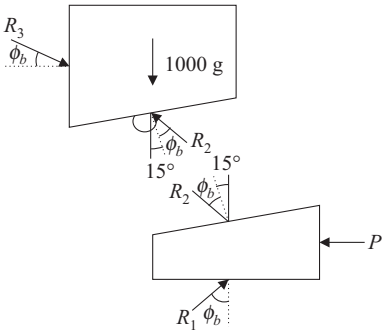
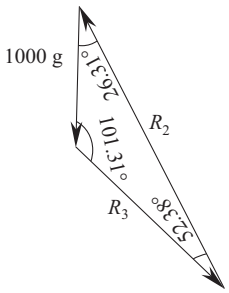
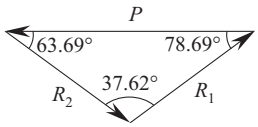
Chapter-7

7.1:







$$\frac{1000\,g}{\sin 52.38°} = \frac{R_2}{\sin 101.31°} \quad \Rightarrow \quad R_2 = 12.14 \text{ kN}$$

$$\frac{R_2}{\sin 78.69°} = \frac{P}{\sin 37.62°} \quad \Rightarrow \quad P = 7.56 \text{ kN}$$

7.5:

$r_m = 75/2 = 37.5$ mm.
$p = 10$ mm
$a = 500$ mm
$W = 30$ kN.
$\phi_s = \tan^{-1}(\mu_s) = 8.53°$

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r}\right] = \tan^{-1}\left[\frac{10}{2\pi(37.5)}\right] = 2.43°$$

(i) $$P = \frac{Wr}{a}\tan(\phi_s + \theta)$$
$$= 435.7 \text{ N}$$
$\theta < \phi_s$, $\therefore$ under self-locking.

(ii) $$P = \frac{Wr}{a}\tan(\phi_s - \theta) = 240.5 \text{ N}$$

(iii) $$\eta = \frac{\tan\theta}{\tan(\phi_s + \theta)} = 21.9\%$$

7.6:

$r = 50$ mm.
$p = 16$ mm
$\eta = 0.2$.

$\phi_s = \tan^{-1}(\mu_s) = 11.31°$

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r}\right] = 2.92°$$

$$\tau_{raise} = Wr \tan(\phi_s + \theta)$$
$$= 1000 \times (0.05) \tan (11.31° + 2.92°)$$
$$= 12.7 \text{ Nm}$$
$$\tau_{lower} = Wr \tan (\phi_s - \theta)$$
$$= 7.37 \text{ N. m}$$
$$\eta = \frac{\tan\theta}{\tan(\theta + \phi_s)} = 20.11\%$$

7.7:

$$p = 10 \text{ mm}$$
$$r = 60 \text{ mm}$$
$$\mu_s = 0.1$$
$$L = 2p = 20 \text{ mm}$$
$$\therefore \quad \phi_s = \tan^{-1}(\mu_s) = 5.71°$$
$$\theta = \tan^{-1}\left[\frac{L}{2\pi r}\right] = 3.04°$$
$$\tau = Wr \tan (\phi_s + \theta)$$
$$100 = W(0.06) \tan (5.71° + 3.04°)$$
$$\Rightarrow \quad W = 1.83 \text{ kN}$$

7.8:

$r_m = 25$ mm $\qquad \phi_s = \tan^{-1}[\mu_s] = 21.8°$

$p = 10$ mm $\qquad \theta = \tan^{-1}\left[\frac{L}{2\pi r_m}\right] = 3.64°$

$\mu_s = 0.4$

(i) $\theta = W \tan (\phi_s + \theta)$
$= (5 \times 10^3) \tan (25.44)$
$= (5 \times 10^3) (0.476)$
$= 2380$ N.

(ii) $\eta = \frac{\tan\theta}{\tan(\theta + \phi_s)} = 13.4\%$

(iii) $\theta = 45° - \frac{\phi_s}{2} = 34.1°$

$$\tan\theta = \frac{L}{2\pi r_m}$$
$$\Rightarrow \quad L = 106.4 \text{ mm}$$
$$\eta_{max} = \frac{1 - \sin\phi_s}{1 + \sin\phi_s} = 45.8\%$$

7.9:

$r_0 = 26$ mm

$p = 6$ mm

$r_1 = 26 - 3 = 23$ mm

$r_m = 24.5$ mm

$\phi_s = \tan^{-1}(0.2) = 11.31°$

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r_m}\right] = 2.23°$$

$\tau = Wr \tan(\phi_s + \theta)$

$\Rightarrow$ $W = 4.24$ kN.

