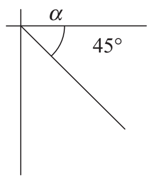
Chapter 13

13.1:

$$y = x^2 + x - 4$$
$$y/_{x=2\text{m}} = 2^2 + 2 - 4 = 2 \text{ m}$$
$$y/_{x=3\text{m}} = 3^2 + 3 - 4 = 8 \text{ m}$$
$$\Delta x = 3 - 2 = 1 \text{ m}$$
$$\Delta y = 8 - 2 = 6 \text{ m}$$
$$\therefore \quad r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{37} = 6.08 \text{ m}$$
$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = 80.54° \text{ to the horizontal}$$

13.2:

$$y = 25 - x^2/2$$
$$y/_{x=0\text{ m}} = 25 \text{ m}$$
$$y/_{x=2\text{ m}} = 25 - 2^2/2 = 23 \text{ m}$$
$$\Delta_x = 2 - 0 = 2 \text{ m}$$
$$\Delta_y = 23 - 25 = -2 \text{ m}$$
$$\therefore \quad r = \sqrt{\Delta x^2 + \Delta y^2} = (\sqrt{2})^2 = 2.83 \text{ m}$$
$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = 45°$$



13.3:

$$x = t^2 \qquad y = 2\,t + 3$$
$$\frac{dx}{dt} = 2\,t \qquad \frac{dy}{dt} = 2$$
$$\frac{d^2x}{dt^2} = 2 \qquad \frac{d^2y}{dt^2} = 0$$
$$x/_{t=2s} = 4 \text{ m} \qquad y = 7\text{m}$$
$$\dot{x} = 4 \text{ m/s} \qquad \dot{y} = 2$$
$$\ddot{x} = 2 \text{ m/s}^2 \qquad \ddot{y} = 0$$

(i) $$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 7^2} = 8.06 \text{ m.at}$$
$$\theta_r = \tan^{-1}\left[\frac{7}{4}\right] = 60.26°$$

(ii) $$v = \sqrt{4^2 + 2^2} = 4.47 \text{ m/s}$$
$$\theta_v = \tan^{-1}\left[\frac{2}{4}\right] = 26.57°$$

(iii) $a = \sqrt{2^2 + 0} = 2 \text{ m/s}^2$

$$\theta_a = \tan^{-1}\left[\frac{0}{2}\right] = 0°$$

13.4:

$$v_x = 4t^2 + 3t + 2 \qquad v_y = 2t^2 - 3$$

$$x = 4\frac{t^3}{3} + 3\frac{t^2}{2} + 2t + x_o \qquad y = \frac{2t^3}{3} - 3t + y_o$$

$$x_o, y_o = 0$$

$$\therefore \quad x = \frac{4t^3}{3} + \frac{3}{2}t^2 + 2t \qquad y = \frac{2}{3}t^3 - 3t$$

$$x/_{t\,=\,3\text{s}} = 55.5 \text{ m} \qquad y/_{t\,=\,3s} = 9 \text{ m}$$

$$\therefore \quad r = 56.2 \text{ m}$$

$$\theta_n = 9.2°$$

$$v_x/_{t\,=\,3\text{s}} = 47 \text{ m} \qquad v_y/_{t\,=\,3s} = 15 \text{ m}$$

$$v = \sqrt{v_x^2 + v_y^2} = 49.3 \text{ m/s}$$

$$\theta_v = \tan^{-1}\left[\frac{15}{47}\right] = 17.7°$$

$$a_x = 8\text{t} + 3 \qquad a_y = 4t$$

$$a_x/_{t\,=\,3\text{s}} = 27 \text{ m/s}^2 \qquad a_y/_{t\,=\,3\text{s}} = 12 \text{ m/s}^2$$

$$\therefore \quad a = \sqrt{a_x^2 + a_y^2} = 29.5 \text{ m/s}^2$$

$$\theta_a = \tan^{-1}\left[\frac{12}{27}\right] = 24°$$

13.5:

$$v_o = 60 \text{ m/s}$$

$$R = 300 \text{ m}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

$$300 = \frac{(60)^2 \sin 2\alpha}{9.81} \qquad \Rightarrow \quad 2\alpha = 54.83° \text{ (or) } 125.17°$$

