Chapter 8

8.1:

S.No.	L_i	$\bar{X}_i$	$\bar{Y}_i$	$L_i\bar{X}_i$	$L_i\bar{Y}_i$
1	5	2.5	0	12.5	0
2	8	0	4	0	32
3	6	3	8	18	48
4	$2\pi(4)$	10	8	80π	64π
5	$\frac{-\pi(4)}{2}$	$10\frac{-2(4)}{\pi}$	$8\frac{-2(4)}{\pi}$	-14.9π	-10.91π
	37.85			235	246.79

$$\bar{x} = \frac{L_i\bar{x_i}}{\Sigma L_i} = \frac{335}{37.85} = 6.21 \text{ cm}$$

$$\bar{y} = \frac{L_i\bar{y_i}}{\Sigma L_i} = \frac{246.79}{37.85} = 6.52 \text{ cm} \approx 6.5 \text{ cm}$$

8.2:

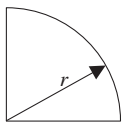
S.No.	L_i	$\bar{X}_i$	$\bar{Y}_i$	$L_i\bar{x}_i$	$L_i\bar{y}_i$
	4	0	3	0	12
	2	1	5	2	10
	$\pi(2)$	4	$\frac{5+2(2)}{\pi}$	8π	39.42
	3	7.5	5	22.5	15
	5	9	2.5	45.	12.5
	20.28			94.63	88.92

8.3:

(a)

r	$r/2$	0	$\frac{r^2}{2}$	0
r	0	$r/2$	0	$r^2/2$
$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	r^2	r^2
$2r + \frac{\pi r}{2}$			$\frac{3r^2}{2}$	$\frac{3r^2}{2}$

$$\bar{x} = \frac{L_i\bar{x_i}}{\Sigma L_i} = \frac{3r^2/2}{2r + \frac{\pi r}{2}} = \frac{3r}{4+\pi}$$

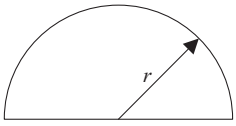


(a)

(b)

L_i	$\bar{y}_i$	$L_i\ \bar{y}_i$
$2r$	0	0
πr	$\frac{2r}{\pi}$	$2r^2$
$r(2+\pi)$		$2r^2$

$$\bar{y} = \frac{L_i\bar{y_i}}{\Sigma L_i} = \frac{2r^2}{r(2+\pi)} = \frac{2r}{2+\pi}$$



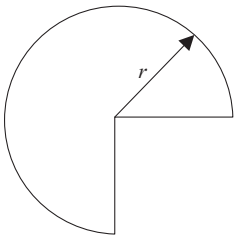
(b)

(c)

$2\pi r$	0	0	0	0
$-\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{-2r}{\pi}$	$-r^2$	r^2
r	$r/2$	0	$\frac{r^2}{2}$	0
r	0	$\frac{-r}{2}$	0	$\frac{-r^2}{2}$
$\frac{3\pi r}{2}+2r$			$\frac{-r^2}{2}$	$\frac{r^2}{2}$

$$\bar{x} = \frac{-r^2/2}{\frac{3\pi r}{2}+2r} = \frac{-r}{4+3\pi}$$

$$\bar{y} = \frac{r}{4+3\pi}$$



(c)

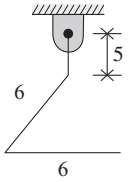
8.4:

5	0	2.5	0	12.5
6.	$\frac{6\cos 60°}{2}$	$5+\frac{6\sin 60°}{2}$	9	45.59
6.	0	5 + 6 sin 60°	0	61.18
17			9	119.27

$$\bar{x} = \frac{9}{17} = 0.53$$

$$\bar{y} = 7.02$$

$$\theta = \tan^{-1}\left[\frac{\bar{x}}{\bar{y}}\right] = \mathbf{4.32°}$$



8.5:

$$\theta = \tan^{-1}\left[\frac{2r/\pi}{r}\right] = \mathbf{32.48°}, = 32.5°$$

8.6:

