Chapter-17

17.1:

$$\theta = 4t^3 - 3t^2 + 2t$$
$$\theta(2) = 4\,(2)^3 - 3(2)^2 + 2(2) = 24 \text{ rad}$$
$$\omega = 12t^2 - 6t + 2$$
$$\omega(2) = 12\,(2)^2 - 6(2) + 2 = 38 \text{ rad/s}$$
$$\alpha = 24t - 6$$
$$\alpha(2) = 24\,(2) - 6 = 42 \text{ rad/s}^2$$

17.2:

$$\theta = t^2 + 2t \qquad \theta(3) = 3^2 + 2(3) = 15 \text{ rad}$$
$$\omega = 2t + 2 \qquad \omega(3) = 2\,(3) + 2 = 8 \text{ rad/s}$$
$$\alpha = 2 \qquad \alpha(3) = 2 \text{ rad/s}^2$$

17.3:

$$\theta = \frac{\pi}{8}t^2 + \frac{\pi}{4}t$$
$$\omega = \frac{\pi}{4}t + \frac{\pi}{4}$$
$$3 = \frac{\pi}{4}t + \frac{\pi}{4} \quad \Rightarrow \quad t = 2.82s$$
$$\theta(2.82) = \frac{\pi}{8}(2.82)^2 + \frac{\pi}{4}(2.82) = 5.34 \text{ red}$$
$$\alpha = \frac{\pi}{4} \text{ rad/s}^2$$

17.4:

$$\alpha = kt$$
$$\omega = \frac{kt^2}{2} + \omega_0$$
$$\theta = \frac{kt^3}{6} + \omega_0 t + \theta_0$$

17.5:

$$\alpha = 3t^2 - 2t$$
$$\omega = \frac{3t^3}{3} - \frac{2t^2}{2} + \omega_0$$
$$= t^3 - t^2 \qquad [\text{since } \omega_0 = 0]$$
$$\theta = \frac{t^4}{4} - \frac{t^3}{3} + \theta_0$$
$$= \frac{t^4}{4} - \frac{t^3}{3} \qquad [\text{since } \theta_0 = 0]$$

17.6:

$$\alpha = 4t^2 - 3t$$

$$\omega = \frac{4t^3}{3} - \frac{3t^3}{3} + \omega_0$$

$$\theta = \frac{4}{3}\frac{t^4}{4} - \frac{3}{2}\frac{t^3}{3} + \omega_0 t + \theta_0$$

$$\omega(3) = \frac{4}{3}(3)^3 - \frac{3}{2}(3)^2 + 2$$

$$= 24.5 \text{ rad/s}$$

$$\theta(3) = \frac{(3)^4}{3} - \frac{1}{2}(3)^3 + 2(3)$$

$$= 19.5 \text{ rad/s}^2$$

17.7:

$$\omega = 2400 \text{ rpm}$$

$$= 2400 \times \frac{2\pi}{60} = 80\ \pi \text{rad/s}$$

17.8:

$$\omega = 4\ \pi \text{ rad/s}$$

$$= \frac{4\pi}{2\pi} \times 60 = 120 \text{ r}_{\text{pm}}$$

17.9:

$$\theta = 32.25 \text{ rev}$$

$$= 32.25 \times 2\pi \text{ rad}$$

$$= 64.5\pi \text{ rad}$$

17.10:

$$\omega_0 = 0$$

$$\omega = 300\ r_{\text{pm}} = 300 \times \frac{2\pi}{60} = 10\pi \text{ rad/s}$$

