Chapter-12

12.1:

$$x = A\cos 2\pi ft$$

$$v = \frac{dx}{dt} = -2\pi f A \sin 2\pi ft$$

$$a = \frac{d_x^2}{d_t^2} = -(2\pi f)^2 A\cos 2\pi ft$$

12.2:

$x = 2t^3 - 8t^2 + 4t - 2$ $\quad x/_{t=3s} = 2(3)^3 - 8(3)^2 + 4(3) - 2 = -8$ m

$v = 6t^2 - 16t + 4$ $\quad v/_{t=3s} = 6(3)^2 - 16(3) + 4 = 10$ m/s

$a = 12t - 16$ $\quad a/_{t=3s} = 12(3) - 16 = 20$ m/s^2

12.3:

$x = t^3 + t^2 + t + 1$ $\quad x/_{t=2s} = (2)^3 + (2)^2 + (2) + 1 = 15$ m

$v = 3t^2 + 2t + 1$ $\quad v/_{t=2s} = 3(2)^2 + 2(2) + 1 = 17$ m/s

$a = 6t + 2$ $\quad a/_{t=2s} = 6(2) + 2 = 14$ m/s^2

12.4:

$x = t^3 + 3t + 5$ $\quad x/_{t=0} = 5$ m $\quad x/_{t=2s} = 2^3 + 3(2) + 5 = 19$ m

$v = 3t^2 + 3$ $\quad v/_{t=0} = 3$ m/s $\quad v/_{t=2s} = 3(2)^2 + 3 = 15$ m/s

$a = 6t$ $\quad a/_{t=0} = 0$ s^2 $\quad a/_{t=2s} = 6(2) = 12$ m/s^2

12.5:

$x = t^4 - 3t^3 + t^2 + 6$ (i) $x/_{t=2s} = 2^4 - 3(2)^3 + 2^2 + 6 = 2$ m

$v = 4t^3 - 9t^2 + 2t$ $\quad v/_{t=2s} = 4(2)^3 - 9(2)^2 + 2(2) = 0$ m/s

$a = 12t^2 - 18t + 2$ $\quad a/_{t=2s} = 12(2)^2 - 18(2) + 2 = 14$ m/s^2

(ii) $V_{ave} = \dfrac{x(3) - x(2)}{3 - 2} = \dfrac{15 - 2}{1} = 13$ m/s

(iii) $a_{ave} = \dfrac{v(3) - v(2)}{3 - 2} = \dfrac{33 - 0}{1} = 33$ m/s^2

(iv) $a = 0 = 12t^2 - 18t + 2$

$\Rightarrow \quad 6t^2 - 9t + 1 = 0$

$\therefore \quad t = \dfrac{9 \pm \sqrt{9^2 - 4(6)(1)}}{12} = \dfrac{9 \pm 7.55}{12} = 1.38$ s (or)

$= 0.12$ s

12.6:

$$x = \frac{t^4}{12} - \frac{2t^3}{3} - \frac{5t^2}{2} + 3t + 4$$

$$v = \frac{4t^3}{12} - \frac{6t^2}{3} - 5\,\frac{2t}{2} + 3$$

$$a = \frac{12t^2}{12} - \frac{12t}{3} - 5$$
$$= t^2 - 4t - 5$$
$$= (t - 5)\,(t + 1)$$
$\Rightarrow$ $t = 5\text{s}$
$x/_{t=5s} = -74.75$ m
$v/_{t=5s} = -30.33$ m/s

12.7:

$$x = t^3 - 5.5\,t^2 + 8t - 6$$
$$v = 3t^2 - 11t + 8$$
$$a = 6t - 11$$

When it reaches momentary rest, $v = 0$

$0 = 3t^2 - 11t + 8 \Rightarrow t = \frac{11 \pm 5}{6}$ $1s, 2.67\,s$

When $a = 0$, $t = 11/6 = 1.83s$
$x = -3.65$ m
$v = -2.08$ m/s

At $t = 0$, $x = -6$ m
$t = 1s$, $x = -2.5$ m

i.e $v = 0$
$t = 1.83s$ $x = -3.65$ m

$\therefore$ Distance travelled $= |-2.5 - (-6)| + |-3.65 - (-2.5)|$
$= 3.5 + 1.15$
$=$ **4.65 m**

12.8:

$$x = \frac{t^3}{3} - 1.5t^2 - 4t + 5$$
$$v = t^2 - 3t - 4$$
$$a = 2t - 3$$
$v = 0 \Rightarrow (t - 4)\,(t + 1)$
$\therefore$ $t = 4s, -1s$
$\therefore$ $x/_{t=4s} = -13.67$ m
$a/_{t=4s} = 5$ m/s^2

