Chapter 7

Solutions

7.1

D : 25 mm diameter of pipe, $V = 3$ m/s
$d = 12$ mm diameter of pipe for the pitot tube

Q_1 : (without pitot-static tube) $= \dfrac{\pi}{4} \times D^2 \times V$ (1)

$$Q_1 = \frac{\pi}{4} \times (25 \times 10^{-3})^2 \times 3 = 1.473 \times 10^{-3} \text{ m}^3/\text{s}$$

If pitot tube is introduced

$$A_2 = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(25 \times 10^{-3})^2 - (12 \times 10^{-3})^2]$$
$$= 3.78 \times 10^{-4} \text{ m}^2$$
$$Q_2 = A_2 V_2; \qquad V_2 = 1.473 \times 10^{-3}/3.78 \times 10^{-4}$$
$$= 3.899 \text{ m/s will be indicated}$$

$$\text{Error} = \frac{3 - 3.899}{3} = -30\%$$

To reduce the error to 1%, the changed velocity must be 3.01 m/s with the pitot tube

$$A_3 = \frac{Q}{V_3} = \frac{1.473 \times 10^{-3}}{3.01} = 4.894 \times 10^{-4} \text{ m}^2$$

$$d_3 = \sqrt{\frac{4A_3}{\pi}} = \sqrt{\frac{4 \times 4.894 \times 10^{-4}}{\pi}} = 2.5 \text{ mm}$$

7.2 For the system of Fig. 7.14

$$F_d = 5\text{N}$$

$$v_f = 80 \text{ km/h} = \frac{80 \times 10^3}{3600} = 22.22 \text{ m/s}$$

$p = 100$ kPa $t = 20$ C, $\rho = 1.1774$ kg/m^3

$\delta = 1$ mm full-scale deflection

$k = ?$ (spring stiffness)

$t = 1.5$ mm thick aluminum, $\rho_{al} = 2600$ kg/m^3

$$F_d = \frac{C_d \, A \rho v_f^2}{2} \qquad (1) \ \{\text{eq. } (7.8)\}$$

F_d: drag force
C_d: drag coefficient = 0.567
A: Projected area
ρ: density
v_f: velocity

$$A = \frac{2 F_d}{C_d \rho v_f^2} = \frac{2 \times 5}{0.567 \times 1.1774 \times 22.22^2} \qquad (2)$$

$$= 0.0303 \text{ m}^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 0.0303}{\pi}} = 0.1964 \text{ m} \qquad (3)$$

$$k = \frac{F_d}{\delta} = \frac{5}{10^{-3}} = 5000 \text{ N/m}$$

$$r_2 = \text{Outer radius of sphere} = \frac{D}{2} = 0.0982 \text{ m}$$

$$r_1 = \text{Inner radius} = r_2 - t = 0.0982 - 0.015$$
$$= 0.0967 \text{ m}$$

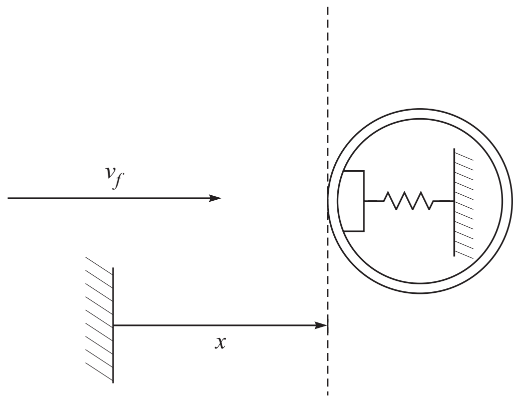
$$V = \frac{4}{3}\pi (r_2^3 - r_1^3) = \frac{4}{3}(0.0982^3 - 0.0967^3)\pi$$
$$= 1.79 \times 10^{-4} \text{ m}^3$$

$$\text{Mass } m = \rho V = 2600 \ 1.79 \times 10^{-4} = 0.465 \text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000}{0.465}} = 103.7 \text{ rad/s (16.5 Hz)}$$

Usable frequency range = 16.5 × 0.2 = 3.3 Hz

Damping



$$-kx + \frac{C_d A\rho}{2}\left(v_f - \frac{dx}{dt}\right)^2 = M\frac{d^2x}{dt^2} \tag{4}$$

{Drag force is always opposite to relative velocity}

$$M\frac{d^2x}{dt^2} + \frac{C_d A\rho}{2}\left(2v_f - \frac{dx}{dt}\right)\frac{dx}{dt} + kx = \frac{C_d A\rho}{2}v_f^2 \tag{5}$$

