GIVE FEEDBACK

CONTINUE >

The bending stress equation looks like this:

$$\sigma_b = \frac{M\,y}{I}$$

The last chapter was all about Bending Moment (M), which is determined by the loads and length of the beam.

In this chapter we look at the Second Moment of Area (I), which is determined by the shape of the beam's cross-section.

Once we have these two values, we simply plug them into the equation (above) to find bending stress.

< BACK GIVE FEEDBACK OK

How second moment of area is related to stiffness

The second moment of area is a special type of area measurement used for bending situations.

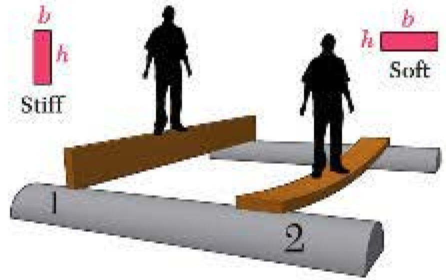
It explains why a plank of wood is much stiffer upright than when laid down flat.

The second moment of area is a measure of the 'efficiency' of a cross-sectional area to resist bending under load.

Both beams have the same **area** and even the same shape.

Beam 1 is stiffer than Beam 2 because it has a higher *second moment of area* I .

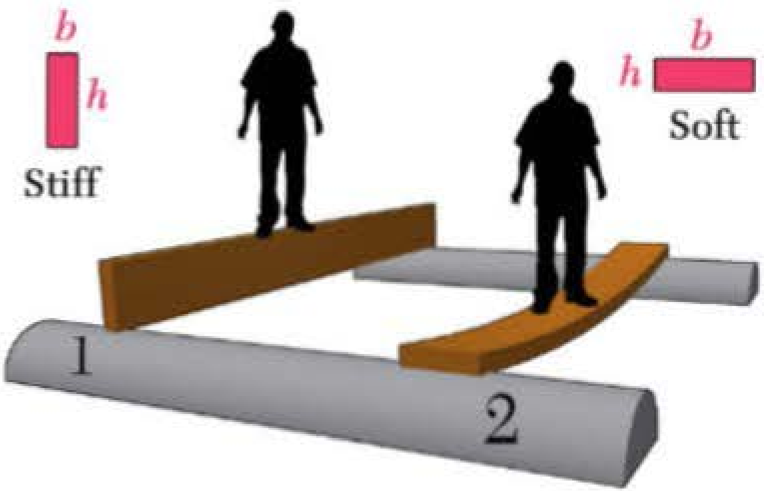
Orientation alters the *second moment of area I*.



GIVE FEEDBACK

OK

Compare Beam 1 with Beam 2 and identify which statements are true.



Check all that apply.

- [ ] They have the same cross-sectional area A
- [ ] They have the same cross-sectional shape b x h
- [ ] They have the same Second Moment of Area I
- [ ] They have the same value for h

The second moment of area for a rectangle is proportional to height cubed.

Note that there is another name for second moment of area.

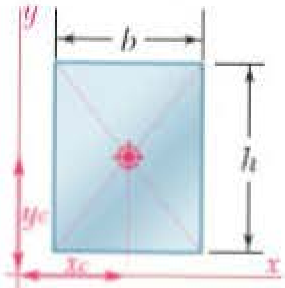
It is also known as "moment of inertia"

But we will avoid this name most of the time. It is too easily confused with mass moment of inertia - which is used for completely different problems.

Second moment of area for a rectangular section

The **second moment of area** (I) of planar shapes is also called the *moment of inertia*.

The equation for a rectangle is given below.

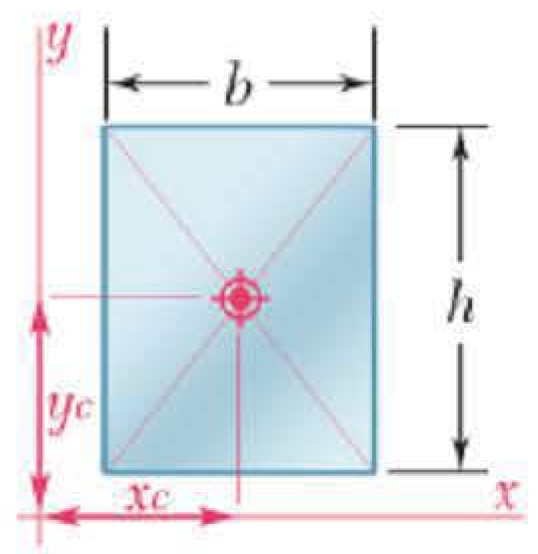
		Area A	*Second moment of area I*
rectangle		$b\,h$	$\dfrac{b\,h^3}{12}$

For more shapes see: Ref: Second Moments of Area for Simple Shapes

GIVE FEEDBACK

OK

Match the variables to their formulas or symbols for the second moment of area for the rectangle shown below.

Rectangle	

Drag statements on the right to match the left.

The area of the cross-section A	$\dfrac{bh^3}{12}$
Second moment of area I	$b \cdot h$
The height of the beam cross-section	h
The width of the beam cross-section	b

The **second moment of area** (I) of planar shapes is also called the ________.

Check all that apply.

- [ ] moment of inertia
- [ ] area
- [ ] mass moment of inertia
- [ ] inertia
- [ ] area moment of inertia

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

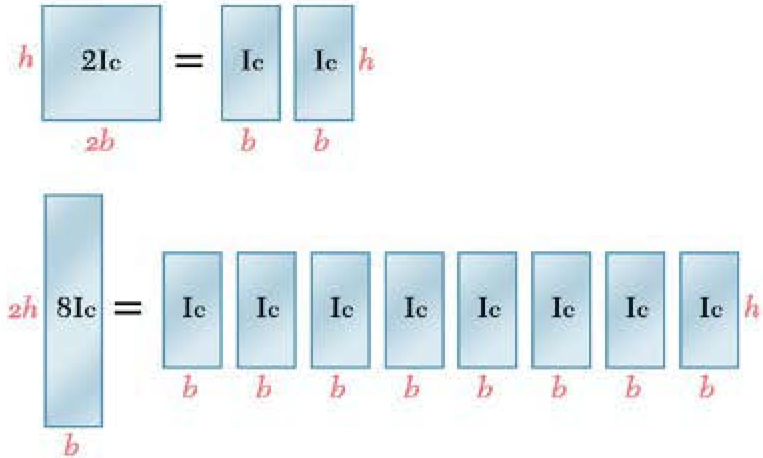
If width is doubled, the second moment of area increases twice.

If height is doubled, the second moment of area increases eight times.

The second moment of area is much more sensitive to depth than width.

If width is doubled, I increases **twice**.

If height is doubled, I increases **eight times**.



GIVE FEEDBACK

CONTINUE >

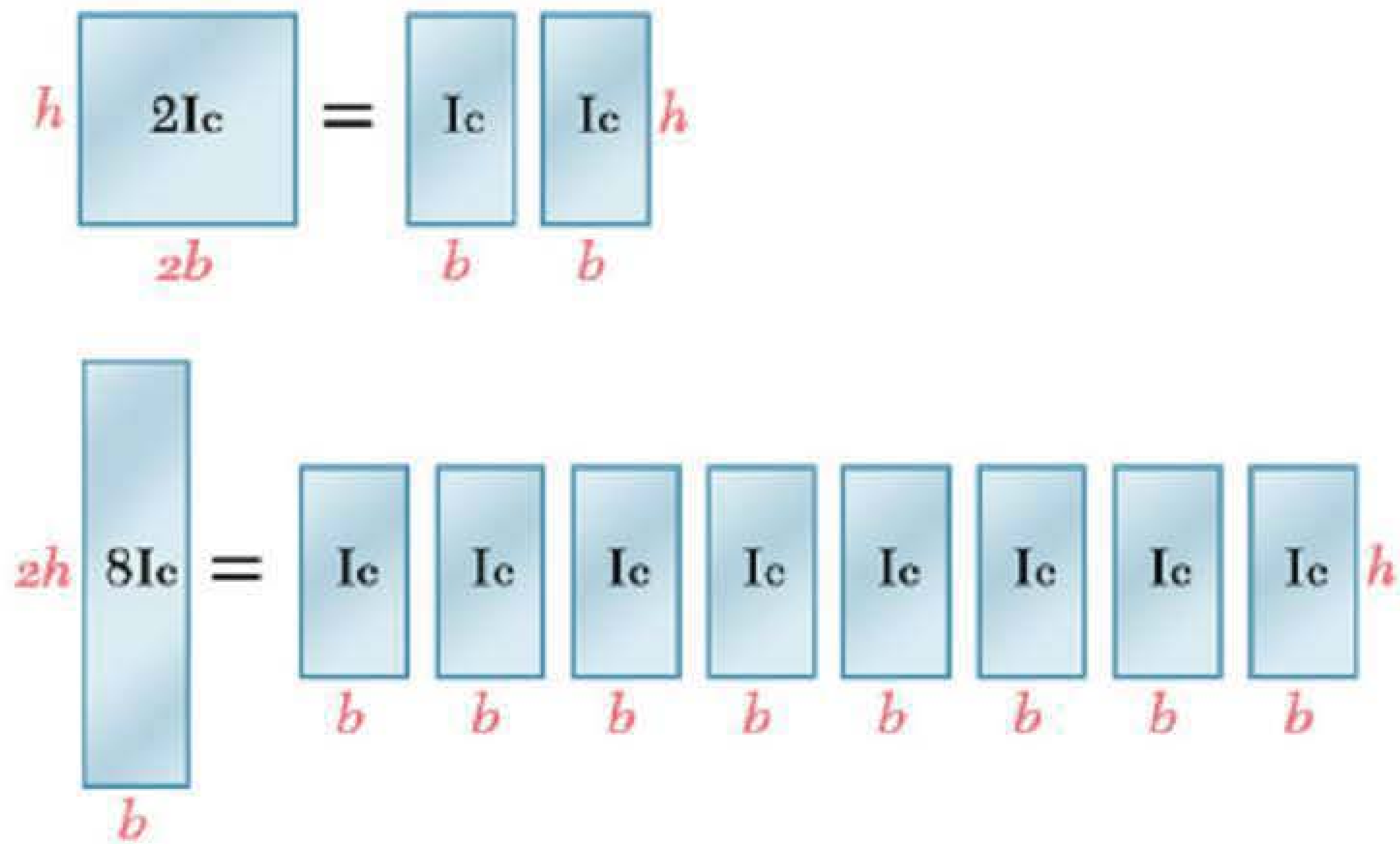
Compare the effect of depth and width on second moment of area

This is why beam depth is emphasised for anything designed to handle bending loads, like these wooden beams in a house structure.

< BACK

GIVE FEEDBACK

OK



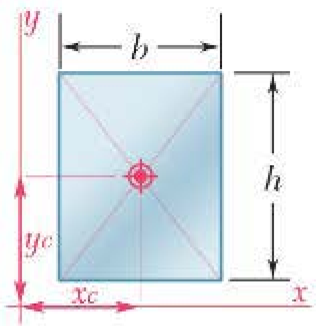
Altering width or height of a beam's cross-section and the effect on the second moment of area: Match the following statements.

Drag statements on the right to match the left.

The second moment of area is much more sensitive to _____ than width.	depth
If width is doubled, I increases ...	x 2
If height is doubled, I increases ...	width
The second moment of area is proportional to the beam's _____	x 8

Example

Determine the moment of inertia of a rectangular area, with base 20 mm and height 30 mm, about its horizontal centroidal axis.



GIVE FEEDBACK

CONTINUE >

Solution

The moment of inertia of a rectangle is given by:

$$I_c = \frac{b\,h^3}{12}$$

Substitute and solve:

$$I_c = \frac{20 \times 30^3}{12}$$
$$= 45{,}000\ \text{mm}^4$$

< BACK GIVE FEEDBACK OK

A bar has a 7 mm square cross-section. Calculate the second moment of area (Ic) about it centroid.

(Round off to nearest integer. Include units as mm^4)

Clear

Clear line

? Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

A beam has a rectangular cross-section of breadth 7 mm and height 10 mm. Calculate the second moment of area (Ic) about its centroid.

(Round off to nearest integer. Include units as mm^4)

± ▾ $\frac{\square}{\square}$ $1\frac{2}{3}$ $\square^2$ ▾ $\sqrt{\square}$ $(\square)$ ▾ Clear

≤ ▾ π $\square \times 10^{\square}$ mm $f(x)$ ▾ $\square_n$ Clear line

? Undo

↵

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint Each hint will reduce the credit received for this question

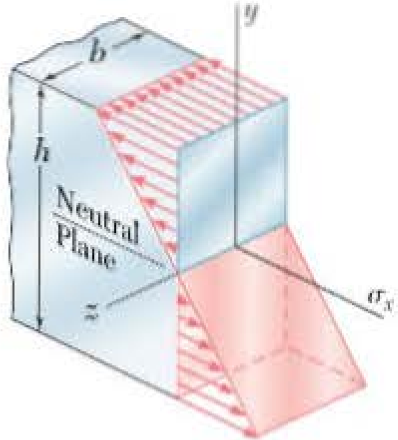
Bending stress is different to direct axial stresses. It is not distributed uniformly over the cross-sectional area of the beam.

In this diagram, the lower half of the beam is in tension and the upper half of the beam is in compression.

Unlike direct tension or compression, bending stress is not distributed uniformly over the cross-sectional area of the beam.

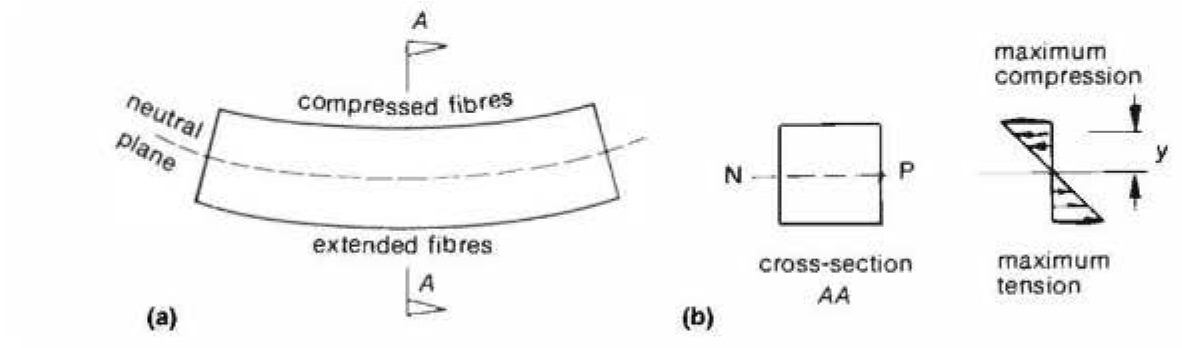
When a beam is subjected to bending, the convex side is extended, while the concave side is compressed (as shown).

Somewhere between the stretched and the compressed areas there is a longitudinal plane along which there is no deformation of length. This plane is known as the **neutral plane (NP)**.



GIVE FEEDBACK

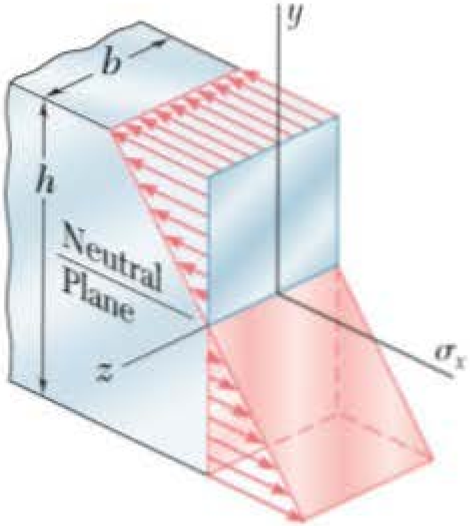
CONTINUE >



(a) *Beam in bending* (b) *Stress distribution*

< BACK GIVE FEEDBACK OK

The neutral plane has (please select)
and is located on the (please select) the beam.



Submit

Do you know the answer?

I KNOW IT

THINK SO

UNSURE

NO IDEA

Here is the formula for bending stress.

For a rectangular beam, the bending stress equals the bending moment times half the depth, divided by the second moment of area.

Determine bending stress in a rectangular section

When a part is in bending stress (like a beam), the cross-section is under a varied stress that goes from tension at the bottom to compression at the top.

The maximum bending stress is found for a rectangular cross-section by:

$$\boxed{\sigma_b = \frac{M\,y}{I}}$$

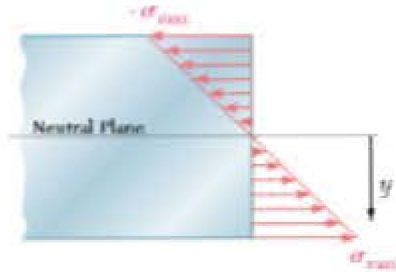
Where:

σ_b = bending stress (MPa)

M = bending moment at a given cross-section (N.mm)

$y = \dfrac{h}{2}$ = half the depth (mm)

$I = \dfrac{b \cdot h^3}{12}$ = second moment of area (mm^4)



Bending stress in a beam. Highest stress occurs with maximum y, tensile on bottom, compressive at the top. There is no axial stress in the middle (centroid).

GIVE FEEDBACK

OK

A rectangular beam is under positive bending moment (sagging).

The bending stress is found for a rectangular cross-section by:

$$\sigma_b = \frac{M\,y}{I}$$

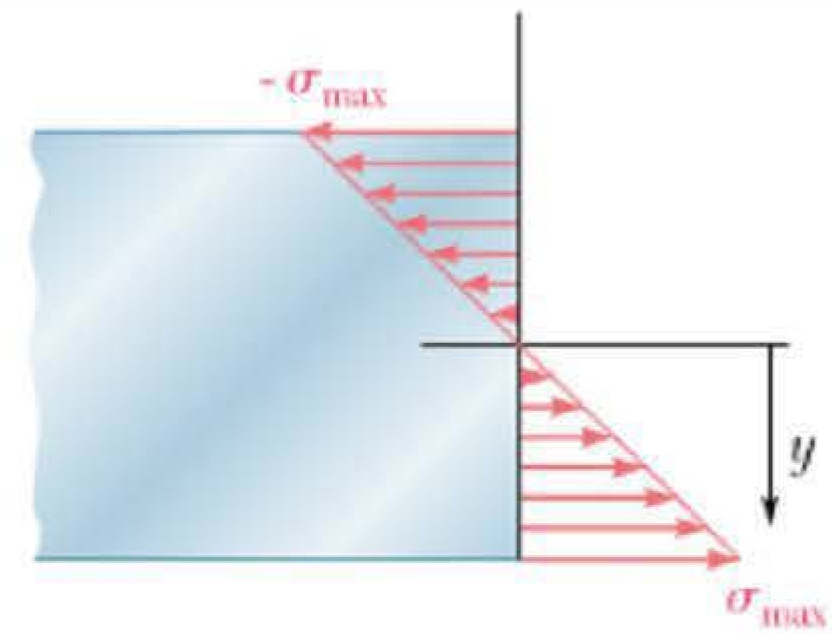
Where:

σ_b = bending stress (MPa)

M = bending moment (N.mm)

y = distance from centroid (mm)

I = second moment of area (mm^4)

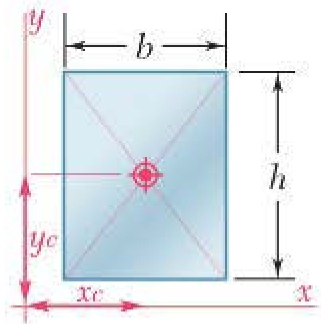


Bending stress in a beam under a positive bending moment.

Drag statements on the right to match the left.

Highest tensile stress occurs at ________.	the bottom of this section
Highest compressive stress occurs at ________.	the centroid of this section
There is zero axial stress at ________.	the upper half of this section
Tensile stress occurs in ________.	the lower half of this section
Compressive stress occurs in ________.	the top of this section

Find the stress in a 20mm wide by 30mm deep rectangular beam when it is under a bending moment of **600 Nm.**



Calculation for second moment of area:
(base *b=20 mm* x height *h=30 mm*).

$$I_c = \frac{20 \times 30^3}{12} = 45{,}000 \text{ mm}^4$$

GIVE FEEDBACK

CONTINUE >

The maximum bending stress for a rectangular cross-section using:

$$\sigma_b = \frac{M\ y}{I}$$

where:

σ_b = bending stress (MPa)

M = bending moment at a given cross-section (N.mm) = **600 x 1000 N.mm**

$y = \frac{h}{2}$ = half the depth (mm) = **30/2 = 15 mm**

$I = \frac{b \cdot h^3}{12}$ = second moment of area (mm^4) = **45000 mm^4**

< BACK GIVE FEEDBACK CONTINUE >

So bending stress is:

$$\begin{aligned}\sigma_b &= \frac{M\ y}{I} \\ &= \frac{(600 \cdot 1{,}000) \cdot 15}{45{,}000} \\ &= 200\ \text{MPa}\end{aligned}$$

That is quite a high stress and will need to be made of a strong material (e.g. steel).

< BACK GIVE FEEDBACK OK

A rectangular bar has a breadth 6 mm and height 8 mm, (which gives a second moment of area of 256 mm^4). It is loaded with a bending moment of 16 Nm. Calculate the highest bending stress in this cross-section.

(Round off to nearest integer. Include units as MPa)

Clear

Clear line

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Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint Each hint will reduce the credit received for this question

The centroid is the geometrical centre of a flat shape.

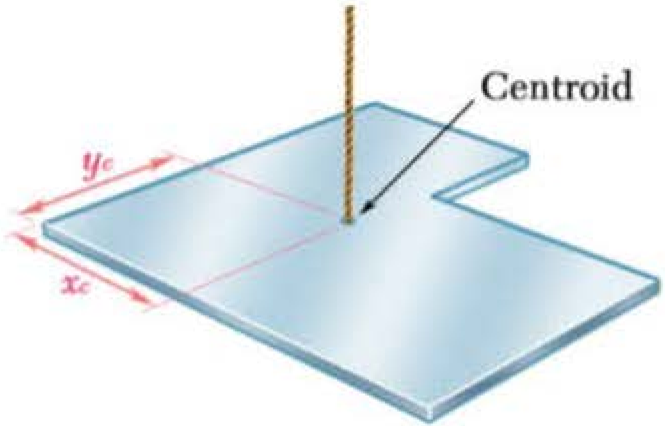
So if you cut the shape out of a plate, the centroid is where the plate will balance.

For simple shapes like circles and squares, the centroid is in the middle.

For more complex shapes we must calculate the centroid.

The **centroid** of a plane area is the geometrical centre of the area distribution.

In other words, if you cut the shape out of a plate, the centroid is where the plate will balance.



Centroid of an area is the balancing point of a plate.

GIVE FEEDBACK

CONTINUE >

Although this analogy is helpful, one should keep in mind that the centroid is a geometrical concept and not a mass-related concept.

The positions of centroids of symmetrical shapes such as circles, squares and rectangles are in the middle. For other shapes the centroids have been determined by integration, or by computer.

For simple centroids see: Ref: Second Moments of Area for Simple Shapes

< BACK GIVE FEEDBACK OK

If you cut any random shape out of a plate, the ______ is where the plate will balance.

Click the correct answer.

- centroid
- middle
- centre
- x axis
- y axis

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Positive bending means the beam will be sagging, the upper half in compression and the lower half in tension.

The location in the middle is called the neutral plane, where there is no axial stress at all.

This middle location is called the centroid.

How the centroid is related to bending stress

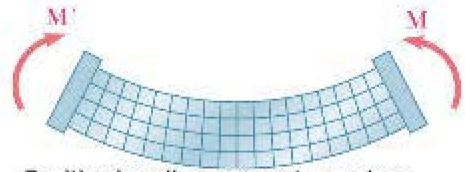
This beam is under a positive bending moment (sagging).

- The **centroid** of the beam's cross-sectional area has no tension or compression.
- The **centroid** along the whole beam is called the **neutral axis**. In 3D it is a plane, so it is also known as the **neutral plane**.

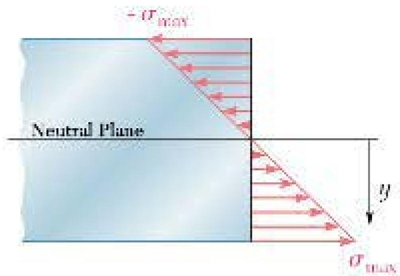
From the bending stress formula:

$$\sigma_b = \frac{M\,y}{I}$$

- Maximum tensile stress σ_{max} is when y is furthest *below* the neutral plane.
- Maximum compressive stress $-\sigma_{max}$ is when y is furthest *above* the neutral plane.



