GIVE FEEDBACK

CONTINUE >

Every object can bend to some extent and even the smallest load will produce a correspondingly small deflection. For example, a person standing on a bridge will cause the bridge to sag just a tiny bit more. Calculating the deflection caused by bending loads is complex so we will use a pre-calculated formula for each type of loading.

When a beam is under a bending load it will deflect. Even under its own weight a beam will bend some amount.

When a beam deflects downwards it is called sagging and an upwards bow is called hogging.

< BACK GIVE FEEDBACK CONTINUE >

But calculating the amount of deflection in bending is not as easy as it sounds. This is because the beam does not necessarily bend in a simple geometric shape, e.g. a circular arc.

To solve bending deflection problems we will find the pre-calculated formula for each type of question, sometimes combining more than one formula to solve the problem. A much greater range of problems can be solved by combining formulas. This is called superposition.

Beam-bending tables are commonly used by engineers to solve these problems. The skill is in finding the right formula and ensuring the numbers are correctly converted to the appropriate units.

< BACK GIVE FEEDBACK OK

There are two main ways an engineer determines the suitability of a component: it must perform satisfactorily in terms of stress and in deflection. In other words, it must be stiff and strong enough to do the job. In this chapter we are investigating the deflection caused by bending, which is the most common type of excessive deflection.

Rigidity under load in the analysis of bending stresses of beams

In the previous chapter, our focus was on bending stresses because of their obvious importance in the design of beams and beam-like machine parts.

However, it can be just as important to make sure the beam is not too flexible under load.

Examples of excess flexibility:

1. A plastered ceiling cracking if the joists supporting it are too flexible (the beam is the ceiling joist)
2. Damaging vibrations developing in rotating machinery if a shaft exhibits excessive flexibility under transverse loads (the beam is the shaft)
3. Machinery with a long cutter that lacks stiffness, causing 'chatter' or vibration of the tool, leaving a poor finish on the part (the beam is the milling cutter)

GIVE FEEDBACK

OK

Which of the following can occur when a beam has sufficient strength but inadequate stiffness?

Check all that apply.

- [ ] Excessive vibration
- [ ] Excessive flexibility
- [ ] Permanent bending
- [ ] Cracking
- [ ] Breaking
- [ ] Excessive movement

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Identify whether the following problems are mostly an issue of either excessive beam stress or deflection.

Drag each item into appropriate category.
Click on an item to send it to the back of the stack.

A lever breaking when overloaded

Excessive beam deflection	Excessive beam stress

Which of the following problems are mostly due to excessive beam stress?

Check all that apply.

- [ ] Permanent bending
- [ ] Cracking
- [ ] Excessive vibration
- [ ] Excessive flexibility
- [ ] Breaking

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Measures that can be used to describe the extent of deformation of a beam

Bending deflection can be measured in two ways. As a radius of the curvature, where a smaller radius means greater deflection, or as a vertical deflection, where a larger number means a greater deflection.

Every object has a certain stiffness. This means there will be deflection whenever there is a load applied.

This means a person walking on a bridge will bend the bridge and cause deflection albeit very minimal.

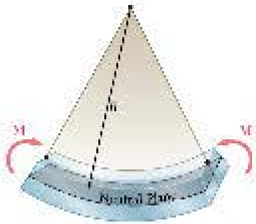
There are two measures that can be used to describe the extent of deformation suffered by a beam subjected to bending.

GIVE FEEDBACK

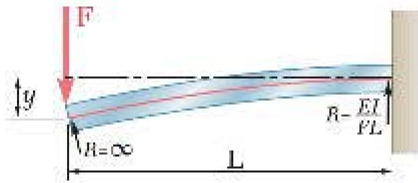
CONTINUE >

Measures that can be used to describe the extent of deformation of a beam

The first is the **radius of curvature**, R of the beam's neutral axis, originally straight but distorted into a curve by the applied loads (image a). The second, and perhaps more useful, is the amount of **deflection**, y of the neutral axis from its original position in the unloaded beam (image b).



a) Radius of curvature (R)



b) Deflection (y) of a beam

In this chapter we are going to use some beam-deflection formulas to determine deflection. This is how engineers solve beam deflection in practical design work.

Select two ways to measure the deformation of a beam under bending.

Check all that apply.

- [ ] Radius of curvature, R
- [ ] Deflection, y
- [ ] Second moment of area, I
- [ ] Height of centroid, y_c
- [ ] Distance from centroid to neutral plane, d

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of the following will cause deflection of a beam? (The beam is written in **bold**.)

Check all that apply.

- [ ] Heavy load lifted by a **gantry crane**
- [ ] A person standing on a **bridge**
- [ ] A train going across the **Sydney Harbour Bridge**
- [ ] An archer drawing a **bow** with an arrow

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

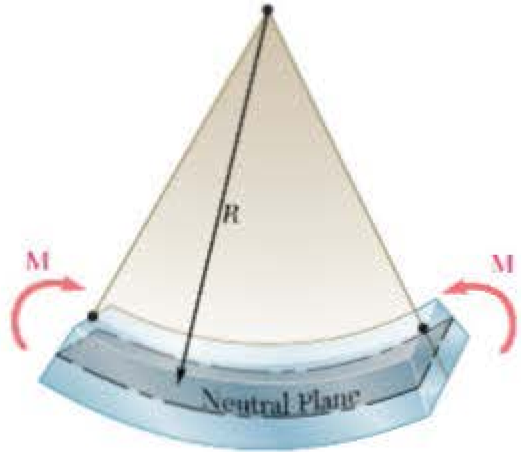
Radius of curvature is rarely constant along the whole beam. If it was, the beam would be bent into a circular arc. Most beams have a variation of radius along their length.

The radius of curvature

When a beam is subjected to bending its shape is distorted into a curve. The **radius of curvature**, R is not necessarily constant along the beam but is related to the magnitude of the bending moment along the beam.

Note that the radius of curvature is the inverse measure of distortion in bending, i.e. smaller radius > tighter bending > more distortion.

For the undistorted (straight) beam the radius is infinite.



Radius of curvature of a beam is circular when the bending moment is constant

GIVE FEEDBACK

OK

A beam without load has (please select) ▼ distortion and the radius of curvature is (please select) ▼.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

For a beam under bending, a smaller radius of curvature means the beam has (please select) ▼ distortion and a (please select) ▼ bending moment.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The radius of curvature becomes smaller as the bending moment increases. The radius of curvature becomes larger as the modulus and second moment of area are increased.

Formula calculating the radius of curvature of a beam

The equation for **radius of curvature** of a beam is:

$$R = \frac{EI}{M}$$

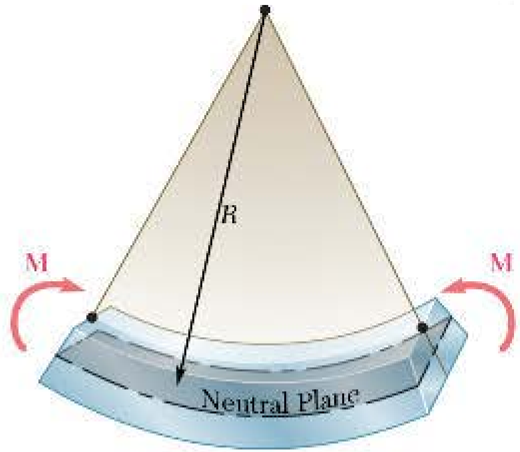
where:

R is the radius of curvature (mm)

M is the bending moment (Nmm)

E is Young's modulus (MPa)

I is the moment of inertia (mm^4)



Radius of curvature is measured to the neutral plane

GIVE FEEDBACK

OK

Match alternative names for the following concepts:

Drag statements on the right to match the left.

Second moment of area	Moment of inertia
Modulus of elasticity	Young's modulus
Centre of area	Centroid
Neutral axis	Neutral plane
Bending	Flexure

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the units to their definitions:

$$R = \frac{EI}{M}$$

Drag statements on the right to match the left.

Radius of curvature	mm
Bending moment	Nmm
Young's modulus	MPa
Moment of inertia	mm^4

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the variables to their definitions:

$$R = \frac{EI}{M}$$

Drag statements on the right to match the left.

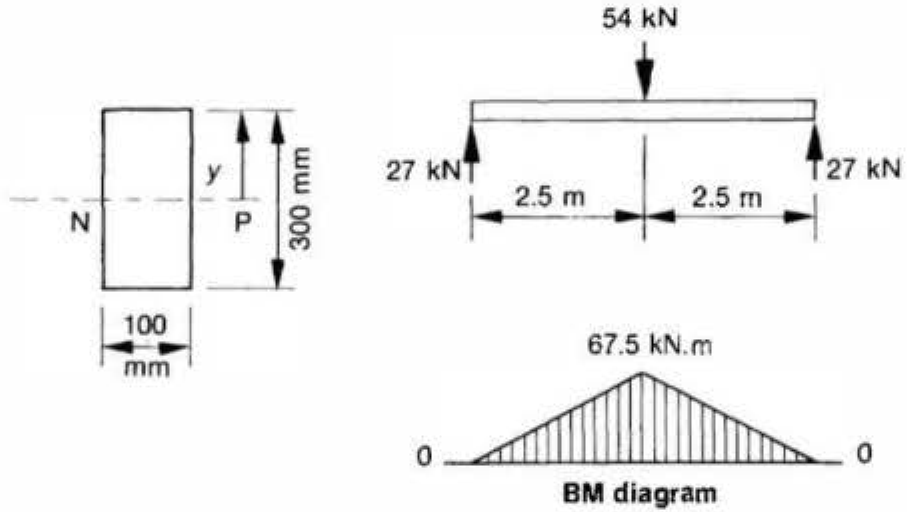
Radius of curvature	R
Bending moment	M
Young's modulus	E
Moment of inertia	I

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

Determine the radius of curvature at the point of maximum bending moment for the beam shown below. Use steel, $E = 200{,}000$ MPa:



GIVE FEEDBACK

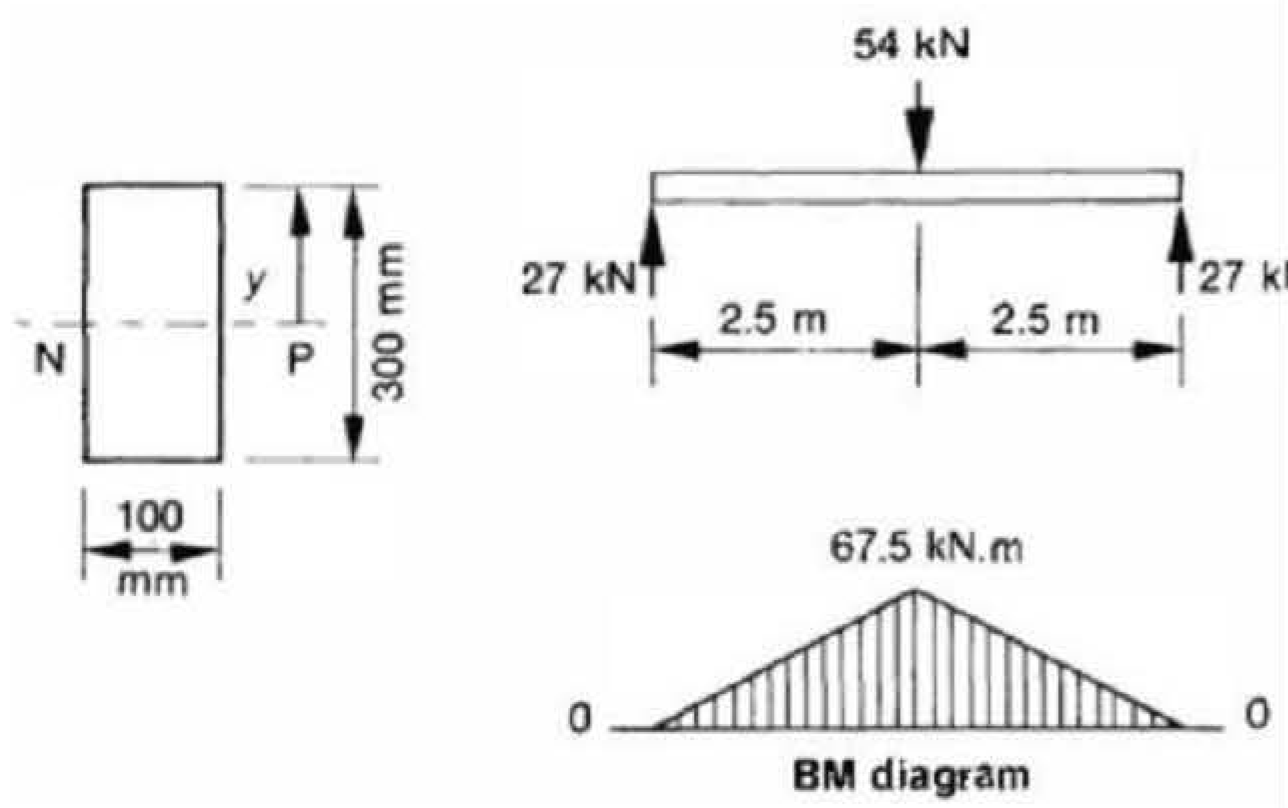
CONTINUE >

Solution

$$
\begin{aligned}
R &= \frac{EI}{M} \\
&= \frac{200{,}000 \text{ MPa} \times 225 \times 10^{6} \text{ mm}^{4}}{67.5 \times 10^{6} \text{ N.mm}} \\
&= 666{,}700 \text{ mm} \\
\therefore R &= 666.7 \text{ m}
\end{aligned}
$$

As expected the radius is very large because under moderate conditions of loading, the curvature produced in a solid beam is relatively small—too small to see by eye.

< BACK GIVE FEEDBACK OK



Determine the radius of curvature at the point of maximum bending moment. Use steel, Modulus = 200,000 MPa:

$$R = \frac{EI}{M}$$

Which equation will determine radius of curvature correctly?

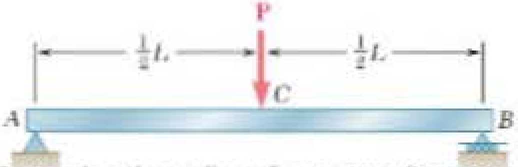
Click the correct answer.

$$\frac{200{,}000\ \text{MPa} \times 67.5 \times 10^6\ \text{N mm}}{225 \times 10^3\ \text{mm}^4}$$

$$\frac{67.5 \times 10^3\ \text{N mm} \times 225 \times 10^6\ \text{mm}^4}{200{,}000\ \text{MPa}}$$

$$\frac{200{,}000\ \text{MPa} \times 225 \times 10^6\ \text{mm}^4}{67.5 \times 10^6\ \text{N mm}}$$

$$\frac{67.5 \times 10^3\ \text{N mm}}{200{,}000\ \text{MPa} \times 225 \times 10^6\ \text{mm}^4}$$



Determine the radius of curvature (in metres) at the point of maximum bending moment for this beam that has a second moment of area of 125 (x 10^6) mm^4. Use steel, E = 200,000 MPa. Length is 8 m and load is 45 kN.
(Include units. Use one decimal place.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

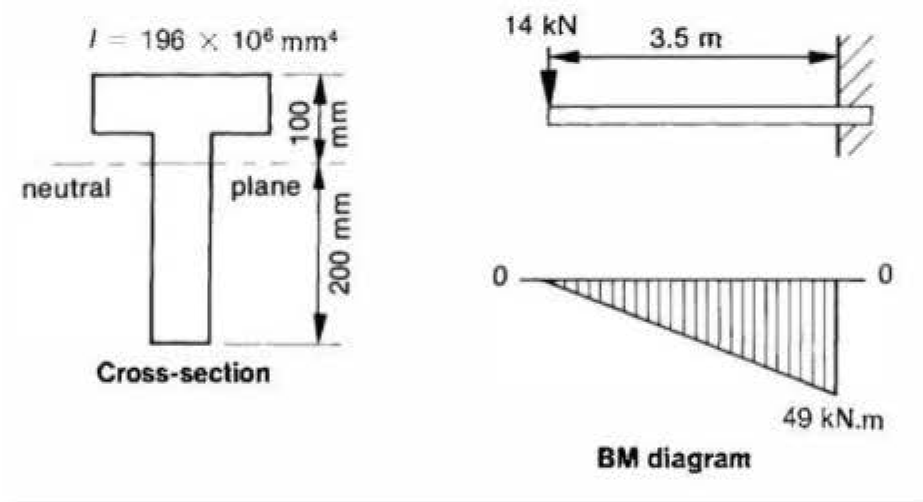
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Example

Determine the radius of curvature at the point of maximum bending moment for the beam shown below. Use concrete, $E = 23{,}000$ MPa.



