Chapter 6

Solutions

6.1 Fig. 6.2 Deadweight gage calibrator

(a) Correction for air buoyancy

$$\rho_{\text{air}} = 1.23 \text{ kg/m}^3$$
$$\rho_{\text{steel}} = 7800 \text{ kg/m}^3$$

For a weight of volume V, the effective weight $W_e = V\,(\rho_{\text{steel}} - \rho_{\text{air}}) = (7800 - 1.23)\, V \times g$ (1)

By using a multiplication factor K, effective weight is given by

$$W_e = \rho_{\text{steel}} \times g \times V \times K = 7800\, K\, Vg \tag{2}$$

Equating (1) and (2)

$$K = \frac{\rho_{\text{steel}} - \rho_{\text{air}}}{\rho_{\text{steel}}} = \frac{7800 - 1.23}{7800} = 0.9984 \tag{3}$$

(b) Oil buoyancy correction

$$F_b = g \times V \times \rho_{\text{oil}} = \frac{\pi D^2}{4} \times L \times \rho_{\text{oil}} \times g \tag{4}$$

$$L = 0.125 \text{ m} \qquad D = 0.005 \text{ m}$$
$$\rho_{\text{oil}} = 840 \text{ kg/m}^3$$

$$F_b = \frac{9.81 \times \pi \times (5 \times 10^{-3})^2 \times 0.125 \times 840}{4},$$

$$F_b = 20 \text{ mN}$$

(c) If air is used in part (b) at 700 kPa at 20°C

$$\rho_{\text{air}} = \frac{\rho}{RT} = \frac{(700 + 100) \times 10^3}{287 \times (273 + 20)} = 9.5 \text{ kg/m}^3$$

$F_b = 0.22$ mN {By using $\rho_{\text{air}} = 9.5 \text{ kg/m}^3$ in equation (4)}

6.2

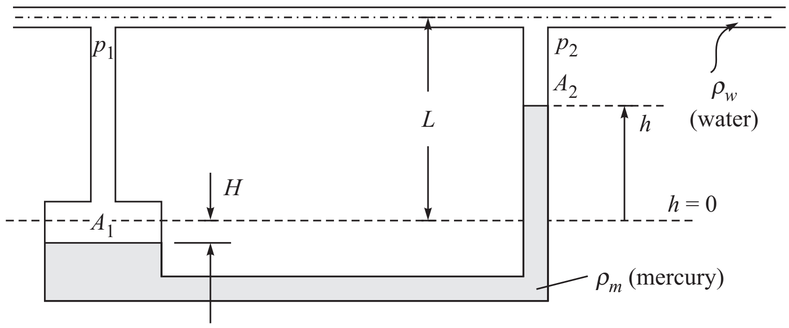

Fig. 1

$$p_1 + (L + H)g\ \rho_w = p_2 + (L - h)\ \rho_w g + (h + H)\rho_m \quad (1)$$

$$h\ A_2 = HA_1 \quad (2)$$

{Same volume has been displaced}

$$p_1 - p_2 = \left(1 + \frac{A_2}{A_1}\right)(\rho_m - \rho_w)gh \quad (2)$$

6.3

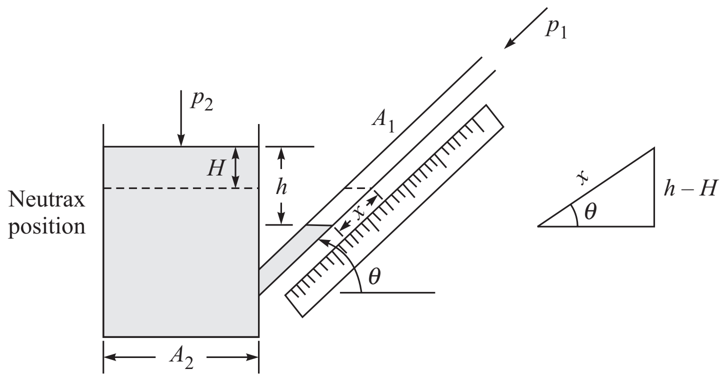

Fig. 1

$$\sin\theta = \frac{h - H}{x} \quad (1)$$

$$HA_2 = A_1 h \quad (2)$$

$$h = H + x\sin\theta$$

$$= h\ \frac{A_1}{A_2} + x\sin\theta \quad (3)$$

$$h\left\{1 - \frac{A_1}{A_2}\right\} = x\sin\theta \quad (4)$$

$$h = \frac{x\sin\theta}{1-\frac{A_1}{A_2}} \tag{5}$$

$$p_1 - p_2 = \rho g h = \frac{\rho g\, x\sin\theta}{1-\frac{A_1}{A_2}} \tag{6}$$

$$A_1 << A_2$$

6.4 This problem refers to manometer dynamics, which has been removed in this version. Hence it is not worked out here.

6.5

$$p = \frac{16Et^4}{3R^4(1-\upsilon^2)}\left[\left(\frac{y_c}{t}\right) + 0.488\left(\frac{y_c}{t}\right)^3\right] \tag{6.3}$$