Linearizing Eq. (5) about v_{f0}

$$M\frac{d^2x}{dt^2} + C_d A\rho V_{f0}\frac{dx}{dt} + kx = \frac{C_d A\rho}{2}v_f^2 \tag{6}$$

$$\zeta = \frac{C_d A\rho v_{f0}}{2\sqrt{kM}} = \frac{0.567 \times 0.0303 \times 1.1774 \times 22.22}{2\sqrt{5000 \times 0.465}}$$

$$= 0.00466$$

7.3

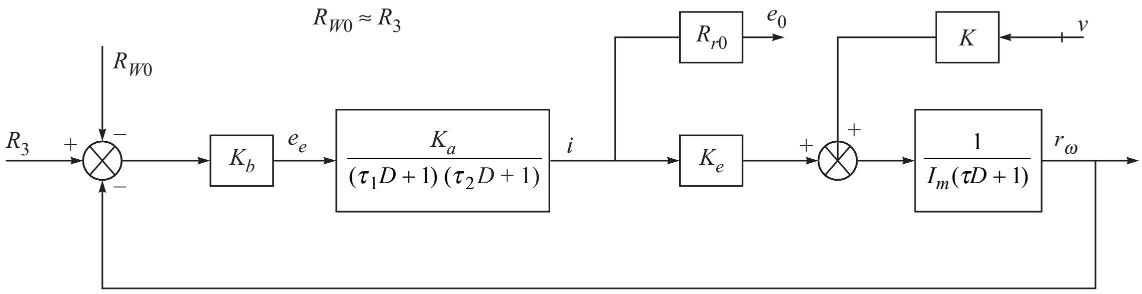


Fig. 1 (Fig. 7.19) Constant-temperature anemometer.

Staring from e_0 and v, the following transfer function is obtained in terms of the differential operator

$$-\left(\frac{e_0}{R_{r_0}} K_e + Kv\right)\left(\frac{1}{I_m(\tau D+1)}\right)\left[\frac{K_b K_a}{\tau_1 \tau_2 D^2 + (\tau_1+\tau_2)D+1}\right] = \frac{e_0}{R_{r_0}} \tag{1}$$

v: Fluid velocity

e_0: Output voltage

Since $R_3 \approx R_{w0}$, error signal is mainly from the feedback loop.

The characteristic equation of (1) is given by

$$\left[\tau_1\tau_2\tau D^3 + (\tau_1\tau_2 + \tau_1\tau + \tau_2\tau)D^2 + (\tau+\tau_1+\tau_2)D + 1 + \frac{K_e K_a K_b}{I_m}\right] e_0 = -\left(\frac{K K_a K_b R_{r_0}}{I_m}\right) v \tag{2}$$

$$\tau = 0.001\text{s} \qquad \tau_1 = \tau_2 = 0.000001$$

$$10^{-15} D^3 + 2.001 \times 10^{-9} D^2 + 1.002 \times 10^{-3} D + K = 0 \tag{3}$$

where $\quad K = 1 + K_{\text{loop}} \quad (4)$

Applying Rouths stability criterion to Eq. (3)

$$\begin{array}{ll} 10^{-15} & 1.002 \times 10^{-3} \\ 2.001 \times 10^{-9} & K \end{array}$$

$$\frac{(2.001) \times 10^{-9} \times 1.002 \times 10^{-3} - K \times 10^{-15})}{2.001 \times 10^{-9}} > 0$$

$$K = \frac{2.001 \times 10^{-9} \times 1.002 \times 10^{-3}}{10^{-15}} \tag{5}$$