GIVE FEEDBACK

CONTINUE >

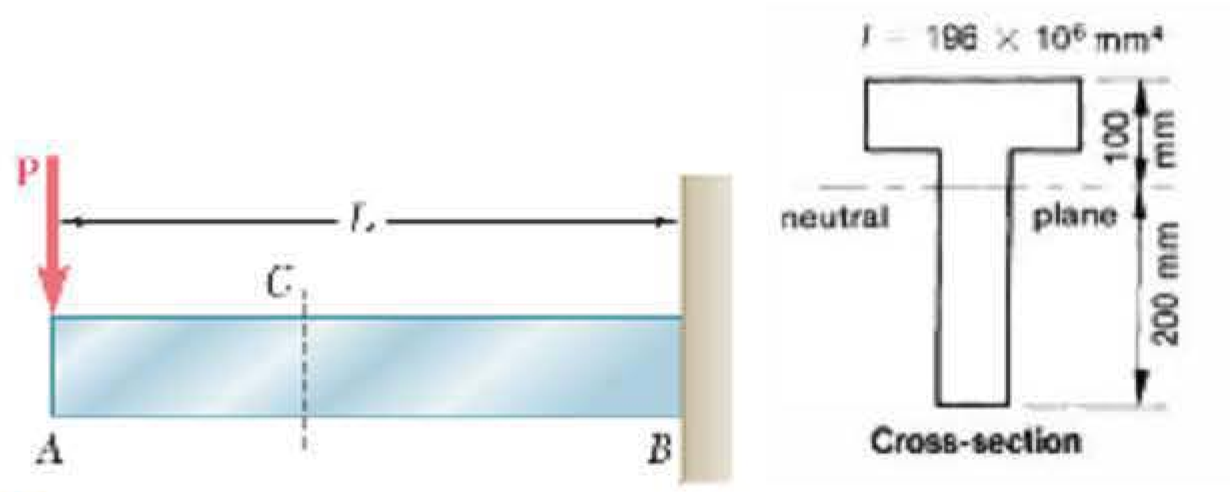
Solution

$$
\begin{aligned}
R &= \frac{EI}{M} \\
&= \frac{23{,}000 \text{ MPa} \times 196 \times 10^{6} \text{ mm}^{4}}{49 \times 10^{6} \text{ N.mm}} \\
&= 92{,}000 \text{ mm} \\
\therefore R &= 92 \text{ m}
\end{aligned}
$$

This radius is still very large but even concrete can be bent without cracking where the bending is very slight.

< BACK GIVE FEEDBACK OK

Determine the radius of curvature (in metres) at the point of maximum bending moment for this beam that has a second moment of area of 196 (x 10^6) mm^4. Use concrete, E = 23,000 MPa. Length is 5 m and load is 13 kN.
(Include units. Use one decimal place.)



Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for t

Factors that cause deflection in beams

Beams are components that endure bending loads.

All beams deflect under load. This deflection is dependent on four factors:

- **Bending moment**, M : An increase in M will *increase* deflection
- **Second moment of area**, I : An increase in I will *decrease* deflection
- **Length of beam**, L : An increase in L will *increase* deflection
- **Modulus**, E : An increase in E will *decrease* deflection

GIVE FEEDBACK

OK

The amount of deflection of a beam under load is dependent on four factors.
Match the increase of each factor with its effect on the amount of beam deflection.

Drag each item into appropriate category.
Click on an item to send it to the back of the stack.

Increase in E modulus of elasticity

Increase deflection	Decrease deflection

Because a beam does not normally bend in a perfectly circular arc, it is not easy to calculate the deflection. This is normally done using calculus, a type of mathematics. We will use pre-calculated formulas for seven common beam loadings.

Groups of beams classified for examining deflection of beams

Various mathematical methods have been used to solve the maximum deflection of certain arrangments of beam bending.

In all cases, deflection is found to be inversely proportional to the radius of curvature where M is dependent on the loads and the length of the beam.

$$\text{Deflection}\,(y) \propto \frac{1}{R} \propto \frac{M}{EI}$$

This means M can vary along the length of the beam. The beam rarely bends in a circular arc but usually in a complicated mathematical curve. This makes deflection (y) quite complex to evaluate, requiring calculus or computer analysis.

Instead of attempting this, we will use solved beam examples which can be viewed here: Beam deflection formulas.

GIVE FEEDBACK

CONTINUE >

Groups of beams classified for examining deflection of beams

In this chapter we examine deflection of beams in the following three groups:

1. Cantilever beams carrying concentrated loads or full-length uniformly distributed loads
2. Simply supported beams with symmetrically located concentrated loads or full-length uniformly distributed loads
3. Simply supported beams with one non-symmetrically located concentrated load

< BACK   GIVE FEEDBACK   OK

Which of the following statements are true regarding bending moment, radius of curvature and deflection?

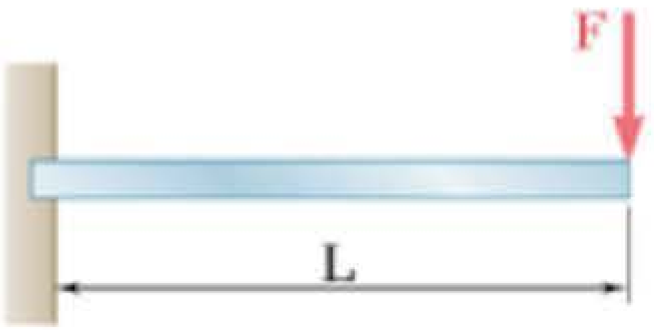
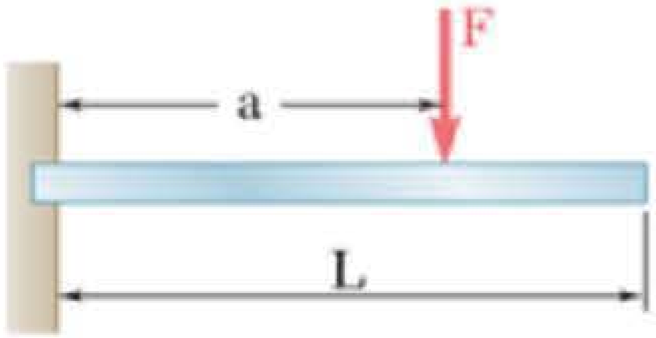
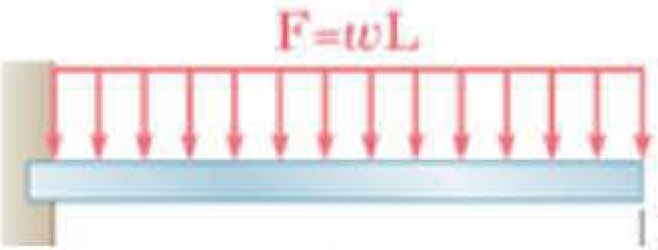
Check all that apply.

- [ ] The radius of curvature varies as bending moment varies along the entire beam
- [ ] The radius of curvature at a point in the beam is inversely proportional to bending moment at that point
- [ ] The deflection at a point in the beam is proportional to the bending moment at that point
- [ ] Radius of curvature is always constant along the entire length of the beam regardless of the loading
- [ ] An elastic beam that bends in a circular arc is under a constant bending moment throughout

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

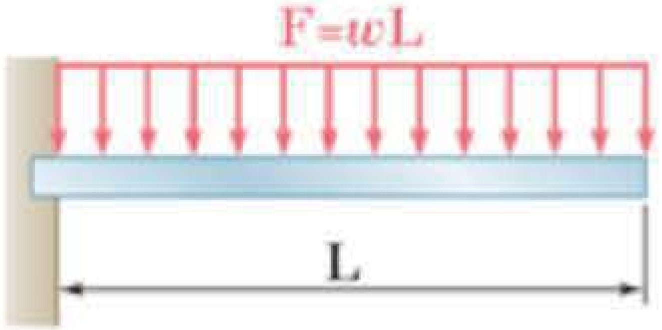
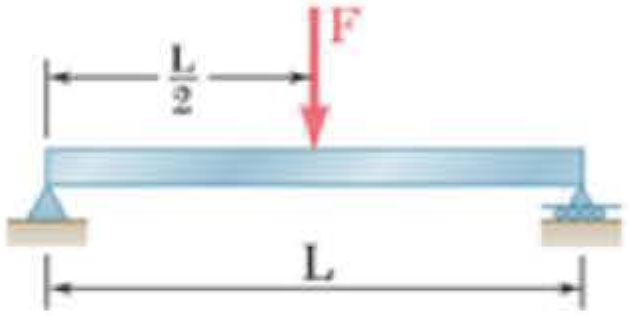
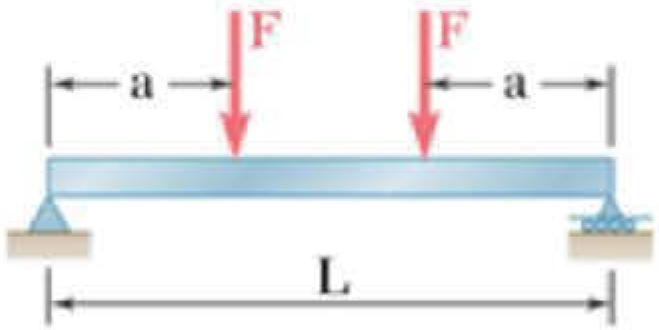
The deflection formulas for the seven types of loaded beams

Case	Beam and load	Maximum deflection	Occurance
1		$y = \dfrac{FL^3}{3EI}$	free end
2		$y = \dfrac{Fa^2(3L - a)}{6EI}$	free end
3		$y = \dfrac{FL^3}{8EI}$	free end

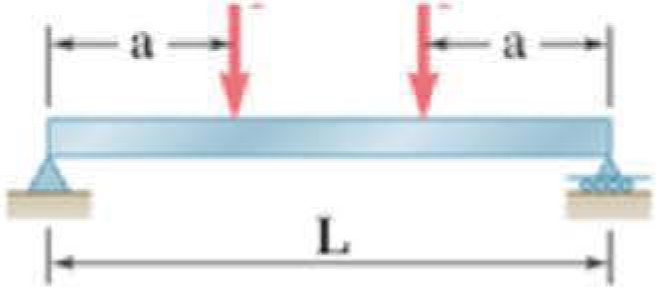
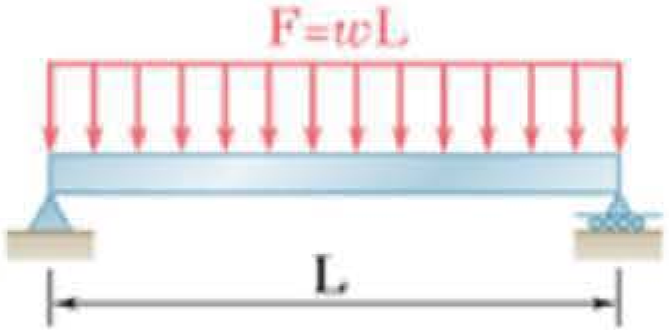
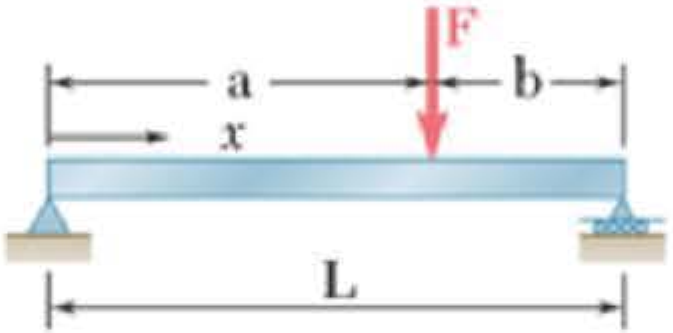
GIVE FEEDBACK

CONTINUE >

The deflection formulas for the seven types of loaded beams

3		$y = \dfrac{FL^3}{8EI}$	free end
4		$y = \dfrac{FL^3}{48EI}$	midspan
5		$y = \dfrac{Fa(3L^2 - 4a^2)}{24EI}$	mid-span

The deflection formulas for the seven types of loaded beams

		$y = \frac{\ldots}{24EI}$	
6		$y = \frac{5FL^3}{384EI}$	mid-span
7		$y = \frac{F a b (a + 2b)\sqrt{3a(a + 2}}{27EIL}$ (where $a > b$)	$x = \frac{\sqrt{a(a + 2b)}}{3}$

GIVE FEEDBACK

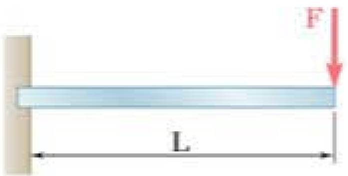
CONTINUE >

Note: To obtain deflection in mm we must use the following units:
F in (N), E in (MPa), I in (mm^4), and L, a and b must be in (mm).

This is a summary of the seven most common beam-deflection formulas. In practice a surprisingly large number of beam-deflection problems encountered by engineers can be solved by the use of these seven formulas.

Additional formulas, which cover more complex cases of irregular loading or a different arrangement of beam supports, can be found in various engineering handbooks.

< BACK GIVE FEEDBACK OK



$$y = \frac{FL^3}{3EI}$$

Doubling the length of the beam will increase the deflection by ______ times.

Click the correct answer.

2

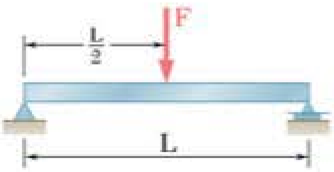
4

8

16

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



$$y = \frac{FL^3}{48\,EI}$$

Doubling the length of this beam will increase the deflection by ______ times.

Click the correct answer.

2

4

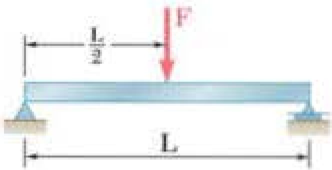
8

16

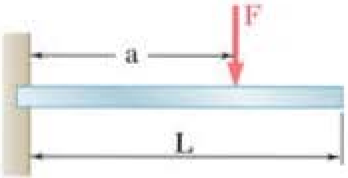
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

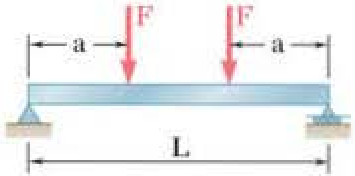
(A)



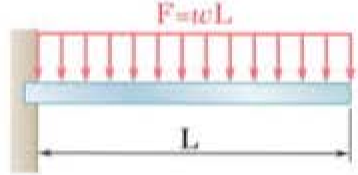
(B)



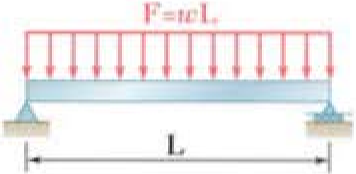
(C)



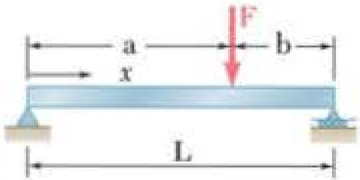
(D)



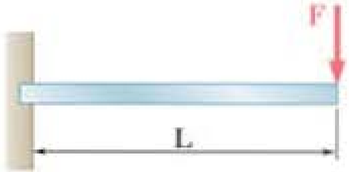
(E)



(F)



(G)



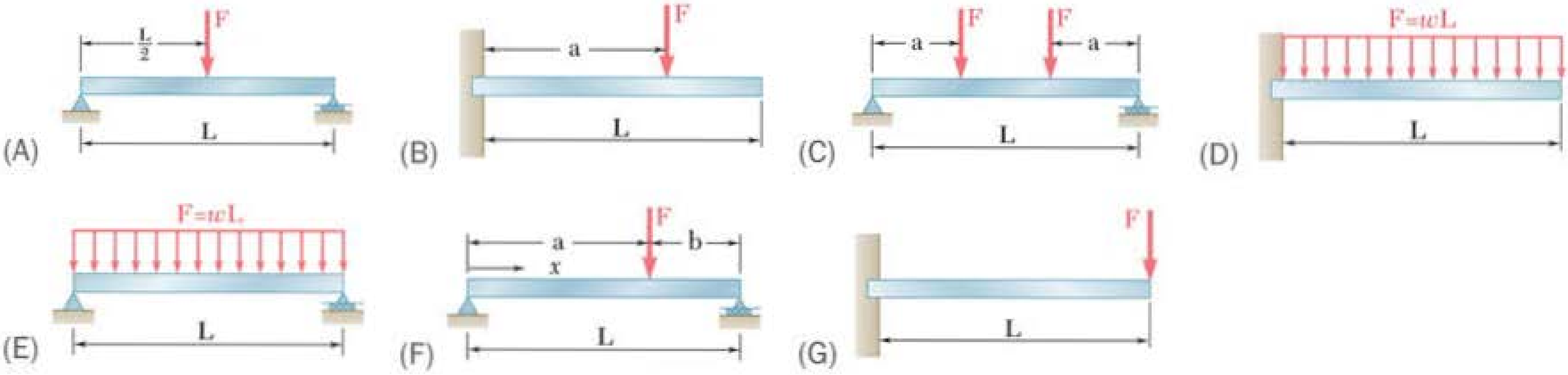
Select all beams where maximum deflection is mid-span.

