# Chapter 18

**18.1:**

$$m = 15 \text{ kg}$$
$$k = 0.25 \text{ cm}$$
$$\therefore \quad \bar{I} = mk^2 = 15(0.25)^2 = 0.9375 \text{ kg m}^2$$
$$w_0 = 1200 \text{ rpm} = 40\pi \text{ rad/s}$$
$$w = 0$$
$$\propto = \frac{w - w_0}{t} = \frac{40\pi - 0}{15} = 8.38 \text{ rad/s}^2$$
$$\therefore \quad M = \bar{I} \propto = 0.9375 \times 8.38 = 7.86 \text{ Nm}$$

**18.2:**

$$w_0 = 3000 \text{ rpm} = 3000 \times \frac{2\pi}{60} = 100\pi \text{ rad/s}$$
$$w = 0$$
$$-M = \bar{I} \propto$$
$$-20 = (20)(0.15)^2 \propto$$
$$\Rightarrow \quad \propto = -44.44 \text{ rad/s}$$
$$\therefore \quad t = \frac{w - w_0}{\propto} = \frac{-100\pi}{-44.44} = 7.07\text{s}$$
$$w^2 = w_0^2 + 2\propto \theta$$
$$\Rightarrow \quad \theta = \frac{0 - (100\pi)^2}{2(-44.44)} = 1110.44 \text{ rad}$$
$$\approx 176.73 \text{ rev.}$$

**18.3:**

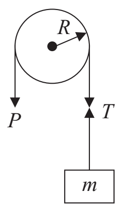
$$50 \times 0.3 \cos 45° = 3(0.35)^2 \propto$$
$$\Rightarrow \quad \propto = 28.86 \text{ rad/s}^2$$

**18.4:**

$$50 \times 0.3 \cos 45° - 20(0.04) = 3(0.35)^2 \propto$$
$$\Rightarrow \quad \propto = 26.68 \text{ rad/s}^2$$

**18.5:**

$$T - mg = ma \quad \ldots(1)$$
$$(P - T)R = \frac{MR^2}{2} \propto$$
$$= \frac{MR}{2} a$$



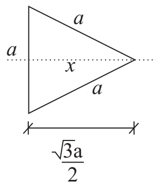
$$P - T = \frac{M}{2}a \;\ldots(2)$$

From (1) & (2),

$$P - mg = \left[m + \frac{M}{2}\right]a$$

$$\Rightarrow \qquad a = \frac{2(P - mg)}{(2m + M)}$$

**18.6:**




$$h = \frac{\sqrt{3}a}{2}$$

$$r_{cm} = \frac{h}{3} = \frac{a}{2\sqrt{3}}$$

$$\bar{I} = \frac{mh^2}{18} = \frac{m}{18}\left(\frac{\sqrt{3}a}{2}\right)^2 = \frac{ma^2}{24}$$

$$I_{axis} = \bar{I} + mr_{cm}^2$$

$$= \frac{ma^2}{24} + m\left[\frac{a}{2\sqrt{3}}\right]^2 = \frac{ma^2}{8}$$

$$mg[r_{cm}] = [I_{axis}]\propto$$

$$mg\left[\frac{a}{2\sqrt{3}}\right] = \frac{ma^2}{8}\propto$$

$$\Rightarrow \qquad \propto = \frac{4a}{\sqrt{3}a}$$

$$a_{cm} = \frac{a}{2\sqrt{3}}\frac{4g}{\sqrt{3}a} = \frac{2}{3}g$$

$$mg - Ay = ma_{cm} = \frac{2m}{3}g$$

$$\Rightarrow \qquad Ay = \frac{mg}{3}$$

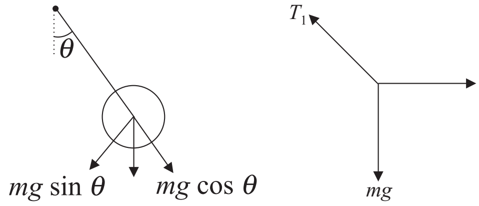
$$\therefore \qquad Ay : mg = 1 : 3$$

**18.7:**

$$mg\cos\theta = T_2$$

(or) $\qquad T_2 = mg\cos\theta$

$$T_1\cos\theta = mg$$



$$\Rightarrow \qquad T_1 = \frac{mg}{\cos\theta}$$

$$\therefore \qquad \frac{T_1}{T_2} = \frac{mg/\cos\theta}{mg\cos\theta} = \frac{1}{\cos^2\theta} = \frac{4}{3}$$