(i) $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{10\pi - 0}{10} = \pi \text{ rad/s}^2$$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 0 + \frac{1}{2}(\pi)(5)^2 = 12.5\pi \text{ rad}$$

$$= 6.25 \text{ nev}$$

$$\omega = \omega_0 + \alpha t$$

$$= 0 + \pi(5) = 5\pi \text{ rad/s}$$

17.11:

$$\omega_0 = 10\pi \text{ rad/s}$$

$$\omega = 0$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 10\pi}{15} = \frac{-2}{3}\pi \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 10\pi(15) + \frac{1}{2}\left(\frac{-2}{3}\pi\right)(15)^2$$

$$= 150\pi - 75\pi = 75\pi \text{ rad}$$

17.12:

$$\omega_0 = 0$$

$$\omega = 7200 \text{ rpm} = 7200 \times \frac{2\pi}{60} = 240\pi \text{ rad/s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{240\pi - 0}{6} = 40\pi \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \frac{1}{2}(40\pi)(6)^2$$

$$= 720\pi \text{ (or) } 360 \text{ rev}$$

When switched-off

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{o - (240\pi)^2}{2(200\pi)}$$

$$= -144\pi \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{o - (240\pi)}{-144\pi} = 1.67 s$$

17.13:

$$\omega_0 = 6000 \text{ rpm} = 6000 \times \frac{2\pi}{60} = 200\pi \text{ rad/s}$$

$$\omega = 10{,}000 \text{ rpm} = 10{,}000 \times \frac{2\pi}{60} = 333.33\pi \text{ rad/s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{333.33\pi - 200\pi}{2.5}$$

$$= 167.55 \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= (200\pi)\,(2.5) + \frac{1}{2}(167.55)\,(2.5)^2$$

$$= 2094.4 \text{ rad}$$

$$= (333 \text{ complete revolutions})$$

17.14:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\frac{\pi}{4} = 0 + \frac{1}{2}\alpha (3)^2$$

$$\Rightarrow \quad \alpha = \frac{\pi}{18} \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t$$

$$= 0 + \frac{\pi}{18}(3)$$

$$= \frac{\pi}{6} \text{ rad/s}$$

17.15:

$$\omega = 45 \text{ rpm} = 45 \times \frac{2\pi}{60} = 4.71 \text{ rad/s}$$

$$v_A = r_A \omega = (1)\,(4.71) = 4.71 \text{ m/s}$$

$$a_A = r_A \omega^2 = (1)\,(4.71)^2 = 22.2 \text{ m/s}^2$$

$$v_B = r_B \omega = (0.4)\,(4.71) = 1.88 \text{ m/s}$$

$$a_B = r_B \omega^2 = (0.4)\,(4.71)^2 = 8.87 \text{ m/s}^2$$

17.16:

$$\omega = 0$$

$$\alpha = \frac{\pi}{4} \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega(4) = 0 + \frac{\pi}{4}(4)$$

$$= \pi \text{ rad/s}$$

***A*:** $v_A = r_A\, \omega = 1\,(\pi) = 3.14 \text{ m/s}$

$$(a_A)_t = r_A\, \alpha = 1\left(\frac{\pi}{4}\right) = 0.785 \text{ m/s}^2$$

$$(a_A)_r = r_A\, \omega^2$$

$$= (1)\,(\pi)^2 = 9.87 \text{ m/s}^2$$

***B*:** $v_B = r_B\omega = (0.4)\,(\pi) = 1.26$ m/s

$$(a_B)_t = r_B\,\alpha = (0.4)\left(\frac{\pi}{4}\right) = 0.314 \text{ m/s}^2$$

$$(a_B)_r = r_B\omega^2$$
$$= (0.4)\,(\pi)^2 = 3.95 \text{ m/s}^2$$

17.17:

$r = 20$ cm $= 0.2$ cm

$t = 2$ s

$\alpha = \pi/4$ rad/s^2

$\omega = \omega_v + \alpha t$

$$= 0 + \frac{\pi}{4}(2) = \frac{\pi}{2} \text{ rad/s}$$

$\therefore$ $v = r\omega = (0.2)\,\dfrac{\pi}{2} = 0.314$ m/s

$$a_t = r\alpha = (0.2)\left(\frac{\pi}{4}\right) = 0.16 \text{ m/s}^2$$

$$a_r = r\omega^2 = (0.2)\left(\frac{\pi}{2}\right) = 0.49 \text{ m/s}^2$$

17.18:

