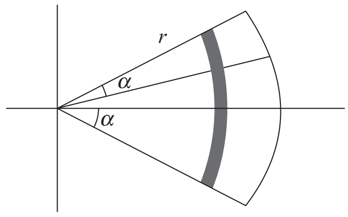
Chapter 9

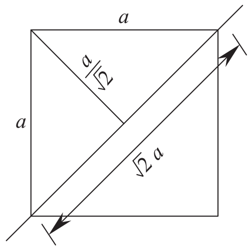
9.1:



$$dA = 2\alpha(r)\,dr$$

$$I_p = \int r^2\,dA = \int_O^R 2\alpha r^3 dr = 2\alpha \frac{r^4}{4}$$

9.2:



$$I_1 = \frac{1}{12}\sqrt{2}\,a\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{1}{12}\frac{a^4}{2}$$

$$I_2 = \frac{1}{12}\cdot\frac{a^4}{2}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

9.3:

$$I = \frac{1}{12}bh^3$$

$$= \frac{1}{2}\sqrt{4r^2 - h^2}\;h^3$$

$$(2r)^2 = b^2 + \mathrm{h}^2$$

$$\sqrt{4r^2 - h^2} = b$$

$$\frac{dI}{dh} = \frac{1}{2}(4r^2 - h^2)^{1/2}\,3h^2 + \frac{1}{2}h^3\frac{1}{2}(4r^2 - h^2)^{-1/2}\,(-2h)$$

$$\Rightarrow \quad \frac{dI}{dh} = 0 \quad \Rightarrow \quad 3\sqrt{4r^2 - h^2} = \frac{h^2}{\sqrt{4r^2 - h^2}} \Rightarrow 3(4r^2 - h^2) = h^2$$

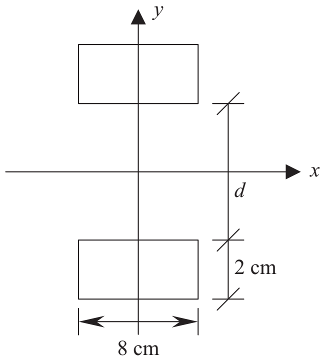
$$12r^2 = 4h^2$$

$$\therefore\ h = \sqrt{3r} \ \text{ and } \ b = r$$

$$I_{\max} = \frac{1}{12}r\left(\sqrt{3}\,r\right)^3$$

$$= \frac{\sqrt{3}}{4}r^4$$

9.4:



$$\bar{I}_{xx} = 2\left[\frac{1}{12}(8)(2)^3 + (8)(2)\left(\frac{d}{2}+1\right)^2\right]$$

$$\bar{I}_{yy} = 2\left[\frac{1}{12}(2)(8)^3\right]$$

$$\bar{I}_{xx} = \bar{I}_{yy}$$

$$\Rightarrow \quad \frac{1}{12}(8)(2)^3 + (8)(2)\left(\frac{d}{2}+1\right)^2 = \frac{1}{12}(2)(8)^3$$

$$(\cancel{8})(\cancel{2})\left(\frac{d}{2}+1\right)^2 = \frac{1}{12}(\cancel{2})(\cancel{8})\ [8^2 - 2^2]$$

$$\left(\frac{d}{2}+1\right)^2 = \frac{60}{12} = 5$$

$$\Rightarrow \quad d = 2.47 \text{ cm}$$

9.5:

$$\bar{I} = 0.11\ R^4 = 0.11\ (5)^4 = 68.75\ \text{cm}^4$$

$$I_{AA} = \bar{I} + \frac{\pi \cdot R^2}{2}\left[4 + \frac{4(5)}{3\pi}\right]^2$$

$$= 68.75 + \frac{\pi(5)^2}{2}\left[4 + \frac{4(5)}{3\pi}\right]^2$$

$$= 1540.6 \text{ cm}^4$$

9.6:

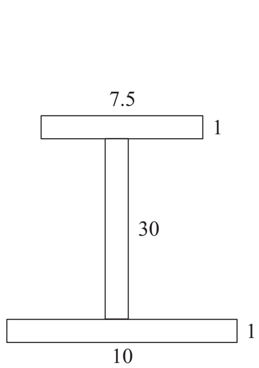
$$I_{xx} = \frac{\pi \cdot R^4}{16} = 0.196\ R^4$$

$$I_{x'x'} = \bar{I} + Ad^2$$

$$= 0.055\ R^4 + \frac{\pi}{4}R^2\left[R - \frac{4R}{3\pi}\right]^2$$

$$= 0.315\ R^4$$

9.7:



$\bar{x} = 5$ cm

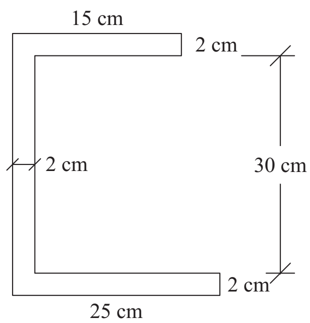
A_i	$\bar{y}_i$	$A_i\bar{y}_i$
10	0.5	5
30	16	480
7.5	31.5	236.25
47.5		721.25

$\Rightarrow \quad \bar{y} = 15.18$ cm

$(\bar{I}_{xx})_i$	$(\bar{I}_{yy})_i$	$A_i(\bar{y}_i - \bar{y})^2$	$A_i(\bar{x}_i - \bar{x})^2$
$\frac{1}{12} \times 10 \times 1^3$ = 0.833	$\frac{1}{12} \times 1 \times 10^3$ = 83.33	$10(0.5 - 15.18)^2$ = 2155.02	0
$\frac{1}{12} \times 1 \times 30^3$ = 2250	$\frac{1}{12} \times 30 \times 1$ = 2.5	$30(16 - 15.18)^2$ = 20.17	0
$\frac{1}{12} \times 7.5 \times 1^3$ 0.625	$\frac{1}{12} \times 1 \times (7.5)^3$ = 35.16	$7.5(31.5 - 15.18)^2$ = 1997.57	0
= 2251.46	120.99	4172.76	0

$$\bar{I}_{xx} = (\bar{I}_{xx})_i + A_i(\bar{y}_i - \bar{y})^2$$
$$= 6424.22 \text{ cm}^4$$
$$\approx 6424.2 \text{ cm}^4$$
$$\bar{I}_{yy} = (\bar{I}_{yy})_i + A_i(\bar{x}_i - \bar{x})^2$$
$$= 120.99 \text{ cm}^4$$
$$\approx 121 \text{ cm}^4$$

9.8:



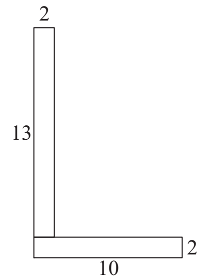
A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
50	12.5	1	625	50
60	1	(15 + 2) = 17	60	1020
30	7.5	32 + 1 = (33)	225	990
140			910	2060

$\bar{x}$ = 6.5 cm $\bar{y}$ = 14.71 cm

S.No	$(\bar{I}_{xx})_i$	$(I_{yy})_i$	$A_i(\bar{y}_i - \bar{y})^2$	$A_i(\bar{x}_i - \bar{x})^2$
1	$\frac{1}{12}(25)(2)^3$ = 16.67	$\frac{1}{12} \times 2 \times (25)^3$ = 2604.17	$50(1 - 14.71)^2$ = 9398.21	$50(12.5 - 6.5)^2$ = 1800
2	$\frac{1}{12} \times 2 \times (30)^3$ = 4500	$\frac{1}{12} \times 30 \times (2)^3$ = 20	$60(17 - 14.71)^2$ = 314.65	$60(1 - 6.5)^2$ = 1815
3	$\frac{1}{12} \times 15 \times (2)^3$ = 10	$\frac{1}{12} \times 2 \times (15)^3$ = 562.5	$30(33 - 14.71)^2$ = 10,035.72	$30(7.5 - 6.5)^2$ = 30
	4526.67	3186.67	19,748.58	3645

$$\bar{I}_{xx} = (\bar{I}_{xx})_i + A_i(\bar{y}_i - \bar{y})^2 = 4526.67 + 19{,}748.58$$
$$= 24,275.25 \text{ cm}^4$$
$$\bar{I}_{yy} = (I_{yy})_i + A_i(\bar{x}_i - \bar{x})^2 = 3186.67 + 3645$$
$$= 6831.67 \text{ cm}^4$$

