Chapter-11

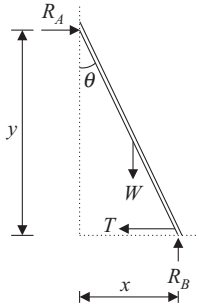
11.1:

$$x^2 + y^2 = l^2$$

$$2x\,\delta x + 2y\,\delta y = 0$$

$$\delta y = -\frac{x}{y}\delta x$$

$$= -\tan\theta\,\delta x$$



Considering positive virtual displacements

$$\delta y = \tan\theta\,\delta x$$

$$-T\,\delta x + W\frac{\delta y}{2} = 0$$

$$\Rightarrow \quad T = \frac{W\tan\theta}{2}$$

11.2:

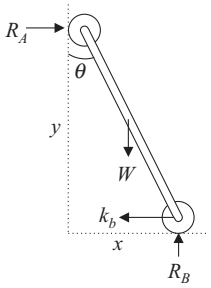
l' = unstratched length of spring

$$\therefore \quad s = l\sin\theta - l'$$

$$x^2 + y^2 = l^2$$

$$2x\,\delta x + 2y\,\delta y = 0$$

$$\Rightarrow \quad \delta y = -\frac{x}{y}\delta x$$



Considering positive virtual displacements,

$$\delta y = \tan\theta\,\delta x$$

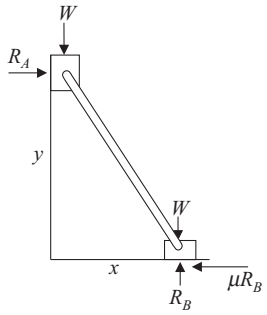
$$-ks\,\delta x + W\frac{\delta y}{2} = 0$$

$$-k(l\sin\theta - l')\,\delta x + W\frac{\delta y}{2} = 0$$

$$k(l\sin\theta - l') = \frac{W}{2}\tan\theta$$

$$\Rightarrow \quad l' = l\sin\theta - \frac{W\tan\theta}{2K}$$

11.3:



$$x^2 + y^2 = l2$$

$$2x\,\delta x + 2y\,\delta y = 0$$

$$\Rightarrow \quad \delta y = -\frac{x}{y}\delta x$$

$$= -\delta x$$

Considering positive virtual displacements,

$$\delta y = \delta x$$

$$W\,\delta y - \mu R_B\,\delta y = 0$$

$$\Rightarrow \quad \mu = \frac{W}{R_B}$$

But $R_B = W + W = 2\,W$

$$\therefore \quad \mu = \frac{1}{2}$$

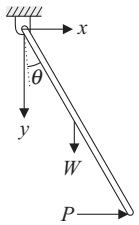
11.4:

$$x^2 + y^2 = l^2$$

$$2x\,\delta x + 2y\,\delta y = 0$$

$$\Rightarrow \quad \delta y = -\frac{x}{y}\,\delta x$$

$$= -\tan\theta\,\delta x$$



Considering positive virtual displacements,

$$\delta y = \tan\theta\,\delta x$$

$$P\,\delta x - W\frac{\delta y}{2} = 0$$

$$\Rightarrow \quad P = \frac{W\tan\theta}{2}$$

11.5:

$$P\,\delta y - W\frac{\delta y}{4} = 0$$

$$\Rightarrow \quad P = \frac{W}{4}$$

11.6:

$$P\,\delta y - W\frac{\delta y}{4} = 0$$

$$\Rightarrow \quad P = \frac{W}{4}$$

11.7:

$$x = l\sin\theta \qquad y = l\cos\theta$$

$$\delta x = l\cos\theta\,\delta\theta \qquad \delta y = -l\sin\theta\,\delta\theta$$

Considering positive virtual displacements,

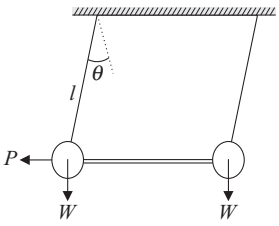
$$\delta x = l\cos\theta\,\delta\theta \qquad \delta y = l\sin\theta\,\delta\theta$$

$$P\,\delta x - W\,\delta y - W\,\delta y = 0$$

$$P\,l\cos\theta\,\delta\theta - 2W\,l\sin\theta\,\delta\theta = 0$$

$$\tan\theta = \frac{P}{2W}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{P}{2W}\right]$$