$\omega_0 = 0$

$\alpha = \pi/2$ rad/s^2

$$\omega_{\max} = 150 \text{ rpm} = 150 \times \frac{2\pi}{60} = 5\pi \text{ rad/s}$$

$\omega = \omega_0 + \alpha t$

$\therefore$ $$t = \frac{\omega - \omega_0}{\alpha} = \frac{5\pi - o}{\pi/2} = 10 \text{ s}$$

(i) At $t = 4$s

$$\omega(4) = 0 + \left(\frac{\pi}{2}\right)(4) = 2\pi \text{ rad/s}$$

$v = r\omega = (0.3)\,(2\pi) = 1.88$ m/s

$a_t = rd = (0.3)\,(\pi/2) = 0.47$ m/s^2

$a_r = r\omega^2 = (0.3)\,(2\pi)^2 = 11.84$ m/s^2

(ii) At $t = 8$ s,

$$\omega(8) = 0 + \left(\frac{\pi}{2}\right)(8) = 4\pi \text{ rad/s}$$

$\therefore$ $v = r\omega = (0.3)\,(4\pi) = 3.77$ m/s

$a_t = r\,\alpha = (0.3)\,(\pi/2) = 0.47$ m/s^2

$a_r = r\omega^2 = (0.3)\,(4\pi)^2 = 47.4$ m/s^2

(iii) At $t = 12$ s,

$\omega = \omega_{max} = 5\pi$ rad/s

& $\alpha = 0$

$\therefore$ $v = r\omega = (0.3)(5\pi) = 4.71$ m/s

$a_t = 0$

$a_r = r\omega^2 = (0.3)(5\pi)^2 = 74$ m/s^2

17.19:

$r_1 = 1$ cm

$r_2 = 2.5$ cm

Since the tape is moving without slipping

$v_1 = v_2$

$r_1 \omega_1 = r_2 \omega_2$

$(1)(30) = (2.5)(\omega_2)$

$\Rightarrow$ $\omega_2 = 12$ rpm

17.20:

$r_B = a$

$\therefore$ $v_B = a\omega$

$r_c = \sqrt{2}\, a$

$\therefore$ $v_c = \sqrt{2}\, a\omega$

$$r_{c.g} = \frac{\alpha}{\sqrt{2}}$$

$\therefore$ $$v_{c.9} = \frac{a\omega}{\sqrt{2}}$$

17.21:

$r_1 = 5$ cm

$r_2 = 10$ cm

$\omega_1 = 50$ rpm

When there is no slipping,

$v_1 = v_2$

$r_1 \omega_1 = r_2 \omega_2$

$(5)(50) = (10)\omega$

$\Rightarrow$ $\omega = 25$ rpm in the clockwise direction

17.22:

$v = 0.25$ m/s

$r = 0.5$ m

$\therefore$ $$\omega = \frac{v}{r} = \frac{0.25}{0.5} = 0.5 \text{ rad/s}$$

$$a_n = r\omega^2 = (0.5)\,(0.5)^2$$
$$= 0.125 \text{ rad/s}^2$$

17.23:

$$a_t = 0.1 \text{ m/s}^2$$

$\therefore$ $v = v_0 + at$

$$= 0 + (0.1)\,(2)$$
$$= 0.2 \text{ m/s}$$

$\therefore$ $\omega = \dfrac{v}{r} = \dfrac{0.2}{0.5} = 0.4$ rad/s

$$\alpha = \frac{a_t}{r} = \frac{0.1}{0.5} = 0.2 \text{ rad/s}^2$$

$$a_n = r\omega^2 = (0.5)\,(0.4)^2 = 0.08 \text{ m/s}^2$$

17.24:

$$v = 10 \text{ m/s}$$
$$t = 3 \text{ s}$$

$\therefore$ $a_t = \dfrac{v - v_o}{t} = \dfrac{10 - 0}{3} = 3.33 \text{ m/s}^2$

$$\omega = \frac{v}{r} = \frac{10}{0.25} = 40 \text{ rad/s}$$

$$a_t = r\,d$$

$\Rightarrow$ $\alpha = a_t/r = \dfrac{3.33}{0.25} = \dfrac{40}{3}$ rad/s^2

17.25:

$$\alpha = \frac{\pi}{50} \text{ rad/s}^2$$

$$\omega_{\max} = \frac{\pi}{4} \text{ rad/s}$$

$\therefore$ Time taken, $t = \dfrac{\omega_{\max}}{\alpha} = 12.5\,s$

(i) At $t = 5\,s$,

$$\omega = \omega_0 + \alpha t = \frac{\pi}{50}(5) = \frac{\pi}{10} \text{ rad/s}$$

$\therefore$ $v = r\omega$

$$= 10\left(\frac{\pi}{10}\right) = \pi \text{ m/s}$$

$$a_t = r\,\alpha = 10\left(\frac{\pi}{50}\right) = 0.2\pi \text{ rad/s}^2$$

$$a_n = r\omega^2 = 10\left(\frac{\pi}{10}\right) = 0.1\,\pi^2 \text{ rad/s}^2$$

(ii) At $t = 10$ s,

$$\omega = \frac{\pi}{50}(10) = 0.2\,\pi \text{ rad/s}$$

$\therefore$ $v = 10\,(0.2)\,\pi = 2\,\pi$ m/s

$a_t = 10\,(\pi/50) = 0.2\pi$ rad/s^2

$a_n = 10\,(0.2\,\pi)^2 = 0.4\pi^2$ rad/s^2

(iii) At $t = 15$, $\omega = \omega_{max} = \pi/4$ rad/s

$$v = (10)\,\frac{\pi}{4} = 2.5\pi \text{ m/s}$$

$a_t = 0$

$$a_n = 10\left(\frac{\pi}{4}\right) = 0.625\,\pi^2 \text{ rad/s}^2$$

17.26:

$\omega = 2\,\pi$ rad/s

Velocity of chain, $v = r\,\omega$

$$= \left(\frac{0.15}{2}\right)(2\,\pi)$$

$$= 0.15\,\pi \text{ red/s}$$

Since there is no slipping,

$$\omega_{\text{rear gear}} = \frac{v}{r_{val\,gear}}$$

$$= \frac{0.15\pi}{(0.06/2)}$$

$$= 5\pi \text{ rad/s}$$

$\therefore$ ω rear wheel $= 5\pi$ rad/s

Since the wheel rolls without slipping,

$v_0 = r\,\omega$

$= (0.3)\,(5\pi)$

$= 4.7$ m/s

17.27:

$v_0 = 5$ m/s

$r = 25$ cm $= 0.25$ m

(i) The absolute velocity of point A is

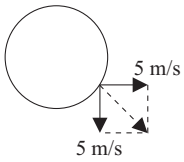
$\vec{v}_A = \vec{v}_0 + \vec{v}_{A/0}$

$0 = \vec{v}_0 + \vec{v}_{A/0}$

$\Rightarrow$ $v_{A/0} = r\,\omega = v_0$

$\therefore$ $\omega = v_0/r = 20$ rad/s

(ii) Point B:

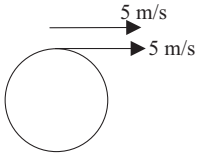


$$\vec{v}_B = \vec{v}_0 + \vec{v}_{B/0}$$

$$v_B = \sqrt{(v_o)^2 + (v_{B/o})^2}$$

$$= \sqrt{5^2 + 5^2} = 7.07 \text{ m/s}$$

(iii)



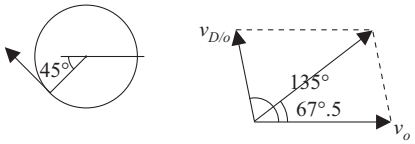
$$\vec{v}_C = \vec{v}_0 + \vec{v}_{C/0}$$

