# Chapter-10

**10.1:**

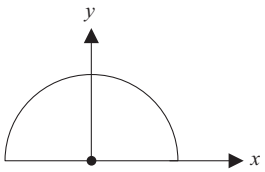
$$I_{xx} = I_{yy} = \frac{\pi R^4}{8}$$

$$(I_{xx})_{\text{mass}} = \rho t \cdot \frac{\pi R^4}{8}$$

$$= \rho t \cdot \frac{\pi \cdot R^2}{2} \cdot \frac{R^2}{4}$$

$$= \frac{\boldsymbol{M \cdot R^2}}{\boldsymbol{4}}$$

$$(I_{zz})_{\text{mass}} = \frac{\boldsymbol{M \cdot R^2}}{\boldsymbol{2}}$$




**10.2:**

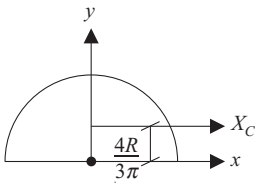
$$\mathrm{I_{xx}} = \frac{\boldsymbol{MR^2}}{\boldsymbol{4}}$$

$$I_{xx} = \frac{\boldsymbol{MR^2}}{\boldsymbol{4}} - M \cdot \left(\frac{4R}{3\pi}\right)^2$$

$$= \frac{MR^2}{4} - MR^2 \cdot \frac{16}{9\pi^2}$$

$$= MR^2 \left[\frac{1}{4} - \frac{16}{9\pi^2}\right]$$

$$= 0.7\, MR^2$$



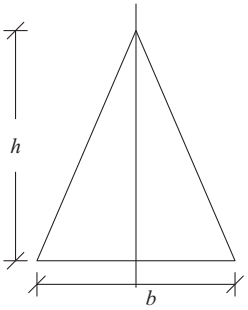

**10.3:**

$$(I_{\text{base}})_{\text{area}} = \frac{1}{12}\, bh^3$$

$$(I_{\text{base}})_{\text{mass}} = \rho t \cdot \frac{1}{12} bh^3$$

$$= \rho t \cdot \frac{1}{2} bh \cdot \frac{1}{6} h^2$$

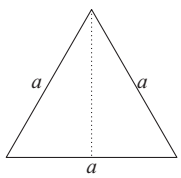
$$= \frac{Mh^2}{6}$$



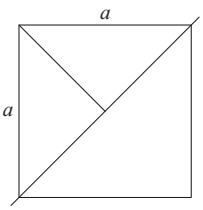

**10.4:**

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2} a$$

$$(I_{\text{base}})_{\text{mass}} = \frac{M}{6}\left[\frac{\sqrt{3}\, a}{2}\right]^2 = \frac{M}{6}\frac{3a^2}{4} = \frac{Ma^2}{8}$$



**10.5:**

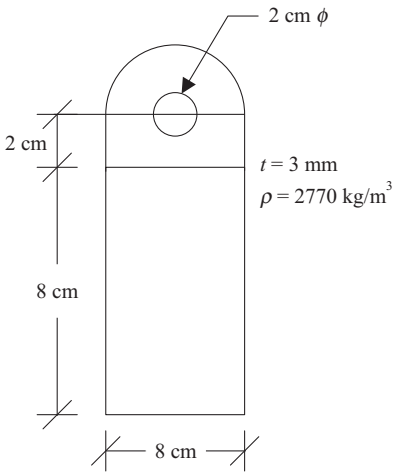



$$(I_{diag})_{area} = \frac{1}{12} \cdot a \cdot a^3 = \frac{a^4}{12}$$

$$(I_{diag})_{mass} = \rho t \cdot \frac{a^4}{12}$$

$$= \frac{M a^2}{12}$$

**10.6:**




$$M_1 = 2770\ [0.08]\ [0.08]\ [0.003] = 0.053 \text{ kg}$$

$$M_2 = 2770\ \frac{\pi}{2}\ (0.04)^2\ (0.003) = 0.0209 \text{ kg}$$

$$M_3 = 2770\ \pi(0.01)^2\ (0.003) = 2.61 \times 10^{-3} \text{ kg}$$

$$I_1 = \frac{M_1 h_1^2}{3} = \frac{0.053 \times (0.08)^2}{3} = 1.13 \times 10^{-4} \text{ kg.m}^2$$

$$I_2 = \frac{M_2 R_2^2}{4} - M_2\left(\frac{4R_2}{3\pi}\right)^2 + M_2\left[0.08 + \frac{4R_2}{3\pi}\right]^2$$