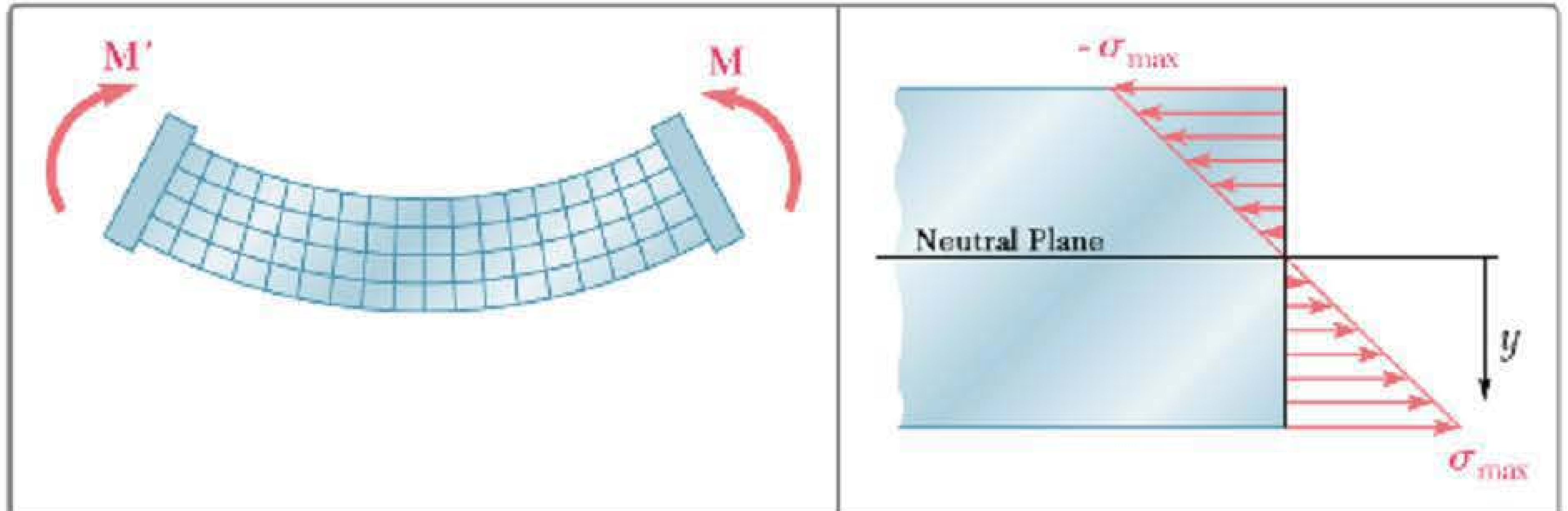
Positive bending moment, sagging.



Axial stresses inside a beam under a positive bending moment.

GIVE FEEDBACK

OK



A beam	A close-up of the same beam

For the beam shown above, match the following statements with the correct words.

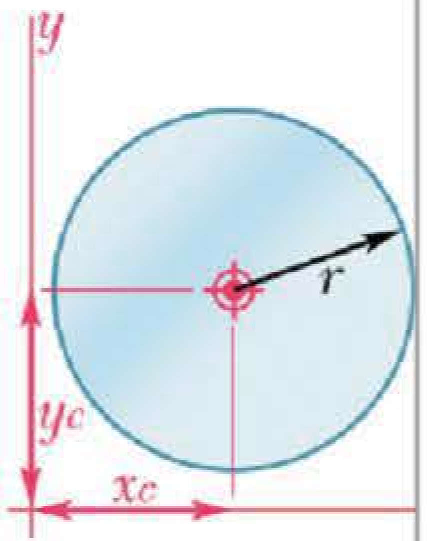
Drag statements on the right to match the left.

This beam is under a ______ bending moment	below
The centroid along the whole length of the beam is called the _____ axis.	positive
In 3D the neutral axis is called the neutral _____	neutral
Maximum tensile stress occurs furthest _____ the neutral plane.	above
Maximum compressive stress occurs furthest _____ the neutral plane.	plane

Here are some simple shapes with their centroids and second moments of area.

Define the centroid for simple planar shapes

The **centroids** of simple planar shapes are given in the table below.
The centroids are in the middle of course. The only surprise is that a triangle is at h/3.

Shape	Diagram	*Area A*	*Second moment of area I**	Centroid
		mm^2	mm^4	mm
circle		$\frac{\pi D^2}{4}$	$\frac{\pi D^2}{64}$	At centre
rectangle		$b h$	$\frac{b h^3}{12}$	At centre. The intersection

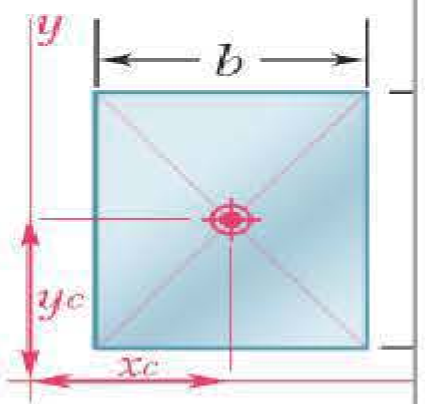
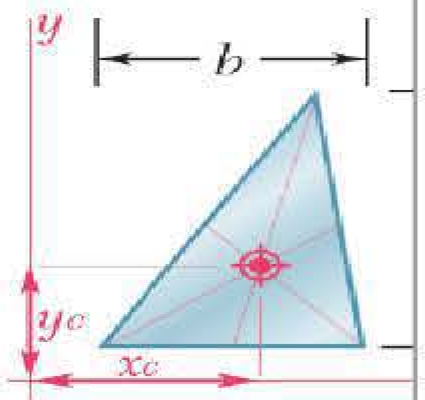
GIVE FEEDBACK

OK

Define the centroid for simple planar shapes

Here are some simple shapes with their centroids and second moments of area.

The **centroids** of simple planar shapes are given in the table below.
The centroids are in the middle of course. The only surprise is that a triangle is at h/3.

Shape	Diagram	*Area A*	*Second moment of area* I*	Centroid
rectangle		$b\,h$	$\frac{b\,h^3}{12}$	At centre. The intersection of diagonals.
triangle of perp height h		$\frac{b\,h}{2}$	$\frac{b\,h^3}{36}$	At the intersection of medians. 1/3 of height h

GIVE FEEDBACK

OK

The centroid of a rectangle of vertical height h is located at _____ from its base.

Click the correct answer.

$\frac{h}{3}$

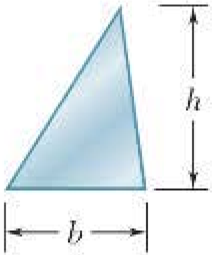
$\frac{h}{2}$

$\frac{2h}{3}$

h

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



This triangular section has height 30 mm and breadth 15 mm. Find the location of the centroid measured from the bottom of the triangle.

(1 decimal place. Include appropriate units)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

The centroid of a triangle of vertical height h is located at _____ from its base.

Click the correct answer.

$\frac{h}{3}$

$\frac{h}{2}$

$\frac{2h}{3}$

h

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The centroid for any shape can be calculated by dividing the area into little bits, and multiplying each bit by its distance from an axis.

We then divide this number by the total area to find the location of the centroid.

We get computers to do this of course.

The **centroid** of a composite shape can be calculated by dividing the area into its simple component parts and then applying the principle of **first moments of area.**

> The first moment of an area about any reference line equals the algebraic sum of the moments of its component areas about the same line.

GIVE FEEDBACK

CONTINUE >

What is the 'moment of area'?

It is the area of the shape multiplied by the distance to its own centre (centroid).

$\Sigma A \cdot y_c$

Total Area Moment = Sum of elemental area moments

$$\Sigma A \cdot y_c = \Sigma(A \cdot y)$$

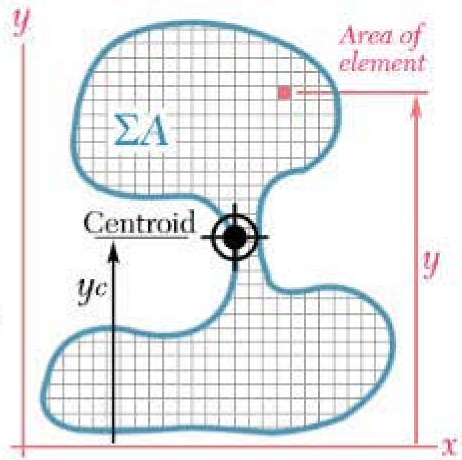
Where:

ΣA = Total area of shape

y_c = y coordinate of the centroid of shape

A = Area of each element

y = y coordinate of each element



< BACK GIVE FEEDBACK OK

The first moment of an area about any reference line equals the algebraic sum of the moments of its component areas about the same line.
The mathematical way of saying this is __________.

Click the correct answer.

- $\Sigma A \cdot y_c = \Sigma(A \cdot y)$
- $\Sigma A \cdot y_c^2 = \Sigma(A \cdot y)^2$
- $\Sigma A \cdot \Sigma y = \Sigma(A \cdot y)$
- $\Sigma A \cdot y = \Sigma(A \cdot y_c)$

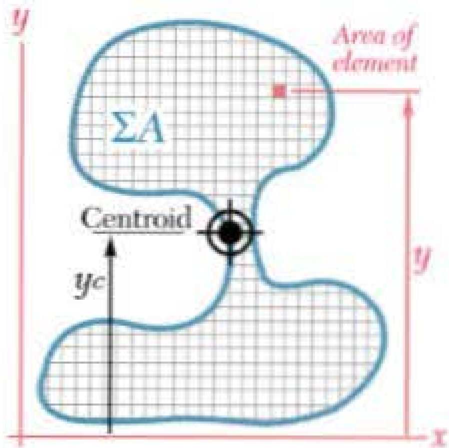
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Total Area Moment=Sum of elemental area moments

$\Sigma A \cdot y_c = \Sigma(A \cdot y)$

Match the symbols with their definitions below.



Drag statements on the right to match the left.

Total area of shape	ΣA
y coordinate of the centroid of shape	y_c
Area of each element	A
y coordinate of each element	y

What does 'first moment of area about an axis' mean?

Click the correct answer.

- The area multiplied by its distance to the axis
- The area multiplied by the square of its distance to the axis
- A force multiplied by the distance to the axis
- The bending moment calculated by a distance to the axis

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Finding the centroid y_c

From the
previous property of first moment of area:

Total area moment = Sum of elemental area moments

$$\Sigma A \cdot y_c = \Sigma(A \cdot y)$$

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma A}$$

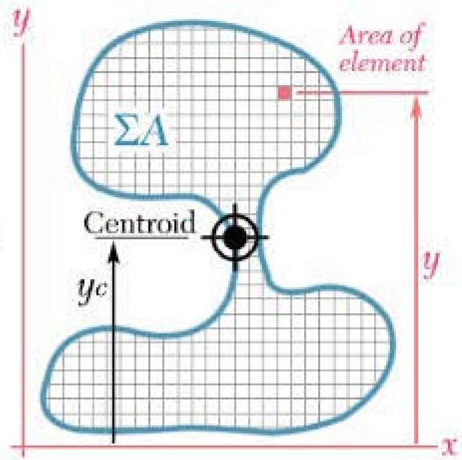
Where:

ΣA = Total area of shape

y_c = y coordinate of the centroid of shape (also $\overline{y}$
)

A = Area of each element

y = y coordinate of each element



GIVE FEEDBACK

CONTINUE >

State the formula for finding the centroid of a composite shape

The same applies in the x direction:

$$x_c = \frac{\Sigma(A\,x)}{\Sigma(A)}$$

Where:

$\Sigma(A)$ is the sum of all the component areas: $A_1 + A_2 + A_3 + ...$

$\Sigma(A\,x)$ is the sum of all the area moments: $A_1 x_1 + A_2 x_2 + A_3 x_3 + ...$

So the centroid (the balancing point of the area) is located at $\{x_c, y_c\}$

< BACK GIVE FEEDBACK OK

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma A}$$

This formula calculates the ________.

Click the correct answer.

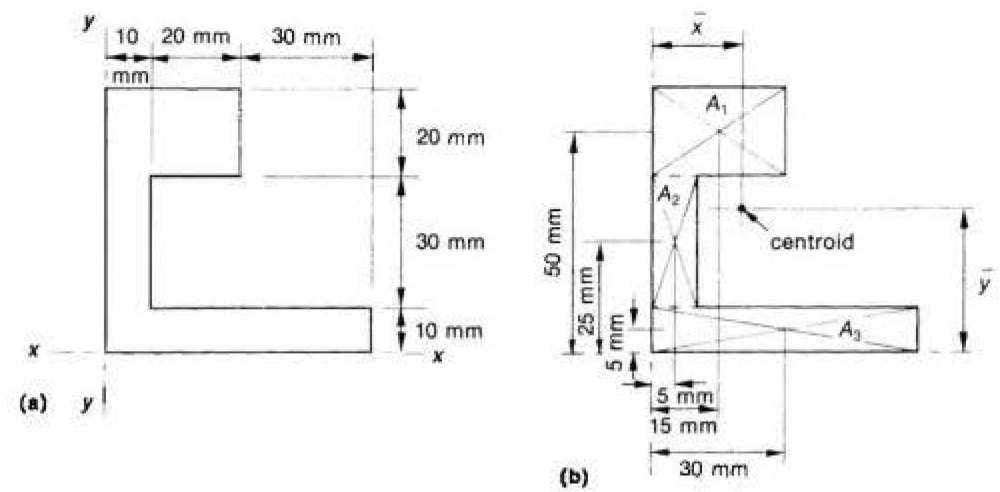
- centroid
- moment of area
- second moment of area
- area

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Example

Locate the centroid of the composite area shown below with respect to the $x - x$ and $y - y$ axes.



Solution

Divide the area into three rectangular elements, A_1, A_2 and A_3, and locate the centroid of each element as in Figure (b).

Calculate each elementary area:

$$A_1 = 30 \times 20 = 600 \text{ mm}^2$$
$$A_2 = 30 \times 10 = 300 \text{ mm}^2$$
$$A_3 = 60 \times 10 = 600 \text{ mm}^2$$
$$(\therefore \text{Total area } \Sigma)(A) = 1{,}500 \text{ mm}^2$$

Calculate the area moments in the $x - x$ direction:

$$A_1 x_1 = 600 \times 15 = 9{,}000 \text{ mm}^3$$
$$A_2 x_2 = 300 \times 5 = 1{,}500 \text{ mm}^3$$
$$A_3 x_3 = 30 \times 10 = 300 \text{ mm}^2$$
$$(\therefore \text{Total area moment } \Sigma)(A\ x) = 28{,}500 \text{ mm}^3$$

< BACK GIVE FEEDBACK CONTINUE >

The position of the centroid with respect to the $y - y$ axis, i.e. located in the $x - x$ direction, is:

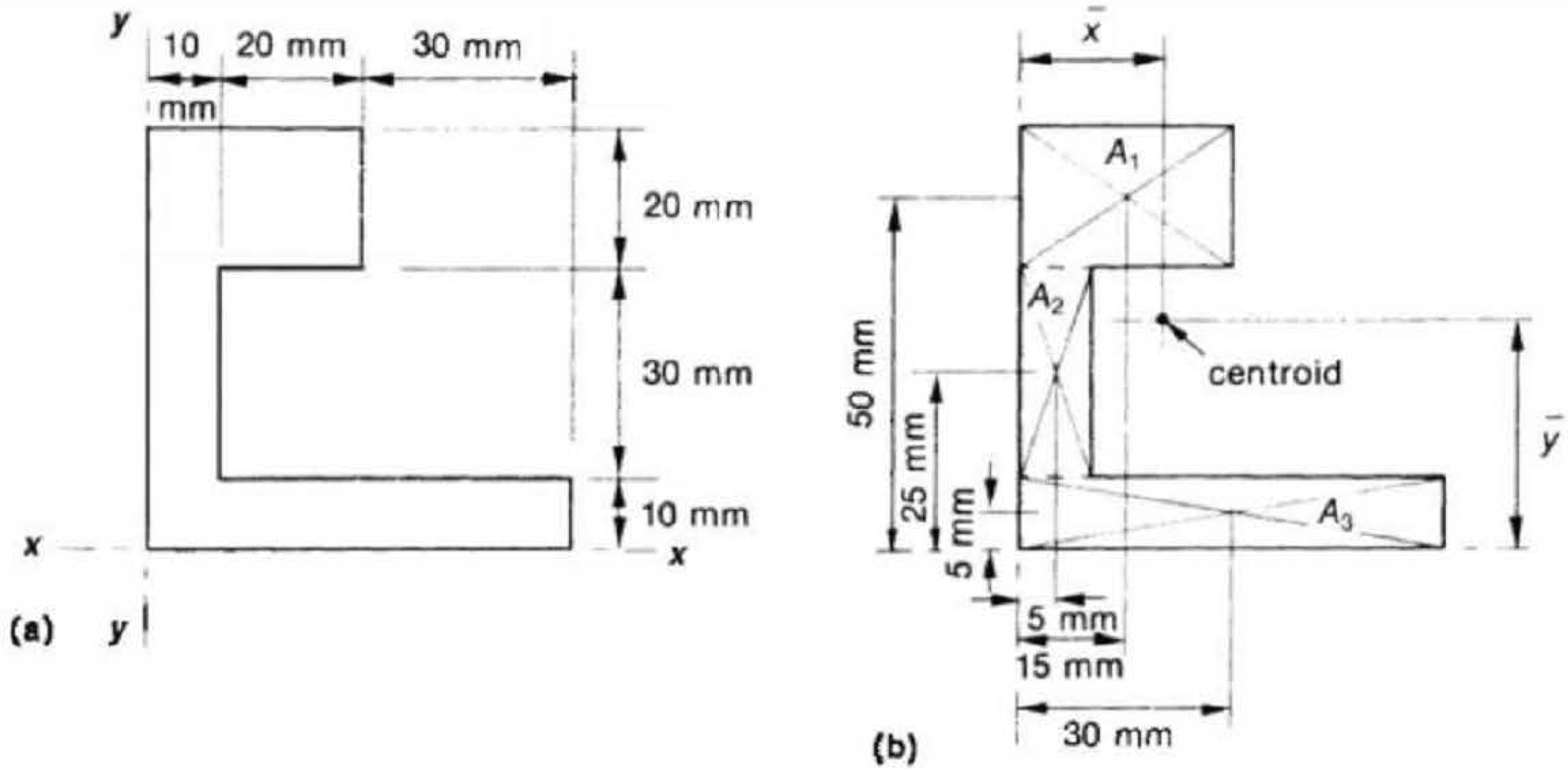
$$\overline{x} = \frac{\Sigma(A\,x)}{\Sigma(A)} = \frac{28{,}500}{1{,}500} = 19\text{ mm}$$

Similarly in the $y - y$ direction:

$$\begin{aligned}\overline{y} &= \frac{\Sigma(A\,y)}{\Sigma(A)} \\ &= \frac{600 \times 50 + 300 \times 25 + 600 \times 5}{600 + 300 + 600} \\ &= 27\text{ mm}\end{aligned}$$

It is obvious from this example that the centroid of a composite area can lie outside the outline of the area itself.

< BACK GIVE FEEDBACK OK



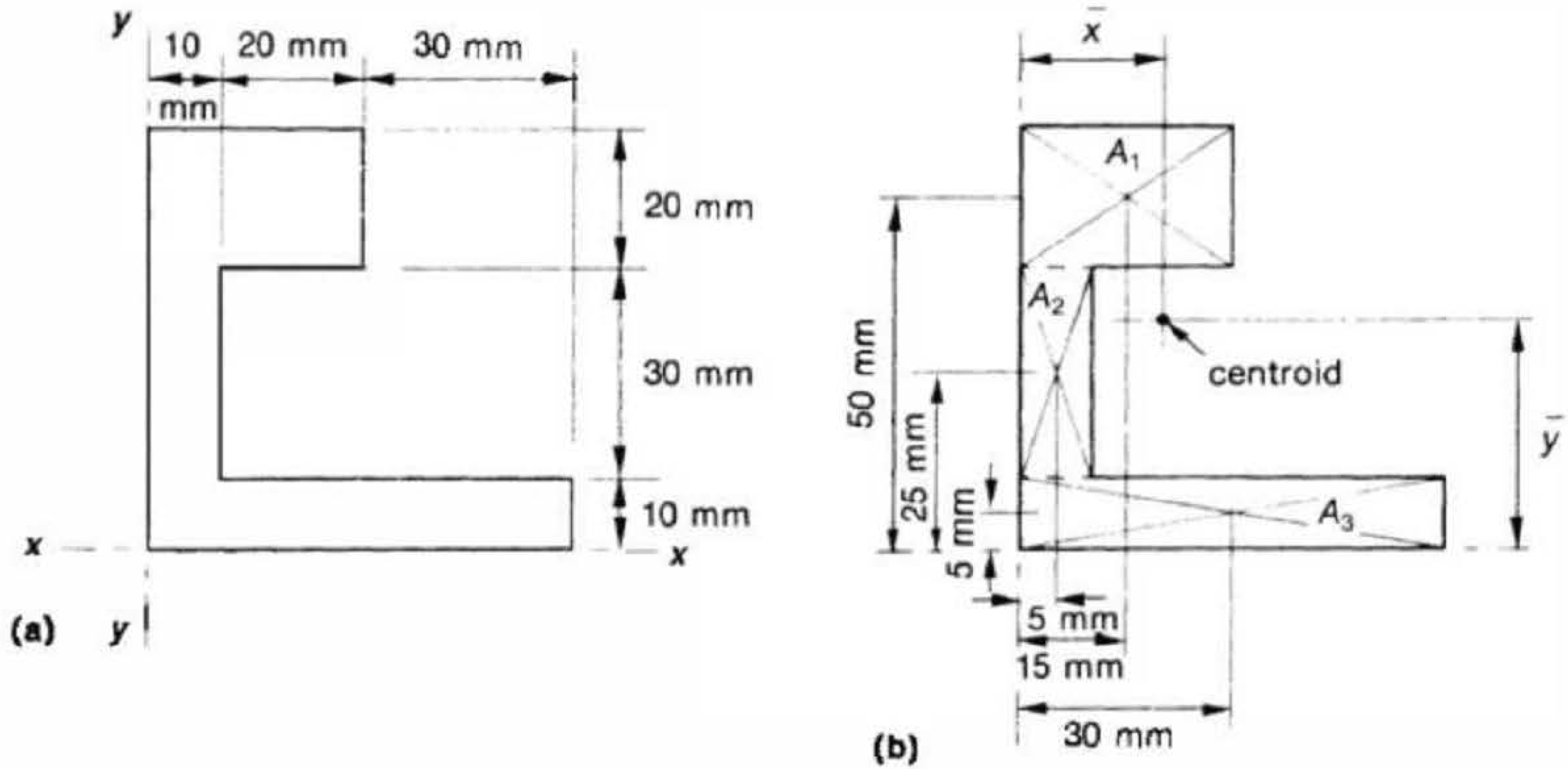
Calculate the centroid in the x direction, using the formula:

$$x_c = \frac{\Sigma(A \cdot x)}{\Sigma(A)}$$

Match the following values for x.

Drag statements on the right to match the left.

x1	5
x2	30
x3	15



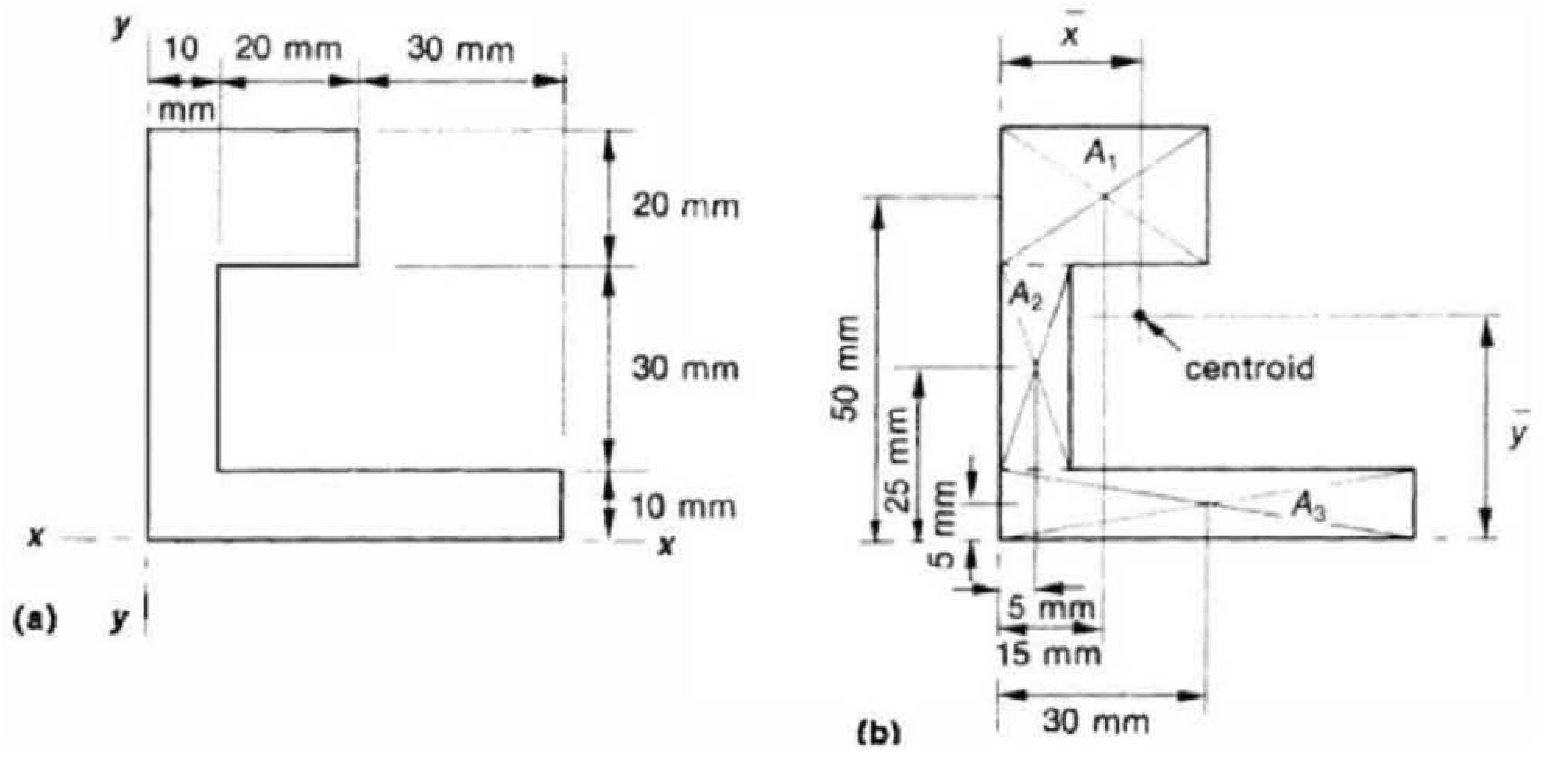
Calculate the centroid in the *y* direction, using the formula:

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

Match the following values for *y*.