9.9:



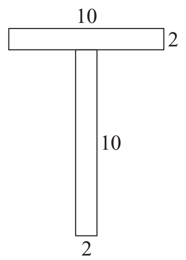
A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
20	5	1	100	20
26	1	2 + 6.5 (= 8.5)	26	221
46			126	241

$\bar{x} = 2.74$ cm $\quad \bar{y} = 5.24$ cm

$(\bar{I}_{xx})_i$	$(\bar{I}_{yy})_i$	$\bar{A}_i(\bar{y}_i - \bar{y})^2$	$A_i(\bar{x}_i - \bar{x})^2$
$\frac{1}{12}(10)(2)^3$ = 6.67	$\frac{1}{12} \times 2 \times (10)^3$ = 166.67	$20(1 - 5.24)^2$ = 359.55	$20(5 - 2.74)^2$ = 102.15
$\frac{1}{12} \times 2 \times (13)^3$ = 366.17	$\frac{1}{12} \times 13x(2)^3$ = 8.67	$26(8.5 - 5.24)^2$ = 276.32	$26(1 - 2.74)^2$ = 78.72
372.84	175.34	635.87	180.87

$$\bar{I}_{xx} = (\bar{I}_{xx})_i + A_i(\bar{y}_i - \bar{y})^2 = 1008.71 \text{ cm}^4$$
$$\bar{I}_{yy} = (\bar{I}_{yy})_i + A_i(\bar{x}_i - \bar{x})^2 = 356.24 \text{ cm}^4$$

9.10:



$\bar{x} = 5$ cm

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
20	5	100
20	11	220
40		320

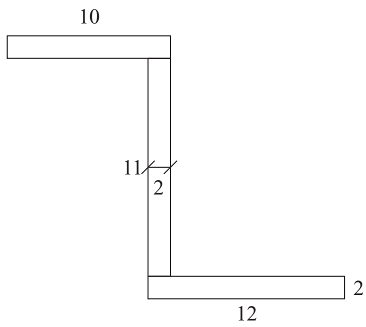
$$\bar{y} = \frac{320}{40} = 8 \text{ cm}$$

$(I_{xx})_i$	$(I_{yy})_i$	$A_i(\bar{y}_i - \bar{y})^2$	$A_i(\bar{x}_i - \bar{x})^2$
$\frac{1}{12}(2)(10)^3$ = 166.67	$\frac{1}{12} \times 10 \times 2^3$ = 6.67	$20(5 - 8)^2$ = 180	0
$\frac{1}{12}(10)\,2^3$ = 6.67	$\frac{1}{12}(2)(10)^3$ = 166.67	$20(11 - 8)^2$ = 180	0
173.34	173.34	360	0

$(\bar{I}_{xx}) = 533.33 \text{ cm}^4$

$(\bar{I}_{yy}) = 173.33 \text{ cm}^4$

9.11: (i)



$\boldsymbol{I_{xx}}$**:**

$$(I_1)_x = \frac{1}{12}(12)(2)^3 + 24(1)^2 = 32$$

$$(I_2)_x = \frac{1}{12}(2)(11)^3 + 22(5.5 + 2)^2 = 1459.33$$

$$(I_3)_x = \frac{1}{12}(10)(2)^3 + 20(11 + 2 + 1)^2 = 3926.67$$

I_{xx} = **5438 cm⁴**

$\boldsymbol{I_{yy}}$**:**

$$(I_1)_x = \frac{1}{12}(2)(12)^3 + 24(8 + 6)^2 = 4992$$

$$(I_2)_x = \frac{1}{12}(11)(2)^3 + 22(10 - 1)^2 = 1789.33$$

$$(I_3)_x = \frac{1}{12}(2)(10)^3 + 20(5)^2 = 666.67$$

I_{yy} = **7448 cm⁴**

9.11: (ii)