Click to highlight.

A, B, C, D, E, F, G

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



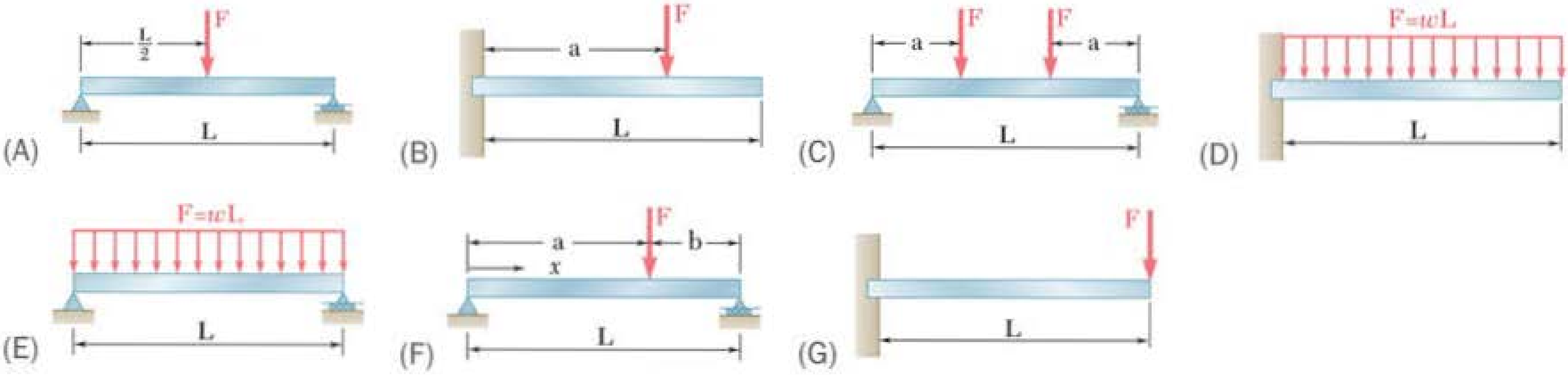
Select all uniformly loaded beams.

Click to highlight.

A, B, C, D, E, F, G

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



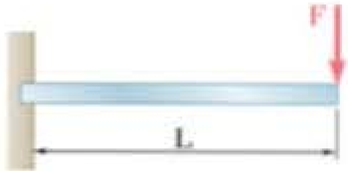
Select all hogging beams.

Click to highlight.

A, B, C, D, E, F, G

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



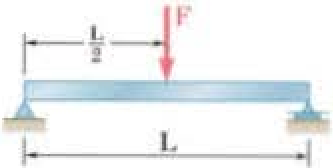
$$y = \frac{FL^3}{3EI}$$

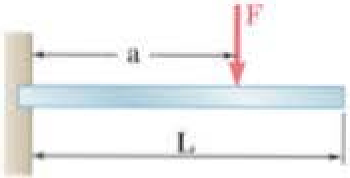
Interpreting this beam deflection formula:

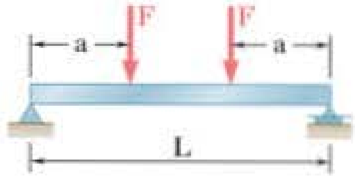
Drag each item into appropriate category.
Click on an item to send it to the back of the stack.

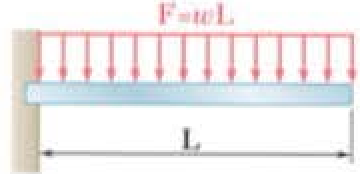
Length is increased

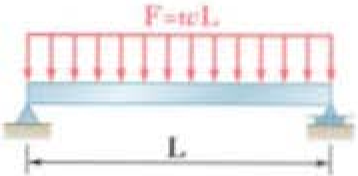
Deflection decreases when	Deflection increases when

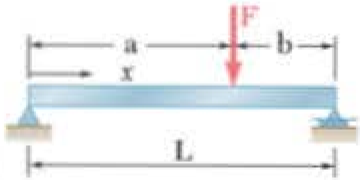
(A) 

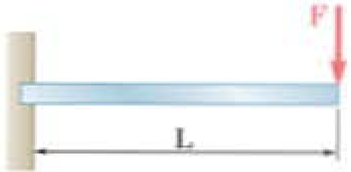
(B) 

(C) 

(D) 

(E) 

(F) 

(G) 

Which of the beam examples could be used to calculate deflection of the milling cutter shown?
(Assume the milling cutter is machining on the side of the cutter, near the tip.)

Click to highlight.

A, B, C, D, E, F, G

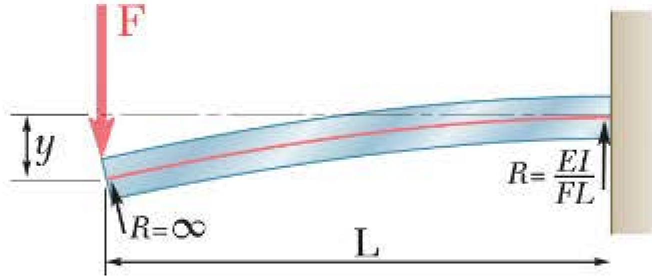
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A cantilever beam will always have the maximum deflection at the end furthest from the wall.

Where the maximum deflection occurs in a cantilever beam

Cantilever beams are represented in Beam deflection formulas by the loading diagrams and formulas in cases 1, 2 and 3.

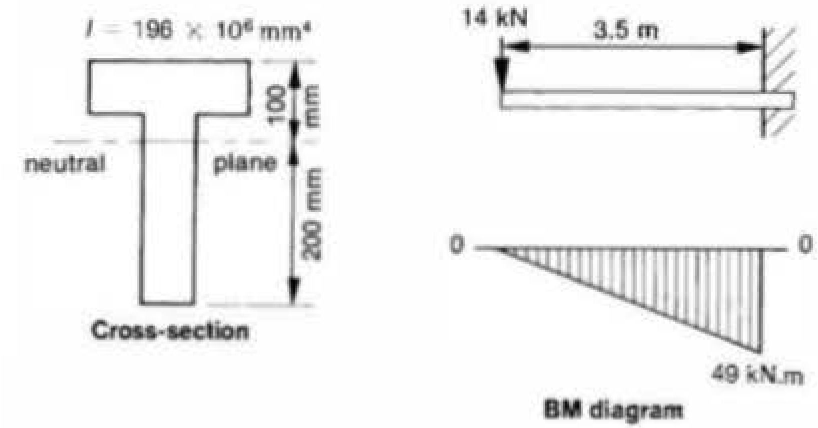


With vertical downward loads, deflection will always be downwards and maximum at the free end of a cantilever beam.
Following inversely to the bending moment, the radius of curvature is tightest at the wall and increases to infinity at the end of the beam where bending moment is zero.
If force F was upwards, the deflection would also be upwards.

GIVE FEEDBACK

OK

Which of the following statements interpret this beam?

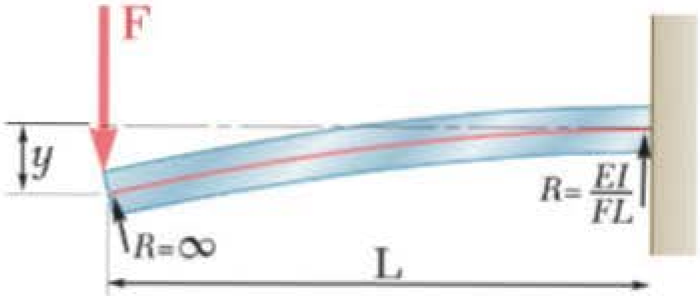


Check all that apply.

- [ ] Bending moment is zero where the 14 kN is applied
- [ ] The smallest radius of curvature occurs at the wall
- [ ] The bending moment is negative which means the beam is in hogging
- [ ] The highest stress is in the top of the beam

Do you know the answer?

Why is the radius infinite at the end of the beam?



Click the correct answer.

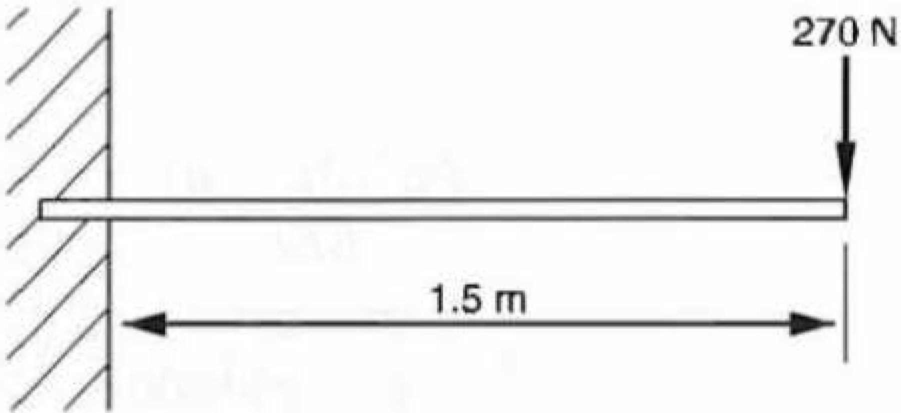
- Bending moment is zero
- Shear force is zero
- Deflection is zero
- Radius of curvature is zero

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

A steel rod ($E = 200{,}000$ MPa), 1.5 m long and 50 mm in diameter, is loaded with a single concentrated load of 270 N at its free end as shown below. Calculate the maximum deflection due to the applied load. The weight of the rod itself may be neglected.



GIVE FEEDBACK

CONTINUE >

Solution

Here we have:

$$F = 270\text{ N}$$
$$L = 1{,}500\text{ mm}$$
$$E = 200{,}000\text{ MPa}$$

and:

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 50^4}{64} = 306.8 \times 10^3\text{ mm}^4$$

Substitute into the formula for the maximum deflection of a cantilever beam with a concentrated load at the free end (Case 1 in Beam deflection formulas):

$$
\begin{aligned}
y &= \frac{FL^3}{3EI} \\
&= \frac{270 \times 1{,}500^3}{3 \times 200{,}000 \times 306.8 \times 10^3} \\
&= 4.95 \text{ mm}
\end{aligned}
$$

This is the amount of downward deflection of the free end of the rod.

< BACK GIVE FEEDBACK OK

A steel rod ($E = 200000$ MPa), 1.5 m long and 45 mm in diameter, is loaded with a single concentrated load of 270 N at its free end. Calculate the maximum deflection.

(Include units. Use two decimal places. Ignore the weight of the rod.)

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π mm

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

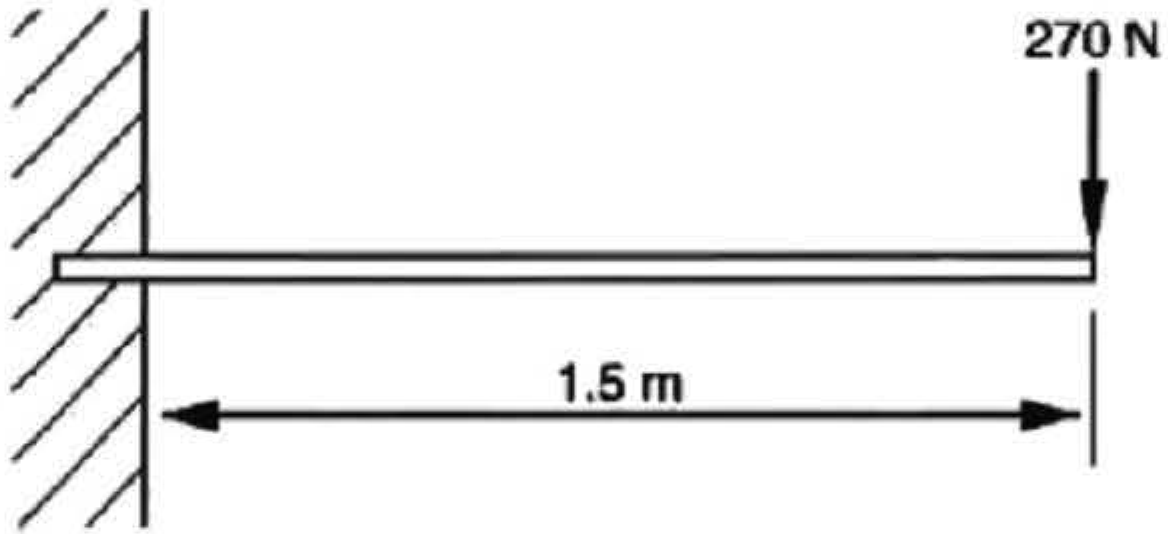
Hint

Each hint will reduce the credit received for this question

Match the values for each variable in the equation for maximum deflection, *y*.

$$y = \frac{F \cdot L^3}{3 \cdot E \cdot I}$$

Use steel, modulus 200 GPa and a beam second moment of area 307 (x 10^3) mm^4.

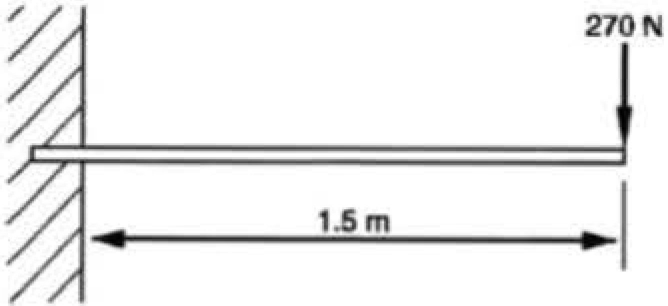


Drag statements on the right to match the left.

F	307,000
L	270
E	200,000
I	1500

Which equation correctly determines the maximum deflection (y)?

Use steel, modulus 200 GPa and beam second moment of area 307 (x 10^3) mm^4.



