GIVE FEEDBACK

CONTINUE >

Bending moment is what bends or breaks a beam. Our job in this chapter is to find it. We need to know the maximum bending moment and where it is located in the beam. We may also need to know the shear force and the bending moment for the entire length of the beam. These are plotted in a shear force diagram and a bending moment diagram.

The most important thing in this chapter is **bending moment**.

Bending moment is what breaks a beam. It also causes a beam to bend.

If we can find the maximum bending moment, we will know how strong the beam has to be to do the job.

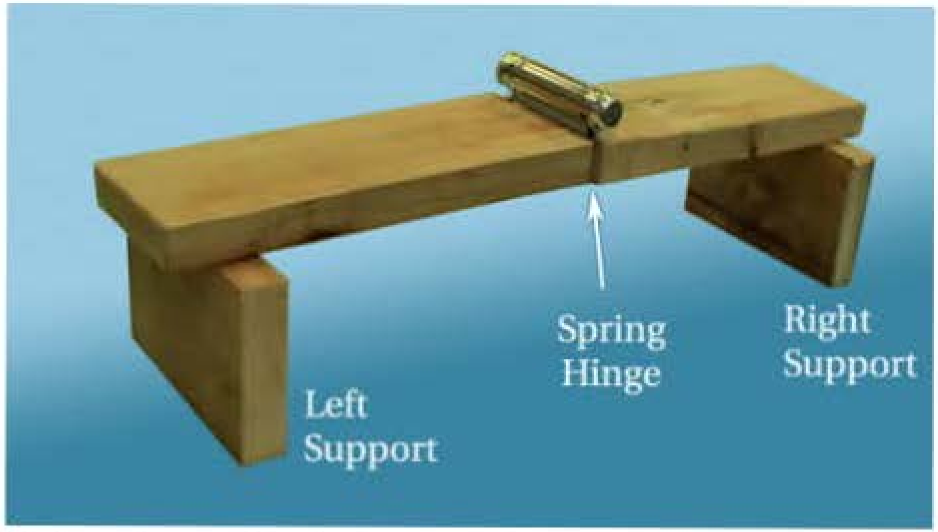
In this section we also introduce the concept of shear force. Shear force is useful in its own right and is also helpful in finding the maximum bending moment in a beam.

We finish this chapter with diagrams showing the shear force and bending moment for the entire length of the beam – the shear force diagram and bending moment diagram.

< BACK GIVE FEEDBACK OK

The bending moment is the torque inside a beam that holds it together. This beam has a spring hinge in the middle. The spring does the job of the bending moment at the mid-section cut. A strong beam means one that can handle a high bending moment.

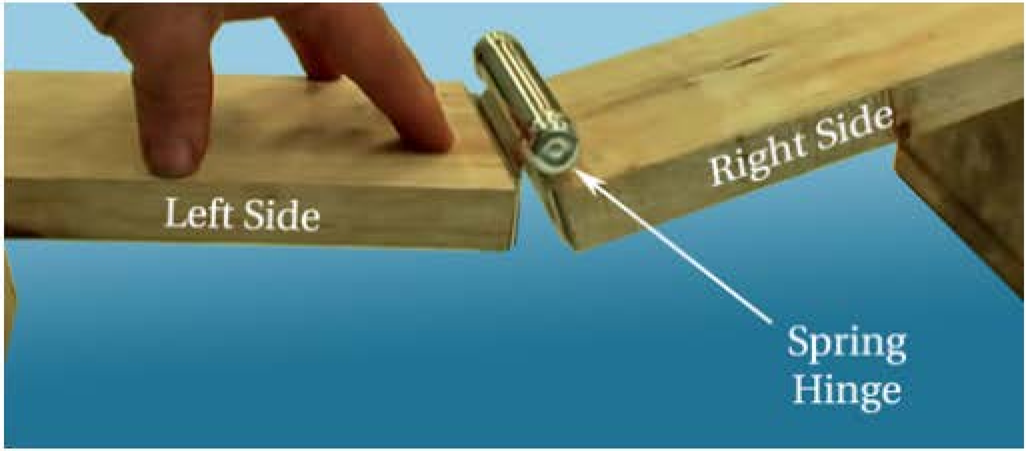
Bending moment is the torque inside a beam that holds it together.



This beam has a spring hinge in the middle. The spring is strong enough to hold the weight of the beam.

GIVE FEEDBACK

CONTINUE >



But when more load is applied, the spring cannot supply enough bending moment to keep the beam from failing.

A strong beam means one that can handle a high bending moment.

< BACK GIVE FEEDBACK OK

A beam that can handle a high bending moment is ______.

Click the correct answer.

- strong
- hinged
- not loaded
- short

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Bending moment occurs everywhere along the beam. In this beam, a hinge replaces the cross-section before it was cut. The spring applies a clockwise moment to the right-hand side and an anticlockwise moment to the left-hand side. Each moment is equal and opposite.

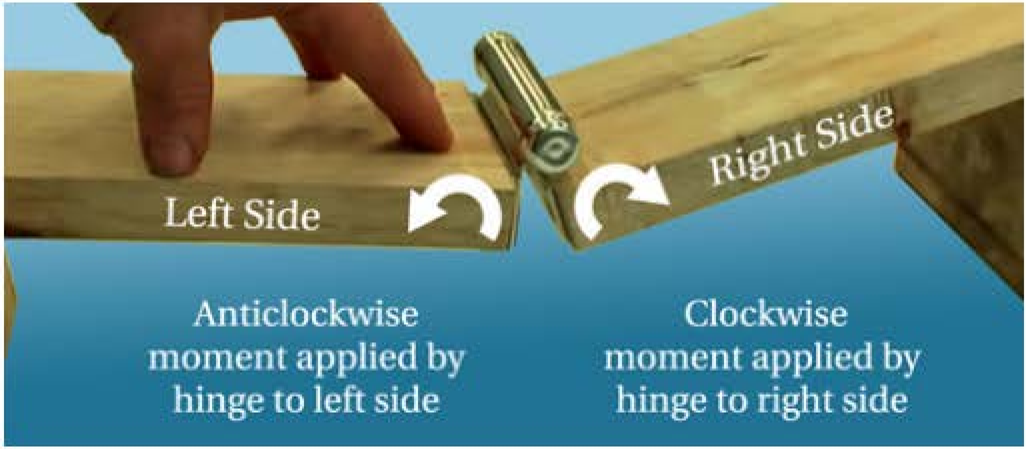
We always treat bending moment as acting at any cross-section along the beam. So bending moment is the torque that holds the beam together at any cross-section along the beam.

Imagine a beam that has been cut through at mid-span and a spring hinge is fitted to hold the two pieces together. The beam can now support itself. When loaded, the hinge applies a torque (or moment) to each side:

- The spring applies a clockwise moment to the right-hand side
- The spring applies a counter-clockwise moment to the left-hand side

GIVE FEEDBACK

CONTINUE >



The section where the cut was made has bending moment that keeps the beam straight. The moment is equal and opposite on each side of the cut, which satisfies equilibrium.

< BACK GIVE FEEDBACK OK

Which statements correctly describe bending moment?

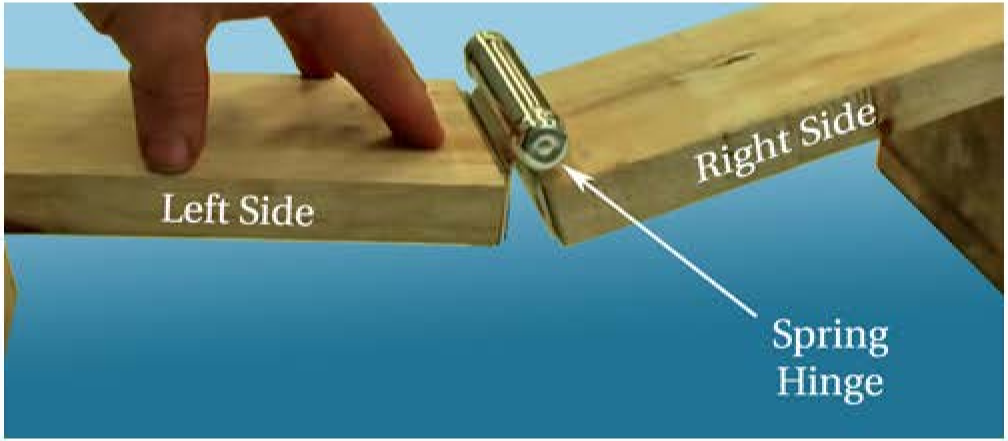
Use mouse to zoom. Click to keep enlarged.

Check all that apply.

- [ ] The spring applies equal and opposite moments to the left and right halves of the beam
- [ ] The bending moment generated by the spring is sufficient to hold the beam straight
- [ ] The spring applies a clockwise moment to the left side
- [ ] If the spring was welded shut and the beam placed back on the supports, the bending moment would increase

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



The spring applies a [Select... ▼] moment to the left side.

The spring applies a [Select... ▼] moment to the right side.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Finding the bending moment for any point in a beam

Bending moment is the first step in determining loads and stresses inside a body. Before we can begin to work inside, we first have to solve all forces outside the body.

This means we must always solve reaction forces for the entire body (non-concurrent forces) as the first in the following steps:

1. Solve equilibrium to get reactions
2. Cut the beam at this location
3. Choose the left or right side of the beam and construct a free body diagram
4. Calculate the moment of all forces around the cut point on that diagram

This will lead to the bending moment.

Note: The calculation of moment for the left side and right side of the cut will arrive at the same number but opposite signs. Equal and opposite moments are applied at the cut section in agreement with Newton's third law—every action has equal and opposite reaction.

GIVE FEEDBACK

OK

Sort these calculation steps into the correct order for finding bending moment at a certain point in a beam.

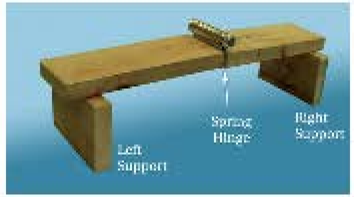
↑↓ Place these in the proper order.