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
24	8 + 6 (= 14)	1	336	24
22	(10 – 1) (= 9)	2 + 5.5 (= 7.5)	198	165
20	5	13 + 1 (= 14)	100	280
66			634	469

$\bar{x} = 9.61$ cm

$\bar{y} = 7.11$ cm

$(\bar{I}_{xx})_i$	$(\bar{I}_{yy})_i$	$A_i(\bar{y}_i - \bar{y})^2$	$A_i(\bar{x}_i - \bar{x})^2$
$\frac{1}{12}(12)(2)^3 = 8$	$\frac{1}{12}(2)(12)^3 = 288$	$24(1 - 7.11)^2 = 895.97$	$24(14 - 9.61)^2 = 462.53$
$\frac{1}{12}(2)(11)^3 = 221.83$	$\frac{1}{12}(11)(2)^3 = 7.33$	$22(7.5 - 7.11)^2 = 3.35$	$22(9 - 9.61)^2 = 8.19$

$\frac{1}{12}(10)\ (2)^3$	$\frac{1}{12}(2)(10)^3$	$20\,(14\ -\ 7.11)^2$	$20(5\ -\ 9.61)^2$
$= 6.67$	$= 166.67$	$= 949.44$	$= 425.04$
236.5	462	1848.76	895.76

$$\bar{I}_{xx} = 2085.26\ \text{cm}^4$$
$$\bar{I}_{yy} = 1357.76\ \text{cm}^4$$

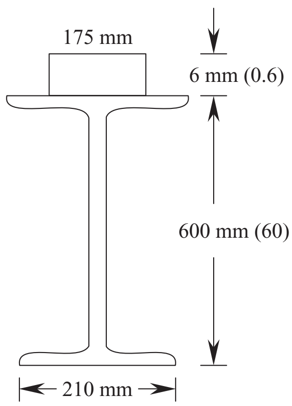
9.12:

$$\bar{I}_{xx} = \bar{I}_{yy} = \frac{1}{12}(15)(15)^3 - \frac{1}{12}(11)(11)^3 = 2998.67\ \text{cm}^4$$

Alternate method:

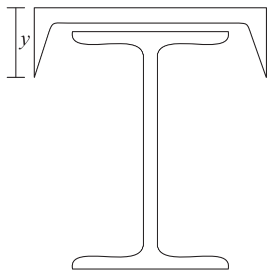
$$(\bar{I}_{xx}) = \bar{I}_{yy} = 2\left[\frac{1}{12}(2)(11)^3 + \frac{1}{12}(15)(2)^3 + (15)(2)(7.5-1)^2\right] = 2998.7\ \text{cm}^4$$

9.13:



$$\bar{y} = \frac{(156.21\times 30) + (17.5\times 0.6)\ (60.3)}{156.21 + (17.5\times 0.6)}$$
$$= 31.91\ \text{cm}$$
$$\bar{I}_{xx} = 91813 + 156.21\ (30 - 31.91)^2$$
$$+ \frac{1}{12} \times 17.5\ (0.6)^3 + (17.5)\ (0.6)\ (60.3 - 31.91)^2$$
$$= 100{,}846.1\ \text{cm}^4$$

9.14:



$$\bar{y} = \frac{(156.21)(30+1.53) + 62.93\,(2.42)}{156.21 + 62.93} = 23.17\ \text{cm}$$
$$\bar{I}_{xx} = 504.8 + 62.93\ (2.42 - 23.17)^2$$
$$+ 91813 + 156.21\ (31.53 - 23.17)^2$$
$$= 130{,}330.60\ \text{cm}^4$$