**18.8:**

$$(P - T_2)r = \frac{m_2 r^2}{2}\propto$$

$$P - T_2 = \frac{m_2}{2}a$$

$$(T_2 - T_1)r = \frac{m_2 r^2}{2}\propto$$

$$T_2 - T_1 = \frac{m_2}{2}a$$

$$T_1 - m_1 g = m_1 a$$

$$P - m_1 g = [m_1 + m_2]a$$

$$\Rightarrow \qquad a = \left[\frac{P - m_1 g}{m_1 + m_2}\right]$$

**18.9:**

$$40\,g - T_1 = 40\,a_1 = 40(0.2)\alpha = 8\alpha \qquad \ldots(1)$$

$$T_2 - 25\,g = 25\,a_2 = 25(0.25)\alpha = 6.25\alpha \qquad \ldots(2)$$

$$T_1(0.2) - T_2(0.25) = 10(0.22)^2\alpha = 0.484\alpha \qquad \ldots(3)$$

$$(1) \times 0.2 \Rightarrow \qquad 8g - 0.2\,T_1 = 1.6\alpha$$

$$(2) \times 0.25 \Rightarrow \qquad 0.25\,T_2 - 6.25g = 1.5625\alpha$$

$$(3) \times 1 \Rightarrow \qquad 0.2\,T_1 - 0.25\,T_2 = 0.484\alpha$$

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$$(8 - 6.25)\,g = 3.6465$$

$$\Rightarrow \qquad \alpha = 4.71 \text{ rad/s}^2$$

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$$\therefore \qquad a_1 = 0.942 \text{ m/s}^2$$

$$a_2 = 1.18 \text{ m/s}^2$$

$$T_1 = 354.72 \text{ N}$$

$$T_2 = 274.7 \text{ N}$$

**18.10:**

$$mg\sin\theta - T = ma \qquad \ldots(1)$$

$$T.r = \frac{Mr^2}{2}\alpha$$

$$\Rightarrow \quad T = \frac{M}{2} r\alpha = \frac{Ma}{2} \quad \ldots(2)$$

$$mg \sin \theta = \left[ m + \frac{M}{2} \right] a$$

$$\therefore \quad a = \frac{mg \sin \theta}{(m + M/2)} = \frac{10 \times 9.81 \times \sin 30^\circ}{(10 + 10/2)} = 3.27 \text{ m/s}^2$$

$$T = \frac{10}{2}(3.27) = 16.35 \text{ N}$$

**18.11:**

$$mg \sin \theta - T_1 = m_1 a \quad \ldots(1)$$

$$T_2 - m_2 g = m_2 a \quad \ldots(2)$$

$$(T_1 - T_2)r = \frac{Mr^2}{2} \alpha$$

$$\Rightarrow \quad T_1 - T_2 = \frac{M}{2} a \quad \ldots(3)$$

$$mg \sin \theta - m_2 g = \left[ m_1 + m_2 + \frac{m_2}{} \right] a$$

$$\Rightarrow \quad a = 2g \frac{[m_1 \sin \theta - m_2]}{2m_1 + 2m_2 + m} = 0.516 \text{ m/s}^2$$

