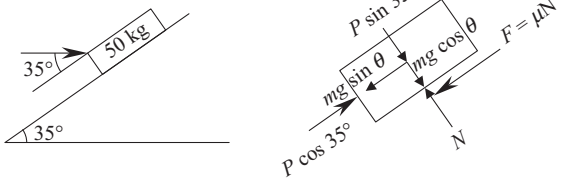
Chapter-6

6.1:

$$\mu p = mg$$

$$\Rightarrow \quad p = \frac{mg}{\mu} = \frac{200}{0.3} = 666.67 \text{ N}$$

6.2:



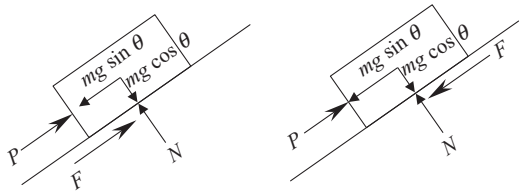
$$N = P \sin 35° + 50g \cos \theta$$

$$P \cos 35° = mg \sin \theta + \mu N$$

$$= 50 \text{ g} \sin \theta + \mu [P \sin 35° + 50 \text{ g} \cos \theta]$$

$$\Rightarrow \quad P = \frac{50\, g[\sin \theta + \mu \cos \theta]}{\cos 35° - \mu \sin 35°} = 513.5 \text{ N}$$

6.3:



$$N = mg \cos \theta \qquad N = mg \cos \theta$$

$$P + F = mg \sin \theta \qquad P = F + mg \sin \theta$$

$$\Rightarrow \quad P = mg \sin \theta - \mu mg \cos \theta \qquad = mg \sin \theta + \mu mg \cos \theta$$

$$\frac{mg[\sin \theta + \mu \cos \theta]}{mg[\sin \theta - \mu \cos \theta]} = \frac{mg}{mg \cos \theta}$$

$$\Rightarrow \quad \sin \theta \cos \theta + \mu \cos^2 \theta = \sin \theta - \mu \cos \theta$$

$$\mu = \frac{\sin \theta [1 - \cos \theta]}{\cos \theta [1 + \cos \theta]}$$

$$= \tan \theta \frac{\sin^2 \theta/2}{\cos^2 \theta/2}$$

$$= \tan \theta \tan^2 \theta/2.$$

6.4:

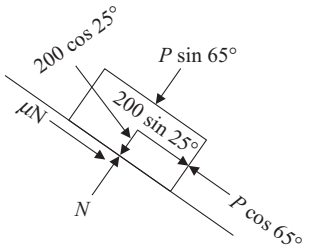
$$N = P \sin 65° + 200 \cos 25°$$

$$P \cos 65° = 200 \sin 2.5° + \mu N$$

$$= 200 \sin 25° + \mu [P \sin 40° + 200 \cos 25°]$$

$$\Rightarrow \quad P = \frac{200[\sin 25° + \mu \cos 25°]}{[\cos 65° - \mu \sin 65°]}$$

$$= 500.4 \text{ N}$$



6.5:

$$N + 100 \sin 20° = 15\, g \cos 30°$$

$$\Rightarrow \quad N = 93.23$$

$$\therefore \quad Fs, \max = \mu N.$$

$$= (0.2)\ (93.23)$$

$$= 18.65 \text{ N}.$$

For equilibrium

$$F = 100 \cos 20° - 15\, g \sin 30°$$

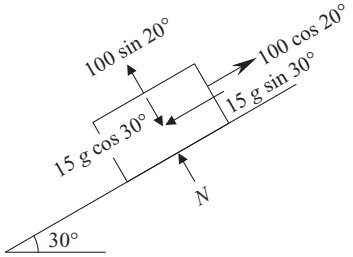
$$= 20.4\ N \text{ (acting down the plane)}.$$

$\sin \theta F > Fs$, max, which cannot be, the block is under motion up the plane.