9.15:

$$\bar{I}_{xx} = \frac{1}{12} \times 0.6 \times (60)^3 + 2\left[\frac{1}{12}\times 25\times (0.6)^3 + 25\times 0.6\times (30.3)^2\right]$$
$$+ 4\ [29.1 + 7.44\ (30 - 1.81)^2]$$
$$= 62{,}109.6\ \text{cm}^4$$

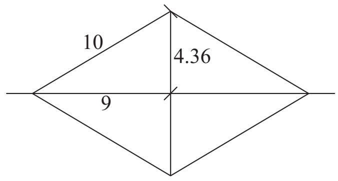
9.16:

$$I_1 = \frac{1}{12}(10)(6.24)^3 - \left[\frac{1}{36}(8)(4.47)^2 + \frac{1}{2}(8)(4.47)\left(1+\frac{4.47}{3}\right)^2\right]$$
$$= 71.77 \text{ cm}^4$$

9.17:

$$I = I_1 - I_2 = \frac{\pi(10)^4}{4} - \frac{\pi(6)^4}{4} = 6836.11 \text{ cm}^4$$

9.18:



$$I = I_1 + I_2 = 2\left[\frac{1}{12}\times 18\times(4.36)^3\right]$$
$$= 248.7 \text{ cm}^4$$

9.19:

$$I = I_1 + I_2 = \frac{1}{3}(4)(4)^3 + \frac{1}{12}(3)(4)^3 = \mathbf{101.33\ cm^4}$$

9.20:

(i) $$I = I_1 - I_2 = \frac{1}{3}(6)(10)^3 - \frac{\pi}{8}(3)^4$$
$$= 1968.2 \text{ cm}^4$$

(ii) $$I = I_1 - I_2 = \frac{1}{3}(6)(10)^3 - \left[0.11(3)^4 + \frac{\pi}{2}(3)^2\left[10 - \frac{4(3)}{3\pi}\right]^2\right]$$
$$= 914.5 \text{ cm}^4$$

9-21:

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
60	5	300
$\frac{-\pi}{2}(3)^2$	$\frac{4(3)}{3\pi}$	-18
45.86		282

$\therefore \quad \bar{y} = \frac{282}{45.86} = 6.15$ cm

$(I_{xx})_i$	$A_i(\bar{y}_i - \bar{y})^2$
$\frac{1}{12}(6)(10)^3$	$60(5-6.15)^2 = 79.35$
$= 500$	
$-0.11(3)^4$	$\frac{-\pi}{2}(3)^2\left(\frac{4}{\pi} - 6.15\right)^2 = -336.22$
$= -8.91$	
491.09	-256.87

$$\bar{I}_{xx} = (I_{xx})\,i + A_i(\bar{y}_i - \bar{y})^2 = 234.22 \text{ cm}^4$$

9.22:

$$(\bar{I}_{xx}) = I_1 - I_2 - I_3$$
$$= \frac{1}{12}(8)(12)^3 - \left[0.11(4)^4 + \frac{\pi\cdot(4)^2}{2}\left(6 - \frac{4(4)^2}{3\pi}\right)\right] \times 2$$
$$= 165.26 \text{ cm}^4$$
$$\bar{I}_{yy} = I_1 - I_2 - I_3$$
$$= \frac{1}{12}(8)^3(12) - \left[\frac{\pi}{8}(4)^4\right] \times 2$$
$$= 310.94 \text{ cm}^4$$

9.23: (i)

$$I_{AB} = \frac{\pi}{8}(5)^4 - \frac{1}{3}[2][2]^3 = 240.1 \text{ cm}^4$$

9.23: (ii)

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{\pi}{2}(5)^2$	$\frac{4(5)}{3\pi}$	$\frac{2}{3}(5)^3$
–4	1	– 4
35.27		79.33

$$\bar{y} = 2.25 \text{ cm}$$
$$\bar{I} = I_{AB} - A\,\bar{y}^{\,2}$$
$$= 61.55 \text{ cm}^4$$

9.24:

$$I_{AB} = \frac{\pi}{8}(5)^4 - \frac{1}{12} \times 10 \times (3)^3 = 222.94 \text{ cm}^4$$

