Chapter 19

19.1:

$T = 4\text{s}$ $\qquad T = \dfrac{2\pi}{\omega}$

$A = 1$ m

$\Rightarrow \quad \omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$ rad/s

$\therefore \quad v_{\max} = A\omega = \dfrac{\pi}{2}$ m/s

$a_{\max} = A\omega^2 = \dfrac{\pi^2}{4}\text{m/s}^2$

19.2:

$\omega = 50$ rpm

$= \dfrac{50 \times 2\pi}{60} = \dfrac{5}{3}\pi$ rad/s = 5.24 rad/s

$a_{\max} = 20\text{m/s}^2$

$A\omega^2 = 20$

(i) $\Rightarrow \quad A = 0.73$ m

$T = \dfrac{2\pi}{\omega} = 1.2$ s

(ii) $v_{\text{extreme position}} = 0$

$v_{\text{mean position}} = V_{\max} = A\omega$

$= 3.83$ m/s

$v_{hdf} = \omega\sqrt{A^2 - x^2}$

$= \omega\sqrt{A^2 + \left(\dfrac{A}{2}\right)^2}$

$= \dfrac{\sqrt{3}}{2}\omega A$

$= 3.31$ m/s

(iii) $\dfrac{v_{\max}}{2} = \omega\sqrt{A^2 - x^2}$

$\dfrac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \quad x = 0.63$ m

19.3:

$$T = 2\text{s}$$

$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad \omega = \pi \text{ rad/s}$$

$$A = 25 \text{ cm}$$

$$v_{max} = A\omega = 25\pi \text{ cm/s}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$= \pi\sqrt{25^2 - 15^2}$$

$$= 20\ \pi \text{ cm/s}$$

19.4:

$$x = a\cos\ \omega t = b\sin\ \omega t$$

$$\frac{dx}{dt} = -a\omega\sin\ \omega t + b\omega\cos\ \omega t$$

$$\frac{d^2x}{dt^2} = -a\omega^2\cos\ \omega t - b\omega^2\sin\ \omega t$$

$$= -\omega^2\ [a\cos\ \omega t + b\sin\ \omega t]$$

$$= -\omega^2\ x$$

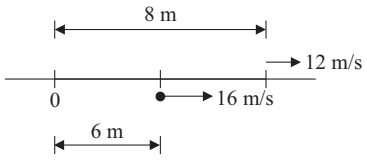
19.5:

$$V = \omega\sqrt{A^2 - x^2}$$

$$16 = \omega\sqrt{A^2 - 6^2}$$

$$12 = \omega\sqrt{A^2 - 8^2}$$

$$\frac{16}{12} = \frac{\sqrt{A^2 - 6^2}}{\sqrt{A^2 - 8^2}}$$



$$\Rightarrow \quad A = 10\text{m}$$

$$\therefore \quad \omega = 2 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \pi s$$

$$V_{max} = A\omega = 20 \text{ m/s}$$

$$A_{max} = A\omega^2 = 40 \text{ m/s}^2$$

19.6:

$$V_{max} = 6 \text{ m/s} = A\omega$$

$$a_{max} = 12 \text{ m/s}^2 = A\omega^2$$

$\Rightarrow \quad \omega = 2$ rad/s

(i) $T = \pi$ s

$A = 3$ m

(ii) $x = A \sin \omega t$

$= 3 \sin (2t)$

$x/_{t=2s} = -2.27$ m

$v/_{t=2s} = 6 \cos (2 \times 2) = -3.92$ m/s

$a/_{t=2s} = -12\sin (2 \times 2) = 9.08$ m/s^2

(iii) $V = -\omega\sqrt{A^2 - x^2}$

$= -2\sqrt{3^2 - (1.5)^2}$

$= -5.2$ m/s

$a = -\omega^2 x$

$= 6$ m/s^2

$t = \frac{1}{\omega} \sin^{-1}\left[\frac{x}{A}\right] = 0.262$s

19.7:

$\omega = \sqrt{\frac{a}{x}} = 5$ rad/s

$\therefore \quad T = 1.26$ s

19.8:

$x/_{t=2s} = 3.54$ m

$v/_{t=2s} = 10\ \pi \cos\left[2nt + \frac{\pi}{4}\right] = 22.2$ m/s

$a/_{t=2s} = -139.6$ m/s^2

$\omega = 2\ \pi$ rad/s

$T = 1$ s

19.9:

$x/_{t=1s} = -2\ m$

$v/_{t=1s} = -4\ \pi \sin\left(\pi t + \frac{\pi}{3}\right) = 10.9$ m/s

$a/_{t=1s} = 19.7$ m/s^2

$\omega = \pi$ rad/s

$T = 2$s

19.10:

$$A = 0.75\text{m}$$
$$\omega = 30 \text{ rpm} = \pi \text{ rad/s}$$
$$mg - R_{\min} = mA\omega^2$$
$$\Rightarrow \quad R_{\min} = 2.4 \text{ N}$$
$$R_{\max} - mg = mA\omega^2$$
$$\Rightarrow \quad R_{\max} = 17.2 \text{ N}$$

19.11:

$$mg - R_{\min} = mA\omega^2$$
$$g = A\omega^2$$
$$\Rightarrow \quad \omega = 4.43 \text{ rad/s}$$
$$= 42.3 \text{ rpm}$$

19.12:

$$\mu mg = mA\omega^2$$
$$\Rightarrow \quad A = \frac{\mu g}{\omega^2}$$
$$= 0.8 \text{ m}$$

19.13:

$$V = A\omega = 4 \text{ m/s}$$
$$mg - R_{\min} = mA\omega^2$$
$$mg = mA\omega^2$$
$$g = A\omega^2$$
$$\Rightarrow \quad \omega = 2.45 \text{ rad/s} = 23.4 \text{ rpm}$$

19.14:

$$A = \left(\frac{b-a}{2}\right)$$
$$v = A\omega$$
$$\Rightarrow \quad \omega = \frac{2v}{(b-a)}$$
$$\therefore \quad T = \frac{\pi(b-a)}{v}$$

19.15:

$$f = \frac{1}{T} = 2 \text{ Hz}$$

$$T = 2\pi\sqrt{\frac{e}{g}}$$

$\Rightarrow$ $e = 6.2$ cm

$\omega = 4\ \pi$ rad/s

$V_{max} = A\omega$

$= (0.5)(4\pi) = 2\pi$ m/s

19.16:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$K_1 = \frac{4\pi^2 m}{T_1^2} \quad \text{and} \quad K_2 = \frac{4\pi^2 m}{T_2^2}$$

$\therefore$ $K_1 - K_2 = 4\pi^2 m\left[\frac{T_2^2 - T_1^2}{T_1^2\ T_2^2}\right]$

19.17:

$mg = Ke$

$\Rightarrow$ $K = 392.4$ N/m

$$e = \frac{1.5 \times 9.81}{392.4} = 3.75 \text{ cm}$$

$$T = 2\pi\sqrt{\frac{e}{g}} = 0.39\text{s}$$

$A = 1$ cm

$$\omega = \sqrt{\frac{g}{e}} = 16.17 \text{ rad/s}$$

$\therefore$ $V_{max} = A\omega = 16.17$ cm/s

19.18:

$\omega = 50$ rpm $= 5.24$ rad/s

$\therefore$ $T = 1.2$s

$$e = \frac{g}{w^2} = 0.36 \text{ m}$$

19.19:

$$k = \frac{k_1 k_2}{k_1 + k_2} = 2.16 \text{ KN/m}$$

$$e = \frac{mg}{k} = 6.81 \text{ cm} \Rightarrow \omega = \sqrt{\frac{g}{e}} = 12 \text{ rad/s}$$

$$T = 2\pi\sqrt{\frac{e}{g}} = 0.52 \text{ s}$$

$$v_{max} = A\omega = 0.6 \text{ m/s}$$

$$a_{max} = A\omega^2 = 7.2 \text{ m/s}^2$$

19.20:

$$k = k_1 + k_2 = 8 \text{ KN/m}$$

$$e = 2.45 \text{ cm}$$

$$T = 2\pi\sqrt{\frac{e}{g}} = 0.1\ \pi \text{ s}$$

$\therefore$ $\omega = 20$ rad/s

$$v_{max} = A\omega = 0.4 \text{ m/s}$$

$$a_{max} = A\omega^2 = 8 \text{ m/s}^2$$

19.21:

$$k_1 = 1.5 \text{ kN/m}$$

$$k_2' = \frac{k_2\, k_3}{k_2 + k_3} = 1.5 \text{ kN/m}$$

$\therefore$ $k = 2$ kN/m

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.444 \text{ s}$$

19.22:

$$k_1 = 3 \text{ kN/m}$$

$$k_2 = \frac{4k}{4 + k}$$

$$T_1 = T_2$$

$\Rightarrow$ $$3 = \frac{4k}{4 + k}$$

$\Rightarrow$ k = 12 kN/m

19.23:

$$k_1 = 6 \text{ kN/m}$$

$$k_2 = \frac{2 \times 6}{8} = 1.5 \text{ kN/m}$$

$$T_1 = T_2$$

$$\frac{m_1}{k_1} = \frac{m_2}{k_2}$$

$\Rightarrow$ $m = 6$ kg

19.24:

(i) $k = 10$ kN/m

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 1.78$ s

(ii) $k = \frac{6 \times 4}{10} = 2.4$ kN/m

$\therefore \quad T = 3.63$ s

(iii) $k = 10$ kN/m

$\therefore \quad T = 1.78$ s

19.26:

$$\omega = 50 \text{ rpm} = \frac{50\pi}{3} \text{ rad/s}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\Rightarrow \quad g = l\omega^2 = 9.87 \text{ m/s}^2$$

19.27:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad l = 0.248 \text{ m}$$

19.30:

$$I = m\left(\frac{l}{2}\right)^2 + ml^2 = \frac{5ml^2}{4}$$

$$d = \frac{m(l/2) + ml}{2m} = \frac{3l}{4}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{5ml^2/4}{2mg(3l/4)}} = 2\pi\sqrt{\frac{5l}{6g}}$$

19.32:

$$\bar{I} = \frac{m(2a)^2}{12} = \frac{ma^2}{3}$$

$$\therefore \quad I_0 = \frac{ma^2}{3} + ma^2 = \frac{4}{3}ma^2$$

$$\therefore \quad L_e = \frac{I_0}{md} = \frac{\frac{4}{3}ma^2}{ma} = \frac{4}{3}a$$

$$T = 2\pi\sqrt{\frac{4a}{3g}}$$

19.33:

(i) $$\bar{I} = \frac{M}{12}[3R^2 + H^2]$$

$$I_0 = \bar{I} + M\left(\frac{H}{2}\right)^2$$

$$= \frac{M}{12}[3R^2 + 4H^2]$$

$$Le = \frac{\frac{M}{12}[3R^2 + 4H^2]}{MH/2} = \frac{3R^2 + 4H^2}{6H}$$

(ii) $$\bar{I} = \frac{MR^2}{2}$$

$$I_0 = \frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}$$

$$Le = \frac{\frac{3MR^2}{2}}{MR} = \frac{3}{2}R$$

$\therefore$ $$\text{ratio} = \frac{3R^2 + 4H^2}{9RH}$$

19.25:

$$E = \frac{mgl}{A\Delta} = \frac{4 \times 9.81 \times 1}{\pi \times (1.5 \times 10^{-3})^2 \times (0.15)}$$

$$= 370.09 \times 10^5 \text{ N/m}^2$$

(i) $$\omega = \sqrt{\frac{g}{\Delta}} = 8.09 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = 0.78 \text{ s}$$

$$v_{max} = A\omega = 1.21 \text{ m/s}$$

(ii) $$T = \frac{2\pi}{\omega} = 0.78\text{s}$$

$$v_{max} = 0.1 \times 8.09 = 0.81 \text{ m/s}$$

(iii) $$x = A\cos\omega t$$

$$-0.15 = 0.2\cos(8.09\,t)$$

$\Rightarrow$ $$t_1 = 0.3 \text{ s}$$

$$v = \omega\sqrt{A^2 - x^2}$$
$$= 8.09\sqrt{(0.2)^2 - (-0.15)^2}$$
$$= 1.07 \text{ m/s}$$
$$t_2 = \frac{v}{g} = 0.11\text{s}$$
$$\therefore \quad t = t_1 + t_2 = 0.41 \text{ s}$$
$$T = 2t = 0.82 \text{ s}$$

19.28:

$T_1 = 1$ s $\qquad g_1 = 9.81$ m/s^2

$T_2 = ?$ $\qquad g_2 = 9.82$ m/s^2

$$\Rightarrow \quad T_2 = \sqrt{\frac{9.81}{9.82}}$$

In 1s, it recoats, $\sqrt{\frac{9.82}{9.81}}$ s.

$$= 1 + \frac{1}{2}\left[\frac{0.01}{9.81}\right]$$

$$\therefore \quad \text{Gain in 1 s} = \frac{1}{2}\left[\frac{0.01}{9.81}\right] = 5.09684 \times 10^{-4} \text{ s}$$

Gain in 1 day = 44.04 s

19.34:

$$M = 1 \text{ kg/m} \times (37.76 \times 10^{-2}) = 37.76 \times 10^{-2} \text{ kg}$$
$$h = \sqrt{(15)^2 - (7.76/2)^2} = 14.49 \text{ cm}$$
$$d = \frac{[1 \times (7.76 \times 10^{-2}) \times (14.19 \times 10^{-2}) + \left[2 \times 1 \times 1.5 \times 10^{-2} \times \frac{14.49 \times 10^{-2}}{2}\right]}{37.76 \times 10^{-2}}$$
$$= 8.73 \text{ cm}$$
$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$
$$\Rightarrow \quad I = \left(\frac{T}{2\pi}\right)^2 Mgd$$
$$= 4.8 \times 10^{-3} \text{ kgm}^2$$