Drag statements on the right to match the left.

y1	5
y2	25
y3	50

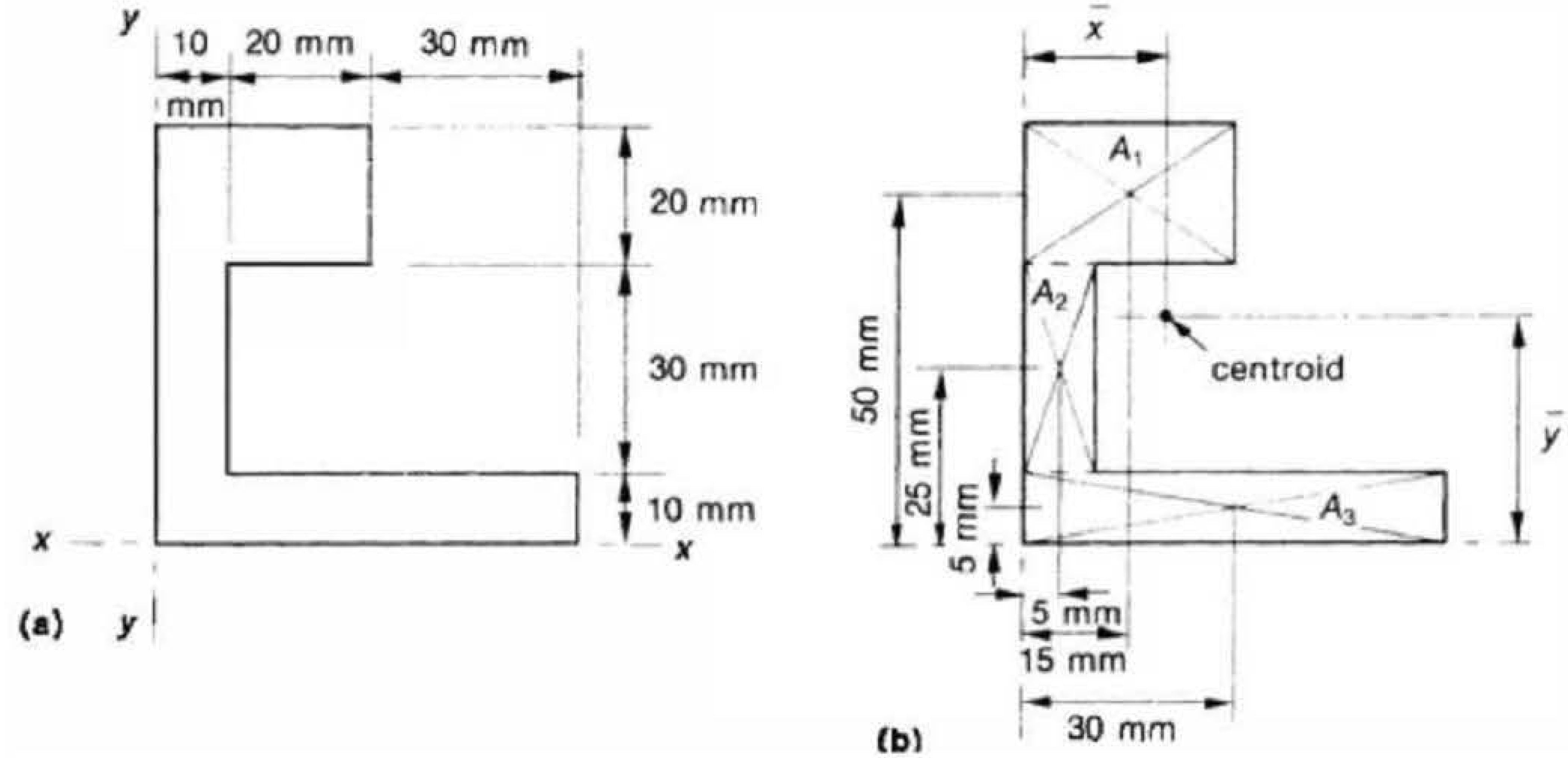


$$? = \frac{600 \times 15 + 300 \times 5 + 600 \times 30}{600 + 300 + 600} = 19\,\mathrm{mm}$$

What does this calculate? (Use every appropriate symbol)

Check all that apply.

- [ ] I_{xx}
- [ ] $\overline{y}$
- [ ] y_c
- [ ] I_c
- [ ] $\overline{x}$
- [ ] x_c



$$? = \frac{600 \times 50 + 300 \times 25 + 600 \times 5}{600 + 300 + 600} = 27\,\text{mm}$$

What does this calculate? (Use every appropriate symbol)

Check all that apply.

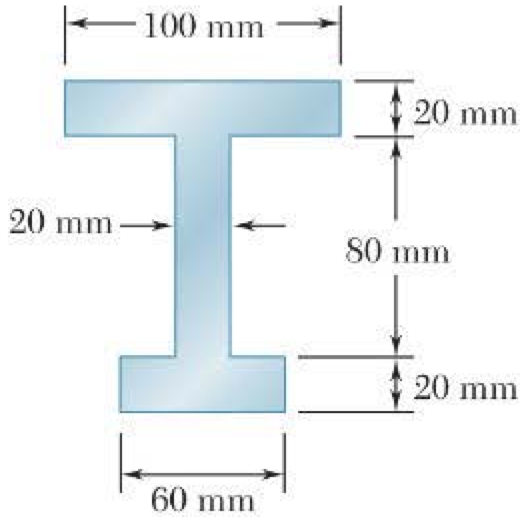
- [ ] x_c
- [ ] $\overline{x}$
- [ ] y_c
- [ ] I_{xx}
- [ ] I_c
- [ ] $\overline{y}$

Calculate the coordinates of the centroid for a symmetrical shape--Example

Calculate the centroid for this section

This section is symmetrical in the *x* axis so the *x* coordinate of the centroid is midway.

We only need to calculate y_c.



Find the centroid	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Find y coordinate of centroid

GIVE FEEDBACK

OK

Calculate the coordinates of the centroid for a symmetrical shape--Example

Find centroid for each element

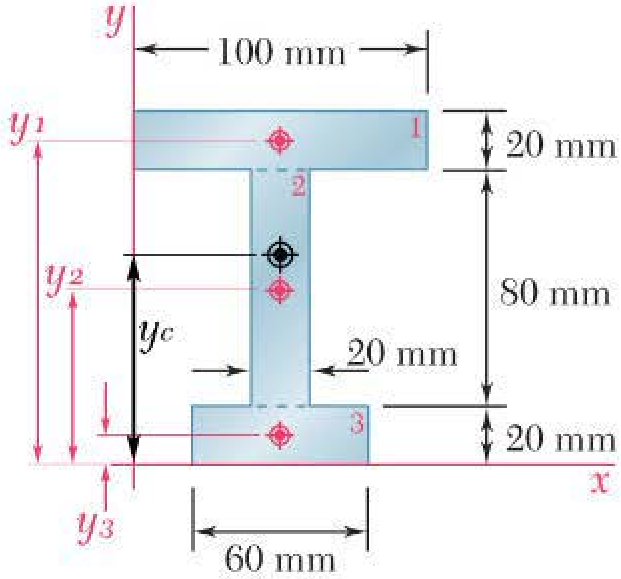
Split the section into three simple elements (1,2 and 3).

Working only in the *y* direction, determine each element centroid:

$$y_1 = 20 + 80 + \frac{20}{2} = 110 \text{ mm}$$

$$y_2 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$



Find the centroid	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Find y coordinate of centroid

GIVE FEEDBACK

OK

Calculate the coordinates of the centroid for a symmetrical shape--Example

Find centroid for each element

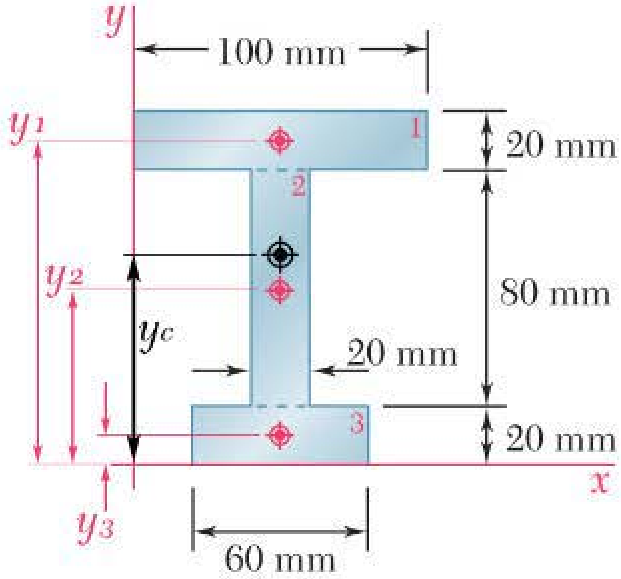
Split the section into three simple elements (1, 2 and 3).

Working only in the *y* direction, determine each element centroid:

$$y_1 = 20 + 80 + \frac{20}{2} = 110 \text{ mm}$$

$$y_2 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$



Find the centroid	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Find y coordinate of centroid

GIVE FEEDBACK

OK

Calculate the coordinates of the centroid for a symmetrical shape--Example

First moment for each element

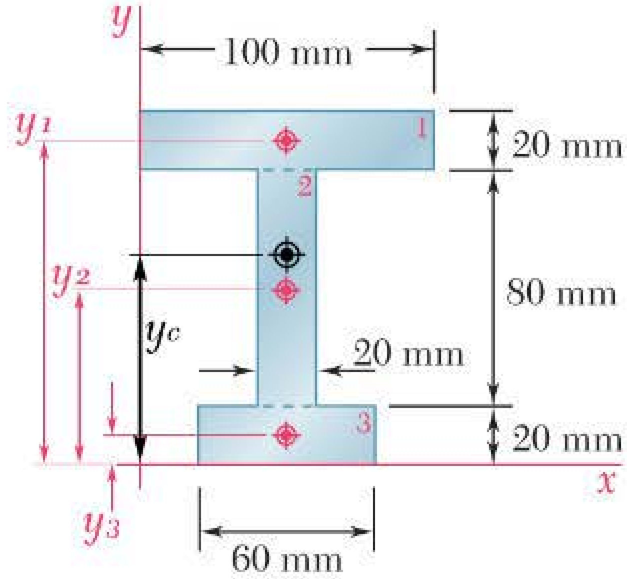
From now on we take all dimensions based on the neutral plane.

$$A\,y_1 = (20 \times 100) \times 110 = 220{,}000 \text{ mm}^3$$

$$A\,y_2 = (20 \times 80) \times 60 = 96{,}000 \text{ mm}^3$$

$$A\,y_3 = (60 \times 20) \times 10 = 12{,}000 \text{ mm}^3$$

$$\Sigma(A\,y) = 220 + 96 + 12$$
$$= 328(\times 10^3) \text{ mm}^3$$



Find the centroid	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Find y coordinate of centroid

GIVE FEEDBACK

OK

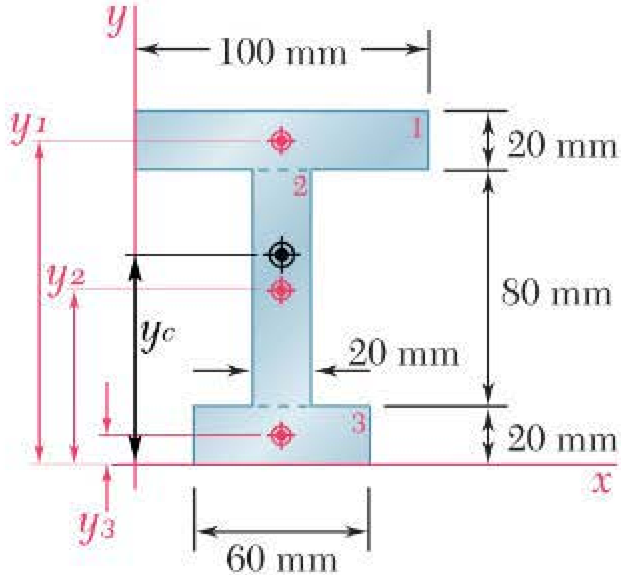
Calculate the coordinates of the centroid for a symmetrical shape--Example

Centroid y_c

$$\Sigma A = 2{,}000 + 1{,}600 + 1{,}200$$
$$= 4{,}800 \text{ mm}^2$$

$$y_c = \frac{\Sigma(A\,y)}{\Sigma A} = \frac{328{,}000}{4{,}800} = 68.333 \text{ mm}$$

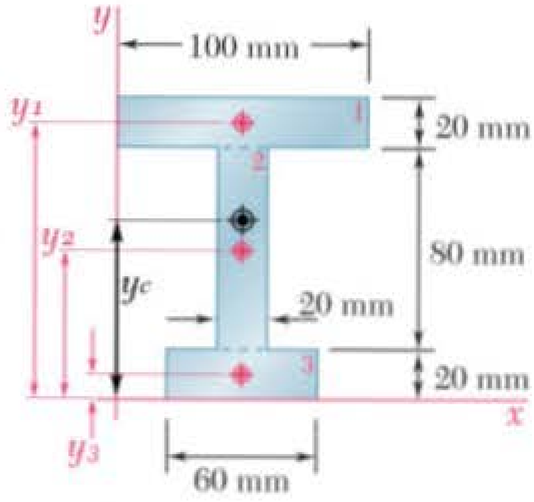
Coordinates of centroid = $\{50, 68.333\}$



Find the centroid	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Find y coordinate of centroid

GIVE FEEDBACK

OK



When calculating the centroid in the y direction, use the formula:

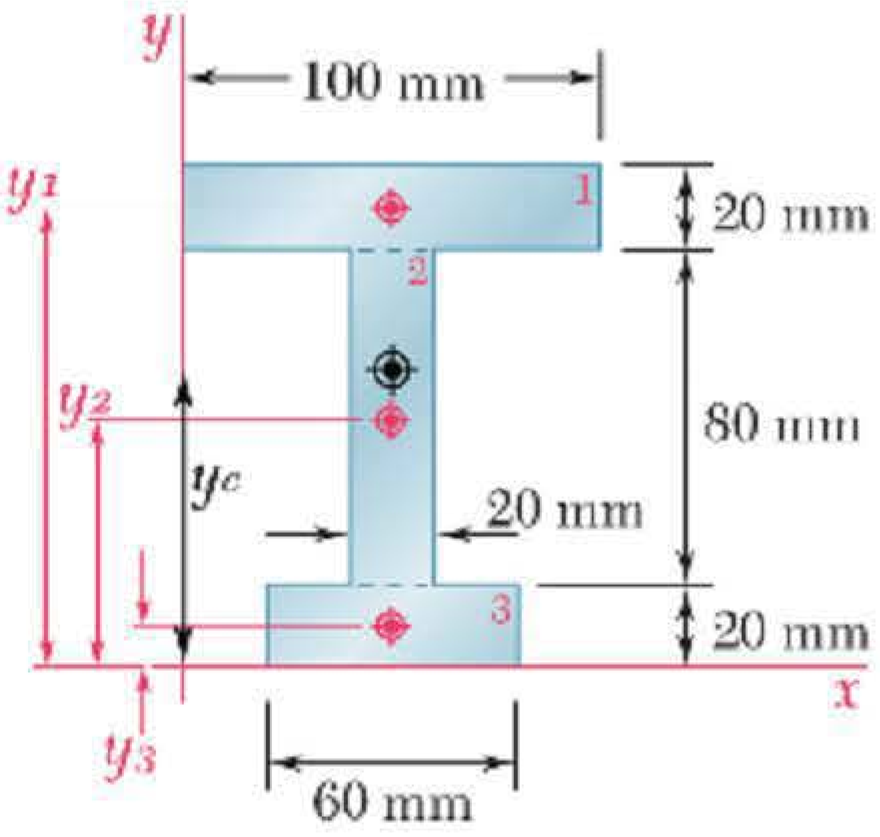
$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

Match the following values for y.

Drag statements on the right to match the left.

$y1$	20+80+10 = 110 mm
$y2$	20+40 = 60 mm
$y3$	10 mm

Do you know the answer?



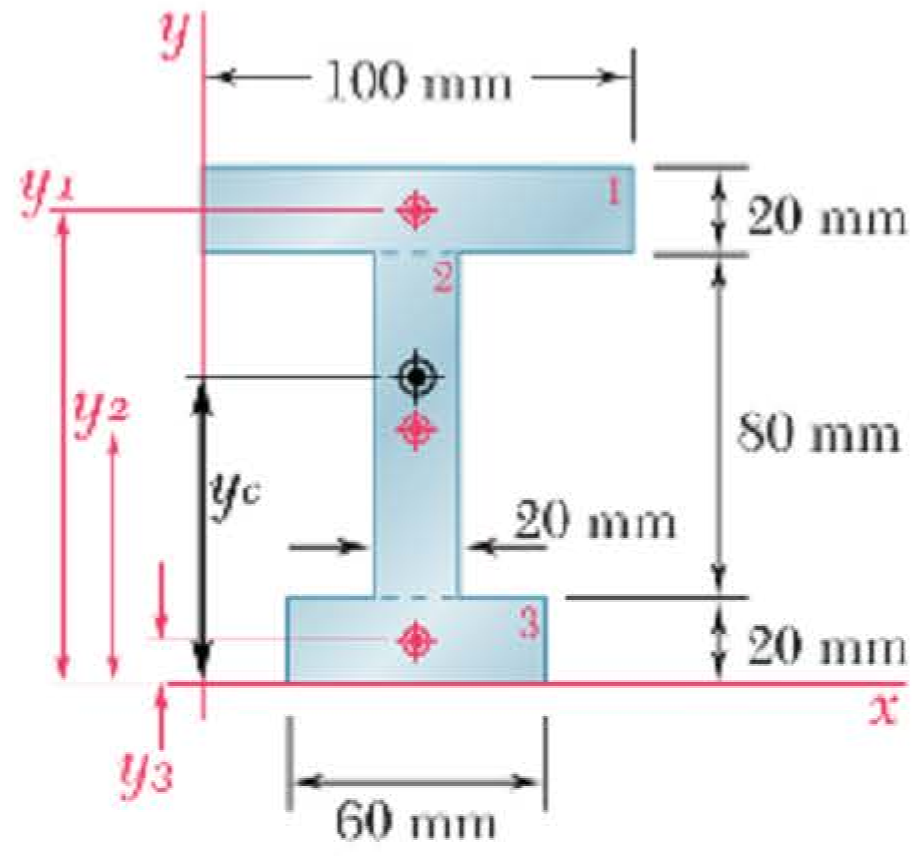
When calculating the centroid in the *y* direction, use the formula:

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

Match the following values for A.

Drag statements on the right to match the left.

A1	$60 \times 20 = 1200$ mm^2
A2	$100 \times 20 = 2000$ mm^2
A3	$2000+1600+1200 = 4800$ mm^2
ΣA	$20 \times 80 = 1600$ mm^2



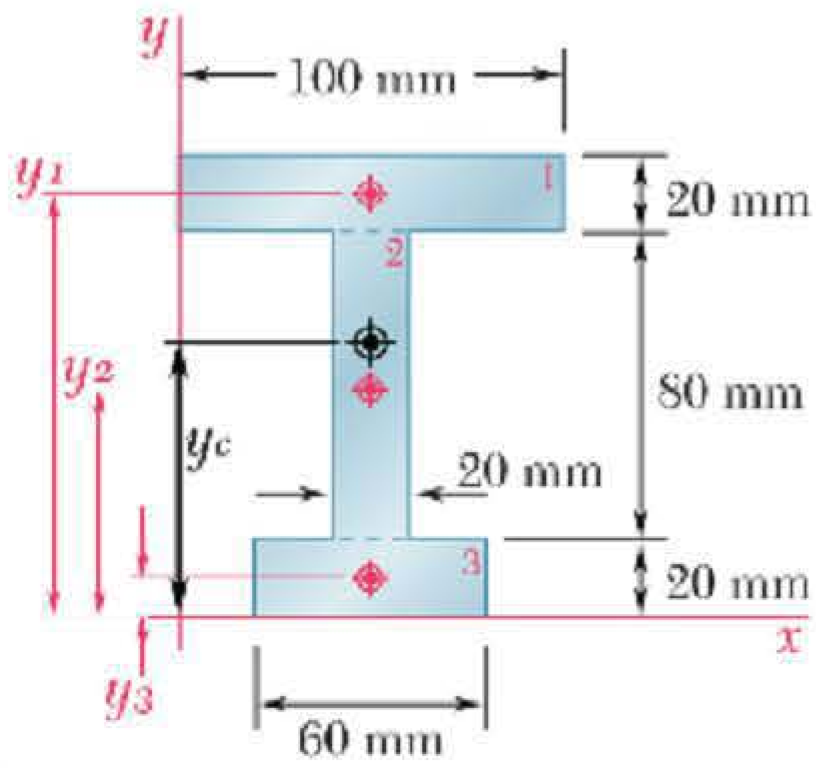
When calculating the centroid in the *y* direction, use the formula:

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

Match the following values for $A\,y$.

Drag statements on the right to match the left.

$A_1 y_1$	220000+96000+12000=328000 mm^3
$A_2 y_2$	2000 x 110 = 220000 mm^3
$A_3 y_3$	1600 x 60 = 96000 mm^3
$\Sigma(A\,y)$	1200 x 10 = 12000 mm^3



When calculating the centroid in the *y* direction, use the formula:

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

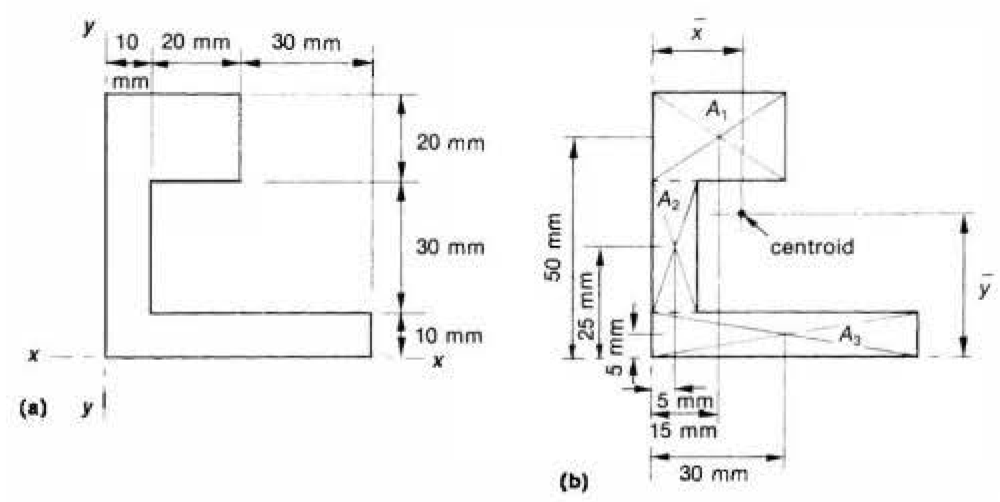
Match the following symbols and formulae.

Drag statements on the right to match the left.

y_c	50mm
ΣA	220000+96000+12000=328000 mm^3
$\Sigma(A\,y)$	328000/48000 = 68.333 mm
x_c	2000+1600+1200 = 4800 mm^2

Example

Use a table format to locate the centroid of the composite area shown below with respect to the $x - x$ and $y - y$ axes.



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Solution

Dividing into three elements, A_1, A_2 and A_3, and with each centroid located as in Figure (b).