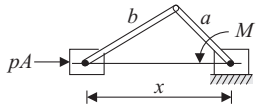
11.8:

$$x = a\cos\theta + \sqrt{b^2 - a^2\sin^2\theta}$$

$$\delta x = \left[-a\sin\theta + \frac{1}{2}(b^2 - a^2\sin^2\theta)^{1/2}(-2a^2\sin\theta\cos\theta)\right]\delta\theta$$

$$= -a\sin\theta\left[1 + \frac{a\cos\theta}{\sqrt{b^2 - a^2\sin^2\theta}}\right]\delta\theta$$

Considering positive virtual displacements,



$$\delta x = a\sin\theta\left[1 + \frac{a\cos\theta}{\sqrt{b^2 - a^2\sin^2\theta}}\right]\delta\theta$$

$$M\delta\theta - pA\,\delta x = 0$$

$$\Rightarrow \quad p = \frac{M}{Aa\sin\theta\left[1 + \frac{a\cos\theta}{\sqrt{b^2 - a^2\sin^2\theta}}\right]}$$

11.9:

$$y_1 = \frac{l}{2}\sin\theta \qquad \delta y_1 = \frac{l}{2}\cos\theta\,\delta\theta$$

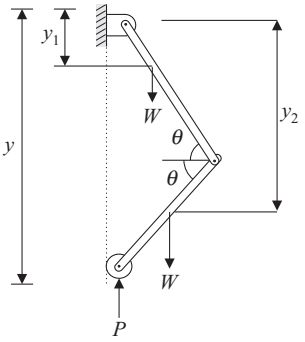
$$y_2 = \frac{3l}{2}\sin\theta \qquad \delta y_2 = \frac{3l}{2}\cos\theta\,\delta\theta$$

$$y = 2l\sin\theta \qquad \delta y = 2l\cos\theta\,\delta\theta$$

$$-P\,\delta y + W\delta y_1 + W\delta y_2 = 0$$

$$-P(2l\cos\theta\,\delta\theta) + W\left(\frac{l}{2}\cos\theta\,\delta\theta\right) + W\left(\frac{3l}{2}\cos\theta\right)\delta\theta = 0$$

$$\Rightarrow \quad P = W$$



11.10:

$$x = l\sin\theta + l\sin\theta + l\sin\theta$$

$$= 3l\sin\theta$$

$$\delta x = 3l \cos\theta\, \delta\theta$$

$$P\delta x - M\delta\theta = 0$$

$$P(3l\cos\theta)\delta\theta - M\delta\theta = 0$$

$$\Rightarrow \quad M = 3Pl\cos\theta$$

11.11:

$$x = 7\frac{l}{2}\sin\theta = 3.5l\sin\theta \qquad \delta x = 3.5l\sin\theta\,\delta\theta$$

$$y = 2\frac{l}{2}\cos\theta = l\cos\theta \qquad \delta y = -l\sin\theta\,\delta\theta$$

Considering positive virtual displacements.

$$\delta x = 3.5\, l\sin\theta\,\delta\theta$$

$$\delta y = l\sin\theta\,\delta\theta$$

$$Q\,\delta x - P\,\delta y = 0$$

$$Q\,(3.5\, l\sin\theta)\delta\theta - P\,(l\sin\theta)\,\delta\theta = 0$$

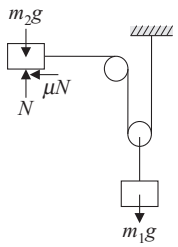
$$\Rightarrow \quad Q = \tan^{-1}\left[\frac{3.5\,Q}{P}\right]$$

11.12:

x = displacement of m_2

y = displacement of m_1

$$= \frac{x}{2}$$



$$m_1 g\,\delta y - \mu N\delta x = 0$$

$$m_1 g\,\frac{\delta x}{2} - \mu m_2 g\,\delta x = 0$$

$$\Rightarrow \quad \mu = \frac{m_1}{2m_2}$$

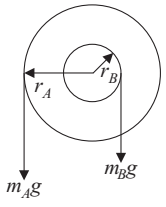
11.13:

$$s_A = r_A\theta \qquad \delta s_A = r_A\,\delta\theta$$

$$s_B = r_B\theta \qquad \delta s_B = r_B\,\delta\theta$$

$$m_B\, g\,\delta s_B - m_A\, g\,\delta s_A = 0$$

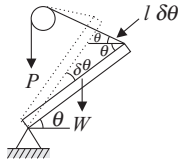
$$m_B\, g\, r_B\,\delta\theta - m_A\, g\, r_A\,\delta\theta = 0$$