$$\alpha = 27.42° \text{ (or) } 62.58°$$

13.6:

$$R = 60 \text{ m}$$

$$T = 3\text{s}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g} = 2\,\frac{v_0^2 \sin\alpha \cos\alpha}{g}$$

$$T = 2\,\frac{v_0 \sin\alpha}{g}$$

$$\frac{R}{T} = v_o \cos\alpha$$

$$\frac{T}{R/T} = \frac{2\tan\alpha}{g}$$

$$\frac{T^2}{R} = \frac{2\tan\alpha}{g}$$

$\Rightarrow$ $\alpha = 36.3°$

$v_o = 24.8$ m/s

13.7:

$h = 2$ m

$$v_o = 54 \text{ kmph} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{(v_0\cos\alpha)^2}$$

$$-2 = x\tan 0 - \frac{1}{2} \times \frac{9.81\,x^2}{(15)^2}$$

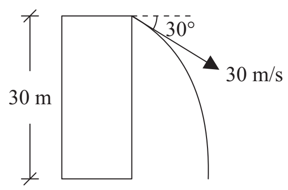
$\Rightarrow$ $x = 9.58$ m

≈ 9.6 m

$x = (v_o \cos\alpha)\,t$

$\Rightarrow$ $t = \dfrac{9.6}{15} = 0.64$ s

13.8:



$(v_o)_x = v_o \cos 30° = 30 \cos 30° = 25.98$ m/s

$(v_o)_y = -v_o \sin 30° = -30 \sin 30° = -15$ m/s

$$-30 = -(v_o)_y\, t - \frac{1}{2}gt^2$$

$$-30 = -15\,t - 4.905\,t^2$$

$$4.905\,t^2 + 15\,t - 30 = 0$$

$$t = \frac{-15 \pm 28.52}{9.81} = 1.38\,\text{s}$$

$x = (v_o)_x\, t = (25.98)(1.38) = 35.9$ m

13.9:

$(v_o)_x = 30$ m/s

$(v_o)_y = 0$

$$-30 = 0 - \frac{1}{2}gt^2$$

$$t = 2.47 \text{ s}$$
$$x = (30)t = 74.1 \text{ m}$$

13.10:

$$(v_o)_x = 30 \cos 30° = 25.98 \text{ m/s}$$
$$(v_o)_y = 30 \sin 30° = 15 \text{ m/s}$$
$$-30 = 15\,t - \frac{1}{2} g t^2$$
$$4.905\,t^2 - 15\,t - 30 = 0$$
$$t = \frac{15 \pm 28.52}{9.81} = 4.44 \text{ s}$$
$$x = (25.98)\,(4.44)$$
$$= 115.35 \text{ m}$$

13.11:

$$(v_o)_x = 10 \text{ m/s} = v_o \cos \alpha$$
$$t = 3.5\text{s} = \frac{2\,v_o \sin \alpha}{g}$$
$$v_o = \frac{10}{\cos 59.78°} = 19.9 \text{ m/s}$$
$$\Rightarrow \qquad \frac{2 \tan \alpha}{g} = \frac{3.5}{10}$$
$$\text{Max. } ht = \frac{v_o^2 \sin^2 \alpha}{2g} = 15.1 \text{ m}$$
$$\alpha = \mathbf{59.8°}$$

13.12:

$$\text{Max. } ht = 15 \text{ m}$$
$$\alpha = 60°$$
$$\frac{v_0^2}{2g} = 15 \;\Rightarrow\; v_o = 17.16 \text{ m/s}$$
$$\text{Max. } ht = \frac{v_o^2 \sin^2 \alpha}{2g} = \frac{(17.16 \times \sin 60°)^2}{2 \times 9.81} = 11.26 \text{ m}$$
$$\frac{v_o^2}{2g} = 11.26 \;\Rightarrow\; v_o = 14.86 \text{ m/s}$$

13.13:

(i) $v_o = 20$ m/s

$\alpha = 30°$

$h = 25$ m

$$y = x \tan \alpha - \frac{1}{2}\frac{gx^2}{v_o^2 \cos^2 \alpha}$$

$$-25 = x \tan 30° - \frac{1}{2} \times 9.81 \frac{x^2}{(20 \times \cos 30°)^2}$$

$$-25 = 0.577\,x - 0.01635\,x^2$$

$$0.01635\,x^2 - 0.577\,x - 25 = 0$$

$$\Rightarrow \quad x = 60.55 \text{ m}$$

$$\approx 60.6 \text{ m}$$

$$t = \frac{x}{v_o \cos \alpha} = \frac{60.6}{20 \times \cos 30°} = 3.5 \text{ s}$$

(ii) $v_y = (v_o)_{y-gt}$

$= 20 \sin 30° - 9.81\,(3.5)$

$= -24.34$ m/s

$v_x = 20 \cos 30° = 17.32$ m/s

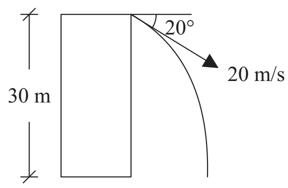
$v = 29.9$ m/s

$$\theta = \tan^{-1} \frac{v_y}{v_x} = 54.6°$$

$$\text{Max. } ht = \frac{v_o^2 \sin^2 \alpha}{2g} = 5.1 \text{ m}$$

$\therefore$ from gr. level $= 25 + 5.1 = 30.1$ m

13.14:



$(v_o)_x = 20 \cos 20° = 18.8$ m/s

$(v_o)_y = -20 \sin 20° = -6.84$ m/s

$$-30 = -(v_o)_y\, t - \frac{1}{2} gt^2$$

$$= -6.84\,(t) - \frac{1}{2} 9.81\, t^2$$

$4.905t^2 + 6.84\,t - 30 = 0$

$t = 1.87$ s

$x = (v_o)_x\, t$

$= 35.16$ m

$v_y = (v_o)_y - gt$

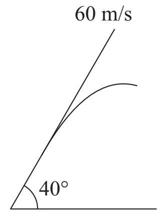
$= -6.84 - 9.81\,(1.87)$

$= -25.18$ m/s

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = 31.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left[\frac{v_y}{v_x}\right] = 53.3°$$

13.15:



$v_o = 60$ m/s $\qquad (v_o)_x = 60 \cos 40° = 45.96$ m/s

$\alpha = 40° \qquad (v_o)_y = 60 \sin 40° = 38.57$ m/s

$$v = \sqrt{v_x^2 + v_y^2}$$

$$50 = \sqrt{(45.96)^2 + v_y^2}$$

$$\Rightarrow \quad v_y = 19.7 \text{ m/s}$$

$$v_y = (v_o)_y - gt$$

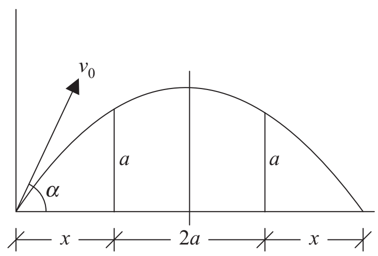
$$\Rightarrow \quad t = 1.92 \text{ s}$$

$$x = (v_o)_x \cdot t = 88.2 \text{ m}$$

$$y = (v_o)_y \cdot t - \frac{1}{2} gt^2$$

$$= 56 \text{ m}$$

13.16:



$$y = x \tan \alpha - \frac{1}{2} \frac{g x^2}{v_o^2 \cos \alpha}$$

$$a = x \tan \alpha - \frac{1}{2} \frac{g x^2}{v_o^2 \cos^2 \alpha}$$

$$\frac{v_o^2 \sin^2 \alpha}{2g} = (a + x) \tan \alpha - \frac{1}{2} \frac{g(a+x)^2}{x_o^2 \cos^2 \alpha}$$

$$\text{Range} = \frac{v_o^2 \sin 2\alpha}{g} = 2\,(a + x)$$

$$\frac{\cancel{v_o^2} \sin^2 \alpha}{\cancel{2}\cancel{g}} = \frac{\cancel{v_o^2} \sin 2\alpha}{\cancel{2}\cancel{g}} \tan \alpha - \frac{1}{\cancel{2}} \cancel{g} \frac{\cancel{v_o^2} \sin^2 2\alpha}{4 \cancel{g^2}\, \cancel{v_o^2} \cos^2 \alpha}$$