$$\therefore \quad F_k = \mu_k N$$

$$= (0.15)\ (93.23)$$

$$= 14\text{N (acting down the plane)}.$$



6.6:

$$N = 600 \cos 30°$$

$$360 = 600 \sin 30° + \mu\, 600 \cos 30°$$

$$= 600 [\sin 30° + \mu \cos 30°]$$

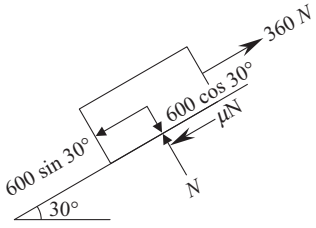
$$\Rightarrow \quad \mu = 0.115$$

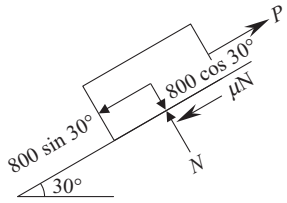
$$P = 840 \sin 30° + \mu\, 840 \cos 30°$$

$$= 503.66 \text{ N}$$

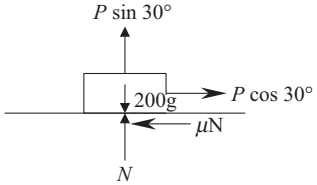
$$\therefore \quad \text{Additional pull} = 503.66 - 360$$

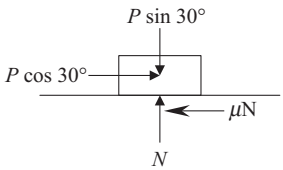
$$= 143.66 \text{ N}.$$





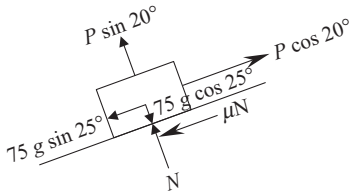
6.7:





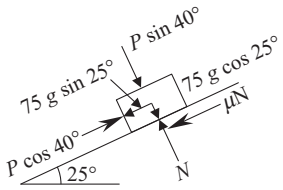
$$N + P\sin 30° = 200\ g$$
$$P\cos 30° = \mu\,[200\ g - P\sin 30°]$$
$$\Rightarrow \quad P = \frac{\mu\, 200\, g}{\cos 30° + \mu \sin 30°} = 495 \text{ N.}$$
$$N = P\sin 30° + 200\ g$$
$$P\cos 30° = \mu\,[200\ g + P\sin 30°]$$
$$\Rightarrow \quad P = \frac{\mu\, 200\, g}{\cos 30° - \mu \sin 30°} = 661.9 \text{ N}$$

6.8:



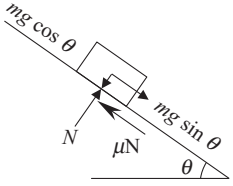
$$N = 75\ g\cos 25° - P\sin 20°$$
$$P\cos 20° = 75\ g\sin 25° + \mu N$$
$$= 75\ g\sin 25° + \mu\,[75\ g\cos 25° - P\sin 20°]$$
$$\Rightarrow \quad P = \frac{75\ g\,[\sin 25° + \mu\cos 25°]}{[\cos 20° + \mu \sin 20°]}$$
$$= 440.74 \text{ N}$$

6.9:



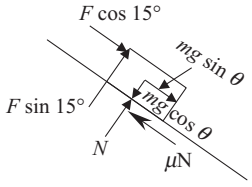
$$N = P\sin 40° + 75\ g\cos 25°$$
$$P\cos 40° = 75\ g\sin 25° + \mu N$$
$$= 75\ g\sin 25° + \mu\,[P\sin 40° + 75\ g\cos 25°]$$
$$\Rightarrow \quad P = \frac{75\ g\,[\sin 25° + \mu\cos 25°]}{[\cos 40° - \mu \sin 40°]}$$
$$= 696.96 \text{ N}$$
$$= 697 \text{ N}$$

6.10:



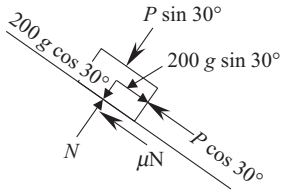
$$N = mg\cos\theta$$
$$mg\sin\theta = \mu N = \mu mg\cos\theta$$
$$\Rightarrow \quad m = \tan\theta$$
$$\therefore \quad \theta = \tan^{-1}[\mu] = \tan^{-1}[0.35] = 19.29°$$
$$F = mg\sin\theta = 2.6 \text{ kN.}$$

6.11:



$$N + F\sin 15° = mg\cos\theta$$
$$\Rightarrow \quad N = mg\cos\theta - F\sin 15°$$
$$F\cos 15° + mg\sin\theta = \mu N = \mu\,[mg\cos\theta - F\sin 15°]$$
$$F\,[\cos 15° + \mu\sin 15°] = mg\,[\mu\cos\theta - \sin\theta]$$
$$\Rightarrow \quad F = 147.18$$
$$\approx 147.2\text{N}$$

