not accurate for ω greater than 0.169 rad/s, which is suitable for measuring battery discharge rates for which it is meant to be used.

9.20 To derive Eq. 9.69

$$\frac{x_0}{Dp_s}(D) = \frac{K}{\tau D + 1} \qquad \text{\{Eq. 9.69\}} \quad (1)$$

p_s: Static pressure

p_c: Chamber pressure

$$p_c V = mRT \qquad (2)$$

V: Chamber volume

R: Gas constant

T: Temperature of the chamber °K

m: mass

$$\frac{dp_c}{dt} = \frac{RT}{V}\frac{dm}{dt} \qquad (3)$$

$$\frac{dm}{dt} = K_c(p_s - p_c) \qquad (4)$$

$$x_0 = K_d(p_s - p_c) \qquad (5)$$

x_0: motion of the diaphragm

From Eqs. (3) and (4)

$$\frac{dp_c}{dt} = \frac{RT}{V} K_c(p_s - p_c) \qquad (6)$$

$$\left(\frac{V}{K_c RT} D + 1\right) p_c = p_s \qquad (7)$$

Let

$$\tau = \frac{V}{K_c RT}\ s \qquad (8)$$

From Eqs. (7) and (8)

$$(\tau D + 1)p_c = p_s \qquad (9)$$

or

$$p_c = \left(\frac{p_s}{\tau D + 1}\right) \qquad (10)$$

From Eqs. (5) and (10)

$$x_0 = K_D\left\{p_s - \frac{p_s}{\tau D + 1}\right\} \qquad (11)$$

$$x_0 = K_D \frac{\tau D p_s}{\tau D + 1} \tag{12}$$

$$\frac{x_0}{K_D \tau D p_s} = \frac{1}{\tau D + 1} \tag{13}$$

$$K = K_D \tau$$

$$\frac{x_0}{D p_s}(D) = \frac{K}{\tau D + 1} \tag{14}$$

Equation (14) is same as Eq. (9.69)

9.21 Sensitivity/response-speed trade-off of Fig. 9.30

The governing equation for Fig. 9.30 is given by

$$\frac{x_0}{D p_s}(D) = \frac{K}{\tau D + 1} \qquad \{\text{Eq. } 9.69\} \quad (1)$$

$$K = \frac{K_d V}{RT K_c} \quad m/Pa/s \tag{2}$$

$$\tau = \frac{V}{RT K_c} \tag{3}$$

From Eqs. (2) and (3)

$$K = K_d \tau \tag{4}$$

If τ is small, speed of response is good but the sensitivity is small

If τ is large, speed of response is poor but the sensitivity is high

However, one can compromise between the above two extreme possibilities by choosing an appropriate value of K_d that gives both high sensitivity and good speed of response.

9.22 Objective: To derive the operating equation of the mechanical filter of Fig. 9.26

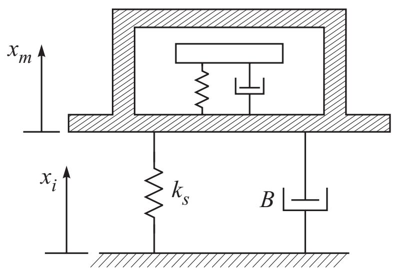


Fig. 1.

To get a simple but reasonably correct model, we can assume that the forces of the accelerometer's spring and damping on the housing are negligible relative to the forces of the rubber on the housing.

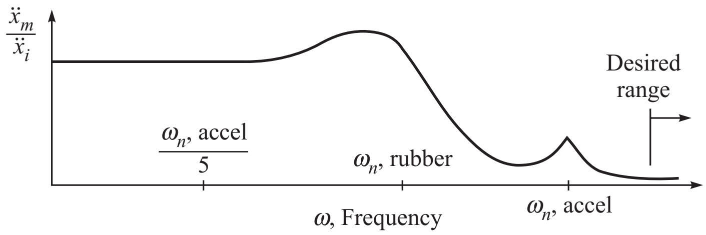
Modeling rubber as a lumped spring and damper

$$K_s(x_i - x_m) + B(Dx_i - Dx_m) = MD^2 x_m \tag{1}$$

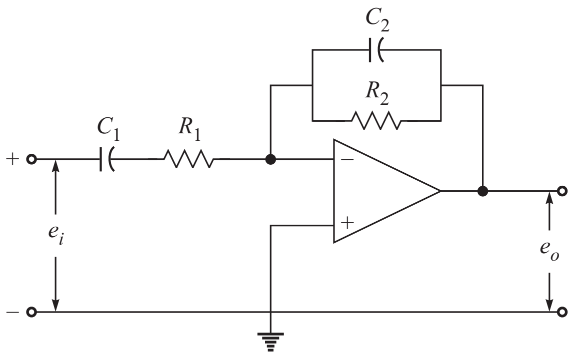
$$\frac{x_m}{x_i}(D) = \frac{BD + K_s}{MD^2 + BD + K_s} = \frac{\tau D + 1}{\frac{D^2}{\omega_n^2} + \frac{2\zeta D}{\omega_n} + 1} \tag{2}$$

Also
$$\frac{\ddot{x}_m}{\ddot{x}_i}(D) = \frac{x_m}{x_i}(D) \tag{3}$$

By properly choosing the rubber composition and shape, we can control the spring constant and damping effects to get the desired mechanical filtering. A conceptual Bode plot shows the desired frequency behavior.



9.23



(a) The amplifier shown in Fig. 1 is an inverting type of amplifier

$$\frac{e_0}{e_i}(D) = -\frac{Z_{fb}}{Z_i} \tag{1}$$

$$Z_{fb} = \frac{R_2/DC_2}{R_2 + \frac{1}{DC_2}} = \frac{R_2}{DR_2C_2 + 1} \tag{2}$$

$$Z_i = R_1 + \frac{1}{DC_1} = \frac{DR_1C_1 + 1}{DC_1} \tag{3}$$

From Eqs. (1), (2) and (3)

$$\frac{e_0}{e_i}(D) = -\frac{R_2 C_1 D}{(R_1 C_1 D + 1)(R_2 C_2 D + 1)} \tag{4}$$

(b) Using the values of part (c), Bode plot is drawn for Eq. (4) and compared against $1/j\omega R_2 C_1$, as shown in Fig. 2.

(c) Part (c) requires SIMULINK.