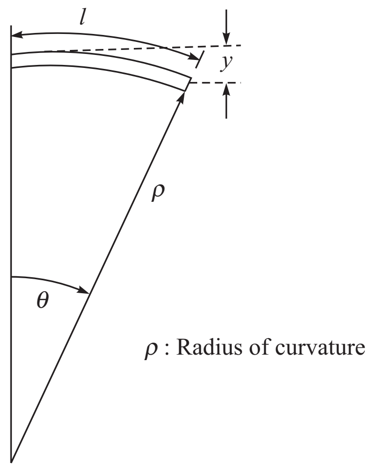
Solutions

8.1



Thickness t = 1 mm
Length l = 50 mm
Width b = 12 mm
ΔT = 30, 60°C
E_1: Young's modulus of invar = 152 GPa
E_2: Young's modulus of brass = 90 GPa
α_1: Coefficient of thermal expansion of invar = 1.7×10^{-6} m/m/°C
α_2: Coefficient of thermal expansion of brass 18.7×10^{-6} m/m/°C

Case (i)

$$\Delta T = 30°\text{C}$$

$$n = \frac{E_1}{E_2} = \frac{152}{90} = 1.688$$

$$n + \frac{1}{n} = 1.688 + \frac{1}{1.688} = 2.28 \approx 2$$

Therefore, Eq. (8.4) can be used

$$\rho = \frac{2t}{3(\alpha_2 - \alpha_1)\Delta T} = \frac{2 \times 1 \times 10^{-3}}{3(18.7 - 1.7) \times 10^{-6} \times 30} = 1.3072 \text{ m} \qquad (1)$$

ρ: Radius of curvature

$$\theta = \frac{l}{P} = \frac{50 \times 10^{-3}}{1.3072} = 0.038 \text{ rad}$$

$$y, \text{deflection} \approx \rho\,(1 - \cos\theta) \qquad (2)$$

$$y = 1.3072\,(1 - \cos 0.038)$$
$$= 0.956 \text{ mm}$$

$$I = \frac{bt^3}{12} \qquad (3)$$

$$I = \frac{12 \times 10^{-3} \times (1 \times 10^{-3})^3}{12} = 10^{-12} \text{ m}^4 \qquad (4)$$

For a cantilever

$$y = \frac{Fl^3}{3EI} \qquad (5)$$

Where 'F' is the force that can cause a deflection 'y'

$$F = \frac{3EIy}{l^3} = \frac{3 \times \left(\frac{E_1 + E_2}{2}\right) I y}{l^3} = \frac{3 \times (152 + 90) \times 10^9 \times 10^{-12} \times 0.956 \times 10^{-3}}{2 \times (50 \times 10^{-3})^3} \qquad (6)$$

$$F = 2.88 \text{ N}$$

Case (ii)

For a 60°C temperature rise

$$\rho = 0.6536 \text{ m} \qquad \text{(from Eq. (1))}$$

$$\theta = \frac{50 \times 10^{-3}}{0.6536} = 0.0765 \text{ rad}$$

$$y = 0.6536\,(1 - \cos 0.0765) \qquad \text{(Eq. 2)}$$
$$= 1.91 \text{ mm}$$

$$F = \frac{3 \times (152 + 90) \times 10^9 \times 10^{-12} \times 1.91 \times 10^{-3}}{2 \times (50 \times 10^{-3})^3} \qquad \text{(Eq. 6)}$$

$$= 5.55 \text{ N}$$

8.2

$$K = 250 \text{ mm/°C}$$

$$\frac{\text{Volume}}{\text{area}} = ?$$

α_V: Differential expansion
coefficient = 0.00016 m^3/m^3/°C

$$y = \frac{\Delta V}{A_C} = \frac{V \times \alpha_V \times \Delta T}{A_C} \tag{1}$$

y: Linear distance moved by the meniscus of the thermometer
ΔV: Change in volume
ΔT: Change in temperature
A_c: Area of cross section

$$K = \frac{y}{\Delta T} = 0.250 \text{ m/°C} = \frac{V \times 0.00016}{A_c} \tag{2}$$

$$\frac{V}{A_c} = \frac{0.250}{0.00016} = 1563 \text{ to obtain the above sensitivity of } 0.250 \text{ m/°C}$$

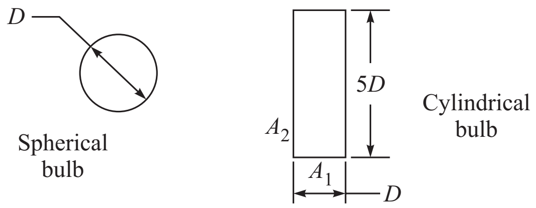


Fig. 2

$$\left.\begin{array}{l} V_s\text{: Volume of spherical bulb} \\ A_s\text{: Area of the spherical bulb} \end{array}\right\} \text{Spherical bulb}$$

$$\frac{V_s}{A_s} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6} \tag{3}$$

Cylindrical bulb
V_c: Volume of the cylindrical bulb
A_c: Area of the cylindrical bulb (curved + one end)

$$\frac{V_c}{A_c} = \frac{\frac{\pi D^2}{4} \times 5D}{5 \times \pi D \times D + \frac{\pi D^2}{4}} = \frac{5D}{21} \tag{4}$$

Assume $V_s = V_c = V$, so that we are comparing spherical and cylindrical bulbs of the same volume

$$\tau_1 = \frac{\rho V_c}{U A_s} \qquad \text{(Spherical bulb) (5)}$$

$$\tau_2 = \frac{\rho V_c}{U A_c} \qquad \text{(Cylindrical bulb) (6)}$$

C: Specific heat, U: heat transfer coefficient (same for both)

$$\frac{\tau_1}{\tau_2} = \frac{A_c}{A_s} = \frac{21}{30} = \frac{7}{10} \qquad (7)$$

From Eqs. (3) and (4) (Cylindrical bulb is slower)

8.3 Using a bimetallic approach to compensate a pressure thermometer has various limitations. In Fig. 8.6, the temperature labelled T_{cap} is a spacewise distribution of temperature along the capillary's length and it is difficult to find an equivalent bimetallic geometry to duplicate this action. A compromise would be to locate a "discrete" bimetallic device at some representative location to sense the average T_{cap}. The bimetallic compensator can be connected exactly the same way as the case compensator.