10	0	5	0	50
8	$\frac{6\sin\theta}{2}$	$\frac{8\sin\theta}{2}$	24 sin θ	32 sin θ
6	$\frac{6\sin\theta}{2}$	$8\sin\theta+\frac{6\cos\theta}{2}$	18 sin θ	49.2
24			33.6	124.8

$$\theta = \tan^{-1}\left[\frac{4}{3}\right] = 53.13°$$

$$\bar{x} = 1.4$$

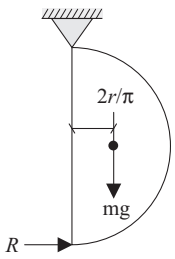
$$\bar{y} = 5.2$$

$$\alpha = \tan^{-1}\left[\frac{\bar{x}}{\bar{y}}\right] = \mathbf{15.07°} \approx 15.1°$$

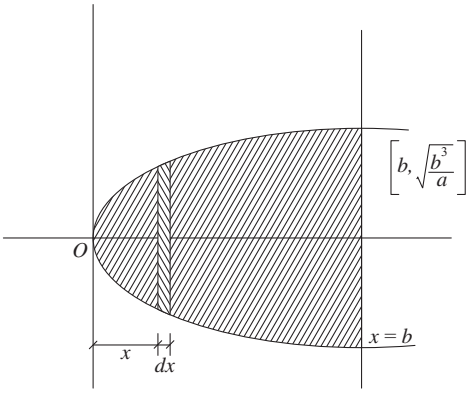
8.7:

$$mg\ \pi r\frac{2r}{\pi} = R(2r)$$

$$\Rightarrow \quad R = \mathbf{mgr}$$



8.8:



When $x = b$, $\quad a\,y^2 = b^3$

$$\Rightarrow \quad y = \sqrt{\frac{b^3}{a}}$$

$$d_A = 2y \cdot dx = 2\,\frac{x^{3/2}}{\sqrt{a}} \cdot dx$$

$$A = \frac{2}{\sqrt{a}} \int_o^b x^{3/2}\,dx = \frac{2}{\sqrt{a}} \cdot \frac{x^{5/2}}{5/2}\bigg|_o^b = \frac{4}{5\sqrt{a}}\,b^{5/2}$$

$$d\,Q_y = x\,d_A = \frac{2}{\sqrt{a}} \cdot x^{5/2}\,dx$$

$$Q_y = \frac{2}{\sqrt{a}} \int_o^b x^{5/2}\,dx = \frac{2}{\sqrt{a}} \cdot \frac{x^{7/2}}{7/2}\bigg|_o^b = \frac{4}{7\sqrt{a}} \cdot b^{7/2}$$

$$\therefore \qquad \bar{x} = \frac{\frac{4}{7\sqrt{a}}\,b^{7/2}}{\frac{4}{5\sqrt{a}}\,b^{5/2}} = \frac{5}{7}b$$

Due to symmetry, $\bar{y} = 0$.

8.9:

(a)

A_i	$\bar{y}_i$	$A_i \bar{y}_i$
35	0.5	17.5
38	20	760
25	39.5	987.5
98		1765

$\bar{y} = 18.01$ cm

≈ 18 cm

(b)

39	19.5	760.5
30	39.5	1185
69		1945.5

$\bar{y} = 28.2$ cm from base

(c)

30	15	0.5	450	15
34.5	0.75	12.5	25.88	431.25
20	10	24.5	200	490
84.5			675.88	936.25

$\bar{x}$ = 8 cm

$\bar{y}$ = 11.08 cm ≈ 11.1 cm

8.10:

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\pi(5)^2/2$	$\frac{4(5)}{3\pi}$	$\frac{2}{3}(5)^3$
$-\pi(3)^2/2$	$\frac{4(3)}{3\pi}$	$-\frac{2}{3}(3)^3$
$\frac{\pi}{2}[5^2-3^2]$		$\frac{2}{3}[5^3-3^3]$

$$\bar{x}=0, \quad \bar{y}=\frac{2/3[5^3-3^3]}{\frac{\pi}{2}[5^2-3^2]}=\mathbf{2.6\ cm}$$