Since they are in the same direction

$$v_C = v_o + v_{C/o}$$

$$= 10 \text{ m/s}$$

(iii) Point D:



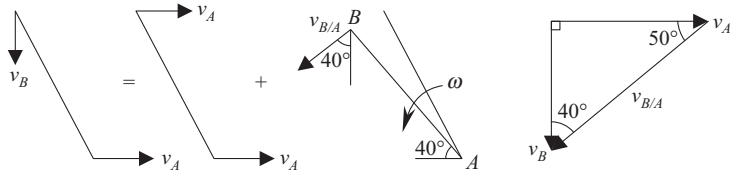
$$\vec{v}_D = \vec{v}_0 + \vec{v}_{D/0}$$

$$v_D = \sqrt{(v_o)^2 + (v_{D/o})^2 + 2(v_o)(v_{D/o})\cos 135^\circ}$$

$$= 3.83 \text{ m/s}$$

at 67.5 to the horizontal

17.28:



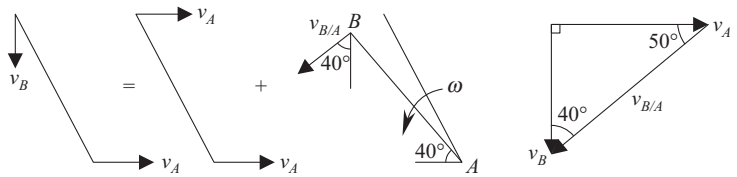
$$\frac{v_A}{\sin 40^\circ} = \frac{v_B}{\sin 50^\circ} = \frac{v_{A/B}}{\sin 90^\circ}$$

$$\Rightarrow \quad v_B = \frac{\sin 50^\circ}{\sin 40^\circ} v_A = 2.38 \text{ m/s}$$

$$v_{B/A} = \frac{\sin 90^\circ}{\sin 40^\circ} v_A = 3.11 \text{ m/s}$$

$$\therefore \quad \omega = \frac{v_{B/A}}{r_{AB}} = \frac{3.11}{1.5} = 2.07 \text{ rad/s} \circlearrowleft$$

17.29:



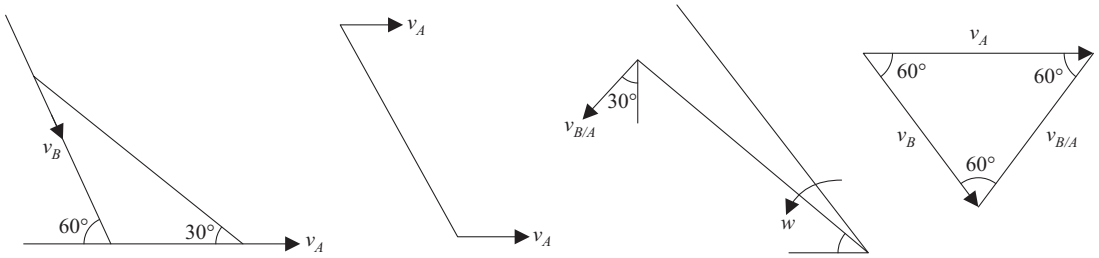
$$\frac{v_A}{\sin 30°} = \frac{v_B}{\sin 60°} = \frac{v_{B/A}}{\sin 90°}$$

$$\Rightarrow \quad v_B = \frac{\sin 60°}{\sin 30°} v_A = 1.732 \text{ m/s} \uparrow$$

$$v_{B/A} = \frac{\sin 90°}{\sin 30°} v_A = 2 \text{ m/s}$$

$$\therefore \quad \omega = \frac{v_{B/A}}{AB} = 1.33 \text{ rad/s}$$

17.30:



Since the included angles are equal,

$$v_A = v_B = v_{B/A} = 2 \text{ m/s}$$

$$\therefore \quad v_B = 2 \text{ m/s}$$

$$\omega = \frac{v_{B/A}}{AB} = \frac{2}{2} = 1 \text{ rad/s}$$

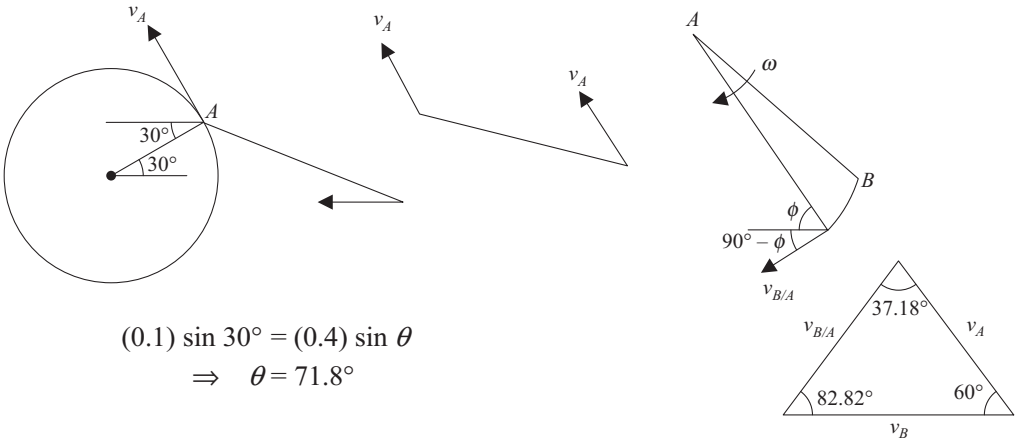
17.31:

$$\omega = 200 \text{ rpm} = 200 \times \frac{2\pi}{60} = 20.94 \text{ rad/s}$$

$$v_A = r_{0A}\, \omega$$

$$= (0.1)\ (20.94)$$

$$= 2.09 \text{ m/s}$$



$$(0.1) \sin 30° = (0.4) \sin \theta$$

$$\Rightarrow \quad \theta = 71.8°$$

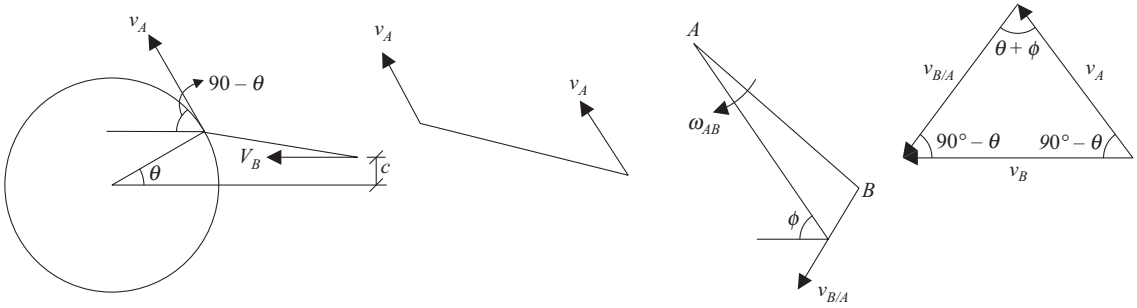
$$\frac{v_A}{\sin 82.82^\circ} = \frac{v_B}{\sin 37.18^\circ} = \frac{v_{B/A}}{\sin 60^\circ}$$

$$\Rightarrow \quad v_B = \frac{\sin 37.18^\circ}{\sin 82.82^\circ} v_A = 1.27 \text{ m/s (to the left)}$$

$$v_{B/A} = \frac{\sin 60^\circ}{\sin 82.82^\circ} v_A = 1.82 \text{ m/s}$$

$$\therefore \quad \omega = \frac{v_{B/A}}{AB} = 4.55 \text{ rad/s} \circlearrowright$$