Click the correct answer.

$$\frac{270 \cdot 1{,}500^3}{3 \cdot 200{,}000 \cdot 307{,}000}$$

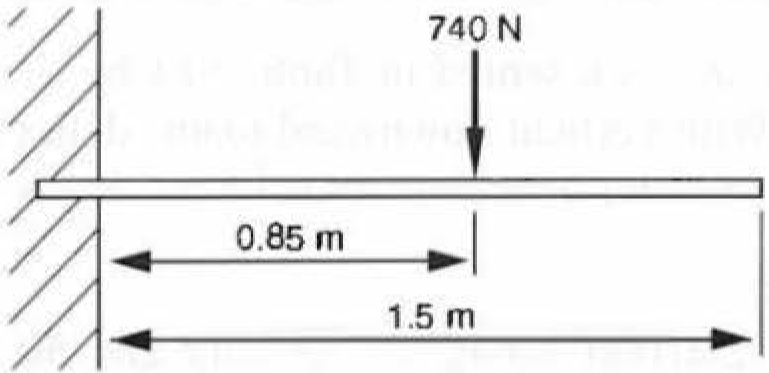
$$\frac{270 \cdot 1.5^3}{3 \cdot 200{,}000 \cdot 307{,}000}$$

$$\frac{270 \cdot 1.5^3}{3 \cdot 200 \cdot 307{,}000}$$

$$\frac{270 \cdot 1.5^3}{3 \cdot 200 \cdot 307}$$

Example

If the steel rod below ($E = 200{,}000$ MPa, $L = 1{,}500$ mm and $I = 306.8 \times 10^3$ mm^4) carries a single load of 740 N located 0.85 m from the support, what is the maximum deflection? Ignore the weight of the rod.



GIVE FEEDBACK

CONTINUE >

Solution

Substitute into the formula for the maximum deflection of a cantilever beam with a concentrated load located at a specified distance ($a = 850$ mm) from the fixed support (Case 2 in Beam deflection formulas):

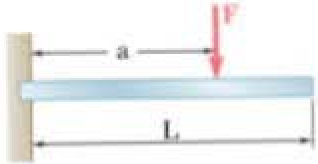
$$
\begin{aligned}
y &= \frac{F a^2 (3L - a)}{6EI} \\
&= \frac{740 \times 850^2 \times (3 \times 1{,}500 - 850)}{6 \times 200{,}000 \times 306.8 \times 10^3} \\
&= 5.3 \text{ mm}
\end{aligned}
$$

This is the amount of maximum deflection of the rod which occurs at its free end.

< BACK GIVE FEEDBACK OK

A steel rod (E = 200000 MPa), 1.5 m long and 45 mm in diameter, is loaded with a single concentrated load of 1750 N at 0.5 m from the wall. The previously calculated I = 201289 mm^4. Calculate the maximum deflection in mm.

(Include units. Use two decimal places. Ignore the weight of the rod.)



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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

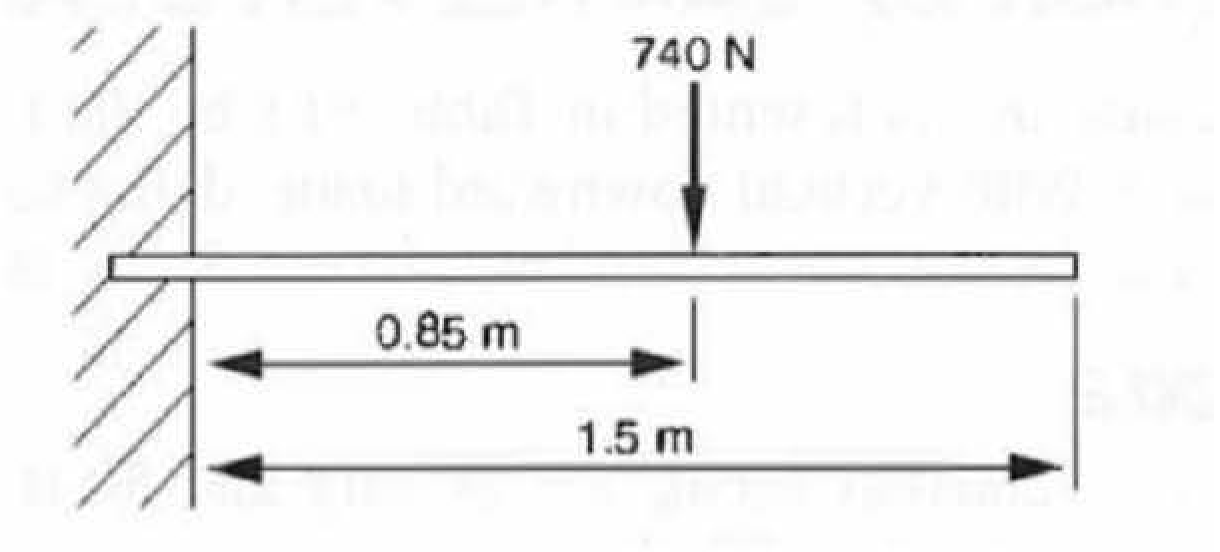
Hint

Each hint will reduce the credit received for this question

Match the values for each variable in the equation for maximum deflection *y*.

$$y = \frac{F \cdot a^2(3L - a)}{6 \cdot E \cdot I}$$

Use aluminium, modulus 70 GPa and a beam second moment of area 307 (x 10^3) mm^4.



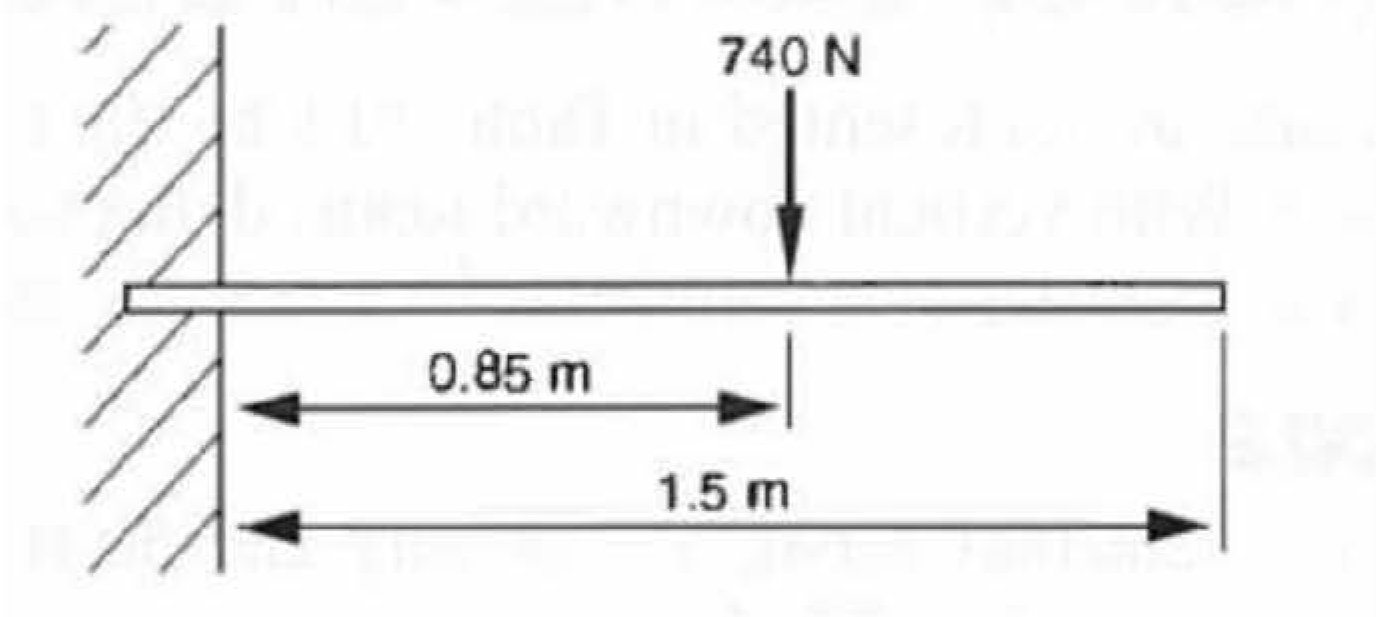
Drag statements on the right to match the left.

F	307,000
a	740
L	850
E	1500
I	70,000

Match the units for each variable in the equation for maximum deflection, *y*.

$$y = \frac{F \cdot a^2(3L - a)}{6 \cdot E \cdot I}$$

Use aluminium, modulus 70 GPa and a beam second moment of area 307 (x 10^3) mm^4.



Drag statements on the right to match the left.

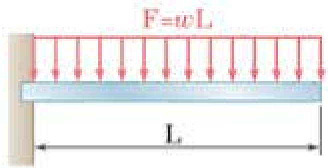
mm	*I*
mm	*L*
N	*E*
MPa	*F*
mm^4	*a*

Example

What is the deflection of the 50mm diameter steel rod with a length of 1.5m?

Assume $E = 200{,}000\ \mathrm{MPa}$ and $I = 306.8 \times 10^3\ \mathrm{mm}^4$.

Hint: The beam is under its own weight but without any other loads. Assume density of steel is $7{,}800\ \mathrm{kg/m}^3$.



GIVE FEEDBACK

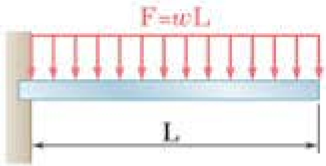
CONTINUE >

Example

What is the deflection of the 50mm diameter steel rod with a length of 1.5m?

Assume $E = 200{,}000\ \mathrm{MPa}$ and $I = 306.8 \times 10^3\ \mathrm{mm}^4$.

Hint: The beam is under its own weight but without any other loads. Assume density of steel is $7{,}800\ \mathrm{kg/m}^3$.



Solution

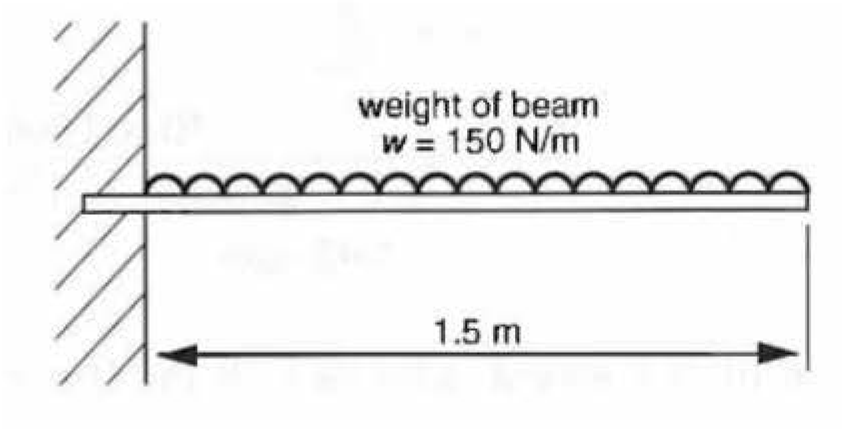
Calculate the weight of the rod:

$$\begin{aligned} F_w &= m\,g \\ &= \frac{\pi \times 0.05^2}{4} \times 1.5 \times 7{,}800 \times 9.81 \\ &= 225.4\ \mathrm{N} \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

Find maximum deflection of cantilever beam with uniformly distributed load over its entire length—Example 2/3

For a rod of constant diameter, the weight is distributed uniformly along its length L. (The equivalent intensity of load distribution in this case is $w = 225.4$ N per 1.5 m, which is 150 N/m) Therefore we have an example of a cantilever beam with a uniformly distributed load over its entire length (Case 3 in Beam deflection formulas):



< BACK GIVE FEEDBACK CONTINUE >

Find maximum deflection of cantilever beam with uniformly distributed load over its entire length—Example 3/3

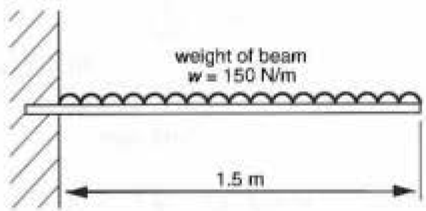
However, the formula for deflection uses the total weight $F_w = 225\ \mathrm{N}$, not the distributed weight $F = 150\ \mathrm{N/m}$.

Choose the appropriate formula and substitute:

$$
\begin{aligned}
y &= \frac{FL^3}{8EI} \\
&= \frac{225.4 \times 1{,}500^3}{8 \times 200{,}000 \times 306.8 \times 10^3} \\
&= 1.55\ \mathrm{mm}
\end{aligned}
$$

This is the amount of deflection at the free end of the rod caused by its own weight.

< BACK GIVE FEEDBACK OK



Maximum deflection $y = \frac{FL^3}{8EI}$

This beam has a second moment of area of 307 x 10^3 mm^4.
The deflection of this steel beam will be calculated in mm using the formula above if we use numerical values as follows:
F is (please select) ▼, L is (please select) ▼,
E is (please select) ▼, and I is (please select) ▼.

Submit

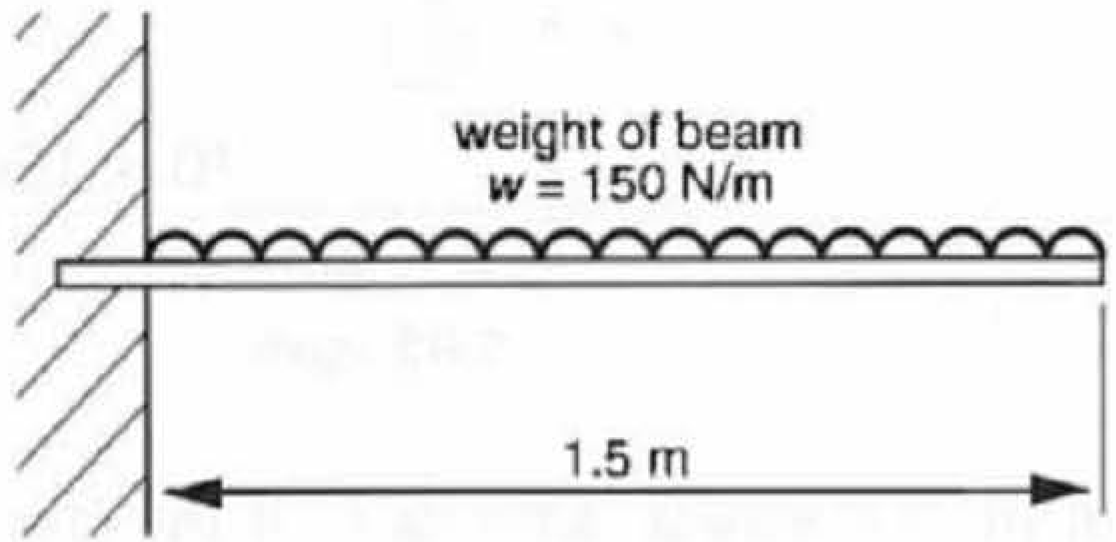
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the values for each variable in the equation for maximum deflection, y.

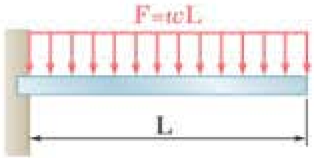
$$y = \frac{F \cdot L^3}{8 \cdot E \cdot I}$$

Use steel, modulus 200 GPa and a beam second moment of area 307 (x 10^3) mm^4.



Drag statements on the right to match the left.

F	F = 150/1000*1.5*1000
L	307 x 1000
E	1.5 x 1000
I	200 x 1000



A steel rod (E = 200,000 MPa, L = 2.5 m, D = 45 mm and I = 201289 mm^4) is loaded by its own weight, without any other loads. Assume density of steel is 7800 kg/m^3. The calculated weight of this rod is 304.2 N.

Calculate the maximum deflection.

(Include units. Use two decimal places.)

+ − · ÷ mm Clear Clear line ? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
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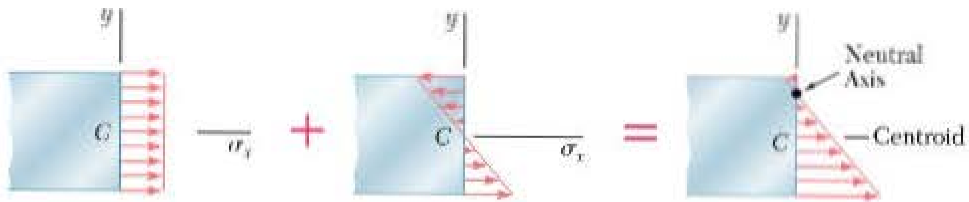
Hint Each hint will reduce the credit received for this question

Some questions will not match a deflection case from our formula sheet but we may be able to combine several formulas to match the question. Adding the forces together will also add the deflections together. This is called the principle of superposition.