Table format to determine centroid

Element	*Area A*	*Distance*		*Area Moment*	
		x	*y*	*Ax*	*Ay*
1	600	15	50	9000	30000
2	300	5	25	1500	7500
3	600	30	5	18000	3000
$\Sigma =$	1500	—	—	28500	45000

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So the centroid is:

$$\overline{x} = \frac{28{,}500}{1{,}500} = 19 \text{ mm}$$

and:

$$\overline{y} = \frac{40{,}500}{1{,}500} = 27 \text{ mm}$$

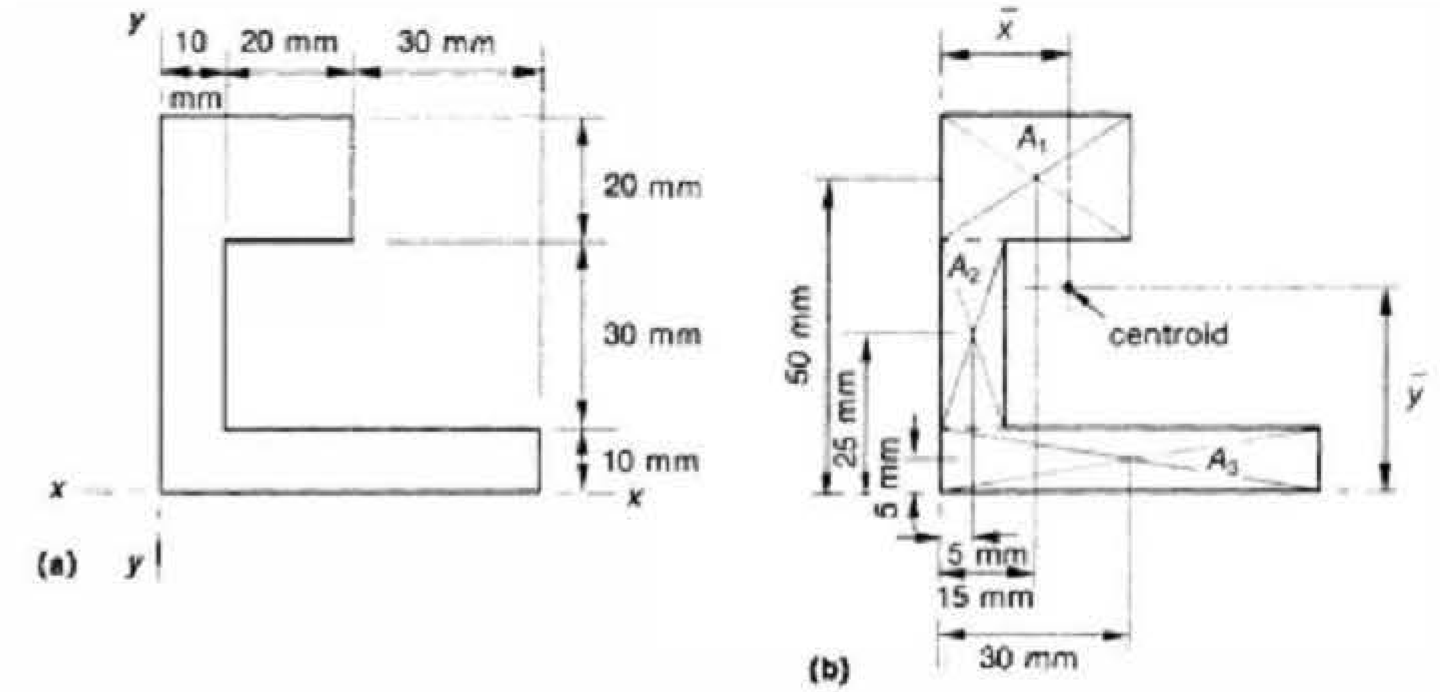


Table format to determine centroid

Element	*Area A*	*Distance*		*Area Moment*	
		x	*y*	*Ax*	*Ay*
1	600	15	50	9000	30000
2	300	5	25	1500	7500
3	600	30	5	18000	3000
Σ=	1500			28500	40,500

Match the following variables to their values listed from the table.

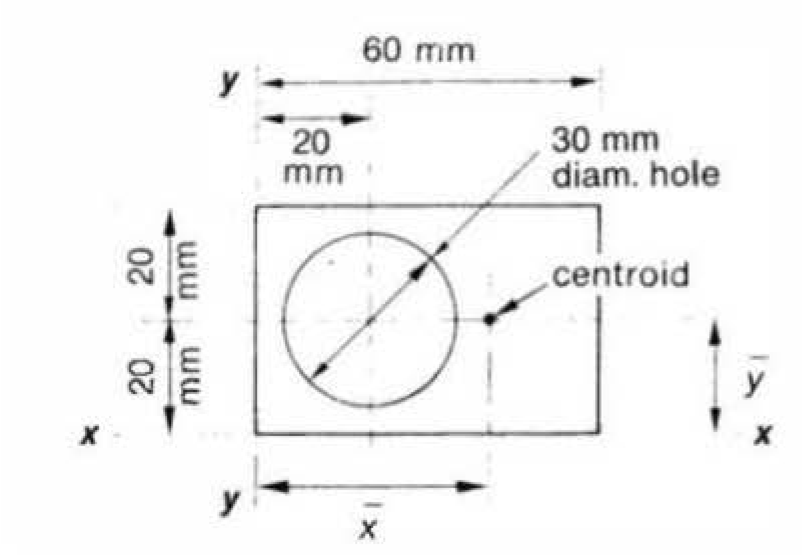
Drag statements on the right to match the left.

y_c	$\frac{28,000}{1,500} = 19$ mm
x_c	$\frac{40,500}{1,500} = 27$ mm
$\Sigma(A\,y)$	28,500 mm^3
$\Sigma(A\,x)$	45,000 mm^3

Example

Locate the centroid of the shape shown below.

In this example, the circle is subtracted from the rectangle.



GIVE FEEDBACK

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Solution

The areas removed, e.g. holes, can be regarded as negative areas.

Area of circle = $-\dfrac{\pi \times 30^2}{4} = -706.86\ \mathrm{mm}^2$

Table format to determine centroid

Element	*Area A*	*Distance*	*Area Moment*
		x	*Ax*
1	2400	30	72000
2	-706.86	20	-14137
$\Sigma =$	1693		57863

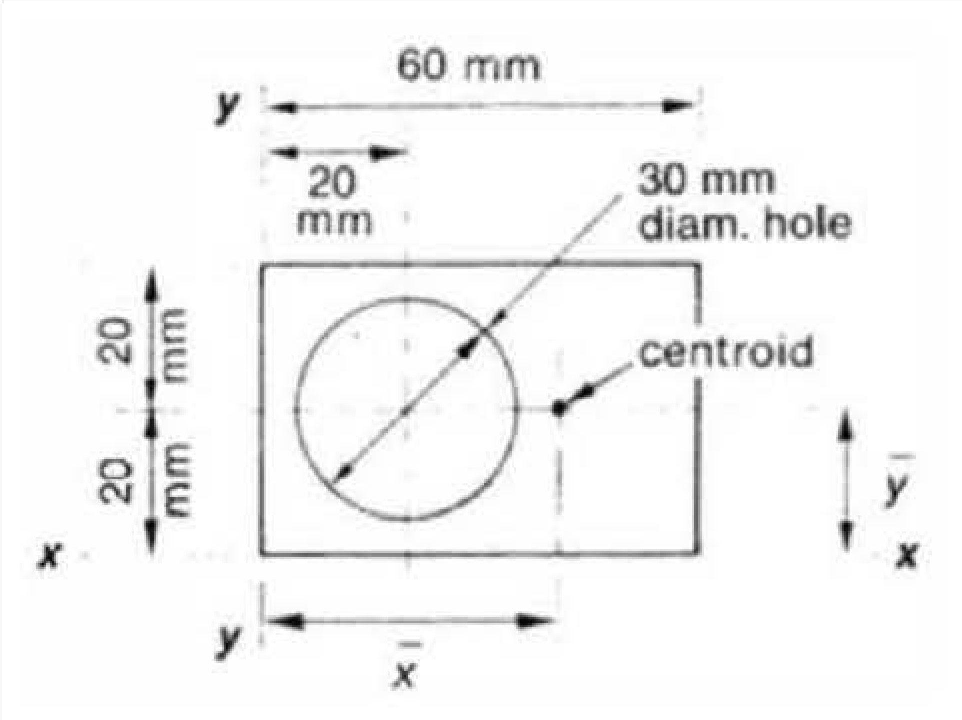
< BACK GIVE FEEDBACK CONTINUE >

So the centroid is:

$$\overline{x} = \frac{57{,}863}{1{,}693} = 34.18\ \mathrm{mm}$$

From symmetry, $\overline{y} = 20\ \mathrm{mm}$, so there is no need to calculate this.

Table format to determine centroid:



Element	*Area A*	*Distance x*	*Area Moment Ax*
1	2400	30	72000
2	-706.86	20	-14137
$\Sigma=$	1693		57863

So the centroid is:

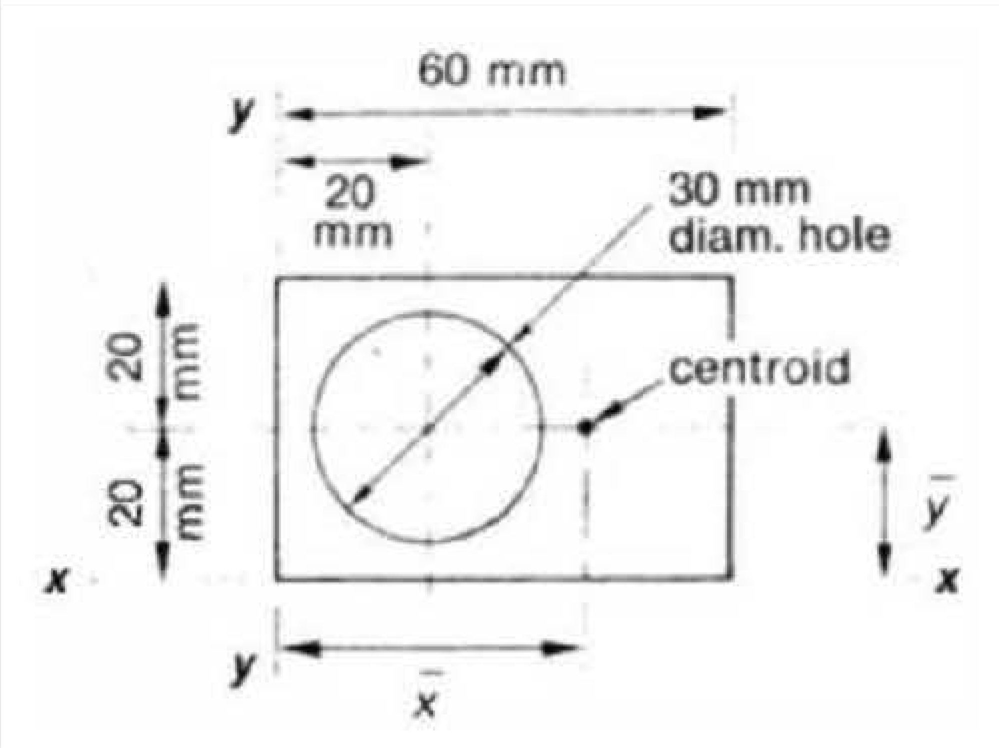
$\bar{x} = \dfrac{57{,}863}{1{,}693} = 34.18\ \text{mm}$ *(From symmetry,* $\bar{y} = 20\ \text{mm}$*, so no calculation required).*

Match the various areas to their numerical values.

Drag statements on the right to match the left.

Area of circle	1693
Area of rectangle	2400
Area of the composite shape	-706.86

Table format to determine centroid:



Element	*Area A*	*Distance x*	*Area Moment Ax*
1	2400	30	72000
2	-706.86	20	-14137
$\Sigma =$	1693		57863

So the centroid is;

$\bar{x} = \dfrac{57{,}863}{1{,}693} = 34.18\ \text{mm}$ *(From symmetry,* $\bar{y} = 20\ \text{mm}$*, so no calculation required).*

Select the correct value for the x coordinate of the centroid of the combined shape.

Click the correct answer.

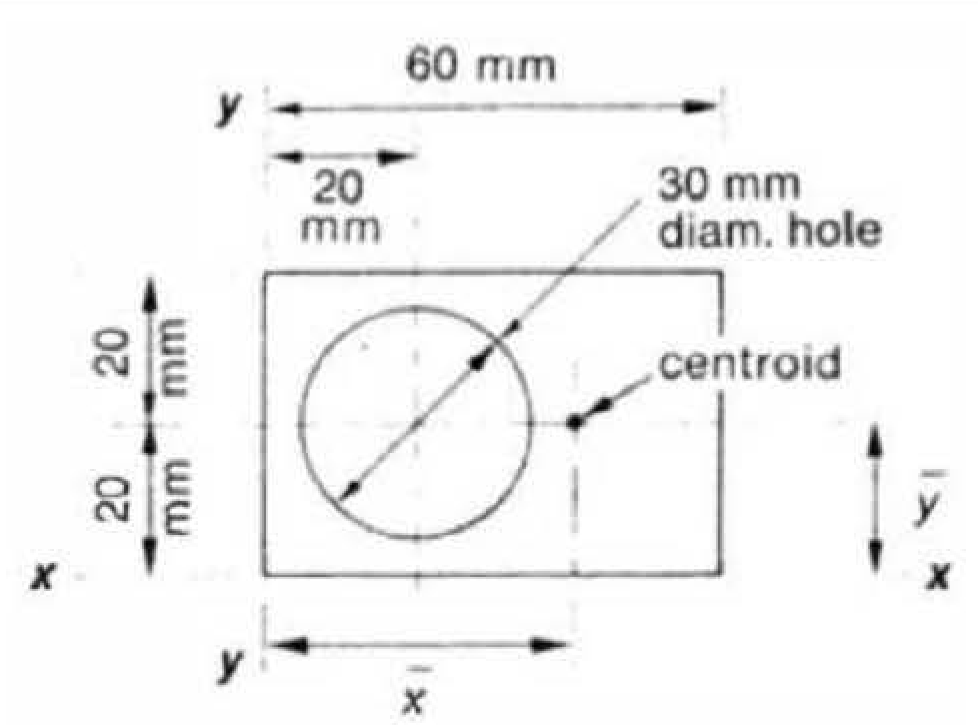
$\dfrac{14{,}137}{706.86} = 20\ \text{mm}$

$\dfrac{72{,}000}{2{,}400} = 30\ \text{mm}$

$20 + 20 = 40\ \text{mm}$

$\dfrac{57{,}863}{1{,}693} = 34.18\ \text{mm}$

Table format to determine centroid:



Element	Area A	Distance x	Area Moment Ax
1	2400	30	72000
2	-706.86	20	-14137
Σ=	1693		57863

So the centroid is:

$\bar{x} = \dfrac{57{,}863}{1{,}693} = 34.18\,\text{mm}$ *(From symmetry, $\bar{y} = 20\,\text{mm}$, so no calculation required).*

Match the various *first area moments* to their numerical values.

Drag statements on the right to match the left.

First area moment for circle	72000
First area moment for rectangle	57863
First area moment for composite shape	-14137

The second moment of area is similar to first moment of area except the distance is squared.

How second moment of area is calculated for any shape

In the name 'second moment of area', the 'second' means squared.

So the cross-section is split into tiles, and the area of each one is multiplied by the square of its distance from the neutral axis.

$$I_c = \Sigma(A \cdot d^2)$$

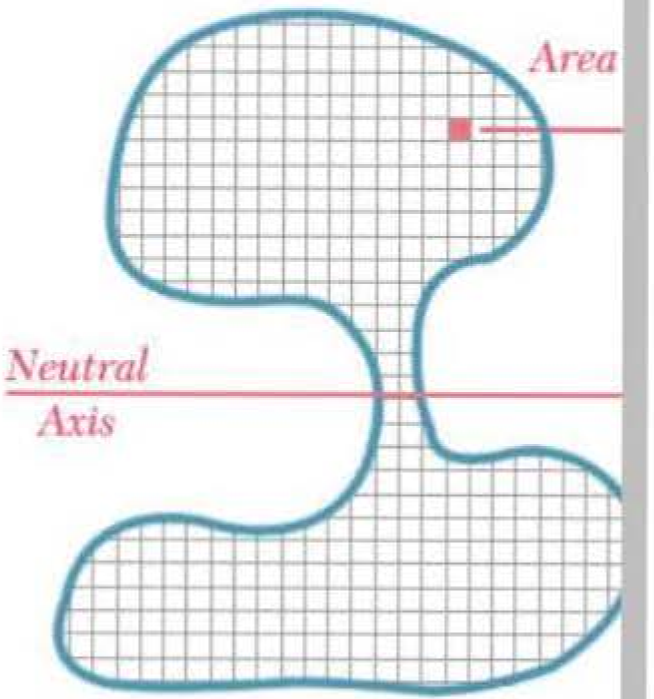
Where:

I_c = centroidal second moment of area

A = area of each element (tile)

d = distance of tile from the neutral axis

This can be calculated using integration (calculus), or by computer (divide into little bits and add them all up).



OK

What does 'second moment of area about an axis' mean?

Click the correct answer.

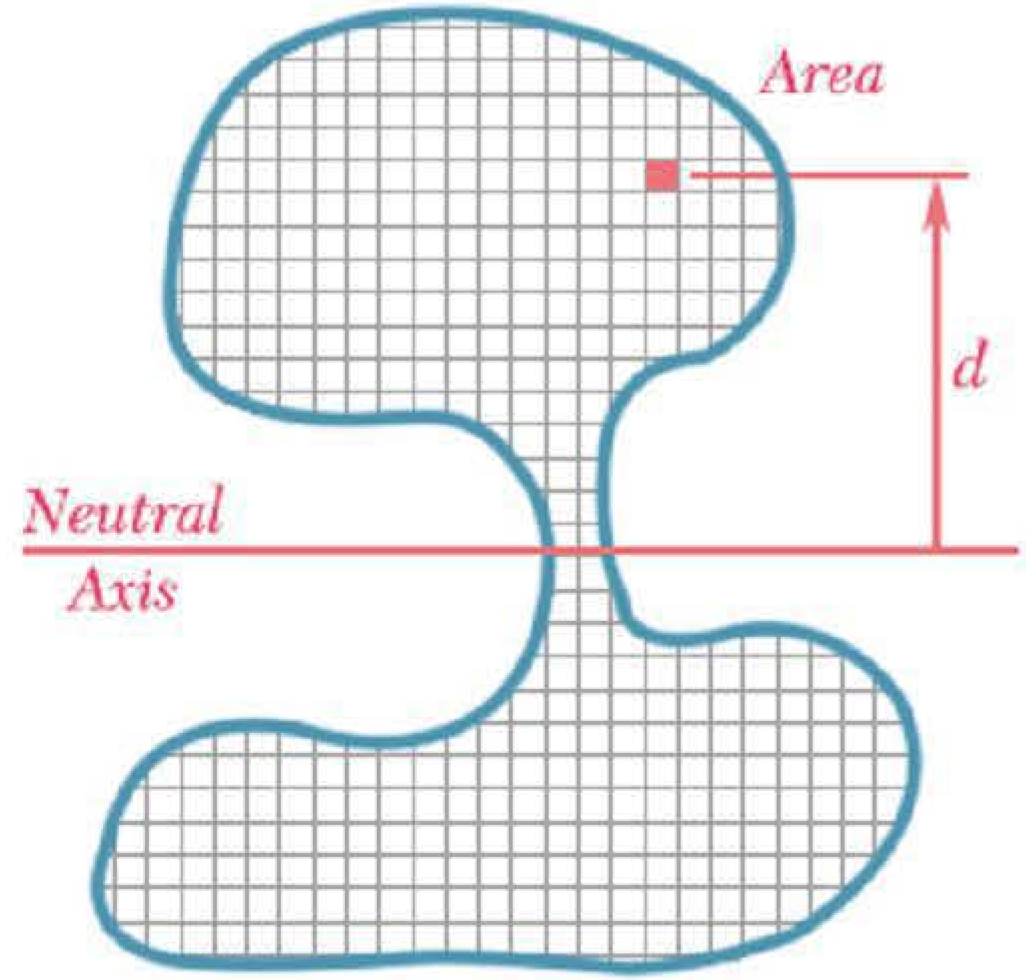
- The area multiplied by its distance to the axis
- The area multiplied by the square of its distance to the axis
- A force multiplied by the distance to the axis
- The bending moment calculated by a distance to the axis

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Match the variables from the following equation with the correct definition.

$$I_c = \Sigma(A \cdot d^2)$$

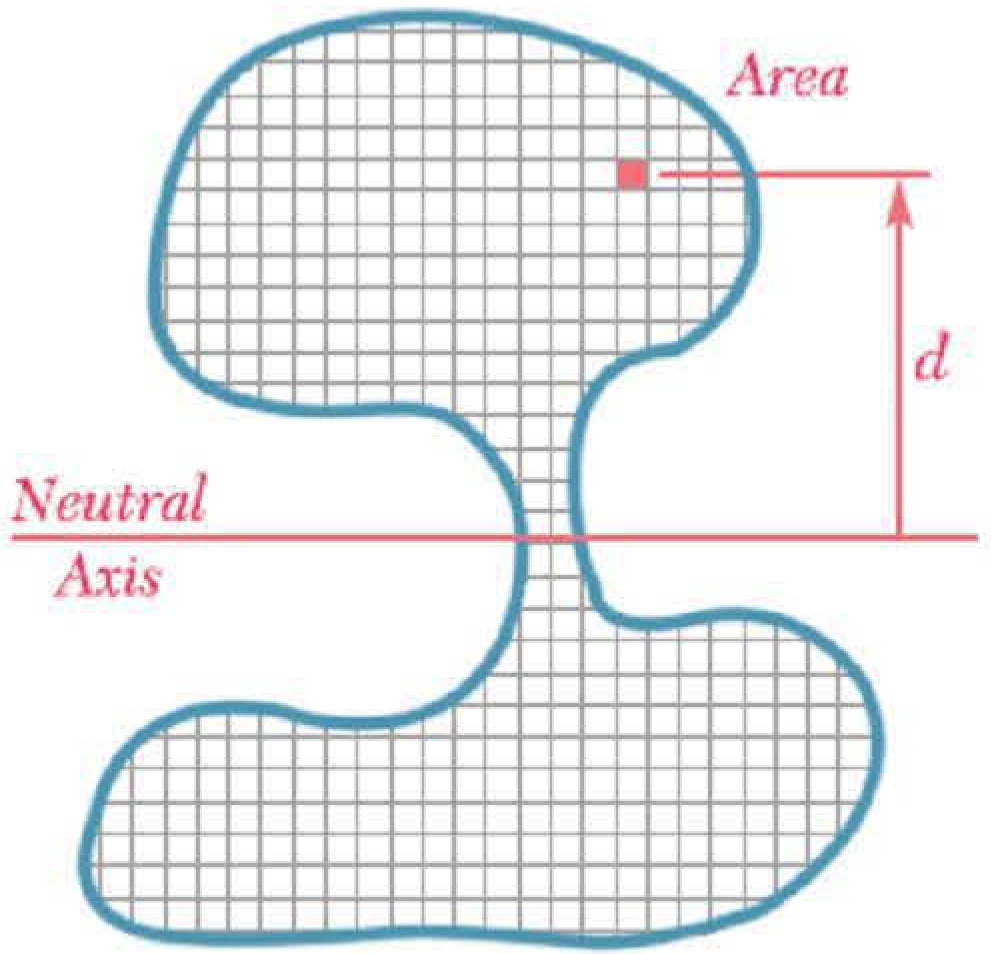


Drag statements on the right to match the left.

area of each element (tile)	I_c
centroidal second moment of area	A
distance of tile from the neutral axis	d

Match the units to the correct variable for the following equation.

$I_c = \Sigma(A \cdot d^2)$



Drag statements on the right to match the left.

A	mm^4
I_c	mm
d	mm^2

Determine the second moment of area about the centroidal axis

The symbol I with subscript c stands for the second moment of area about the centroidal axis.

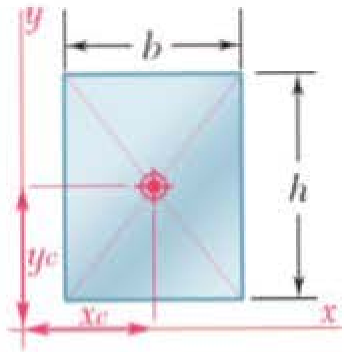
This is how the shape will behave under pure bending.

Earlier we found the second moment of area for a rectangular section, with base 30 mm x height 20 mm.

$$I_c = \frac{b \times h^3}{12} = \frac{20 \times 30^3}{12} = 45{,}000 \text{ mm}^4$$

The symbol I_c stands for **the second moment of area about the centroidal axis.**

This is how this section behaves in positive bending, the upper half in compression, the lower half in tension. The centroid (neutral plane) has zero axial stress.



The centroid is in the centre of a rectangle

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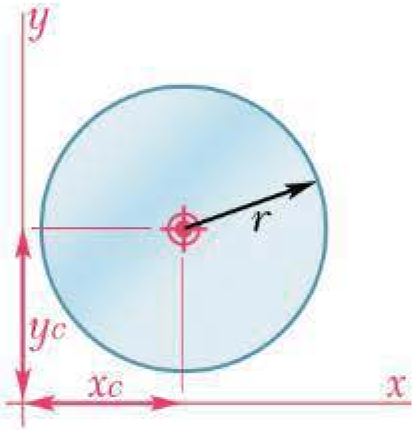
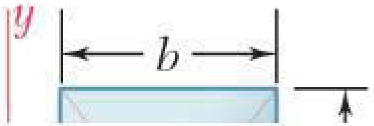
The same applies for the other simple shapes.