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{\pi}{2}(5)^2$	$\frac{4(5)}{3\pi}$	$\frac{2}{3}(5)^3$
$-\frac{1}{2}(10)^5(3)$	1	– 15
24.27		68.33

$$\bar{y} = 2.82 \text{ cm}$$
$$\bar{I}_{xx} = I_{AB} - A\,(\bar{y})^2 = 222.94 - (24.27)(2.82)^2$$
$$= 29.94 \text{ cm}^4$$

9.25:

$$\bar{I}_{xx} = \frac{1}{12}(10)(10)^3 - \frac{\pi}{4}(2.5)^4 = 802.65 \text{ cm}^4$$
$$I_{\text{BASE}} = 802.65 + [10 - 10 \times \pi(2.5)^2]\,[5]^2 = 2811.78 \text{ cm}^4$$

9.26:

$$I_{AB} = \frac{1}{12}[10][6.24]^3 - \frac{1}{12}[10][3]^3 = 179.98 \text{ cm}^4$$
$$\approx 180 \text{ cm}^4$$

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{1}{2}(10)(6.24)$	$\frac{6.24}{3}$	64.9
$-\frac{1}{2}(10)(3)$	1	– 15
16.2		49.9

$$\bar{y} = 3.08 \text{ cm}$$
$$\bar{I} = I_{AB} - A(\bar{y})^2$$
$$= 179.98 - 16.2\,(3.08)^2 = 26.3 \text{ cm}^4$$

9.27:

$$I_{AB} = I_1 - I_2 = \frac{1}{12}\left[10 + \frac{6}{\tan 35°}\right][6]^3 - \frac{1}{12}\left[\frac{6}{\tan 35°}\right]6^3 = 180 \text{ cm}^4$$

$$\bar{y} = \frac{\frac{1}{2}\left(10 + \frac{6}{\tan 35°}\right)(6)\times\frac{6}{3} - \frac{1}{2}\left(\frac{6}{\tan 35°}\right)(6)\frac{6}{3}}{\frac{1}{2}\left(10 + \frac{6}{\tan 35°}\right)6 - \frac{1}{2}\left(\frac{6}{\tan 35°}\right)6} = \mathbf{2\ cm}$$

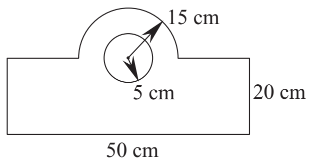
$$\bar{I} = I_{AB} - [A]\,[\bar{y}]^2 = 180 - \frac{1}{2}\left[\left(10 + \frac{6}{\tan 35°}\right)6 - \left(\frac{6}{\tan 35°}\right)6\right][2]^2$$
$$= 60 \text{ cm}^4$$

9.28:

$$I_{AB} = I_1 - I_2 = \frac{1}{3}(10)(6)^3 - \frac{1}{3}(4)(2)^3 = 709.33 \text{ cm}^4$$

9.29:

$$2\left\{\frac{1}{12}\big[(5)\,(40)\big]^3\right\} + \frac{1}{3}(30)(40)^3 - \left[\frac{\pi}{4}(7.5)^4 + \pi(7.5)^2(20)^2\right]$$
$$= 620\ 162.5 \text{ cm}^4$$



9.30:

$$\frac{1}{3}[50]\ [20]^3 + 0.11\,(15)^4 + \frac{\pi}{2}(15)^2\left[20 + \frac{4(15)}{3\pi}\right]^2 - \left[\frac{\pi}{4}(5)^4 + \pi(5)^2(20)^2\right]$$
$$= 352691 \text{ cm}^4$$