$\therefore$ $\tau = Wr \tan(\phi_s - \theta) = 16.6$ N.m

7.10:

$$W = P \times \frac{\pi d^2}{4} = 3 \times 10^6 \times \frac{\pi}{4}(0.15)^2 = 53.01 \text{ kN}$$

$r_0 = 30$ mm

$p = 6$ mm

$r_i = 27$ mm

$r_m = 28.5$ mm

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r_m}\right] = 1.92°$$

$\phi_s = \tan^{-1}[\mu_s] = 11.31°$

$\tau = Wr \tan(\phi_s + \theta)$

$= 355.2$ Nm

7.11:

$r_0 = 30$ mm

$r_i = 20$ mm

$p = 2(r_0 - r_i) = 20$ mm

$r_m = 25$ mm

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r_m}\right] = 7.26°$$

$\phi_s = \tan^{-1}[0.2] = 11.31°$

(i) $\tau = Wr \tan(\phi_s + \theta)$

$= 168$ N.m.

(ii) $\tau = Wr \tan(\phi_s - \theta)$

$= 35.4$ N.m.

(iii) $$\eta = \frac{\tan\theta}{\tan(\phi_s + \theta)} = 37.92\%$$

7.13:

$$\ln\frac{T_2}{T_1} = \mu\beta$$

$$\ln(n) = \mu\beta$$

$\therefore$ $$\text{No. of turns} = \frac{\ln n}{2\pi\mu}$$

7.14:

$$\frac{T_2}{T_1} = e^{\mu_s\beta}$$

$$= e^{(0.2)(3\pi)}$$

$$= 6.586$$

$T_1 = 100$ kg $\Rightarrow$ $T_2 = 658.6$ kg.

$T_2 = 100$ kg $\Rightarrow$ $T_1 = 15.2$ kg.

7.15:

$$\frac{T_2}{T_1} = e^{\mu_s\beta}$$

$$= e^{(0.2)(2\pi)}$$

$$= 3.514$$

$T_1 = 100$ kg $\Rightarrow$ $T_2 = 351.4$ kg

$T_2 = 100$ kg $\Rightarrow$ $T_1 = 28.5$ kg

7.16:

$$\alpha = \sin^{-1}\left[\frac{4}{15}\right] = 15.47°$$

$\therefore$ $$\beta = 180° + 2\alpha + 2\alpha$$

$$= 241.88°$$

$$= 4.22\ \mu\text{d}$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta} = 2.33$$

$T_1 = 50$ kg $\Rightarrow$ $T_2 = 116.5$ kg

$T_2 = 50$ kg $\Rightarrow$ $T_1 = 21.5$ kg

7.17:

$$\frac{r_1}{l_1} = \frac{r_2}{l_2} \qquad \Rightarrow \qquad l_1 = 0.8\, l_2$$

$\therefore \quad l_1 + l_2 = 27$ cm

$\therefore \quad l_2 = 15$ cm

& $\quad l_1 = 12$ cm

$$\alpha = \sin^{-1}\left[\frac{r_1}{l_1}\right] = 19.47°$$

$$\beta = 180° + (4 \times 19.47°) = 257.88°$$

$$= 4.5 \text{ rad.}$$

$$\therefore \quad \frac{T_2}{T_1} = e^{\mu_s \beta} = 2.46$$

$T_1 = 100$ kg $\Rightarrow$ $T_2 = 246$ kg $= 2413$ N

$T_2 = 100$ kg $\Rightarrow$ $T_1 = 40.65$ kg $= 399$ N

7.18:

$$\frac{r_1}{l_1} = \frac{r_2}{l_2} \qquad \Rightarrow \qquad l_2 = 15 \text{ cm}$$