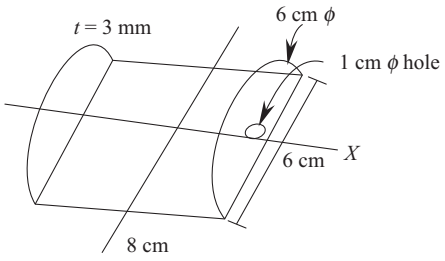
$$= 1.99 \times 10^{-4} \text{ kg.m}^2$$

$$(-)\ I_3 = \frac{M_3 R_3^2}{4} + M_3[0.1]^2$$

$$= 2.62 \times 10^{-5} \text{ kg m}^2$$

$$I = I_1 + I_2 + I_3 = 2.86 \times 10^{-4} \text{ kgm}^2$$

**10.7:**




$$M_1 = 7850 \times (0.08)\ (0.06)\ (0.003) = 0.113 \text{ kg}$$

$$M_2 = 7850\ \frac{\pi}{2}(0.03)^2 \times (0.003) = 0.033 \text{ kg}$$

$$M_3 = 7850\ \pi(0.005)^2\ (0.003) = 1.85 \times 10^{-3} \text{ kg}$$

$$I_{xx} = \frac{M_1\ (0.06)^2}{12} + 2\frac{M_2\ (0.03)^2}{2} - 2\left[\frac{M_3\ (0.005)^2}{2} + M_3\ (0.015)^2\right]$$

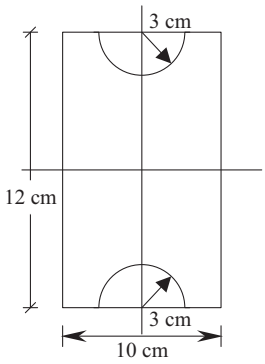
$$= 6.3 \times 10^{-5} \text{ kgm}^2$$

$$I_{yy} = \frac{M_1 (0.08)^2}{12} + 2\left[\frac{M_2 (0.03)^2}{4} - M_2\left(\frac{4(0.03)}{3\pi}\right)^2 + M_2\left[(0.04)^2 + \frac{(4 \times 0.03)^2}{3\pi}\right]\right]$$

$$-2\left[\frac{M_3 (0.005)^2}{4} + M_3[(0.015)^2 + (0.04)^2]\right]$$

$$= 1.74 \times 10^{-4} \text{ kg.m}^2$$

$$I_{zz} = \frac{M_1}{12}\left[(0.08)^2 + (0.06)^2\right] + 2\left[\frac{M_2(0.03)^2}{4} + M_2(0.04)^2\right]$$
$$-2\left[\frac{M_3(0.005)^2}{4} + M_3(0.04)^2\right]$$
$$= 2.09 \times 10^{-4}\ \text{kg.m}^2$$




**10.8:**

$t = 4$ mm

$\rho = 7850\ \text{kg/m}^3$

$M_1 = (7850)\ (0.1)\ (0.12)\ (0.004) = 0.3768$ kg

$M_2 = (7850)\ \frac{\pi}{2}\ (0.03)^2\ (0.004) = 0.044$ kg

$$I_{xx} = \frac{M_1(0.12)^2}{12} - 2\left[\frac{M_2(0.03)^2}{4} - M_2\left(\frac{4(0.03)^2}{3\pi}\right) + M_2\left(0.06 - \frac{4(0.03)}{3\pi}\right)^2\right]$$
$$= 2.5 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$$

$$I_{yy} = \frac{M_1(0.1)^2}{12} - 2\left[\frac{M_2(0.03)^2}{4}\right] = 2.94 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$$

$$I_{zz} = \frac{M_1}{12}[(0.12)^2 + (0.1)^2] - 2\left[\frac{M_3(0.03)^2}{4} - M_2\left(\frac{4(0.03)}{3\pi}\right)^2 + M_2\left(0.06 - \frac{4(0.03)}{3\pi}\right)^2\right]$$
$$= 5.44 \times 10^{-4}\ \text{kg m}^2$$