8.4

T_{gas}: Gas temperature °C
T_i: Inlet temperature to water tube°C
T_0: Outlet temperature °C
$\dot{m}$: Mass flow rate of water, kg/s.
C_p: Specific heat
From the energy balance

$$\left[T_{\text{gas}} - \left(\frac{T_i + T_0}{2}\right)\right] UA = \dot{m}\, C_p\, (T_0 - T_i) \qquad (1)$$

U: Convective heat transfer coefficient
A: Area of the heat exchanger
U could change for different operating conditions and A is uncertain.

T_0 and T_i can be obtained from thermocouples. If U is available for one of the flow rates, its value for other flow rates can be extrapolated.

$$T_{\text{gas}} = \frac{\dot{m} c_p (T_0 - T_i)}{UA} + \frac{(T_i + T_0)}{2UA} \qquad (2)$$

8.5 For a system to have dynamic response, there must be some energy storage in the system. The simplest energy storage model would be to assume heating of the metal tube and the water contained in the tube. In addition, it is assumed that the tube metal and water experience the same temperature rise dT_0.

$$\left[T_{\text{gas}} - \frac{(T_i + T_0)}{2}\right] UA - \dot{m}\, C_p\, (T_0 - T_i) = (M_m\, C_m + M_w\, C_w)\frac{dT_0}{dt} \qquad (1)$$

M_m: Mass of the metal tube
C_m: Specific heat of the metal tube
M_w: Mass of water contained in the tube
C_w: Specific heat of water

$$(\tau D + 1)\ T_0 = \frac{T_{\text{gas}}}{\dfrac{UA}{2} + \dot{m} C_w} - T_i \tag{2}$$

$$\tau = \frac{M_m C_m + M_w C_w}{\dfrac{UA}{2} + \dot{m} C_w} \tag{3}$$

By inserting and withdrawing a shutter interposed between the sensor and the gas flow source, the above system can be subjected to a step response. A linear increase in temperature on a $\log_e$ scale would confirm the first-order behavior

8.6 Steady-state relation between hot-gas temp as input and thermocouple voltage as an output for Fig. 8.13 (b)

Here we assume that the outer shell has perfect insulations, so that all the heat taken from the hot gas stream is absorbed by the cooling water. In addition to the gas thermocouple shown, we need sensors for the inlet and outlet water temperatures, water flow rate and gas flow rate

$$(T_{\text{gas}} - T_{\text{meas}})\, \dot{m}_{\text{gas}}\, C_{\text{gas}} = (T_{w_{\text{out}}} - T_{w_{\text{in}}})\ \dot{m}_w\, C_w$$

$\dot{m}_{\text{gas}}$: Gas flow rate, kg/s

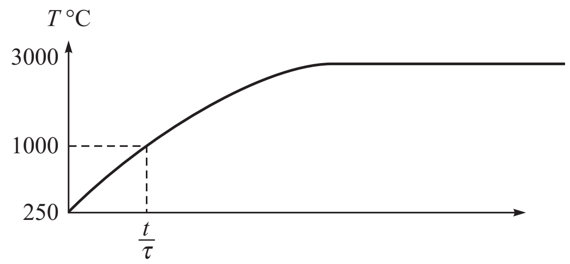
$\dot{m}_w$: Water flow rate, kg/s

C_{gas}: Specific heat of gas J/kg/°C
C_w: Specific heat of water J/kg/°C

$$T_{\text{gas}} = \frac{(T_{w_{\text{out}}} - T_{w_{\text{in}}})\, \dot{m}_w\, C_w}{\dot{m}_{\text{gas}}\, C_{\text{gas}}} + T_{\text{meas}}$$

T_{meas}: Temperature measured by the thermocouple

8.7 In Fig. 8.14, $\tau = 2.8$ s $T_{\text{gas}} = 3000$°C and the thermocouple damage limit is 1000°C. How long can the cooling be left off if the steady-state cooled thermocouple temperature is 250°C?



$$\frac{1000-250}{3000-250} = 1 - e^{-t/\tau} \qquad (1)$$

$$0.2727 = 1 - e^{-t/\tau} \qquad (2)$$

$$-\,t/\tau = \ln(1 - 0.2727) \qquad (3)$$

$$t = 0.318 \times 2.8 \text{ s} \qquad (4)$$

$$= 0.89 \text{ s}$$

8.8

Fig. 8.21*d* $\quad R_1 = R_2 = 10000\ \Omega \quad R_4 = 1000\ \Omega$

Temperature range 0 – 400°C

Termometer 1000 Ω at 200°C

$$e_i = 20 \text{ V}$$

From Fig. 8.19, $\quad \dfrac{R_3}{R_0} = 1.15$ at 200°C

If $\quad R = 1000\ \Omega \quad R_0 = \dfrac{1000}{1.15} = 870\ \Omega$

Assuming linear relationship

$$R_3 = R_0\,(1 + a_1\,T) \qquad \{8.15\}\ (1)$$

$$R_3 = R_0 \text{ at } T = 0$$

$$R_3 = 1000\ \Omega \text{ at } T = 200°\text{C}$$

$$1000 = 870\,(1 + a_1 \times 200) \qquad (2)$$

$$a_1 = \frac{0.15}{200} \qquad (3)$$

From Eqs. (1) and (3)

$$R_3(T) = 870\left(1 + \frac{0.15}{200}\,T\right) \qquad (4)$$

$$e_0(t) = \left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)e_i \qquad (5)$$

Equation (5) is plotted on the next page

Table 1: Temperature versus bridge output of the resistance thermometer

Temperature Degree C	*Output voltage, Volts eq. (5)*
0	−0.22
20	−0.20
40	−0.17
60	−0.15
80	−0.13

Table 1: Cond.