$$0 \le \frac{y_c}{t} \le 1$$

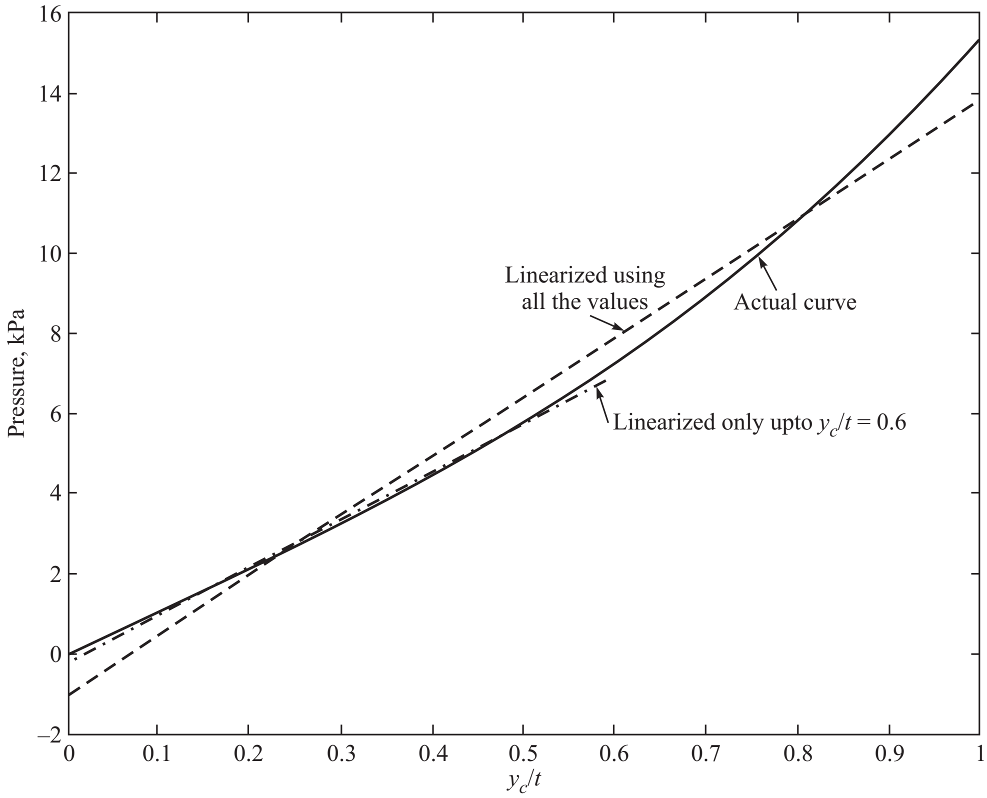

Fig. 1

E = 180 GPa (Young's modulus)
t = 1 mm
R = 100 mm
v = 0.26 (Poisson's ratio)

For various values of $\frac{y_c}{t}$, pressure p is computed. $\frac{y_c}{t}$ v_s pressure (kPa) is shown in Fig. 1

Due to the presence of cubic terms, there is a non-linearity in the pressure-deflection relationship, which is not desirable.

A best-fit line shows that the non-linearity is not severe, but still unacceptable.

A best-fit line choosing values only between $0 < \frac{y_c}{t} < 0.6$ shows that in this range, the linearity is acceptable but the pressure range is limited to 6 kpa.

6.6

$$p_{\max} = 700 \text{ kPa}$$

Figure 6.11

$$f_n = 10 \text{ Hz (minimum)}$$

Nonlinearity 3%

$$\text{full scale output} = 10 \text{ mV (minimum)}$$

Diaphragm material, stainless steel

Strain gage $R = 350\ \Omega$, gage factor = 2 size 8 mm & 8 mm

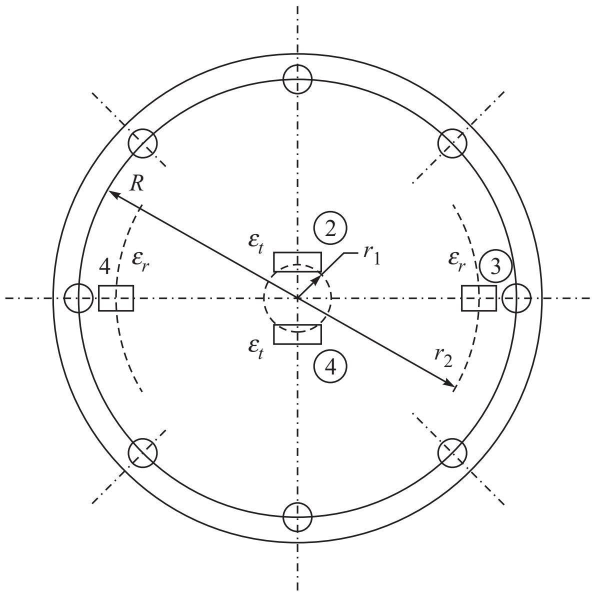

Fig. 1

The diaphragm is of radius R and thickness t.

There are four strain gages 1, 2, 3 and 4 mounted as shown in Fig. 1. Gages 2 and 4 that are near the center measure the tangential strain; gages 3 and 4 at the periphery measure the radial strain. Minimum value of r_1 is 4 mm and maximum value of r_2 is R – 4 mm, based on the strain gage size.

The diaphragm is designed based on the following considerations.

(1) Minimum natural frequency 10 Hz
(2) Maximum non-linearity < 3%
(3) Space requirements
(4) Minimum output voltage 10 mV
(5) Failure due to compressive load.

The maximum radius is assumed to be 75 mm and the maximum thickness is assumed to be 5 mm.

The thickness and radius are designed based on the above five considerations and a feasible region is established. That leaves the designer with enough flexibility to choose any design from the feasible region.

(1) Minimum natural frequency

From Eq. (6.8)

$$\omega_n = \frac{10.21}{R^2}\sqrt{\frac{Et^2}{12\rho_d(1-v^2)}} \tag{1}$$

$$E = 200 \text{ GPa}$$
$$\rho_d = 7800 \text{ kg/m}^3$$
$$v = 0.3 \text{ Poisson's ratio}$$
$$\omega_n = 2\pi 10 \text{ rad/s}$$

$$2\pi\, 10 = \frac{10.21}{R^2}\sqrt{\frac{200\times 10^9\, t^2}{12\times 7800\times(1-0.3^2)}} \tag{2}$$

$$\frac{t}{R^2} > 0.004 \tag{3}$$

Equation (3) is plotted for various values of R and the corresponding value of t. The value of t is so negligible that it coincides with the horizontal axis. So this is one of the easiest conditions that can be met with any design.

