Chapter 4

4.1:

$M_o = 2 \text{ kN} \times (0.03)$
$= 60 \text{ N.m anticlockwise}$

4.2:

$M_A = 5 \text{ kN} \times 5$
$= 25 \text{ kN.m anticlockwise}$

4.3:

$M_o = -60 \cos 20 \times 0.6 + 60 \sin 20 \times 0.1$
$= 31.8 \text{ N.m clockwise}$

4.4:

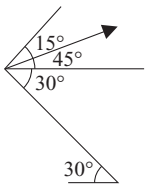
$M_A = -800 \sin 60 \times 3 + 800 \cos 60 \times 0.3$
$= 1958.5 \text{ N.m (clockwise)}$
$M_B = 800 \sin 60 \times 2 + 800 \cos 60 \times 0.3$
$= 1505.6 \text{ N.m anticlockwise}$

4.5:

$M_A = 500 \sin 30 \times 1$
$= 250 \text{ N.m clockwire}$
$M_o = 500 \sin 30 \times 1 + 500 \cos 30 \times 1.5$
$= 899.5 \text{ N.m clockwise}$

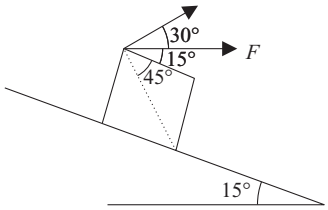
4.6:

$M_A = 1500 \cos 15 \times 6$
$= 8.69 \text{ kN.m}$



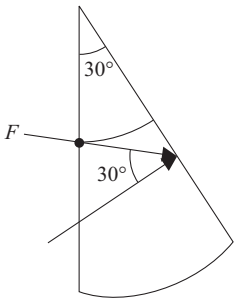
4.7:

$M_B = F \cos 30°.a\sqrt{2} = \mathbf{1.225}\boldsymbol{Fa}$



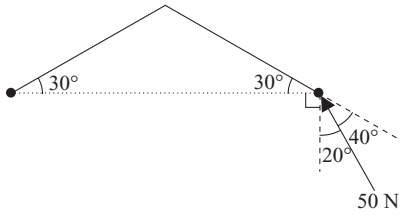
4.8:

$$M_A = F \cos 30° \times \frac{0.75}{\cos 30°}$$
$$= 0.75F$$



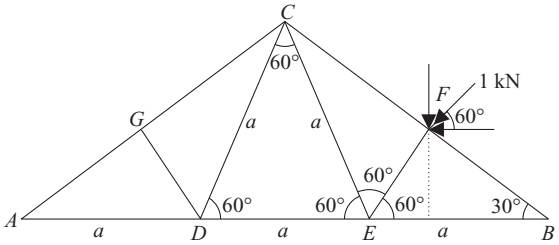
4.9:

$M_o = 50 \cos 20° [0.75 \cos 30 \times 2]$
$= 61.03$ N.m



4.10:

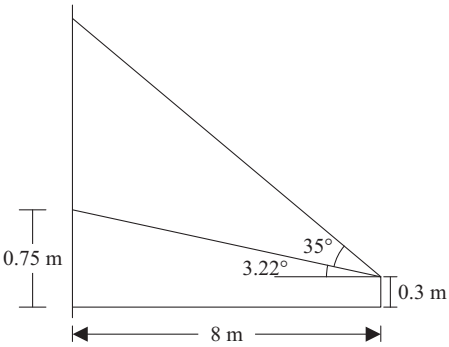
$M_A = -1 \sin 60° [2a + a \sin 30° \cdot \cos 60°]$
$+ 1 \cos 60° [a \sin 30° \sin 60°]$
$= \mathbf{1.732}\boldsymbol{a}$ kN.m
$M_B = 1 \sin 60° [a - a \sin 30° \cdot \cos 60°]$
$+ 1 \cos 60° [a \sin 30° \sin 60°]$
$= \mathbf{0.866}\boldsymbol{a}$ kN.m



4.11:

$$\theta = \tan^{-1}\left[\frac{0.75 - 0.3}{8}\right] = 3.22°$$

$M_A = 4000 \sin 3.22° \times 8 + 4000 \cos 3.22° \times 0.3$
$= 20.74$ kN.m



4.12:

C_1: $2500 \cos 30° \times 8.6 = 18.62$ kN.m
C_2: $2000 \cos 45° \times 8.3 = 11.74$ kN.m
C_3: $1000 \cos 60° \times 8 = 4$ kN.m

4.13:

$$\vec{r} = 4\vec{i} + 3\vec{j}$$
$$\vec{F} = -500\vec{k}$$
$$\vec{M}_o = \vec{r} \times \vec{F} = (4\vec{i} + 3\vec{j}) \times (-500\vec{k})$$
$$= 2000\vec{j} - 1500\vec{i}$$
$$= 500[-3\vec{i} + 4\vec{j}]$$