$l_1 = 12$ cm

$$\alpha = \sin^{-1}\left[\frac{5}{15}\right] = 19.47°$$

$$\beta = 257.88° = 4.5 \text{ rad}$$

$$\therefore \quad \frac{T_2}{T_1} = e^{\mu_s \beta} = 2.46$$

$T_1 = 100$ kg, $\Rightarrow$ $T_2 = 246$ kg $= 2413$ N

$T_2 = 100$ kg, $\Rightarrow$ $T_1 = 40.65$ kg $= 399$ N

7.19:

$$T_1 = \mu_s mg = 0.2 \times 200 \times 9.81$$

$$= 392.4 \text{ N}$$

$$\frac{W}{T_1} = e^{\mu_s \beta} = e^{(0.2)(\pi/2)} = 1.37$$

$$\Rightarrow \quad W = 537.6 \text{ N}$$

7.20:

$$\frac{l_1}{l_2} = e^{\mu_s \beta}$$

$$= e^{(0.2)(\pi)} = 1.874$$

$$\frac{l_1}{l_1 + l_2} = \frac{1.874}{2.874}$$

$\Rightarrow \quad l_1 = 0.651$

7.21:

$r_1 = 300$ mm $\qquad N_1 = ?$

$r_2 = 200$ mm $\qquad N_2 = 300$ rpm

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

$\Rightarrow \quad N_1 = 200$ rpm

7.22:

$r_1 = 1$ m $\qquad N_1 = 200$ rpm

$r_2 = 0.5$ m $\qquad N_2 = ?$

(i) $$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

$\Rightarrow \quad N_2 = 400$ rpm

(ii) $$\frac{N_2}{N_1} = \frac{r_1}{r_2}\left(1 - \frac{P}{100}\right)$$

$\Rightarrow \quad N_2 = 384$ rpm

7.23:

$N_1 = 500$ rpm

$d_1 = 800$ mm $\qquad d_3 = 600$ mm

$d_2 = 500$ mm $\qquad d_4 = 400$ mm

(i) $$\frac{N_2}{N_1} = \frac{d_1}{d_2}\frac{d_3}{d_4} = \frac{800}{500} \times \frac{600}{400}$$

$\Rightarrow \quad N_2 = 1200$ rpm

(ii) $$\frac{N_2}{N_1} = \frac{d_1}{d_2}\frac{d_3}{d_4}\left(1 - \frac{P}{100}\right)$$

$\Rightarrow \quad N_2 = 1164$ rpm

7.24:

$t = 6$ mm

$d_1 = 5$m $\qquad N_1 = 200$ rpm

$d_2 = 3$m $\qquad N_2 = ?$

(i) $$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

$\Rightarrow \quad N_2 = 333.33$ rpm

(ii) $$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t} = \frac{5.006}{3.006}$$

$\Rightarrow$ $N_2 = 333.07$ rpm

7.25:

$d_1 = 500$ mm

$d_2 = 300$ mm

$l = 1.8$ m

(i) $$\alpha = \sin^{-1}\left[\frac{r_1 - r_2}{l}\right] = 3.2°$$

driver pulley $= 180° + 2\alpha = 186.4°$

driven pulley $= 180° - 2\alpha = 173.6°$

(ii) $$\alpha = \sin^{-1}\left[\frac{r_1 + r_2}{l}\right] = 12.8°$$

driver & driven pulleys $= 180° + 2\alpha = 205.6°$

7.26:

$d_1 = 40$ cm $\qquad l = 60$ cm

$d_2 = 30$ cm $\qquad \mu_s = 0.15$

(i) $$L = 2l + \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{l}$$

$= 2.3$ m

(ii) $$L = 2l + \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{l}$$

$= 2.5$ m

7.27:

$N_1 = 110$ rpm.

$(N_2)_1 = 50$ rpm.