8.11:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
πr^2	r	0	πr^3	0
$-\pi(r/2)^2$	$\frac{3r}{2}$	0	$-\frac{3}{8}\pi r^3$	0
$\pi r^2\left[1-\frac{1}{4}\right]$			$\pi r^3\left[1-\frac{3}{8}\right]$	

$$\bar{x}=\frac{\pi r^3\,(5/8)}{\pi r^2\,(3/4)}=r\,\frac{5}{6}=\frac{5}{6}r$$

For r = 5 cm, $\bar{x}$ = 4.17 cm, $\bar{y}$ = 0

8.12:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
12.5	3.33	1.67	41.63	20.88
$-\frac{\pi(2)^2}{2}$	$2.5+\frac{4(2)}{3\pi}\frac{1}{\sqrt{2}}$	$2.5-\frac{4(2)}{3\pi}\frac{1}{\sqrt{2}}$		
= – 6.28	= 3.1	= 1.9	– 19.47	– 11.93
6.22			22.16	8.95

$\bar{x}$ = 3.56 cm ≈ 3.6 cm $\bar{y}$ = 1.44 cm ≈ 1.4 cm

8.13:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
9	1.5	1.5	13.5	13.5
$\frac{-\pi(3)^2}{4}$	$\frac{4(3)}{3\pi}\left[3-\frac{4(3)}{3\pi}\right]$	–9		–12.23
$= -7.07$	$= \frac{4}{\pi}$	$= 1.73$		
1.93			4.5	1.27

$\bar{x} = 2.33$ cm $\quad \bar{y} = 0.66$ cm

8.14:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
$\pi(0.4)^2$	0	0	0	0
$-\pi(0.1)^2$	0.2	0	$-\pi(0.1)^2(0.2)$	0

$\bar{x} =$ **–1.33 cm** $\quad \bar{y} = 0$ cm

8.15:

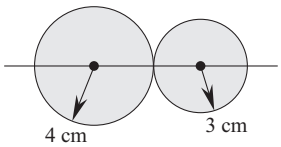
10	0.5	5
$\frac{\pi(4)^2}{2}$	$1+\frac{4(4)}{3\pi}$	67.8
$\frac{-\pi(3)^2}{2}$	$1+\frac{4(3)}{3\pi}$	–32.14
$\frac{\pi(0.5)^2}{2}$	$1+\frac{4(0.5)}{3\pi}$	0.48
$-\pi(0.15)^2$	1	–0.07
21.32		41.07

$\bar{y} = 1.93$ cm $\quad \approx 1.9$ cm

8.16:

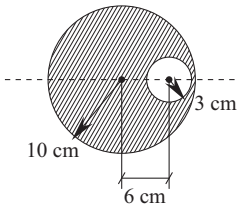
A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$\pi(4)^2$	–4	$-\pi(4)^3$
$\pi(3)^2$	3	$\pi(3)^3$
$\pi[4^2+3^2]$		$-\pi[4^3-3^3]$

$\bar{x} = 1.48$ cm



8.17:

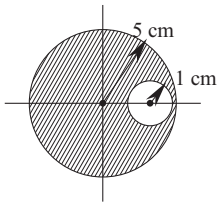
A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$\pi(10)^2$	0	0
$-\pi(3)^2$	6	$-\pi(3)^2$ 6.
$\pi(10^2-3^2)$		$-\pi(3)^2$ 6



$$\bar{x} = \frac{-\pi(3)^2.6}{\pi(10^2-3^2)} - 0.593 \text{ cm}$$

8.18:

A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$\pi(5)^2$	0	0
$-\pi(1)^2$	x	$-\pi(x)$
$\pi(5^2-1^2)$		$-\pi(x)$

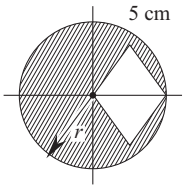


$$\Rightarrow \qquad \bar{x} = \frac{-\pi.x}{\pi(5^2-1^2)} = -0.1 \text{ cm.}$$

$$\Rightarrow \qquad x = 2.4 \text{ cm}$$

8-19:

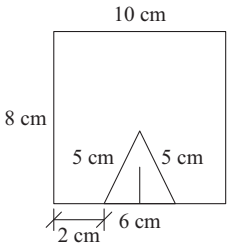
A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$\pi(r)^2$	0	0
$-\left(\frac{r}{\sqrt{2}}\right)^2$	$\frac{r}{2}$	$\frac{-r^3}{4}$
$r^2\left[\pi-\frac{1}{2}\right]$		$-r^3/4$



$$\bar{x} = \frac{-r^3/4}{r^2[\pi - 1/2]} = \frac{-r}{4\left[\pi-\frac{1}{2}\right]} = \mathbf{0.095r}$$

8.20: (a)

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
80	4	320
$-\frac{1}{2}(6)(4)$ $=-12$	$\frac{4}{3}$	–16
68		304



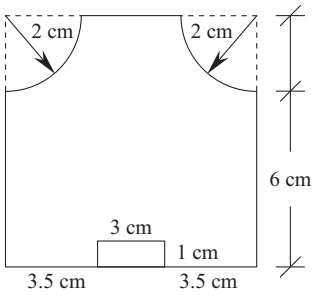
$\therefore$ $\bar{y} = 4.47$ cm from base

≈ 4.5 cm from base

$\bar{x} = 0$

8.20: (b)

A_i	$\bar{y}_i$	$A_i \bar{y}_i$
60	3	180
-3	0.5	-1.5
$\dfrac{-\pi(2)^2}{4}$ $= -3.14$	$6 - \dfrac{4(2)}{3\pi}$ $= 5.15$	-16.18
-3.14	5.15	-16.18
50.72		146.14



$\bar{y} = 2.88$ cm from base $\approx \boxed{2.9 \text{ cm}}$

$\bar{x} = 0$

8.21: (a)

A_i	$\bar{x}_i$	$A_i \bar{x}_i$
24	3	72
$\dfrac{\pi(2)^2}{2}$	$6 + \dfrac{4(2)}{3\pi}$	43.03
$-\dfrac{1}{2} \times (4)(3)$	1	-6
24.28		109.03

$\bar{x} = 4.49$ cm ≈ 4.5 cm

$\bar{y} = 0$ due to symmetry.

8.21: (b)

A_i	$\bar{y}_i$	$A_i \bar{y}_i$
24	1.5	36
$\dfrac{1}{2}(8)(3)$ $= 12$	$3 + 1$ $= 4$	48
$-\pi(1)^2$	3	-3π
32.86		74.56

$\bar{y} =$ **2.27 cm** ≈ 2.3 cm $\quad \bar{x} = 0$ due to symmetry

8.22: (a)

A_i	$\bar{y}_i$	$A_i \bar{y}_i$
$\dfrac{\pi(5)^2}{2}$	$\dfrac{4(5)}{3\pi}$	$\dfrac{2}{3}(5)^3$
-8	1	-8
31.27		75.33

$\bar{y} =$ **2.41 cm** ≈ 2.4 cm $\quad \bar{x} = 0$ due to symmetry

8.22: (b)

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{\pi(4)^2}{2}$	$\frac{4(4)}{3\pi}$	$\frac{2}{3}(4)^3$
$-\frac{1}{2}(8)(2)$	$\frac{2}{3}$	$\frac{-16}{3}$
17.13		37.33

$\Rightarrow$ $\bar{y} = 2.18 \text{ cm} \approx 2.2 \text{ cm}$

$\bar{x} = 0$ due to symmetry

8.23: (a)

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
$\frac{\pi}{4}(5)^2$	$\frac{4(5)}{3\pi}$	$\frac{4(5)}{3\pi}$	$\frac{1}{3}(5)^3$	$\frac{1}{3}(5)^3$
–4	1	1	–4	–4
15.63			37.67	37.67

$\bar{x} = \bar{y} = 2.41 \text{ cm}$

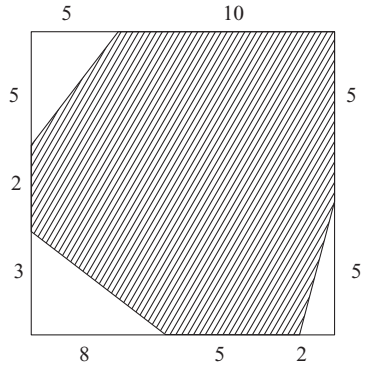
8.23: (b)