Shape	Diagram	*Area A*	*Second moment of area I**	Centroid
		mm^2	mm^4	mm
circle		$\frac{\pi D^2}{4}$	$\frac{\pi D^2}{64}$	At centre
rectangle		$b\,h$	$\frac{b\,h^3}{12}$	At centre. The intersection of diagonals

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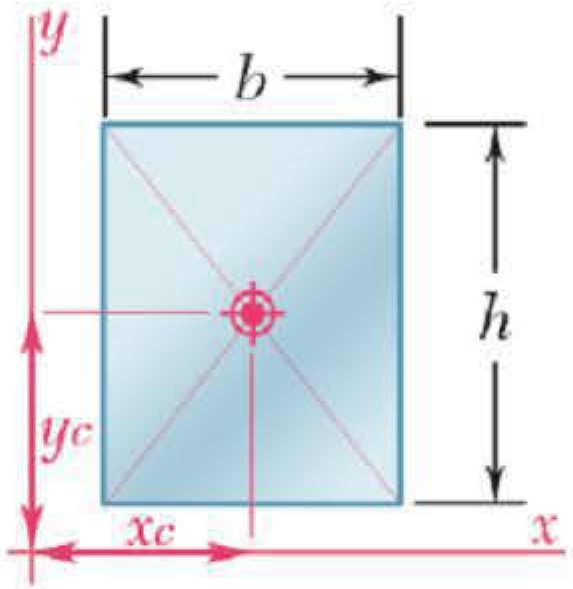
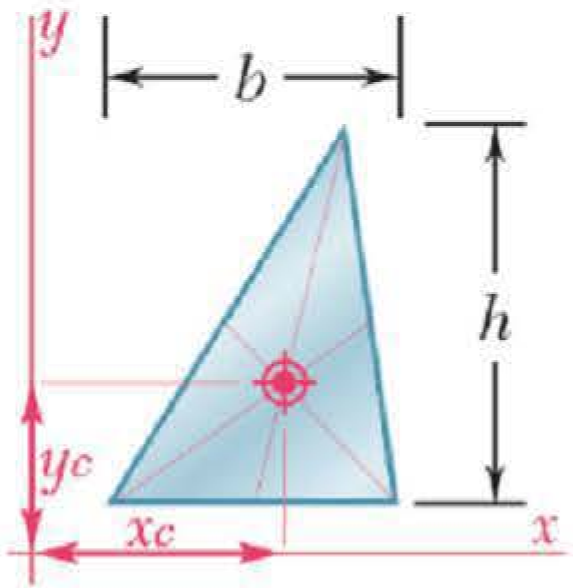
rectangle		$b\,h$	$\frac{b\,h^3}{12}$	At centre. The intersection of diagonals.
triangle of perp height h		$\frac{b\,h}{2}$	$\frac{b\,h^3}{36}$	At the intersection of medians. 1/3 of height h

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For the symbol **lc**, the **l** stands for the (please select)
and the **c** stands for the (please select) axis.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A shaft has a diameter of 29 mm. What is the second moment of area?

(Round off to nearest integer. Include units as mm^4)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

So long as the centroids are at the same location, the second moments of area can be added or subtracted directly.

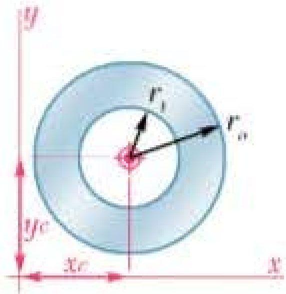
How to calculate second moment of area for hollow shapes--Example

If the centroid of various area elements in a scross-section are in the same plane, the second moments of area of each element can be directly added or subtracted.

This includes hollow objects, and can quite often be used for symmetrical objects.

Example

To calculate this pipe, we simply substract any voids as negative second moments of area.



$$I = I_o - I_i$$
$$= \frac{\pi D_o^4}{64} - \frac{\pi D_i^4}{64}$$
$$= \frac{\pi \times (D_o^2 - D_i^2)}{64}$$

GIVE FEEDBACK

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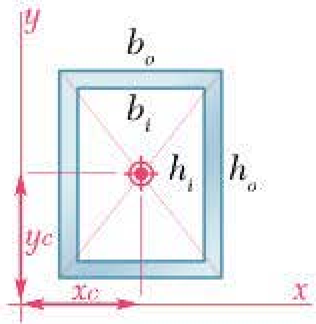
To calculate this RHS (rolled hollow section), we again substract any voids as negative second moments of area.



$$I = I_o - I_i$$
$$= \frac{b_o \times h_o^3}{12} - \frac{b_i \times h_i^3}{12}$$

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A hollow beam has a rectangular cross-section of breadth 11 mm and height 15 mm. Wall thickness is 3. Calculate the second moment of area (Ic) in mm^4, about the centroid.

(Round off to nearest integer. Include units)

Clear

Clear line

? Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

A hollow shaft has a diameter of 15 mm. Wall thickness is 2.
Calculate the second moment of area (Ic) about the centroid.

(Round off to nearest integer. Include units as mm^4)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

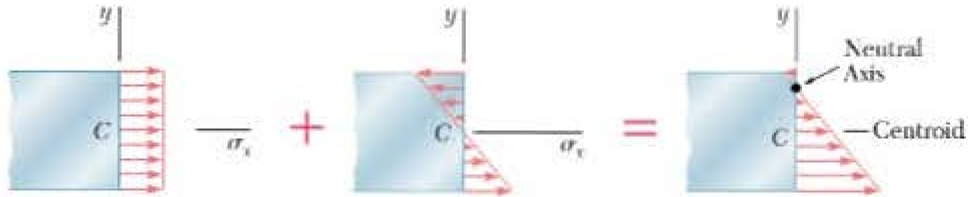
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

If pure bending is given additional tension the neutral plane will move above the centroid.

The neutral axis (or neutral plane) can be moved away from the centroid. For example, by a combination of bending and tension, the neutral axis is moved *above* the centroid.



GIVE FEEDBACK

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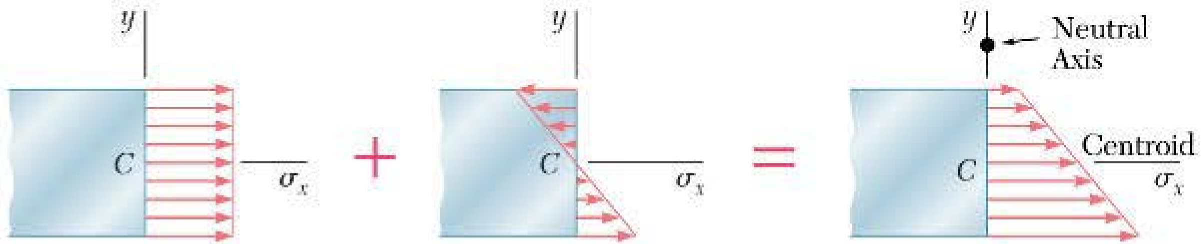
If the tension is high enough compared to the bending stress, it is even possible to move the neutral axis completely off the section.



This situation can occur in a combined section such as a floor truss. The lower chord is under some bending but a lot more tensile stress. The neutral axis can be a significant distance from the centroid of the member.

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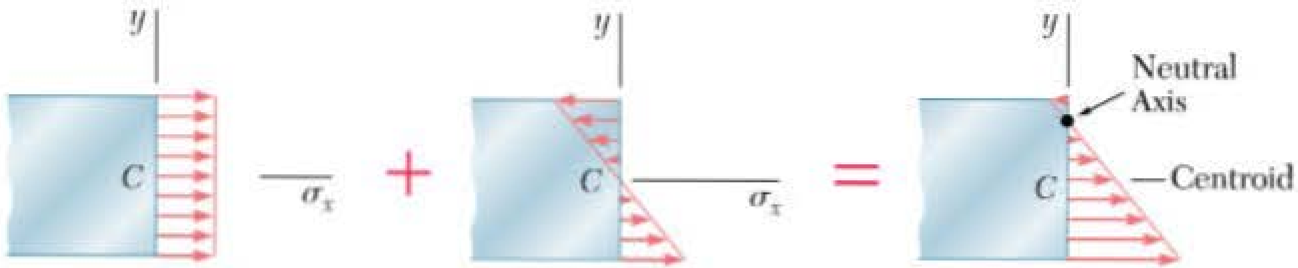
A beam is under bending.



If tension is added, what happens?

Check all that apply.

- [ ] The neutral plane is moved above the centroid
- [ ] The tensile stress in the beam is increased
- [ ] The compressive stress in the beam is increased
- [ ] The centroid is moved higher upward
- [ ] The neutral plane remains at the centroid

Do you know the answer?

The parallel axis theorem must be used whenever the neutral plane does no coincide with the centroid.

State the formula for the second moment of area about a parallel axis

When the neutral plane is off the centroid, the second moment of area is increased by the square of the distance.

The formula for the second moment of area about another axis is the **parallel axis theorem:**

$$I = I_c + A\,d^2$$

Where:

I is the second moment of area about the neutral plane

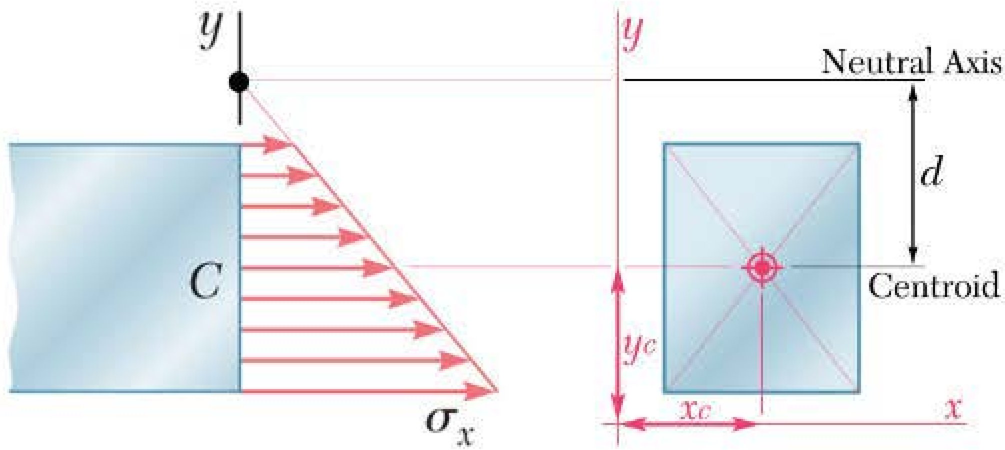
I_c is the second moment of area about the centroid

A is the area

d is the distance between the centroid and the neutral plane

GIVE FEEDBACK

CONTINUE >



Bending where the neutral axis has been shifted by a distance d above the centroid.

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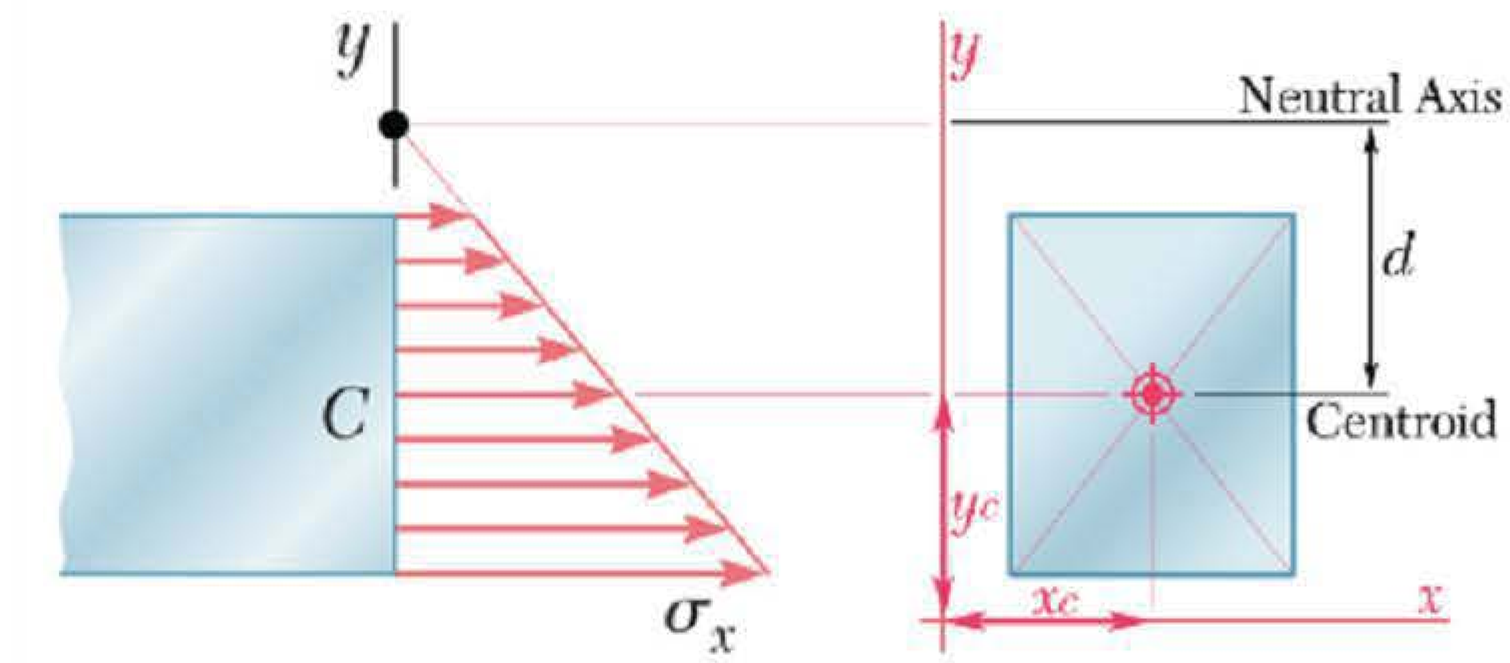
GIVE FEEDBACK

OK

$$I = I_c + A d^2$$



Match each variable to its definition.

Drag statements on the right to match the left.

The second moment of area about the neutral plane	d
The second moment of area about the centroid	I
The area	A
The distance between the centroid and the neutral plane	I_c

$$I = I_c + A\,d^2$$

What is this formula used for?

Check all that apply.

- [ ] When the neutral plane does not go through the area's centroid
- [ ] When the area is under both bending moment and compressive load
- [ ] When a beam is under bending and the neutral plane runs through the centroid
- [ ] When trying to find the centroid

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The usual way the neutral plane of a particular shape gets shifted from its centroid is when the shape is joined with other shapes to form a combined beam.

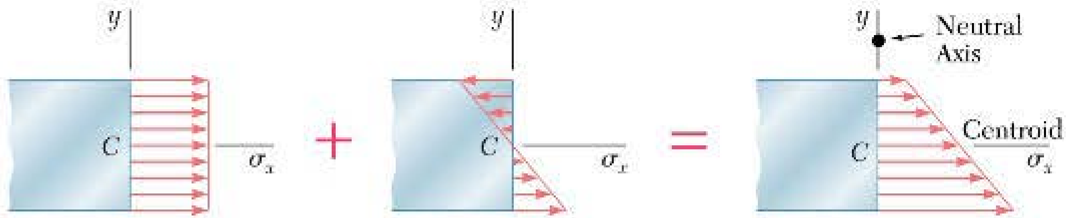
How does the neutral axis get shifted away from the centroid?

Consider these truss beams.

GIVE FEEDBACK

CONTINUE >

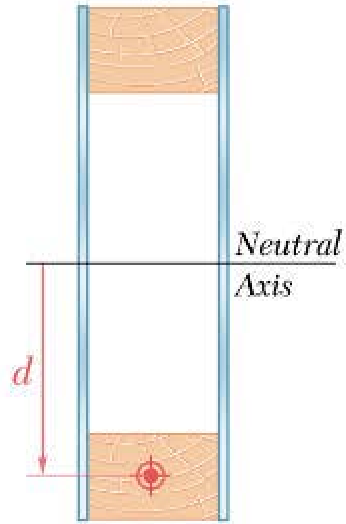
Each beam bends as one piece, which means the neutral axis of the composite beam is halfway up in the metal truss. For the bottom wooden chord, the stress would be like this (below), but even more exaggerated:



When multiple elements are tied together to form a beam, the neutral axis is at the centroid of the *combined* shape.

This means the second moment of area of *each element* must be converted using the parallel axis theorem.

$$I = I_c + A\ d^2$$

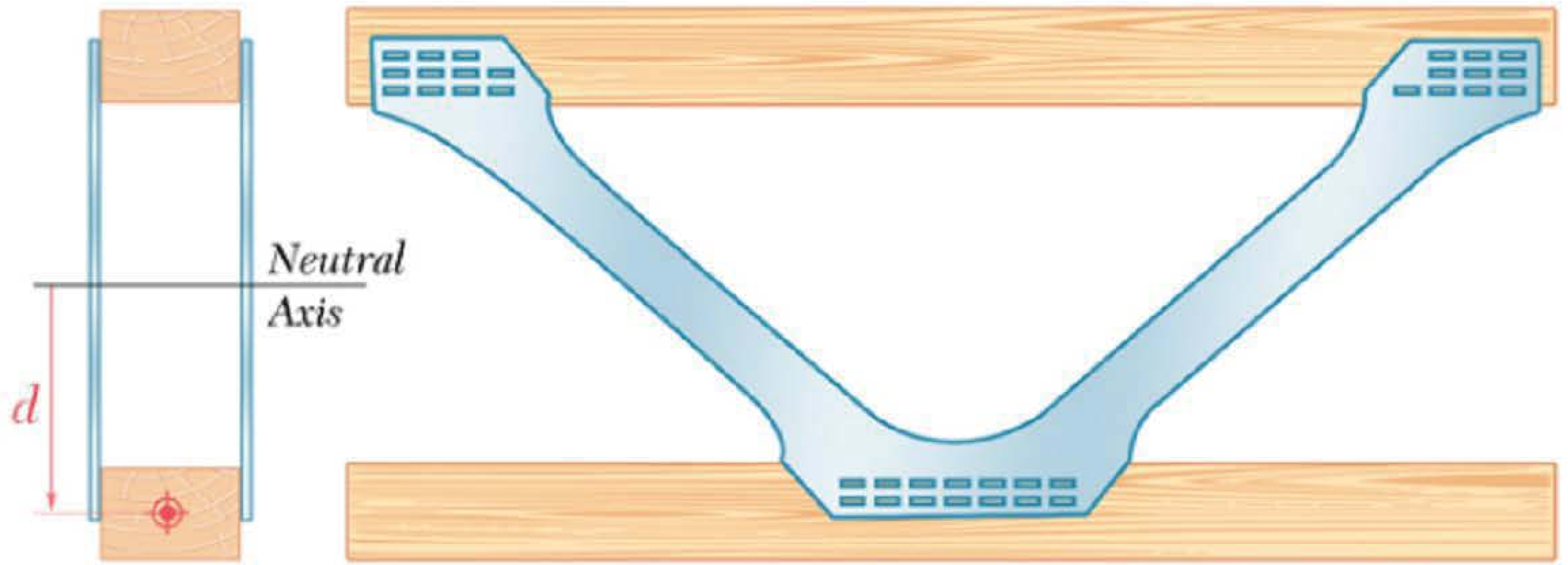


Notes:

In this example, the $A\ d^2$ term is much larger than the elemental second moment of area I_c.

This is the whole idea of a truss. Place the area as far away from the neutral plane as possible, to significantly increase the second moment of area I.

< BACK GIVE FEEDBACK OK



This represents a floor truss.

If the metal truss sides are removed, which equation best explains the stiffness of the remaining members?

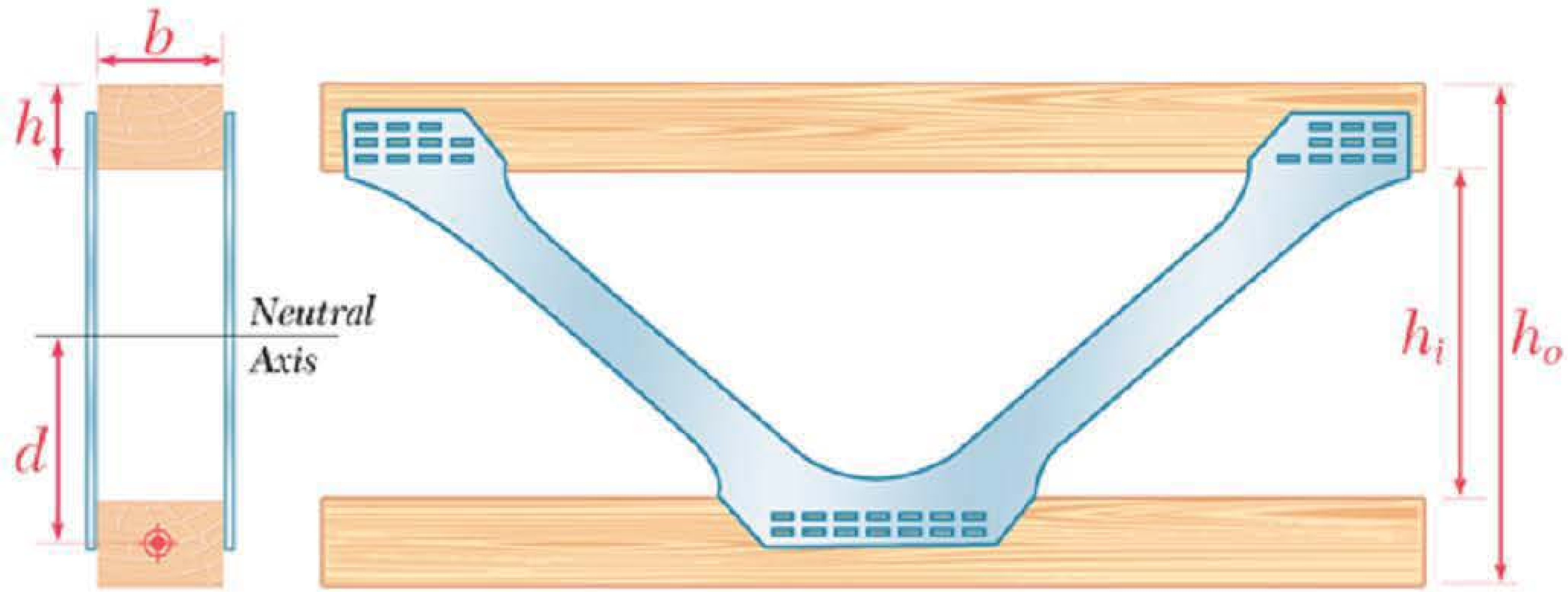
Click the correct answer.

$$x_c = \frac{\Sigma(A\,x)}{\Sigma A}$$

$$I = I_c + A\,d^2$$

$$y_c = \frac{\Sigma(A\,y)}{\Sigma A}$$

$$I_c = \frac{b \cdot h^3}{12}$$



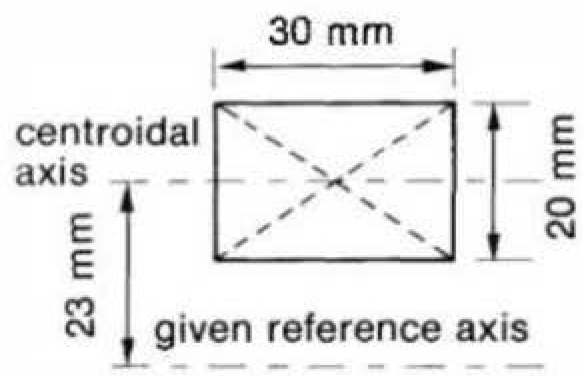
This represents a short section of a floor truss.
The Second Moment of Area can be determined by ________.

Check all that apply.

- [ ] $I = \dfrac{b \cdot (h_o^3 - h_i^3)}{12}$
- [ ] $I = I_c + A\,d^2$
- [ ] $I = \dfrac{b \cdot h^3}{12}$
- [ ] $I = \dfrac{\pi \cdot d^4}{64}$

Example

This example has a rectangular element 30 mm wide and 20 mm high, bending about an axis that is 23 mm away from the centroid. Find I.



Solution

The moment of inertia about the centroidal axis of the rectangular area is:

$$
\begin{aligned}
I_c &= \frac{b\,h^3}{12} \\
&= \frac{30 \times 20^3}{12} \\
&= 20{,}000 \text{ mm}^4
\end{aligned}
$$

< BACK GIVE FEEDBACK CONTINUE >

The parallel axis term is:

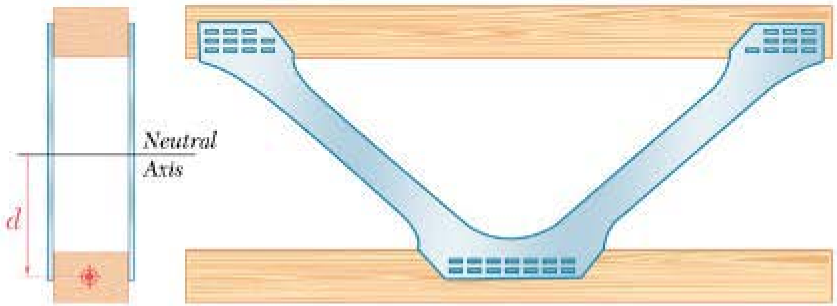
$$\begin{aligned} A\,d^2 &= (30 \times 20) \times 23^2 \\ &= 317{,}400\ \mathrm{mm}^4 \end{aligned}$$

Therefore, the required moment of inertia about the given reference axis is:

$$\begin{aligned} I &= I_c + A\,d^2 \\ &= 337{,}400\ \mathrm{mm}^4 \end{aligned}$$

Example

Determine the moment of inertia of the lower chord (75 mm wide by 50 mm tall) if the floor truss is 400 mm high.