- Solve equilibrium to get all reaction forces
- Chose the location along the beam where bending moment is to be determined
- Cut the beam at that point and take the left (or right) side as a free body diagram
- Calculate the moment of all forces around the cut point on that left (or right) diagram

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

How do we know if the cut section of the beam is in equilibrium?



Use mouse to zoom. Click to keep enlarged.

Check all that apply.

- [ ] Because the moments applied to left and right sides are equal and opposite
- [ ] Because it is in the centre of the span
- [ ] Because a spring hinge has been fitted there
- [ ] Because the beam is not collapsing

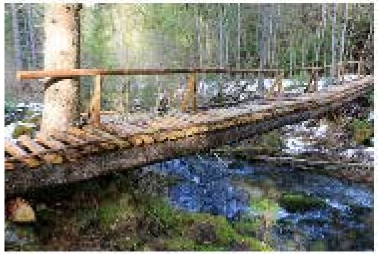
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Bending moment involves opposite moments, one positive and one negative. We define a positive bending moment by the bend of the beam. A positive bending moment causes the beam to deflect downwards. This is called sagging, like this sagging log bridge. A negative bending moment causes the beam to deflect upwards. This is called hogging, like this weightlifting bar.

Positive bending moment

Bending moment at a cross-section in a beam is a pair of opposite moments, one positive and one negative. We define a positive bending moment by the bend of the beam. A positive bending moment causes the beam to deflect downwards. This is called sagging, like the log bridge sagging under its own weight in the first image below. A negative bending moment causes the beam to deflect upwards. This is called hogging, like the weightlifter's bar in the second image below.



GIVE FEEDBACK

OK

Regarding positive and negative bending moment, match the descriptions with the words.

Drag statements on the right to match the left.

Word that describes the shape of a beam under a negative bending moment	Hogging
Word that describes the shape of a beam under a positive bending moment	Sagging
Direction in which a positive bending moment deflects the beam	Down
Direction in which a negative bending moment deflects the beam	Up

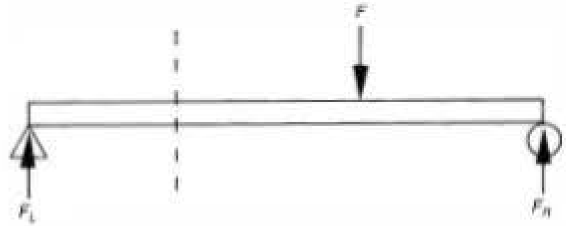
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Bending moment can be caused by any combination of applied forces or moments. Sagging indicates positive bending moment. A more accurate way to define positive bending moment is when the curvature of the beam faces upward.

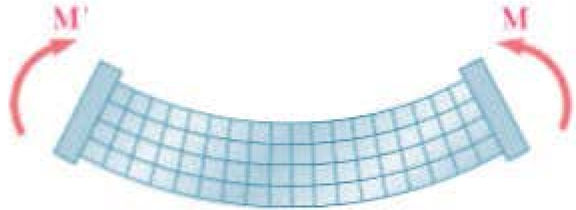
Upward curvature as positive bending moment

Bending moment (BM) can be caused by forces:



(a)

or moments:

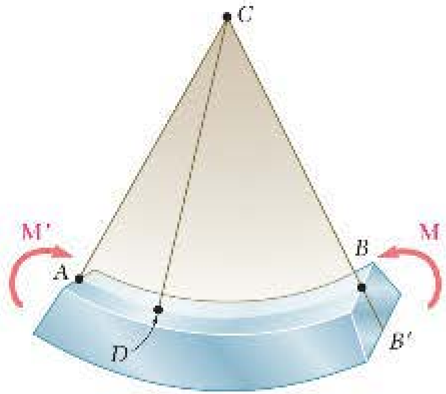


Regardless of whether the BM is created by forces or moments, whenever the beam sags downwards it has a positive BM.

GIVE FEEDBACK

CONTINUE >

A more accurate way to define positive BM is to identify sections of the beam where the curvature faces upward.



This means that any part of the beam that has upward-facing curvature is under positive BM while downward-facing curvature indicates negative BM.

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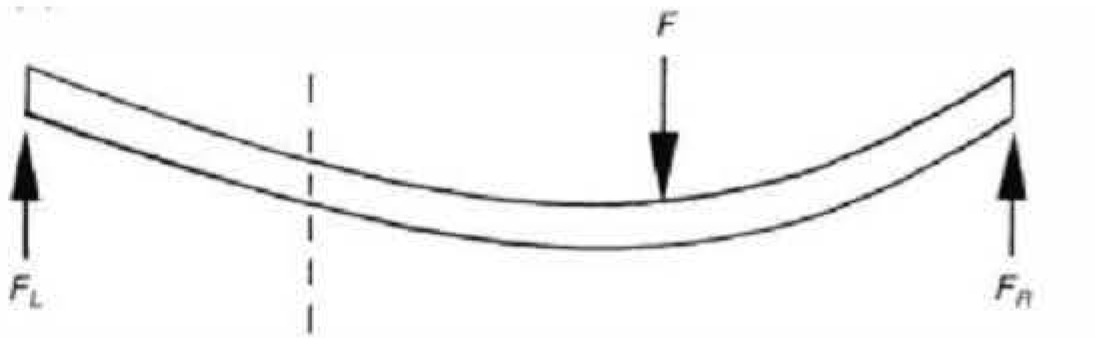
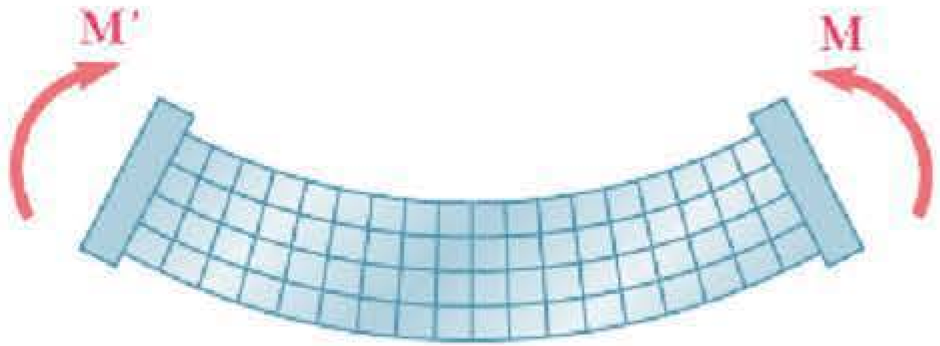
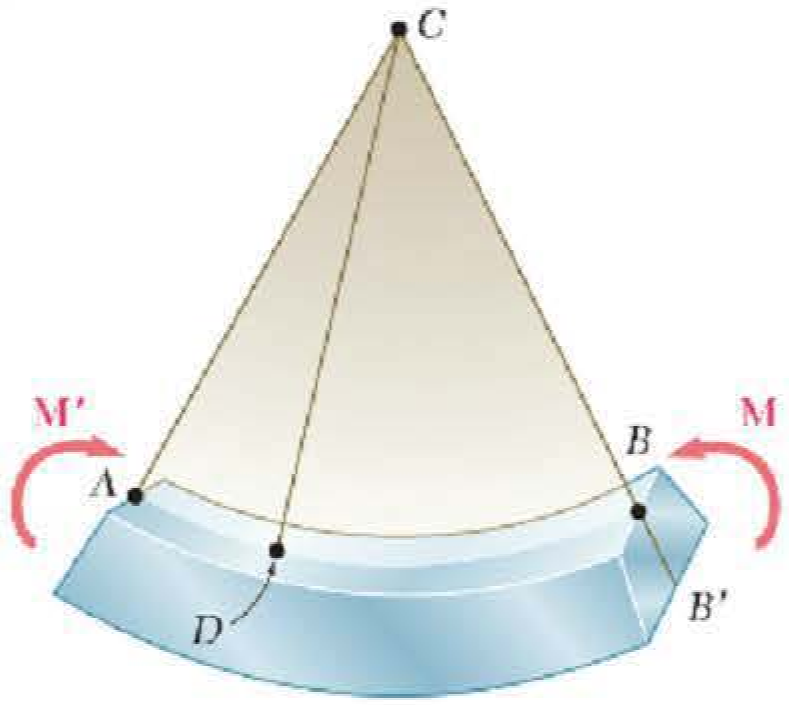
GIVE FEEDBACK
OK



(a)



(b)



(c)

Which of these diagrams represent a positive bending moment?

Click the correct answer.

- The left half of (a), (b) and (c)
- Only (b) and (c)
- All three
- Only (a)
- The right half of (a), (b) and (c)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of these diagrams represent a positive bending moment?

Click the correct answer.

- All three
- Only (b) and (c)
- Only (a)
- The left half of (a), (b) and (c)
- The right half of (a), (b) and (c)

Do you know the answer?

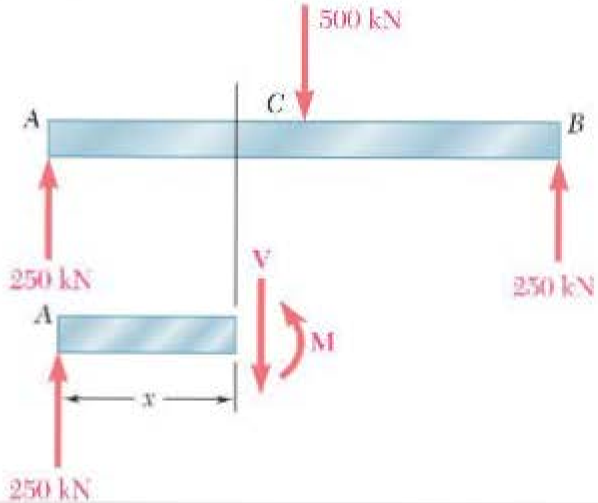
I KNOW IT | THINK SO | UNSURE | NO IDEA

Here is an example for calculating the bending moment at some point in a beam. Keep in mind that the reaction forces must be found before you can address bending moment. In other words, external equilibrium must be solved before focusing on the interior of the body.