12.9:

$$a = 3t$$
$$v = \frac{3t^2}{2} + c_1$$

At $t = 0,\ v = 0 \Rightarrow c_1 = 0$

$\therefore \quad x = \frac{3}{2}\frac{t^3}{3} + c_2$

At $t = 0,\ x = 2m, \Rightarrow c_2 = 2$

$\therefore \quad x = \frac{t^3}{2} + 2$

$x/_{t=4s} = 34$ m

$v/_{t=4s} = \frac{3}{2}t^2 = 24$ m/s

$a/_{t=4s} = 12$ m/s^2

12.10:

$a = 3t^2 - 4t^3$

$v = \frac{3t^3}{3} - \frac{4t^4}{4} + c_1$

$= t^3 - t^4 + c_1$

At $t = 0,\ \text{v} = 0 \Rightarrow c_1 = 0$

$\therefore \quad \boxed{v = t^3 - t^4}$

$x = \frac{t^4}{4} - \frac{t^5}{5} + c_2$

At $t = 1\text{s}, \quad x = 0,$

$0 = \frac{1}{4} - \frac{1}{5} + c_2$

$= \frac{1}{20} + c_2 \Rightarrow c_2 = \frac{-1}{20}$

$\boxed{x = \frac{t^4}{4} - \frac{t^5}{5} - \frac{1}{20}}$

12.11:

$a = kt - 1$

$v = \frac{kt^2}{2} - t + c_1$

At $t = 0,\ v = 0,$

$\Rightarrow \quad 0 = c_1$

$v = \frac{kt^2}{2} - t$

At $t = 2s, \quad v = 2$ m/s

$2 = \frac{k(2)^2}{2} - 2$

$$2 = 2k - 2$$

$$\boxed{k = 2}$$

$$v = t^2 - t$$

$$x = \frac{t^3}{3} - \frac{t^2}{2} + c_2$$

At $t = 0,\ x = 0 \Rightarrow c_2 = 0$

$\therefore$ $$x = \frac{t^3}{3} - \frac{t^2}{2}$$

$x/_{t=3s} = 4.5$ m

$v/_{t=3s} = 6$ m/s

$a/_{t=3s} = 2t - 1 = 5$ m/s^2

12.12:

$$a = kt^2 - 3$$

$$v = \frac{k\,t^3}{3} - 3t + c_1$$

At $t = 0,\ v = 0 \Rightarrow c_1 = 0$

At $t = 3s,\ v = 9 \quad \Rightarrow \quad 9 = \frac{k}{3}.(3)^3 - 3\ (3)$

$$\Rightarrow \quad k = 2$$

$\therefore$ $$v = \frac{2}{3}\,t^3 - 3t$$

$$x = \frac{2}{3}\frac{t^4}{4} - 3\frac{t^2}{2} + c_2$$

At $t = 0, x = 0 \Rightarrow c_2 = 0$

$x/_{t=2s} = -3.33$ m

$x/_{t=2s} = -0.67$ m/s

$a/_{t=2s} = 2t^2 - 3$

$= 5$ m/s^2

12.13:

$$a = \frac{1}{v}$$

$$\frac{dv}{dt} = \frac{1}{v}$$

$$v\,dv = dt$$

$$\frac{v^2}{2} = t + c_1$$

At $t = 2s, \quad v = 2$ m/s, $\Rightarrow\ c_1 = 0$

$$v^2 = 2t$$

$$v = \sqrt{2}\, t^{1/2} = \sqrt{2t}$$

$$x = \sqrt{2} \cdot \frac{t^{3/2}}{3/2} = \frac{2\sqrt{2}}{3} t^{3/2}$$

12.14:

$$a = kv$$

$$\frac{dv}{dt} = kv$$

$$\frac{dv}{v} = k\, dt$$

$$ln\ v = kt + c_1$$

At $t = 0, \quad v = v_0 \ \Rightarrow \ c_1 = ln\ v_0$

$$ln\ \frac{v}{v_o} = kt$$

$$\Rightarrow \quad v = v_0\, e^{kt}$$

$$x = \frac{v_o\, e^{kt}}{k} + c_2$$

At $t = 0, x = 0 \Rightarrow c_2 = -\frac{v_o}{k}$

$$\therefore \quad x = \frac{v_o}{k}[e^{kt} - 1]$$

12.15:

$$a = g - kv$$

$$\frac{dv}{dt} = g - kv$$

$$\frac{dv}{g - kv} = dt$$

$$\frac{-1}{k} ln(g - kv) = t + c_1$$

At $t = 0, \quad v = 0 \ \Rightarrow \ c_1 = -\frac{1}{k}\ ln\ g$

$\therefore$

$$\frac{1}{k} ln\ \frac{g}{g - kv} = t$$

$$\frac{g}{g - kv} = e^{kt}$$

$$1 - \frac{k}{g} v = e^{-kt}$$

$$\frac{g}{k}[1 - e^{-kt}] = v$$

$$x = \frac{g}{k}\left[t + \frac{e^{-kt}}{k}\right] + c_2$$

At $t = 0,\ x = 0 \Rightarrow$

$$C_2 = \frac{-g}{k^2}$$

$$x = \frac{g}{k}\ t + \frac{g}{k^2}\ e^{-kt} - \frac{g}{k^2}$$

$$= \frac{g}{k}t - \frac{g}{k^2}[1 - e^{-kt}]$$

12.16:

$$a = \frac{-1}{x^2}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} = \frac{-1}{x^2}$$

$$vdv = -\frac{1}{x^2}dx$$

$$\frac{v^2}{2} = \frac{1}{x} + c_1$$

At $t = \frac{4}{3}s,\ x = 2$, and $v = 1$ m/s

$$\frac{1}{2} = \frac{1}{2} + c_1 \Rightarrow c_1 = 0$$

$$\frac{v^2}{2} = \frac{1}{x}$$

$$v = \sqrt{2}\,x^{-1/2}$$

$$\frac{dx}{dt} = \sqrt{2}\,x^{-1/2}$$

$$\frac{dx}{x^{-1/2}} = \sqrt{2}\,dt$$

$$x^{3/2} = \frac{3}{2}\sqrt{2}\,t = \frac{3}{\sqrt{2}}t$$

12.17:

$V_o = 150$ kmph $= 41.67$ m/s

$a = -2.5$ m/s^2

$v = 1$ kmph $= 0.28$ m/s

$$v = v_o + at$$

$$0.28 = 41.67 - 2.5\,t \Rightarrow t = 16.56s$$

$$s = V_o t + \frac{1}{2}at^2 = 41.07\,(16.56) - \frac{1}{2}(2.5)\,(16.56)^2 = 347.26 \text{ m}$$

12.18:

$$s = 100 \text{ m}$$

$$V_o = 0$$

$$a = 2.2 \text{ m/s}^2$$

$$s = V_o t + \frac{1}{2}at^2$$

$$100 = 0 + \frac{1}{2}(2.2)\,t^2 \Rightarrow t = 9.53 \text{ s}$$

12.19:

$$v = 54 \text{ kmph} = 15 \text{ m/s}$$

$$a = 1 \text{ m/s}^2$$

$$t = 10 \text{ s}$$

$$v = V_o + at$$

$$= 15 + (1)\,(1.0) = 25 \text{ m/s} = 90 \text{ hmph}$$

12.20:

$$V_o = 60 \text{ kmph} = 16.67 \text{ m/s}$$

$$s = 150 \text{ m}$$

$$v^2 = V_o^2 + 2\,as$$

$$0 = (16.67)^2 + 2\,(a)\,(150)$$

$$\Rightarrow \quad a = -0.926 \text{ m/s}^2$$

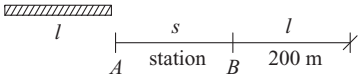
12.21:

$$a = 1 \text{ m/s}^2$$

$$v = 72 \text{ kmph} = 20 \text{ m/s}$$

$$t = 10 \text{ s}$$

(i) $s = vt - \frac{1}{2}at^2$



$$= 20\,(10) - \frac{1}{2}(1)\,(10)^2 = 150 \text{ m}$$

(ii) $v = V_o + at$

$$v_o = 20 - (1)\,(10) = 10 \text{ m/s} = 36 \text{ kmph}$$

12.22:

$$s = 200 \text{ m}$$

$$t = 10 \text{ s}$$

$$a = 1 \text{ m/s}^2$$

$$s = v_o t + \frac{1}{2} at^2$$

$$200 = v_o (10) + \frac{1}{2}(1)(10)^2$$

$\Rightarrow$ $v_o = 15$ m/s

$$v = v_o + at$$

$$= 15 + (1)(10) = 25 \text{ m/s}$$

12.23:

$v_o = 80$ m/s

s = 1200 m

$v = 0$

(i) $v^2 = V_o^2 + 2$ as

$0 = 80^2 + 2\ (a)\ (1200)$

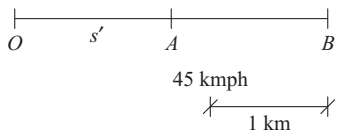
$\Rightarrow$ $a = -2.67$ m/s^2

(ii) $v/_{t=5} = v_o + at$

$= 80 + (-2.67)\ (5)$

$= 66.65$ m/s

12.24:



$v = 80$ kmph = 22.22 m/s

$t = 20\,s$

$s + l = (22.22)\ 20$

$200 + l = 444.4$

$\Rightarrow$ $l = 244.4$ m

12.25:

$v = 1$ m/s

$a = 0.3$ m/s^2

$s_1 = 1\ (t + 30)$

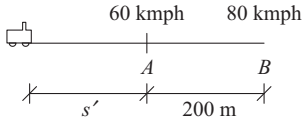
$$s_2 = (1)\ t + \frac{1}{2}(0.3)t^2$$

$$s_1 = s_2 \Rightarrow t + 30 = t + \frac{1}{2}(0.3)\,t^2$$

$$\frac{60}{0.3} = t^2 \Rightarrow t = 14.14s$$

$s_1 = 1\,(t + 30) = 44.14$ m

12.26:



$V_A = 60$ kmph = 16.67 m./s

$V_B = 80$ kmph = 22.22 m/s

$V_B^2 = V_A^2 + 2a\,s$

$$a = \frac{(22.22)^2 - (16.67)^2}{2(200)} = 0.54 \text{ m/s}^2$$

$$V_B = V_A + at$$

$$\Rightarrow \quad t = \frac{22.22 - 16.67}{0.54} = 10.28 \text{ s}$$

$$V_A^2 = V_o^2 + 2a\,s'$$

$$16.67^2 = 0 + 2\,(0.54)\,s'$$

$$\Rightarrow \quad s' = 257.3 \text{ m}$$

$$V_A = V_o + at'$$

$$\Rightarrow \quad t = 16.67/0.54 = 30.87 \text{ s}$$

12.27:

$v_o = 0$

$v_A = 45$ kmph $= 12.5$ m/s

$v_B = 60$ kmph $= 16.67$ m/s

$$v_B^2 = v_A^2 + 2\,a\,(s)$$

$$a = \frac{(16.67)^2 - (12.5)^2}{2(1000)} = 0.061 \text{m/s}^2$$

$$v_B = v_A + at$$

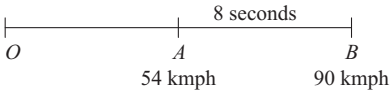
$$t = \frac{16.67 - 12.5}{0.061} = 68.36 \text{ s}$$

$$v_A^2 = Vo^2 + 2a\,s'$$

$$s' = \frac{12.5^2 - 0}{2(0.061)} = 1280.74 \text{ m}$$

$$= 1.28 \text{ km}$$

12.28:



$v_A = 54$ kmph $= 15$ m/s

$v_B = 90$ kmph $= 25$ m/s

$$v_B = v_A + at$$

$$\Rightarrow \quad a = \frac{25 - 15}{8} = 1.25 \text{ m/s}^2$$

$$v_B^2 = v_A^2 + 2\,as$$

$$\Rightarrow \quad s = \frac{25^2 - 15^2}{2(1.25)} = 160 \text{ m}$$

12.29:

$v_1 = 10$ m/s $\qquad v_2 = 15$ m/s

$x_1 = 5$ m $\qquad x_2 = 15$ m

$s = 15 - 5 = 10$ m

$v_2^2 = v_1^2 + 2$ as

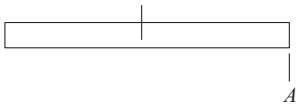
$15^2 = 10^2 + 2$ a (10)

$\Rightarrow$ a = 6.25 m/s^2

$v_1^2 = v_0^2 + 2a\,(x_1 - x_0)$

$10^2 = v_0^2 + 2\,(6.25)\,(5)$

$\Rightarrow$ $v_0 = 6.12$ m/s



12.30:

$v_i = u$ (i) $v = u + at$

$v_f = v$ $\Rightarrow$ $a = \dfrac{v-u}{t}$

$s = l$ (ii) $v^2 = u^2 + 2al$

t $\Rightarrow$ $l = \dfrac{v^2-u^2}{2a} = \dfrac{(v+u)\,(v-u)}{2\cdot a}$

$= \dfrac{v+u}{2}\cdot t$

(iii) $v_m^2 = u^2 + 2a\dfrac{l}{2}$

$= u^2 + al$

$= u^2 + \left[a\cdot\dfrac{v+u}{2}\cdot t\right]$

$= u^2 + a\cdot\dfrac{v+u}{2}\cdot\dfrac{v-u}{a}$

$= \dfrac{2u^2 + v^2 - u^2}{2}$

$= \dfrac{v^2 + u^2}{2}$

$\Rightarrow$ $v_m = \sqrt{\dfrac{v^2+u^2}{2}}$

12.31:

$v_{o1} = 0$ $v_{o2} = 0$

$a_1 = 1$ m/s^2 $a_2 = 1.2$ m/s^2

t $t - 4$

$s_1 = V_{o1}\,t + \dfrac{1}{2}a_1\,t^2$ $s_2 = v_{o2}\,t_2 + \dfrac{1}{2}a_2\,t_2^2$