4.14:

$$\vec{F} = 200\left[\frac{5\vec{i} + 3\vec{j} + 3\vec{k}}{\sqrt{5^2 + 3^2 + 3^2}}\right] = \frac{200}{\sqrt{43}}[5\vec{i} + 3\vec{j} + 3\vec{k}]$$
$$\vec{r} = 6\vec{i} + 3\vec{j} + 4\vec{k}$$

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & +3 & 4 \\ 5 & +3 & 3 \end{vmatrix} \frac{200}{\sqrt{43}} = \frac{200}{\sqrt{43}}[\vec{i}(9-12) - \vec{j}(18-20) + \vec{k}\ (18-15)]$$

$$= \frac{200}{\sqrt{43}}[-3\vec{i} + 2\vec{j} + 3\vec{k}] = \frac{-600\vec{i}}{\sqrt{43}} + \frac{400\vec{j}}{\sqrt{43}} + \frac{600\vec{k}}{\sqrt{43}}$$

$$\vec{M}_A = [(6\vec{i} + 3\vec{j} + 4\vec{k}) - (6\vec{i})] \times \vec{F}$$

$$= \frac{200}{\sqrt{43}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 4 \\ 5 & 3 & 3 \end{vmatrix} = \{\vec{i}[(9-12)] - \vec{j}[0-20] + \vec{k}[0-15]\} \frac{200}{\sqrt{43}}$$

$$= [-3\vec{i} + 20\vec{j} - 15\vec{k}] \frac{200}{\sqrt{43}}$$

$$= \frac{-600\vec{i} + 4000\vec{j} - 3000\vec{k}}{\sqrt{43}}$$

$$M_x = -\frac{600}{\sqrt{43}} \text{ N.m}$$

4.15:

$$\vec{M}_{O/A} = 10\vec{k} \times \left[100\left(\frac{-4\vec{i} + 4\vec{j} - 10\vec{k}}{\sqrt{132}}\right)\right]$$

$$= \frac{1000}{\sqrt{132}}[-4\vec{j} - 4\vec{i}]$$

$$= \frac{-4000\vec{i} - 4000\vec{j}}{\sqrt{132}}$$

$$\vec{M}_{O/B} = 8\vec{k} \times \left[150\left(\frac{5\vec{i} - 3\vec{j} - 8\vec{k}}{\sqrt{98}}\right)\right]$$

$$= \frac{1200}{\sqrt{98}}[5\vec{j} + 3\vec{i}]$$

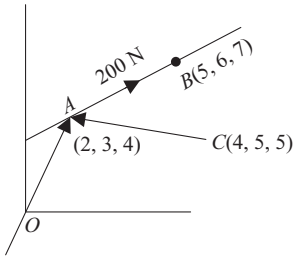
$$= \frac{3000\vec{i} + 6000\vec{j}}{\sqrt{98}}$$

4.16:

$$\overrightarrow{AB} = 3\vec{i} + 3\vec{j} + 3\vec{k}$$

$$\hat{n}_{AB} = \frac{3\vec{i} + 3\vec{j} + 3\vec{k}}{3\sqrt{3}}$$

$$\vec{F} = \frac{200}{\sqrt{3}}[3\vec{i} + 3\vec{j} + 3\vec{k}]$$



$$\vec{r} = \overrightarrow{OA} = 2\vec{i} + 3\vec{j} + 4\vec{k}$$

$$\vec{r}_{CA} = -2\vec{i} - 2\vec{j} - \vec{k}$$

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix} \frac{200}{\sqrt{3}}$$

$$= [\vec{i}(3-4) - \vec{j}(2-4) + \vec{k}(2-3)] \frac{200}{\sqrt{3}}$$

$$= [-\vec{i} + 2\vec{j} - \vec{k}] \frac{200}{\sqrt{3}}$$

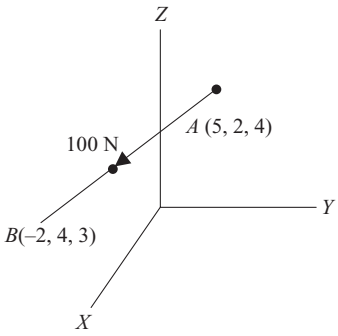
$$= \frac{-200\vec{i} + 400\vec{j} - 200\vec{k}}{\sqrt{3}}$$

$$\vec{M}_C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -2 & -1 \\ 1 & 1 & 1 \end{vmatrix} \frac{200}{\sqrt{3}}$$