(2) Maximin non-linearity

From Eq. (6.3)

$$p = \frac{16Et^4}{3R^4(1-v^2)}\left\{\frac{y_c}{t} + 0.488\left(\frac{y_c}{t}\right)^3\right\} \tag{4}$$

Non-linearity is contributed by the cubic term.

Therefore, condition for non-linearity becomes

$$\frac{0.488\left(\frac{y_c}{t}\right)^3}{\frac{y_c}{t}} < 3\% \tag{5}$$

$$\left(\frac{y_c}{t}\right) < \sqrt{\frac{0.03}{0.488}} \tag{6}$$

$$\frac{y_c}{t} < 0.25 \tag{7}$$

Substituting Eq. (7) in Eq. (4), and using the maximum value of pressure

$$700 \times 10^3 = \frac{16 \times 200 \times 10^9 \times t^4}{3R^4(1 - 0.3^2)} \{0.25 + 0.488 \times 0.25^3\}$$

$$\frac{t}{R} = 0.0390 \tag{8}$$

For various values of R, Eq. (8) is plotted to determine the feasible region that ensures (maximum) non-linearity < 3%

(3) Space requirements

Since the strain gages need some space, one cannot place the strain gages very close to the center or periphery. Therefore, 15 mm radius at the center and 10 mm at the periphery are not used as feasible regions this is just a thumb rule based on strain gage dimensions.

(4) Minimum output voltage

Equation 6.4 a for radial stress is given by

$$\sigma_r = \frac{3pR^2}{8t^2} v \left[\left(\frac{1}{v} + 1\right) - \left(\frac{3}{v} + 1\right)\left(\frac{r}{R}\right)^2\right] \tag{9}$$

Equation 6.4 b for tangential stress is given by

$$\sigma_t = \frac{3pR^2 v}{8t^2} \left[\left(\frac{1}{v} + 1\right) - \left(\frac{1}{v} + 3\right)\left(\frac{r}{R}\right)^2\right] \tag{10}$$

$$\sigma_{r(r=r1\,(4\text{ mm}))} = 3.4125 \times 10^5 \left(\frac{R}{t}\right)^2 - \frac{13.86}{t^2} \tag{11}$$

$$\sigma_t^{r1} = 3.4125 \times 10^5 \left(\frac{R}{t}\right)^2 - \frac{7.98}{t^2} \tag{12}$$

$$\sigma_r^{r2}\ (r_2 = R - 4\ \text{mm}) = -\,525000\left(\frac{R}{t}\right)^2 + 6930\ \frac{R}{t^2} - \frac{13.86}{t^2} \tag{13}$$

$$\sigma_t^{r2} = -157500\left(\frac{R}{t}\right)^2 + 3.99 \times 10^3\ \frac{R}{t^2} - \frac{7.98}{t^2} \tag{14}$$

The above stresses expressed in Eqs. (11) through (14) will have to be converted to strains using biaxial stress strain relations

Equation (6.6) is given by

$$\varepsilon_r = \frac{\sigma_r - v\sigma_t}{E} \tag{15}$$

$$\varepsilon_t = \frac{\sigma_t - v\sigma_r}{E} \tag{16}$$

$$\varepsilon_r^{r2} = -\,2.388 \times 10^{-6}\left(\frac{R}{t}\right)^2 + 2.8665 \times 10^{-8}\ \frac{R}{t^2} - \frac{5.733\times 10^{-11}}{t^2} \tag{17}$$

$$\varepsilon_t^{r1} = 1.1944 \times 10^{-6}\left(\frac{R}{t}\right)^2 - \frac{1.911\times 10^{-11}}{t^2} \tag{18}$$

Let us assume a gage current (max) of 30 mA.

For 350 Ω gage resistance, $e_{ex} = 30 \times 10^{-3}(350 + 350) = 21\text{V}$

$$e_0 = \frac{r}{(1+r)^2}\ e_{ex} \times s_g \times p_1 \times \varepsilon_1 \tag{19}$$

$r = 1$ For equal gage resistances

s_g: Gage factor = 2

p_1: Bridge factor

ε_1: Strain of the reference gage.

$$p_1\varepsilon_1 = 2(\varepsilon_t^{r1} - \varepsilon_r^{r2})$$

$$e_0 = \frac{1}{4} \times 21 \times 2 \times 2\ (\varepsilon_t^{r1} - \varepsilon_r^{r2}) \tag{20}$$

$$e_0 = 7.5246 \times 10^{-5}\left(\frac{R}{t}\right)^2 - 6.0237 \times 10^{-6}\ \frac{R}{t^2} + \frac{1.2039\times 10^{-9}}{t^2} \tag{21}$$

$$e_0 > 10\ \text{mV} \tag{22}$$

Using Eqs. (21) and (22), t is obtained for various values of R.

(5) Failure due to compressive load

$$\sigma_{\text{design}} = 200\ \text{MPa}$$

Maximum normal stress at $r = R = 525000\ \dfrac{R^2}{t^2}$

$$200 \times 10^6 = 525000\ \frac{R^2}{t^2}$$

$$t = R\ \sqrt{\frac{525000}{200 \times 10^6}}$$

$$t = 0.05R \tag{23}$$

Equation (23) is plotted for various values of R and the coresponding values of t. Figure 2 gives the feasible region.