9.31:

$$I_{xx} = \frac{\pi}{4}(8)^4 - \frac{\pi}{4}(4)^4 = 3015.93 \text{ cm}^4$$

$$I_{yy} = \frac{\pi}{4}(8)^4 - \left[\frac{\pi}{4}(4)^4 + \pi(4)^4\right] = 2211.68 \text{ cm}^4$$

$$\approx 2211.7 \text{ cm}^4$$

9.32:

$$\bar{I}_{xx} = 3015.93 \text{ cm}^4$$

$$\bar{x} = \frac{\pi(8)^2\,(0) - \pi(4)^2\,(4)}{\pi(8^2 - 4^2)} = -1.33 \text{ cm from}$$

$$\bar{I}_{yy} = \frac{\pi}{4}(8)^4 + \pi(8)^2\,(1.33)^2 - \left[\frac{\pi}{4}(4)^4 + \pi(4)^2\,(4 + 1.33)^2\right]$$

$$= 1943.6 \text{ cm}^4$$

9.33:

$$I_{AB} = \frac{\pi}{8}(5)^4 - \frac{\pi}{8}(3)^4 = 213.63 \text{ cm}^4$$

9.34:

$$\bar{y} = \frac{\frac{1}{2}(60)(60)(20) - 20(10)(30)}{\frac{1}{2}(60)60 - 20(10)} = 18.75 \text{ cm}$$

$$\bar{I}_{xx} = \frac{1}{36}(60)(60)^3 + \frac{1}{2}(60)(60)\,[20 - 18.75]^2$$

$$- \left[\frac{1}{12}(10)(20)^3 + (10)(20)(30 - 18.75)^2\right]$$

$$= 330,\,833.33 \text{ cm}^4$$

$$k_x = \sqrt{\frac{\bar{I}_{xx}}{A}} = 14.4 \text{ cm}$$

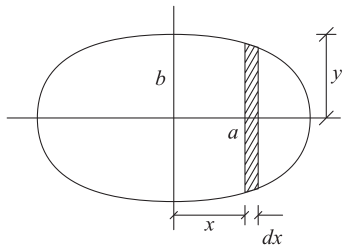
9.35:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \quad \frac{a^2 - x^2}{a^2} = \frac{y^2}{b^2}$$

$$dA = 2y\,dx = 2\cdot\frac{b}{a}\sqrt{a^2 - x^2}\cdot x^2\,dx$$

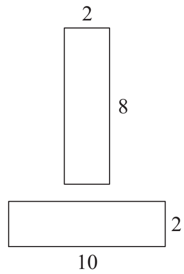
From Example 9.4, we can get

$$\bar{I}_{yy} = \frac{\Pi b a^3}{4} \;\;\&\;\; \bar{I}_{xx} = \frac{\Pi a b^3}{4}$$



$$\therefore \qquad \bar{I}_p = \frac{\Pi ab}{4}(a^2 + b^2)$$

9.36:



20	1	20
16	6	96
36		116

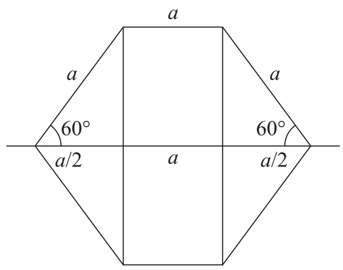
$\bar{y} = 3.22$ cm

$\frac{1}{12}(10)(2)^3 = 6.67$	$\frac{1}{12}(2)(10)^3 = 166.67$	$20(1 - 3.22)^2 = 98.568$	0
$\frac{1}{12}(2)(8)^3 = 85.33$	$\frac{1}{12}(8)(2)^3 = 5.33$	$16\,(6 - 3.22)^2 = 123.65$	0
92	172	222.22	0

$\bar{I}_{xx} = 314.22$ cm^4 $\qquad \therefore \qquad \bar{I}_p = \bar{I}_{xx} + \bar{I}_{yy} = 486.22$ cm^4

