Chapter-14

14.1:

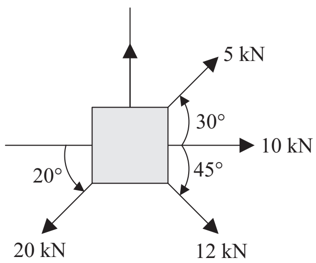
$m = 5$ kg

$F_x = 10 + 5 \cos 30° + 12 \cos 45° - 20 \cos 20°$

$= 4.02$ kN

$F_y = 5 \sin 30° + 15 - 12 \sin 45° - 20 \sin 20°$

$= 2.17$ kN



$a_x = \frac{F_x}{m} = 0.804 \text{ m/s}^2 \qquad a_y = \frac{F_y}{m} = 0.434 \text{ m/s}^2$

$\therefore \quad a = \sqrt{a_x^2 + a_y^2} = 0.91 \text{ m/s}^2$

$\theta = \tan^{-1}\left[\frac{0.434}{0.804}\right] = 28.4°$

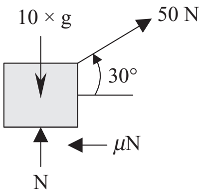
14.2:

$N + 50 \sin 30° - 10 \times g = 0$

$\Rightarrow \quad N = 73.1$ N

$50 \cos 30° - \mu N = ma$

$50 \cos 30° - \mu(73.1) = 10 \times a$



$\Rightarrow \quad a = 2.87 \text{ m/s}^2$

$\therefore \quad v(5) = v_o + a\,(5)$

$= 14.35$ m/s

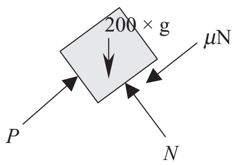
$s(5) = v_o\,(5) + \frac{1}{2}a(5)^2$

$= 35.88$ m

14.3:

$N - 200 \times g \cos 25° = 0$

$\Rightarrow \quad N = 1778.2$ N



(i) **When moving with constant velocity:**

$P - 200 \times g \sin 25 - \mu N = 0$

$\Rightarrow \quad P = 1273.7$ N

(ii) **When moving with constant acceleration:**

$P - 200 \times g \sin 25 - \mu N = ma$

$\Rightarrow \quad P = 200 \times g \sin 25 + (0.25)\,(1778.2) + (200 \times 0.5)$

$= 1373.7$ N

14.4:

$\vec{F} = 5t\,\vec{i} + 2t^2\,\vec{j}$

$\therefore \quad \vec{a} = \frac{\vec{F}}{m} = 2.5\,t\,\vec{i} + t^2\,\vec{j}$

$\therefore \quad \vec{v} = 1.25\, t^2\, \vec{i} + \dfrac{t^3}{3}\, \vec{j}$

$\vec{r} = 1.25 \dfrac{t^2}{3}\, \vec{i} + \dfrac{t^4}{12}\, \vec{j}$

$\vec{a}\,(2) = 5\,\vec{i} + 4\,\vec{j} \qquad \therefore \quad |\vec{a}\,| = 6.4 \text{ m/s}^2; \quad \theta_a = 38.66°$

$\vec{v}\,(2) = 5\,\vec{i} + 2.67\,\vec{j} \qquad \therefore \quad |\vec{v}\,| = 5.67 \text{ m/s}; \quad \theta_v = 28.1°$

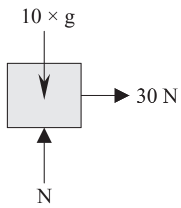
$\vec{r}\,(2) = 3.33\,\vec{i} + 1.33\,\vec{j} \qquad \therefore \quad |\vec{r}\,| = 3.59 \text{ m}; \quad \theta_r = 21.77°$

14.5:

$m = 10$ kg

$F = 30$ N

$N = 10 \times g = 98.1$ N



$30 = ma$

$\Rightarrow \quad a = 3 \text{ m/s}^2$

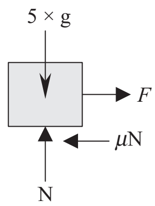
$v(3) = 5 + 3\,(3)$

$= 14$ m/s

$s(3) = 5(3) + \dfrac{1}{2}(3)(3)^2$

$= 28.5$ m

14.6:



$F = 20$ N

$m = 5$ kg

$F - f = ma$

$F - \mu N = ma$

$20 - (0.25)(5 \times 9.81) = 5 \times a$

$\Rightarrow \quad a = 1.55 \text{ m/s}^2$

(i) $\quad v = v_o + at$

$v(5) = 0 + (1.55)\,(3) = 4.65$ m/s

(ii) $\quad s = v_o t + \dfrac{1}{2} a t^2$

$s(5) = 0 + \dfrac{1}{2}(1.55)\,(3)^2 = 6.98$ m

$-\mu N = ma$

$\Rightarrow \quad a = -\mu g = -2.45 \text{ m/s}^2$

$v^2 = v_o^2 + 2\,as$

$0 = (4.65)^2 + 2\,(-2.45)s$

$\Rightarrow \quad s = 4.41$ m

$v = v_o + at$

$0 = 4.65 + (-2.45)t$

$\Rightarrow \quad t = 1.9s$

14.7:

$m = 3$ tons

$v_o = 60$ kmph $= 16.67$ m/s

$s = 150$ m

$v^2 = v_o^2 + 2as$

$$\Rightarrow \quad a = \frac{0-(16.67)^2}{2(150)} = -0.926 \text{ m/s}^2$$

$\therefore$ Braking force $= ma$

$= 2778$ N $= 2.8$ kN

$v = v_o + at$

$$\Rightarrow \quad t = \frac{0-16.67}{-0.926} = \mathbf{18}\,\boldsymbol{s}$$

14.8:

$m = 10$ tons

$v = 54$ kmph $= 15$ m/s

$t = 10$ s

$v = v_o + at$

$$\Rightarrow \quad a = \frac{15}{10} = 1.5 \text{ m/s}^2$$

$f = (1$ kN/ton$)\,(10$ tons$) = 10$ kN

$P - f = ma$

$\Rightarrow \quad P = f + ma$

$= (10 \times 10^3) + (10 \times 10^3)\,(1)$

$= 20 \times 10^3$

$=$ **20 kN**

14.9:

$m = 20$ tons

$v = 200$ kmph $= 55.56$ m/s

$s = 300$ m

$f = (1$ kN/ton$)\,(20) = 20$ kN

$v^2 = v_o^2 + 2as$

$$\Rightarrow \quad a = \frac{(55.56)^2}{2\,(300)} = 5.14 \text{ m/s}^2$$

$P - f = ma$

$\Rightarrow \quad P = 122.8$ kN

14.10:

$m = 2$ tons

$v_o = 60$ kmph $= 16.67$ m/s

$t = 5$ s

$v = v_o + at$

$0 = 16.67 + a\,(5)$

$\Rightarrow \quad a = -3.334 \text{ m/s}^2$

$\therefore$ Braking force $= ma = 6.67$ kN.

4.11:

$m = 2$ tons

$v_o = 0$

$v_o = 60$ kmph $= 16.67$ m/s

$t = 15$ s

$f = (0.5 \text{ kN/ton})\,(2) = 1$ kN

$v = v_o + at$

$\Rightarrow \quad a = +1.11 \text{ m/s}^2$

(i) $P - f = ma$

$\Rightarrow \quad P = (1 \times 10^3) + (2 \times 10^3)\,(1.11) = 3.22$ kN

(ii) when switched off:

$-f = \text{ma}'$

$\Rightarrow \quad a' = -0.5 \text{ m/s}^2$

$$\therefore \quad t = \frac{0 - v_o}{a} = \frac{0 - 16.67}{-0.5} = 33.34 \text{ s}$$