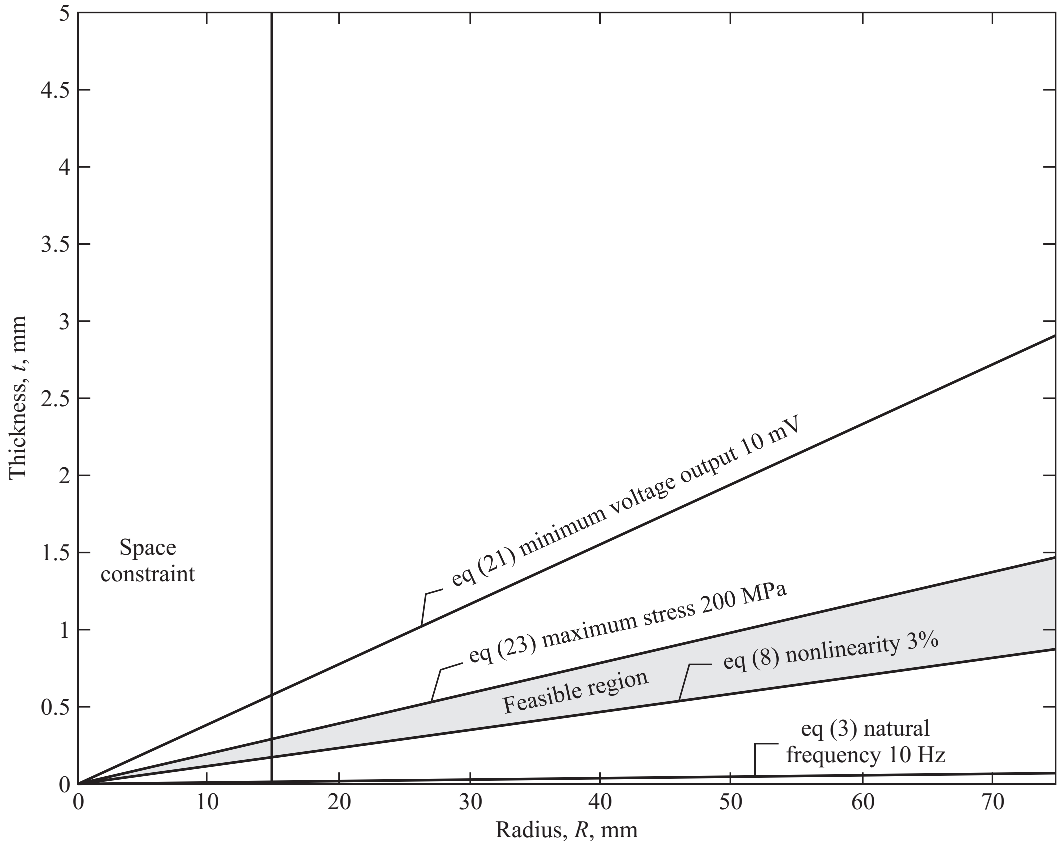

Fig. 2

6.7 Radius of the diaphragm $R = 75$ mm

Radius at which radial strain gages are fixed

$$r_r = 63 \text{ mm}$$

Radius at which tangential strain gages are fixed r_t = 13 mm

Young's modulus	E = 190 GPa
Poisson's ratio	ν = 0.26
Density of diaphragm material	ρ = 8350 kg/m^3
R: Gage resistance = 120 Ω	
Gage factor = 2.0	
Battery voltage	e_{ex} = 5V, t = 1.25 mm

(a) Sensitivity in mV/MPa
From Eq. (6.4a)

$$\sigma_{r_1}(r = 13\text{ mm}) = \frac{3p\times(75\times10^{-3})^2\times0.26}{8\times(1.25\times10^{-3})^2}\left[\frac{1}{0.26}+1-\left(\frac{3}{0.26}+1\right)\left(\frac{13}{75}\right)^2\right]$$

$$= 1.568\times10^3 p \tag{1}$$

From (6.4b)

$$\sigma_{t_1}(r = 13\text{ mm}) = \frac{3p\,(75\times10^{-3})^2\times0.26}{8\times(1.25\times10^{-3})^2}\left[\left(\frac{1}{0.26}+1\right)-\left(\frac{1}{0.26}+3\right)\left(\frac{13}{75}\right)^2\right]$$

$$= 1.6288\times10^3 p \tag{2}$$

From (6.4a)

$$\sigma_{r_2}(r = 63\text{ mm}) = \frac{3p\times(75\times10^{-3})^2\times0.26}{8\times(1.25\times10^{-3})^2}\left[\frac{1}{0.26}+1-\left(\frac{3}{0.26}+1\right)\left(\frac{63}{75}\right)^2\right]$$

$$= -1.4043\times10^3 p \tag{3}$$

From (6.4)(b)

$$\sigma_{t_2}(r = 63\text{ mm}) = \frac{3p\times(75\times10^{-3})^2\times0.26}{8\times(1.25\times10^{-3})^2}\left[\frac{1}{0.26}+1-\left(\frac{1}{0.26}+3\right)\left(\frac{63}{75}\right)^2\right]$$

$$= 5.4432p \tag{4}$$

From Eq. (6.6), tangential strain at $r = r_t$ = 13 mm

$$\varepsilon_t = \frac{p(1.6288\times10^3-0.26\times1.568\times10^3)}{190\times10^9} \tag{5}$$

$$\varepsilon_t = 6.43\times10^{-9}p\ \{\text{at } r = r_t = 13\text{ mm}\}$$

$$\varepsilon_r\ (r = r_r = 63\text{ mm})$$

$$= \frac{p(-1.4043\times10^3-0.26\times5.4432)}{190\times10^9} \tag{6}$$

$$\varepsilon_r = -7.40\times10^{-9}p$$

$$e_{ex} = 5\text{V}$$

$$e_0 = \frac{r}{(1+r)^2}\ e_{ex}\times S_g\times2\times(\varepsilon_t-\varepsilon_r) \tag{7}$$

$$S_g = \text{gage factor } r = 1 \text{ \{equal resistance\}}$$
$$= 2$$

$$e_0 = \frac{1}{4} \times 5 \times 2 \times 2[6.43 \times 10^{-9} - (-7.40 \times 10^{-9})]p$$

$$\frac{e_0}{p} = 6.915 \times 10^{-8} \text{ volts/pa} = 69.15 \frac{\text{mV}}{\text{MPa}}$$