$K = 2005$ or $K_{\text{loop}} = 2004$ is the margin of stability

$$K_{\text{loop}} = \frac{K_c K_b K_a}{I_m} \tag{6}$$

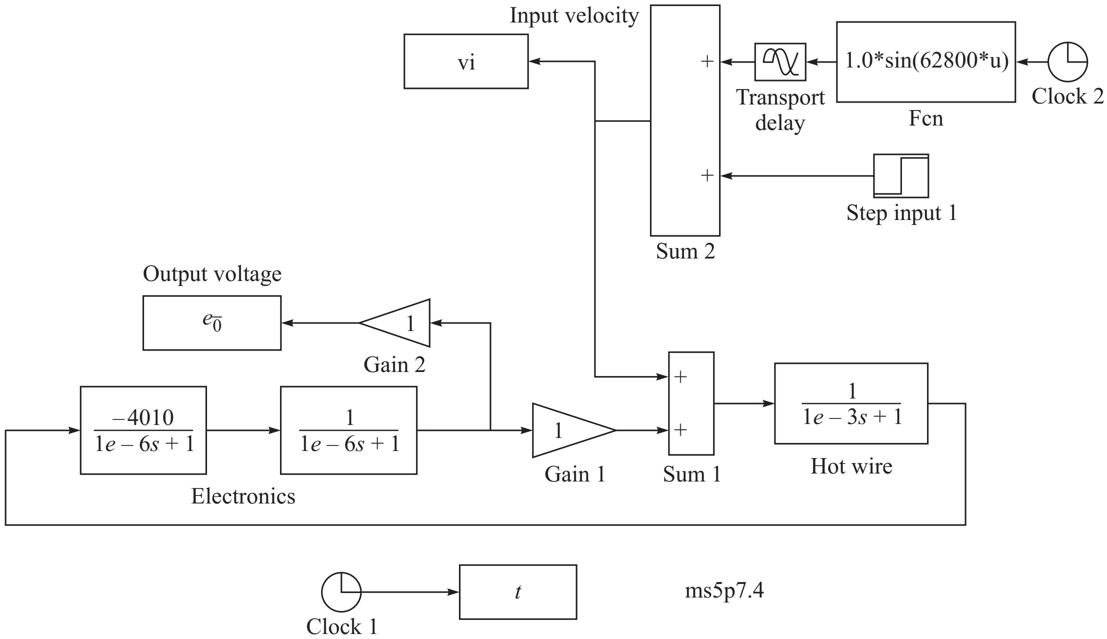
$$K_{\text{loop}} = \frac{1}{5} \times 2005$$

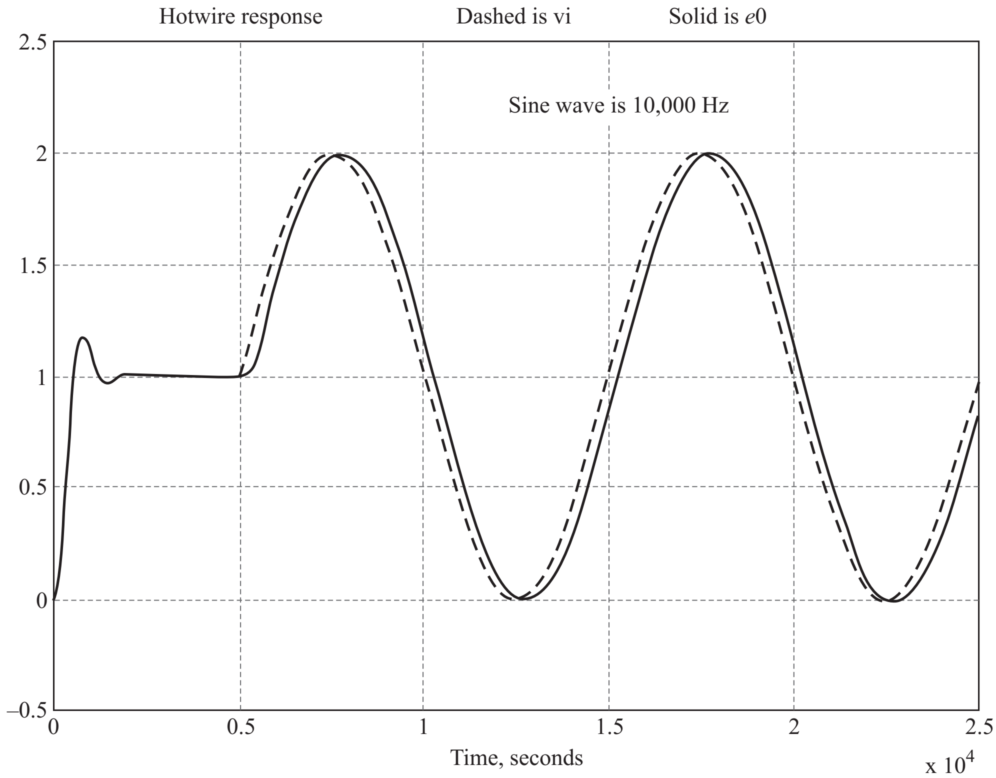
$$\tau_{ct} = \frac{\tau}{1 + \frac{1}{5} \times 2005} = \frac{0.001}{1 + \frac{1}{5} \times 2005} = 2.48 \times 10^{-6} \text{ s} \qquad (7.36)\ (7)$$

$$\%\text{ improvement} = 1 - \frac{\tau - \tau_{ct}}{\tau} = \frac{0.001 - 2.48 \times 10^{-6}}{0.001}$$

$$= 0.248\%$$

7.4 This problem can be easily solved using SIMULINK.


Input velocity
vi
+
+
Transport delay
1.0*sin(62800*u)
Fcn
Clock 2
Step input 1
Sum 2
Output voltage
e0
1
Gain 2
−4010
1e − 6s + 1
1
1e − 6s + 1
Electronics
1
Gain 1
Sum 1
1
1e − 3s + 1
Hot wire
t
Clock 1
ms5p7.4