$v^2 = v_o^2 + 2\,as$

$$\Rightarrow \quad s = \frac{0 - (16.67)^2}{2\,(-0.5)} = 277.9 \text{ m}$$

14.12:

$m = 500$ kg

$v_o = 45$ kmph $= 12.5$ m/s

$v = 0$

$t = 6$ s

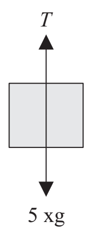
$$\therefore \quad a = \frac{v - v_o}{t} = -2.083 \text{ m/s}^2$$

$-\mu mg = ma$

$$\Rightarrow \quad \mu = \frac{-2.083}{-9.81} = 0.212$$

$f = \mu mg = 1.04$ kN

14.13:



$m = 5$ kg

$T_{allow} = 60$ N

$T_{allow} - mg = ma$

$\Rightarrow \quad a = \frac{T_{\text{allow}}}{m}\ 9$

$= 2.19\ \text{m/s}^2$

14.14:

$m = 2$ tons

(i) $T - mg = ma$

$(23 \times 10^3) - (2 \times 10^3)\,(9.81) = (2 \times 10^3)\,a$

$\Rightarrow \quad a = 1.69\ \text{m/s}^2$

(ii) $T - mg = ma$

$\Rightarrow \quad a = -\ 0.81\ \text{m/s}^2$

(iii) $mg - T = ma$

$\Rightarrow \quad a = 1.81\ \text{m/s}^2$

(iv) $mg - T = ma$

$\Rightarrow \quad a = -\ 0.69\ \text{m/s}^2$

14.15:

$M = 600$ kg

$m = n \times 60$

$v = 3$ m/s

$t = 2$s

$T_{allow} = 12$ kN

$a = \frac{v - v_o}{t} = 1.5\ \text{m/s}^2$

$T - (M + m)\,g = (M + m)\,a$

$T = (M + m)\,(g + a)$

$12 \times 10^3 = (600 + 60n)\,(9.81 + 1.5)$

$\Rightarrow \quad n = 7.68$

$\therefore$ No. of persons = 7

14.16:

$m = 6$ tons

$v = 4$ m/s

$T = 50$ kN

$mg - T = -\ ma$

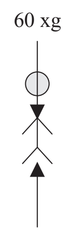
$\Rightarrow \quad a = -\left[g - \frac{T}{m}\right] = -\ 1.48\ \text{m/s}^2$

$\therefore \quad v^2 = v_o^2 + 2\,as$

$0 = (4)^2 + 2\,(-1.48)\,s$

$\Rightarrow \quad s = 5.41$ m

14.17:



$m = 60$ kg

$R - mg = ma$

(i) $80 \times g - 60 \times g = 60 \times a$

$\Rightarrow \quad a = 3.27\ \text{m/s}^2$ (accelerating upwards)

(ii) $50 \times g - 60 \times g = 60 \times a$

$\Rightarrow \quad a = -1.635\ \text{m/s}^2$ decelerating upwards or accelerating downwards.

$\approx -1.64\ \text{m/s}^2$

(iii) $0 - 60 \times g = 60 \times a$

$\Rightarrow a = -g$ accelerating downwards or freely falling

14.18:

$T = 50$ N

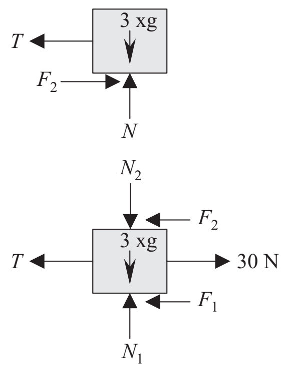
$a = 1$ m/s^2

$T - mg = ma$

$\Rightarrow \quad m = \dfrac{T}{g+a} = 4.63$ kg

$\therefore \quad W = mg = 45.4$ N

14.19:



$N_2 = 3 \times g$

$\therefore \quad F_2 = \mu N_2 = 0.45 \times g$

$T - F_2 = m_2\, a$

$T = 0.45 \times g + 3a$ ...(1)

$N_1 - N_2 - 5 \times g = 0$

$\Rightarrow \quad N_1 = 8g$

$F_1 = \mu N_1 = 1.2 \times g$

$-30 - T - F_1 - F_2 = m_1 a$

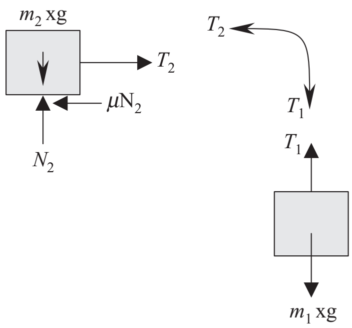
$T = 30 - 1.2 \times g - 0.45 \times g - 5a$ ... (2)

$0.45 \times g + 3a = 30 - 1.2 \times g - 0.45 \times g - 5a$

$\Rightarrow \quad a = 1.18$ m/s^2

$\therefore \quad T = 7.95$ N

14.20:



Block m_1:

$m_1 \times g - T_1 = m_1\, a$

$\Rightarrow \quad T_1 = m_1\, [g - a]$

Block m_2:

$N_2 = m_2 \times g$

$T_2 - \mu N_2 = m_2\, a$

$\Rightarrow \quad T_2 = \mu\, m_2 \times g + m_2 a$

$$\frac{T_1}{T_2} = e^{\mu \cdot \frac{\pi}{2}} = 1.369$$

$$\frac{m_1\,[g-a]}{m_2\,[\mu g + a]} = 1.369$$

$$\frac{5[9.81 - a]}{8[9.81 \times 0.2 + a]} = 1.369$$

$\Rightarrow \quad a = 1.73$ m/s^2

$\therefore$ $T_1 = 5\ [9.81 - 1.73] = 40.4$ N

$T_2 = 8\ [0.2 \times 9.81 + 1.73]$

$= 29.5$ N

14.21:

$m_1 = 8$ kg

$m_2 = 2$ kg

$m_3 = 3$ kg

$m_1 g - T_1 = m_1 a_1$ ...(a)

$T_1 - 2T_2 = 0$ ...(b)

$T_2 - m_2 g = m_2 [a_2 + a_1]$ ...(c)

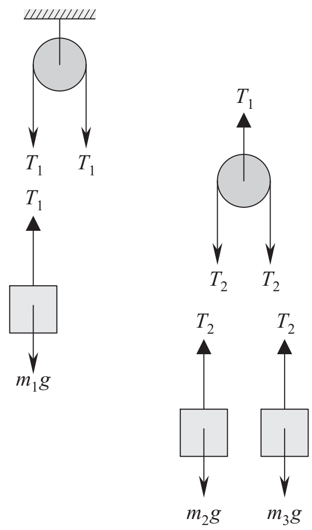
$m_3 g - T_2 = m_3 [a_2 - a_1]$ ...(d)