Previously we combined two separate loadings by simply adding their stresses together.



Pure tension + pure bending = tension bending

This is known as the **principle of superposition.**

The resultant effect of several loads acting on a beam is the sum of the contributions from each of the loads applied individually.

GIVE FEEDBACK

CONTINUE >

We can do the same with deflections provided the beam remains elastic. The deflections at the same point on the beam may be computed separately and then added together.

This allows us to combine multiple cases in the Beam deflection formulas, making sure we calculate deflection at the same point. For cantilevers (Cases 1, 2, 3) this is the end of the beam. For simply supported beams (Cases 4, 5, 6) it is the middle.

Note: Case 7 has a special location for maximum deflection, so it is the odd one out and cannot be combined with Cases 4, 5 and 6.

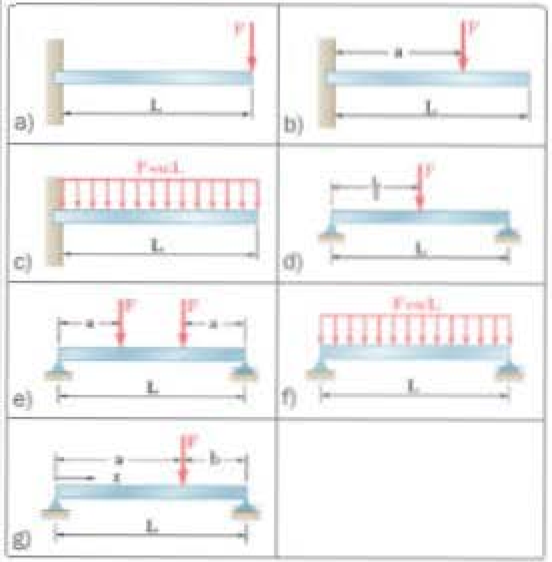
We want to use the principle of superposition to solve a complex beam-bending problem.
For each separate deflection calculation, which of the following requirements are compulsory?

Check all that apply.

- [ ] Beams must be the same material
- [ ] Loads on each beam must be the same
- [ ] Length of each beam must be the same
- [ ] Beams must have the same cross-section
- [ ] Deflection must be measured at the same point in the beam
- [ ] Deflections of each beam must be the same

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



SMALL MEDIUM LARGE

Match each diagram to the correct equation for maximum deflection.
Refer to the Beam deflection formulas.

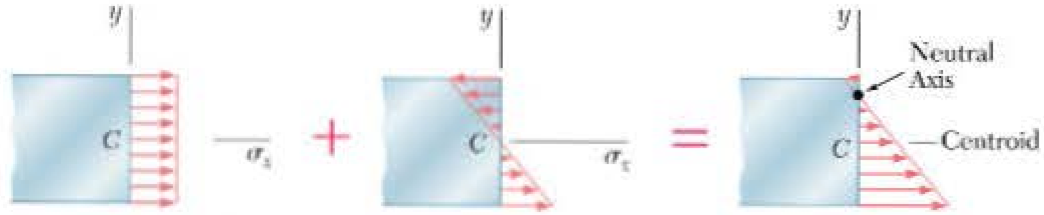
Drag statements on the right to match the left.

a)	$y = \frac{FL^3}{3EI}$
b)	$y = \frac{Fa^2(3L - a)}{6EI}$
c)	$y = \frac{FL^3}{8EI}$
d)	$y = \frac{FL^3}{48EI}$
e)	$y = \frac{Fa(3L^2 - 4a^2)}{24EI}$
f)	$y = \frac{5FL^3}{384EI}$
g)	$y = \frac{Fab(a + 2b)\sqrt{3a(a + 2b)}}{27EIL}$

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

We can combine two separate loadings by adding their stresses together.



Pure tension + pure bending = tension bending

This is known as the principle of ______.

Click the correct answer.

superposition

moments

equilibrium

bending

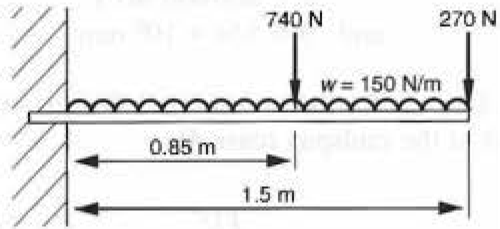
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The principle of superposition works in this case because each of the previous questions have maximum deflection at the same location—the free end of the beam.

Example

Find the maximum deflection of the steel rod ($E = 200{,}000$ MPa, 1.5 m long and 50 mm in diameter) with two concentrated loads acting on the rod, including the weight of the beam itself (150 N/m):



Finding second moment of area (I):

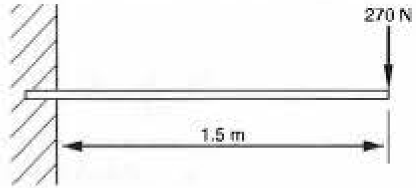
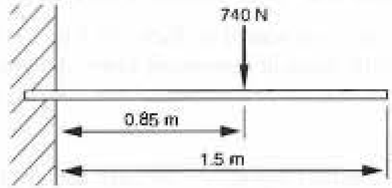
$$I = \frac{\pi D^4}{64} = \frac{\pi \times 50^4}{64} = 306.8 \times 10^3 \text{ mm}^4$$

GIVE FEEDBACK

CONTINUE >

Solution

By superposition of three cases for bending of a cantilever:

$y_1 = \dfrac{FL^3}{3EI}$ $= \dfrac{270 \times 1{,}500^3}{3 \times 200{,}000 \times 306.8 \times 10^3}$ $= 4.95$ mm	
$y_2 = \dfrac{Fa^2(3L - a)}{6EI}$ $= \dfrac{740 \times 850^2 \times (3 \times 1{,}500 - 850)}{6 \times 200{,}000 \times 306.8 \times 10^3}$ $= 5.3$ mm	

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GIVE FEEDBACK
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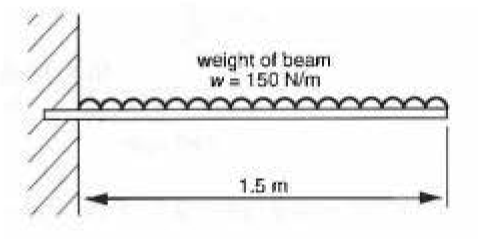
Determine the weight of the beam (F_w), then use equation for distributed load on a cantilever.	$F_w = m\,g$ $= \dfrac{\pi \times 0.05^2}{4} \times 1.5 \times 7{,}800 \times 9.81$ $= 225.4\ \mathrm{N}$
$y_3 = \dfrac{FL^3}{8\,EI}$ $= \dfrac{225.4 \times 1{,}500^3}{8 \times 200{,}000 \times 306.8 \times 10^3}$ $= 1.55\ \mathrm{mm}$	

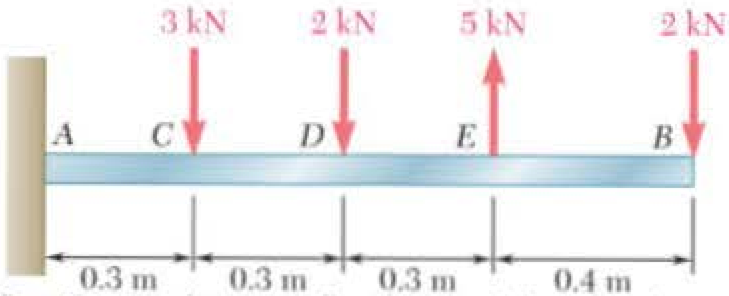
< BACK GIVE FEEDBACK CONTINUE >

This total downward deflection of the free end of the rod is the sum of the separate loads:

Deflection due to 270 N load: $y_1 = 4.95\ \mathrm{mm}$
Deflection due to 740 N load: $y_2 = 5.3\ \mathrm{mm}$
Deflection due to own weight: $y_3 = 1.55\ \mathrm{mm}$

Therefore the deflection of the free end of the rod under combined loading conditions, including its own weight, is found by summation of the individual results:

$$\begin{aligned} y &= 4.95 + 5.3 + 1.55 \\ &= 11.8\ \mathrm{mm} \end{aligned}$$



Can the maximum deflection for this beam be solved using the deflection formulas? Why or why not?

Click the correct answer.

Yes, by combining Case 1 and Case 2

No, the formulas only work if the forces are downward

Yes, since the forces are evenly spaced they can be treated as a distributed load

No, there are more than three forces

Do you know the answer?

I KNOW IT

THINK SO

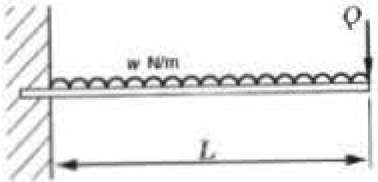
UNSURE

NO IDEA

A steel rod (E = 200,000 MPa, L = 2.5 m, D = 50 mm and I = 306796 mm^4) weighs 375.6 N. The deflection at the end of the beam is 12 mm due to the beam weight, and 23.3 mm due to the 275 N force Q.

Calculate the total deflection at the end of the beam.

(Include units. Use two decimal places.)



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Click and type your answer here

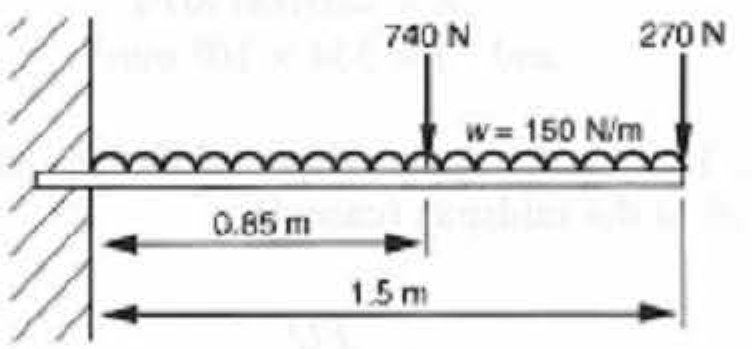
CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

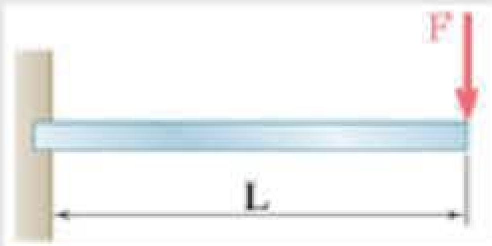
Hint Each hint will reduce the credit received for this question

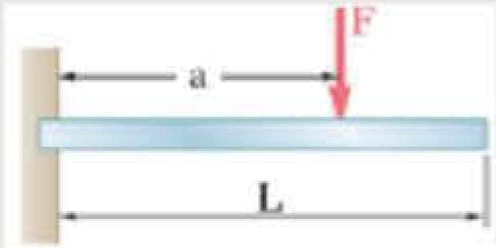
If you were being asked to determine the maximum deflection of this cantilever beam shown below;

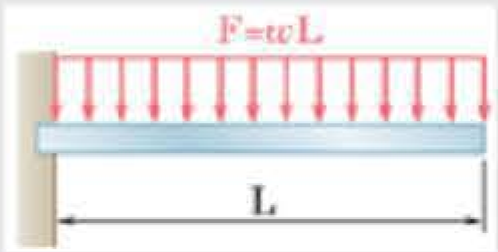


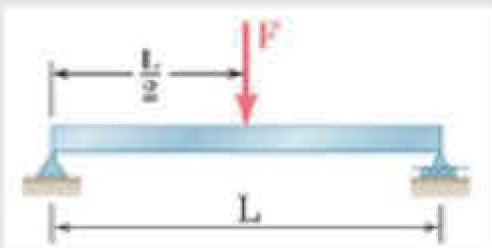
Select the appropriate bending deflection cases for solving this problem by **superposition.**

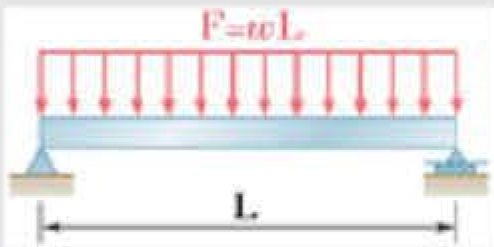
Check all that apply.

- [ ] 

- [ ] 

- [ ] 

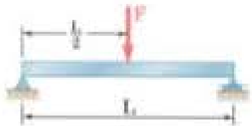
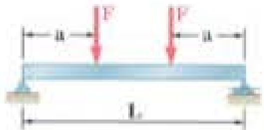
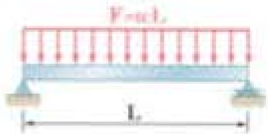
- [ ] 

- [ ] 

These simply supported beams all have maximum deflection at mid-span. Since these mid-span locations match, they can be combined with each other using superposition.

Where maximum deflection occurs in a symmetrically loaded, simply supported beam

Simply supported beams with symmetrical loading are represented in the [Beam deflection formulas] by the loading diagrams and formulas in Cases 4, 5 and 6:

4		$y = \dfrac{FL^3}{48\,EI}$	mid-span
5		$y = \dfrac{Fa(3L^2 - 4a^2)}{24\,EI}$	mid-span
6		$y = \dfrac{5FL^3}{384\,EI}$	mid-span

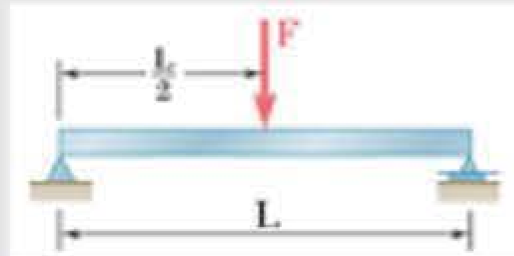
With vertical downward loads, deflection is always downwards and maximum deflection is at the mid-span.

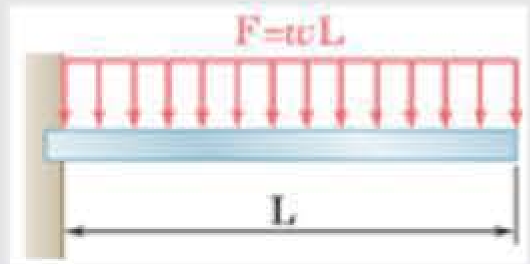
GIVE FEEDBACK

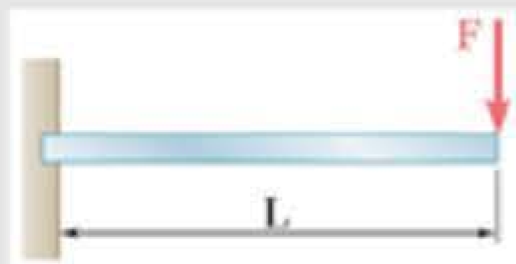
OK

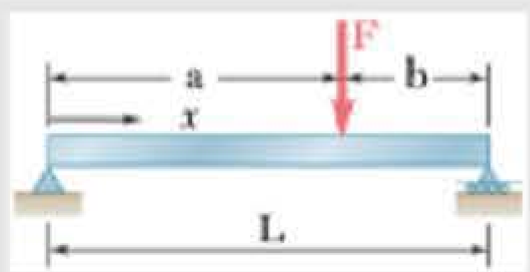
Select all simply supported beams.

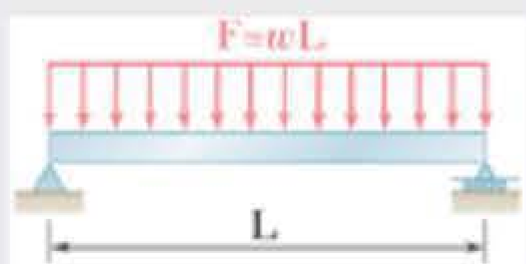
Check all that apply.