**10.9:**

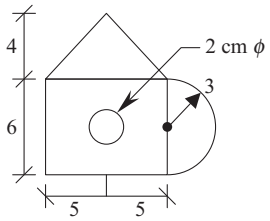
$t = 4$ mm $\quad \rho = 7850\ \text{kg/m}^3$

$M_1 = 7850 \times (0.1)\ (0.06)\ (0.004) \quad = 0.188$ kg

$M_2 = 7850 \times \frac{1}{2}(0.1)\ (0.04)\ (0.004) \quad = 0.063$ kg

$M_3 = 7850 \times \frac{\pi}{2}\ (0.03)^2\ (0.004) \quad = 0.044$ kg

$M_4 = 7850 \times \pi(0.01)^2\ (0.004) \quad = 9.865 \times 10^{-3}$ kg




$$I_1 = \frac{M_1 h_1^2}{3} = 2.26 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$$

$$I_2 = \frac{M_2 h_2^2}{18} + M_2\left[0.06 + \frac{0.04}{3}\right]^2 = 3.44 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$$

$$I_3 = \frac{M_3 R_3^2}{4} + M_3 R_3{}^2 = 4.95 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$$

$$I_4 = \frac{M_4 R_4^2}{4} + M_A(0.03)^2 = 9.13 \times 10^{-6}\ \text{kg}\cdot\text{m}^2$$

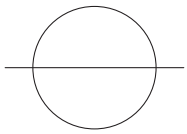
$I = I_1 + I_2 + I_3 - I_4$

$= 6.1 \times 10^{-4}\ \text{kg.m}^2$

**10.10:**

$$\bar{I} = \frac{2}{5} MR^2$$

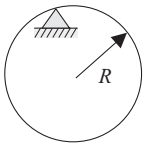
$$I_{\text{tangent}} = \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$$



**10-11:**

$$\bar{I}_{zz} = MR^2$$

$$I_{zz} = MR^2 + MR^2 = 2\ MR^2$$

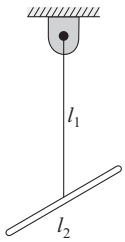



**10.12:**

$$I_o = \frac{(ml_1)l_1^2}{12} + (m\ l_1)\ (l_1/2)^2 + \frac{(ml_2)(l_2)^2}{12} + (m\,l_2)\ (l_1)^2$$

$$= m\left[\frac{l_1^3}{12} + \frac{l_1^3}{4} + \frac{l_2^3}{12} + l_2\, l_1^2\right]$$

$$= \frac{m}{12}\left[4\ l_1{}^3 + l_2{}^3 + 12\, l_2\ l_1{}^2\right)$$




**10.13:**

$$\bar{I} = \frac{M}{12}[3R^2 + H^2]$$

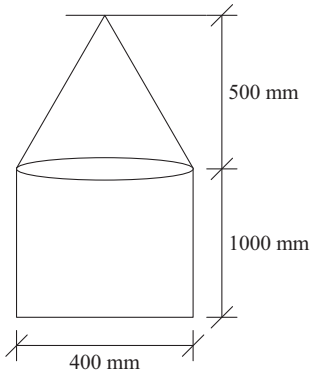
$$I = \frac{M}{12}[3R^2 + H^2] + M\left(\frac{H}{2}\right)^2$$

$$= M\left[\frac{R^2}{4} + \frac{H^2}{12} + \frac{H^2}{4}\right]$$

$$= M\left[\frac{\boldsymbol{R^2}}{\boldsymbol{4}} + \frac{\boldsymbol{H^2}}{\boldsymbol{3}}\right]$$