Hotwire response
Dashed is vi
Solid is e0
Sine wave is 10,000 Hz
2.5
2
1.5
1
0.5
0
−0.5
0
0.5
1
1.5
2
2.5
Time, seconds
x 10^4

7.5 Equation for flow rate is given by

$$Q_t = \frac{A_2 f}{\sqrt{1-\left(\frac{A_2 f}{A_1 f}\right)^2}} \sqrt{\frac{2(p_1 - p_2)}{\rho}} \quad (1)\ \{\text{Eq. 7.40}\}$$

Change of temperature affects the orific diameter and the pipe diameter

Let
$$C_1 = \frac{A_2 f}{\sqrt{1-\left(\frac{A_2 f}{A_1 f}\right)^2}} \quad (2)$$

$$A_{2f} = \frac{\pi d_{2f}^2}{4} \qquad A_{1f} = \frac{\pi d_{1f}^2}{4}$$

d_{2f}: diameter of the orifice
d_{1f}: diameter of the pipe
$d'_{2f} = d_{2f}(1 + \alpha_2\ \Delta T)$, ΔT = change of temperature
$d'_{1f} = d_{1f}(1 + \alpha_1\ \Delta T)$

$$C_1 = \frac{(d_{2f}\ (1+\alpha_2\,\Delta T)^2}{\sqrt{1-\left\{\frac{d_{2f}\ (1+\alpha_2\,\Delta T)}{d_{1f}\ (1+\alpha_1\,\Delta T)}\right\}^4}} \quad (3)$$

For $d_{2f} = 25$ mm, $d_{1f} = 50$ mm
$\alpha_2 = 5.145 \times 10^{-6}$ mm/mm/°C
$\alpha_1 = 3.44 \times 10^{-6}$ mm/mm/°C

ΔT	$C_1 \times 10^6$
0	2581.988
5	2582.124
10	2582.260
15	2582.395
20	2582.531
25	2582.667
30	2582.802
35	2582.938
40	2583.074

For a 40°C change in temperature, change in flow rate will be by a factor of 1.00042

7.6 Q : flow rate = $\frac{0.2}{60} \times 10^{-6} = 3.33 \times 10^{-9}$ m^3/s

Δp : Pressure drop = 750 Pa
Re : Reynold's number = 500
ρ : Density of water = 1000 kg/m^3
μ : Viscosity of water = 0.001 Pa – s

$$\text{Re} = \frac{\rho D V}{\mu} \tag{1}$$

where D is the tube diameter

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \tag{2}$$

where V is the velocity

From Eqs. (1) and (2)

$$\text{Re} = \frac{4\rho Q}{\pi \mu D} \tag{3}$$

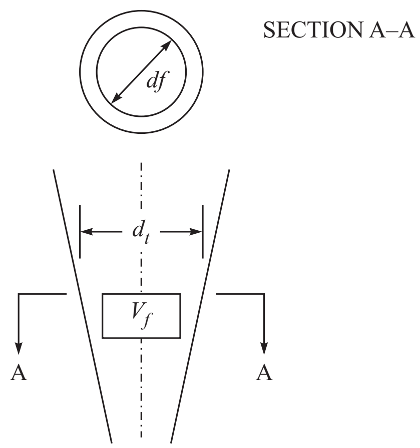
$$D = \frac{4\rho Q}{\text{Re}\,\pi\mu} = \frac{4\times 1000\times 3.33\times 10^{-9}}{500\times \pi \times 0.001} = 0.0085 \text{ mm}$$

The above diameter is ridiculously small and impractical. Therefore, assuming a reasonably small diameter, length is designed based on pressure drop. Let $D = 0.5$ mm, which results in a Reynold's number much less than 500.

$$L = \frac{\pi D^4 \Delta p}{128 \mu Q} = \frac{\pi \times (0.5\times 10^{-3})^4 \times 750}{128\times 0.001\times 3.33\times 10^{-9}} = 0.345 \text{ m} \qquad \text{(Eq. 7.52)}$$

The above length is reasonable.