(b) Natural frequency in vacuum
From Eq. (7-8)

$$\omega_n = \frac{10.21}{\text{R}^2}\sqrt{\frac{E t^2}{12\rho_d(1-v^2)}} = 10.21 \times \frac{(1.25\times10^{-3})}{(75\times10^{-3})^2}\sqrt{\frac{190\times10^9}{12\times8350\times(1-0.26^2)}} \tag{8}$$

$$\omega_n = 3235.60 \text{ rad/s} \quad (515 \text{ Hz})$$

(c) $p_{\max}$ for 2% non-linearity
From Eq. (5) of prob. 6.6

$$\frac{0.488\left(\frac{y_c}{t}\right)^3}{\frac{y_c}{t}} \le 2\% \tag{9}$$

$$\left(\frac{y_c}{t}\right) \le \sqrt{\frac{0.02}{0.488}} \tag{10}$$

$$\frac{y_c}{t} \le 0.202$$

From Eq. (6.3)

$$p_{\max} = \frac{16\times190\times10^9\times(1.25\times10^{-3})^4}{3\times(75\times10^{-3})^4(1-0.26^2)} \times \{0.202 + 0.488 \times 0.202^3\}$$

$$p_{\max} = 17.3 \text{ kPa}$$

6.8

L: Length 250 mm
d_i: diameter 8 mm
μ: viscosity 0.07 Pa-s
(a) First order system 95% accuracy within 0.05s

$$\tau_{\text{step}} = \frac{t_{95\%}}{3} = \frac{0.05}{3} = 0.017\text{s} \tag{1}$$

(b) Steady-state error 15 kPa for a ramp input 700 kpa/s

$$K = 700 \text{ kPa/s} \quad p_{ss_{error}} = 15 \text{ kPa}$$

$$\tau_{ramp} = \frac{p_{ss_{error}}}{K} = \frac{15\times 10^3}{700\times 10^3} = 0.0214\text{s} \tag{2}$$

(c) Sinusoidal input, accuracy = 90%
frequency f = 20 Hz

$$\frac{1}{\sqrt{(2\pi f \tau_{sine})^2+1}} = 0.9 \tag{3}$$

$$\tau_{sine} = \frac{1}{2\times\pi\times 20}\sqrt{\left(\frac{1}{0.9}\right)^2-1} \tag{4}$$

$$= 0.00385\text{s}$$

τ_{sine} is the smallest. Therefore $\tau = 0.00385$ must be used to satisfy all the conditions.

$$\tau = \frac{128\mu L c_{vp}}{\pi d_t^4} \tag{5}$$

C_{vp} = Compliance {volume change per unit pressure change}

$$C_{vp} = \frac{\tau\pi d_t^4}{128\mu L} = \frac{0.00385\times\pi\times(8\times 10^{-3})^4}{128\times 0.07\times 0.250} = 22 \text{ cm}^3\text{/MPa}$$

6.9 For a flush-diaphragm installation
ω_n for air and water

$$\rho_{air} = 1.23 \text{ kg/m}^3 \quad \rho_{water} = 1000 \text{ kg/m}^3$$
E = 200 GPa (Young's modulus of steel)

$$\omega = \frac{10.21}{CR^2}\sqrt{\frac{Et^2}{12\rho_d(1-\nu^2)}} \tag{1}$$

$$C = \sqrt{1+0.669\frac{\rho_f}{\rho_d}\frac{R}{t}} \tag{2}$$

$$\frac{\omega_{n_{air}}}{\omega_{n_{water}}} = \frac{C_{water}}{C_{air}} \tag{3}$$

$$C_{water} = \sqrt{1+\frac{0.669\times 1000}{7800}\left\{\frac{R}{t}\right\}} \tag{4}$$

$$\sqrt{1 + 0.085\frac{R}{t}} \tag{5}$$

$$C_{\text{air}} = \sqrt{1 + \frac{0.669 \times 1.23}{7800}\left\{\frac{R}{t}\right\}} \tag{6}$$

$$\sqrt{1 + 0.000105\frac{R}{t}} \tag{7}$$

$$\frac{\omega_{n_{\text{air}}}}{\omega_{n_{\text{water}}}} = \sqrt{\frac{1 + 0.085\frac{R}{t}}{1 + 0.000105\frac{R}{t}}} \tag{8}$$

Therefore, natural frequency when used with water is much less than used with air.

6.10 For the differential-pressure installation, expression for time constant of liquid-filled, heavily damped, slow acting system. The liquid is the same on both sides.