GIVE FEEDBACK

CONTINUE >

Solution

This will need the parallel axis theorem:

$$I = I_c + A\,d^2$$

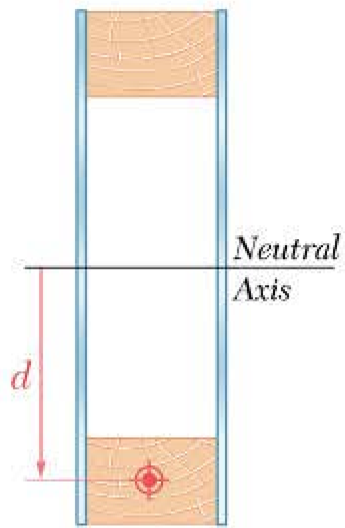
Where:

$$I_c = \frac{b \cdot d^3}{12} = \frac{75 \cdot 50^3}{12} = 781{,}250 \text{ mm}^4$$

Big number. But wait until you see the next one!

Find distance *d*:

$$d = \frac{400}{2} - \frac{50}{2} = 175 \text{ mm}$$



< BACK GIVE FEEDBACK CONTINUE >

So the parallel axis term is:
$A\,d^2 = (50 \cdot 75) \cdot 175^2 = 114{,}843{,}750 \text{ mm}^4$

That is 147 times larger the I_c term!

The total for the lower chord is:

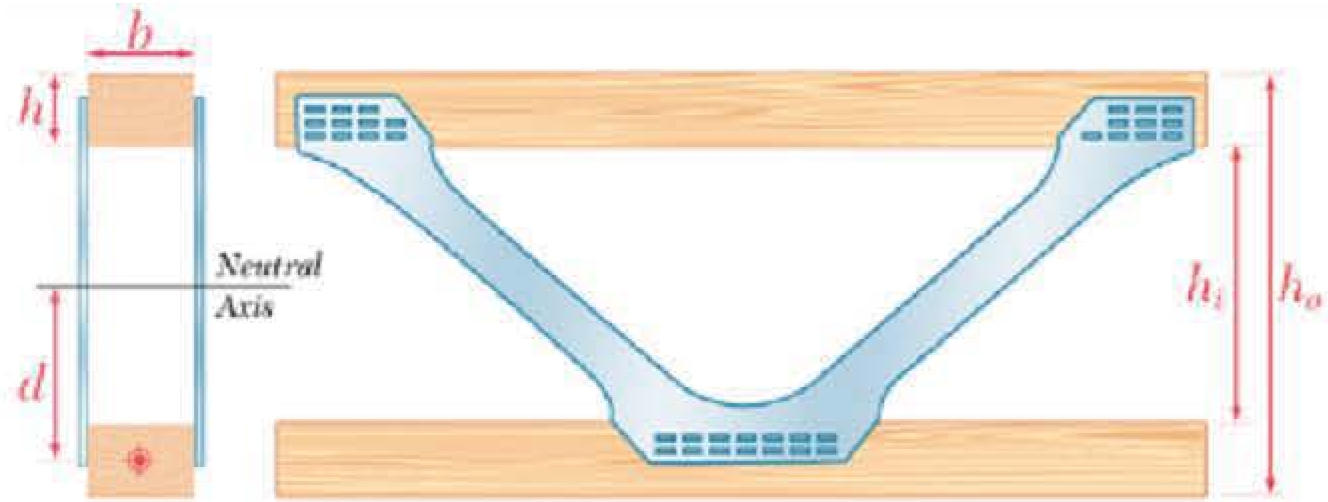
$I = 781{,}250 + 114{,}843{,}750 = 115{,}625{,}000 \text{ mm}^4$

So the total for both chords is:
$I_{tot} = 2 \cdot 115{,}625{,}000 = 231{,}250{,}000 \text{ mm}^4$

Note:
These number are huge, which is quite common with second moment of area. Engineers often work in millions of mm^4
For example, the above total would be: $I_{tot} = 115.625 (\times 10^6) \text{ mm}^4$

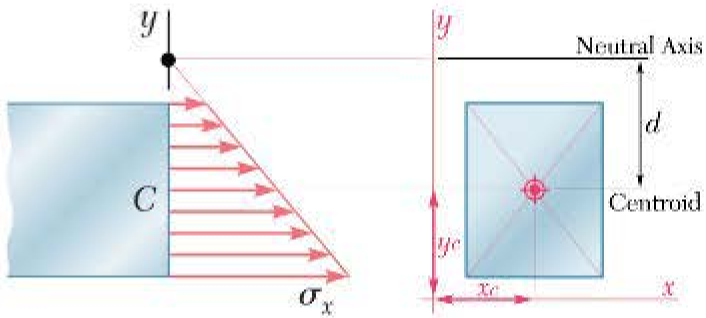
< BACK GIVE FEEDBACK OK

Calculation of the second moment of inertia of the lower chord ($b = 75\,\text{mm}$ and $h = 50\,\text{mm}$) of this floor truss, where $h_o = 400\,\text{mm}$: Match the variables to the appropriate equations.



Drag statements on the right to match the left.

I_c	$50 \times 75 = 3{,}750\,\text{mm}^2$
A	$781{,}250 + 114{,}843{,}750 = 115{,}625{,}000\,\text{mm}^4$
d	$(50 \cdot 75) \cdot 175^2 = 114{,}843{,}750\,\text{mm}^4$
$A\,d^2$	$\dfrac{75 \cdot 50^3}{12} = 781{,}250\,\text{mm}^4$
I	$\dfrac{400}{2} - \dfrac{50}{2} = 175\,\text{mm}$



This beam is 25 mm wide and 35 mm high. If the neutral plane is 25 mm above the centroid, calculate the second moment of area.

(Round off to nearest integer. Include units)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

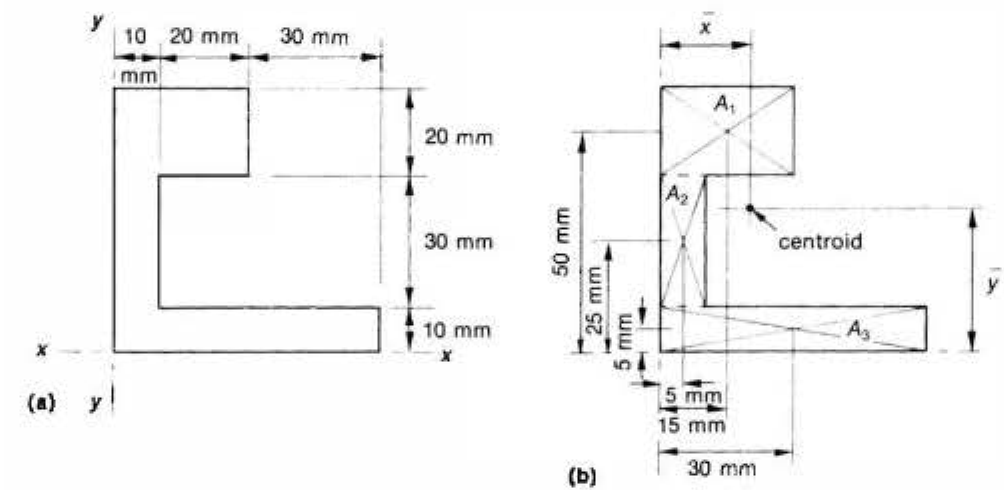
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Example

For the composite area given below, determine the moment of inertia about its horizontal centroidal axis. The centroid was found earlier, at $x_c = 19$ and $y_c = 27$. (Note $x_c = \overline{x}$)



GIVE FEEDBACK

CONTINUE >

Solution

The moment of inertia of a composite area about its centroidal axis is equal to the sum of the individual moments of inertia of its component areas relative to the common centroidal axis:

$$I = I_1 + I_2 + I_3$$

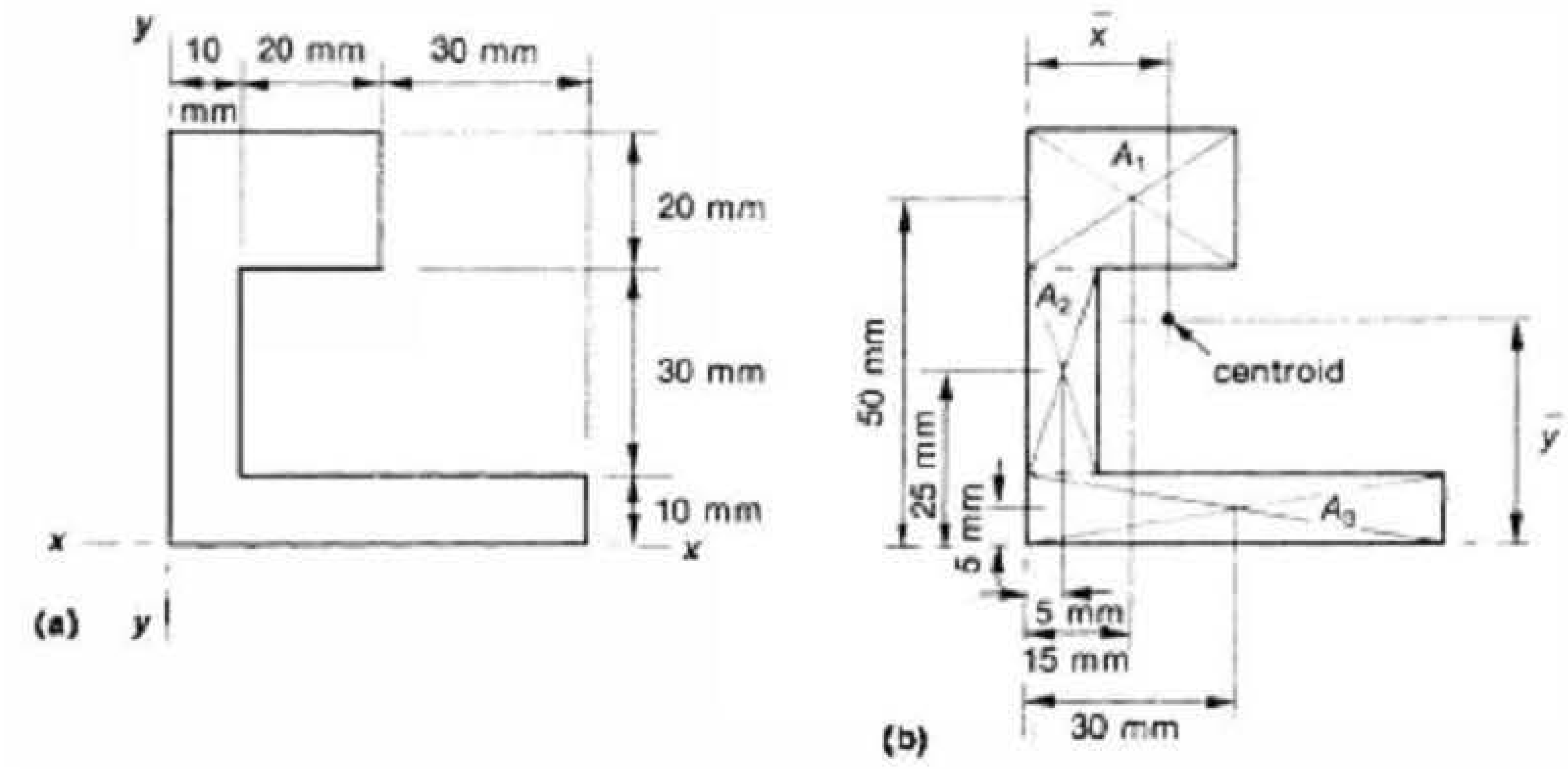
But we can only add them if they have the **same neutral axis**, which means we must use the parallel axis theorem on each element before we can add them.

Using the tabulated method:

Element	Area A	Distance from centroid d	Centroidal moment of inertia I_c	Transfer term $A\,d^2$	Transferred moment of inertia $I = I_c + A\,d^2$
1	30×20 $= 600$	23	$\frac{30 \times 20^3}{12}$ $= 20{,}000$	600×23^2 $= 317{,}400$	337 400
2	30×10 $= 300$	2	$\frac{10 \times 30^3}{12}$ $= 22{,}500$	300×2^2 $= 1{,}200$	23 700
3	60×10 $= 600$	22	$\frac{60 \times 10^3}{12}$ $= 5{,}000$	600×22^2 $= 290{,}400$	295 400
Total moment of inertia of the area about its centroidal axis $\Sigma(I) = 656{,}500\ \mathrm{mm}^4$					

So each element has a centroidal second moment of area of only 20 000, 22 500 and 5000 mm^4. But when they are combined into a single beam, the second moment becomes 656 500.

There is a massive increase due to the parallel axis term $A\,d^2$.



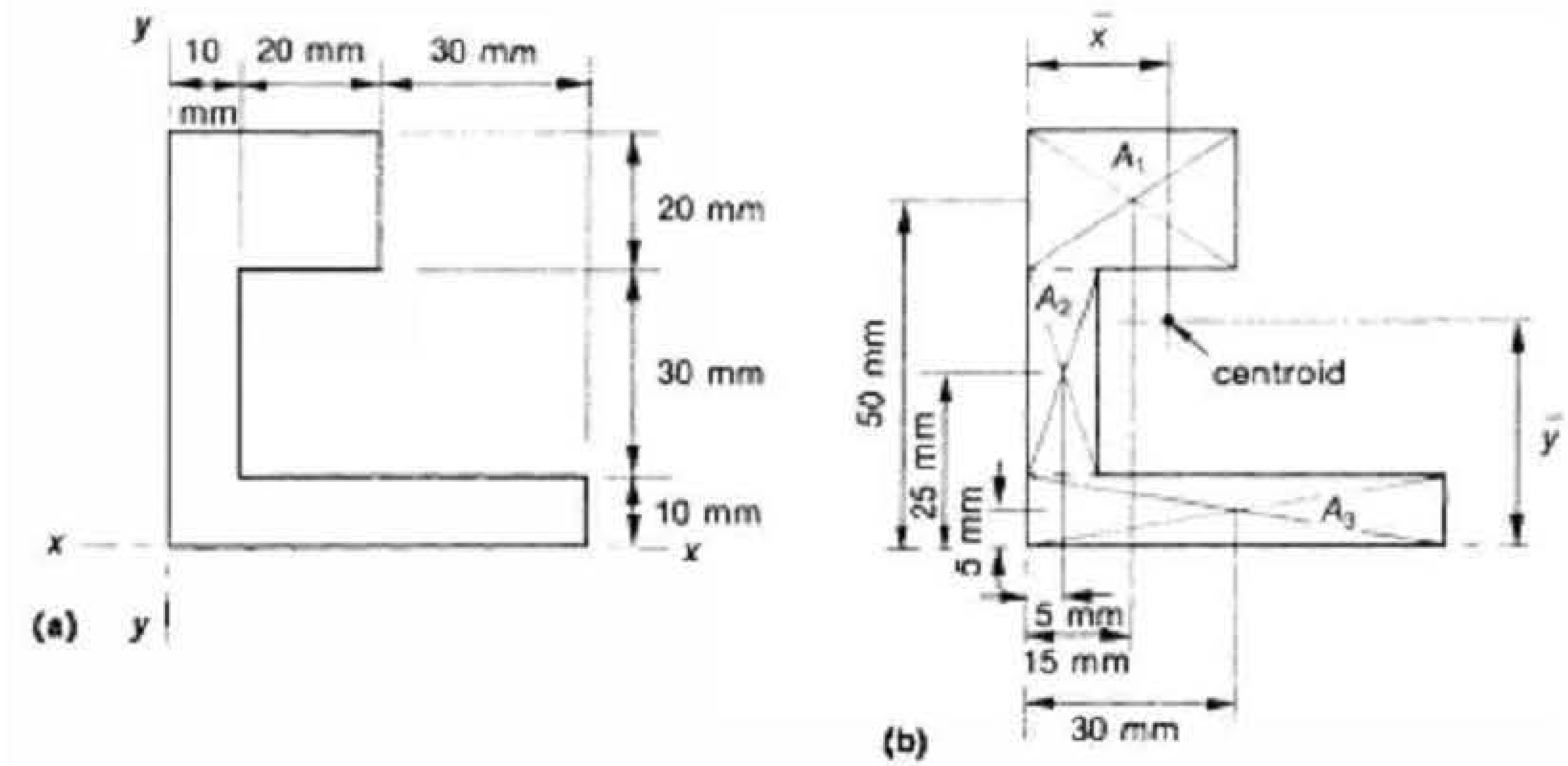
Using the tabulated method:

Element	Area A	Distance from centroid d	Centroidal moment of inertia I_c	Transfer term $A\,d^2$	Transferred moment of inertia $I = I_c + A\,d^2$
1	$30 \times 20 = 600$	23	$\frac{30 \times 20^3}{12} = 20{,}000$	$600 \times 23^2 = 317{,}400$	337 400
2	$30 \times 10 = 300$	2	$\frac{10 \times 30^3}{12} = 22{,}500$	$300 \times 2^2 = 1{,}200$	23 700
3	$60 \times 10 = 600$	22	$\frac{60 \times 10^3}{12} = 5{,}000$	$600 \times 22^2 = 290{,}400$	295 400

Compare elements in this cross-section.

Drag statements on the right to match the left.

The element with the largest area	Element 3
The element with the largest centroidal second moment of area	Element 1
The element with the largest second moment of area about the combined centroid	Element 2
The element with the smallest centroidal second moment of area	Element 1 and Element 3



Using the tabulated method:

Element	Area A	Distance from centroid d	Centroidal moment of inertia I_c	Transfer term $A\,d^2$	Transferred moment of inertia $I = I_c + A\,d^2$
1	$30 \times 20 = 600$	23	$\frac{30 \times 20^3}{12} = 20{,}000$	$600 \times 23^2 = 317{,}400$	337 400
2	$30 \times 10 = 300$	2	$\frac{10 \times 30^3}{12} = 22{,}500$	$300 \times 2^2 = 1{,}200$	23 700
3	$60 \times 10 = 600$	22	$\frac{60 \times 10^3}{12} = 5{,}000$	$600 \times 22^2 = 290{,}400$	295 400

Match the following **totals** for this cross-section.

Drag statements on the right to match the left.

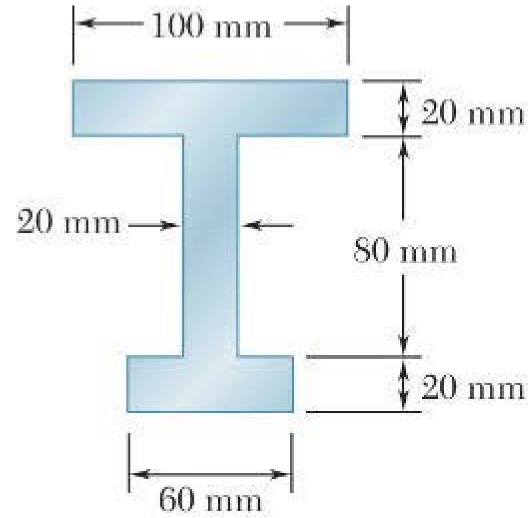
Total area	This should not be added together
Total second moment of area	656500
Total centroidal second moment of area	1500

Solve second moment of area for a combined section

Find the second moment of area $I_{x\,x}$ for this section.

$I_{x\,x}$ means the neutral plane is horizontal (parallel to the x axis).

We also know that the neutral plane will be at the centroid for the combined shape.



Find the second moment of area $I_{x\,x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

Solve second moment of area for a combined section

Find centroid for each element

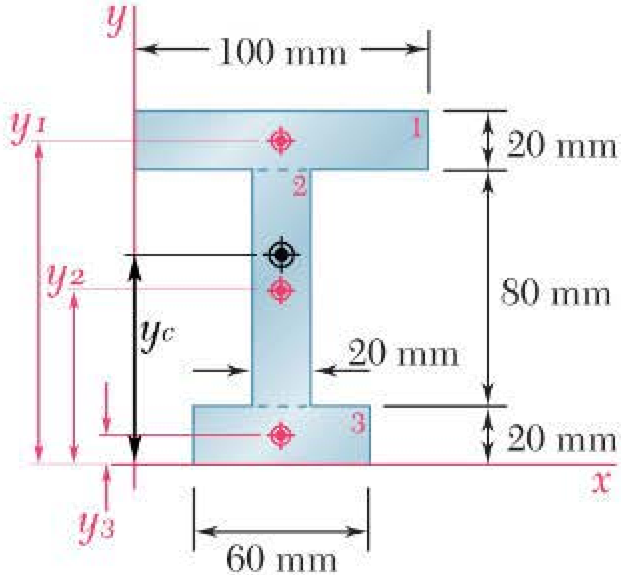
Split the section into three simple elements (1, 2 and 3).

Working only in the y direction, determine each element's centroid:

$$y_1 = 20 + 80 + \frac{20}{2} = 110 \text{ mm}$$

$$y_2 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$



Find the second moment of area I_{xx}	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

Solve second moment of area for a combined section

First moment for each element

From now on we take dimensions from the neutral plane (N.P.)

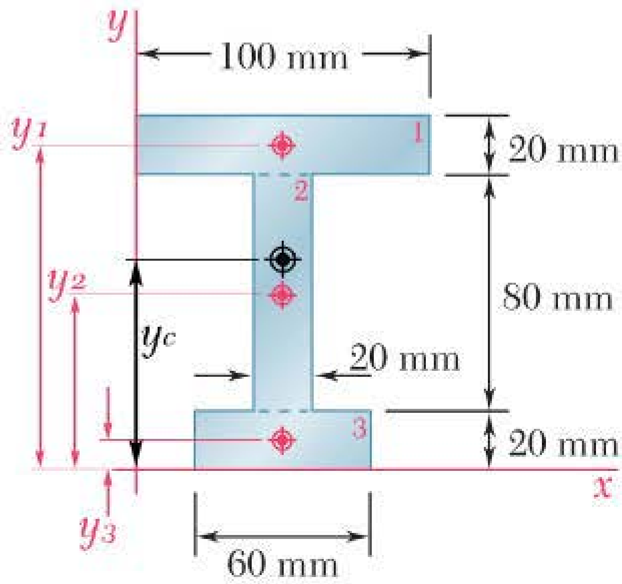
$A\,y_1 = (20 \times 100) \times 110 = 220{,}000\ \text{mm}^3$

$A\,y_2 = (20 \times 80) \times 60 = 96{,}000\ \text{mm}^3$

$A\,y_3 = (60 \times 20) \times 10 = 12{,}000\ \text{mm}^3$

$\Sigma (A\,y) = 220 + 96 + 12$ thousand

$= 328 (\times 10^3)\ \text{mm}^3$



Find the second moment of area $I_{x\,x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

Solve second moment of area for a combined section

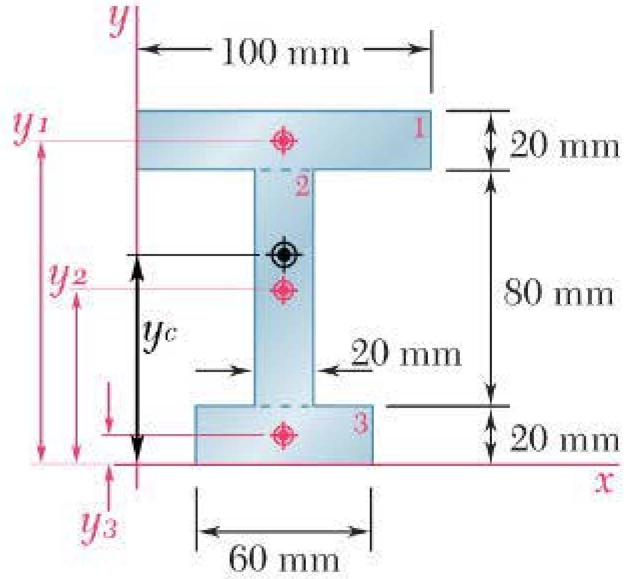
Find centroid (yc)

$$y_c = \frac{\Sigma(A \cdot y)}{\Sigma(A)}$$

$$= \frac{(220{,}000 + 96{,}000 + 12{,}000)}{(2{,}000 + 1{,}600 + 1{,}200)}$$

$$= \frac{328{,}000}{4{,}800}$$

$$= 68.33 \text{ mm}$$



Find the second moment of area $I_{x\ x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

Solve second moment of area for a combined section

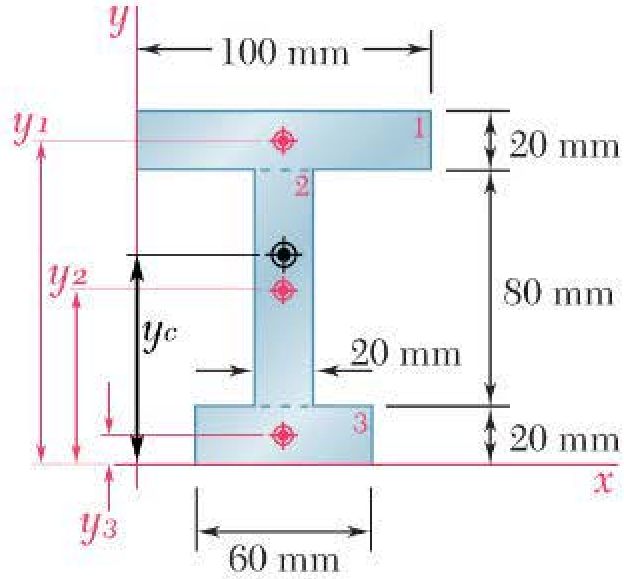
Centroidal second area moments:

$$I_c = \frac{b \times d^3}{12}$$

$$I_{c1} = \frac{100 \times 20^3}{12} = 66{,}667 \text{ mm}^4$$

$$I_{c2} = \frac{20 \times 80^3}{12} = 853{,}333 \text{ mm}^4$$

$$I_{c3} = \frac{60 \times 20^3}{12} = 40{,}000 \text{ mm}^4$$



Find the second moment of area $I_{x\,x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