100	−0.11
120	−0.09
140	−0.06
160	−0.04
180	−0.02
200	0.00
220	0.02
240	0.04
260	0.06
280	0.09
300	0.11
320	0.13
340	0.15
360	0.17
380	0.19
400	0.21

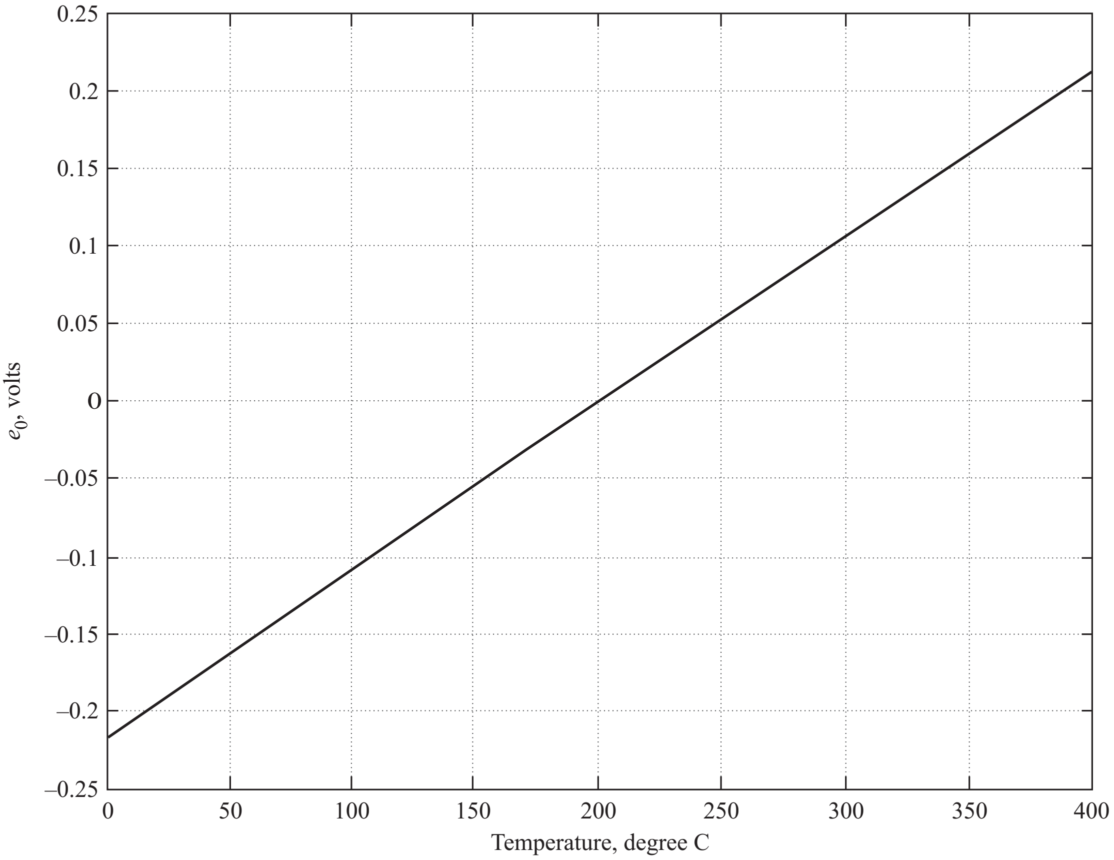


Fig. 1 Plot of temperature versus bridge output of the resistance thermometer using data of Table 1.

Note that the output voltage is zero at 200°C at which the bridge was balanced

8.9

$$\begin{aligned} R &= 500\ \Omega,\ \text{resistance} \\ I &= 5\ \text{m}A,\ \text{current} \\ A &= 3\ \text{cm}^2,\ \text{surface area} \\ U_{\text{air}} &= 9\ \text{W/m}^{2\circ}\text{C, convection heat transfer coefficient in air} \\ U_{\text{water}} &= 600\ \text{W/m}^{2\circ}\text{C, convection heat transfer coefficient in water} \end{aligned}$$

For air

electrical heat generation = heat transfer rate

$$\begin{aligned} I^2R &= U_{\text{air}}\, A\,\Delta T \\ (5\times10^{-3})^2\times 500 &= 9\times 3\times 10^{-4}\times\Delta T \\ \Delta T &= 4.62^\circ\text{C} \end{aligned}$$

For water

$$\begin{aligned} I^2R &= U_{\text{water}}\, A\,\Delta T \\ (5\times10^{-3})^2\times 500 &= 600\times 3\times 10^{-4}\Delta T \\ \Delta T &= 0.07^\circ\text{C} \end{aligned}$$

8.10

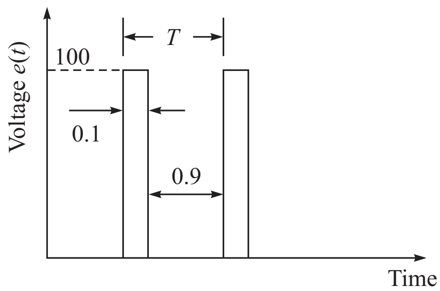


Fig. 1: Voltage versus time.