6.12:



$$N = 200\,g\cos 30° + P\sin 30°$$

$$200\,g\sin 30° = P\cos 30° + \mu N$$

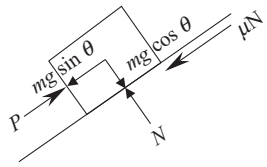
$$= P\cos 30° + \mu\,[200\,g\cos 30° + P\sin 30°]$$

$$\Rightarrow \quad P = \frac{200\,g\,[\sin 30° - \mu\cos 30°]}{[\cos 30° + \mu\sin 30°]}$$

$$= 371.08 \text{ N}$$

$$\approx 371.1 \text{ N}$$

6.13:



$$Ph = mg\left[\cos\theta\cdot\frac{50}{2} + \sin\theta\cdot\frac{75}{2}\right]$$

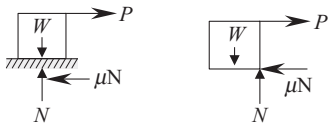
$$N = mg\cos\theta$$

$$P = mg\sin\theta + \mu N$$

$$= mg\sin\theta + \mu mg\cos\theta$$

$$\therefore \quad h = \frac{\cos\theta\cdot\frac{50}{2} + \sin\theta\cdot\frac{75}{2}}{\sin\theta + 0.2\cos\theta} = 60 \text{ cm}$$

6.14:



$$N = W$$

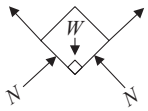
$$P = \mu W$$

$$= 0.3 \times 100 \times 9.81 = 294.3 \text{ N}$$

$$P.a = W.\frac{a}{2}$$

$$\Rightarrow \quad P = 490.5 \text{ N}$$

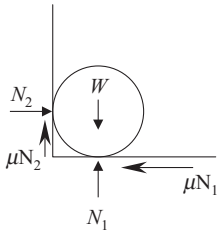
6.15:



$$N = \frac{W}{\sqrt{2}}$$

$$P = 2\mu N = 2\mu\frac{W}{\sqrt{2}} = \sqrt{2}\,\mu mg$$

6.16:



$$N_1 + \mu N_2 = W$$

$$N_2 = \mu N_1$$

$$N_1 + \mu^2 N_1 = W$$

$$\Rightarrow \quad N_1 = \frac{W}{1+\mu^2}$$

$$N_2 = \frac{\mu W}{1+\mu^2}$$

$$M = (\mu N_1)r + (\mu N_2)r$$

$$= \mu r(N_1 + N_2)$$

$$= \frac{\mu W r}{1+\mu^2}[1 + \mu]$$

6.17:

$$M = \mu N_1 r + \mu N_2 r$$

$$= \mu r (N_1 + N_2)$$

$$N_1 = N_2 \sin 30°$$

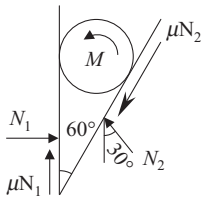
$$N_2 \cos 30° = W$$

$$\Rightarrow \quad N_2 = \frac{2}{\sqrt{3}} W$$

$$\therefore \quad N_1 = \frac{W}{\sqrt{3}}$$

$$M = \mu r \left[\frac{W}{\sqrt{3}} + \frac{2}{\sqrt{3}} W\right]$$

$$= \sqrt{3}\, w \mu r$$



6.18:

$$F_1 = F_2 = k\Delta = 10^3 \times 0.05 = 50 \text{ N}$$

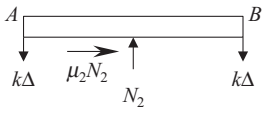
$$N_2 = F_1 + F_2 = 100 \text{ N}$$

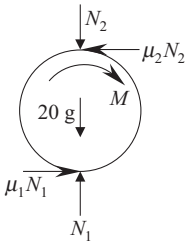
$$N_1 = N_2 + 20 \text{ g} = 296.2 \text{ N}$$

$$M = \mu_1 N_1 r + \mu_2 N_2 r$$

$$= [0.2 \times 296.2 + 0.15 \times 100]0.15$$

$$= 11.14 \text{ N.m}$$





6.19:

$$N_A = W \cos \theta$$

$$W \sin \theta = \mu_A N_A + T$$

$$= \mu_A W \cos \theta + T$$

$$\Rightarrow \quad T = W [\sin \theta - \mu_A \cos \theta] \ldots(1)$$

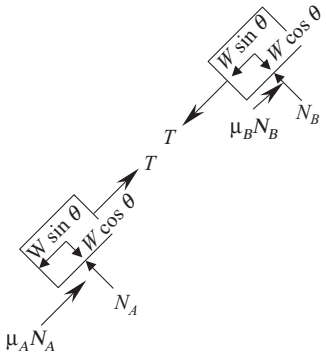
$$N_B = W \cos \theta$$

$$T + W \sin \theta = \mu_B N_B = \mu_B W \cos \theta$$

$$\Rightarrow \quad T = W [\mu_B \cos \theta - \sin \theta] \ldots(2)$$

From (1) & (2)