$$T_1 = 43.9 \text{ N}$$

$$T_2 = 41.3 \text{ N}$$

**18.12:**

$$100 \times 1.5 \cos 30^\circ = \bar{I} \alpha$$

$$= \frac{ml^2}{12} \alpha = \frac{20}{12}(3)2\alpha$$

$$\Rightarrow \quad \alpha = 8.66 \text{ rad/s}^2$$

$$w^2 = w_0^2 + 2\alpha\theta$$

$$= 0 + 2(8.66)\left(\frac{\pi}{6}\right) = 9.07$$

$$\therefore \quad \text{KE} = \frac{1}{2} \bar{I} w^2$$

$$= \frac{1}{2}\left[\frac{20}{12}(3)^2\right](9.07) = 68 \text{ J}$$

**18.13:**

$$Fn = \rho a v^2$$

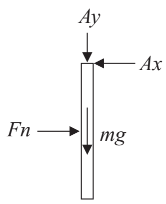
$$= (1000)\frac{\pi}{4}(0.006)^2\ (15)^2 = 6.36 \text{ N}$$

$$[Fn]\frac{h}{2} = I_{\text{axis}}\ \alpha$$

$$= \frac{mh^2}{3}\alpha$$

$$Fn = \frac{2}{3}mh\alpha$$

$$\Rightarrow \qquad \alpha = \frac{6.36\times 3}{2\times 2\times 0.5} = 9.54 \text{ rad/s}^2$$




$$a_{\text{cm}} = \frac{h}{2}\alpha = (0.25)(9.54) = 2.4 \text{ m/s}^2$$

$$mg + Ay = 0$$

$$\Rightarrow \qquad Ay = -mg = -19.6 \text{ N}$$

$$Fn - Ax = ma_{\text{cm}}$$

$$\Rightarrow \qquad Ax = Fn - ma_{\text{cm}}$$

$$= 6.36 - (2)(2.4)$$

$$= 1.56 \text{ N}$$

**18.15:**

$$m_2 g - T_2 = m_2 a \qquad \ldots(1)$$

$$T_1 = m_1 a \qquad \ldots(2)$$

$$(T_2 - T_1)r = \frac{mr^2}{2}\alpha$$

$$T_2 - T_1 = \frac{m}{2}r\alpha = \frac{m}{2}\alpha \qquad \ldots(3)$$

$$m_2 g = \left[m_1 + m_2 + \frac{m}{2}\right]\alpha$$

$$25\ g = \left[20 + 25 + \frac{10}{2}\right]\alpha$$

$$\Rightarrow \qquad a = \frac{g}{2} \text{ m/s}^2$$

$$\therefore \qquad T_1 = 98.1 \text{ N}$$

$$T_2 = 122.6 \text{ N}$$

**18.16:**

$$mgr = mr^2\,\alpha$$

$$\alpha = \frac{g}{r}$$

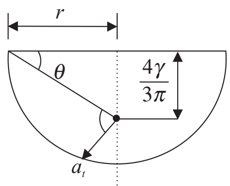
$\therefore$ $$a_{cm} = r\alpha = g$$

$$mg - Ay = ma_{cm} = mg$$

$\Rightarrow$ $$Ay = 0$$

and $$Ax = 0$$

**18.17:**




$$\theta = \tan^{-1}\left[\frac{4r/3\pi}{r}\right] = \tan^{-1}\left[\frac{4}{3\pi}\right] = 23°$$

$$\bar{I}_{xx} = 0.07\,mr^2$$

$$\bar{I}_{yy} = 0.25\text{ mr}^2$$

$\therefore$ $$\bar{I}_{zz} = \bar{I}_{xx} + \bar{I}_{yy} = 0.32\text{ mr}^2$$

$\therefore$ $$I_A = \bar{I}_{zz} + m\left[\sqrt{\left(\frac{4r}{3\pi}\right)^2 + r^2}\right]^2$$

$$= 1.5\,mr^2$$

$$\Sigma\,MA = I_A\,\alpha$$

$$mgr = (1.5\,mr^2)\alpha$$

$\Rightarrow$ $$\alpha = \frac{g}{1.5r} = 65.4\text{ rad/s}^2$$

$$A_{cm} = r_{cm}\,\alpha = \left[\sqrt{\left(\frac{4r}{3\pi}\right)^2 + r^2}\right]\alpha$$