$$E_{\text{rms}} = \frac{\sqrt{\int_0^T e^2\,dt}}{T} \tag{1}$$

$$\begin{aligned} T &= 0.1 + 0.9 = 1\ \text{sec} \\ e &= 100 \end{aligned}$$

$$E_{\text{rms}} = \frac{\sqrt{100^2 \times 0.1}}{1} = 31.62 \text{ volts} \tag{2}$$

$$E_{\text{peak}} = 100 \text{ V}$$

$$\text{Peak to rms} = \frac{100}{31.62} = 3.16$$

Average heating power, $$W = \frac{E_{\text{rms}}^2}{R} \tag{3}$$

$$R = 500\ \Omega$$

$$W = \frac{31.62^2}{500} = 2 \text{ W} \tag{4}$$

8.11

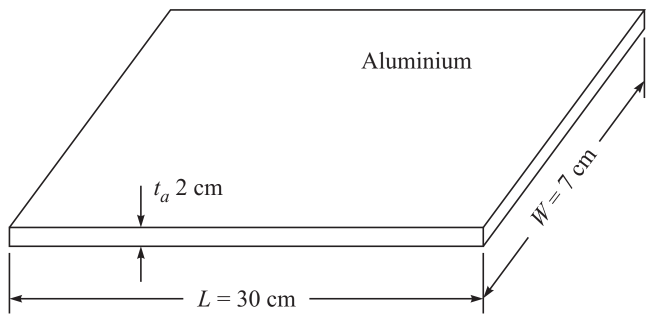


Fig. 1: Aluminium block

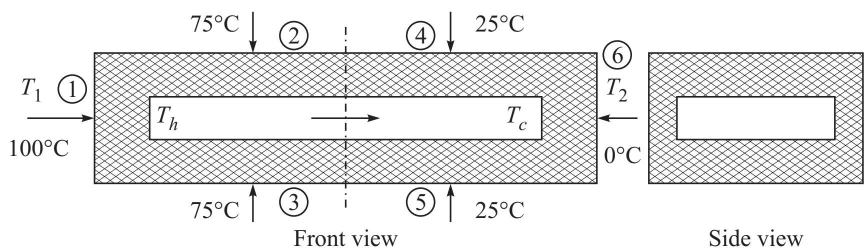


Fig. 2: Aluminium block surrounded by foam

Aluminium block of Fig. 1 is enclosed on all sides by a 2 cm thick foam, as shown in Fig. 2, one end is at 100°C and the other end is at 0°C. The purpose of the isothermal block is to minimize temperature gradient in the aluminium block. Heat flow into the foam is due to convection, and then conduction occurs through the foam and then along the length of alumin. block.

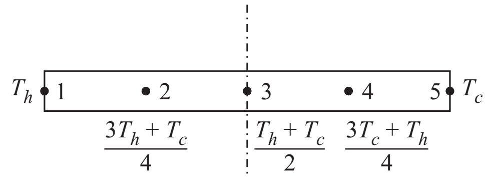


Fig. 3: Nodal temperatures of the alumin block

The temperature distribution along the alumin. block covered with foam is assumed as shown in figure 3. The aluminium block is divided into two parts at the centre (node 3) and conservation of energy for each lump gives T_h and T_c

k_a: Coefficient of thermal conductivity for aluminium = 170 W/m^2 – °C/m

k_f: Foam 0.026 W/m^2 – °C/m

h: Heat transfer coefficient 5.0 W/m^2 °C

a_1 = Area (1) of figure 2 = $t_a \times$ W = 0.0014 m^2

$a_6 = a_1$ = 0.0014 m^2

$$a_2 = \frac{L}{2}(\mathrm{W} + t_f) = \frac{0.30}{2}(0.07 + 0.02) = 0.0135 \text{ m}^2$$

$a_4 = a_2$ = 0.0135 m^2

$a_3 = a_2$ = 0.0135 m^2

$a_5 = a_2$ = 0.0135 m^2

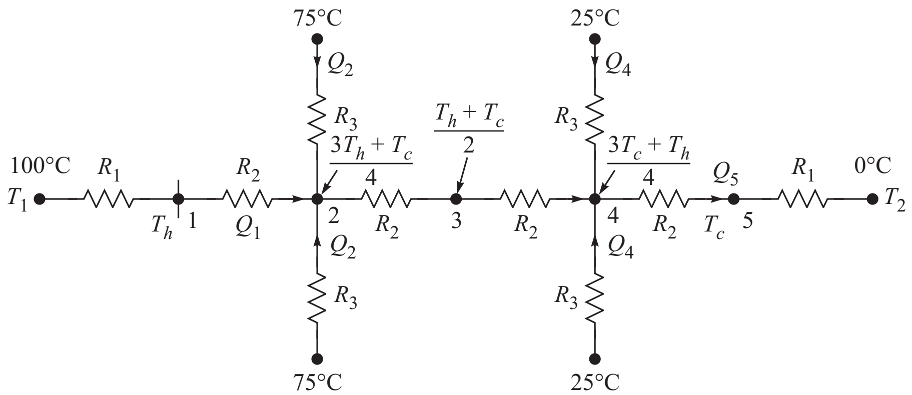


Fig. 4: Thermal resistance circuit of Fig. 2.

(Node numbers same as Fig. 3)

$$R_1 = \frac{1}{ha_1} + \frac{t_f}{k_f\, a_1} = \frac{1}{a_1}\left(\frac{1}{h} + \frac{t_f}{k_f}\right) \tag{1}$$

$$= \frac{1}{0.0014}\left(\frac{1}{5} + \frac{0.02}{0.026}\right) = 692.31$$

$$R_2 = \frac{L}{4}\frac{1}{k_a\,a_1} = \frac{0.30}{4\times170\times0.0014} = 0.315 \tag{2}$$

$$R_3 = \frac{1}{a_2}\left(\frac{1}{h}+\frac{t_f}{k_f}\right) = \frac{1}{0.0135}\left(\frac{1}{5}+\frac{0.02}{0.026}\right) = 71.8 \tag{3}$$