$$\bar{I} = \frac{MR^2}{2}$$

$$I = \frac{MR^2}{2} + MR^2 = \boldsymbol{\frac{3}{2} MR^2}$$




**10.14:**

$$\rho = 4000\ \text{kg.m}^3$$

$$M_1 = \rho \frac{1}{3} \pi R_1^2 H_1$$

$$= 4000 \times \frac{1}{3}\pi \times (0.2)^2\ (0.5) = 83.78\ \text{kg}$$

$M_2 = \rho\, \pi\, R_2{}^2 H_2$

$= 4000 \times \pi \times (0.2)^2\ (1)$

$= 502.65$ kg

$$I_1 = \frac{3}{5} M_2 \left[\frac{R_1^2}{4} + H_1^2\right] = \frac{3}{5} \times 83.78 \left[\frac{(0.2)^2}{4} + (0.5)^2\right] = 13.07 \text{ kg}\cdot\text{m}^2$$

$$I_2 = \frac{M_2}{12}[3R_2{}^2 + H_2{}^2] = + M^2[1]^2 \frac{502.65}{12}\ [3\ (0.2)^2 + 1^2] + 502.65\ (1)^2 = 549.56 \text{ kg}$$

$I_o = I_1 + I_2 = 562.6 \text{ kg}\cdot\text{m}^2$

**10.15:**

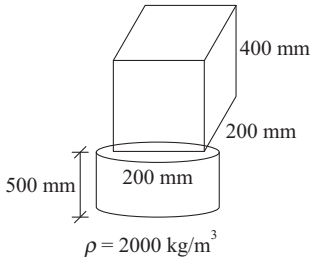
$M_1 = \rho V_1 = 2000 \times 0.2 \times 0.2 \times 0.4 = 32$ kg

$M_2 = \rho V_2 = 2000 \times \pi \times (0.15)^2\ 0.5 = 70.69$ kg

$\bar{I}_{zz} = (\bar{I}_{zz})_1 + (\bar{I}_{zz})_2$

$$= \frac{M_1}{12}[(0.2)^2 + (0.2)^2] + \frac{M_2}{2}[(0.15)^2]$$

$= 1.008 \text{ kg}\cdot\text{m}^2$




**10.16:**

$\rho_1 = 7850 \text{ kg/m}^3$

$\rho_2 = 11{,}375 \text{ kg/m}^3$

$$M_1 = \rho_1 \frac{1}{3}\ \pi R_1{}^2 H_1$$

$$= \rho_1 \frac{1}{3}\ \pi \times (0.1)^2\ (0.3)$$

$= 24.66$ kg

$M_2 = (\rho_2 - \rho_1)\ \pi \times (0.04)^2\ (0.05)$

$= 0.86$ kg

$$I_1 = \frac{3}{10} M_1 R_1{}^2$$

$$I_2 = (\rho_1\ V_2)\ \frac{R_2^2}{2} = M'_2\ \frac{R_2^2}{2}$$

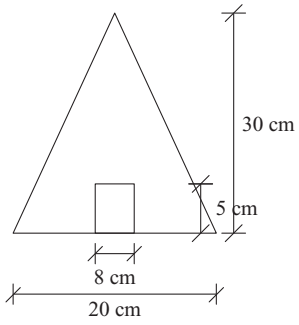
$$I_3 = (\rho_2\ V_2)\ \frac{R_2^2}{2} = M''_2\ \frac{R_2^2}{2}$$

$I = I_1 - I_2 + I_3$

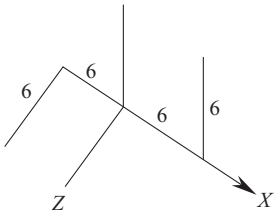
$$= \frac{3}{10} M_1\ R_1{}^2 + M_2 \frac{R_2^2}{2}$$

$= 0.075 \text{ kg.m}^2$

$= 7.5 \times 10^{-2} \text{ kg}\cdot\text{m}^2$



**10.17:**




$$M_1 = M_3 = 0.15 \times \frac{6}{24} = 0.0375 \text{ kg}$$

$$M_2 = 0.15 \times \frac{12}{24} = 0.075 \text{ kg}$$

$$I_{zz} = I_1 + I_2 + I_3$$

$$= [0.0375 \times (0.06)^2] + \left\{\frac{0.075}{12}[0.12]^2\right\} + \left\{\frac{0.0375 \times (0.06)^2}{12}\right\}$$