(c) $\div m_2$ $\Rightarrow$ $\dfrac{T_2}{m_2} - g = a_2 + a_1$

(d) $\div m_3$ $\Rightarrow$ $g - \dfrac{T_2}{m_3} = a_2 - a_1$

$$T_2\left[\frac{m_2 + m_3}{m_2 m_3}\right] - 2g = 2a_1$$

$$T_2\left[\frac{m_2 + m_3}{m_2 m_3}\right] = 2\ [g + a_1]$$



From (a),

$T_1 = m_1 [g - a_1]$

From (b),

$$m_1[g - a_1] = 4\ [g + a_1]\frac{m_2 m_3}{m_2 + m_3}$$

$$8\ [g - a_1] = 4[g + a_1]\ \frac{2 \times 3}{5}$$

$$8g - 8a_1 = 4.8g + 4.8a_1$$

$\Rightarrow$ $a_1 = \dfrac{3.2\,g}{12.8} = \dfrac{g}{4}$

Substituting this in eq. (a),

$$T_1 = m_1[g - a_1] = 8\left[g - \frac{g}{4}\right] = 6g$$

$\therefore$ $T_2 = 3g$

Substituting this in eq. (c),

$3g - 2g = 2\ [a_2 + a_1]$

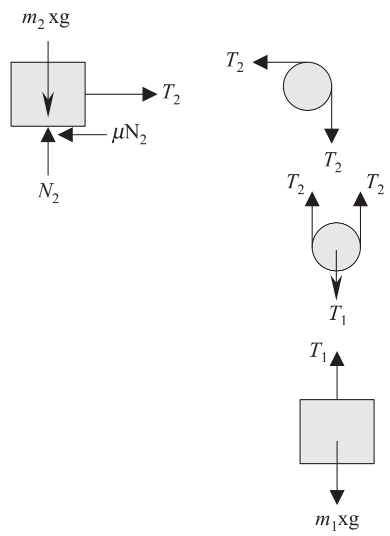
$\Rightarrow$ Accln. of 2 kg block $= \dfrac{g}{2}$

Substituting in eq. (d),

$3g - 3g = 3\,[a_2 - a_1]$

$\Rightarrow$ Accln. of 3 kg black = 0.

14.22



$$2T_2 = T_1$$

$$a_1 = \frac{a_2}{2}$$

If $a_1 = a$, $a_2 = 2a$

$$N_2 = m_2 \times g$$

$$T_2 - \mu N_2 = m_2\, a_2$$

$$\Rightarrow \quad T_2 = \mu m_2 \times g + 2\, m_2\, a$$

$$= m_2\, [\mu g + 2a]$$

$$m_1 \times g - T_1 = m_1\, a_1$$

$$\Rightarrow \quad T_1 = m_1 \times g - m_1\, a$$

$$= m_1\, [g - a]$$

since $2T_2 = T_1$

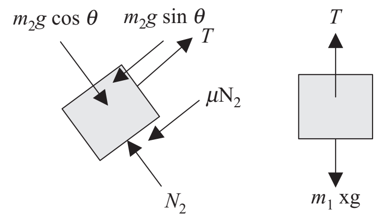
$$2\, m_2\, [\mu g + 2a] = m_1\, [g - a]$$

$$\Rightarrow \quad a = 0.88 \text{ m/s}^2$$

$$\therefore \quad a_1 = 0.88 \text{ m/s}^2 \;\;\&\;\; a_2 = 1.76 \text{ m/s}^2$$

$$\therefore \quad T_1 = 44.65 \text{ N} \;\;\&\;\; T_2 = 22.33 \text{ N}$$

14.23:



For impending motion of m_2 down the incline,

$$T = m_2\, g \sin\theta - \mu mg \cos\theta$$

$$= 25.65 \text{ N}$$

For equilibrium of m_1, $T = m_1 \times g = 58.86$ N.

Hence, we conclude that the block of mass m_2 moves up the plane.

$$m_1 \times g - T = m_1 a$$

$$\Rightarrow \quad T = m_1\, [g - a] \qquad \text{...(a)}$$

$$N_2 = m_2\, g \cos\theta$$

$$T - m_2\, g \sin\theta - \mu m_2\, g \cos\theta = m_2 a$$

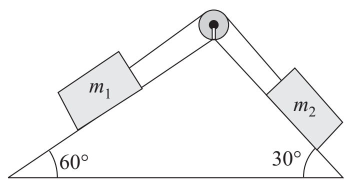
$$\Rightarrow \quad T = m_2\, [g\,(\sin\theta + \mu \cos\theta) + a] \qquad \text{...(b)}$$