Let the heat flow rates be as shown in Fig. 4

$$Q_1 = \frac{T_1 - \dfrac{3T_h+T_c}{4}}{(R_1+R_2)} = \frac{4T_1 - 3T_h - T_c}{4(R_1+R_2)} \tag{4}$$

$$Q_2 = \frac{75 - \dfrac{3T_h+T_c}{4}}{R_3} = \frac{300 - 3T_h - T_c}{4R_3} \tag{5}$$

$$Q_3 = \frac{\dfrac{3T_h+T_c}{4} - \dfrac{T_h+T_c}{2}}{R_2} = \frac{T_h - T_c}{4R_2} \tag{6}$$

$$Q_1 + 2\,Q_2 = Q_3 \tag{7}$$

(from Fig. 4)

$$\frac{4T_1 - 3T_h - T_c}{4(R_1+R_2)} + \frac{300 - 3T_h - T_c}{2R_3} = \frac{T_h - T_c}{4R_2} \tag{8}$$

$$\left[\frac{1}{4R_2} + \frac{3}{4(R_1+R_2)} + \frac{3}{2R_3}\right]T_h + \left[-\frac{1}{4R_2} + \frac{1}{4(R_1+R_2)} + \frac{1}{2R_3}\right]T_c = \frac{100}{R_1+R_2} + \frac{150}{R_3} \tag{9}$$

From Fig. 4,

$$Q_3 + 2\,Q_4 = Q_5 \tag{10}$$

$$Q_3 = \frac{T_h - T_c}{4R_2} \quad \text{(from Eq. (6))} \tag{11}$$

$$Q_4 = \frac{25 - \dfrac{(3T_c+T_h)}{4}}{R_3} = \frac{100 - 3T_c - T_h}{4R_3} \tag{12}$$

$$Q_5 = \frac{\dfrac{3T_c+T_h}{4} - 0}{R_1+R_2} = \frac{3T_c+T_h}{4(R_1+R_2)} \tag{13}$$

From Eqs. (10) through (13)

$$\frac{T_h - T_c}{4R_2} + \frac{100 - 3T_c - T_h}{2R_3} = \frac{3T_c + T_h}{4(R_1 + R_2)} \tag{14}$$

$$\left[\frac{1}{4R_2} - \frac{1}{2R_3} - \frac{1}{4(R_1 + R_2)}\right] T_h + \left[-\frac{1}{4R_2} - \frac{3}{2R_3} - \frac{3}{4(R_1 + R_2)}\right] T_c = \frac{-50}{R_3} \tag{15}$$

Solving the simultaneous equations represented by Eqs. (9) and (15)

$$\begin{Bmatrix} T_h \\ T_c \end{Bmatrix} = \begin{bmatrix} 0.8153 - 0.7860 \\ 0.7860 - 0.8153 \end{bmatrix}^{-1} \begin{Bmatrix} 2.2337 \\ -0.6964 \end{Bmatrix}$$

T_h = 50.4823°C
T_c = 49.5222
ΔT = 0.9601°C

$$\frac{\Delta T}{T} = \frac{0.9601}{100} = 0.0096°\text{C}/°\text{C}$$

The assumed temperature difference at the ends of 100°C is rarely encountered in practice therefore the design is satisfactory for small variations of temperature at the ends.

8.12 Objective: percentage of radiated power above 10 μm to the total power

$$W_\lambda = \frac{C_1}{\lambda^5 (e^{c_2/\lambda T} - 1)} \qquad \{\text{Eq. 8.19}\} \tag{1}$$

C_1 = 37413 W μm^4/cm^2
C_2 = 14388 μm·K
λ: Wavelength of radiation, μm
T: Absolute temperature of black body, K
W_λ = Hemispherical spectral radiant intensity W/cm^2 – μm
Given λ_1 = 10 μm Let λ_2 = 500 μm (upper limit of wavelength)
T = 400 K

$$W_{\lambda_1 - \lambda_2} = \int_{10}^{500} \frac{37413}{\lambda^5 (e^{14388/\lambda 400} - 1)} d\lambda \tag{2}$$

From numerical methods

$$W_{\lambda_1 - \lambda_2} = 0.0752 \text{ W/cm}^2 \tag{3}$$

Total radiated power at 400 K

$$W_t = 5.67 \times 10^{-12}\, T^4 \qquad \{\text{Eq. 8.21}\} \tag{4}$$

$$W_t = 5.67 \times 10^{-12} \times 400^4 = 0.1452 \text{ W/m}^2 \tag{5}$$

$$\text{Percentage of radiated power} = \frac{W_{\lambda_1 - \lambda_2}}{W_t} = \frac{0.0752}{0.1452} = 52\%$$

8.13 Objective: disadvantage of a large time constant in a thermal radiation detector using a chopper. A slow detector requires a slow chopping rate if we want the signal to reach the correct value in each chopping cycle. Slow chopper speed means that a slow low-pass filter must be used to reduce the ripple to tolerable values. The slow filter means that the overall instrument will have a slow response speed to changing temperatures.

8.14

$p_{\text{static}} = 690$ kPa
$p_{\text{stag}} = 900$ kPa
Recovery factor $r = 0.8$
$t_w = 38°\text{C}$
$t_r = 200°\text{C}$
Radius of probe. $r = 6$ mm, diameter $D = 12$ mm
Length of probe. $L = 0.3$ m
Thermal conductivity $k = 600$ W/m^2 °C
Surface convection coefficient h, 60 W/m^2 °C/m
Radiation effects are negligible

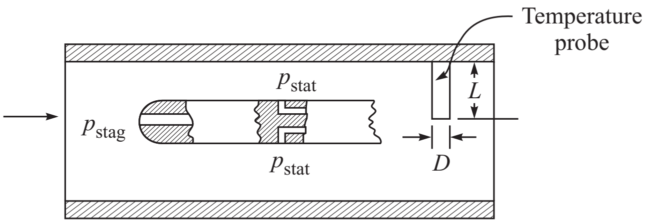


Fig. 1: Subsonic air flow in a duct.