$(N_2)_2 = 70$ rpm,

$(N_2)_3 = 90$ rpm

$r_1 = 80$ mm

(i) $$\frac{(N_2)_1}{N_1} = \frac{r_1}{r_2}$$

$\Rightarrow$ $r_2 = 176$ mm

$r_1 + r_2 = 256$ mm

$$\frac{(N_2)_2}{N_1} = \frac{r_3}{r_4} = \frac{7}{11}$$

$$r_3 + r_4 = 256$$

$$\Rightarrow \quad r_3 = 99.55 \text{ mm}$$

$$r_4 = 156.45 \text{ mm}$$

$$\frac{(N_2)_3}{N_1} = \frac{r_5}{r_6} = \frac{9}{11}$$

$$r_5 + r_6 = 256 \text{ mm}$$

$$\Rightarrow \quad r_5 = 115.2 \text{ mm}$$

$$r_6 = 140.8 \text{ mm}$$

7.29:

$$\beta = \frac{165}{180} \times \pi = 2.88 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$T_2 = 2 \text{ kN} \qquad \Rightarrow \qquad T_1 = 1.124 \text{ kN}$$

(i) $\tau = (T_2 - T_1)r = 263$ N.m

(ii) $P = (T_2 - T_1)v$

$$= (T_2 - T_1)\frac{\pi dN}{60} = 8.3 \text{ kW}$$

7.30:

$p = 150$ mm

$t = 10$ mm

$m = 1200 \text{ kg/m}^3 \times 0.15 \times 0.01 = 1.8 \text{ kg/m}$

$P = 10$ kW

$d_1 = 600$ mm

$d_2 = 400$ mm

$N_2 = 400$ rpm

$l = 2m$

$$P = (T_2 - T_1)v = (T_2 - T_1)\frac{\pi dN}{60}$$

$$10 \times 10^3 = (T_2 - T_1)\pi \times \frac{0.4 \times 400}{60}$$

$$\Rightarrow \quad T_2 - T_1 = 1193.7 \text{ N}$$

$$\alpha = \sin^{-1}\left[\frac{r_1 - r_2}{l}\right] = 2.87°$$

$$\beta = 180° - 2\alpha = 174.26°$$

$$= 3.04 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = 2.14$$

$\therefore \quad T_1 = 1047.11$ N

$T_2 = 2240.81$ N

$\therefore \quad \sigma_{max} = \frac{T_2}{bt} = 1.49$ N/mm^2

(ii) $T_{max} = T_2 + mv^2$

$$= 2240.81 + 1.8 \times \left(\pi \times \frac{0.4 \times 400}{60}\right)^2$$

$= 2367.14$ N

$\therefore \quad \sigma_{max} = 1.58$ N/mm^2

7.31:

$$\alpha = \sin^{-1}\left[\frac{r_1 + r_2}{l}\right] = 14.48°$$

$\beta = 180° + 2\alpha = 208.96°$

$= 3.65$ rad

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = 2.49$$

$\therefore \quad T_1 = 801.14$ N

$T_2 = 1994.84$ N

$\therefore \quad \sigma_{max} = 1.33$ N/mm^2

(ii) $T_{max} = 2121.17$ N

$\therefore \quad \sigma_{max} = 1.41$ N/mm^2

7.32:

$\tau = 400$ Nm

$\beta = 250°$

$= 4.36$ rad

$\mu_s = 0.3$

$r = 0.2$ m

$\tau = (T_2 - T_1)r$

$400 = (T_2 - T_1) \times 0.2$

$\Rightarrow \quad T_2 - T_1 = 2000$ N

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$= 3.7$

$\therefore \quad T_1 = 740.74$ N

$T_2 = 2740.74$ N

$T_2 \times 50 = P \times 400$

$\Rightarrow \quad P = 342.6$ N

7.33:

$T_1 \times 50 = P \times 400$

$\Rightarrow \quad P = 92.6$ N

7.34:

$r = 300$ mm

$\tau = 300$ N.m.

$\mu_s = 0.2$

$\tau = (T_2 - T_1)r$

$300 = (T_2 - T_1)(0.3)$

$\Rightarrow \quad T_2 - T_1 = 1000$ N

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$= e^{(0.2)(1.5\pi)}$$

$$= 2.57$$

$\Rightarrow \quad T_1 = 636.9$ N

$T_2 = 1636.9$ N

$T_2 \times 10 + T_1 \times 20 = P \times 80$

$\Rightarrow \quad P = 363.9$ N

7.35:

$T_2 \times 20 + T_1 \times 10 = P \times 80$

$\Rightarrow \quad P = 488.9$ N