Example

The following beam is 8 m long with a 500 kN force applied at the centre. The reaction forces are 250 kN at each end.

Find the bending moment at a cross-section x m from the left end.



GIVE FEEDBACK

CONTINUE >

Solution

1. Solve static equilibrium (done)
2. Cut the beam at the location of interest (done)
3. Choose a side and construct a free body diagram (done)
4. Solve equilibrium of moments for the free body diagram

Analyse the free body diagram of the left-hand side.

Let: BM = moment applied by section to the left-hand side

ΣM_F = moments caused by all external loads on the left-hand side

$$\Sigma M_F - \text{BM} = 0$$

$$\begin{aligned} \text{BM} &= \Sigma M_F \\ &= 250\ \text{kN} \cdot x\ \text{m} \\ &= 250 \cdot x\ \text{kN m} \end{aligned}$$

This is the moment applied to the body (by the cross-section) at any point x from the left end. The BM is positive because the beam will tend to sag (upward curvature).

< BACK GIVE FEEDBACK CONTINUE >

To find the BM at the centre (4 m from the left end):

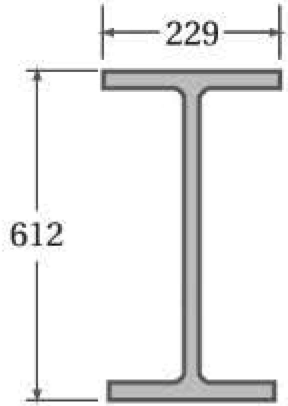
Let: $x = 4\,\text{m}$

$$\begin{aligned}\text{BM} &= \Sigma M \\ &= 250 \cdot 4\,\text{kN m} \\ &= 1{,}000\,\text{kN m}\end{aligned}$$

This number represents the required strength of the beam.

< BACK GIVE FEEDBACK CONTINUE >

This is a very strong beam. It has to be – a load of $500\ \mathrm{kN}$ (51 tonnes) is applied in the middle of this $8\ \mathrm{m}$-long beam. This would take a large universal beam (610 UB 125) almost to the yield point (280 MPa).



610 UB 125

This is something we will be able to calculate when we deal with bending stress.

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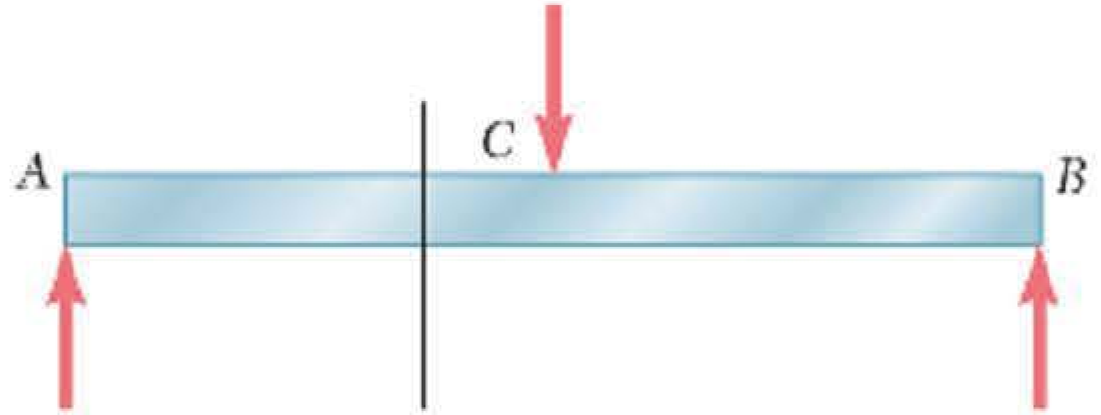


A beam is 7.4 m long. How much force can be applied mid-span (in kN) if the beam can handle a bending moment of 92 kNm?

(Minimum one decimal place. Include units.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

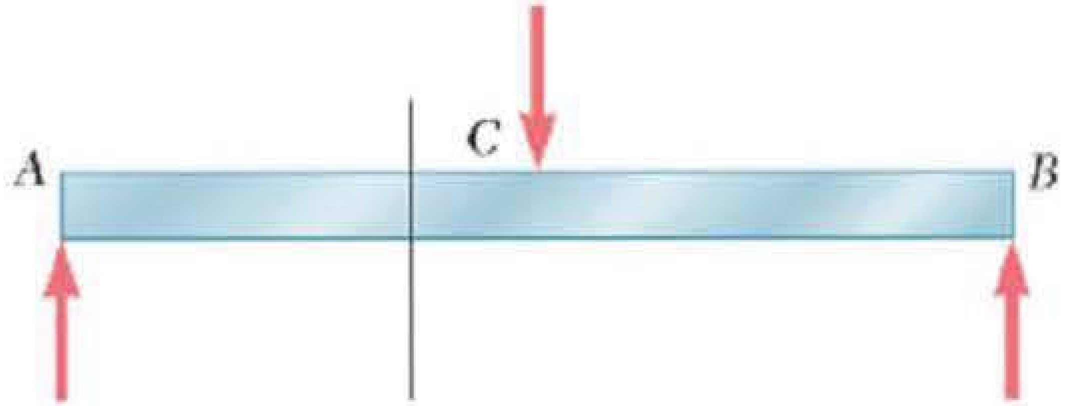
SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question



A beam is 5.3 m long with a 56 kN force applied at the centre. Find the bending moment at mid-span in kNm.

(Minimum one decimal place. Include units and plus/minus sign for positive/negative BM.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

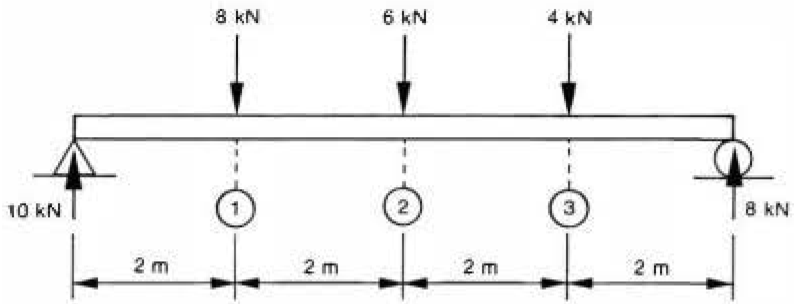
- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

Calculate the bending moments for different cross-sections of a beam

For the beam and loading shown, determine the bending moments at each of the three applied forces.



The initial step is to solve reactions.

These have been solved—10 kN left reaction and 8 kN right reaction.

If this was not the case, we would have to solve equlibrium of non-concurrent forces to find left and right reactions.

Determining bending moment at three points along a beam	Location 1	Location 2	Location 3	Summary

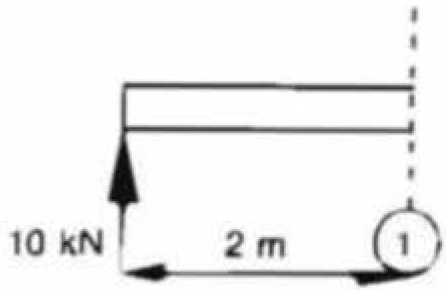
GIVE FEEDBACK

OK

Calculate the bending moments for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 1)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of moments for the free body diagram

We will start from the left-hand side and consider all forces to the left of the cross-section.

Bending moment at cross-section 1:

$$
\begin{aligned}
BM_1 &= \Sigma M \\
&= 10\,\text{kN} \times 2\,\text{m} \\
&= 20\,\text{kN.m}
\end{aligned}
$$

Determining bending moment at three points along a beam	Location 1	Location 2	Location 3	Summary

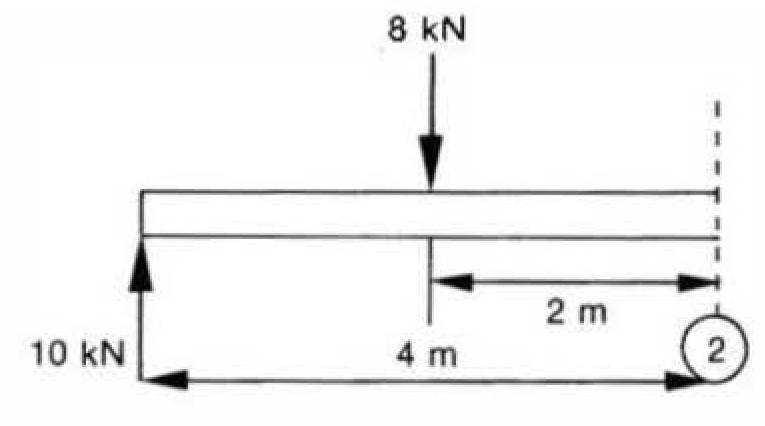
GIVE FEEDBACK

OK

Calculate the bending moments for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 2)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of moments for the free body diagram

Bending moment at cross-section 2:

$$
\begin{aligned}
BM_2 &= \Sigma M \\
&= 10\ \mathrm{kN} \times 4\ \mathrm{m} - 8\ \mathrm{kN} \times 2\ \mathrm{m} \\
&= 24\ \mathrm{kN.m}
\end{aligned}
$$

Determining bending moment at three points along a beam	Location 1	Location 2	Location 3	Summary

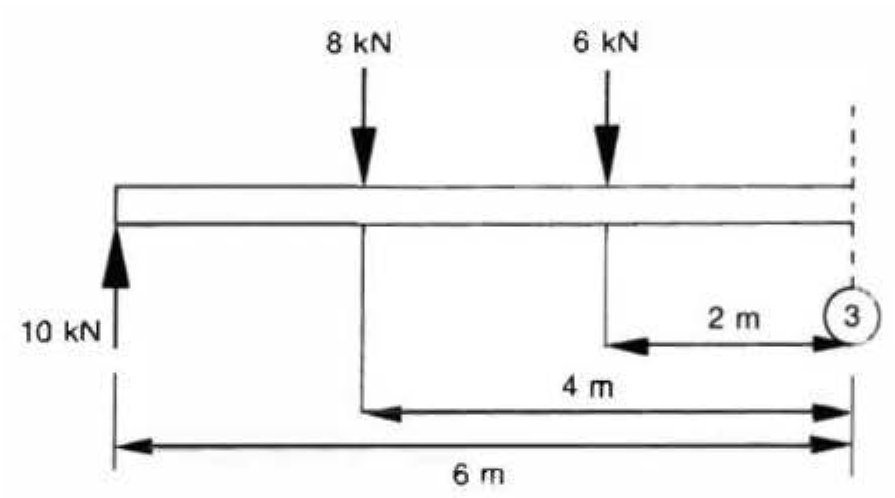
GIVE FEEDBACK

OK

Calculate the bending moments for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 3)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of moments for the free body diagram