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
$\frac{\pi}{4}(5)^2$	$\frac{4(5)}{3\pi}$	$\frac{4(5)}{3\pi}$	$\frac{1}{3}(5)^3$	$\frac{1}{3}(5)^3$
$-\frac{\pi}{4}(3)^2$	$\frac{4(3)}{3\pi}$	$\frac{4(3)}{3\pi}$	$\frac{1}{3}(3)^3$	$\frac{1}{3}(3)^3$
$\frac{\pi}{4}[5^2 - 3^2]$			$\frac{1}{3}[5^3 - 3^3]$	$\frac{1}{3}[5^3 - 3^3]$

$\bar{x} = \bar{y} = 2.6 \text{ cm}$

8.24:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
150	7.5	5	1125	750
$-\frac{1}{2}(5)(5)$ $= -12.5$	$\frac{5}{3}$ $= 1.67$	$\left(10 - \frac{5}{3}\right)$ $= 8.33$	–20.88	–104.13
$-\frac{1}{2}(3)(8)$ $= -12$	$\frac{8}{3}$ $= 2.67$	$\frac{3}{3} = 1$	– 32.04	– 12
$-\frac{1}{2}(2)(5)$	$15 - \frac{2}{3}$	$\frac{5}{3} = 1.67$	–71.65	–8.35



$=-5$	$=14.33$		
120.5		1000.43	625.52

$\bar{x} = 8.3$ cm

$\bar{y} = 5.19$ cm

8.25:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
$\frac{\pi}{4}(10)^2$	$\frac{4(10)}{3\pi}$	$\frac{4(10)}{3\pi}$	$\frac{1}{3}(10)^3$	$\frac{1}{3}(10)^3$
-10	$\frac{4.5}{\sqrt{2}}$	$\frac{4.5}{\sqrt{2}}$	$\frac{-45}{\sqrt{2}}$	$\frac{-45}{\sqrt{2}}$
68.54			$\frac{1}{3}(10)^3 - \frac{45}{\sqrt{2}}$	$\frac{10^3}{3} - \frac{45}{\sqrt{2}}$

$$\bar{x} = \bar{y} = \frac{301.51}{68.54} = 4.4 \text{ cm}$$

8.26:

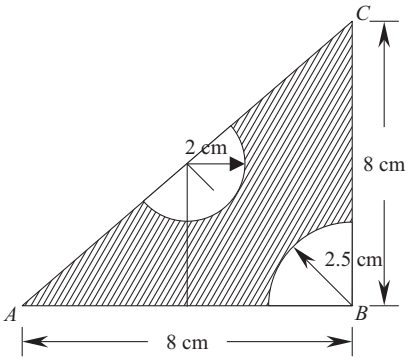
A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{\pi}{2}(5)^2$	$\frac{4(5)}{3\pi}$	$\frac{2}{3}(5)^3$
$-\frac{1}{2}\times 7\times 3$ $=-10.5$	$1+\frac{3}{3}=2$	-21
28.77		62.33

$$\bar{y} = \frac{62.33}{28.77} = 2.17 \text{ cm} \qquad \bar{x} = 0 \text{ due to symmetry}$$

8.27:

Due to symmetry, $\bar{x} = \bar{y}$ about corner B

A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$\frac{1}{2}\times 8\times 8 = 32$	$\frac{8}{3}$	256/3
$-\frac{1}{4}\pi(2.5)^2$	$\frac{4(2.5)}{3\pi}$	$\frac{-(2.5)^3}{3}$
$-\frac{1}{2}\pi(2)^2$	$4-\frac{4(2)}{3\pi}\frac{1}{\sqrt{2}}$	–21.36
20.81		58.77

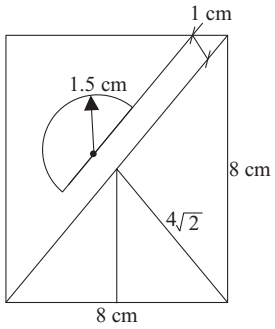


$$\bar{x} = \bar{y} = \frac{58.77}{20.81} = 2.82 \text{ cm}$$

$$\approx 2.8 \text{ cm}$$

8.28:

Due to symmetry, $\bar{x} = \bar{y}$ about corner D



A_i	$\bar{x}_i$	$A_i\bar{x}_i$
$8 \times 8 = 64$	4	256
$-\frac{1}{2}\pi(1.5)^2$	$\left[4\sqrt{2}+1+\frac{4(1.5)}{3\pi}\right] = 7.3$	– 25.8
60.47		230.2

$$\bar{x} = \bar{y} = 3.81 \text{ cm}$$

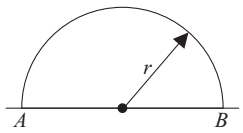
$$\approx 3.8 \text{ cm}$$

8.29:

A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
120	3	10	360	1200
$\frac{1}{2}(12)(6)$ $= 36$	$6+\frac{6}{3}$ $= 8$	$\frac{12}{3}$ $=4$	288	144
156			648	1344

$\bar{x} = 4.15$ cm $\qquad \bar{y} =$ **8.62 cm**

8.30:



$$S \cdot A = 2\pi \cdot L\bar{y}$$

$$(4\pi r^2) = 2\pi(\pi r)\bar{y}$$

$$\Rightarrow \quad \boxed{\bar{y} = \frac{2r}{\pi}}$$

8.31:

L_i	$\bar{y}_i$	$L_i\bar{y}_i$
a	$\frac{\sqrt{3}a}{4}$	$\frac{\sqrt{3}a^2}{4}$
a	$\frac{\sqrt{3}a}{4}$	$\frac{\sqrt{3}a^2}{4}$
$2a$		$\frac{\sqrt{3}}{2}a^2$

$$\bar{y} = \frac{\frac{\sqrt{3}}{2}a^2}{2a} = \frac{\sqrt{3}}{4}a$$

$$SA = 2\pi L\ \bar{y}$$

$$= 2\pi(2a)\frac{\sqrt{3}}{4}a$$

$$= \sqrt{3}\cdot\pi a^2$$

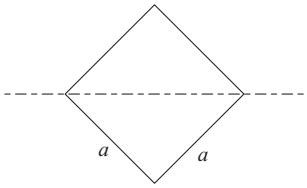
$$\text{vol} = 2\pi\ A\cdot\bar{y}$$

$$= 2\pi\frac{1}{2}\ a\ \frac{\sqrt{3}a}{2}.\frac{1}{3}\ \frac{\sqrt{3}a}{2}$$

$$= \frac{\pi\cdot a^3}{4}$$

8.32:

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
$\frac{a^2}{2}$	$\frac{1}{3}\cdot\frac{a}{\sqrt{2}}$	$\frac{a^3}{6\sqrt{2}}$



$$\bar{y} = \frac{a^3/6\sqrt{2}}{a^2/2}$$

$$= \frac{a}{3\sqrt{2}}$$

$$V = 2\pi\cdot A\cdot\bar{y}$$

$$= 2\pi\left[\frac{a^2}{2}\right]\frac{a}{3\sqrt{2}}$$

$$= \frac{\pi a^3}{3\sqrt{2}}$$

$$= \frac{\pi a^3}{\sqrt{18}}$$

8.33:

A_i	$\bar{x}_i$	$A_i\bar{x}_i$
πr	$5+\frac{2r}{\pi}$	$\pi r\left[5+\frac{2r}{\pi}\right]$
$2r$	5	$10r$
$[\pi+2]r$		$r\,[5\pi+2r]+10r$
		$= r\,[5\pi+2r+10]$

$$\bar{y} = \mathbf{6.17\ cm}$$

$$S\cdot A = 2\pi A\cdot y$$

$= 2\pi \cdot r\,[5\pi + 2r + 10]$

$= \mathbf{597.68\ cm^2}$

$\approx 597.7\ \text{cm}^2$

$V = 2\pi A\,\bar{y}$

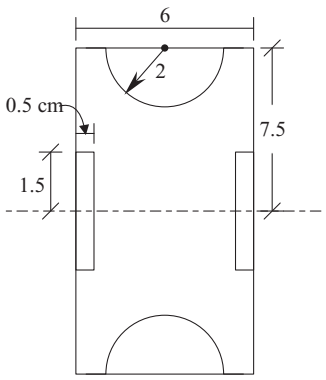
$= 2\pi\left(\frac{\pi r^2}{2}\right)\left[5 + \frac{4r}{3\pi}\right]$