7.7 For a rotameter, to derive Eq. (7.53) based on the simplified model.



d_f: Diameter of the float (constant)
d_t : Diameter of the tube (variable)

V_f : Volume of the float
ρ_f : Density of float
ρ_{ff} : Density of fluid
A_f : Area of float (constant) $\pi d_f^2/4$

A_t : Area of tube (variable) $\dfrac{\pi d_t^2}{4}$

Weight of the float-buoyant force = net pressure

$$V_f \rho_f g - V_f \rho_{ff} g = \Delta p A_f$$

$$\Delta p = \frac{g(V_f \rho_f - V_f \rho_{ff})}{A_f} \tag{1}$$

$$\Delta p = \frac{gV_f}{A_f}(\rho_f - \rho_{ff}) \tag{2}$$

Equation (7.40) for the orifice

$$Q_t = \frac{A_{2f}\, C_d}{\sqrt{1-\left(\dfrac{A_{2f}}{A_{rf}}\right)^2}} \sqrt{\frac{2\Delta p}{\rho_{ff}}} \tag{3}$$

A_{2f}: Orifice area
A_{1f}: Pipe area
C_d : Discharge coefficient

$$A_{2f} = A_t - A_f \tag{4}$$
$$A_{1f} = A_t \tag{5}$$

From Eqs. (2) to (5)

$$Q_t = \frac{C_d (A_t - A_f)}{\sqrt{1-\left(\dfrac{A_t - A_f}{A_t}\right)^2}} \sqrt{\frac{2gV_f}{A_f}\,\frac{\rho_f - \rho_{ff}}{\rho_{ff}}} \tag{6}$$

7.8 To derive the condition for minimizing error due to change of fluid density

$$\hat{m} = K_1(A_t - A_f)\sqrt{\rho_{ff}(\rho_f - \rho_{ff})} \tag{1}$$

$$\frac{\partial \dot{m}}{\partial P_{ff}} = \frac{1 K_1 (A_t - A_f)}{2\sqrt{\rho_{ff}(\rho_f - \rho_{ff})}}(\rho_f - 2\rho_{ff}) \tag{2}$$

ρ_{ff}: Density of fluid ρ_f: density of float

$$\frac{\partial \dot{m}}{\partial P_{ff}} = 0 \quad \Rightarrow \quad \rho_f = 2\rho_{ff} \tag{3}$$

Equation (3) means that if the density of float is twice the density of fluid, error due to its variation will be minimum.

To prove this point further, let $\rho_{ff}^{\,0}$ be the nominal density and ρ_{ff} density variation around this nominal value.

Let
$$\frac{\rho_{ff}}{P_{ff}^{0}} = x \tag{4}$$

and
$$\frac{\rho_{f}}{\rho_{ff}^{0}} = y \tag{5}$$

From Eqs. (1), (4) and (5)

$$\dot{m} = K_1\ (A_t - A_f)\ \rho_{ff}^{\,0} \sqrt{x(y-x)} \tag{6}$$

The ideal value of $\sqrt{x(y-x)}$ is 1. Therefore, error is given by

$$err = 1 - \sqrt{x(y-x)} \tag{7}$$

For various values of $x\{0.95 \text{ to } 1.05\}$ and $y\{1.4 \text{ to } 3\}$ Eq. (7) is plotted. The results are presented in Table 1. It can be clearly seen that for any density variation, when the density of float is twice that of the fluid, error is the least.

Table 1: Error due to density variations of the fluid for various densities of the float (Eq. 7)

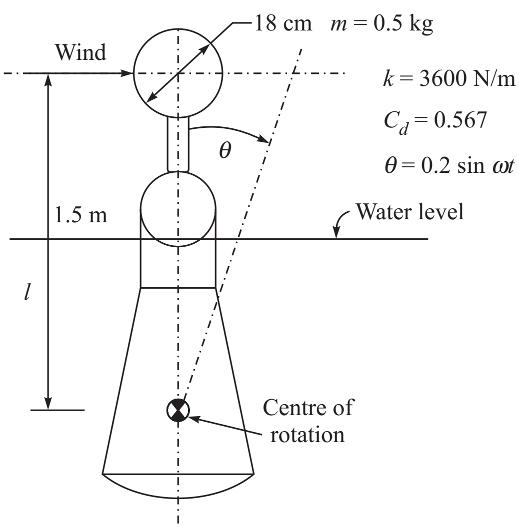
Ratio of density of the float to the density of fluid	Density variations of the fluid around the nominal value $x = P_{ff}/P_{ff}^{0}$ (eq. 4)											
	y\x	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05
	1.4	0.35	0.35	0.36	0.37	0.38	0.38	0.39	0.40	0.41	0.42	0.42
	1.6	0.21	0.22	0.22	0.23	0.23	0.24	0.25	0.25	0.26	0.27	0.27
	1.8	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.14	0.14	0.15
	2	0.01	0.01	0.00	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.04
	2.2	0.11	0.11	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.07	0.07
	2.4	0.20	0.20	0.20	0.19	0.19	0.18	0.18	0.17	0.17	0.17	0.16
	2.6	0.29	0.29	0.28	0.28	0.28	0.27	0.27	0.26	0.26	0.26	0.25
	2.8	0.37	0.37	0.37	0.36	0.36	0.35	0.35	0.35	0.34	0.34	0.34
	3	0.45	0.45	0.45	0.44	0.44	0.44	0.43	0.43	0.42	0.42	0.42