Bending moment at cross-section 3:

$$BM_3 = \Sigma M$$
$$= 10\,\text{kN} \times 6\,\text{m} - 8\,\text{kN} \times 4\,\text{m} - 6\,\text{kN} \times 2\,\text{m}$$
$$= 16\,\text{kN.m}$$

Determining bending moment at three points along a beam	Location 1	Location 2	Location 3	Summary

GIVE FEEDBACK

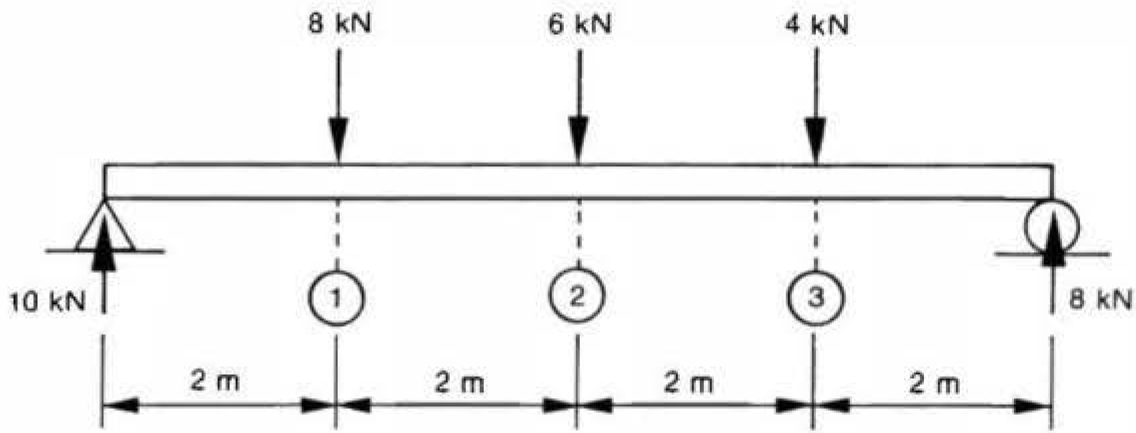
OK

Calculate the bending moments for different cross-sections of a beam

$BM_1 = \Sigma M$
$= 10\ kN \times 2\ m$
$= 20\ kN.m$

$BM_2 = \Sigma M$
$= 10\ kN \times 4\ m - 8\ kN \times 2\ m$
$= 24\ kN.m$

$BM_3 = \Sigma M$
$= 10\ kN \times 6\ m - 8\ kN \times 4\ m - 6\ kN \times 2\ m$
$= 16\ kN.m$



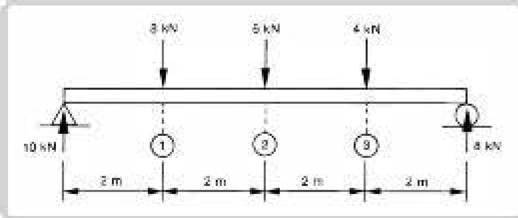
It turns out that if we always work with the left-hand side as the free body diagram and take clockwise moments as positive, then the bending moment will automatically have the correct sign.

Determining bending moment at three points along a beam	Location 1	Location 2	Location 3	Summary

GIVE FEEDBACK

OK

The sum of all moments on the left side of point 2 is (+24 kNm). Which of the following statements are true?



Use mouse to zoom. Click to keep enlarged.

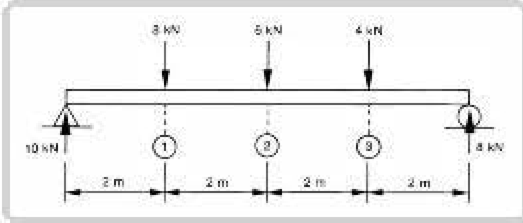
Check all that apply.

- [ ] The BM at point 2 is (+24 kNm)
- [ ] The BM at point 2 is (–24 kNm)
- [ ] The sum of all moments on the right side of point 2 is (+24 kNm)
- [ ] The sum of all moments on the right side of point 2 is (–24 kNm)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The sum of all moments on the left side of point 3 is (+16 kNm). Which of the following statements are true?



Use mouse to zoom. Click to keep enlarged.

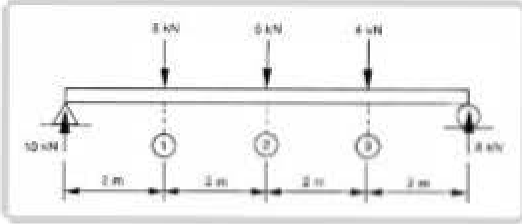
Check all that apply.

- [ ] The BM at point 3 is (+16 kNm)
- [ ] The BM at point 3 is (–16 kNm)
- [ ] The sum of all moments on the right side of point 3 is (+16 kNm)
- [ ] The sum of all moments on the right side of point 3 is (–16 kNm)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which statements are true regarding bending of this beam?



Use mouse to zoom. Click to keep enlarged.

Check all that apply.

- [ ] The BM at the left-end support is zero
- [ ] The BM is highest at point 2
- [ ] The BM at the right-end support is 16 kNm
- [ ] The sum of moments at any point along the beam is zero
- [ ] The bending moment at any point along the beam is zero

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Often we need to find the maximum bending moment because this is what we need to design for. But that might mean we have to pick many points to calculate bending moment, and how can we be sure we didn't miss it? This is why we draw a bending moment diagram.

We can readily calculate the BM at a certain point in a beam by taking a section and summing the moments for the beam on one side of that cut (e.g. the left side).

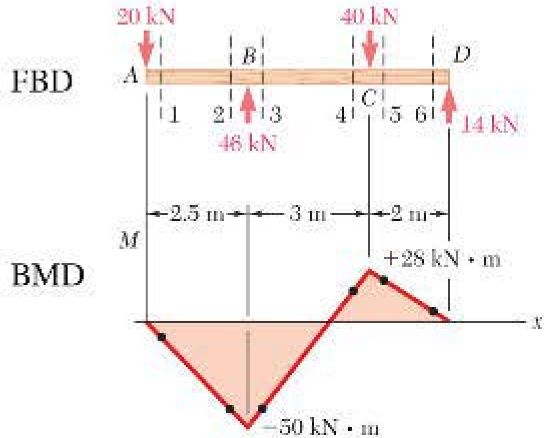
That's fine if you know which point to look at but there are times we want to know the value of BM along the entire length of the beam. Here are some of those times:

1. When we are not sure where the maximum BM occurs and need to check the whole beam (e.g. a complex beam with many forces, moments or distributed loads applied)
2. When the strength of the beam is being optimised to match the BM everywhere (e.g. optimised structural components for minimum weight or material, such as aircraft landing gear, large excavator arm, large concrete bridge, racing and sports components)
3. When full data for the BM is required so modifications can be validated (e.g. a component of a machine or a structural beam needs a pipe to pass through it—where is a safe place to drill the hole?)

GIVE FEEDBACK

CONTINUE >

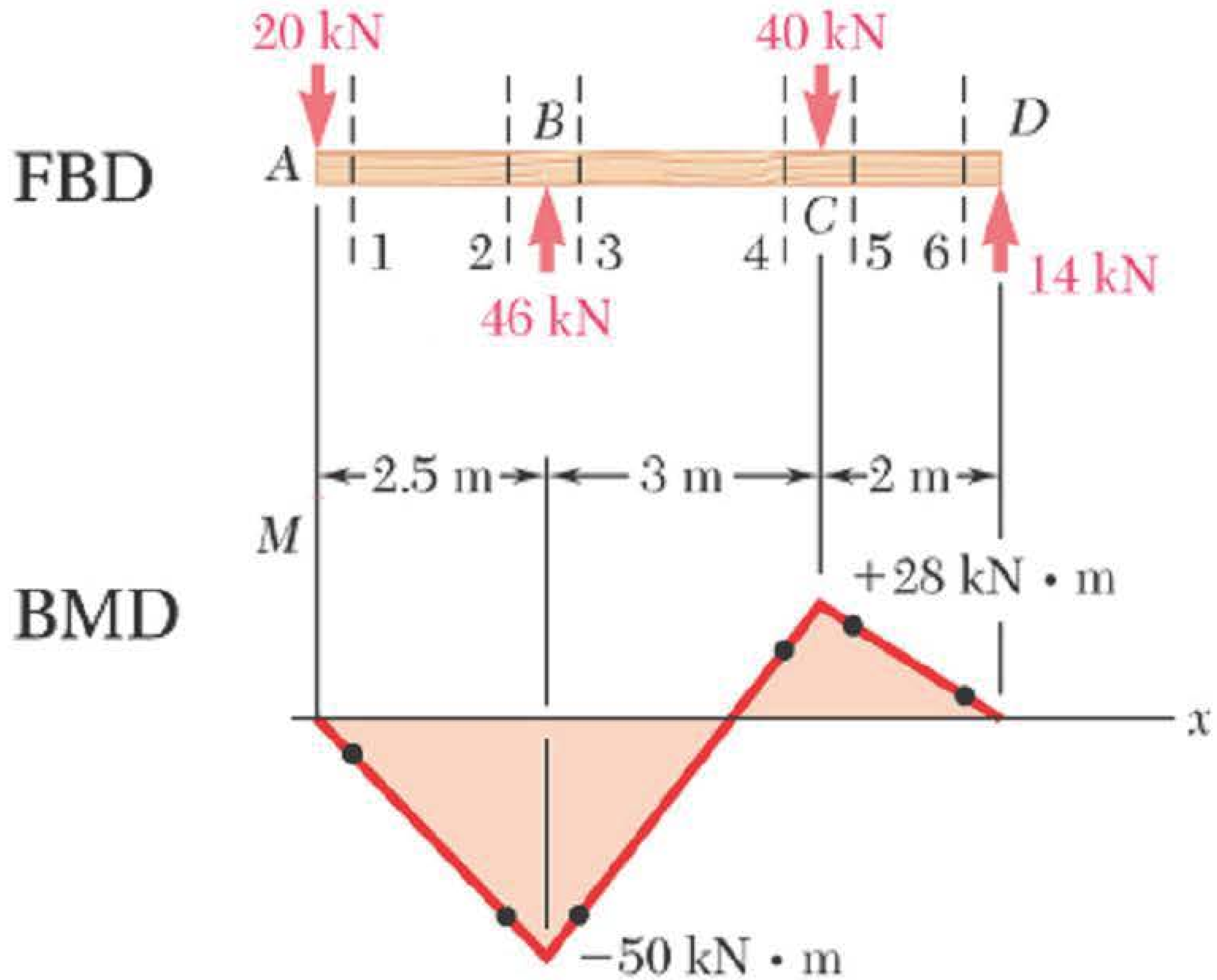
The plot of BM along the entire length of a beam is called a bending moment diagram (BMD). It is usually drawn to line up with the free body diagram (FBD), as shown below.