17.32:



$$a \sin \theta = b \sin \phi + c$$

$$v_A = a\,\omega$$

$$= \frac{(b \sin \phi + c)}{\sin \theta} \omega.$$

$$\frac{v_A}{\sin(90^\circ - \phi)} = \frac{v_B}{\sin(\theta - \phi)} = \frac{v_{B/A}}{\sin(90^\circ - \theta)}$$

$$\Rightarrow \quad v_B = \frac{\sin(\theta + \phi)}{\sin(90^\circ - \phi)} v_A$$

$$= \frac{\sin(\theta + \phi)}{\sin(90^\circ - \phi)} \left(\frac{b \sin \phi + c}{\sin \theta} \right) \omega$$

$$= \frac{\sin(\theta + \phi)}{\cos \phi \sin \theta} (b \sin \phi + c)\, \omega$$

$$= \frac{\sin \theta + \phi + \cos \theta \sin \phi}{\cos \phi \sin \theta} (b \sin \phi + c)\, \omega$$

$$= (1 + \tan \phi \cos \theta)\,(b \sin \phi + c)\omega$$

$$v_{B/A} = \frac{\sin(90^\circ \square \square \square)}{\sin(90^\circ \square \square \square)} v_A$$

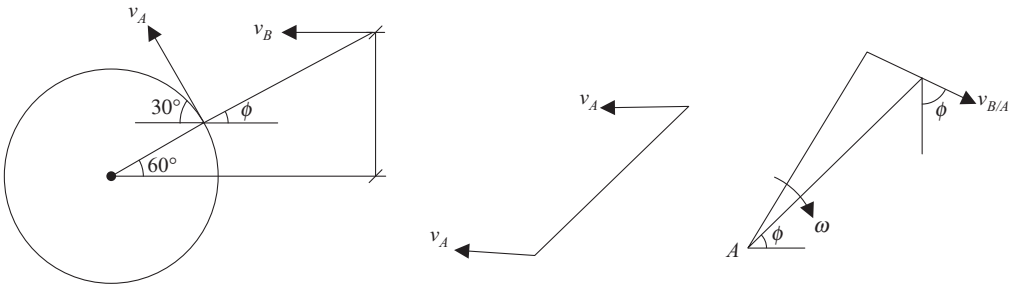
$$= \frac{\cos \square}{\cos \square} \frac{(b \sin \square + c)}{\sin \square} \omega$$

$$= \cot\theta\,[b\tan\phi + c\sec\phi]\omega$$

$$\omega_{AB} = \frac{v_{B/A}}{b}$$

$$= \cot\theta\left[\tan\phi + \frac{c}{b}\sec\phi\right]\omega$$

17.33:



$$\sin\phi = \frac{15 - 7.5\sin 60°}{20} = 0.425$$

$$\Rightarrow \quad \phi = 25.15°$$

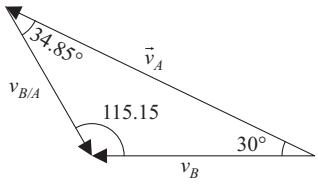
$$\omega = 100 \text{ rpm}$$

$$= 100 \times \frac{2\pi}{60} = 10.47 \text{ rad/s}$$

$$v_A = r_{0A}\,\omega$$

$$= (0.075)\,(10.47)$$

$$= 0.785 \text{ m/s}$$



$$\frac{v_A}{\sin 115.15°} = \frac{v_B}{\sin 34.85°} = \frac{v_{B/A}}{\sin 30°}$$

$$\Rightarrow \quad v_B = \frac{\sin 34.85°}{\sin 115.15°}\,v_A = 0.496 \text{ m/s to the left}$$