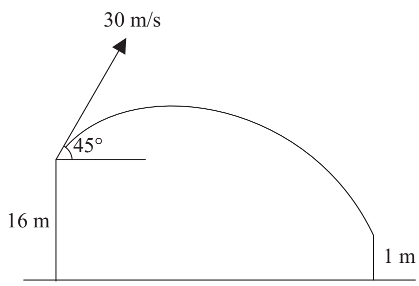
$$\sin^2 \alpha = 2 \sin \alpha \cos \alpha \tan \alpha - \frac{\cancel{4} \sin^2 \alpha \cos^2 \cancel{\alpha}}{\cancel{4} \cos^2 \cancel{\alpha}}$$

$$= 2 \sin^2 \alpha - \sin^2 \alpha$$

$$a = x \tan \alpha \frac{-1}{2} g \frac{x^2}{x_o^2 \cos^2 \alpha}$$

$$a = (2a + x) \tan \alpha \;\frac{-1}{2} g \frac{(2a + x)^2}{x_o^2 \cos^2 \alpha}$$

13.17:



$v_o = 30$ m/s

$\alpha = 45°$

$y = -0.6$ m

$$-0.6 = x \tan 45° - \frac{1}{2}\frac{(9.81)x^2}{(30 \cos 45)^2}$$

$$0.0109\,x^2 - x - 0.6 = 0$$

$$x = 92.34 \text{ m}$$

$$t = \frac{x}{v_o \cos\alpha} = 4.35 \text{ s}$$

$$v_y = (v_o)_y - gt = -21.46 \text{ m/s}$$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = 30.2 \text{ m/s}$$

$$\theta = \tan^{-1}\left[\frac{v_y}{v_x}\right] = 45.3°$$

13.18:

$$v_o = 20 \text{ m/s}$$

$$\alpha = 60° \qquad v_o = 20 \text{ m/s at } 60°$$

$$\text{Range} = \frac{v_o^2 \sin 2\alpha}{g} = 35.3 \text{ m}$$

13.19:

$$(v_o)_x = 20 \cos 60° + 5 = 15 \text{ m/s}$$

$$(v_o)_y = 20 \sin 60° = 17.32 \text{ m/s}$$

$$T = \frac{2\,v_o \sin\alpha}{g} = 3.53\text{s}$$

Horizontal distance = 52.95 m

Distance between = (52.95 – 5 × 3.53) = 35.3 m

$$v_y = (v_o)_y = 17.32 \text{ m/s}$$

$$v_x = 15 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 22.9 \text{ m/s}$$

$$\alpha = \tan^{-1}\left[\frac{v_y}{v_x}\right]$$

$$= 49.1°$$

$$52.95 - 35.3 = 17.65 \text{ m}$$

13.20:

$$v_o = 60 \text{ m/s}$$

$$\alpha = 50°$$

$$\text{Range} = \frac{v_o^2 \sin 2\alpha}{g} = 361.4 \text{ m}$$

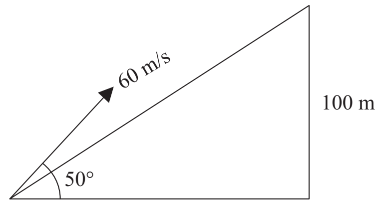
$$\rightarrow \text{Range} = 361.4 - 50 = 311.4 \text{ m}$$

$$= \frac{v_o^2 \sin 2\alpha}{g}$$

$\Rightarrow$ $2\alpha = 58.06°$ (or) $121.94°$

$\alpha = 29.03°$ (or) $60.97°$

13.21:

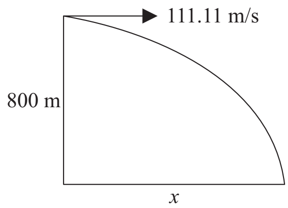


$$100 = x \tan 50° - \frac{1}{2}\frac{(9.81)\,x^2}{(60\cos 50°)^2}$$

$$3.298 \times 10^{-3} \cdot x^2 - 1.192\,x + 100 = 0$$

$$x = \frac{1.192 \pm 0.319}{2 \times 3.298 \times 10^{-3}} = 132.4 \text{ m}$$

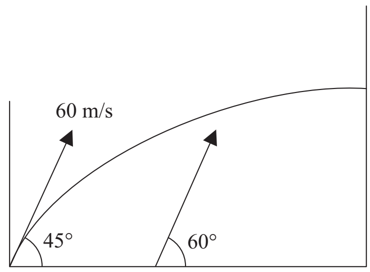
13.22:



$$-800 = x \cdot \tan(0) - \frac{1}{2}\frac{(9.81)\,x^2}{(111.11\cos 0)^2}$$

x = **1.42 km**

13.23:



$$R_{\max} = \frac{v_o^2 \sin^2\alpha}{2g} = \frac{60^2 \sin^2 45°}{2 \times 9.81} = 91.74 \text{ m}$$

$$x = \frac{v_o^2 \sin 2\alpha}{2g} = \frac{60^2 \sin 90°}{2 \times 9.81} = 183.5 \text{ m}$$

$$y = x \tan\alpha - \frac{1}{2}\frac{g\,x^2}{v_o^2 \cos^2\alpha}$$

$$91.74 = x \tan 60° - \frac{1}{2}\frac{9.81 \times x^2}{(60 \times \cos 60)^2}$$

$$5.45 \times 10^{-3}\, x^2 - 1.732\,x + 91.74 = 0$$

$$x = \frac{1.732 \pm 1}{2 \times 5.45 \times 10^{-3}} = 67.16 \text{ m}$$

$\therefore$ Distance to be moved = 183.5 – 67.16 = 116.3 m

13.24:

$\beta = 30°$

$v_o = 15$ m/s

$\alpha = 50°$

$$t = \frac{2v_o}{g\cos\beta}\,[\sin(\alpha - \beta)]$$

$$= 1.21 \text{ s}$$

$$R = \frac{v_o^2}{g\cos^2\beta}[\sin(2\alpha - \beta) - \sin\beta]$$

$$= 13.45 \text{ m}$$

$$R_{\max} = \frac{v_o^2}{g(1+\sin\beta)} = 15.29 \text{ m}$$

$$\alpha = 45° + \frac{\beta}{2} = 60°$$

13.25:

$$v_o = 15 \text{ m/s}$$

$$\alpha = 20°$$

$$\beta = 30°$$

$$t = \frac{2\,v_o}{g\cos\beta}[\sin(\alpha + \beta)]$$

$$= 2.71 \text{ s}$$

$$R = \frac{v_o^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$$

$$= 44.03 \text{ m}$$

$$R_{\max} = \frac{v_o^2}{g(1-\sin\beta)} = 45.9 \text{ m}$$

$$\alpha = 45° - \frac{\beta}{2} = 30°$$

13.26:

$$y = 3x^2$$

$$\frac{dy}{dx} = 6\text{x}$$

$$\frac{d^2y}{dx^2} = 6$$

$$\therefore \qquad \rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{[1+(6x)^2]^{3/2}}{6}$$

$$\therefore \qquad \rho/_{x=2} = \frac{[1+(12)^2]^{3/2}}{6} = 291 \text{ m}$$

$$a_t = 0$$

$$a_n = \frac{v^2}{\rho} = \frac{(15)^2}{291} = 0.773 \text{ m/s}^2$$

$$\therefore \quad a = \sqrt{a_t^2 + a_n^2} = 0.733 \text{ m/s}^2$$

13.27:

$$x = 2t + 1$$

$$y = t^2$$

$$\dot{x} = 2 \qquad \dot{y} = 2t$$

$$\ddot{x} = 0 \qquad \ddot{y} = 2$$

$$\therefore \quad \rho = \frac{[\dot{x}^2 + \dot{y}^2]^{3/2}}{\dot{x}\,\ddot{y} - \ddot{x}\,\dot{y}} = \frac{[(2)^2 + (2t)^2]^{3/2}}{(2)\,(2) - (0)\,(2t)}$$

$$= \frac{[4 + 4t^2]^{3/2}}{4}$$

$$\rho(2) = \frac{[4 + 4(2)^2]^{3/2}}{4} = 22.36 \text{ m}$$

13.28:

$$y = x \tan \alpha - \frac{1}{2} g \frac{x^2}{v_o^2 \cos^2 \alpha}$$

$$\frac{dy}{dx} = \tan \alpha - \frac{1}{2} g \cdot \frac{2x}{v_o^2 \cos^2 \alpha}$$

$$= \tan \alpha - \frac{gx}{v_o^2 \cos^2 \alpha}$$

$$\frac{d^2y}{dx^2} = \frac{-g}{v_o^2 \cos^2 \alpha}$$

At the highest point, we know $\frac{dy}{dx} = 0$

$$\therefore \quad \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{v_o^2 \cos^2 \alpha}{g} = \frac{(20)^2\,(\cos 30)^2}{9.81} = 30.6 \text{ m}$$

13.29:

when $x = 10$ m, $\quad$ when $x = 30$ m

$$\frac{dy}{dx} = 0.25 \qquad \frac{dy}{dx} = -\,0.404$$

$$\therefore \quad \rho = \frac{[1 + (0.25)^2]^{3/2}}{1/30.6} = 33.5 \text{ m} \qquad \therefore \quad \rho = 38.4 \text{ m}$$

13.30:

$$y^2 = 4\,ax$$

(or) $$y = 2\sqrt{a}\;x^{1/2}$$

$$\frac{dy}{dx} = \sqrt{a}\,x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}\sqrt{a}\,x^{-3/2}$$

(i) **At $x = a$**

$$\frac{dy}{dx} = 1 \quad \& \quad \frac{d^2y}{dx^2} = \frac{-1}{2a}$$

$$\therefore \qquad \rho = \frac{[1+(1)^2]^{3/2}}{-1/2a} = -5.66\ a$$

(ii) **At $x = 2a$**

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad \& \quad \frac{d^2y}{dx^2} = \frac{-1}{4\sqrt{2}\,a}$$

$$\therefore \qquad \rho = \frac{\left[1+\left(\frac{1}{\sqrt{2}}\right)^2\right]^{3/2}}{-1/4\sqrt{2}a} = -10.4a$$

13.31:

$$y = e^x$$

$$\frac{dy}{dx} = e^x \qquad \Rightarrow \qquad \left.\frac{dy}{dx}\right|_{x=2} = e^2$$

$$\frac{d^2y}{dx^2} = e^x \qquad \Rightarrow \qquad \left.\frac{d^2y}{dx^2}\right|_{x=2} = e^2$$

$$\therefore \qquad \rho = \frac{[1+(e^2)^2]^{3/2}}{e^2} = 56.1\ \text{m}$$

13.32:

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 6x$$

(i) At $x = 1$ m,

$$\frac{dy}{dx} = 3 \quad \& \quad \frac{d^2y}{dx^2} = 6$$

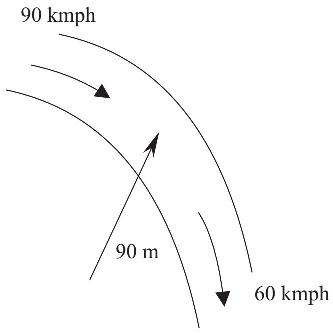
$$\therefore \qquad \rho = \frac{[1+(3)^2]^{3/2}}{6} = 5.27\ \text{m}$$

(ii) At $x = 2$m,

$$\frac{dy}{dx} = 12 \quad \& \quad \frac{d^2y}{dx^2} = 12$$

$$\therefore \qquad \rho = \frac{[1+(12)^2]^{3/2}}{12} = 145.5 \text{ m}$$

13.33:



$$v_o = 90 \text{ kmph} = 25 \text{ m/s}$$

$$v = 60 \text{ kmph} = 16.67 \text{ m/s}$$

$$\therefore \qquad a_t = \frac{v - v_o}{t} = \frac{16.67 - 25}{10} = -0.833 \text{ m/s}^2$$

$$v(5) = v_o + a_t\,(t)$$

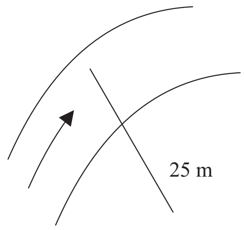
$$= 25 + (-\,0.833)\,(5) = 20.835 \text{ m/s}$$