$$\frac{p_{\text{stag}}}{p_{\text{stat}}} = \left[1 + \frac{k-1}{2}\left(\frac{V}{C}\right)^2\right]^{\frac{k}{k-1}} \qquad \text{\{Eq. 7.5\}} \quad (1)$$

$$M = \frac{V}{C} = \sqrt{\left[\left(\frac{p_{\text{stag}}}{p_{\text{stat}}}\right)^{\frac{k-1}{k}} - 1\right]\frac{2}{k-1}} \qquad (2)$$

M = Mach number
k = 1.4 for air

$$M = \sqrt{\left[\left(\frac{900}{690}\right)^{\frac{1.4-1}{1.4}} - 1\right]\frac{2}{1.4-1}} \qquad (3)$$

$M = 0.63$

The temperature probe. is modeled in the form of a fin.

$$mL = \sqrt{\frac{hcL}{kA}} \tag{4}$$

C = Circumference

A = Area, L = length

$$mL = \sqrt{\frac{60 \times \pi \times 0.012 \times 0.3}{600 \times \frac{\pi \times 0.012^2}{4}}} = 3.16 \tag{5}$$

$$T_r = T_f + \frac{(T_w - T_f)}{\cos h mL} \tag{6}$$

$$T_f = \frac{T_r - \frac{T_w}{\cos h mL}}{1 - \frac{1}{\cos h mL}} \tag{7}$$

$$T_f = \frac{(200 + 273) - \frac{(38 + 273)}{\cos h\, 3.16}}{1 - \frac{1}{\cos h\, 3.16}} = 488\ K = 215\ °\text{C} \tag{8}$$

$$T_{\text{stat}} = \frac{T_f}{1 + r\left(\frac{k-1}{2}\right)M^2} \tag{9}$$

r = recovery factor

$$T_{\text{stat}} = \frac{488}{1 + 0.8\left(\frac{1.4-1}{2}\right)0.63^2} = 459\ K \tag{10}$$

$$t_{\text{stat}} = 459 - 273 = 186\ °\text{C} \tag{11}$$

8.15

d = 0.75 mm

Copper/constantan thermocouple

Pressure p = 100 kPa (1)

Temperature t = 38 °C, T = 38 + 273 = 311 K (2)

Velocity, V = 30 m/s (3)

Objective: to estimate the time constant of the thermocouple

Density and specific heat values of constantan are hard to find. Therefore they are calculated based on percentage of copper (57%) and nickel (43%) in constantan

Density of copper $\rho_c = 9392 \text{ kg/m}^3$ (4)

Density of nickel $\rho_n = 9330 \text{ kg/m}^3$ (5)

Density of constantan $\rho_{cn} = 0.57\ P_c + 0.43\ P_n$ (6)

$$\rho_{cn} = 0.57 \times 9392 + 0.43 \times 9330$$

$$= 9365 \text{ kg/m}^3 \quad (7)$$

Average density of copper and constantan

$$\rho = \frac{P_c + P_{cn}}{2} = \frac{9392 + 9365}{2} = 9379 \text{ kg/m}^3 \quad (8)$$

Specific heat of copper $c_c = 382\ J\text{/kg/°C}$ (9)

Specific heat of nickel $c_{en} = 457\ J\text{/kg/°C}$ (10)

Specific heat of constantan, $c_{cn} = 0.57\ c_c + 0.43\ c_{en}$ (11)

$$= 0.57 \times 382 + 0.43 \times 457$$

$$c_{cn} = 414\ J\text{/kg/°C} \quad (12)$$

Average specific heat of copper constantan

$$c = \frac{c_c + c_{cn}}{2} = \frac{382 + 414}{2} \quad (13)$$

$$= 398\ J\text{/kg/°C}$$

R = Gas constant of air = 287 J/kg – K (14)

Density of air, $\rho_{\text{air}} = \dfrac{p}{RT} = \dfrac{100 \times 10^3}{287 \times 311} = 1.120 \text{ kg/m}^3$ (15)

$$C_1 = \text{Velocity of sound} = \sqrt{kRT} \quad (16)$$

k = 1.4 for air, ratio of specific heats

$$C_1 = \sqrt{1.4 \times 287 \times 311} = 353.5 \text{ m/s} \quad (17)$$

Mach number, $M = \dfrac{V}{c_1} = \dfrac{30}{353.5} = 0.085$ (18)

Flow mass velocity, $G = P_{\text{air}}\ V$ (19)

$$= 1.120 \times 30 = 33.6 \text{ kg/m}^2 - \text{s}$$

$$T_{\text{stag}} = T + \frac{(k-1)}{2} M^2 \quad (20)$$

T_{stag} = Stagnation temperature

$$T_{stas} = 311 + \frac{(1.4-1)}{2}\ 0.085^2$$
$$\approx 311\ K \tag{21}$$

Based on the above data, there are two equations that can be used to calculate the time constant of a thermocouple. Unfortunately, they have been excluded in this edition. The equations in the earlier edition are in British units. Since they are empirical in nature, they cannot be easily converted into SI units. Therefore, the above data are converted into British units as follows

$$\rho = \frac{9365}{16.799} = 558.29\ \text{lbm}/ft^3 \tag{22}$$

$$C = \frac{414 \times 0.456}{(1052 \times 1.8)} = 0.099\ \text{Btu/lbm °F} \tag{23}$$

$$d = 0.75 \times 10^{-3} \times 40 = 0.03'' \tag{24}$$

$$G = \frac{33.6 \times 0.3^2}{0.456} = 6.63\ \text{lbm}/ft^2 - \text{s} \tag{25}$$

$$T = \frac{311}{0.55} = 565.45\ \text{°R} \approx T_{stag}$$

$$\tau_1 = \frac{3500\,\rho\, c\, d^{1.25}\, G^{-15.8/\sqrt{T_{stag}}}}{T_{stag}}\ \text{sec} \tag{26}$$