← Mini. error

↑

$y = \dfrac{P_f}{P_{ff}^{0}}$ eq. (5)

7.9



$$F_d = \frac{C_d A \rho v_f^2}{2} \qquad (1)\ \{\text{Eq. (7.8)}\}$$

F_d : Drag force
C_d : Drag coefficient (0.567 given)
A : Projected area
ρ : Density
v_f: Velocity

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.18^2}{4} = 0.02545 \text{ m}^2 \qquad (2)$$

$$\rho = 1.204 \text{ kg/m}^3 \text{ (density of air)}$$

$$F_d = \frac{0.567 \times 0.02545 \times 1.204\, v_f^2}{2} = 0.008687\ v_f^2 \qquad (3)$$

For $v_f = 10$ m/s (say)

$$F_d = 0.008687 \times 100 = 0.8687 \text{ N} \qquad (4)$$

v_r, velocity due to rocking

$$= l\dot{\theta} \qquad (5)$$

$$= 1.5 \times 0.2\ \omega \cos \omega t$$

If $\omega = 2$ rad/s

$$v_r = 1.5 \times 0.2 \times 2 \cos \omega t = 0.6 \cos 2t \qquad (6)$$

The gravity component along the x-axis

$$F_g^x = mg\,\theta = \text{mg } 0.2 \sin 2t = 0.5 \times 9.81 \times 0.2 \sin 2t$$

$$= 0.981 \sin 2t \qquad (7)$$

This is larger than the drag force

The force due to the angular acceleration

$$f_\alpha^x = ml\alpha = 0.5 \times 1.5\ (-\ 0.2 \times 4\ \sin 2t)$$
$$= -0.6\ \sin 2t \tag{8}$$

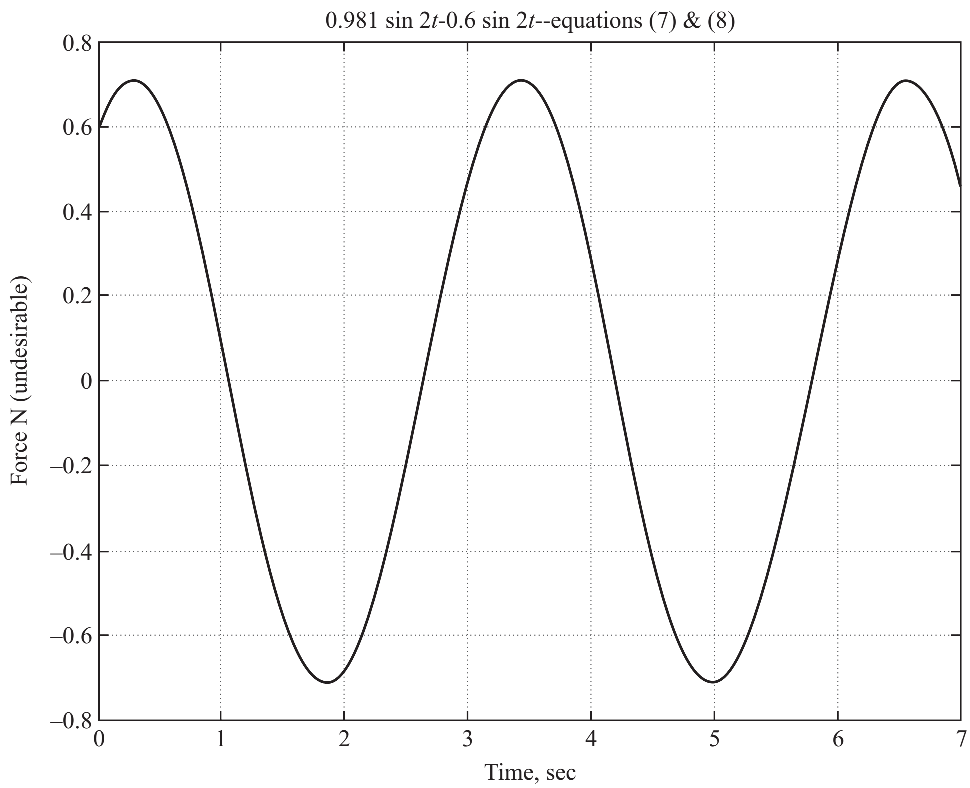
The rocking velocity represented by Eq. (6) is negligible for wind speeds of 10 m/s. Therefore, Eqs. (7) and (8) are important.