$$v_{B/A} = \frac{\sin 30°}{\sin 115.15°}\,v_A = 0.434 \text{ m/s}$$

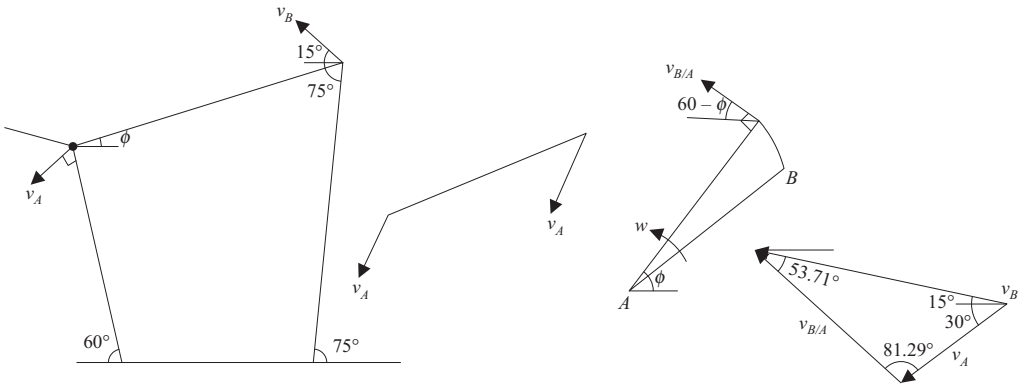
$$\therefore \quad \omega_{AB} = \frac{v_{B/A}}{AB} = \frac{0.434}{0.2} = 2.17 \text{ rad/s}\ \circlearrowright$$

17.34:

$$\omega = 60 \text{ rpm} = 2\pi \text{ rad/s}$$

$$v_A = r_{0,A}\,\omega = (0.05)\,(2\pi) = 0.314 \text{ m/s}$$

$$\sin\phi = \frac{12\sin 75° - 5\sin 60°}{20} = 21.29°$$



$$\frac{v_A}{\sin 53.71°} = \frac{v_B}{\sin 81.29°} = \frac{v_{B/A}}{\sin 45°}$$

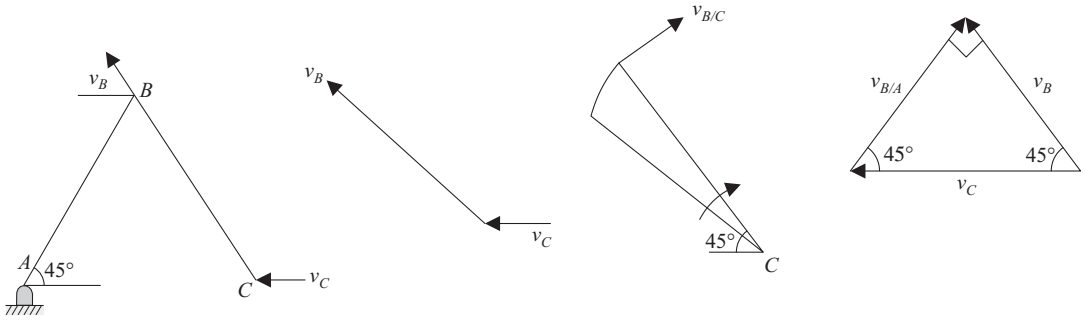
$$\Rightarrow \quad v_B = \frac{\sin 81.29°}{\sin 53.71°} v_A = 0.385 \text{ m/s}$$

$$\omega_{02B} = v_B / o_2 B = 3.21 \text{ rad/s}$$

$$v_{B/A} = \frac{\sin 45°}{\sin 53.71°} v_A = 0.275 \text{ m/s}$$

$$\therefore \quad \omega_{AB} = \frac{v_{B/A}}{AB} = 1.38 \text{ rad/s}$$

17.35:



$$\frac{v_C}{\sin 90°} = \frac{v_B}{\sin 45°} = \frac{v_{B/C}}{\sin 45°}$$

$$\Rightarrow \quad v_B = \frac{\sin 45°}{\sin 90°} v_C = 0.707 \text{ m/s}$$

$$v_{B/C} = 0.707 \text{ m/s}$$

$$\therefore \quad \omega_{BC} = \frac{v_{B/C}}{BC} = \frac{0.707}{0.5} = 1.414 \text{ rad/s} \curvearrowright$$

$$\omega_{AB} = \frac{v_B}{AB} = \frac{0.707}{0.5} = 1.414 \text{ rad/s} \curvearrowleft$$