From (a) & (b),

$$a = 0.43 \text{ m/s}^2$$

$$T = 56.3 \text{ N}$$

14.24:



$$m_1 = 30 \text{ kg}$$

$$m_2 = 25 \text{ kg}$$

Since $m_1 g \sin \theta_1 > m_2\, g \sin \theta_2$, block m_1 moves down the incline.

$$m_1\, g \sin 60° - T = m_1\, a$$

$$\Rightarrow \quad T = m_1\, [g \sin 60° - a] \qquad \text{...(a)}$$

$$T - m_2\, g \sin 30° = m_2\, a$$

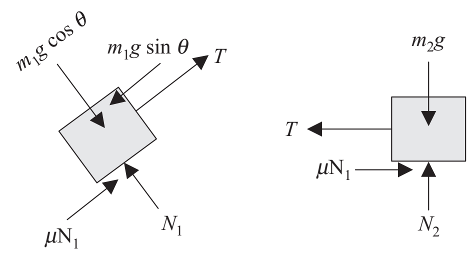
$$\Rightarrow \quad T = m_2\, [g \sin 30° + a] \qquad \text{...(b)}$$

$$\therefore \quad m_1\, [g \sin 60° - a] = m_2\, [g \sin 30° + a]$$

$$\Rightarrow \quad a = 2.404 \text{ m/s}^2$$

$$T = 182.7 \text{ N}$$

14.25:



Block-I:

$$\Sigma F_y = 0$$

$$\Rightarrow \quad N_1 = m_1\, g \cos \theta$$

$$\Sigma F_x = m_1\, a$$

$$\Rightarrow \quad m_1\, g \sin \theta - \mu m_1\, g \cos \theta - T = m_1\, a$$

Block–II:

$$\Sigma F_y = 0$$

$$\Rightarrow \quad N_2 = m_2\, g$$

$$\Sigma F_x = 0$$

$$T - \mu m_2\, g = m_2\, a$$

Adding the two equations,

$$m_1\, g \sin \theta - \mu m_1\, g \cos \theta - \mu m_2\, g = (m_1 + m_2)\, a$$

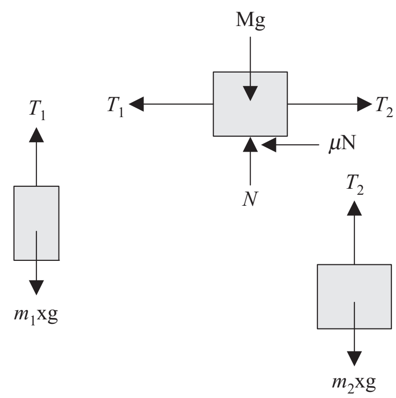
$$\Rightarrow \quad a = \frac{m_1 g\, (\sin \theta - \mu \cos \theta) - \mu m_2 g}{m_1 + m_2}$$

$$= 0.33 \text{ m/s}^2$$

$$T = m_2[\mu g + a]$$

$$= \mathbf{11.5\ N}$$

14.26:



Since $m_2 > m_1$, the block on the table move to the right.

$$m_2 \times g - T_2 = m_2\, a$$

$$T_2 - \mu Mg - T_1 = Ma$$

$$T_1 - m_1 \times g = m_1\, a$$

Adding up,

$$m_2 \times g - m_1 \times g - \mu M g = [m_1 + m_2 + M]a$$

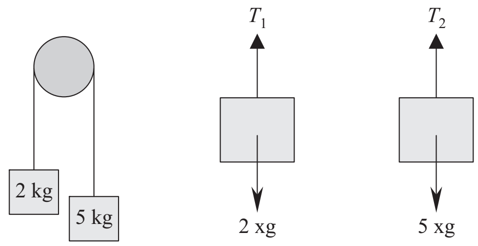
$$\Rightarrow \quad a = \frac{[m_2 - m_1 - \mu M]}{m_1 + m_2 + M} g$$