The three major sources of error in measuring wind velocity can be summarized as follows:

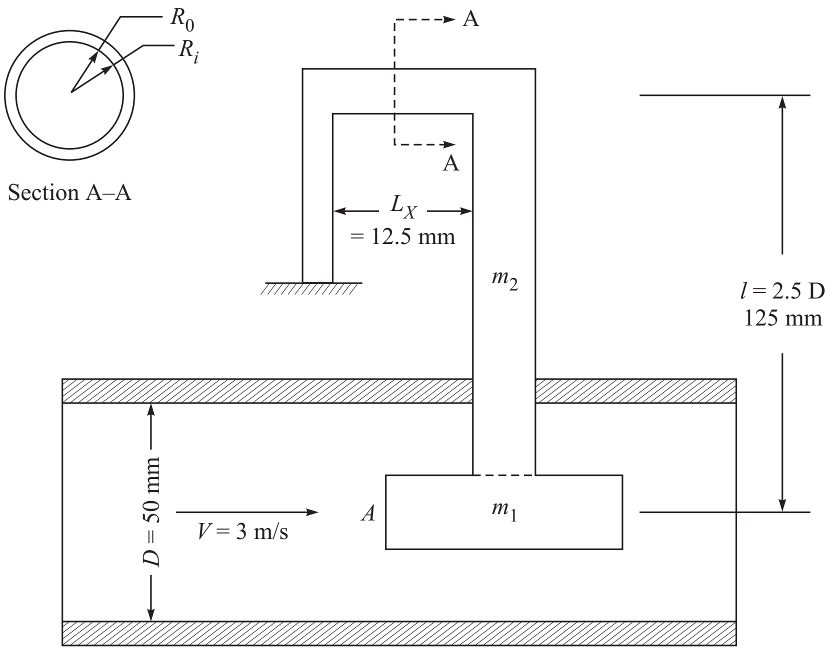
(1) Rocking causes a relative velocity of the still air with respect to the sensor, which cannot be distinguished from an actual wind
(2) Rocking causes a tilt angle which puts a component of the spheapostrophe weight along the force sensors sensitive axis
(3) Angular acceleration of the rocking causes a horizontal component of translation velocity and a reaction to this cannot be distinguished from a wind force.

$$F_d^{vr} = 0.008687 \times 0.6^2\ \cos^2 2t \tag{9}$$
$$= 0.003127\ \cos^2 2t$$

Only for wind velocities of 0.6 m/s, Eq. (9) will be comparable. Therefore, Eqs. (7) and (8) are plotted. The graph is presented as follows.



7.10



Assumptions

(1) Water flowing through a 50 mm diameter pipe
(2) Velocity V = 3 m/s
(3) $l = 2.5D = 125$ mm
(4) $L_X = 12.5$ mm
(5) $m_1 = 22$ gms
(6) $m_2 = 7$ gms
(7) Projected area of the drag $A = 625$ mm^2
(8) F_d : Drag force = 14 N
(9) E: Young's modulus of elasticity 70 GPa (alumin)
(10) ρ: Density of water 1000 kg/m^3
(11) σ: Stress = 120 MPa Corresponding to $\varepsilon = 1500$ µs

$$I = \frac{\pi}{4}(R_0^4 - R_i^4) \tag{1}$$

$$\sigma = \frac{M_c}{I} = \frac{F_d \times l R_0}{\frac{\pi}{4} R_0^4 (1 - k^4)} \tag{2}$$

where $$k = \frac{R_i}{R_0}$$

$$1 - k^4 = \frac{4 F_d l}{\pi \sigma R_0^3}$$

$$k = \left[1 - \frac{4 F_d l}{\pi \sigma R_0^3}\right]^{1/4} \tag{4}$$

Let $R_0 = 5$ mm

$$k = \left[1 - \frac{4 \times 4 \times 0.125}{\pi \times 120 \times 10^6 \times 0.005^3}\right]^{1/4} = 0.9606 \tag{5}$$

$$R_i = k R_0 = 0.9606 \times 5 = 4.803 \text{ mm}$$

$$t = R_0 - R_i = 5 - 4.803 = 0.197 \text{ mm}$$

$$K_s\text{: Stiffness} = \frac{M}{\Delta\theta} = \frac{1}{\int_0^{L_X} \frac{1}{EI} dx} = \frac{EI}{L_X} \tag{6}$$

$$I = \frac{\pi}{4}(R_0^4 - R_i^4) = \frac{\pi}{4}(0.005^4 - 004803^4) \tag{7}$$

$$= 7.2909 \times 10^{-11} \text{ m}^4$$

From Eqs. (6) and (7)

$$K_s = \frac{70 \times 10^9 \times 7.2909 \times 10^{-11}}{0.0125} = 408.3 \frac{N - m}{\text{rad}} \tag{8}$$