{Eq. 8.91, Fifth edition 2004}

where ρ = Average density of two thermocouple materials, lbm/ft^3
C = Average specific heat of two thermocouple

materials, Btu/lbm °F

d = wire diameter, in
G = Flow mass velocity lbm/ft^2 – s
T_{stag} = Stagnation temperature R

From Eqs. (22) through (26)

$$\tau_1 = \frac{3500 \times 558.29 \times 0.099 \times 0.03^{1.25}\, 6.63^{\frac{-15.8}{\sqrt{565.45}}}}{565.45} \tag{27}$$
$$= 1.1724\ \text{s}$$

Another equation for M = 0.1 to 0.9, which is marginally applicable to this problem

$$\tau_2 = \frac{4.05\,\rho c d^{1.50}\left\{1 + \left(\frac{k-1}{2}\right) M^2\right\}^{0.25}}{p^{0.5}\, M^{0.5}\, T_{stag}^{0.18}} \tag{28}$$

{Eq. 8.92, fifth edition 2004}

ps: For Eq. (28), p is in atmospheres that is 1 for this problem

$$\tau_2 = \frac{4.05 \times 558.29 \times 0.099 \times 0.03^{1.50} \left\{1 + \left(\frac{1.4-1}{2}\right) 0.085^2\right\}^{0.25}}{1^{0.5} \times 0.085^{0.5} \times 565.45^{0.18}} \tag{29}$$

$\tau_2 = 1.2360$ s

τ_1 and τ_2 are close to each other

8.16 The text does not give complete temperature voltage tables for thermocouples, but we can estimate the needed K_e values from Fig. 8.10. We see in this figure that the sensitivities of copper/constantan and iron. Constantan thermocouples are actually close to each other. Thus K_e for various combinations will be nearly same.

The Gardon gage sensitivity, however, depends on the thermal conductivity k; small k gives larger sensitivity because the temperature difference between centre and edge (for a given heat flux) will be greater if k is smaller

$$k \alpha \; \frac{1}{\text{Sensitivity}} \tag{1}$$

The time constant, however, is given by

$$\tau \alpha \; \frac{\rho c}{k} \tag{2}$$

Therefore, a sensitive material will have a large time constant. Based on the values of P, C and k, the relative sensitivities and relative response speeds of iron, copper and constantan are presented in Table 1.

Table 1: Sensitivity and speed of response of materials used in Gardon gages

Material	*Density P, kg/m³*	*Specific heat, C J/kg°C*	*Thermal Conductivity W/m°C*	*Sensitivity*	*Speed of response*
Iron	7897	452	73	5.3	3.76
Copper	8890	392	386	1.0	20.41
Constantan	8897	414	19.5	19.8	1.0

Sensitivity $\propto \frac{1}{k}$. Therefore, it is normalized with respect to copper

$\tau \alpha \frac{Pc}{k}$. Therefore, speed of response is normalized with respect to constantan

8.17 Objective: Test setup for evaluating the step response of temperature sensors exposed to air flows of different velocities and temperatures

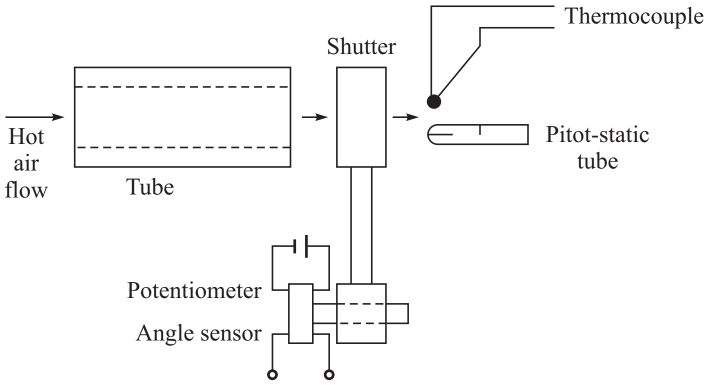


Fig. 1: Step function testing of temperature sensors

First, we need a source of air flow that can be adjusted over the range of velocities of interest. It can be sent to a pressure regulator and then to a small stagnation tank with a discharge nozzle. Else, various types of blowers and compressors can be used to generate the air flow. A hair dryer can also be used.

Second, a shutter mechanism is needed to suddenly apply the air flow as a step input on the temperature sensor. The shutter could be either manually or electrically activated. Potentiometer angle sensors are used to measure the speed of shutter closing.

Third, Pitot-static tube is used to measure velocities closer to the thermocouple.

Last, the thermocouple that can measure temperature of the flow

It is very difficult to manually apply step input. Very fine wire thermocouple is fast enough to actually show turbulent fluctuations in the flow, where as the fatter wires smooth these out to give a steady average value after the exponential step response.

8.18 As this problem is related to heat-flux sensors, this has been left out of this edition.

8.19 As this problem is related to heat-flux sensors, it has been left out of this edition

8.20 Linearized dynamic analysis of venturi pneumatic pyrometer

Let Q be the volume flow rate

The pressure drop for hot gases is given by

$$Q = K_Q \sqrt{\Delta p_h \rho_h} \qquad (1)$$

K_Q: Calibration factor

Δp_h: Pressure drop of hot gases

ρ_h: Density of hot gases

The corresponding equation for the cold side is given by

$$Q = K_Q \sqrt{\Delta p_c \rho_c} \qquad (2)$$

Because, both venturies carry the same mass flow rate.