We can find the BM at any point along the beam (points 1 to 6 above) by reading them off the graph (BMD).

< BACK GIVE FEEDBACK OK

A bending moment diagram is necessary when:



Check all that apply.

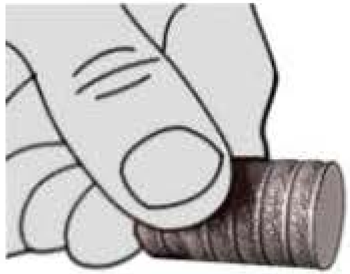
- [ ] Full data for the BM is required so modifications can be validated
- [ ] We want to find the BM at one particular location in the beam
- [ ] We are not sure where the maximum BM occurs and need to check the whole beam
- [ ] The strength of the beam is being optimised to match the BM everywhere

Shear force is how much a beam wants to slide apart vertically. If the beam could slide without bending we would see the effect of shear force in the beam. Shear force is a shortcut to plotting a bending moment diagram.

Why shear force is important

Shear force is how much a beam wants to slide apart vertically.

Here we are imagining only the sliding aspect of the beam not the bending. So imagine the beam as a horizontal stack of slippery magnets. They can slide but cannot bend, as depicted in the moving image below.



GIVE FEEDBACK

CONTINUE >

This is rare in reality because bending stresses are much higher than shear stresses. Initially shear force seem very different to bending moment.

There are two reasons we might need to know the shear force along the beam:

1. The shear force can be used as a shortcut to plotting a bending moment diagram
2. Sometimes a beam can be overloaded by shear force (although bending is more common)

< BACK GIVE FEEDBACK OK

Shear force is how much a beam wants to (please select) ▼ in the (please select) ▼ direction.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Shear force is how much a beam wants to slide apart vertically. Which other statements are true about shear force?

Check all that apply.

- [ ] For beams, bending moment is usually much more important than shear force
- [ ] Shear force is magnetic
- [ ] Shear force makes it easier to plot a bending moment diagram
- [ ] A real beam or component cannot fail due to shear force

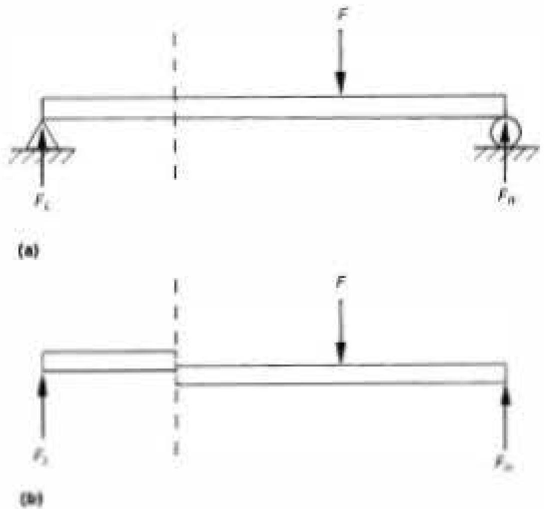
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Shear force exists—you just don't see it happen. It is a hidden force inside the beam. In this example a beam has a single load applied on top and support reactions on each end. An imaginary cut through the beam (shown as a dotted line) will result in the left portion tending to slide upwards. This is shear force.

Shear force in a loaded beam

Shear force can occur in an ordinary beam that is subjected to loads.



(a) Beam cut by imaginary transverse section
(b) Left-hand portion tends to move up relative to right-hand portion

GIVE FEEDBACK

CONTINUE >

Shear force in a loaded beam

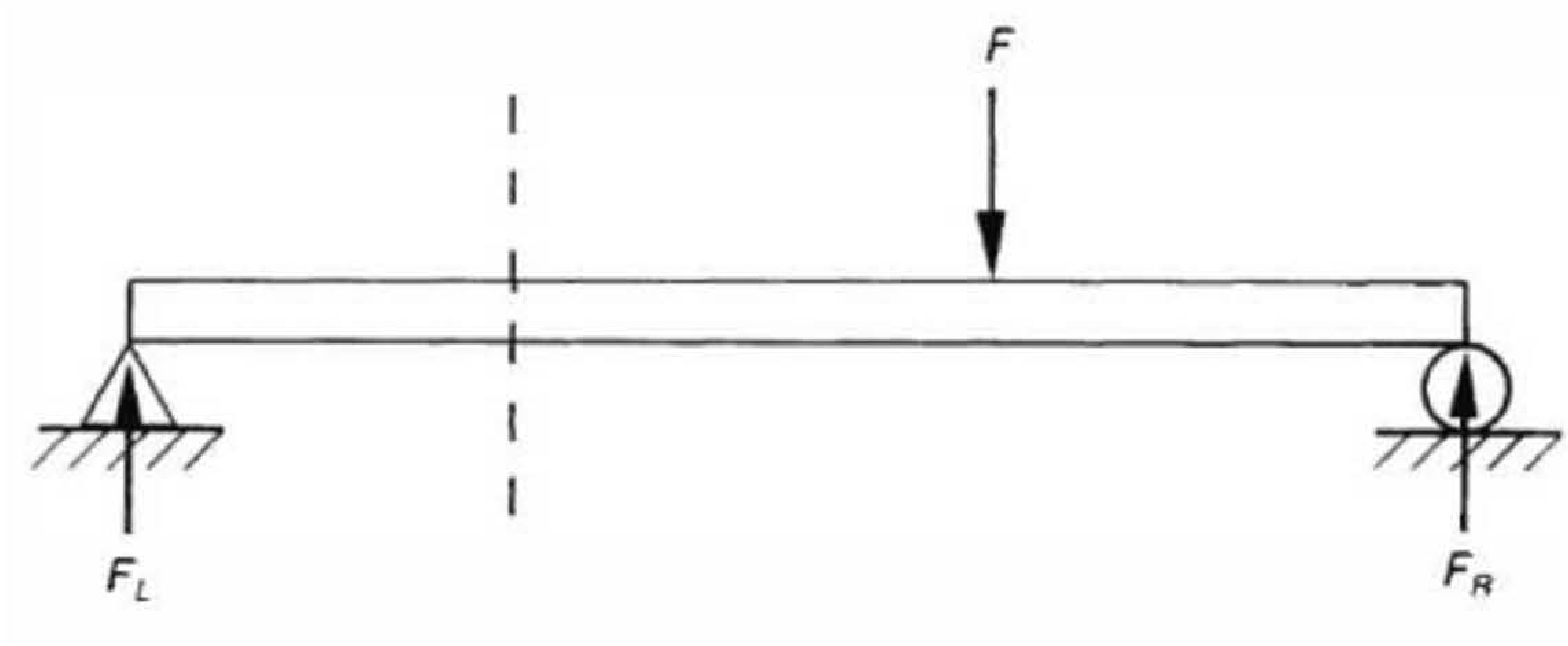
Consider two portions of a beam cut by an imaginary section transverse to the beam as shown in Figure (a).

The effect of the unbalanced forces on each part of the beam is to move the left-hand portion upwards relative to the right-hand portion, as shown in Figure (b) above.

The reason that we never actually observe such movement between parts of a solid beam is because internal forces exist within the material of the beam, in the plane of the imaginary cross-section, which resist the tendency for such movement. The magnitude of the internal force at any given cross-section depends on the sum of the external forces acting on each portion of the beam to one side of the cross-section. This internal resistance force is called shear force.

The magnitude of the shear force at any cross-section of a beam is equal to the algebraic sum of all external forces, i.e. loads and reactions, acting on either portion of the beam to one side of the section only.

< BACK GIVE FEEDBACK OK



The magnitude of the shear force at the imaginary cut section (dotted line) of this beam is (please select) ▼ and this shear force is (please select) ▼ direction.

Submit

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

We find the shear force in exactly the same way as we do for bending moments except that we are dealing with equilibrium of vertical forces rather than moments.

Procedure for determining shear force at a location in a beam

To find the shear force at any location in a beam, follow these steps:

1. Solve static equilibrium
2. Cut the beam at the location of interest
3. Choose a side and construct a free body diagram
4. Solve equilibrium of forces for the free body diagram

You may notice that the first three steps are exactly the same as the procedure for finding bending moment but that step 4 is equilibrium of vertical forces (rather than equilibrium of moments).

GIVE FEEDBACK

OK

Sort these calculation steps into the correct order for finding shear forces at a certain point in a beam.