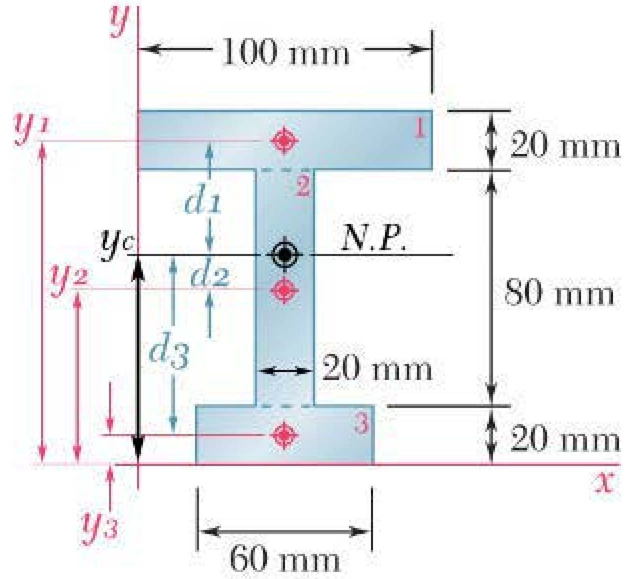
Solve second moment of area for a combined section

Parallel axis terms: $A\ d^2$

$$A_1 d_1^2 = (100 \times 20) \times (110 - 68.333)^2$$
$$= 3{,}472{,}222\ \text{mm}^2$$

$$A_2 d_2^2 = (20 \times 80) \times (60 - 68.333)^2$$
$$= 111{,}111\ \text{mm}^2$$

$$A_3 d_3^2 = (60 \times 20) \times (10 - 68.333)^2$$
$$= 4{,}083{,}333\ \text{mm}^2$$



Find the second moment of area $I_{x\ x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

Solve second moment of area for a combined section

Total second moment of area

$$I = I_c + A\,d^2$$

$I_1 = 66{,}667 + 347{,}222 = 3{,}538{,}889\ \text{mm}^4$

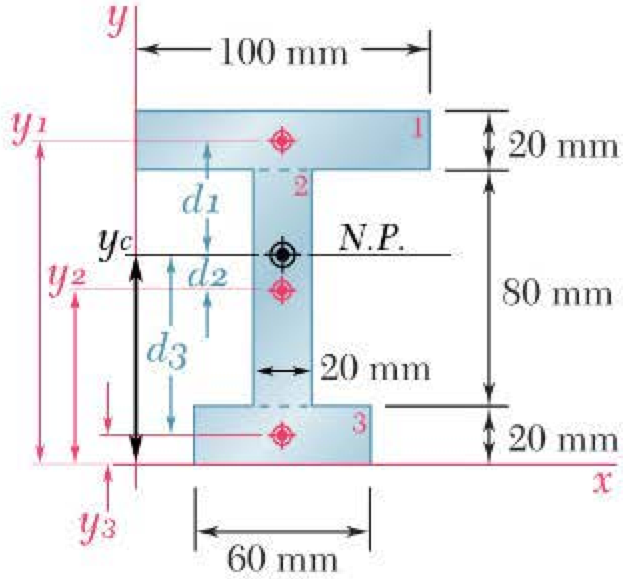
$I_2 = 853{,}333 + 111{,}111 = 964{,}444\ \text{mm}^4$

$I_3 = 40{,}000 + 4{,}083{,}333 = 4{,}123{,}333\ \text{mm}^4$

Total

$I = I_1 + I_2 + I_3$

$\quad = 8{,}626{,}667\ \text{mm}^4$



Find the second moment of area I_{xx}	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

OK

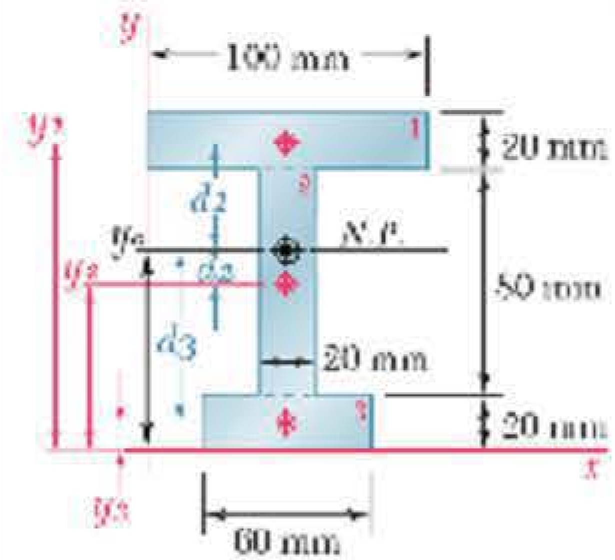
Solve second moment of area for a combined section

	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	Ic	d	Ad2	I
2		mm	mm	mm	mm2	mm3	mm4	mm	mm4	mm4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

Find the second moment of area $I_{x\ x}$	Find centroid for each element	Calculate first moment for each element	Find y coordinate of centroid	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment

GIVE FEEDBACK

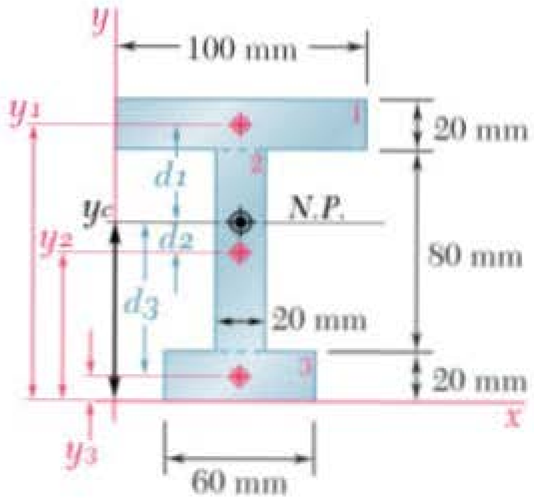
OK

A beam cross section. In order to calculate the second moment of area, the section is split into 3 elements. Match the following descriptions of each element.	

	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	Ic	d	Ad2	I
2		mm	mm	mm	mm2	mm3	mm4	mm	mm4	mm4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

Drag statements on the right to match the left.

Element with the highest centroidal second moment of area	Element 3
Element with the highest area	Element 3
Element with the highest second moment of area about the neutral axis.	Element 2
Element that is furthest away from the neautral plane	Element 1



If $y_c = 68.333$ mm, match the values of **d** used in the parallel axis theorem.

Drag statements on the right to match the left.

d_1	(20+80+10) - 68.333 = 48.667 mm
d_2	68.333 - (20+40) = 8.333 mm
d_3	68.333 - 10 = 58.333 mm

Do you know the answer?

As we already know, there is no axial stress at the neutral plane, and stress increases with the distance from the neutral plane.

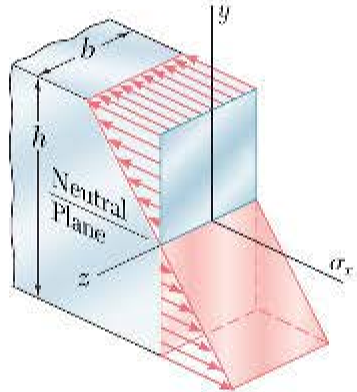
Although we have only looked at rectangular beams, the same rule applies to any beam cross-section.

Stress distribution in a beam cross-section

Regardless of the shape of the cross-section, the neutral plane always passes through its centroid.

The greater the distance of a particular point or element from the neutral plane, the greater the stress.

1. There is no axial stress at the neutral plane.
2. The maximum tensile stress occurs in the extreme fibre on the convex side of the beam.
3. The maximum compressive stress occurs in the extreme fibre on the concave side of the beam.



GIVE FEEDBACK

OK

To revise key words and knowledge of bending, match the following definitions and terms.

Drag statements on the right to match the left.

The maximum tensile stress occurs on the ______ side of the beam	convex
The maximum compressive stress occurs on the ______ side of the beam	concave
y_c is the same as _____	$\overline{y}$
Location of zero bending stress can be labelled as _____	N.P.
$\overline{x}$ is the same as _____	x_c

Do you know the answer?

With any beam cross-section, we also use the same equation for bending stress.

To use this equation we must find Bending Moment (M) and Second Moment of Area (I).

The rest is easy.

How a bending problem is solved

A bending problem is based on the equation for bending stress:

$$\sigma_b = \frac{M\,y}{I}$$

In order to complete this formula we need to first solve M and I

1. To find bending moment M as we did in the previous chapter, we can use a moment equation for the maximum if the location is known, or we can use the shear force diagram to locate the maximum.
2. To find second moment of area I as we did in this chapter, we must calculate using a simple centroidal element or use the parallel axis theroem for combined area elements.
3. y is simply the distance from the neutral plane that we want to find stress—most often it is the distance to the upper or lower extremity.

GIVE FEEDBACK

OK

$$\sigma_b = \frac{M\,y}{I}$$

In order to complete this formula we need to first solve M and I

Drag statements on the right to match the left.

The shear force diagram might be used to help find ____.	M
The parallel axis theorem might be needed in order to calculate ____.	I

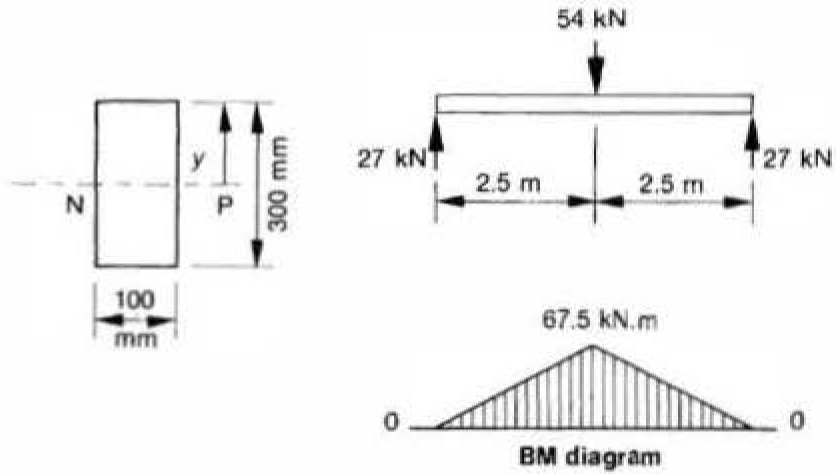
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Here is a problem taking it all the way from free body diagram and bending moment diagram, through section properties and finally solving the bending stress.

Example

A beam of rectangular cross-section, 300 mm deep by 100 mm wide, is subjected to a positive bending moment of 67.5 kN.m. Determine the maximum value of bending stress.



GIVE FEEDBACK

CONTINUE >

Solution

The cross-section of the beam is as shown above.

Moment of inertia:

$$I = \frac{b\,h^3}{12}$$
$$= \frac{100 \times 300^3}{12}$$
$$= 225 \times 10^6 \text{ mm}^4$$

Distance to the extreme fibre:

$$y = 150 \text{ mm}$$

Bending moment:

$$M = 6.75 \text{ kN.m}$$
$$= 6.75 \times 10^6 \text{ N.mm}$$

< BACK GIVE FEEDBACK CONTINUE >

Bending stress:

$$\sigma_b = \frac{M\,y}{I}$$

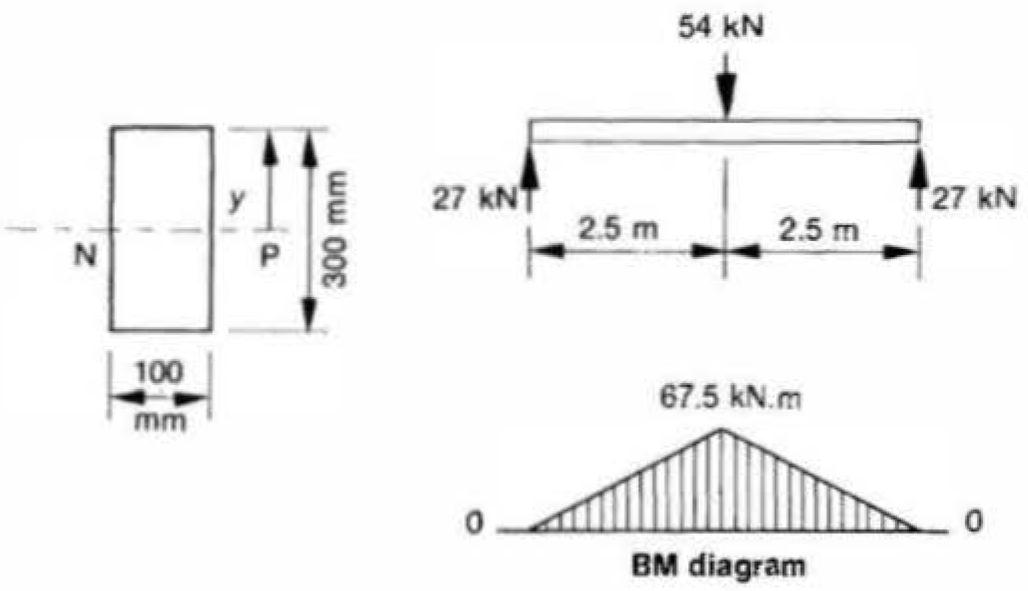
$$= \frac{67.5 \times 10^6\ \mathrm{N.mm} \times 150\ \mathrm{mm}}{225 \times 10^6\ \mathrm{mm}^4}$$

$$= 45\ \mathrm{MPa}$$

In this case, because of the symmetrical cross-section, the extreme fibres on the tension and compression sides are at the same distance from the neutral plane. Therefore the answer represents the maximum compressive stress in the top fibre as well as the maximum tensile stress in the bottom fibre.

< BACK GIVE FEEDBACK OK

A beam of rectangular cross-section, 300 mm deep by 100 mm wide, is subjected to a positive bending moment of 67.5 kN.m.



Moment of inertia:

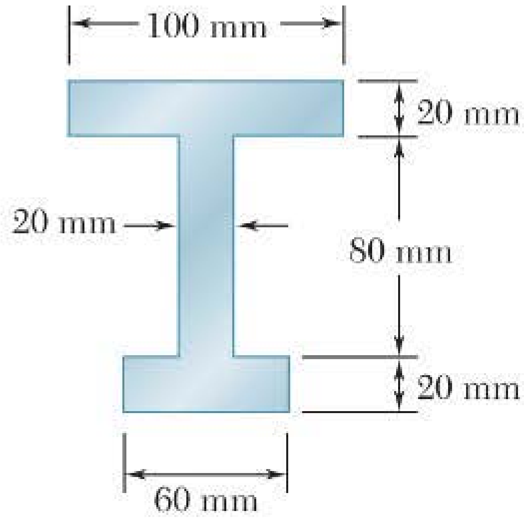
$$I = \frac{b\,h^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6\ \mathrm{mm}^4$$

Determine the maximum value of bending stress.

Click the correct answer.

Solve bending stress for a combined section

Find the maximum tensile stress of this 3 m long beam with a load of 20 kN at midspan. Ignore its own weight.



Bending stress is:

$$\sigma_b = \frac{M\,y}{I}$$

Since this is in positive bending (sagging) the maximum tensile stress will occur at the bottom.

Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

Find centroid for each element

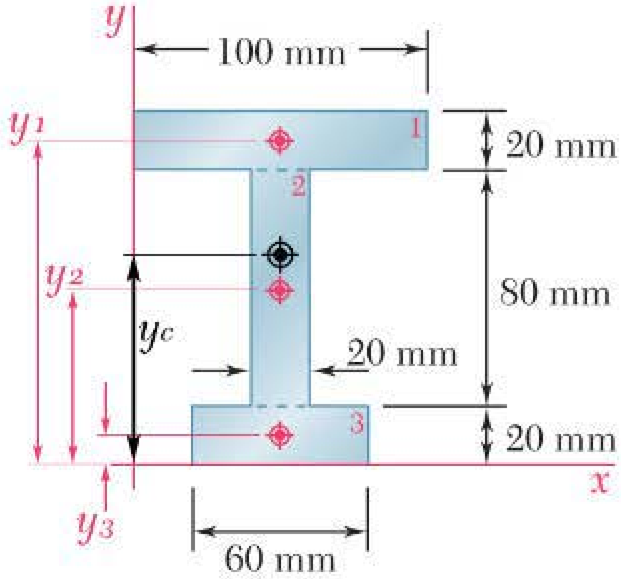
Split the section into three simple elements (1, 2 and 3).

Working only in the y direction, determine each element centroid:

$$y_1 = 20 + 80 + \frac{20}{2} = 110 \text{ mm}$$

$$y_2 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

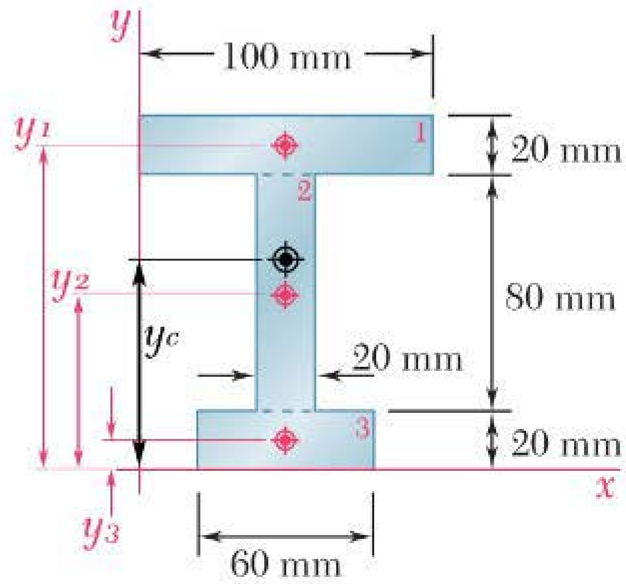
Solve bending stress for a combined section

Find centroid

$$y_c = \frac{(\Sigma A\, y)}{\Sigma A}$$

$$y_c = \frac{328{,}000}{4{,}800}$$
$$= 68.333 \text{ mm}$$

This is measured using the x and y axes shown in the diagram. The shape is symmetrical in the x direction, so
$x_c = 50$ mm.



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

First moment for each element

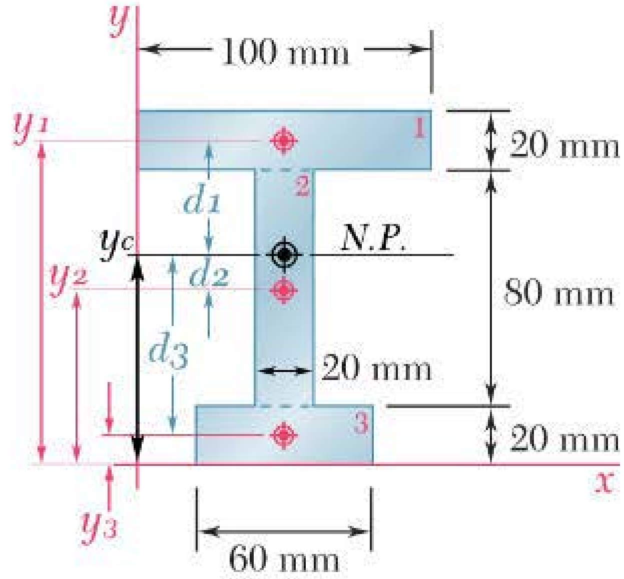
From here on we base dimensions from the neutral plane (N.P.)

$$A\,y_1 = (20 \times 100) \times 110 = 220{,}000 \text{ mm}^3$$

$$A\,y_2 = (20 \times 80) \times 60 = 96{,}000 \text{ mm}^3$$

$$A\,y_3 = (60 \times 20) \times 10 = 12{,}000 \text{ mm}^3$$

$$\Sigma(A\,y) = 220 + 96 + 12$$
$$= 328\,(\times 10^3) \text{ mm}^3$$



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

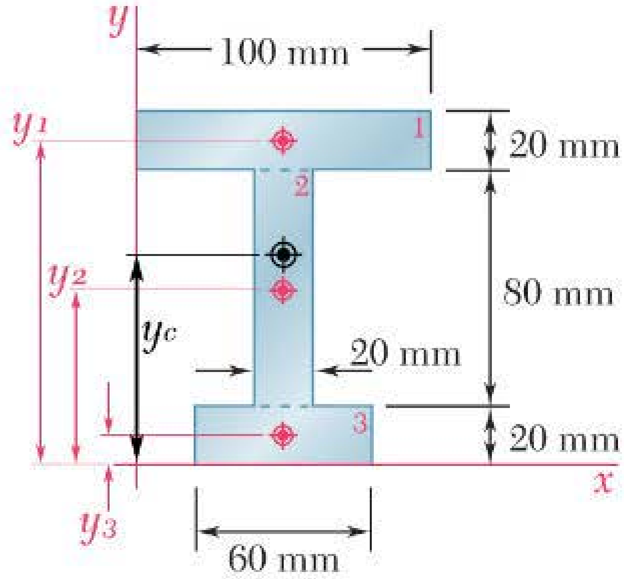
Centroidal second area moments:

$$I_c = \frac{b \times d^3}{12}$$

$$I_{c1} = \frac{100 \times 20^3}{12} = 66{,}667 \text{ mm}^4$$

$$I_{c2} = \frac{20 \times 80^3}{12} = 853{,}333 \text{ mm}^4$$

$$I_{c3} = \frac{60 \times 20^3}{12} = 40{,}000 \text{ mm}^4$$



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

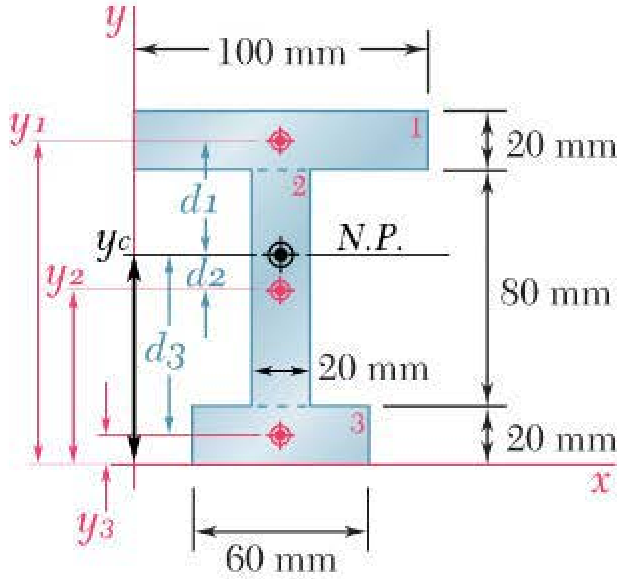
Solve bending stress for a combined section

Parallel axis terms: $A\ d^2$

$$A_1 d_1^2 = (100 \times 20) \times (110 - 68.333)^2$$
$$= 3{,}472{,}222\ \text{mm}^2$$

$$A_2 d_2^2 = (20 \times 80) \times (60 - 68.333)^2$$
$$= 111{,}111\ \text{mm}^2$$

$$A_3 d_3^2 = (60 \times 20) \times (10 - 68.333)^2$$
$$= 4{,}083{,}333\ \text{mm}^2$$



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

Total second moment of area

$I = I_c + A\,d^2$

$I_1 = 66{,}667 + 347{,}222 = 3{,}538{,}889 \text{ mm}^4$

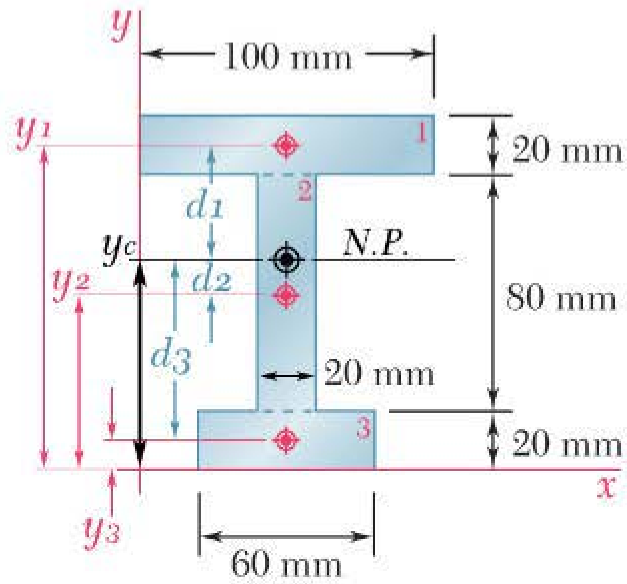
$I_2 = 853{,}333 + 111{,}111 = 964{,}444 \text{ mm}^4$

$I_3 = 40{,}000 + 4{,}083{,}333 = 4{,}123{,}333 \text{ mm}^4$

Total

$I = I_1 + I_2 + I_3$

$= 8{,}626{,}667 \text{ mm}^4$



Solve for maximum bending stress	Find centroid for each element	Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	Ic	d	Ad2	I
2		mm	mm	mm	mm2	mm3	mm4	mm	mm4	mm4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	68.333				

Solve for maximum bending stress

Find centroid for each element

Find centroid y_c

Calculate first moment for each element

Calculate centroidal second area moments

Calculate parallel axis terms

Calculate total second area moment

Tabulated calculation for total second area moment

Find th maximu bendin mome

GIVE FEEDBACK

OK

Solve bending stress for a combined section

Now find the maximum stress of this 3 m long beam with a load of 20 kN at midspan. Ignore its own weight.