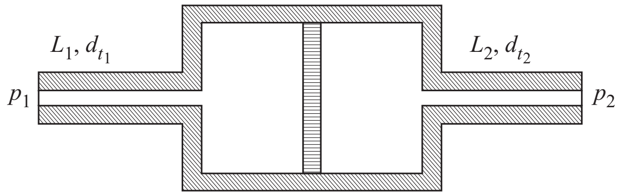

Fig. 1

$$p_1 - p_{m_1} = \frac{32\,\mu\,L_1\,\bar{v}_1}{d_{t_1}^2} \tag{1}$$

p_{m_1}: Dynamic pressure in system 1

$\bar{v}_1$: Average velocity in system 1

$$p_2 - p_{m_2} = \frac{32\,\mu\,L_2\,\bar{v}_2}{d_{t_2}^2} \tag{2}$$

Volume flow rate in system 1 $= \dfrac{dV_1}{dt} = \dfrac{\pi\,d_{t_1}^2}{4}\,\bar{v}_1$ (3)

Volume flow rate in system 2 $= \dfrac{dV_2}{dt} = \dfrac{\pi\,d_{t_2}^2}{4}\,\bar{v}_2$ (4)

Substituting average velocities in Eqs. (1) and (2) into Eqs. (3) and (4)

$$\frac{dV_1}{dt} = \frac{\pi d_{t_1}^4}{4} \frac{(p_1 - p_{m_1})}{32 \mu L_1} = \frac{\pi d_{t_1}^4 (p_1 - p_{m_1})}{128 \mu L_1} \tag{5}$$

$$\frac{dV_2}{dt} = \frac{\pi d_{t_2}^4 (p_2 - p_{m_2})}{128 \mu L_2} \tag{6}$$

Since
$$d_{p_m} = \frac{dV}{c_{vp}} \tag{7}$$

$$\frac{dV}{dt} = C_{vp} \frac{d}{dt} (p_{m_1} - p_{m_2}) \tag{8}$$

C_{vp}: Compliance
From Eqs. (5) and (8)

$$C_{vp} \frac{d}{dt} (p_{m_1} - p_{m_2}) = \frac{\pi d_{t_1}^4 (p_1 - p_{m_1})}{128 \mu L_1} \tag{9}$$

From Eqs. (6) and (8)

$$C_{vp} \frac{d}{dt} (p_{m_1} - p_{m_2}) = \frac{-\pi d_{t_2}^4 (p_2 - p_{m_2})}{128 \mu L_2} \tag{10}$$

From Eq. (9)

$$p_1 = \frac{128 \mu L_1 C_{vp} \dfrac{d}{dt} (p_{m_1} - p_{m_2})}{\pi d_{t_1}^4} + p_{m_1} \tag{11}$$

From Eq. (10)

$$p_2 = \frac{-128 \mu L_2 C_{vp} \dfrac{d}{dt} (p_{m_1} - p_{m_2})}{\pi d_{t_2}^4} + p_{m_2} \tag{12}$$

Subtracting Eqs. (11) and (12)

$$p_1 - p_2 = \frac{128 \mu C_{vp}}{\pi} \left\{ \frac{L_1}{d_{t_1}^4} + \frac{L_2}{d_{t_2}^4} \right\} \frac{d}{dt} (p_{m_1} - p_{m_2}) + p_{m_1} - p_{m_2} \tag{13}$$

$$\Delta p = \tau \frac{d}{dt} \Delta p_m + \Delta p_m \tag{14}$$

where
$$\tau = \frac{128 \mu c_{vp}}{\pi} \left\{ \frac{L_1}{d_{t_1}^4} + \frac{L_2}{d_{t_2}^4} \right\} \tag{15}$$

6.11 Figure *P* 6.2 fast acting model

For a single system, the equivalent mass is given by

$$M_e = \frac{\pi \rho L d_p^4}{3d_t^2} \tag{1}$$

d_p: Diameter of the piston

By applying equation (1) to a differential pressure system, similar to the previous problem,

$$M_e = \pi \rho d_p^4 \left[\frac{L_1}{3d_{t_1}^2} + \frac{L_2}{3d_{t_2}^2}\right] \tag{2}$$

The natural frequency of a single transducer-tubing system is given by

$$\omega_n = \sqrt{\frac{1}{\frac{1}{w_{n_1 t}^2} + \frac{16 \rho L C_{vp}}{3\pi d_t^2}}} \tag{3}$$

$\omega_{n_1 t}$: Transducer natural frequency.

The natural frequency of a differential transducer is given by

$$\omega_n = \sqrt{\frac{1}{\frac{1}{\omega_{n_1 t}^2} + \frac{16 \rho c_{vp}}{3\pi}\left(\frac{L_1}{d_{t_1}^2} + \frac{L_2}{d_{t_2}^2}\right)}} \tag{4}$$

6.12

$$\omega_{n,t} = 5000 \text{ Hz} \quad C_{vp} = 0.000713 \ \frac{cc}{\text{kPa}}$$

$$\rho = 1130 \text{ kg/m}^3 \quad \mu = 0.024$$

$$d_t = 5 \text{ mm}, L = 1.5 \text{ m}$$

$$\omega_n = \sqrt{\frac{1}{\frac{1}{(2\pi \times 5000)^2} + \frac{16 \times 1130 \times 0.000713 \times 10^{-6}}{3 \times \pi \times 1000} \times \frac{1.5}{(5 \times 10^{-3})^2}}} \tag{1}$$

$\omega_n = 17.34$ Hz (Natural frequency of the tubing)

$$\zeta = \frac{16\sqrt{\frac{3}{\pi}}\,\mu\sqrt{\frac{Lc_{vp}}{\rho}}}{d_t^3} \tag{2}$$

$$\frac{16\sqrt{\dfrac{3}{\pi}} \times 0.024 \times \sqrt{\dfrac{1.5 \times 0.000713 \times 10^{-9}}{1130}}}{(5 \times 10^{-3})^3} \tag{3}$$

ζ = 0.0912 (damping factor)