Therefore, $$\frac{\Delta p_c}{\Delta p_h} = \frac{p_h}{p_c} \qquad (3)$$

Since the venturi pressure drops are designed to be very small, the absolute pressure stays nearly the same all along the tube. For a constant pressure p, density is given by

$$\rho = \frac{p}{RT} \tag{4}$$

Regarding dynamic response, consider a step change in inlet temperature T_h. This causes an instantaneous change in the inlet density and thus Δp_h. The differential pressure sensor can be modeled as a first order system. It results in the measured value of Δp_h, Δp_{hm}

The main function of this pyrometer is the heat exchanger, since the objective is to cool the hot gases to a level that can be measured using a temperature sensor. A reasonable model for the heat exchanger is given by

$$\frac{T_c}{T_h}(s) = K_T\, e^{-\tau_{dt}s} \tag{5}$$

The dead time τ_{dt} is the residence time of the hot gas in the tube, the time it takes for the fluid to move from the T_h location to the T_c location.

The cooled fluid results in the change of density to ρ_c that results in the differential pressure Δp_c for the cold gases. The temperature sensor can be modeled as a first-order system that results in T_{cm} (measured value of cooled gas) the differential pressure Δp_c can also be measured using a first-order system that results in Δp_{cm} (measured value of Δp_c).

Now we have the measured values of T_c, Δp_c, Δp_h, which are respectively T_{cm}, Δp_{cm} and Δp_{hm}. From Eq. 8.13, T_{hm} is given by

$$T_{hm} = KT_{cm}\frac{\Delta p_{hm}}{\Delta p_{cm}} \tag{6}$$

Thus we can measure the temperature of hot gases using the linearized dynamic model of the venturi pneumatic pyrometer

The linearized model is shown in Fig. 1

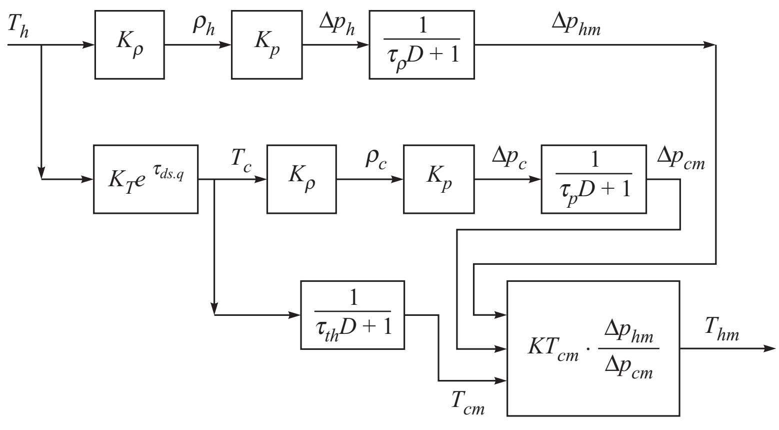


Fig. 1: Linearized model of the venturi pneumatic pyrometer

8.21 Junction semiconductor sensors

In the series connected devices, all three must be carrying the same current. This current will have to be the current generated by the coolest of three. Because, this the only situation where the other two devices are able to produce this same current at some available "supply" voltage. That is the total supply voltage is 15 volts to ground, but each device will have its own "supply" voltage, the voltage across its terminals. The sum of these 3 individual supply voltages, plus the iR drop through the resistor must always add up to 15 volts. To allow the two hotter devices to produce a current lower than "normal" for their temperature, the supply voltage on each must drop enough to get into the "non-horizontal" portion of the curves in Fig. 8.28 C. Thus the voltage drops across each device self-adjust to find this equilibrium condition

The circuit with the 3 devices in parallel is easier to explain. Each device produces a current in the "normal" way, and the 3 currents are summed and passed through the resistor. The sum of currents and thus the resistor voltage drop is proportional to the average current and thus the average temperature.

8.22 Objective: To derive Eq. (8.32)

Since the FET gate draws essentially no current, the circuit simplifies to a current source driving the parallel combination of C_S and R_L (Fig. 8.36 C)

Impedance of the capacitor is given by

$$Z_C = \frac{1}{c_s D} \tag{1}$$

Impedance of the resistor is given by

$$Z_R = R_L \tag{2}$$

Net impedance of the above two parallel elements

$$Z(D) = \frac{\frac{1}{c_s D} R_L}{\frac{1}{c_s D} + R_L} = \frac{R_L}{R_L C_s D + 1} \tag{3}$$

Voltage output e_g is given by

$$e_g = i_s \, Z(D) = i_s \left(\frac{R_L}{R_L C_s D + 1} \right) \tag{4}$$

$$i_s = K_P \frac{dT}{dt} \qquad \{8.28\} \; (5)$$

Where K_P is a sensitivity constant

By modeling the thermal system as a detector of thermal capacitance C_t connected to a constant temperature heat sink of thermal resistance R_t, absorbed radiant power as W walts, for small perturbations.

$$Wdt - \frac{T-0}{R_t} dt = C_t \, dT \qquad \{8.29\} \ (6)$$

$$(\tau_t \, D + 1) \; T = R_t \, W \qquad (7)$$

$$\tau_t = R_t \, C_t \qquad (8)$$

From Eqs. (5) and (7)

$$i_s = K_P D \left(\frac{R_t W}{\tau_t D + 1} \right) = - \frac{\dfrac{e_0}{R_f}}{R_f C_f \; D + 1} \qquad \{8.30\} \ (9)$$

$$\frac{e_0}{W} (D) = - \frac{K_e D}{(\tau_t D + 1)(\tau_e D + 1)} \qquad (10)$$

$$K_e = K_p \, R_t \, R_f \qquad (11)$$

$$\tau_e = R_f \, C_f \qquad (12)$$

$$\frac{i_s}{W} = \frac{K_p R_t D}{(\tau_t D + 1)} \qquad (13)$$

From Eqs. (4) and (13)

$$e_g = \frac{K_p R_t R_L D}{(R_L C_s D + 1)(\tau D + 1)} \qquad (14)$$