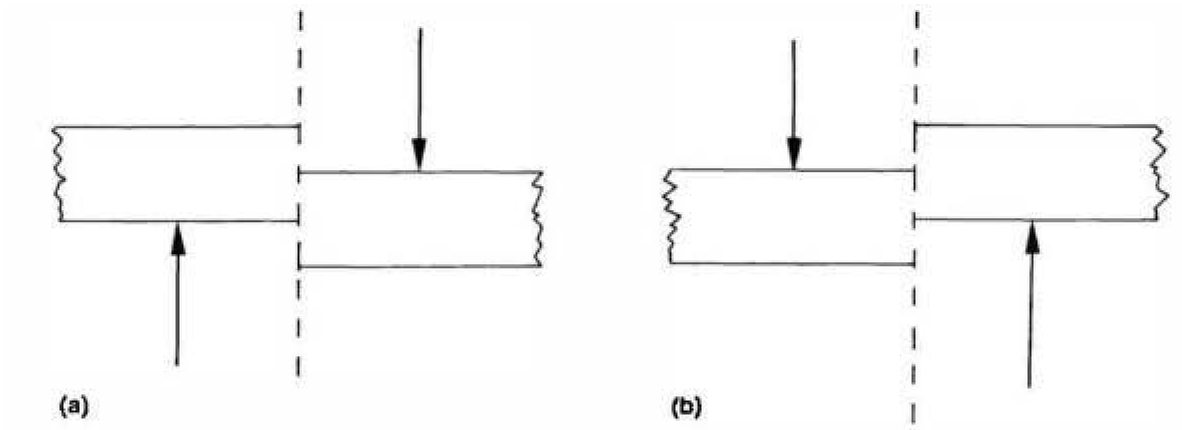
↑↓ Place these in the proper order.

- Solve equilibrium to get all reaction forces
- Choose the location along the beam where bending moment is to be determined
- Cut the beam at that point and take the left (or right) side as a free body diagram
- Calculate the sum of all forces on that left (or right) free body diagram

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

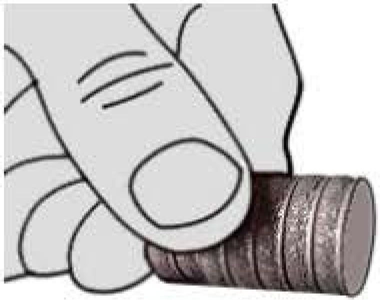
Consider a beam. Any location where the beam is cut can either have a positive shear force, as in Figure (a) below, or a negative shear force, as in Figure (b).



GIVE FEEDBACK

CONTINUE >

This matches the animation below, where a positive shear force has the left side up and right side down. (↑ + ↓)



< BACK GIVE FEEDBACK CONTINUE >

The sign convention for shear forces

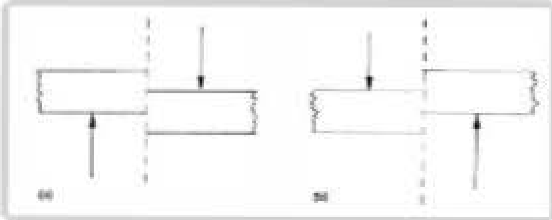
It also agrees with the definition of positive shear stress in the interactive animation below.

Video object isn't supported in c++ version

This animation demonstrates that a positive shear force (up on the left-hand-side $\uparrow + \downarrow$) equates to a positive shear stress ($\tau > 0$).

< BACK | GIVE FEEDBACK | OK

Match the labels to the correct description relating to shear force.



Use mouse to zoom. Click to keep enlarged.

Drag statements on the right to match the left.

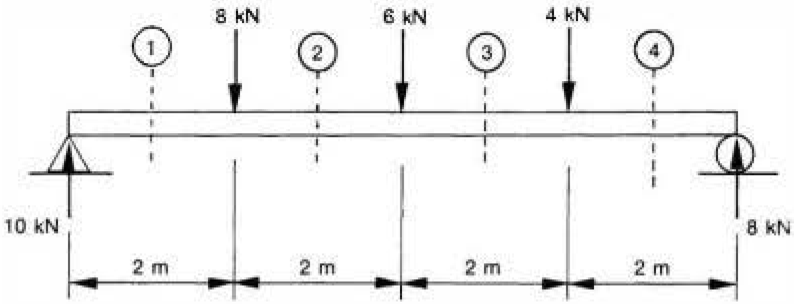
Positive shear force	(a)
Negative shear force	(b)
Zero shear force	(not shown)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Calculate the shear force for different cross-sections of a beam

For the beam and loading shown, determine the shear force at each of the three applied forces.



The initial step is to solve reactions.

These have been solved—10 kN left reaction and 8 kN right reaction.

If this was not the case, we would have to solve equlibrium of non-concurrent forces to find left and right reactions.

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

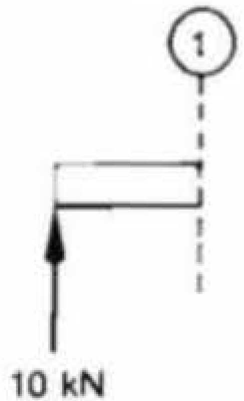
GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 1)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of forces for the free body diagram

We will start from the left-hand side and consider all forces to the left of the cross-section.

Bending moment at cross-section 1:

$SF_1 = \Sigma F$

$= 10$ kN (positive shear force)

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

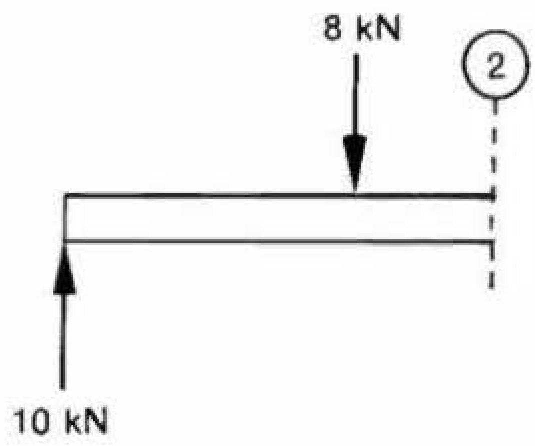
GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 2)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of forces for the free body diagram

Bending moment at cross-section 2:

$$\begin{aligned} SF_2 &= \Sigma F \\ &= 10\text{ kN} - 8\text{ kN} \\ &= 2\text{ kN (positive shear force)} \end{aligned}$$

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

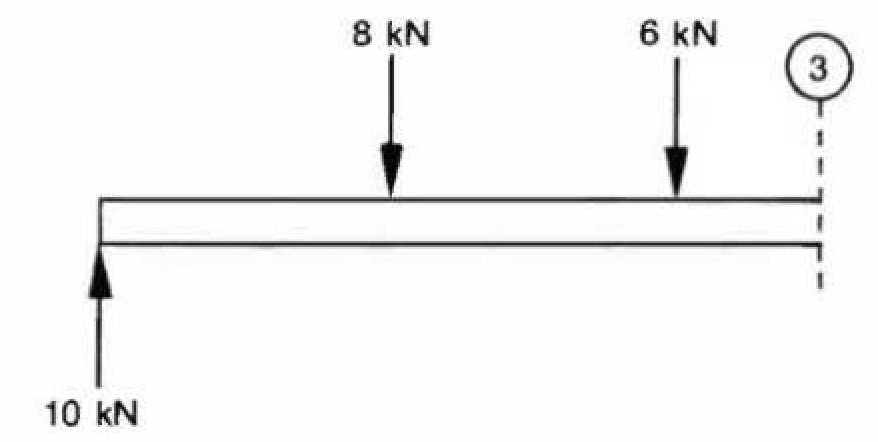
GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 3)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of forces for the free body diagram

Bending moment at cross-section 3:

$$SF_3 = \Sigma F$$
$$= 10\,\mathrm{kN} - 8\,\mathrm{kN} - 6\,\mathrm{kN}$$
$$= -4\,\mathrm{kN}$$ (negative shear force)

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

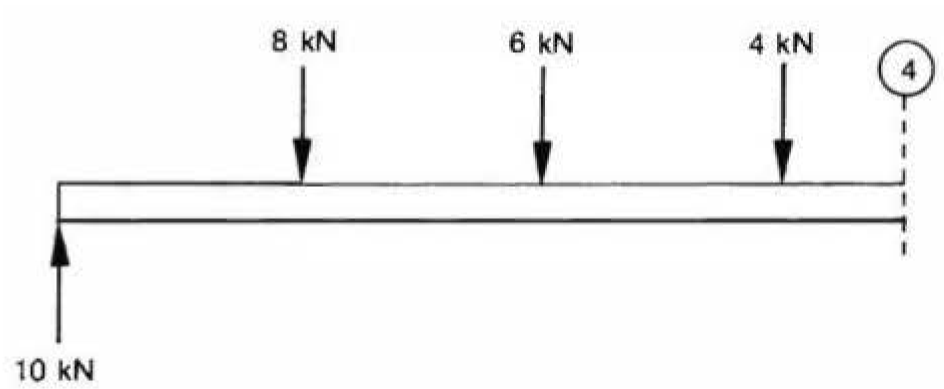
GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam

Step 1. Cut the beam at the point in interest (point 3)

Step 2. Choose a side and construct a free body diagram (left side)



Step 3. Solve equilibrium of forces for the free body diagram

Bending moment at cross-section 4:

$$SF_3 = \Sigma F$$
$$= 10\,\text{kN} - 8\,\text{kN} - 6\,\text{kN} - 4\,\text{kN}$$
$$= -8\,\text{kN} \text{ (negative shear force)}$$

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam

$SF_1 = \Sigma F$

$= 10\,\text{kN}$ (positive shear force)

$SF_2 = \Sigma F$

$= 10\,\text{kN} - 8\,\text{kN}$

$= 2\,\text{kN}$ (positive shear force)