6.13 Internal volume of the pressure pickup V = 0.07 cm^3. Same as 6.12 except that air at 700 kPa and 38°C is used.

Tube volume $$V_t = \frac{\pi d_t^2}{4} \times L = \frac{\pi \times 0.5^2 \times 150}{4} = 29.45 \text{ cm}^3 \tag{1}$$

C: Speed of sound $= \sqrt{\gamma RT}$

$$= \sqrt{1.4 \times 287 \times (273 + 38)} \tag{2}$$

= 353.5 m/s

$$\omega_n = \frac{C}{L\sqrt{\dfrac{V}{V_t} + \dfrac{1}{2}}} = \frac{353.5}{1.5\sqrt{\dfrac{0.07}{29.45} + 0.5}} = 332.49 \text{ rad/s} \tag{3}$$

ω_n = 332.49/2/π = 53 Hz (natural frequency)

μ: Viscosity of air at 38°C = 1.86 × 10^{-5} N – s/m^2

ρ = 1.1774 kg/m^3

at 300 K

$$\zeta = \frac{16\mu L}{d_t^2\sqrt{\gamma p\rho}}\sqrt{\frac{1}{2} + \frac{V}{V_t}} \tag{4}$$

$$= \frac{16 \times 1.86 \times 10^{-5} \times 1.5}{(5 \times 10^{-3})^2\sqrt{1.4 \times 700 \times 10^3 \times 1.1774}} \times \sqrt{\frac{1}{2} + \frac{0.07}{29.45}} \tag{5}$$

= 0.0117 (Damping factor)

$$\lambda = \frac{C}{f} \quad \lambda\text{: Wavelength, m}$$

C: Velocity m/s

f: Hz

The wavelength at the natural frequency

$$\lambda = \frac{C}{f_n} = \frac{353.5}{53} = 6.66 \text{ m}$$

L = 1.5 m, tube length

$$\frac{\lambda}{L} = \frac{6.66}{1.5} = 4.44. \text{ Expected value of } \frac{\lambda}{L} = 10$$

$$\frac{\lambda}{L} = 4.44 \text{ can give results within 10\%.}$$

6.14

prob. 6.13

$$L = 25 \text{ mm} \quad d_t = 2.5 \text{ mm}$$

$$V = 0.07 \text{ cm}^3, \quad p = 700 \text{ kPa}, \quad t = 38°\text{C}$$

V_t: Tube volume

$$= \frac{\pi \times 0.25^2}{4} \times 2.5 = 0.1227 \text{ cm}^3$$

$$C = 353.5 \text{ m/s}$$

$$\omega_n = \frac{353.5}{0.025 \times \sqrt{\dfrac{0.07}{0.1227} + 0.5}} = 13666.5 \text{ rad/s}$$

$$f_n = \frac{13666.5}{2\pi} = 2175 \text{ Hz}$$

$$\zeta = \frac{16 \times 1.86 \times 10^{-5} \times 0.025}{\left[(2.5) \times 10^{-3}\right]^2 \times \sqrt{1.4 \times 700 \times 10^3 \times 1.1774}} \times \sqrt{\frac{1}{2} + \frac{0.07}{0.1227}}$$

$$= 0.0011 \text{ (Damping factor)}$$

$$\lambda = \frac{C}{f_n} = \frac{353.5}{2175} = 0.1625$$

$$\frac{\lambda}{L} = \frac{0.1625}{0.025} = 6.5 \text{ (Acceptable)}$$

6.15 Resistance change of 100Ω coils of manganin and gold chrome for 350 MPa pressure and 38°C temperature changes. {use the Table on page 357}

(1) Manganin

$$p_s = 2.45 \times 10^{-8} \; \Omega/\Omega/\text{kPa} = 24.5 \times 10^{-12} \; \Omega/\Omega/\text{Pa}$$

$$\Delta R_p = p_s \, \Delta p \times R = 24.5 \times 10^{-12} \times 350 \times 10^6 \times 100$$

$$= 0.8575 \; \Omega$$

$$t_s = 3 \times 10^{-6} \; \Omega/\Omega/°\text{C}$$

$$\Delta R_t = t_s \, \Delta_t \, R = 3 \times 10^{-6} \times 38 \times 100 = 0.0114 \; \Omega$$

$$\Delta R = \Delta R_p + \Delta R_t = 0.8575 + 0.0114 = 0.8689 \; \Omega$$

(2) Gold chrome

$$p_s = 9.76 \times 10^{-9} \; \Omega/\Omega/\text{kPa} = 9.76 \times 10^{-12} \; \Omega/\Omega/\text{Pa}$$

$$\Delta R_p = 9.76 \times 10^{-12} \times 350 \times 10^6 \times 100 = 0.3416\ \Omega$$
$$t_s = 1.44 \times 10^{-6}\ \Omega/\Omega/°C$$
$$\Delta R_t = 1.44 \times 10^{-6} \times 38 \times 100 = 0.005472\ \Omega$$
$$\Delta R = 0.3416 + 0.005472 = 0.3470\ \Omega$$

Error due to temperature changes is less than 2% in both cases

6.16 Design of capillary leak for a microphone

$$f_{\max} = 10 \text{ Hz amplitude error} = 10\% \text{ at } 10 \text{ Hz}$$
$$L = 25 \text{ mm} \quad d_t = ? \quad V = 8 \text{ cm}^3 \text{ (internal volume)}$$