{Equivalent stiffness}

J: Rotary moment of inertia

$$= m_1 l^2 + m_2 \frac{l^2}{3}$$

$$= 0.022 \times 0.125^2 + \frac{0.007 \times 0.125^2}{3}$$

$$= 3.8 \times 10^{-4} \text{ kg} - m^2$$

$$f_n\text{: natural frequency} = \frac{1}{2\pi}\sqrt{\frac{K_s}{J}} \tag{9}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{408.3}{3.8 \times 10^{-4}}} = 165 \text{ Hz}$$

Drag coefficient

$$C_d = \frac{2F}{A\rho V^2} = \frac{2 \times 14}{625 \times 10^{-6} \times 1000 \times 3^2} \tag{10}$$

$$C_d = 4.97$$

7.11

Fluid resistance $$R_f \triangleq \frac{\text{Pressure drop}}{\text{Volume rate flow}} \tag{1}$$

Fluid inertance $$I_f \triangleq \frac{\text{Pressure drop}}{\text{Time rate of change of volume rate}} \tag{2}$$

Newton's law for the mass of fluid within the laminar flow element

$$(p_a - p_b)\, A - (Q\, R_f)\, A = I_f \frac{dQ}{dt} \tag{3}$$

For the pressure sensor model

$$\frac{p_1 - p_2}{(p_a - p_b)}(D) = \frac{1}{\tau_{\text{sens}} D + 1} \tag{4}$$

$$\frac{p_1 - p_2}{Q}(D) = \frac{R_f}{A} \frac{\left(\frac{I_f}{R_f} D + 1\right)}{(\tau_{\text{sens}} D + 1)} = \frac{K(\tau_{\text{lam}} D + 1)}{(\tau_{\text{sens}} D + 1)} \tag{5}$$

where $$K = \frac{R_f}{A} \tag{6}$$

$$\tau_{\text{lam}} = \frac{I_f}{R_f} \tag{7}$$

7.12

$$A_{2f} \text{ (orifice area)} = \frac{\pi \times 0.01^2}{4} = 7.854 \times 10^{-5} \text{ m}^2 \tag{1}$$

$$A_{1f} \text{ (pipe area)} = \frac{\pi \times 0.025^2}{4} = 4.909 \times 10^{-4} \text{ m}^2 \tag{2}$$

$$V = 5 \text{ m/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$Q = AV = 4.909 \times 10^{-4} \times 5 = 2.4545 \times 10^{-3} \text{ m}^3/\text{s} \tag{3}$$

$$Q = \frac{A_2 f}{\sqrt{1 - \left(\frac{A_2 f}{A_1 f}\right)^2}} \sqrt{\frac{2 \Delta p}{\rho}} \tag{4}$$

$$\frac{\frac{Q^2}{A_2 f^2}\left\{1 - \left(\frac{A_2 f}{A_1 f}\right)^2\right\}\rho}{2} = \Delta p \tag{5}$$

$$\Delta p = \frac{\left(\dfrac{2.4545\times10^{-3}}{7.854\times10^{-5}}\right)^2\left\{1-\left(\dfrac{7.854\times10^{-5}}{4.909\times10^{-4}}\right)^2\right\}\times1000}{2} \quad (6)$$

$$\Delta p = 476 \text{ kPa}$$

$$\Delta p = \rho_m\, g h \quad (7)$$

$$h = \frac{\Delta p}{\rho_m g} = \frac{476\times10^3}{13600\times9.81} = 3.57 \text{ m of mercury (unreasonable) height} \quad (8)$$

7.13

$$d_2 = 30 \text{ mm}$$

$$d_1 = 80 \text{ mm}$$

$$h = 0.3 \text{ m of mercury}$$

$$\rho_{\text{crude}} = 800 \text{ kg/m}^3 \quad \rho_m = 13600 \text{ kg/m}^3 \text{ (mercury)}$$

$$Q = ?$$

$$\Delta p = \rho_m\, g h \quad (1)$$

$$= 13600 \times 9.81 \times 0.3$$

$$\Delta p = 40 \text{ kPa}$$

$$Q = \frac{A_2}{\sqrt{1-\left(\dfrac{A_2}{A_1}\right)^2}}\sqrt{\frac{2\Delta p}{\rho_{\text{crude}}}} \quad (2)$$

$$A_2 = \frac{\pi\times0.03^2}{4} = 7.0686 \times 10^{-4} \text{ m}^2 \quad (3)$$

$$A_1 = \frac{\pi\times0.08^2}{4} = 0.005027 \text{ m}^2 \quad (4)$$

$$Q = \frac{7.0686\times10^{-4}}{\sqrt{1-\left(\dfrac{7.0686\times10^{-4}}{0.005027}\right)^2}}\times\sqrt{\frac{2\times40\times10^3}{800}} \quad (5)$$

$$= 0.0072 \text{ m}^3/\text{s}$$