$SF_3 = \Sigma F$

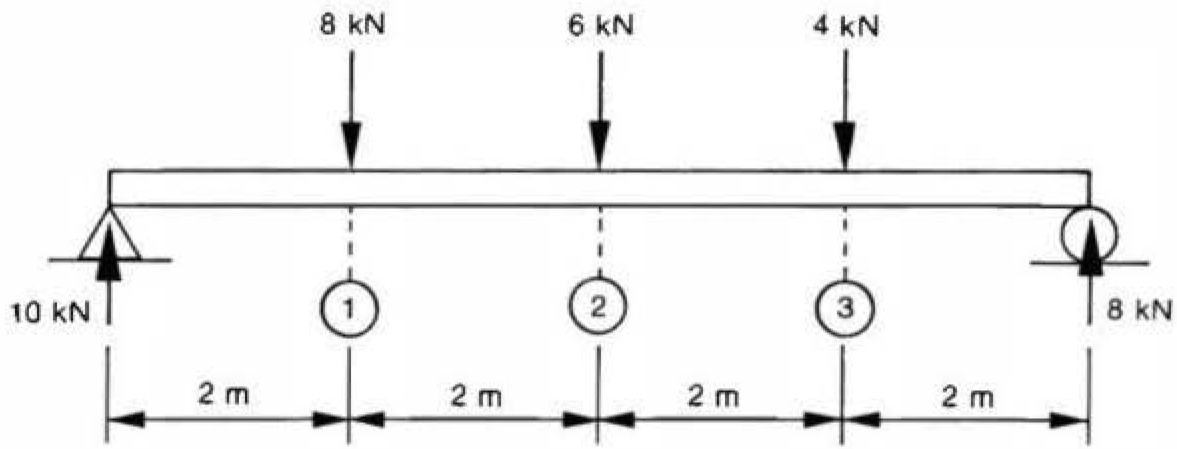
$= 10\,\text{kN} - 8\,\text{kN} - 6\,\text{kN}$

$= -4\,\text{kN}$ (negative shear force)

$SF_4 = \Sigma F$

$= 10\,\text{kN} - 8\,\text{kN} - 6\,\text{kN} - 4\,\text{kN}$

$= -8\,\text{kN}$ (negative shear force)



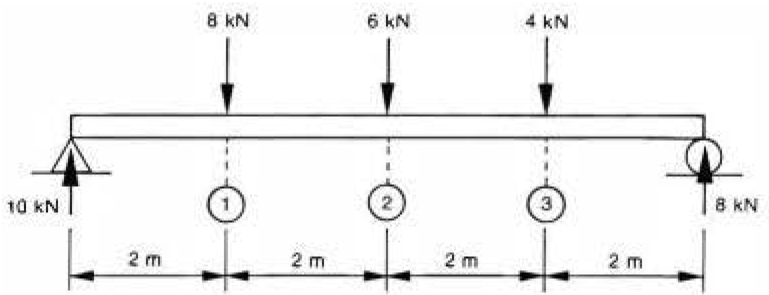
It turns out that if we always work with the left-hand side as the free body diagram, the shear force will automatically have the correct sign.

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

GIVE FEEDBACK

OK

Calculate the shear force for different cross-sections of a beam



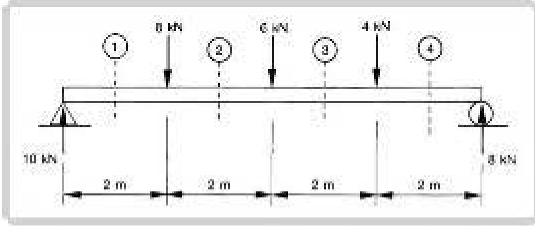
Left-hand side	Right-hand side	Shear force
$\Sigma F_{0\text{-}1} = 10\,\text{kN}$	$\Sigma F_{0\text{-}1} = 8 - 4 - 6 - 8 = -10\,\text{kN}$	+ 10 kN
$\Sigma F_{1\text{-}2} = 10 - 8 = 2\,\text{kN}$	$\Sigma F_{1\text{-}2} = 8 - 4 - 6 = -2\,\text{kN}$	+ 2 kN
$\Sigma F_{2\text{-}3} = 10 - 8 - 6 = -4\,\text{kN}$	$\Sigma F_{2\text{-}3} = 8 - 4 = 4\,\text{kN}$	-4 kN
$\Sigma F_{3\text{-}4} = 10 - 8 - 6 - 4 = -8\,\text{kN}$	$\Sigma F_{3\text{-}4} = 8\,\text{kN}$	-8 kN

Determining shear force at three points along a beam	Location 1	Location 2	Location 3	Location 4	Summary	Compare left-hand side to right-hand side

GIVE FEEDBACK

OK

The sum of all vertical forces on the left side of point 2 is (+2 kN). Which of the following statements are true?



Use mouse to zoom. Click to keep enlarged.

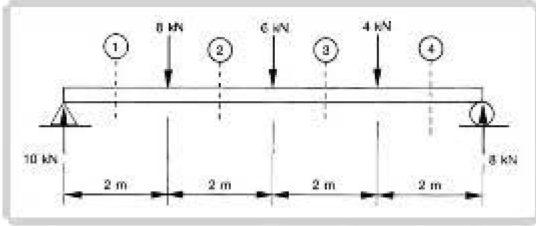
Check all that apply.

- [ ] The shear force at point 2 is (+2 kN)
- [ ] The shear force at point 2 is (–2 kN)
- [ ] The sum of all forces on the right side of point 2 is (+2 kN)
- [ ] The sum of all forces on the right side of point 2 is (–2 kN)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

The sum of all vertical forces on the left side of point 3 is (−4 kN). Which of the following statements are true?



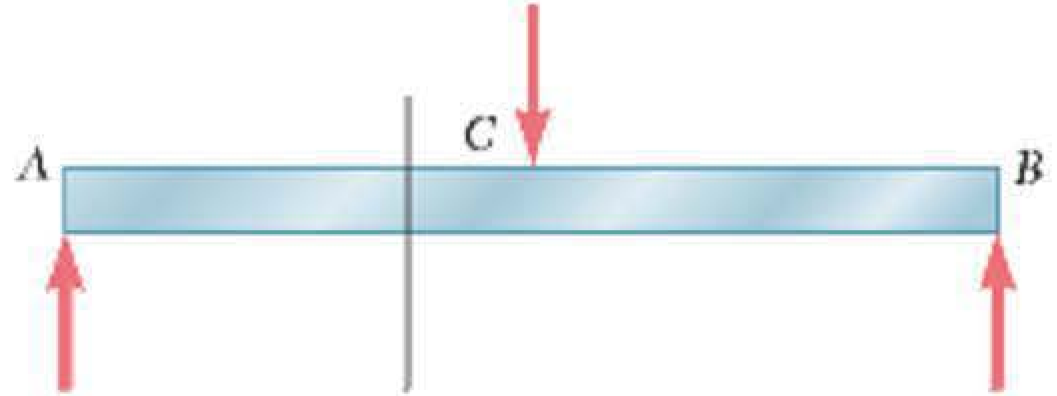
Use mouse to zoom. Click to keep enlarged.

Check all that apply.

- [ ] The shear force at point 3 is (−4 kN)
- [ ] The shear force at point 3 is (+4 kN)
- [ ] The sum of all forces on the left side of point 3 is (+4 kN)
- [ ] The sum of all forces on the right side of point 3 is (+4 kN)

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



A beam is 5.1 m long with a 59 kN force applied at the centre. Find the shear force at the line shown (at a distance of 2.04 m from the left end).

(Minimum one decimal place. Include units and correct +/- sign.)

Clear

Clear line

?

Undo

Click and type your answer here

CHALLENGE

SUBMIT

SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

A shear force diagram is simple to construct because shear force in the beam does not change where there are no applied external forces.

A graph of all the shear forces along the entire beam is called a shear force diagram.

It involves plotting the values obtained by calculation against the distance measured along the beam.

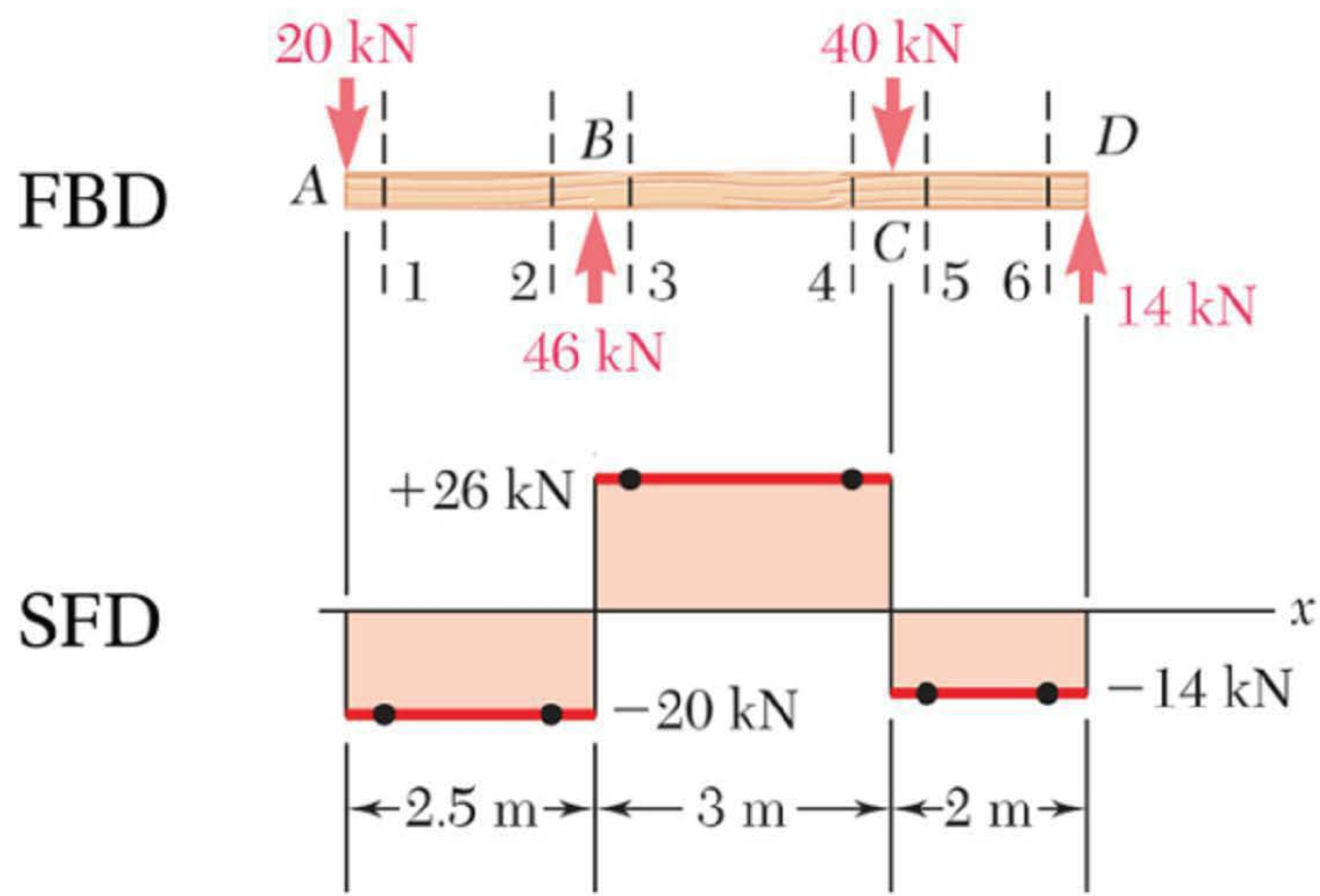
This is simple because the shear force is constant between each external force.