$= \pi^2 r^2\left[5 + \frac{4r}{3\pi}\right]$

$= \mathbf{557.23\ cm^3} \approx 557.2\ \text{cm}^3$

8.34:

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
45	3.75	168.75
$-\frac{\pi(2)^2}{2}$	$7.5 - \frac{4(2)}{3\pi}$	−41.762
= −6.28	= 6.65	
−0.75	0.75	−0.5625
−37.97		126.43



$\therefore \quad \bar{y} = \frac{126.43}{37.97} = 3.33\ \text{cm}$

$V = 2\pi(A)\,(\bar{y})$

$= 2\pi(37.97)\,(3.33)$

$= \mathbf{794.4\ cm^3}$

8.35:

A_i	$\bar{y}_i$	$A_i\bar{y}_i$
16	1 + 2/2 = 2	32
$\frac{1}{2}8(2)$	3 + 2/3	29.36
= 8	= 3.67	
24		61.36

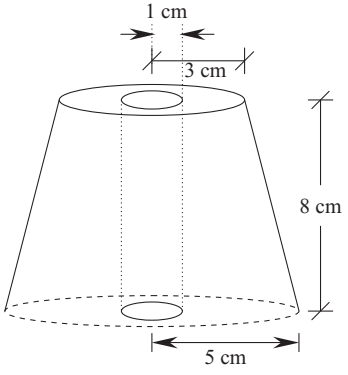
$\bar{y} = 2.56\ \text{cm}$

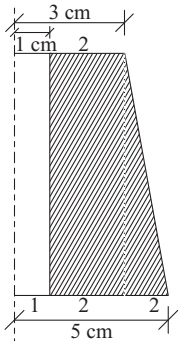
$\therefore \quad V = 2\pi(A)\,(\bar{y})$

$= 2\pi(24)\,(2.56)$

$= \mathbf{386.04\ cm^3}$

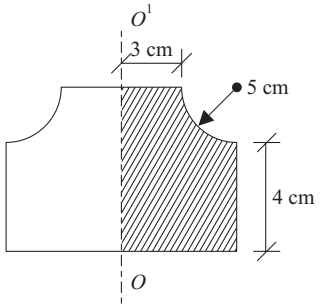
$\approx 386\ \text{cm}^3$





8.36:

L_i	$\bar{x}_i$	$L_i \bar{x}_i$
3	1.5	4.5
3	8	32
8	4	32
$\frac{\pi(5)}{2}$ = 7.85	$8 - \frac{2(5)}{\pi}$ = 4.82	37.84
22.85		106.34



$\bar{x} = 4.65$ cm

$S \cdot A = 2\pi L \quad \bar{x} = 2\pi(106.34)$

$= 668.15 \text{ cm}^2$

$\approx 668.2 \text{ cm}^2$

A_i	$\bar{x}_i$	$A_i \bar{x}_i$
72	4	288
$-\frac{\pi(5)^2}{4}$ = –19.63	$8 - \frac{4(5)}{3\pi}$ = 5.88	– 115.52
52.37		172.58

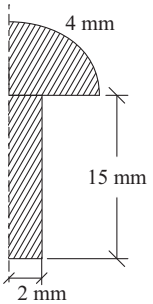
$V = 2\pi A \bar{y}$

$= 2\pi(172.58)$

$= 1084.4 \text{ cm}^3$

8.37:

A_i	$\bar{x}_i$	$A_i \bar{x}_i$
30	1	30
$\frac{\pi(4)^2}{4}$ = 12.57	$\frac{4(4)}{3\pi}$	$\frac{1}{3}(4)^3$ =21.33
42.57		51.33