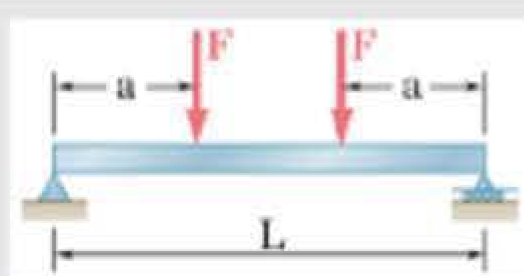
- [ ] 

- [ ] 

- [ ] 

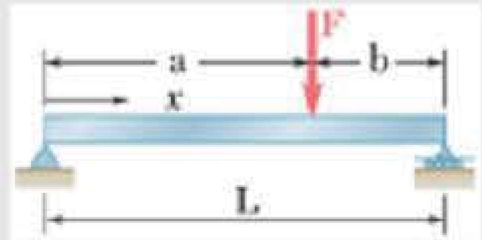
- [ ] 

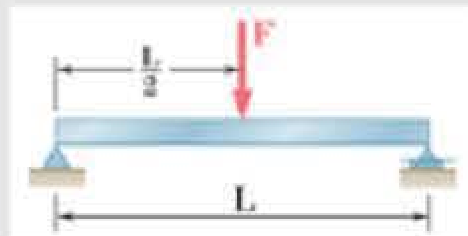
- [ ] 

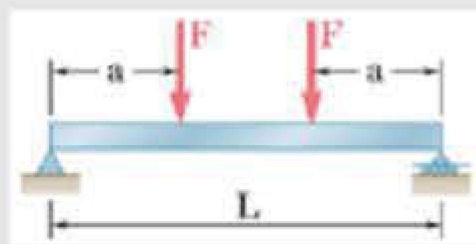
- [ ] 

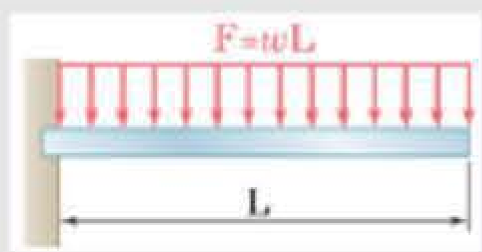
Select all symmetrically loaded, simply supported beams.

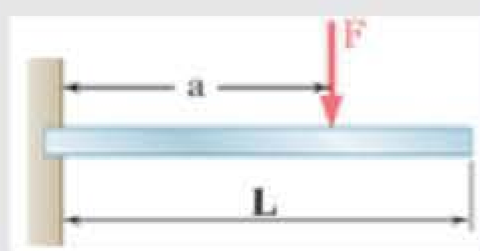
Check all that apply.

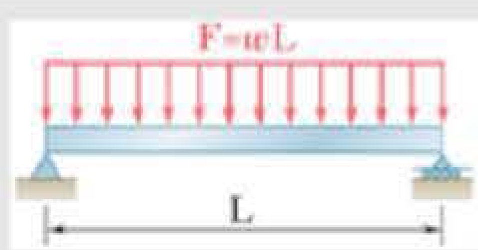
- [ ] 

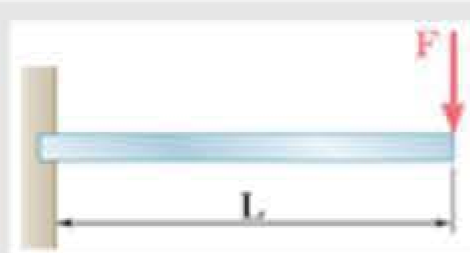
- [ ] 

- [ ] 

- [ ] 

- [ ] 

- [ ] 

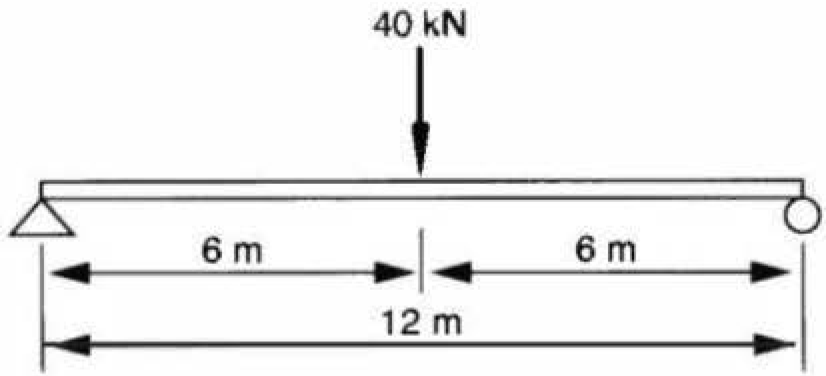
- [ ] 

Here is a deflection calculation for a simply supported beam. For these beams the maximum deflection is mid-span, six metres from the end.

Calculate deflection at midspan of symmetrically loaded, simply supported beam with concentrated load—Example

Example

A steel structural beam ($E = 200{,}000\ \mathrm{MPa}, I = 554 \times 10^{6}\ \mathrm{mm}^{4}$) is 12 m long between simple supports at each end and carries a concentrated load of 40 kN at its mid-span point, as shown below. Calculate the deflection due to this load.



GIVE FEEDBACK

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Solution

Here we have:

$$\begin{aligned} F &= 40{,}000 \text{ N} \\ L &= 12{,}000 \text{ mm} \\ E &= 200{,}000 \text{ MPa} \\ I &= 554 \times 10^{6} \text{ mm}^{4} \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

Calculate deflection at midspan of symmetrically loaded, simply supported beam with concentrated load—Example 3/3

Substitute into the formula for the maximum deflection of a simply supported beam with a concentrated load at the mid-span (Case 4 [Beam deflection formulas]):

$$
\begin{aligned}
y &= \frac{FL^3}{48EI} \\
&= \frac{40{,}000 \times 12{,}000^3}{48 \times 200{,}000 \times 554 \times 10^6} \\
&= 13 \text{ mm}
\end{aligned}
$$

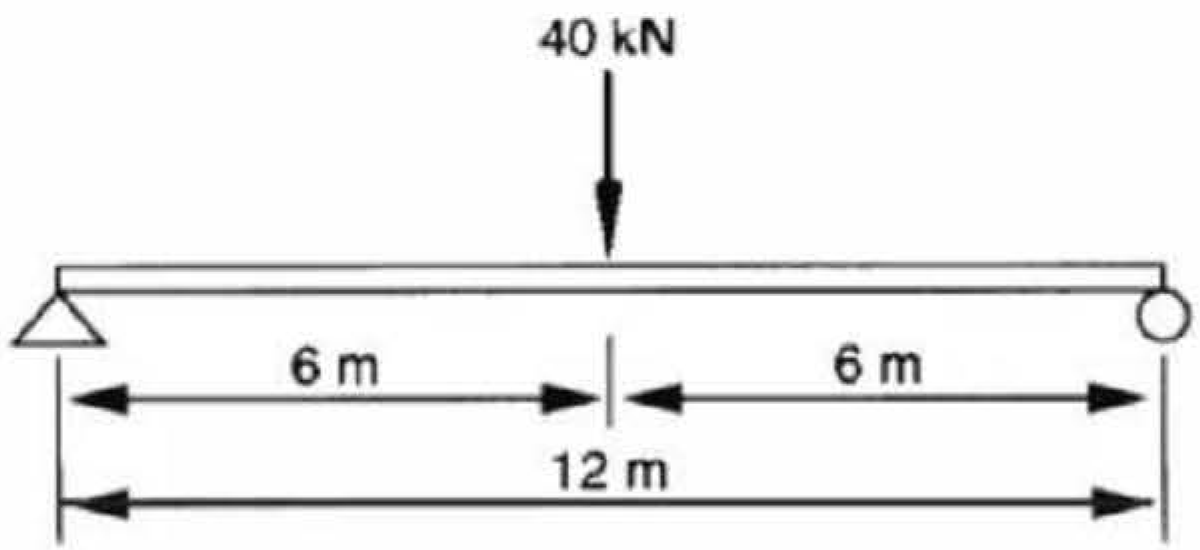
This is the maximum deflection at the mid-point.

< BACK GIVE FEEDBACK OK

Match the values for each variable in the equation for maximum deflection *y*.

$$y = \frac{F \cdot L^3}{48 \cdot E \cdot I}$$

Use steel, modulus 200 GPa and a beam second moment of area 554 (x 10^6) mm^4.

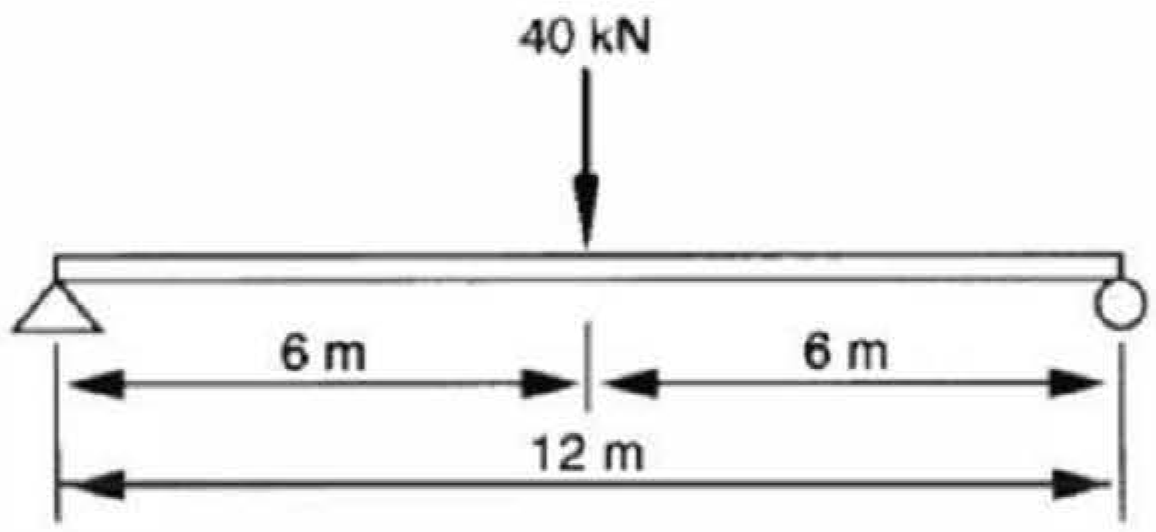


Drag statements on the right to match the left.

F	12 000
L	40 000
E	554 000 000
I	200 000

Which equation correctly determines the maximum deflection *y*?

Use steel, modulus 200 GPa and beam second moment of area 554 (x 10^6) mm^4.



Click the correct answer.

$$y = \frac{40{,}000 \cdot 12{,}000^3}{48 \cdot 200{,}000 \cdot 554{,}000{,}000}$$

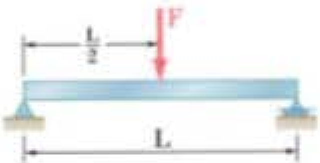
$$y = \frac{40 \cdot 12^3}{48 \cdot 200 \cdot 554}$$

$$y = \frac{40 \cdot 12^3}{48 \cdot 200 \cdot 554{,}000{,}000}$$

$$y = \frac{40{,}000 \cdot 12^3}{48 \cdot 200{,}000 \cdot 554{,}000{,}000}$$

A simply supported steel beam (I = 554 x 10^6 mm^4, E = 200 GPa) is 12 m long and carries a concentrated load of 40 kN at mid-span. Calculate max deflection in mm.

(Include units. Use minimum 2 decimal places)



± ▾ | $\frac{\square}{\square}$ | $\square^2$ ▾ | $\sqrt{\square}$ | $(\square)$ ▾ | Clear

≤ ▾ | π | $\square \times 10^{\square}$ | mm | ▾ | $\overline{\square}$ | Clear line

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Click and type your answer here. Show your work.

CHALLENGE | SUBMIT | SHOW ANSWER

INSTRUCTIONS

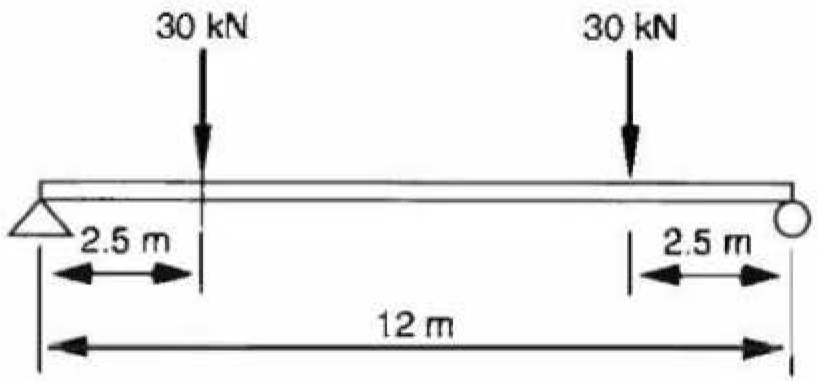
- You must show intermediate steps for full credit, one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Example

If a steel beam ($E = 200{,}000\ \mathrm{MPa}$, $L = 12{,}000\ \mathrm{mm}$, and $I = 554 \times 10^6\ \mathrm{mm}^4$) carries two symmetrically located concentrated loads of 30 kN each, located 2.5 m from the supports as shown below, what is the maximum deflection due to this loading?



GIVE FEEDBACK

CONTINUE >

Solution

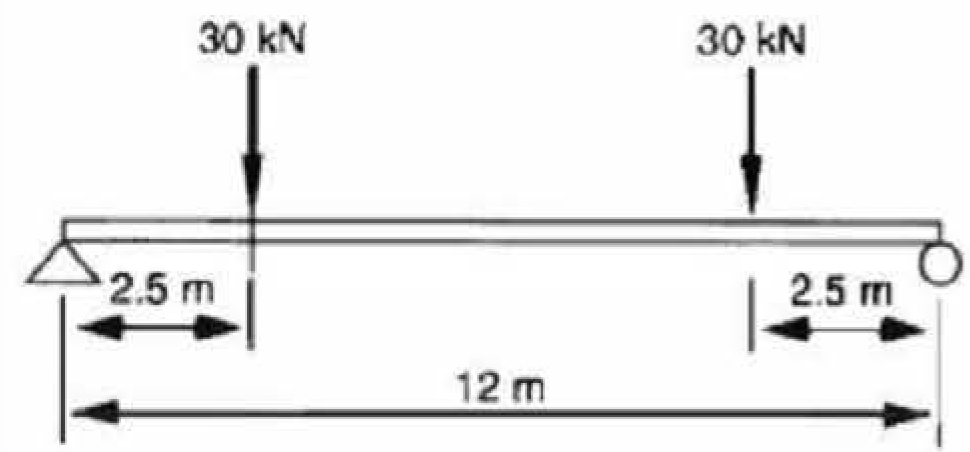
Substitute into the equation for Case 5 (two symmetrical concentrated loads, see Beam deflection formulas):

$$\begin{aligned} y &= \frac{F\,a\,(3L^2 - 4a^2)}{24EI} \\ &= \frac{30{,}000 \times 2{,}500 \times (3 \times 12{,}000^2 - 4 \times 2{,}500^2)}{24 \times 200{,}000 \times 554 \times 10^6} \\ &= 11.5\ \text{mm} \end{aligned}$$

This is the maximum deflection, which occurs at mid-span.

< BACK GIVE FEEDBACK OK

A simply supported steel beam (I = 554 x 10^6 mm^4) is 12 m long and carries two symmetrically located, concentrated loads of 30 kN at 2.5 m from each end.



Using the equation for maximum deflection $y = \dfrac{F\,a(3L^2 - 4a^2)}{24EI}$, which of the following equations is numerically correct for determining deflection y in mm?