$$\Rightarrow \quad m_A = \frac{m_B\, r_B}{r_A}$$

$$= 3.33 \text{ kg}$$

11.14:



$$P\delta y_1 - W\delta y_2 = 0$$

$$\tan\phi = \frac{3}{4}$$

$$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$

$$\delta y_1 = l\,\delta\theta \cos\,[90° - 30° - 36.87°]$$

$$= l\cos 23.13°\,\delta\theta$$

$$y_2 = \frac{l}{2}\sin\theta$$

$$\delta y_2 = \frac{l}{2}\cos\theta\,\delta\theta$$

$$P[l\cos 23.13°]\delta\theta = W\left[\frac{l}{2}\cos\theta\right]\delta\theta$$

$$\Rightarrow \quad P = \frac{W}{2}\frac{\cos\theta}{\cos 23.13°}$$

$$= \frac{28.5\times 6\times 9.81\times\cos 30°}{2\times\cos 23.13°}$$

$$= 790\text{ N}$$

11.15:

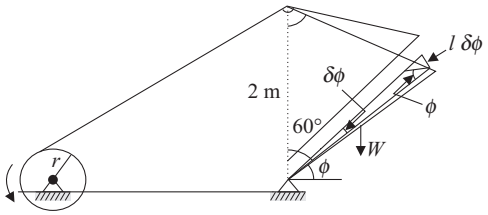
$$M\,\delta\theta - W\delta y = 0$$

$$\text{Shortening of string} = r\,\delta\theta$$

$$r\,\delta\theta = l\,\delta\phi\cos 30°$$

$$y = \frac{l}{2}\sin\phi$$

$$\delta y = \frac{l}{2}\cos\phi\,\delta\phi$$



$$M\,\delta\theta - W\frac{l}{2}\cos\phi\,\delta\phi = 0$$

$$M\,\delta\theta - W\frac{l}{2}\cos\phi\,\frac{r\delta\theta}{l.\cos 30°} = 0$$

$$\Rightarrow \quad M = \frac{W}{2}\cdot\frac{\cos\phi}{\cos 30°}\,r$$

$$= \frac{W}{2}\,\frac{\cos 30°}{\cos 30°}\,r$$

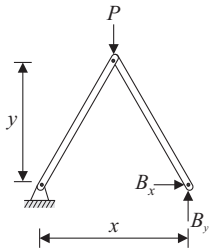
$$= \frac{100 \times 2 \times 9.81 \times 0.15}{2}$$

$$= 147.15 \text{ N}$$

11.16:

$x = 2\,l \sin\theta$ $\qquad$ $\delta x = 2\,l \cos\theta\,\delta\theta$

$y = l \cos\theta$ $\qquad$ $\delta y = -\,l \sin\theta\,\delta\theta$



Considering positive virtual displacements,

$$\delta x = 2\,l \cos\theta\,\delta\theta$$

$$\delta y = l \sin\theta\,\delta\theta$$

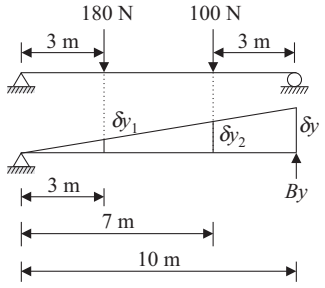
$$P\delta y + B_x\,\delta x = 0$$

$$P\,[l \sin\theta]\delta\theta + B_x\,[2\,l \cos\theta]\delta\theta = 0$$

$$\Rightarrow \qquad B_x = -\frac{P \tan\theta}{2}$$

Similarly, $\qquad A_x = \frac{P \tan\theta}{2}$

11.17:



$$\delta y_1 = \frac{3}{10}\delta y$$

$$\delta y_2 = \frac{7}{10}\delta y$$

$$By\,\delta y - 180\,\delta y_1 - 100\,\delta y_2 = 0$$

$$By\,\delta y - 180\left(\frac{3}{10}\delta y\right) - 100\left(\frac{7}{10}\right)\delta y = 0$$

$$\Rightarrow \qquad By = 124 \text{ N}$$

Similarly, $\qquad Ay = 156\text{N}.$