For a condenser microphone

$$\frac{f_d}{\dfrac{p_i}{A_d}}(D) = \frac{\tau D}{\tau D + 1} \tag{1}$$

$$0.9 = \frac{\omega\tau}{\sqrt{(\omega\tau)^2 + 1}}$$

$$\omega\tau = \frac{1}{\sqrt{\left(\dfrac{1}{0.9}\right)^2 - 1}} = 2.06$$

$$\tau_{\max} = \frac{2.06}{2\pi f_{\max}} = \frac{2.06}{2\pi \times 10} = 0.03228\text{s}$$

$$\tau = \frac{128\,\mu L V}{\pi E_a d_t^4} \qquad (2)\ \{\text{Eq. } 6.16\}$$

μ = Viscosity, d_t: Diameter of tube
L: Length
V: Volume
E_a: Bulk modulus = kp {k(or γ)} ratio of specific heats
p, pressure

$$k = 1.4 \text{ (Ratio of specific heats)}$$
$$p = 100 \text{ kPa}$$
$$\mu = 1.86 \times 10^{-5} \text{ Pa–s}$$

$$0.0328 = \frac{128 \times 1.86 \times 10^{-5} \times 0.025 \times 8 \times 10^{-6}}{\pi \times 1.4 \times 100 \times 10^3 \times d_t^4} \tag{3}$$

$$d_t = 0.43 \text{ mm}$$

$$2 \text{ cycles/hr} = 3.49 \times 10^{-3} \text{ rad/s}$$

$$\text{Amplitude ratio} = \frac{3.49 \times 10^{-3} \times 0.0328}{\sqrt{(3.49 \times 10^{-3} \times 0.0328)^2 + 1}}$$

$$= 0.000114$$

Atmospheric pressure changes hardly affect the microphone performance.

6.17 Formula for natural frequency of a tubing made of two sections of different lengths and diameters

Let L_1 be the length and d_{t_1} diameter of the first section, and L_2, dt_2 corresponding values of the second section.

Equivalent
$$M_e = \left(\frac{L_1}{d_{t_1}^2} + \frac{L_2}{d_{t_2}^2}\right)\frac{\pi \rho d_p^4}{3} \quad (1)$$

$$\omega_n = \sqrt{\frac{1}{\frac{1}{\omega_{n_1 t}^2} + \frac{16\rho C_{vp}}{3\pi}\left(\frac{L_1}{d_{t_1}^2} + \frac{L_2}{d_{t_2}^2}\right)}} \quad (2)$$

$w_{n_1 t}$: Natural frequency of the transducer.

6.18 A gas bubble of volume V_g trapped at the transducer diaphragm. A gas bubble in the liquid contributes additional compliance, which adds directly to the compliance of the diaphragm. The gas compression process is modeled as adiabatic, since the frequencies involved are usually high enough to prevent much heat transfer from taking place

$$E_a = 1.4\text{p for air}$$

$$C_{vp_{\text{gas}}} = \frac{V_{\text{gas}}}{kp}$$

Since gases are so compressible, even small bubbles can have a big effect on system natural frequency.

6.19 The solutions for this problem will be posted later.

6.20

d_t (tube diameter) = 2 mm
V (transducer volume) = 10 cm^3
p (pressure) = 500 kPa L = 2 m
t (temperature) = 60°C
μ (viscosity) at 60°C 2.072×10^{-5} Pa–s
ρ (density) at 60°C   0.9980 kg/m^3

Tube volume
$$V_t = \frac{\pi d_t^2}{4} \times L = \frac{\pi \times 0.2^2 \times 200}{4} = 6.28 \text{ cm}^3$$

C: Speed of sound $= \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times (273 + 60)}$

365.78 m/s

$$\omega_n = \frac{C}{L\sqrt{\frac{V}{V_t} + \frac{1}{2}}} = \frac{365.78}{2\sqrt{\frac{10}{6.28} + \frac{1}{2}}} = 126.44 \text{ rad/s}$$

$$f_n = 20.12 \text{ Hz}$$

$$\zeta = \frac{16\mu L}{d_t^2\sqrt{rp\rho}}\sqrt{\frac{1}{2} + \frac{V}{V_t}} = \frac{16 \times 2.072 \times 10^{-5} \times 2}{(2 \times 10^{-3})^2\sqrt{1.4 \times 500 \times 10^3 \times 0.9980}} \times \sqrt{\frac{1}{2} + \frac{10}{6.28}}$$

$$= 0.286$$

6.21 Acoustic filter to attenuate sharp pressure transients of air above 30 Hz. Therefore, this is a low-pass filter of magnitude

$$M = \frac{1}{\sqrt{1 + (\omega\tau)^2}} \tag{1}$$

$$V = 0.7 \times 10^{-6} \text{ m}^3$$

$$\mu = 1.86 \times 10^{-5} \text{ Pa–s}$$

Given $M = 0.4$ at 30 Hz

$$0.4 = \frac{1}{\sqrt{1 + (2\pi)^2 30^2 \tau^2}} \tag{2}$$

$$\tau = \sqrt{\frac{\left[\left(\frac{1}{0.4}\right)^2 - 1\right]}{(2\pi 30)^2}} = 0.0122\text{s} \tag{3}$$

$$\tau = \frac{128\mu LV}{\pi E_a d_t^4} \tag{4}$$

{Eq. 6.16 see also Prob. 6.16}

$\{E_a = kp = 1.4 \times 100 \times 10^3\}$

$$\frac{L}{d_t^4} = \frac{\pi E_a \tau}{128\mu V} = \frac{\pi \times 1.4 \times 100 \times 10^3 \times 0.0122}{128 \times 1.86 \times 10^{-5} \times 0.7 \times 10^{-6}} \tag{5}$$

$\frac{L}{d_t^4} = 3.2197 \times 10^{12}$ will attenuate 40% at 30 Hz and much more at higher frequencies

For $L = 1$ m, $d_t = 0.75$ mm