$= \frac{1}{2} a_1 t^2$ $= v_{02} (t - 4) + \frac{1}{2} a_2 (t - 4)^2$

$= \frac{1}{2} a_2 (t - 4)^2$

When car B overtakes car A,

$s_1 = s_2 \Rightarrow$

$\frac{1}{2} a_1 t^2 = \frac{1}{2} a_2 (t - 4)^2$

$t^2 = 1.2 (t - 4)^2 = 1.2 (t^2 + 16 - 8t)$

$\Rightarrow \quad 0.2 t^2 + 19.2 - 9.6 t = 0$

$t^2 - 48t + 96 = 0$

$t = \frac{48 \pm \sqrt{48^2 - 4(96)}}{2}$

$t = 45.91$ s (or) 2.09 s

$s_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} 1(45.91)^2 = 1.054$ km

12.32:

$v_1 = 60$ kmph $= 16.67$ m/s $v_{o2} = 0$

(t) $(t - 3)$

$a_1 = 0$ $a_2 = 2$ m/s^2

$s_1 = v_1 t$ $s_2 = v_{o2} t_2 + \frac{1}{2} a_2 t_2^2$

$= v_{o2} (t - 3) + \frac{1}{2} a_2 (t - 3)^2$

$= \frac{1}{2} a_2 (t - 3)^2$

When car overtakes the truck, $s_1 = s_2 \Rightarrow$

$16.67 t = \frac{1}{2} 2 (t - 3)^2$

$= t^2 + 9 - 6t$

$t^2 - 22.67 t + 9 = 0 \Rightarrow t = 22.27s$ (or) 0.40 s

$s_1 = v_1 t = 16.67 (22.27) = 371.2$ m

12.33:

$v_1 = 54$ kmph $= 15$ m/s $v_2 = 72$ kmph $= 20$ m/s

t $(t - 2)$

$s_1 = v_1 t$ $s_2 = v_2 (t - 2)$

When car overtakes the truck, $s_1 = s_2 \Rightarrow$

$v_1 t = v_2 (t - 2)$

$$15\,t = 20\,(t-2)$$

$\therefore \quad t = 8\text{s}$

$$s_1 = 15(8) = 120 \text{ m}$$

12.34:

$v_{o1} = 0$ $\qquad$ $v_{o2} = 45$ kmph $= 12.5$ m/s

$t = 15s$ $\qquad$ $t = 15$ s

$$s_1 = v_{o1}\,t + \frac{1}{2}a_1\,t_2 \qquad s_2 = v_{o2}\,t$$

$$= \frac{1}{2}a_1\,t^2$$

when car overtakes truck, $\qquad s_1 = s_2 \Rightarrow$

$$\frac{1}{2}a_1\,(15)^2 = 12.5\,(15)$$

$$a_1 = \frac{25}{15} = 1.67 \text{ m/s}^2$$

$$s_2 = v_{o2}\,t = 12.5\,(15) = 187.5 \text{ m}$$

$$v_1 = v_{01} + a_1\,t = 0 + (1.67)\,(15) = 25.05 \text{ m/s}$$

$$\approx 90 \text{ kmph}$$

12.35:

$v_{o1} = 45$ kmph $= 12.5$ m/s $\qquad$ $v_{o2} = 60$ kmph $= 16.67$ m/s

$s = 300$ m

$a_1 = -0.5$ m/s^2 $\qquad$ $s_2 = 16.67\,(0.7)$

$s_1 = 12.5\,(0.7)$ $\qquad$ $= 11.67$ m

$= 8.75$ m

$s = 279.58$ m

$$v_1^{\,2} = v_{o1}^{\,2} + 2a_1 s_1'$$

$$0 = (12.5)^2 + 2\,(-0.5)\,(s_1')$$

$\Rightarrow \quad s_1' = 156.25$ m

$\therefore \quad s_2' = 123.33$ m

$$v_2^{\,2} = v_0^{\,2} + 2a_2\,s_2'$$

$$\Rightarrow \quad a_2 = \frac{0-(16.67)^2}{2\,(123.33)} = 1.13 \text{ m/s}^2$$

12.36:

$v_{o1} = 15$ m/s $\qquad$ $v_{o2} = 10$ m/s

$s = 100$ m

$a_1 = 0$ $\qquad$ $a_2 = 0.5$ m/s^2

$s_1 = 100 + 15\ t$ $\qquad$ $s_2 = 10t + \frac{1}{2}(0.5)\ t^2$

when car overtakes truck, $\qquad$ $s_1 = s_2 \Rightarrow$

$$100 + 15\ t = 10\ t + \frac{1}{2}(0.5)\ t^2$$

$$0.25\ t^2 - 5\ t - 100 = 0$$

$$t = 32.36 s$$

$$s_1 = 100 + 15\ (32.36) = 585.4 \text{ m}$$

$$v_2 = v_{o2} + a_2\ t = 10 + 0.5\ (32.36) = 26.18 \text{ m/s}$$

12.37:

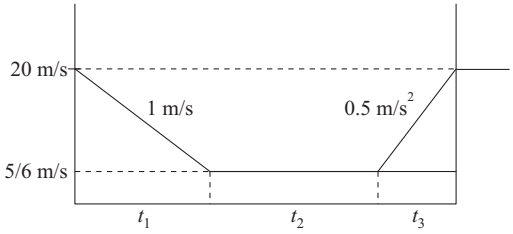
$v_1 = 72$ kmph $= 20$ m/s

$a_2 = -1$ m/s^2

$v_2 = 3$ kmph $= \frac{5}{6}$ m/s

$t_2 = 120\ s$

$a_3 = 0.5$ m/s^2



$$t_1 = \frac{-20 + 5/6}{-1} = 19.17 s$$

$$s_1 = \frac{1}{2}\left[20 + \frac{5}{6}\right][19.17] = 199.69 \text{ m}$$

$$s_2 = \frac{5}{6} \times 120 = 100 \text{ m}$$

$$t_3 = \frac{20 - 5/6}{0.5} = 38.33 s$$

$$s_3 = \frac{1}{2}\left[20 + \frac{5}{6}\right][38.33] = 399.27 \text{ m}$$

$\therefore$ $\quad s = s_1 + s_2 + s_3 = 698.96$ m

$= 699$ m

$t = t_1 + t_2 + t_3 = 177.5 s$

Time taken to travel with constant velocity $= \frac{698.9}{20} = 34.95 s$

$\therefore$ Time lost $= 177.5 - 34.95$

$= 142.6$ s

12.38:

$v_o = 0$

$a = 1$ m/s^2

$v = 60$ kmph $= 16.67$ m/s

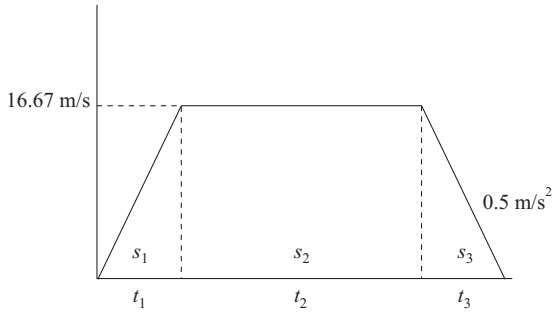
$s = s_1 + s_2 + s_3 = 5000$ m

$t_1 = \dfrac{16.67}{1} = 16.67\,\text{s}$

$s_1 = \dfrac{1}{2}\,16.67 \times 16.67 = 138.94$ m

$t_3 = \dfrac{16.67 - 0}{0.5} = 33.34s$

$s_3 = \dfrac{1}{2}(16.67)(33.34) = 277.89$ m



$\therefore \quad s_2 = 5000 - 138.94 - 277.89 = 4583.17$ m

$\therefore \quad t_2 = 274.94$ s

$\therefore \quad T = t_1 + t_2 + t_3 = 16.67 + 274.94 + 33.34 = 324.95 \approx 325$ s

12.39:

$v_o = 0$

$a = 1\ \text{m/s}^2$

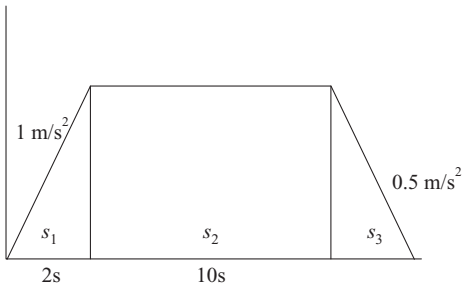
$v_1 = (1)\ (2) = 2$ m/s

$s_1 = \dfrac{1}{2}(2)(2) = 2$ m

$s_2 = 2 \times 10 = 20$ m

$t_3 = \dfrac{2}{0.5} = 4$ s

$s_3 = \dfrac{1}{2} \times 2 \times 4 = 4$ m



$\therefore \quad H = s_1 + s_2 + s_3 = 2 + 20 + 4 = 26$ m

$T = t_1 + t_2 + t_3 = 2 + 10 + 4 = 16$ s

12.40:

$t_4 = \dfrac{20}{0.5} = 40\,\text{s}$

$\therefore \quad T = t_1 + t_2 + t_3 + t_4 = 305\,\text{s}$

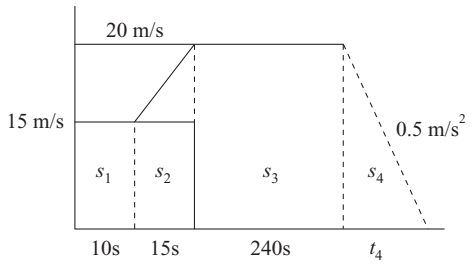
$\therefore \quad s_1 = 15(10) = 150$ m

$s_2 = \dfrac{1}{2}\,15\ [20 + 15] = 262.5$ m

$s_3 = 20 \times 240 = 4800$ m

$s_4 = \dfrac{1}{2} \times 20 \times 40 = 400$ m

$\therefore \quad s = s_1 + s_2 + s_3 + s_4$



$= 5612.5$ m

$= 5.613$ km

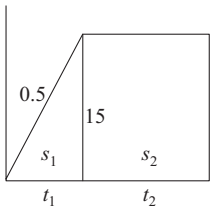
12.41:

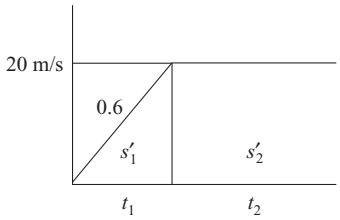
$s = 6000$ m $\qquad v_{o2} = 0$

$v_{o1} = 0$ $\qquad a_2 = 0.6$ m/s^2

$a_1 = 0.5$ m/s^2 $\qquad v_2 = 72$ kmph $= 20$ m/s

$v_1 = 54$ kmph $= 15$ m/s





$$t_1 = \frac{15}{0.5} = 30 \text{ s} \qquad t'_1 = \frac{20}{0.6} = 33.33\ s$$

$$s_1 + s_2 = \frac{1}{2}15 \times 30 + 15\ (t - 30) \qquad s_1' + s_2' = \frac{1}{2}20 \times 33.33 + 20 \times (t - 33.33)$$

$$= 15\ t - 225 \qquad = 20\ t - 333.3$$

$$s_1 + s_2 + s_1' + s_2' = 6000$$

$$35\ t - 558.3 = 6000$$

$$t = 187.38 \text{ s}$$

$$s_1 + s_2 = 15\ (187.38) - 225$$

$$= 2.59 \text{ km}$$

12.42:

$v_o = 20$ m/s

(i) $v/_{t=2} = v_o - g\ t = 20 - 9.81\ (2) = 0.38$ m/s

$v/_{t=3} = 20 - 9.81\ (3) - 9.43$ m/s

(ii) $h_{\max} = \dfrac{v_0^2}{2g} = 20.39$ m

(iii) $t = \dfrac{v_1}{g} = 2.04$ s

(iv) $T = 2\ t = 4.08$ s

12.43:

$v_o = 25$ m/s

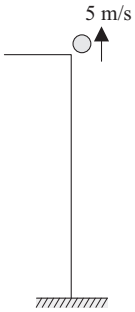
(i) $h_{\max} = \dfrac{v_0^2}{2g} = \dfrac{25^2}{2 \times 9.81} = 31.86$ m

(ii) $t = \dfrac{v_o}{g} = \dfrac{25}{9.81} = \mathbf{2.55}\,\text{s}$

(iii) $v = v_o - gt$

$-10 = 25 - 9.81\ t$

$\Rightarrow \quad t = \mathbf{3.57}\,\text{s}$



12.44:

$T = 6\text{s}$

$t = T/2 = 3\ \text{s}$

Time to reach maximum point from top of building

$$= \frac{5}{9.81} = 0.51s$$

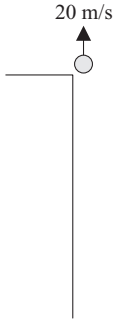
Time to scale the bldg = 3 – 0.51 = 2.49s

(i) $H = v_o t - \dfrac{1}{2}gt^2$

$$= vt + \frac{1}{2}gt^2$$

$$= 5\ (2.49) + \frac{1}{2}(9.81)(2.49)^2$$

$$= 42.86\ \text{m}$$



(ii) $v = v_o - gt$

$\Rightarrow \quad v_o = 5 + 9.81\ (2.49) = 29.43\ \text{m/s}$

(iii) $h = \dfrac{v^2}{2g} = \dfrac{5^2}{2 \times 9.81} = 1.27\ \text{m}$

12.45:

$$s = h + 20\ t - \frac{1}{2}gt^2$$

when it reaches ground, $s = 0 \Rightarrow$

$$-h = 20\ t - \frac{1}{2}gt^2$$

$$h = 22.63\ \text{m}$$

12.46:

$$s = v_o t + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.81 \times 3^2 = 44.15\ \text{m}$$

$$v = v_o + g\ t = 0 + 9.81\ (3) = 29.43\ \text{m/s}$$

12.47:

$v_{o1} = 20\ \text{m/s}$ $\qquad$ $v_{o2} = 30\ \text{m/s}$

$s_1 = 20\ t - \dfrac{1}{2}gt^2$ $\qquad$ $s_2 = 30\ (t-2) - \dfrac{1}{2}g(t-2)^2$

when they meet, $s_1 = s_2$

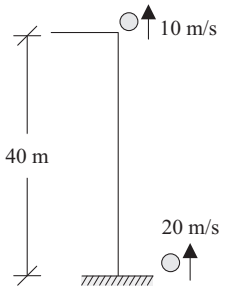
$$\therefore \quad 20\,t - \frac{1}{2}(9.81)\,t^2 = 30\,(t-2) - \frac{1}{2}(9.81)\,(t-2)^2$$

$$\frac{1}{2}(9.81)\,[t^2 + 4 - 4t - t^2] + 20t - 30t + 60 = 0$$

$$-19.62\,t + 19.62 - 10t + 60 = 0$$

$$\Rightarrow \quad t = 2.69 \text{ s} \approx 2.7s$$

$$s_1 = 20\,(2.7) - 4.905\,(2.7)^2 = 18.2 \text{ m}$$



12.48:

$$s_1 = 40 + 10\,t - \frac{1}{2}gt^2$$

$$s_2 = 20\,t - \frac{1}{2}gt^2$$

(i) when they meet,

$s_1 = s_2 \Rightarrow$

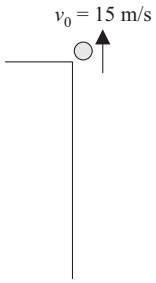
$$40 + 10t - \frac{1}{2}gt^2 = 20\,t - \frac{1}{2}gt^2$$

$$t = 4s$$

$$s_2 = 20\,(4) - 4.905\,(4)^2 = 1.52 \text{ m}$$

(ii) $v_1 = v_{o1} - gt = 10 - 9.81\,(4) = -29.24$ m/s

$v_2 = v_{o2} - gt = 20 - 9.81\,(4) = -19.24$ m/s



12.49:

$$s_1 = 15\,t - \frac{1}{2}gt^2$$

$$s_2 = 0 + \frac{1}{2}g(t-2)^2$$

$$-15\,t + \frac{1}{2}gt^2 = \frac{1}{2}g\,(t-2)^2$$

$$-15\,t + 4.905\,t^2 = 4.905\,(t^2 + 4 - 4\,t)$$

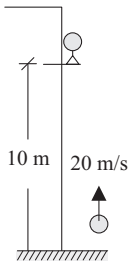
$$= 4.905\,t^2 + 19.62 - 19.62\,t$$

$$t = 4.25\text{s}$$

$$s_2 = \frac{1}{2}(9.81)(2.25)^2 = 24.83 \text{ m}$$

$v_1 = 15 - 9.81\,(4.25) = -26.69$ m/s

$v_2 = 0 + 9.81\,(2.25) = 22.07$ m/s



12.50:

(i) $v^2 = v_o^2 - 2\,gs$

$= 20^2 - 2\,(1.81)\,(10) = 203.8$

$\Rightarrow \quad v = 14.28$ m/s ↑

$v = v_o - gt$

$\frac{14.28 - 20}{-9.81} = t = 0.58$ s

(ii) when it moves down:

$v = 14.28$ m/s ↑

$\therefore \quad t = \frac{-14.28 - 20}{-9.81} = 3.5$ s

12.51:

$v_o = 0$

$a = 1.2$ m/s^2

$s = 30$ m

$s = v_o\, t + \frac{1}{2} a t^2$

$30 = \frac{1}{2}(1.2)\, t^2$

$\Rightarrow \quad t = 7.07$ s

$v = v_o + at$

$= 0 + (1.2)\,(7.07)$

$= 8.48$ m/s

$v_o = 8.48$ m/s

$s = v_o\, t - \frac{1}{2} g t^2$

$-30 = 8.48\, t - 4.905\, t^2$

$4.905\, t^2 - 8.48\, t - 30 = 0$

$\Rightarrow \quad t = 3.5$ s

$v = v_o - gt$

$= 8.48 - 9.81\,(3.5)$

$= -25.9$ m/s

12.52:

$a = 0.5$ m/s^2

$t = 30$ s

$v = v_o + at$

$= 0 + 0.5\,(30) = 15$ m/s

$s = v_o\, t + \frac{1}{2} a t^2$

$= 0 + \frac{1}{2}(0.5)\,(30)^2$

$= 225$ m

Ball:

$s = v_o\, t - \frac{1}{2} g t^2$

$-225 = 15\, t - 4.905\, t^2$

$4.905\, t^2 - 15\, t - 225 = 0$

$\Rightarrow \quad t = 8.47$ s

$v = v_o - gt$

$= 15 - 9.81\,(8.47) = -68.1$ m/s