$$= \frac{200}{\sqrt{3}} [\vec{i}(-2+1) - \vec{j}(-2+1) + \vec{k}(-2+2)]$$

$$= \frac{200}{\sqrt{3}} [-\vec{i} + \vec{j}]$$

4.17:



$$\vec{r} = \overrightarrow{OA} = 5\vec{i} + 2\vec{j} + 4\vec{k}$$

$$\vec{F} = 100\left[\frac{-7\vec{i} + 2\vec{j} - \vec{k}}{\sqrt{54}}\right]$$

$$\vec{M}_o = \vec{r} \times \vec{F} = \frac{100}{\sqrt{54}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 2 & 4 \\ -7 & 2 & -1 \end{vmatrix}$$

$$= \frac{100}{\sqrt{54}} [\vec{i}(-2-8) - \vec{j}(-5+28) + \vec{k}(10+14)]$$

$$= \frac{100}{\sqrt{54}} [-10\vec{i} - 23\vec{j} + 24\vec{k}]$$

$$\vec{M}_z = \vec{M}_o \cdot \vec{k} = \frac{100}{\sqrt{54}}.24 = \frac{2400}{\sqrt{54}} \text{ N.m}$$

4.18:

$$\vec{r}_1 = 4\vec{i} + 3\vec{j} \qquad \vec{r}_2 = \vec{i} + 2\vec{j}$$

$\vec{F}_1 = 100\ [\cos 30°\vec{i} - \sin 30°\vec{j}]$ $\quad$ $\vec{F}_2 = F[-\cos 10°\vec{i} + \sin 10°\vec{j}]$

$\vec{M}_1 = 100\ [-4 \sin 30° - 3 \cos 30°]$ $\quad$ $\vec{M}_2 = F[\sin 10° + 2 \cos 10°]$

$= -100\ [4 \sin 30° + 3 \cos 30°]$

$$M_1 = M_2 \quad \Rightarrow \quad F = 214.5 \text{ N}$$

4.19:

$\vec{M}_o = 100 \times 1 \sin 50° - 50 \times 0.5 \cos 50°$

$= 60.5$ N.m (anticlockwise)

4.20:

$\vec{M}_o = 20 \times 0.3 = 6$ N.m

4.21:

$\vec{M}_o = 10 \cos 10° \times 0.04 = 0.39$ N.m

4.22:

$\vec{M}_o = 100 \sin 20° \times 2 - 50 = 18.4$ N.m (anticlockwise)

4.23:

$\vec{F} = 100\ [\cos 60°\vec{i} + \sin 60°\vec{j}]$

$\overrightarrow{OA} = 4\vec{i} + 3\vec{j}$

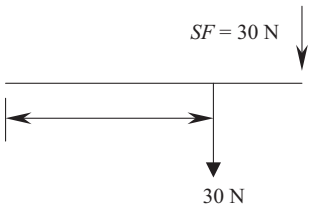
$$M_o = 100 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & 0 \\ \cos 60° & \sin 60° & 0 \end{vmatrix} = 100[4 \sin 60 - 3 \cos 60] = 196.4 \text{ N.m}$$

$\overrightarrow{M_B} = 100 \cos 60 \times 3 = 150$ N.m

$\overrightarrow{M_C} = 100 \sin 60 \times 4 = 346.4$ N.m

4.24:

$$\frac{40 \times 4 - 30 \times 2}{30} = \mathbf{3.33\ m}$$



4.25:

$\Sigma F = 30$ N $\uparrow$

$\Sigma M = 15 \times 2 = 30$ N.m

4.26:

A: $\Sigma F = P$ $\quad$ $\Sigma M = M + \dfrac{PL}{4}\ \downarrow$

B: $\Sigma F = P$ $\quad$ $\Sigma M = M - \dfrac{3PL}{4}\ \downarrow$

4.27:

A: $\Sigma F = 5 + (2 \times 3) = 11$ kN $\quad$ $\Sigma M = 5 \times 2 + 6 \times 4.5 = 37$ kNm

B: $\Sigma F = 11$ kN $\quad$ $\Sigma M = 5 \times 4 + 6 \times 1.5 = 29$ kNm

4.28:

$$\Sigma F_x = -\frac{60}{\sqrt{2}} + 150 \cos\theta = 98.8 \text{ N} \qquad \theta = \tan^{-1}\left[\frac{1}{2}\right]$$

$$\Sigma F_y = 100 + 150 \cos\theta + \frac{50}{\sqrt{2}} = 202.4 \text{ N}$$

$$M_o = -7.5 + \frac{50}{\sqrt{2}} \times 0.2 + \frac{50}{\sqrt{2}} \times 0.1 + 100 \times 0.2 + 150 \sin\theta \times 0.3 - 150 \cos\theta \times 0.3$$

$$= 2.98 \text{ N.m}$$

4.29:

$$\Sigma F_x = \frac{40}{\sqrt{2}} - \frac{25}{\sqrt{2}} = 10.61 \text{ N}$$

$$\Sigma F_y = 50 - \frac{40}{\sqrt{2}} - \frac{25}{\sqrt{2}} = 4.04 \text{ N}$$

$$\Sigma M_A = 50 \times 0.02 - \frac{40}{\sqrt{2}} \times 0.01 - \frac{40}{\sqrt{2}} \times 0.03 - \frac{25}{\sqrt{2}} \times 0.05 + \frac{25}{\sqrt{2}} \times 0.04$$