$$\sin \theta - \mu_A \cos \theta = \mu_B \cos \theta - \sin \theta$$



$$2 \sin \theta = (\mu_A + \mu_B) \cos \theta$$

$$\tan \theta = \frac{\mu_A + \mu_B}{2} = \frac{5}{12}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{5}{12}\right]$$

6.20:

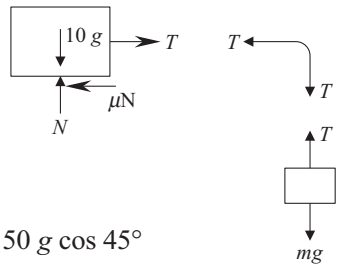
$$T = mg$$

$$N = 10\ g$$

$$T = \mu N = \mu\ 10\ g$$

$$\therefore \quad mg = \mu\ 10\ g$$

$$\Rightarrow \quad m = 2 \text{ kg}$$



6.21:

$$N = 50\ g \cos 45°$$

$$T + \mu N - 50\ g \sin 45° = 0$$

$$T + \mu\ [50\ g \cos 45°] - 50\ g \sin 45° = 0$$

$$T = 50\ g\ [\sin 45° - \mu \cos 45°]$$

$$= 28.3 \times g$$

$$\therefore \quad m = 28.3 \text{ kg}$$

$$N = 50\ g \cos 45°$$

$$T - \mu N - 50\ g \sin 45° = 0$$

$$T = \mu N + 50\ g \sin 45°$$

$$= m\ 50\ g \cos 45° + 50\ g \sin 45°$$

$$= 50\ g\ [\mu \cos 45° + \sin 45°]$$

$$= 42.43 \times g$$

$$\therefore \quad m = 42.43 \text{ kg}$$

6.22:

$$N_1 = N_2 + 100\ g \cos 30°$$

$$100\ g \sin 30° = F_1 + F_2$$

$$= \mu N_1 + \mu N_2$$

$$= \mu\ (N_1 + N_2)$$

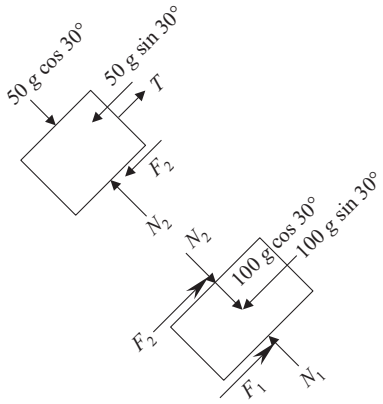
$$= \mu\ [50\ g \cos 30° + 100\ g \cos 30° + 50\ g \cos 30°]$$

$$= 200\ g \cos 30°\ \mu$$

$$\Rightarrow \quad \mu = \frac{\tan 30°}{2}$$

$$= 0.29$$

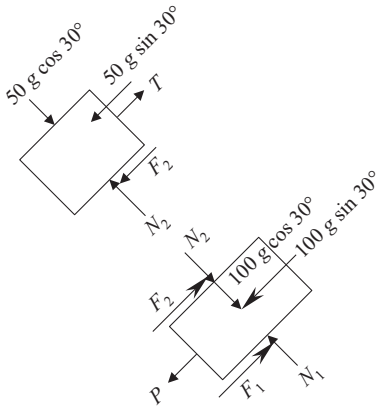
$$T = 50\ g \sin 30° + \mu\ 50\ g \cos 30°$$



$= 50\ g\ [\sin 30° + \mu \cos 30°]$

$= 368$ N

6.23:



$N_2 = 50\ g \cos 30°$

$N_1 = N_2 + 100\ g \cos 30°$

$= 150\ g \cos 30°$

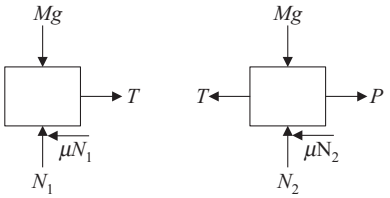
$P + 100\ g \sin 30° = F_1 + F_2$

$= \mu_1 N_1 + \mu_2 N_2$

$= 0.33 \times 150\ g \cos 30° + 0.3 \times 50\ g \cos 30°$

$\Rightarrow \quad P = 57.5$ N

6.24:



Block-1

$N_1 = Mg$

$T_1 = \mu N_1$

$= \mu Mg$

Block-2

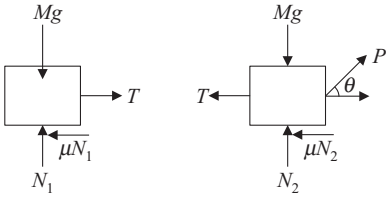
$N_2 = Mg$

$P = T + \mu N_2$

$= \mu Mg + \mu Mg$

$= 2\mu Mg$

6.25:



Block-1

$N_1 = Mg$

$T = \mu N_1$

$= \mu Mg$

Block-2

$N_2 + P \sin \theta = Mg$

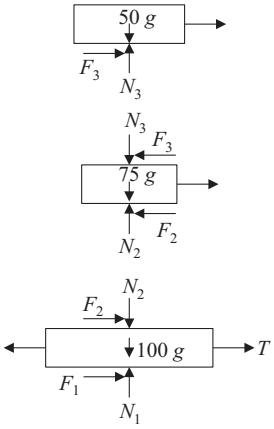
$P \cos \theta = T + \mu N_2$

$= T + \mu\ [Mg - P \sin \theta]$

$= \mu Mg + \mu\ [Mg - P \sin \theta]$

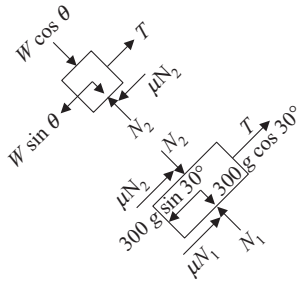
$$\Rightarrow \quad P = \frac{2\mu Mg}{\cos \theta + \mu \sin \theta}$$

6.26:



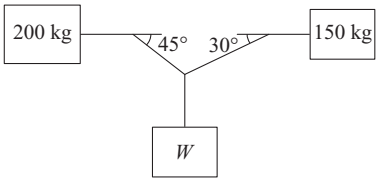
$$N_3 = 50\ g$$
$$F_3 = \mu.50\ g$$
$$N_2 = N_3 + 75\ g$$
$$= 125\ g$$
$$F_2 = \mu.125\ g$$
$$T = F_2 + F_3$$
$$= \mu.175\ g$$
$$N_1 = N_2 + 100\ g$$
$$= 225\ g$$
$$F_1 = \mu.225\ g$$
$$P = T + F_1 + F_2$$
$$= \mu.175\ g + \mu.225\ g + \mu.125\ g$$
$$= \mu\ 525\ g$$
$$= 1287.6\ \text{N}$$

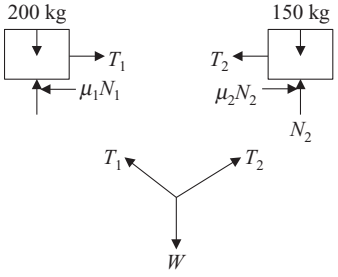
6.27:



$$N_1 = 300\ g \cos 30° + N_2 \qquad N_2 = W \cos 30°$$
$$300\ g \sin 30° = \mu N_1 + \mu N_2 + T \qquad T = W \sin\theta + \mu N_2$$
$$300\ g \sin 30° - \mu\,[300\ g \cos 30° + 2\ W \cos 30°]$$
$$= W \sin\theta + \mu W \cos 30°$$
$$300\ g\,[\sin 30° - \mu \cos 30°] = W\,[\sin\theta + 3\ \mu \cos 30°]$$
$$\Rightarrow \qquad W = 725.8\ \text{N}$$

6.28:





Block-A:

$$N_1 - 200\ g = 0 \Rightarrow N_1 = 1962\ \text{N}$$
$$T_1 - \mu N_1 = 0$$
$$\therefore \qquad T_1 = \mu_1\ (1962)$$