Before we find bending stress:

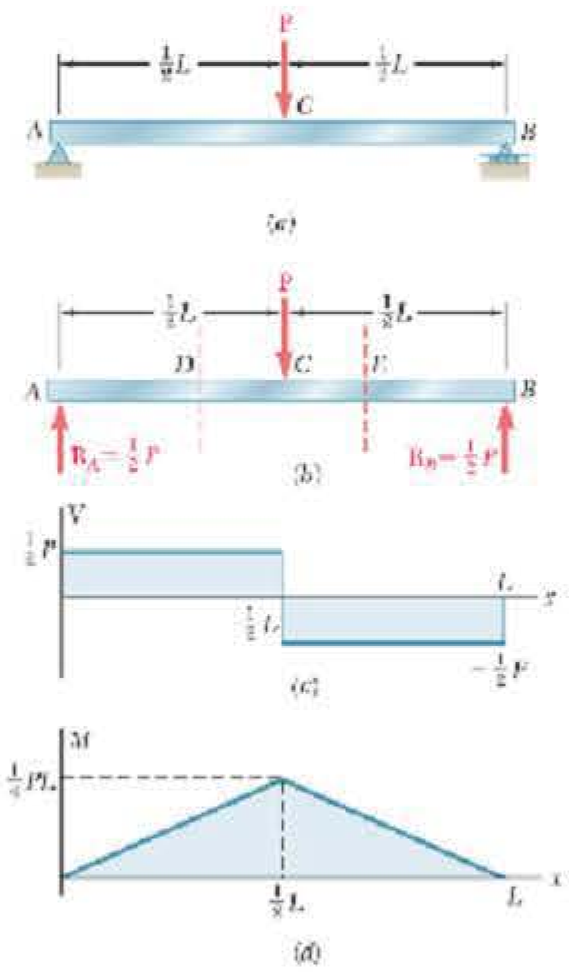
$$\sigma_b = \frac{M\,y}{I}$$

Still need to calculate M:

$$M = \frac{PL}{4}$$

$$= \frac{(20{,}000 \cdot 3{,}000)}{4}$$

$$= 15{,}000 \text{ N mm}$$



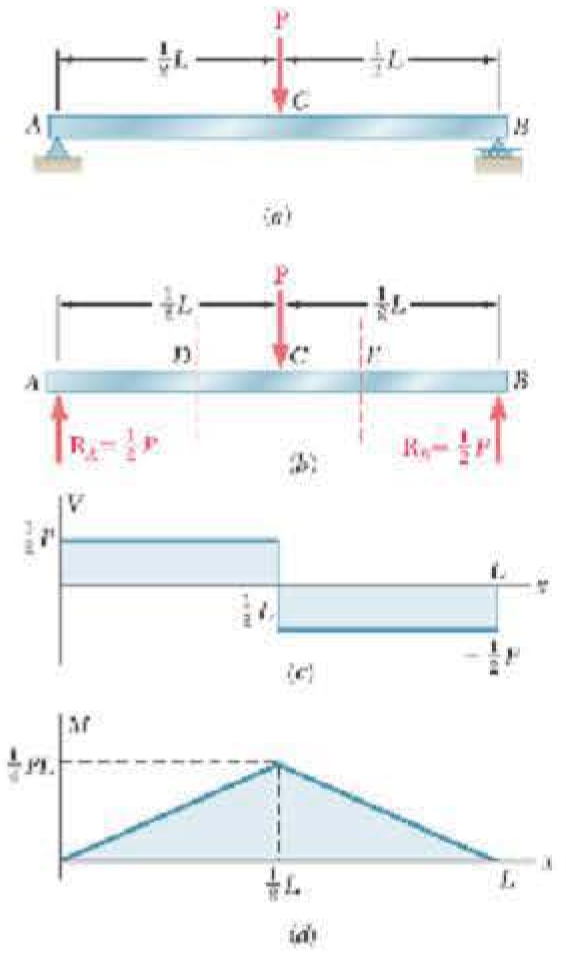
Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	Find the maximum bending moment	Find the maximum bending stress

GIVE FEEDBACK

OK

Solve bending stress for a combined section

Now find the maximum stress of this 3 m long beam with a load of 20 kN at midspan. Ignore its own weight.



Now we can use the equation at last!

$$\sigma_b = \frac{M\,y}{I}$$

$$= \frac{(15{,}000 \cdot 68.333)}{8{,}626{,}667}$$

$$= 104.33\ \text{MPa}$$

Done.

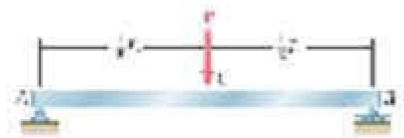
Find centroid y_c	Calculate first moment for each element	Calculate centroidal second area moments	Calculate parallel axis terms	Calculate total second area moment	Tabulated calculation for total second area moment	**Find the maximum bending moment**	**Find the maximum bending stress**

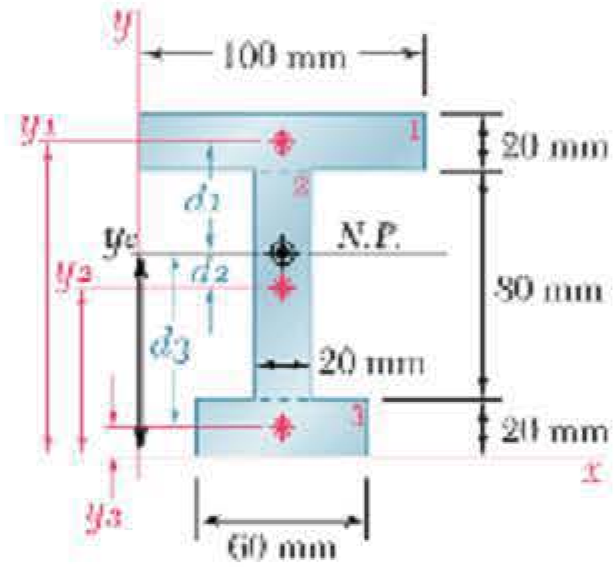
GIVE FEEDBACK

OK

Procedure for finding the maximum **tensile** stress for this 3m long beam with a load of 20 kN at midspan. Ignore its own weight.

Follow the order in the table and include final stress at the end.





	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	I_c	d	Ad^2	I
2		mm	mm	mm	mm^2	mm^3	mm^4	mm	mm^4	mm^4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

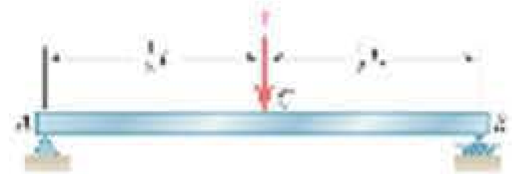
↑↓ **Place these in the proper order.**

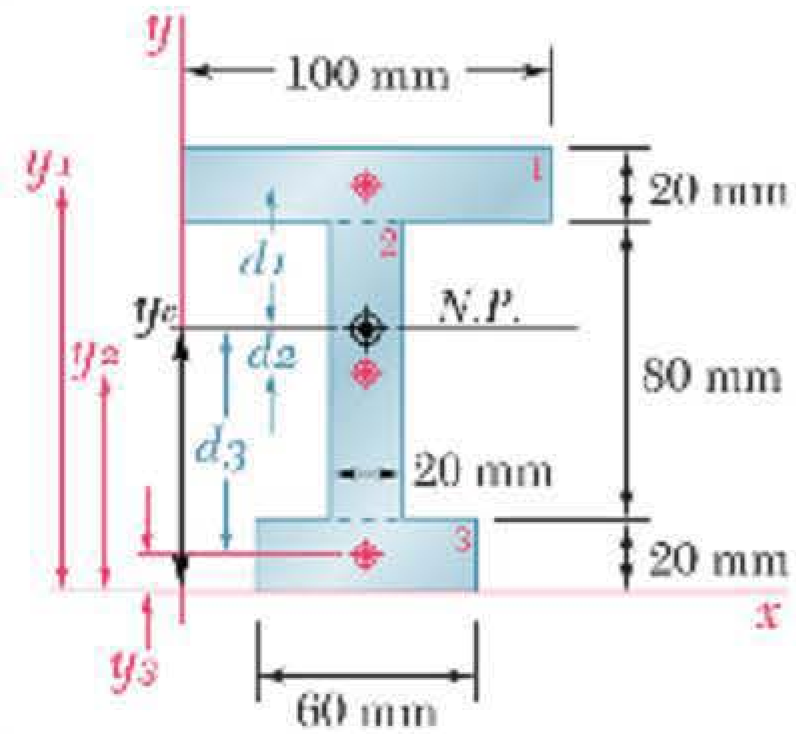
- Find second moment of area for each element (but DO NOT add them up)
- Find the distances of each element's centroid from the combined centroid. $d = y_c - y_{1c}$
- Calculate bending moment M and solve for bending stress in $\sigma_b = \dfrac{M\,y}{I}$
- Find centroid $y_c = \dfrac{\Sigma(A\,y)}{\Sigma A}$
- Split the section into three elements. Find total area ΣA, then total first moment of area $\Sigma(A\,y)$
- Apply the parallel axis theorem to each element: $I_1 = I_c + A\,d^2$ then add second moment of area for each element $I_{tot} = I_1 + I_2 + I_3$

Find maximum **compressive** stress for this 3m long beam with a load of 20 kN at midspan. Ignore its own weight.

(Note: each formula is numerically accurate)

$$\sigma_b = \frac{M\,y}{I}$$





	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	I_c	d	Ad^2	I
2		mm	mm	mm	mm^2	mm^3	mm^4	mm	mm^4	mm^4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

Click the correct answer.

$$\frac{(10{,}000 \cdot 1.5)\,\text{N mm} \cdot (120 - 68.3333)\,\text{mm}}{8{,}626{,}667\ \text{mm}^4} = 0.08984\,\text{MPa}$$

$$\frac{(10{,}000 \cdot 1.5 \cdot 1{,}000)\,\text{Nmm} \cdot (120 - 68.3333)\,\text{mm}}{8{,}626{,}667\ \text{mm}^4} = 89.84\,\text{MPa}$$

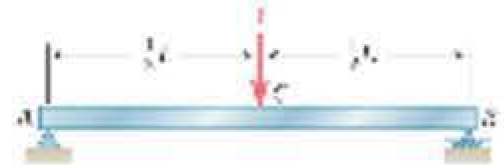
$$\frac{(10{,}000 \cdot 1.5 \cdot 1{,}000)\,\text{Nmm} \cdot 68.3333\,\text{mm}}{8{,}626{,}667\ \text{mm}^4} = 118.8\,\text{MPa}$$

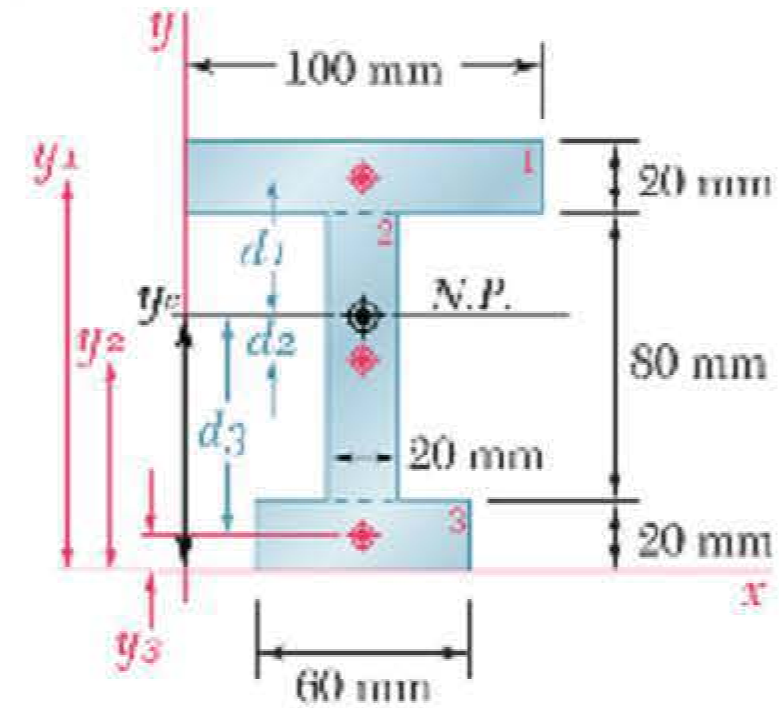
$$\frac{(10{,}000 \cdot 1.5)\,\text{N mm} \cdot 68.3333\,\text{mm}}{8{,}626{,}667\ \text{mm}^4} = 0.1188\,\text{MPa}$$

Find maximum **tensile** stress for this 3m long beam with a load of 20 kN at midspan. Ignore its own weight.

(Note: each formula is numerically accurate)

$$\sigma_b = \frac{M y}{I}$$





	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	I_c	d	Ad^2	I
2		mm	mm	mm	mm^2	mm^3	mm^4	mm	mm^4	mm^4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

Click the correct answer.

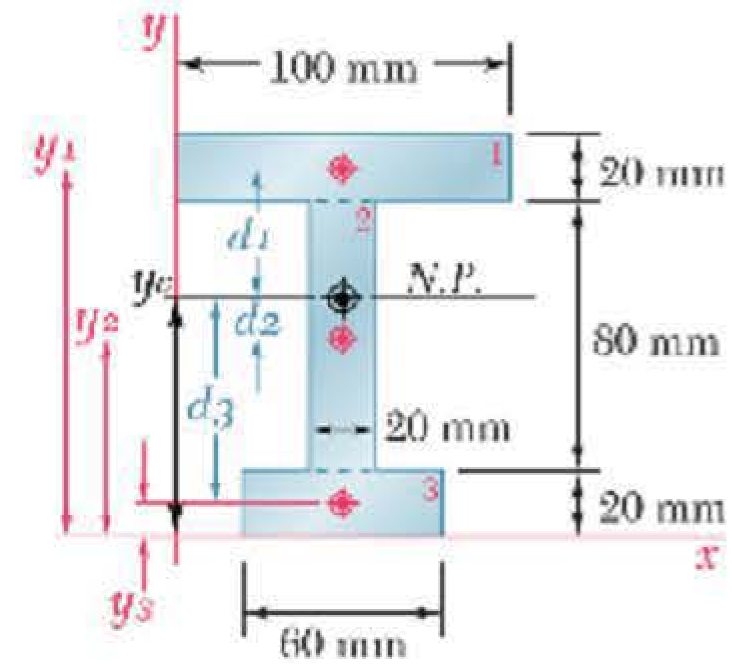
$$\frac{(10{,}000 \cdot 1.5)\,\mathrm{N\,mm} \cdot (120 - 68.3333)\,\mathrm{mm}}{8{,}626{,}667\,\mathrm{mm}^4} = 0.08984\,\mathrm{MPa}$$

$$\frac{(10{,}000 \cdot 1.5 \cdot 1{,}000)\,\mathrm{N\,mm} \cdot 68.3333\,\mathrm{mm}}{8{,}626{,}667\,\mathrm{mm}^4} = 118.8\,\mathrm{MPa}$$

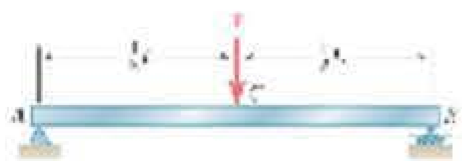
$$\frac{(10{,}000 \cdot 1.5)\,\mathrm{N\,mm} \cdot 68.3333\,\mathrm{mm}}{8{,}626{,}667\,\mathrm{mm}^4} = 0.1188\,\mathrm{MPa}$$

$$\frac{(10{,}000 \cdot 1.5 \cdot 1{,}000)\,\mathrm{N\,mm} \cdot (120 - 68.3333)\,\mathrm{mm}}{8{,}626{,}667\,\mathrm{mm}^4} = 89.84\,\mathrm{MPa}$$

Find maximum stresses for this 3m long beam with a load of 20 kN at midspan. Ignore its own weight.



$$\sigma_b = \frac{M\,y}{I}$$



	A	B	C	D	E	F	G	H	I	J
1		b	d	y	A	Ay	I_c	d	Ad^2	I
2		mm	mm	mm	mm^2	mm^3	mm^4	mm	mm^4	mm^4
3	#1	100	20	110	2000	220000	66667	-41.667	3472222	3538889
4	#2	20	80	60	1600	96000	853333	8.333	111111	964444
5	#3	60	20	10	1200	12000	40000	58.333	4083333	4123333
6	Σ				4800	328000				**8626667**
7					y_c	**68.333**				

Drag statements on the right to match the left.

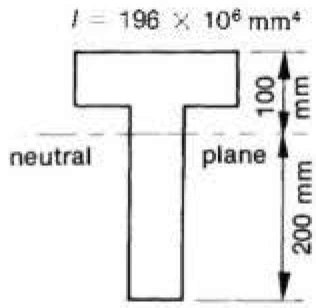
M	$3{,}538{,}889 + 964{,}444 + 4{,}123{,}333 = 8{,}626{,}667$
I	$\frac{20{,}000\ \text{N}}{2} \times \frac{3\ \text{m}}{2} = 1{,}500\ \text{Nm}$
y for tensile stress	$(20 + 80 + 20) - 68.333 = 51.667$
y for compressive stress	$\frac{328{,}000}{4{,}800} = 68.333$

This question shows a neutral plane that is not halfway up the cross-section.

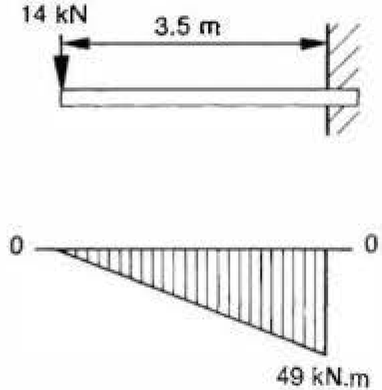
This results in a higher stress at the bottom than at the top, because distance y is greater.

Example

For the cantilever beam shown below, determine the maximum value of stress.



Cross-section



BM diagram

GIVE FEEDBACK

CONTINUE >

Solution

Maximum bending moment occurs at the fixed end of the beam.

$$\begin{aligned} M &= 14\,\mathrm{kN} \times 3.5\,\mathrm{m} \\ &= 49\,\mathrm{N.m} \\ &= 49 \times 10^{6}\,\mathrm{N.mm} \end{aligned}$$

This is negative bending moment. Therefore the convex or tension fibre is on the top and the concave or compression fibre is on the bottom. Hence, for maximum tension, $y_t = 100$ mm, and for maximum compression, $y_c = 200$ mm.
Maximum stresses can now be calculated.

Tensile stress in the top fibre:

$$\sigma_t = \frac{M\,y_t}{I}$$

$$= \frac{49 \times 10^6\ \mathrm{N.mm} \times 100\ \mathrm{mm}}{196 \times 10^6\ \mathrm{mm}^4}$$

$$= 25\ \mathrm{MPa}$$

Compressive stress in the bottom fibre:

$$\begin{aligned}\sigma_c &= \frac{M\,y_c}{I}\\ &= \frac{49 \times 10^6\ \mathrm{N.mm} \times 200\ \mathrm{mm}}{196 \times 10^6\ \mathrm{mm}^4}\\ &= 50\ \mathrm{MPa}\end{aligned}$$

Such non-symmetrical cross-sections are sometimes used for beams made from materials which are stronger in compression than they are in tension, such as concrete.
However, concrete is good for 50 MPa compression, but not 25 MPa tensile stress. So if this cantilevered beam was made of concrete it would have a concentration of steel reinforcing at the top to absorb the tensile forces.

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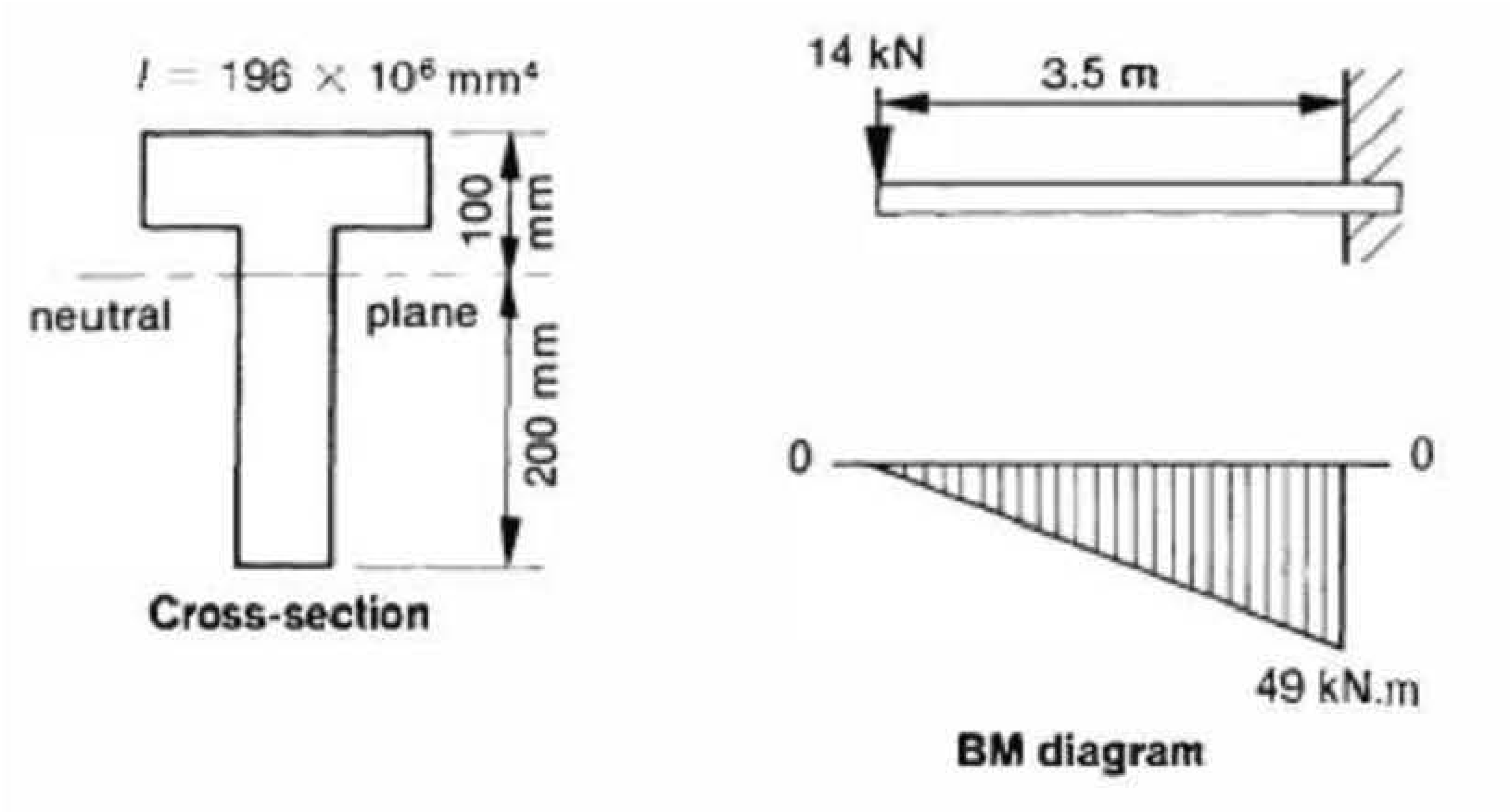

Maximum bending moment occurs at the fixed end of the beam (at the wall).

$M = 14\,\text{kN} \times 3.5\,\text{m} = 49\,\text{Nm} = 49 \times 10^6\,\text{Nmm}$

Find the maximum tensile stress σ_t and the maximum compressive stress σ_c.

Check all that apply.

- [ ] $\sigma_c = \dfrac{49 \times 10^6\,\text{N.mm} \times 200\,\text{mm}}{196 \times 10^6\,\text{mm}^4} = 50\,\text{MPa}$
- [ ] $\sigma_t = \dfrac{49 \times 10^6\,\text{N.mm} \times 200\,\text{mm}}{196 \times 10^6\,\text{mm}^4} = 50\,\text{MPa}$
- [ ] $\sigma_c = \dfrac{49 \times 10^6\,\text{N.mm} \times 100\,\text{mm}}{196 \times 10^6\,\text{mm}^4} = 25\,\text{MPa}$
- [ ] $\sigma_t = \dfrac{49 \times 10^6\,\text{N.mm} \times 100\,\text{mm}}{196 \times 10^6\,\text{mm}^4} = 25\,\text{MPa}$