$I_{yy} = 172$ cm^4

9.37:



$$2\left[\frac{1}{3}(a)(a\sin 60°)^3\right] + 4\left[\frac{1}{12}\left(\frac{a}{2}\right)(a\sin 60°)^3\right]$$

$$= a^4\left[\frac{2}{3}\left(\frac{\sqrt{3}}{2}\right)^3 + \frac{4}{12}\,\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^3\right]$$

$$= a^4\left(\frac{\sqrt{3}}{2}\right)^3\left[\frac{2}{3} + \frac{1}{6}\right]$$

$$= \frac{5}{6}\,\frac{3\sqrt{3}}{8}\cdot a^4$$

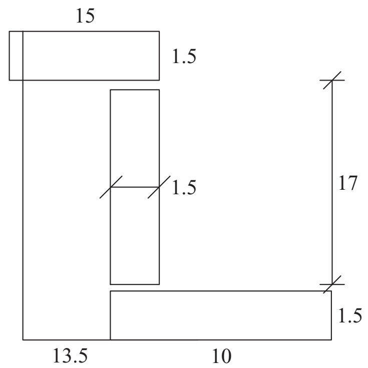
$$= \frac{5\sqrt{3}}{16}\,a^4$$

9.38:

$$\frac{r^4}{8} - \left[0 + \frac{\pi}{2}\left(\frac{r}{2}\right)^2\left(\frac{r}{2}\right)\frac{4(r/2)}{3\pi}\right]$$

$$= \frac{r^4}{8} - \frac{r^4}{24} = \frac{3-1}{24}r^4 = \frac{r^4}{12}$$

9.39:



22.5	7.5	19.25	168.75	433.125
25.5	14.25	10	363.375	255.00
15	18.5	0.75	277.5	11.2
63			809.625	699.375

$\bar{x} = 12.85$ cm

$\bar{y} = 11.1$ cm

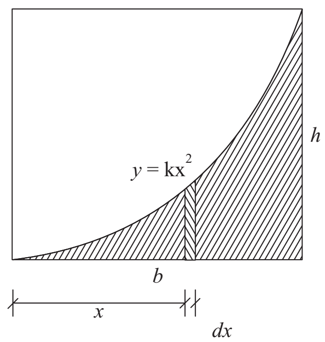
0   $22.5\ (7.5 - 12.85)\ (19.25 - 11.1) = -981.06$

0   $25.5\ (14.25 - 12.85)\ (10 - 11.1) = -39.27$

0   $15\ (18.5 - 12.85)\ (0.75 - 11.1) = -877.16$

-1897.5 cm^4

9.40:



$$dA = y\,dx$$

$$d\,I_{xy} = x \cdot \frac{y}{2} \cdot y\,dx$$

$$\Rightarrow \quad I_{yy} = \frac{1}{2} x y^2 dx$$

$$= \frac{1}{2} \cdot \frac{h^2}{b^4} \cdot \frac{b^6}{6}$$

$$\boxed{= \frac{1}{12} b^2 h^2}$$

9.41:

$$I_{uu} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$= 3.25 \times 10^4 + 0.9\ 5x\ 10^4 \cos 80° - 1.5 \times 10^4 \ \sin 80°$$

$$= 1.94 \times 10^4 \text{ mm}^4$$

$$I_{uu} = 3.25 \times 10^4 - 0.95 \times 10^4 \cos 80° + 1.5 \times 10^4 \sin 80°$$

$$= 4.56 \times 10^4 \text{ mm}^4$$

9.42:

$$I_{\text{ave}} = \frac{3.3 + 2}{2} = 2.65 \times 10^4 \text{ mm}^4$$

$$R = \sqrt{\left(\frac{3.3 - 2}{2}\right)^2 + (1.5)^2} = 1.63 \times 10^4 \text{ mm}^4$$

$$I_{max} = 4.285 \times 10^4 \text{ mm}^4$$
$$I_{min} = 1.02 \times 10^4 \text{ mm}^4$$
$$\tan 2\theta = \frac{1.5}{0.65} == 66.57°$$
$$\theta_1 = 33.3° \quad \theta_2 = 123.3°$$

9.43:

$$I_{xx} = \frac{\pi r^4}{8}$$
$$I_{yy} = \frac{\pi r^4}{8}$$
$$I_{uu} = \frac{\pi r^4}{8} + 0 + 0$$