Block-B:

$$N_2 - 150\ g = 0 \Rightarrow N_2 = 1471.5\ \text{N}$$

Weight W:

$$T_1 \sin 45° + T_2 \sin 30° = W$$
$$T_1 \cos 45° = T_2 \cos 30°$$
$$\therefore \qquad T_1 = 360.4\ \text{N}$$
$$\therefore \qquad W = 402\ \text{N}$$
$$\text{Also,} \qquad \mu_1 = 0.184$$

$$T_2 - \mu_2 N_2 = 0$$

$\therefore$ $$T_2 = 0.2\ (1471.5) = 2943\ \text{N}.$$

6.29:

$$N_1 - N_2 = 0 \Rightarrow N_1 = N_2$$

$$\mu_1 N_1 + \mu_2 N_2 = W \Rightarrow (\mu_1 + \mu_2)N = W$$

$$N_2\, l \sin\theta + \mu_2 N_2\, l \cos\theta = W \frac{l}{2} \cos\theta$$

$$N_2 [\sin\theta + \mu_2 \cos\theta] = \frac{W}{2} \cos\theta$$

$\therefore$ $$N_1 = N_2 = N = \frac{W \cos\theta}{2[\sin\theta + \mu_2 \cos\theta]} = \frac{W}{\mu_1 + \mu_2}$$

$$(\mu_1 + \mu_2) \cos\theta = 2 \sin\theta + 2\mu_2 \cos\theta$$

$$(\mu_1 - \mu_2) \cos\theta = 2 \sin\theta$$

$$\tan\theta = \left[\frac{\mu_1 - \mu_2}{2}\right]$$

$$\theta = \tan^{-1}\left[\frac{\mu_1 - \mu_2}{2}\right]$$

6.30:

$$\mu_A = \mu_B = 0.3$$

$$\Sigma Fy = 0 \Rightarrow$$

$$N_A + N_B \cos\theta + \mu N_B \sin\theta - W = 0 \quad \text{(a)}$$

$$\Sigma Fx = 0 \Rightarrow$$

$$\mu N_A + \mu N_B \cos\theta - N_B \sin\theta = 0 \quad \text{(b)}$$

$$\Sigma N_B = 0 \Rightarrow$$

$$-N_A .l \cos 45° + \mu N_A\, l \sin 45° + W \frac{l}{2} \cos 45° = 0$$

$$-N_A + \mu N_A + \frac{W}{2} = 0$$

$\Rightarrow$ $$N_A = \frac{W/2}{1-\mu} = 0.714\ \text{W} \quad \text{(c)}$$

$$N_B (\cos\theta + \mu \sin\theta) = W - N_A \quad \text{(d)}$$

$$N_B (\sin\theta - \mu \cos\theta) = \mu N_A \quad \text{(e)}$$

$\Rightarrow$ $$\frac{\cos\theta + \mu \sin\theta}{\sin\theta - \mu \cos\theta} = \frac{W - 0.714W}{0.3\,(0.714)W} = 1.335$$

$$\cos\theta + 0.3 \sin\theta = 1.335 \sin\theta - 0.4 \cos\theta$$

$$1.4 \cos\theta = 1.035 \sin\theta$$

$$\Rightarrow \quad \tan\theta = 1.353$$
$$\theta = 53.53^\circ \approx 53.5^\circ$$

6.31:

$$\Sigma F_x = 0 \Rightarrow$$
$$N_A - N_B \sin 30^\circ - \mu N_B \cos 30^\circ = 0 \quad \text{(a)}$$
$$\Sigma F_y = 0 \Rightarrow$$
$$\mu N_A + N_B \cos 30^\circ - \mu N_B \sin 30^\circ - W = 0 \quad \text{(b)}$$
$$\Sigma M_B = 0 \Rightarrow$$
$$-N_A\, l \sin\theta - \mu N_A\, l \cos\theta + W\frac{l}{2}\cos\theta = 0$$
$$N_A\,[\sin\theta + \mu\cos\theta] = \frac{W}{2}\cos\theta \quad \text{(c)}$$

From (a),

$$N_A - N_B\,(\sin 30^\circ + 0.2\cos 30^\circ) = 0$$
$$N_A - 0.673\,N_B = 0 \quad \text{(d)}$$
$$\Rightarrow \quad N_A = 0.673\,N_B$$

From (b),

$$0.2\,N_A + N_B\,(\cos 30^\circ - 0.2\sin 30^\circ) - W = 0$$
$$0.2\,N_A + 0.766\,N_B - W = 0 \quad \text{(e)}$$
$$N_B = 1.11\,W$$
$$N_A = 0.747\,W$$

From (c),

$$0.747\,W\,[\sin\theta + 0.2\cos\theta] = \frac{W}{2}\cos\theta$$
$$\Rightarrow \quad 0.747\sin\theta = 0.3506\cos\theta$$
$$\Rightarrow \quad \theta = 25.1^\circ$$