Click the correct answer.

$$y = \frac{30{,}000 \cdot 2{,}500(3 \cdot 12{,}000^2 - 4 \cdot 2{,}500^2)}{24 \cdot 200 \times 10^3 \cdot 554 \times 10^3}$$

$$y = \frac{30{,}000 \cdot 2{,}500(3 \cdot 12{,}000^2 - 4 \cdot 2{,}500^2)}{24 \cdot 200 \times 10^3 \cdot 554 \times 10^6}$$

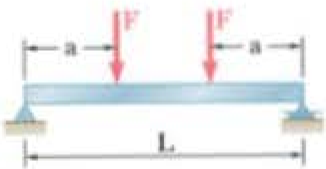
$$y = \frac{30 \cdot 2{,}500(3 \cdot 12{,}000^2 - 4 \cdot 2{,}500^2)}{24 \cdot 200{,}000 \cdot 554 \times 10^6}$$

$$y = \frac{30{,}000 \cdot 2.5(3 \cdot 12^2 - 4 \cdot 2.5^2)}{24 \cdot 200{,}000 \cdot 554 \times 10^6}$$

$$y = \frac{30{,}000 \cdot 2{,}500(3 \cdot 12{,}000^2 - 4 \cdot 2{,}500^2)}{24 \cdot 200 \times 10^6 \cdot 554 \times 10^6}$$

A simply supported steel beam (I = 554 x 10^6 mm^4 and E = 200 GPa) is 12 m long and carries two symmetrically located concentrated loads of 40 kN at 3 mm from each end. Calculate maximum deflection in mm.

(Include units. Use two decimal places.)



± ▾ | $\frac{\square}{\square}$ | $1\frac{2}{3}$ | $\square^2$ ▾ | $\sqrt{\square}$ | (□) ▾ | Clear

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

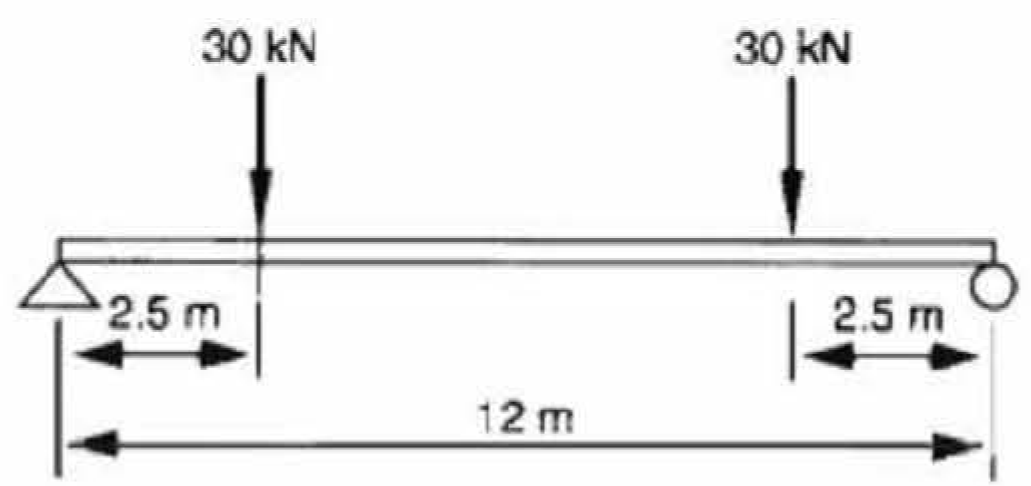
INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

A simply supported steel beam ($I = 554 \times 10^6$ mm^4) is 12 m long and carries two symmetrically located, concentrated loads of 30 kN at 2.5 m from each end.



Using the equation for maximum deflection $y = \dfrac{Fa(3L^2 - 4a^2)}{24EI}$, which of the following variables are numerically correct for determining deflection y in mm?

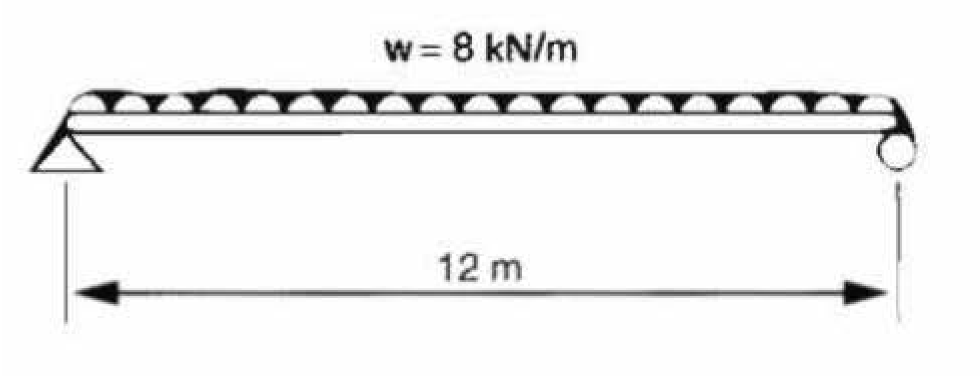
Check all that apply, and then click "Submit answer(s)".

- [ ] $a = 2.5$
- [ ] $E = 200\,000$
- [ ] $L = 12$
- [ ] $F = 30\,000$
- [ ] $I = 554 \times 10^6$

Submit answer(s)

Example

If a simply supported steel beam ($E = 200{,}000\ \mathrm{MPa}$, $L = 12{,}000\ \mathrm{mm}$ and $I = 554 \times 10^6\ \mathrm{mm}^4$) was used to carry a uniformly distributed load of 8 kN/m over its entire length (below), what would be the mid-span deflection?



GIVE FEEDBACK

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Solution

If the load intensity is $w = 8\,\mathrm{kN/m}$, then the total load on the beam is:

$$
\begin{aligned}
f &= 8\,\mathrm{kN/m} \times 12\ \mathrm{m} \\
&= 96\ \mathrm{kN} \\
&= 96{,}000\ \mathrm{N}
\end{aligned}
$$

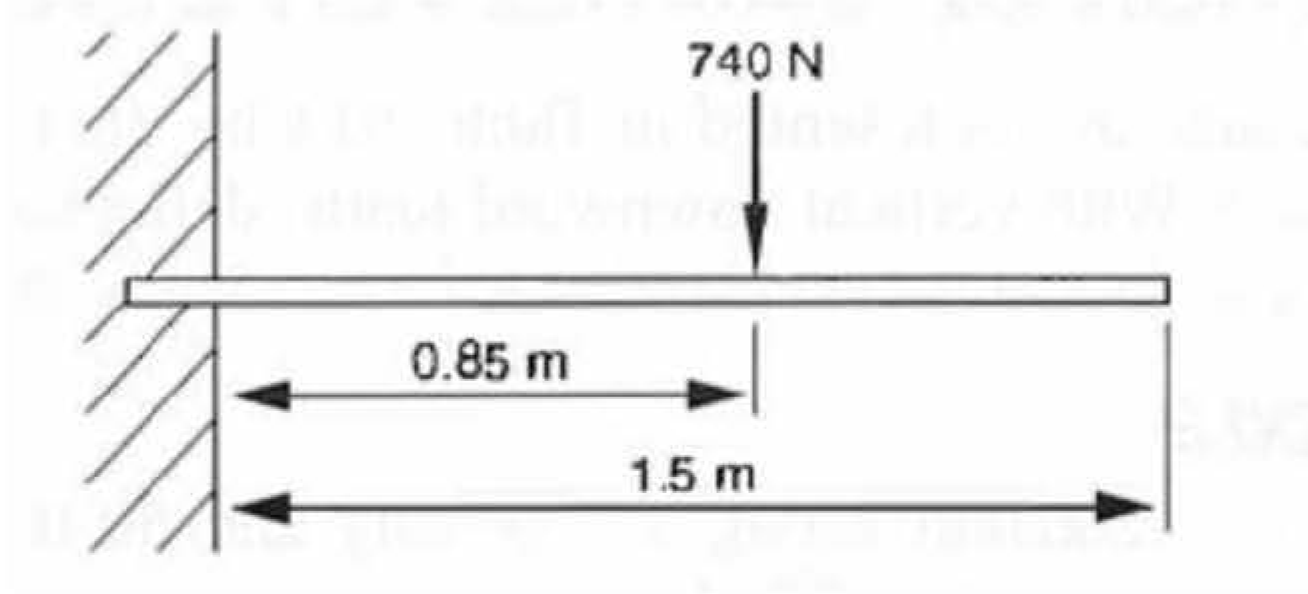
Substitute into the equation for Case 6 (Beam deflection formulas):

$$
\begin{aligned}
y &= \frac{5FL^3}{384EI} \\
&= \frac{5 \times 96{,}000 \times 12{,}000^3}{384 \times 200{,}000 \times 554 \times 10^6} \\
&= 19.5\ \mathrm{mm}
\end{aligned}
$$

Here again, this is the maximum deflection, which occurs at mid-span.

< BACK GIVE FEEDBACK OK

Match the values for each variable in the equation for maximum deflection *y* (in mm).
Use aluminium, modulus 70 GPa and a beam second moment of area 307 (x 10^3) mm^4.



$$y = \frac{F \cdot a^2(3L - a)}{6 \cdot E \cdot I}$$

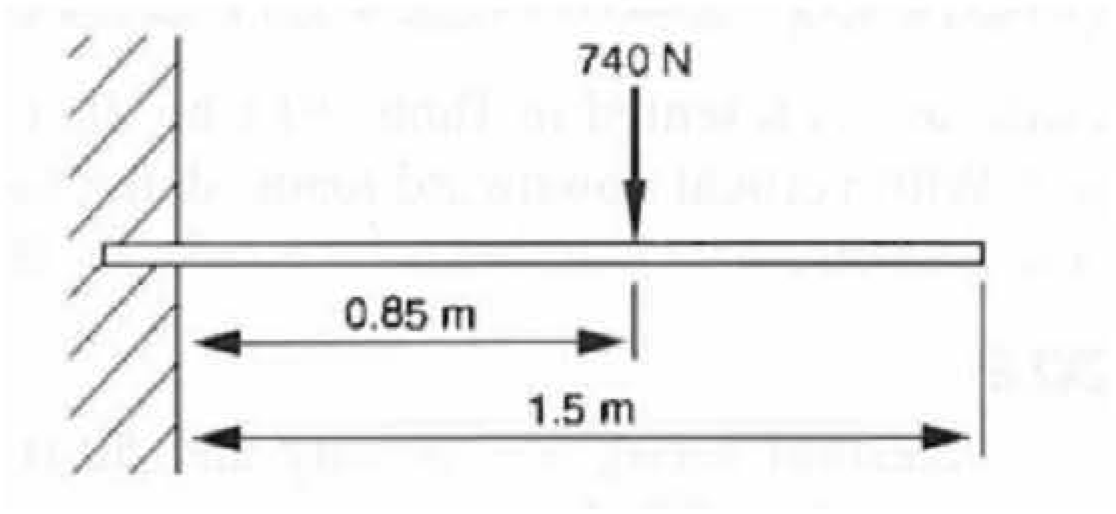
Drag statements on the right to match the left.

850	*F*
1500	*E*
740	*a*
70 000	*L*
307 000	*I*

Match the units for each variable in the equation for maximum deflection y:

$$y = \frac{F \cdot a^2(3L - a)}{6 \cdot E \cdot I}$$

Use aluminium, modulus 70 GPa and a beam second moment of area 307 (x 10^3) mm^4.

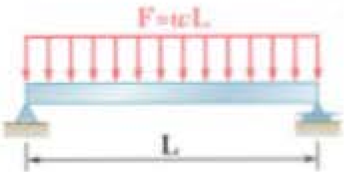


Drag statements on the right to match the left.

mm	*I*
mm	*a*
N	*L*
MPa	*E*
mm^4	*F*

A simply supported steel beam (I = 554 x 10^6 mm^4) carries a distributed load of 7.5 kN/m for the entire length of the 12 m beam. Calculate the maximum deflection in mm.

(Include units. Use two decimal places.)



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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

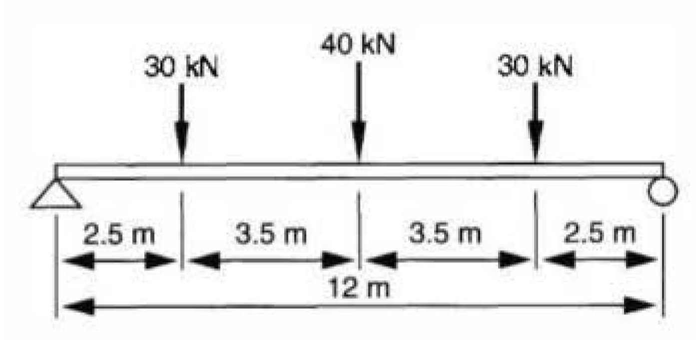
Each hint will reduce the credit received for this question

Calculate deflection at mid-span of symmetrically loaded, simply supported beam with three concentrated loads—Example

The principle of superposition works in this case. The preceding questions have maximum deflection at the same location—the middle of the beam.

Example

Determine the maximum deflection for a simply supported steel beam ($E = 200{,}000$ MPa, $L = 12{,}000$ mm and $I = 554 \times 10^6$ mm^4) that carries three symmetrical loads as shown below.



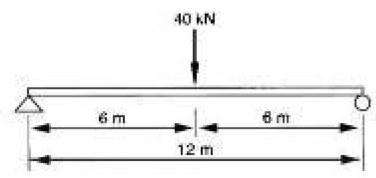
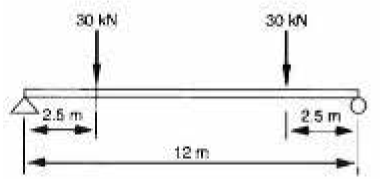
GIVE FEEDBACK

CONTINUE >

Calculate deflection at mid-span of symmetrically loaded, simply supported beam with three concentrated loads—Example

Solution

We can use superposition of loads to solve this problem, one as a mid-span load of 40 kN and one as two symmetrical loads of 30 kN.

$y = \frac{FL^3}{48\,EI}$ $= \frac{40{,}000 \times 12{,}000^3}{48 \times 200{,}000 \times 554 \times 10^6}$ $= 13$ mm	
$y = \frac{F\,a\,(3L^2 - 4a^2)}{24\,EI}$ $= \frac{30{,}000 \times 2{,}500 \times (3 \times 12{,}000^2 - 4 \times 2{,}500^2)}{24 \times 200{,}000 \times 554 \times 10^6}$ $= 11.5$ mm	

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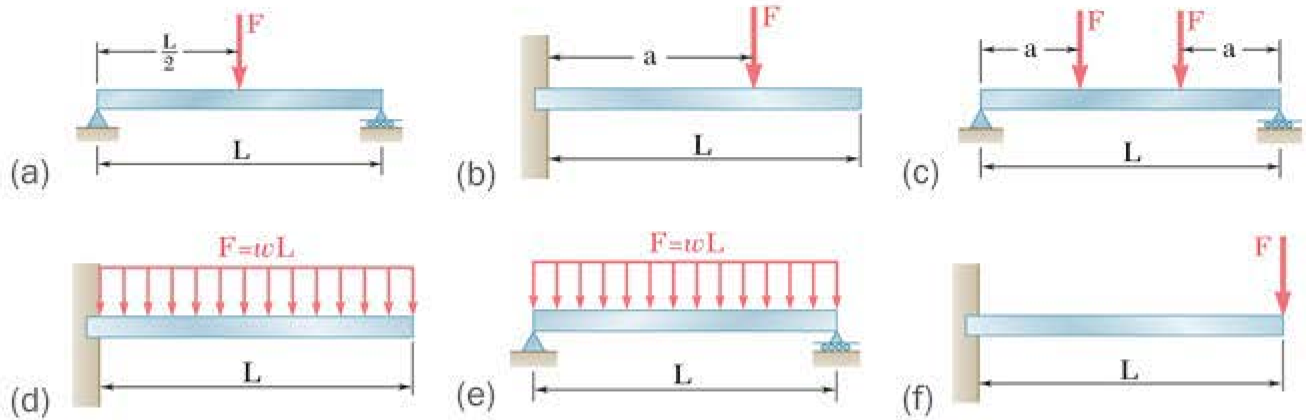
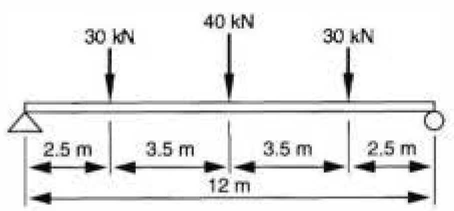
So the total deflection for the combined loading can be easily found by superposition:

$$\begin{aligned} \text{Deflection } y &= 13\ \mathrm{mm} + 11.5\ \mathrm{mm} \\ &= 24.5\ \mathrm{mm} \end{aligned}$$



Which of the 6 bending cases above are needed to solve (by superposition) this problem below?