$= 1.25 \text{ m/s}^2$

$T_2 = m_2 [g - a] = 42.8 \text{ N}$

$T_1 = m_1 [g + a] = 33.2 \text{ N}$

14.27:



$5 \times g - T_2 = 5a$

$\Rightarrow \quad T_2 = 5 [g - a]$ ...(a)

$T_1 - 2 \times g = 2a$

$\Rightarrow \quad T_1 = 2 [g \times a]$ ...(b)

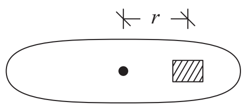
$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{\mu \cdot \pi} = 1.874$$

$$\therefore \quad \frac{5[g-a]}{2[g+a]} = 1.874$$

$\Rightarrow \quad a = 1.4 \text{ m/s}^2$

$T_1 = 22.42 \text{ N}$ and $T_2 = 42.05 \text{ N}$

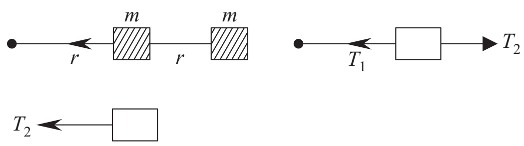
14.28:



$$\frac{mv^2}{r} = \mu mg$$

$\Rightarrow \quad v = \sqrt{\mu gr}$

14.29:



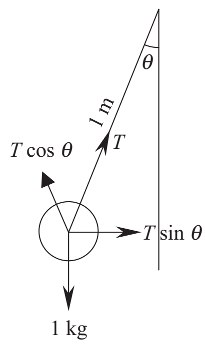
$$(T_1 - T_2) = \frac{mv^2}{r} \quad \Rightarrow \quad T_1 = 3mr\,\omega^2$$

$$= \frac{mr^2\omega^2}{r} \qquad T_2 = \frac{mv^2}{2r}$$

$$= mr\,\omega^2 \qquad = \frac{m(2r\omega)^2}{2r}$$

$$= 2mr\,\omega^2$$

14.30:



$T_{max} = 2 \times g = 19.62 \text{ kN}$

$$T_{max} = \frac{mv^2}{r}$$

$$\Rightarrow \quad v^2 = \frac{19.62 \times 1}{0.5}$$

$\therefore \quad v = 6.26 \text{ m/s}$

14.31:

$\omega = 60 \text{ rpm}$

$= 2\pi \text{ rad/s}$

$$T \cos \theta = mg$$
$$T \sin \theta = mr\,\omega^2$$
$$= ml \sin \theta\, \omega^2$$
$$\Rightarrow \quad T = ml\,\omega^2$$
$$= 1 \times 1 \times (2\pi)^2$$
$$= 39.5 \text{ N}$$
$$\theta = \cos^{-1}\left[\frac{mg}{T}\right]$$
$$= 75.6°$$

14.32:

$$T_{\max} = 50 \text{ N}$$
$$\Rightarrow \quad 50 = m_1\omega^2$$
$$\therefore \quad \omega = 7.07 \text{ rad/s}$$
$$= 67.5 \text{ rpm}$$
$$\theta = \cos^{-1}\left[\frac{mg}{T}\right]$$
$$= 78.7°$$

14.33:

$$m = 1 \text{ gm}$$
$$r = 1 \text{ cm}$$
$$\omega = 100 \text{ rpm} = 10.47 \text{ rad/s}$$
$$R \cos \theta = mg \qquad \text{...(a)}$$
$$R \sin \theta = m[(R - r) \sin \theta]\, \omega^2$$
$$\Rightarrow \quad R = m\,(R - r)\,\omega^2 \qquad \text{...(b)}$$
$$\therefore \quad \text{(a)} \div b \Rightarrow \cos \theta = \frac{g}{(R - r)\omega^2}$$
$$= \frac{9.81}{(0.14)\,(10.47)^2}$$
$$\therefore \quad \theta = \mathbf{50.3°}$$