$$= \left[\sqrt{\frac{16}{9\pi^2} + 1}\right]r\alpha$$

$$= 7.105\text{ m/s}^2$$

$$A_x = m\,a_t\sin\theta$$

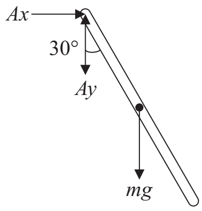
$$= (0.5)\,(7.105)\sin 23°$$

$$= 1.39\text{ N}$$

$$Mg - Ay = m\,a_t\cos\theta$$

$\Rightarrow$ $$Ay = 1.63\text{ N}$$

**18.18:**




$$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}\alpha$$

$$\Rightarrow \quad \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

$$a_{cm} = r_{cm}\,\alpha \quad = \frac{l}{2}.\frac{3}{2}\frac{g}{l}\sin\theta$$

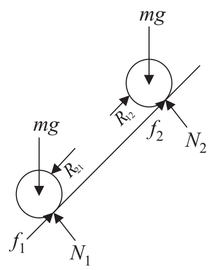
$$= \frac{3}{4}g\sin\theta = \frac{3}{8}g$$

$$mg - Ay = ma$$

$$\Rightarrow \quad Ay = mg - m\frac{3}{8}g$$

$$= \frac{5}{8}mg$$

**18.19:**




*Cylinder*

$$N_1 = mg\cos\theta$$

$$mg\sin\theta + R_{21} - f_1 = ma$$

$$f_1\,r = \frac{mr^2}{2}\alpha$$

$$\Rightarrow \quad f_1 = \frac{m}{2}\alpha$$

$$\therefore \quad mg\sin\theta + R_{21} = \frac{3}{2}ma \quad \ldots(1)$$

*Sphere*

$$N_2 = mg\cos\theta$$

$$mg\sin\theta - R_{12} - f_2 = ma$$

$$f_2\,r = \frac{2}{5}mr^2\,\alpha$$

$$\Rightarrow \quad f_2 = \frac{2}{5}ma$$

$$\therefore \quad mg\sin\theta - R_{12} = \frac{7}{5}ma \quad \ldots(2)$$

Since $\quad R_{12} = R_{21} \Rightarrow$

$$2mg\sin\theta = \left[\frac{3}{2} + \frac{7}{5}\right]ma = \frac{29}{10}\text{ma}$$

$$\Rightarrow \qquad a = \frac{20}{29} g \sin \theta$$

$$mg \sin \theta - \frac{7}{5} m \frac{20}{29} g \sin \theta = R_{12}$$

$$\Rightarrow \qquad R_{12} = R_{21} = \frac{1}{29} mg \sin \theta$$

**18.20:**

$$a_s = \frac{5}{7} g \sin \theta$$

$$a_h = \frac{1}{2} g \sin \theta$$

$$s = v_0 t + \frac{1}{2} at^2$$

$$s_1 = 0 + \frac{1}{2} \frac{5}{7} g \sin \theta t^2$$

$$= \frac{5}{14} g \sin \theta t^2$$

$$s_2 = 0 + \frac{1}{2} \frac{1}{2} g \sin \theta t^2$$

$$= \frac{1}{4} g \sin \theta t^2$$

$$\frac{s_1}{s_2} = \frac{5}{14} \frac{4}{1} = \frac{10}{2}$$

If $s_1 = 10$ m, then $s_2 = 7$ m

**18.21:**

$$a_c = \frac{2}{3} g \sin \theta$$

$$a_h = \frac{1}{2} g \sin \theta$$

$$s_1 = 0 + \frac{1}{2} \frac{2}{3} g \sin \theta t^2$$

$$= \frac{1}{2} g \sin \theta t^2$$

$$s_2 = 0 + \frac{1}{2} \frac{1}{2} g \sin \theta t^2$$

$$= \frac{1}{4} g \sin \theta t^2$$

$$\frac{s_1}{s_2} = \frac{1}{3}\frac{4}{1} = \frac{4}{3}$$

If $s_1 = 10$ m, $s_2 = 10.\frac{3}{4} = 7.5$ m

**18.22:**

$$s_s = \frac{1}{2}\left(\frac{5}{7} g \sin\theta\right)t^2 = \frac{5}{14} g \sin\theta\, t^2 \ldots(1)$$

$$s_c = \frac{1}{2}\left(\frac{2}{3} g \sin\theta\right)t^2 = \frac{1}{3} g \sin\theta\, t^2 \qquad \ldots(2)$$

$$s_h = \frac{1}{2}\left(\frac{1}{2} g \sin\theta\right)t^2 = \frac{1}{4} g \sin\theta\, t^2 \qquad \ldots(3)$$