$$\therefore \qquad a_n = \frac{v^2}{\rho} = 4.823 \text{ m/s}^2$$

$$\therefore \qquad a = \sqrt{a_t^2 + a_n^2} = 4.89 \text{ m/s}^2 \approx 4.9 \text{ m/s}^2$$

$$\theta = \tan^{-1}\frac{a_n}{a_t} = 80.2°$$

13.34:



$$a_t = 1 \text{ m/s}^2$$

$$s = \frac{\pi r}{2} = \frac{\pi}{2}(25) = 12.5\ \pi\text{m.}$$

$$v_B^2 = v_A^2 + 2\,a_t\,s$$

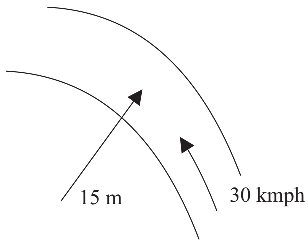
$$= 0 + 2\,(1)\,(12.5\,\pi)$$

$$\Rightarrow \qquad v_B = 8.86 \text{ m/s}$$

$$\therefore \qquad a_n = \frac{v_B^2}{\rho} = \frac{(8.86)^2}{25} = 3.14 \text{ m/s}^2$$

$$\therefore \qquad a = \sqrt{a_t^2 + a_n^2} = 3.3 \text{ m/s}^2$$

13.35:

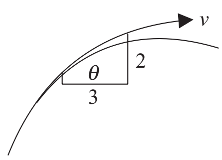


$$v = 30 \text{ kmph} = 8.33 \text{ m/s}$$

$$a_t = 0$$

$$a_n = \frac{v^2}{\rho} = \frac{[8.33]^2}{15} = 4.63 \text{ m/s}^2$$

13.36:



$$v = 10 \text{ m/s}$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x \ \ \& \ \ \frac{d^2y}{dx^2} = 2$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{[1+(2x)^2]^{3/2}}{2} = \frac{[1+4x^2]^{3/2}}{2}$$

$$\rho(1/3) = \frac{[1+(4/9)]^{3/2}}{2} = 0.868 \text{ m}$$

Slope of curve at $x = 1/3$ m is:

$$\frac{dy}{dx} = \frac{2}{3} = \tan\theta$$

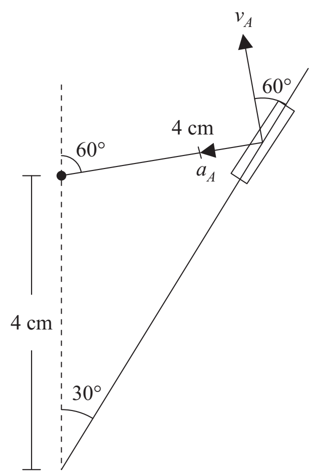
$\Rightarrow$ $\theta = 33.69°$

$\therefore$ $v_x = v\cos\theta$ & $v_y = v\sin\theta$

$= 8.32$ m/s $= 5.55$ m/s

$$a_t = 0, \quad a_n = \frac{v^2}{\rho} = \frac{10^2}{0.868} = 115.21 \text{ m/s}^2$$

13.37:



$\omega = 200$ rpm

$= 200 \times \dfrac{2\pi}{60} = 20.94$ rad/s

$v_A = r_{o2A}\ \omega = 83.76$ cm/s

$a_A = r_{o2A}\ \omega^2 = 1753.93$ cm/s^2

Components of velocity along radial & transverse directions:

$v_r = v_A \cos 60°$ & $v_\theta = v_A \sin 60°$

$= 41.88$ cm/s $= 72.54$ cm/s

$v_\theta = r\dot{\theta}$ $v_r = \dot{r} = 41.88$ cm/s

$72.54 = (12.49)\dot{\theta}$

$\Rightarrow$ $\dot{\theta} = 5.81$ rad/s

Components of acceleration along radial & transverse directions:

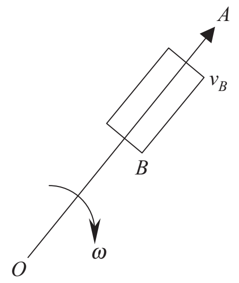
$a_r = -a_A \sin 60°$ & $a_\theta = a_A \cos 60°$

$= -1518.95$ cm/s^2 $= 876.97$ cm/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$876.97 = (6.93)(\ddot{\theta}) + 2(41.88)(10.47)$

13.38:



$v_B = 3$ m/s

$\omega = 30$ rpm $= 30 \times \dfrac{2\pi}{60} = 3.14$ rad/s

$r_B = 1.5$ m

$v_r = 3$ m/s

$v_\theta = r_B\,\omega = 4.71$ m/s

$\therefore \quad \vec{v} = 3\,\hat{e}_r + 4.71\,\hat{e}_\theta$

$\therefore \quad v = 5.58$ m/s & 57.5° to the crank

$$\vec{a} = [r - r\dot{\theta})^2]\,\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\,\dot{\theta}]\,\hat{e}_\theta$$
$$= [0 - 1.5\,(3.14)^2]\,\hat{e}_r + [\,(1.5)\,(0) + 2(3)\,(3.14)]\hat{e}_\theta$$
$$= -\,14.79\,\hat{e}_r + 18.84\,\hat{e}_\theta$$

$a = 23.95$ m/s^2 at 51.9°

13.39:

$v_B = 2$ m/s $\qquad \dot{\theta} = 2$ rad/s

$a_B = 1$ m/s^2 $\qquad \ddot{\theta} = 1.5$ rad/s^2

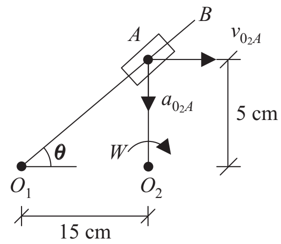
$$\vec{v} = v_r\,\hat{e}_r + v_\theta\hat{e}_\theta$$
$$= \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$
$$= 2\,\hat{e}_r + (1.5)\,(2)\,\hat{e}_\theta$$

$v = 3.61$ m/s at 56.3° to the crank

$$\vec{a} = [\dot{r} - r\dot{\theta}^2]\,\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\,\hat{e}_\theta$$
$$= [1 - (1.5)\,(2)^2]\,\hat{e}_r + [(1.5)\,(1.5) + 2\,(2)\,(2)]\,\hat{e}_\theta$$
$$= -5\,\hat{e}_r + 10.25\,\hat{e}_\theta$$

$\therefore \quad a = 11.4$ m/s^2 at 64° to the crank

13.40:



$\omega = 10$ rad/s

$$v_{o_2A} = r_{o_2A}\,\omega$$
$$= (5)\,(10)$$
$$= 50 \text{ cm/s}$$

$$\theta = \tan^{-1}\left[\frac{5}{15}\right] = 18.43°$$

$\therefore$ Components of velocity of A along radial & transverse directions

$v_r = v_{o_2A}\cos\theta = 47.44$ cm/s

$v_\theta = v_{o_2A}\sin\theta = 15.81$ cm/s

We know $\quad v_r = \dot{r} \qquad$ & $\quad v_\theta = r\dot{\theta}$

$$\therefore \quad \dot{\theta} = \frac{v_\theta}{r_{o_1A}} = \frac{15.81}{\sqrt{15^2 + 5^2}} = 1 \text{ rad/s}$$

and $\dot{r} = 47.44$ cm/s

$$a_{o_2A} = r_{o_2A}\ \omega^2 = 500 \text{ cm/s}^2$$

$\therefore$ Components of acceleration of A along radial & transverse directions,

$$a_r = -\,a_{o_2A} \sin\,\theta = -\,158.1 \text{ cm/s}^2$$

$$a_\theta = a_{o_2A} \cos\,\theta = 474.36 \text{ cm/s}^2$$

We know $a_r = \ddot{r} - r(\dot{\theta})^2$ & $a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta}$

$$\Rightarrow \quad 474.36 = \left(\sqrt{15^2 + 5^2}\right)\ddot{\theta} + 2(47.44)(1)$$

$$\Rightarrow \quad \ddot{\theta} = 24 \text{ rad/s}^2$$