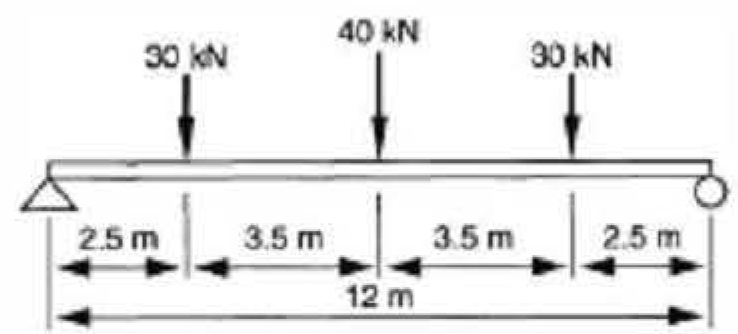
A simply supported steel beam with three symmetrical loads, but ignoring weight of the beam.



Click to highlight.

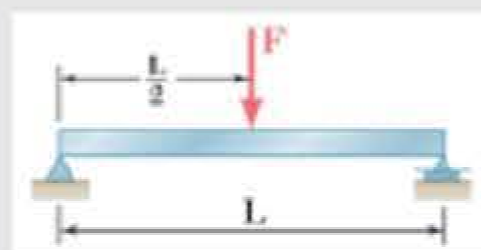
A, B, C, D, E, F, G

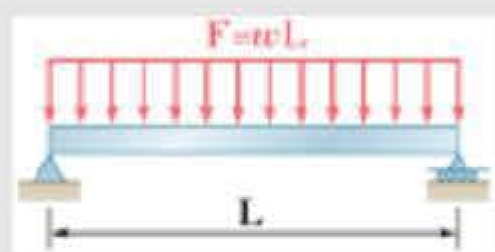
A simply supported steel beam ($E = 200$ GPa, $L = 12$ m and $I = 554 \times 10^6$ mm^4) has to carry three symmetrically located loads as shown below. Ignore the weight of the beam.

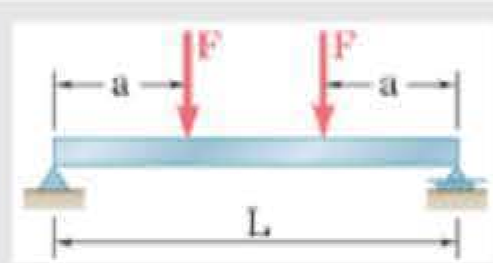


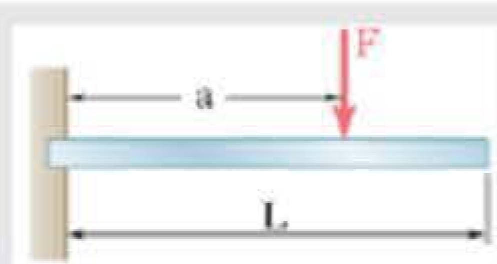
Select the appropriate bending deflection cases for solving this problem by superposition.

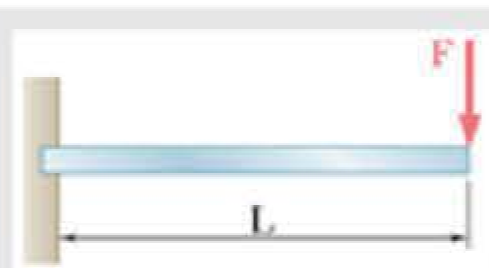
Check all that apply.

- [ ] 

- [ ] 

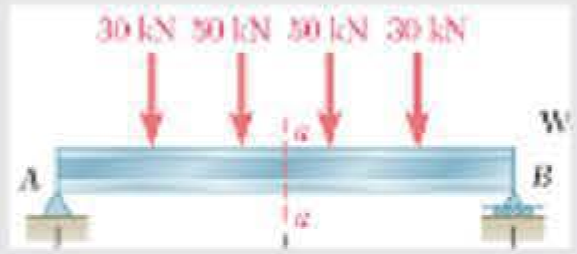
- [ ] 

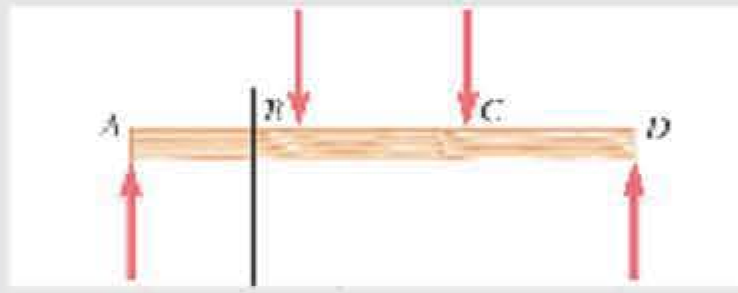
- [ ] 

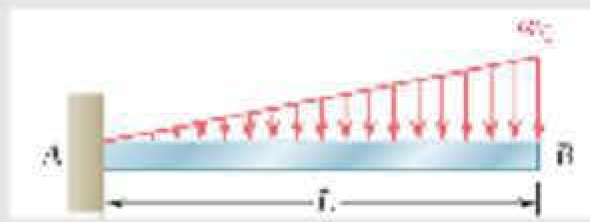
- [ ] 

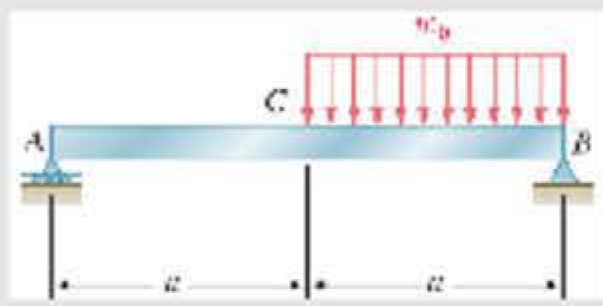
The seven deflection cases would be suitable for solving which type of question(s)?
Include superposition of loads if necessary.

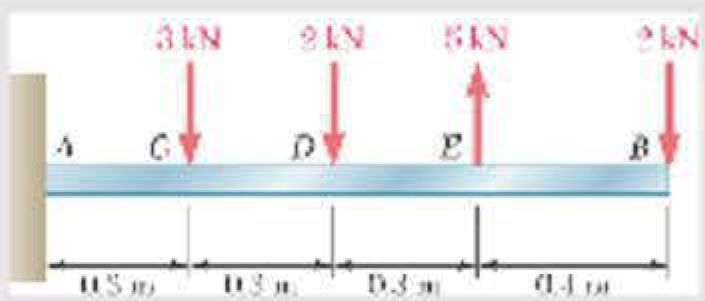
Check all that apply.

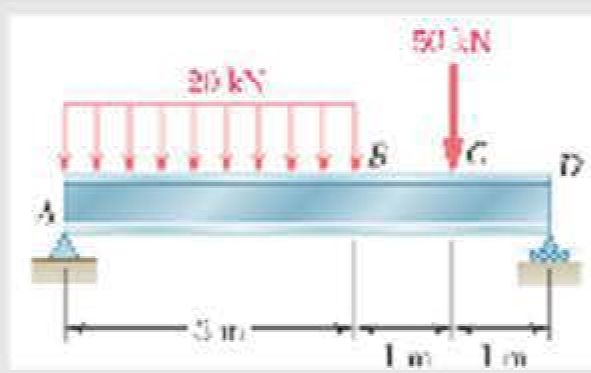
- [ ] 

- [ ] 

- [ ] 

- [ ] 

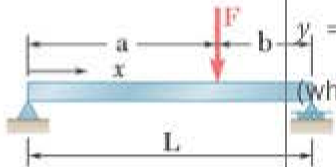
- [ ] 

- [ ] 

This off-centre loading of a simply supported beam is the odd one out. It cannot be combined with any other formula because the maximum deflection is not located at mid-span.

Where maximum deflection occurs in a non-symmetrically loaded, simply supported beam

When the loading on a beam is non-symmetrical, solutions are complicated by the fact that the position (x) of the maximum deflection is not immediately predictable. It does not correspond to the mid-span or to the position of the load, but lies somewhere between these two points and has to be located by a separate calculation.

Case	Beam and load	Maximum deflection	Occurance
7		$y = \frac{F\,a\,b\,(a+2b)\sqrt{3a\,(a+2b)}}{27\,EIL}$ (where $a > b$)	$x = \sqrt{\frac{a\,(a+2b)}{3}}$

Note: To obtain deflection in mm we must use the following units:

F in (N), E in (MPa), I in (mm^4), and L, a and b must be in (mm).

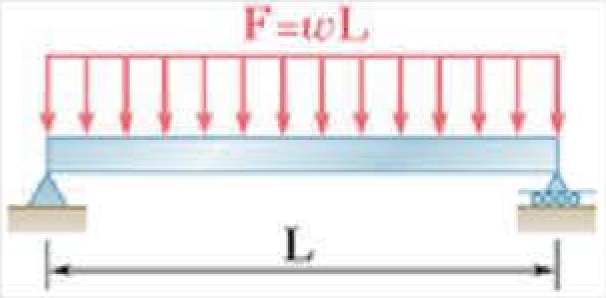
Superposition will not work with this formula because the location of the maximum deflection is not fixed at mid-span but changes with location of the force (see also Beam deflection formulas).

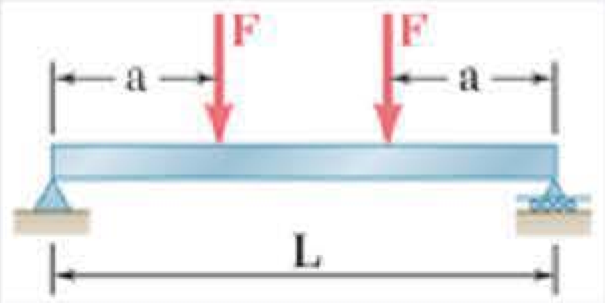
GIVE FEEDBACK

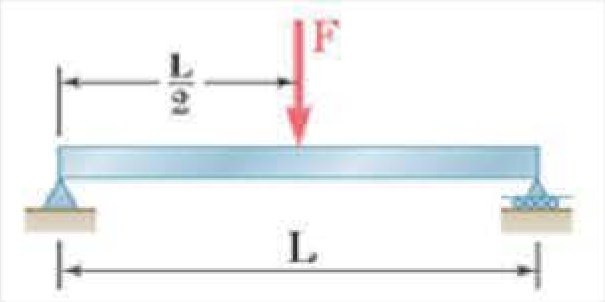
OK

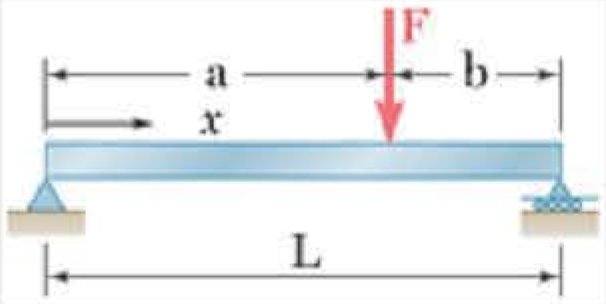
Which of the following simply supported beams cannot be used in superposition with the others?

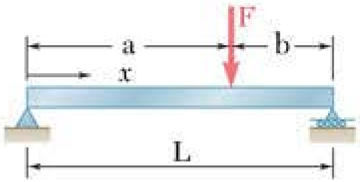
Click the correct answer.











Superposition will not work with non-symmetrical loading because the maximum deflection:

Click the correct answer.

- Is not fixed at mid-span
- Cannot be calculated
- Can only be calculated at mid-span
- Is so small it is negligable

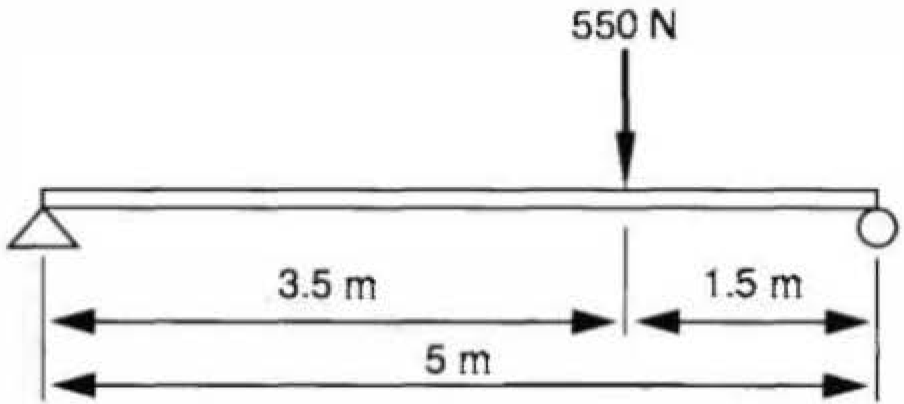
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate maximum deflection for simply supported beam due to single non-symmetrical concentrated load—Example 4

Example

A simply supported timber beam, 5 m long and 100 mm deep $\times$ 50 mm wide, carries a single concentrated load of 550 N located 1.5 m from one of the supports (below). Assuming $E = 12{,}000$ MPa, calculate and locate the maximum deflection of the beam.



GIVE FEEDBACK

CONTINUE >

Calculate maximum deflection for simply supported beam due to single non-symmetrical concentrated load—Example 4

Solution

Examine the formulas for Case 7 in the Beam deflection formulas table:

$$y = \frac{F\,a\,b\,(a + 2b)\sqrt{3a\,(a + 2b)}}{27\,EIL} \qquad x = \sqrt{\frac{a\,(a + 2b)}{3}}$$

There is a condition: $a > b$

Therefore we must choose:

$$\begin{aligned} a &= 3.5 \text{ m} = 3{,}500 \text{ mm} \\ b &= 1.5 \text{ m} = 1{,}500 \text{ mm} \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

Calculate maximum deflection for simply supported beam due to single non-symmetrical concentrated load—Example 3/4

Calculate the moment of inertia of the rectangular cross-section:

$$
\begin{aligned}
I &= \frac{b\,h^{3}}{12} \\
&= \frac{50 \times 100^{3}}{12} \\
&= 4.167 \times 10^{6}\ \text{mm}^{4}
\end{aligned}
$$

Now substitute and evaluate:

$$
\begin{aligned}
y &= \frac{F\,a\,b\,(a + 2b)\sqrt{3a\,(a + 2b)}}{27\,EIL} \\
&= \frac{550 \times 3{,}500 \times 1{,}500\,(3{,}500 + 2 \times 1{,}500)\sqrt{3 \times 3{,}500\,(3{,}500 + 2 \times 1{,}500)}}{27 \times 12{,}000 \times 4.167 \times 10^{6} \times 5{,}000}
\end{aligned}
$$

∴ Maximum deflection $y = 23\ \text{mm}$

< BACK

GIVE FEEDBACK

CONTINUE >

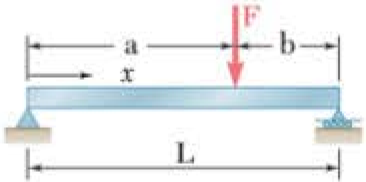
Calculate maximum deflection for simply supported beam due to single non-symmetrical concentrated load—Example 4

Now we can locate the position along the beam where the maximum deflection would occur:

$$
\begin{aligned}
x &= \sqrt{\frac{a(a+2b)}{3}} \\
&= \sqrt{\frac{3{,}500(3{,}500+2\times 1{,}500)}{3}} \\
&= 2{,}753\ \mathrm{mm}
\end{aligned}
$$

The distance from the left-hand support (where dimension a is also measured from) is:

$$x = 2.75\ \mathrm{m}$$



A simply supported timber beam is 12 m long with a concentrated load of 40 kN at 8.4 m from one end. Find the location **x** (in mm) of maximum deflection.

(Include units. Round off to nearest mm.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question