$V = 2\pi A \bar{x}$

$= 2\pi (51.33)$

$= 322.52 \text{ mm}^3$

$= 322.52 \times 10^{-9} \ m^3$

$\therefore$ Mass $= \rho V$

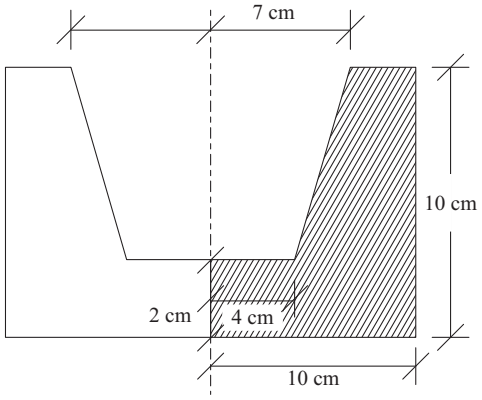
$= (7850)\ (322.52 \times 10^{-9})$

$= 2.53 \times 10^{-3}$ kg

$\therefore$ No-of rivets $= 5 \text{ kg} / 2.53 \times 10^{-3}$

$= 1976$ pieces

8.38:



L_i	$\bar{x}_i$	$L_i \bar{x}_i$
4	2	8
3	7 + 1.5 (= 8.5)	25.5
10	10	100
10	5	50
8.54	4 + 1.5 = 5.5	46.97
35.54		230.47

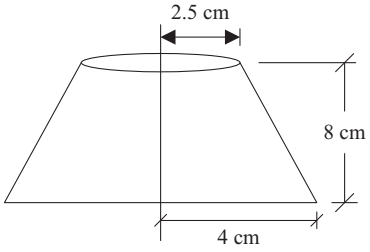
$$S \cdot A = 2\pi L \bar{x}$$
$$= 2\pi(230.47)$$
$$= \mathbf{1448.1\ cm^2}$$

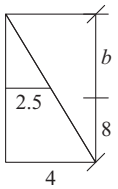
A_i	$\bar{x}_i$	$A_i \bar{x}_i$
100	5	500
–32	2	–64
$\frac{-1}{2}(3)(8)$ = – 12	4 + 3/3 = 5	–60
56		376

$$V = 2\pi A \cdot \bar{y}$$
$$= 2\pi(376)$$
$$= 2362.48\ \text{cm}^3$$
$$\approx 2362.5\ \text{cm}^3$$

8.39:

$$\frac{2.5}{h} = \frac{4}{8+h}$$
$$20 + 2.5h = 4h$$
$$h = \frac{20}{1.5} = 13.33\ \text{cm}$$





V_i	Z_i	$V_i Z_i$
$\frac{1}{3}\pi(4)^2(8 + 13.33)$ = 357.39	$\frac{1}{4}(8 + 13.33)$ = 5.33	1904.89
$-\frac{1}{3}\pi(2.5)^2(13.33)$ = –87.24	$8 + \frac{1}{4}13.33$ = 11.33	–988.43
270.15		916.46

$$\bar{y} = \frac{916.46}{270.15} = \mathbf{3.4\ cm}$$

8.40:

V_i	$\bar{x}_i$	$V_i \bar{x}_i$
$\pi(2)^2\ 10$	5	200π
$\pi(1)^2\ 8$	$(10 + 4)$ $= 14$	112π
48π		$312\ \pi$

$$\bar{x} = \frac{312\ \pi}{48\ \pi} = 6.5 \text{ cm}$$

8.41:

V_i	$\bar{z}_i$	$V_i \bar{z}_i$
$\pi(5)^2\ 10$	5	1250π
$\frac{2}{3}\pi(5)^3$	$10 + \frac{3}{8}(5)$ $= 11.875$	989.58π
$-\frac{2}{3}\pi(5)^3$	$\frac{3}{8}(5)$ $= 1.875$	-156.25π
250π		2083.33π

$$\bar{Z} = \frac{2083.33\ \pi}{250\ \pi} = \mathbf{8.33\ cm}$$

8.42:

m_i	$\bar{x}_i$	$m_i \bar{x}_i$
$(7850)\ \pi(2)^2\ 10$	5	$(7850)\ 200\pi$
$(11{,}370)\ \pi(1)^2\ 8$	14	$(11{,}370)\ 112\ \pi$
$(404960)\ (\pi)$		$(2843440)\ \pi$

$$\Rightarrow \quad \bar{x} = \frac{(2843440)\ \pi}{(404960)\ \pi}$$

$$= \mathbf{7.02\ cm}$$

$$\approx 7 \text{ cm}$$