$$+ 0.0375\ [(0.06)^2 + (0.03)^2]$$

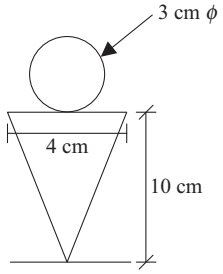
$$= 4.05 \times 10^{-4} \text{ kg.m}^2$$

$$I_{xx} = \left\{\frac{0.0375 \times (0.06)^2}{12} + 0.0375 \times (0.03)^2\right\} + \frac{0.0375 \times (0.06)^2}{12} + 0.0375 \times (0.03)^2$$

$$= 9 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

$I_{yy}$ = same as $I_{zz}$

**10.18:**




$$M_1 = \rho \frac{1}{3} \pi (0.02)^2 (0.1)$$

$$= 0.329 \text{ kg}$$

$$M_2 = \rho \frac{4}{3}\ \pi (0.015)^3$$

$$= 0.111 \text{ kg}$$

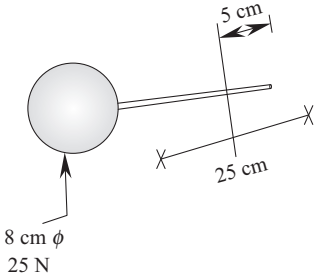
$$I_{AA} = I_1 + I_2$$

$$= \frac{3}{5} M_1 \left[\frac{R_1^2}{4} + H_1^2\right] + \frac{2}{5} M_2\ R^2{}_2 + M_2\ [0.115]^2$$

$$= \frac{3}{5} \times 0.329 \left[\frac{(0.02)^2}{4} + (0.1)^2\right] + \frac{2}{5}(0.11)\,(0.015)^2 + (0.111)\,(0.115)^2$$

$$= 3.47 \times 10^{-3} \text{ kg.m}^2$$

**10.19:**




$$M_1 = \frac{25}{9.81} \text{kg} \quad M_2 = \frac{10}{9.8}\ \text{kg}$$

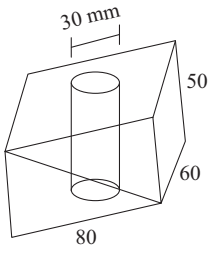
$$I_0 = I_1 + I_2$$

$$= \left[\frac{2}{5} \times \left(\frac{25}{9.81}\right)(0.04)^2 + \frac{25}{9.81}(0.24)^2\right]$$

$$+ \left(\frac{10}{9.81}\right)\frac{(0.25)^2}{12} + \left(\frac{10}{9.81}\right)(0.075)$$

$$= 0.16 \text{ kg.m}^2$$

**10.20:**




$$M_1 = \rho\,(0.08)\,(0.06)\,(0.05) = 1.884 \text{ kg}$$

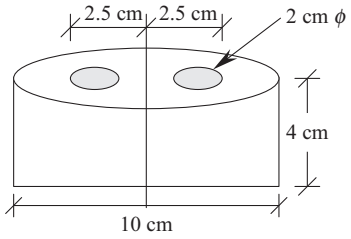
$$M_2 = \rho\pi(0.015)^2\,(0.05) = 0.277 \text{ kg}$$

$$I = I_1 - I_2$$

$$= \frac{M_1}{12}\left[(0.08)^2 + (0.06)^2\right] - \frac{M_2\,(0.015)^2}{2}$$

$$= 1.54 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

**10.21:**




$$M_1 = \rho\pi(0.05)^2\,(0.04) = 2.466 \text{ kg}$$

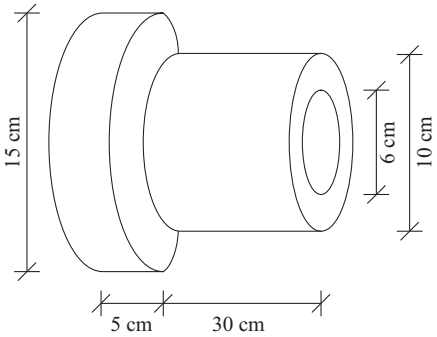
$$M_2 = \rho\pi\,(0.01)^2\,(0.04) = 0.099 \text{ kg}$$

$$I_{zz} = \frac{M_1 R_1^2}{2} - 2\left[\frac{M_2 R_2^2}{2} + M_2\,(0.025)^2\right]$$