From (1), $g \sin\theta\, t^2 = \dfrac{14 s_s}{5}$

$$\therefore \quad s_c = \frac{1}{3}\frac{14 s_s}{5} = \frac{14}{15} s_s = 0.933\, s_s$$

$$s_h = \frac{1}{4}\frac{14 s_s}{5} = \frac{14}{20} s_s = 0.7\, s_s$$

$\therefore$ cylinder is at $(1 - 0.933)s_s = 0.067\, s_s$

hoop is at $(1 - 0.7)\, s_s = 0.3\, s_s$

**18.24:**

$$v = 60 \text{ kmph} = 16.67 \text{ m/s}$$

$$\text{K.E.} = \frac{1}{2} mv^2 + \frac{1}{2} IW^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2}(mk^2)\frac{v^2}{r^2}$$

$$= \frac{1}{2} mv^2\left[1 + \frac{k^2}{r^2}\right]$$

$$= 868.4 \text{ J}$$

**18.25:**

$$W_0 = 200 \text{ rpm} = 200\frac{2\pi}{60} = \frac{20}{3}\pi \text{ rad/s}$$

$$w = 300 \text{ rpm} = 10\,\pi \text{ rad/s}$$

$$t = 6\text{s}$$

$$\alpha = \frac{w - w_0}{t} = \frac{10 - \dfrac{20\pi}{3}}{6} = 1.745 \text{ rad/s}^2$$

$$\theta = \frac{w^2 - w_0^2}{2\alpha} = 157/11 \text{ rad}$$

$\therefore$ $$M = I\alpha = (25)(0.2)2(1.745) = 1.745 \text{ Nm}$$

$\therefore$ $$W = M\theta = 274.2 \text{ J}$$

$$\Delta\text{K.E} = \frac{1}{2}I\,(w^2 - w_0^2) = 274.2 \text{ J}$$

**18.26:**

$$w = \frac{2\pi \text{ rad}}{24 \text{ hrs.}} = \frac{2\pi \text{ rad}}{24 \times 3600s} = 7.27 \times 10^{-5} \text{ rad/s}$$

$$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}(6 \times 10^{24})(6370 \times 10^3)^2 = 9.738 \times 10^{37} \text{ kg.m}^2$$

$$H_G = \bar{I}w = 7.08 \times 10^{33} \text{ kg. m}^2\text{/s}$$

$$\text{K.E} = \frac{1}{2}\,\bar{I}w^2 = 2.57 \times 10^{29} \text{ J}$$

**18.27:**

$$2g - T = 2a = 2(0.06)\alpha \qquad \ldots(1)$$

$$T(0.06) = 3(0.1)^2\alpha$$

$\Rightarrow$ $$T = \frac{3(0.1)^2}{0.06}\alpha \qquad \ldots(2)$$

$$2g - \frac{3(0.1)^2}{0.06}\alpha = 2(0.06)\alpha$$

$\Rightarrow$ $$\alpha = \frac{2g}{0.62} = 31.65 \text{ rad/s}^2$$

$\therefore$ $$\alpha = (0.06)(31.65) = 1.9 \text{ m/s}^2.$$

$$v^2 = v_0^2 + 2as$$

$$= 0 + 2\,(1 - 9)(3)$$

$\Rightarrow$ $$v = 3.38 \text{ m/s}$$

**18.29:**

$$100(2\pi r)^5 = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\,\frac{mr^2}{2}\,\frac{v^2}{r^2}$$

$$= \frac{3}{4}mv^2$$

$$100\,[2\pi \times 0.1]^5 = \frac{3}{4} \times 10 \times J^2$$

$\Rightarrow$ $$v = 6.5 \text{ m/s}$$