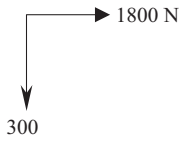
$$= -30.81 \text{ N.cm}$$

4.30:

$$R = 1824.8 \text{ N} \qquad \Sigma M = 200 \times 0.15 = 30 \text{ N.m}$$

$$\bar{x} = \frac{30}{300} = 0.1 \text{ m or } 10 \text{ cm}$$

$$\bar{y} = \frac{30}{1800} = \mathbf{1.67\ cm}$$



4.31:

$$\Sigma F_y = 700 - 500\ \frac{3}{5} = 400 \text{ N}$$

$$\Sigma F_x = 500\ \frac{4}{5} = 400 \text{ N} \qquad \therefore\ F = 400\sqrt{2} \text{ N}$$

$$\Sigma M = \mathbf{0}$$

4.32:

$$\Sigma F_y = \frac{L}{2}[W_1 + W_2]$$

$$\Sigma M =$$

4.33:

$$F = \frac{4}{2}[157 + 196] = 706 \text{ kN}$$

$$M = \frac{4}{3}\left[\frac{157 + 2(196)}{157 + 196}\right] \times 706$$

$$= 1464 \text{ kN.m}$$

4.34:

$$\Sigma F_x = -\frac{5}{\sqrt{2}} = 3.54 \text{ N}$$

$$\Sigma F_y = -10 - 3 - \frac{5}{\sqrt{2}} = 16.54 \text{ N}$$

$$R = \mathbf{16.91\ kN}$$

$$\Sigma M_A = -(10 \times 2) - 10 - \left(\frac{5}{\sqrt{2}} \times 5\right) - 3 \times 7$$

$$= 68.68 \text{ kN.m}$$

$$\bar{x} = \frac{68.68}{16.54} = \mathbf{4.15\ m}$$

4.35:

$$\Sigma F_x = 40 \cos 30° = 34.64 \text{ N} \qquad \bar{x} = \frac{-10}{-20} = 0.5 \text{ m}$$

$$\Sigma F_y = -40 \sin 30° = -20 \text{ N} \qquad \bar{y} = -\left(\frac{-10}{34.64}\right) = \mathbf{0.29\ m}$$

$$\Sigma M = -40 \sin 30° \times 1 + 10 \times 1$$

$$= -10 \text{ Nm}$$

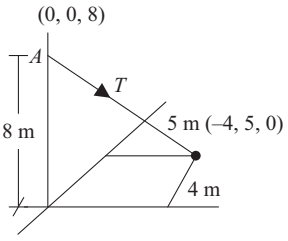
4.36:

$$\vec{F} = 100\left[\frac{-4\vec{i} + 5\vec{j} - 8\vec{k}}{\sqrt{105}}\right]$$

$$\vec{r}_{OA} = 8\vec{k}$$

$$\therefore \quad \vec{M}_o = \vec{r}_{OA} \times \vec{F} = \frac{800}{\sqrt{105}}[-4\vec{j} - 5\vec{i}]$$

$$= \frac{-4000\vec{i}}{\sqrt{105}} - \frac{3200\vec{j}}{\sqrt{105}}$$



4.37:

$$M = P \times 2d = 2Pd$$

4.38:

$\vec{r}_i$	$\vec{F}_i$	
$4\vec{i} + 2\vec{j}$	$15\vec{k}$	$-60\vec{j} + 30\vec{i}$
$5\vec{i} + 5\vec{j}$	$-20\vec{k}$	$+100\vec{j} - 100\vec{i}$
$2\vec{i} + 8\vec{j}$	$-10\vec{k}$	$20\vec{j} - 80\vec{i}$
$(x\vec{i} + y\vec{j})$	$-15\vec{k}$	$60\vec{j} - 150\vec{i}$

$$15x\vec{j} - 15y\vec{i} = 60\vec{j} - 150\vec{i}$$

$$\bar{x} = \frac{60}{15} = 4 \text{ m.}$$

$$\bar{y} = \frac{150}{15} = \mathbf{10\ m.}$$