$$= \frac{2.466 \times (0.05)^2}{2} - 2\left[\frac{0.099 \times (0.01)^2}{2} + 0.099\,(0.025)^2\right]$$

$$= 2.95 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

**10.22:**




$$M_1 = \rho\pi\,(0.075)^2\,(0.05) = 6.94 \text{ kg}$$

$$M_2 = \rho\pi\,\left[(0.05)^2 - (0.03)^2\right]\,[0.3] = 11.84 \text{ kg}$$

$$I = I_1 + I_2$$

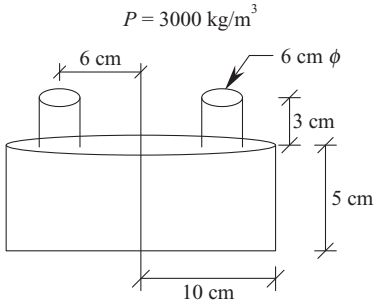
$$= \frac{M_1 R_1^2}{2} + \frac{M_2^2}{2}\,[R_2'^2 + R_2''^2]$$

$$= \frac{6.94 \times (0.075)^2}{2} + \frac{11.84}{2}[(0.05)^2 + (0.03)^2]$$

$$= 0.0396 \text{ kg.m}^2$$

$$= 3.96 \times 10^{-2} \text{ kg.m}^2$$

**10.23:**




$$M_1 = \rho \cdot \pi \cdot (0.1)^2\,(0.05) = 4.712 \text{ kg}$$

$$M_2 = \rho \cdot \pi \cdot (0.03)^2\,(0.03) = 0.254 \text{ kg}$$

$$I = I_1 + 2\,(I_2)$$

$$= \frac{M_1 R_1^2}{2} + 2\left[\frac{M_2 R_2^2}{2} + M_2\,(0.06)^2\right]$$

$$= \frac{4.712 \times (0.1)^2}{2} + 2\left[0.254\,\frac{(0.03)^2}{2} + 0.254\,(0.06)^2\right]$$

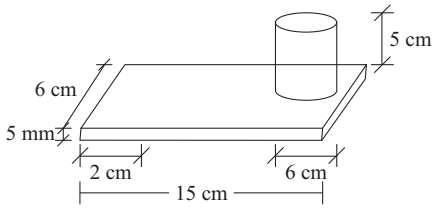
$$= 0.0256 \text{ kg.m}^2$$

$$= 2.56 \times 10^{-2} \text{ kg.m}^2$$

**10.24:**

$\rho_1 = 7850 \text{ kg/m}^3$

$\rho_2 = 6400 \text{ kg/m}^3$




$M_1 = 7850\,(0.15)\,(0.06)\,(0.005)$

$= 0.353 \text{ kg}$

$M_2 = 6400\,(\pi)\,(0.03)^2\,(0.05)$

$= 0.905 \text{ kg}$

$$I_1 = \frac{M_1 l_1^2}{12} + M_1 d_1^2$$

$$= \frac{(0.353)(0.15)^2}{12} + (0.353)(0.055)^2$$

$$= 1.73 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_2 = \frac{M_2 R_2^2}{2} + M_2 d_2^2$$

$$= \frac{(0.905)(0.03)^2}{2} + (0.905)\,(0.1)^2$$

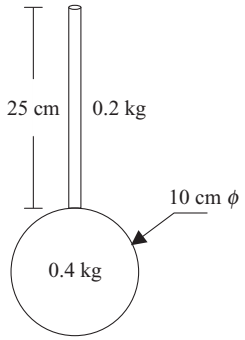
$$= 9.46 \times 10^{-3} \text{ kg.m}^2$$

$I = I_1 + I_2$

$= 11.2 \times 10^{-3} \text{ kg.m}^2$

$= 1.12 \times 10^{-2} \text{ kg.m}^2$

**10.25:**




$$I_1 = \frac{(0.2)(0.25)^2}{12} + (0.2)\,(0.125)^2 + \frac{(0.4)(0.06)^2}{2} + (0.4)\,(0.3)^2$$

$$= 4.07 \times 10^{-2} \text{ kg.m}^2$$