The shear force diagram becomes a set of horizontal straight lines with step changes under each load.

GIVE FEEDBACK

CONTINUE >

The following example shows a free body diagram with the matching shear force diagram below it. Note the horizontal lines between forces where there is no change in shear force.



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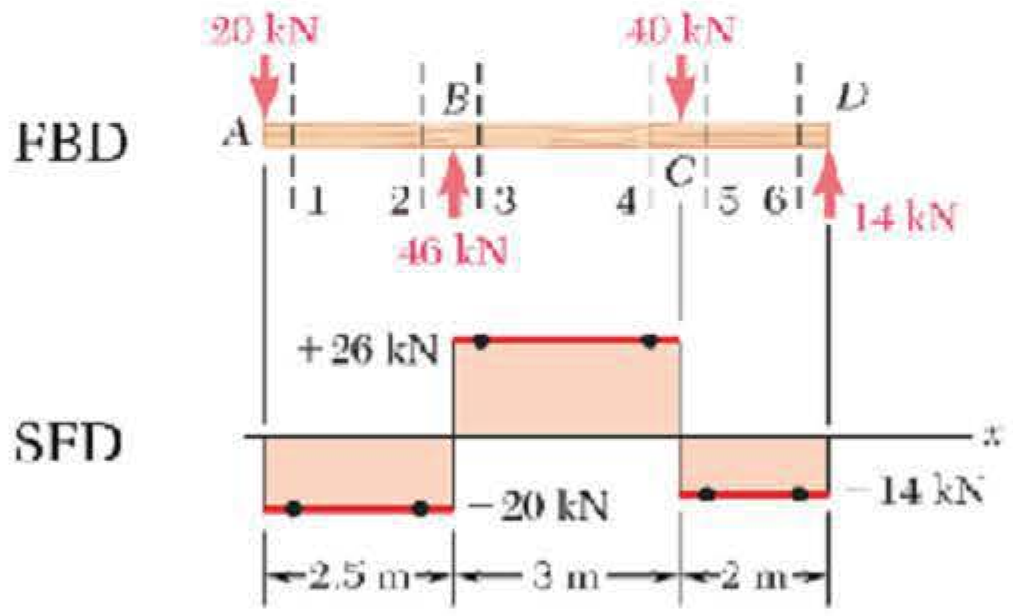
Which of the following statements are true about this shear force diagram?

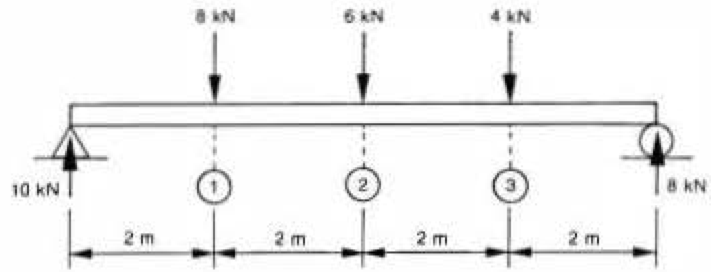


Check all that apply.

- [ ] None of the points (1 to 6) have the same shear force
- [ ] Points 1 and 2 have the same shear force
- [ ] Points 3 and 4 have the same shear force
- [ ] Points 2 and 3 have the same shear force

Sketching a shear force diagram is easy. It is best if you arrange the beam so that a free end is on the left. A free end means free to pivot not rigidly mounted into a wall, for instance. Then you simply plot the shear force from left to right, taking vertical steps as you meet each applied force. The reason we set it up from left to right is so the force directions (up or down) match the sign of the shear force.

Example

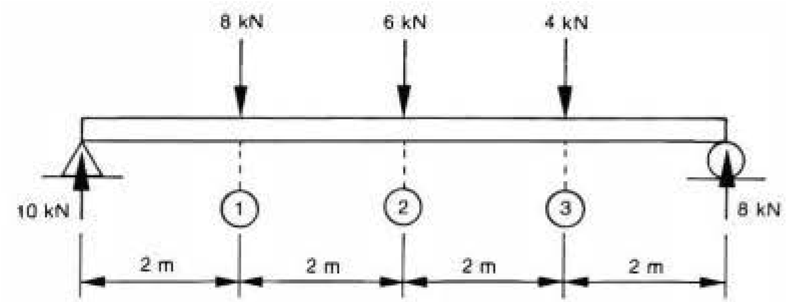


Draw the shear force diagram for this beam.

GIVE FEEDBACK

CONTINUE >

Example

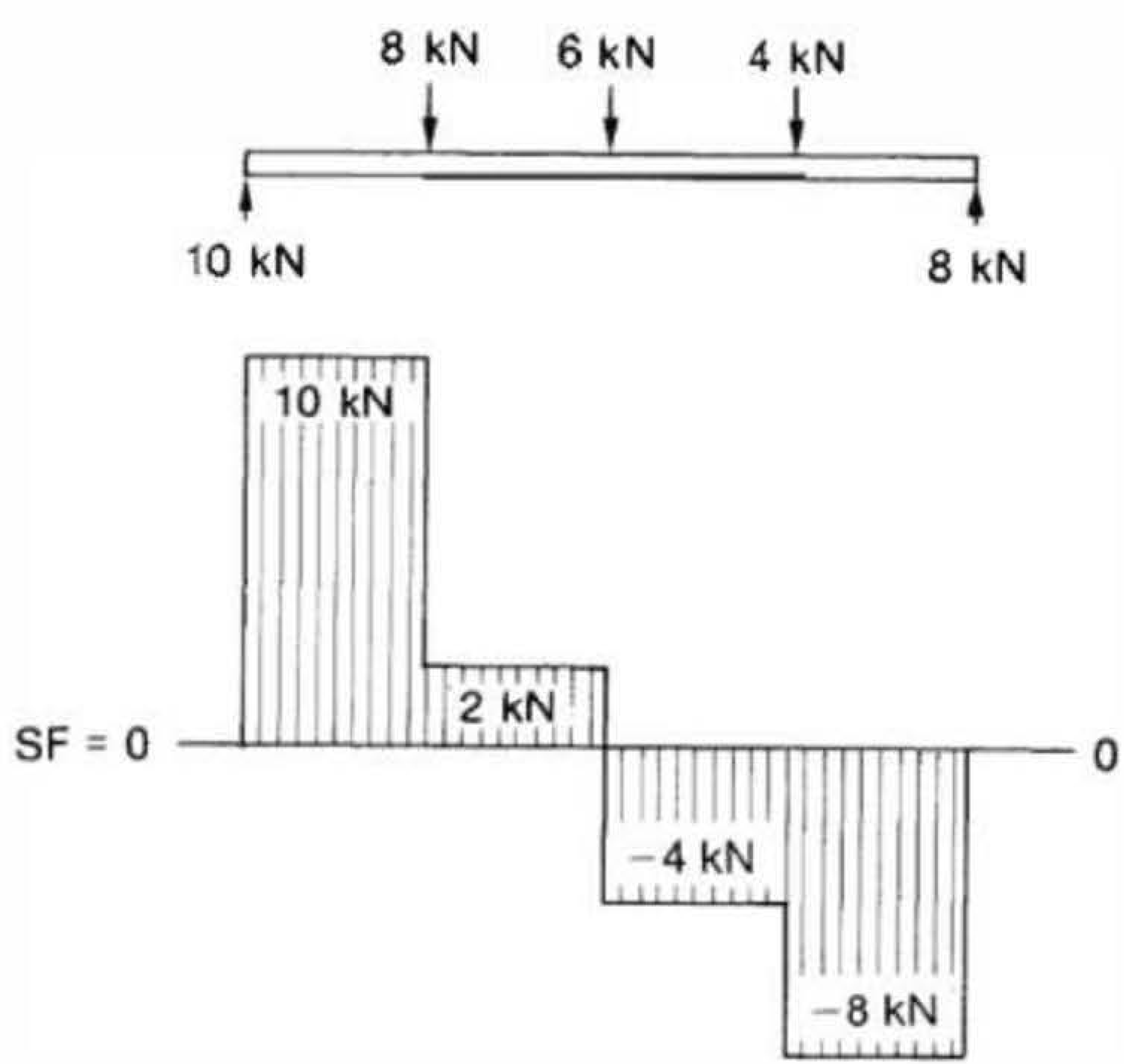


Draw the shear force diagram for this beam.

Solution

The shear force diagram should be plotted directly below the free body diagram of the beam, the horizontal distance representing the length of the beam.

< BACK GIVE FEEDBACK CONTINUE >



Start on the left free end (where SF = 0) and draw a vertical line to scale representing 10 kN. This is positive shear so it is plotted upwards.

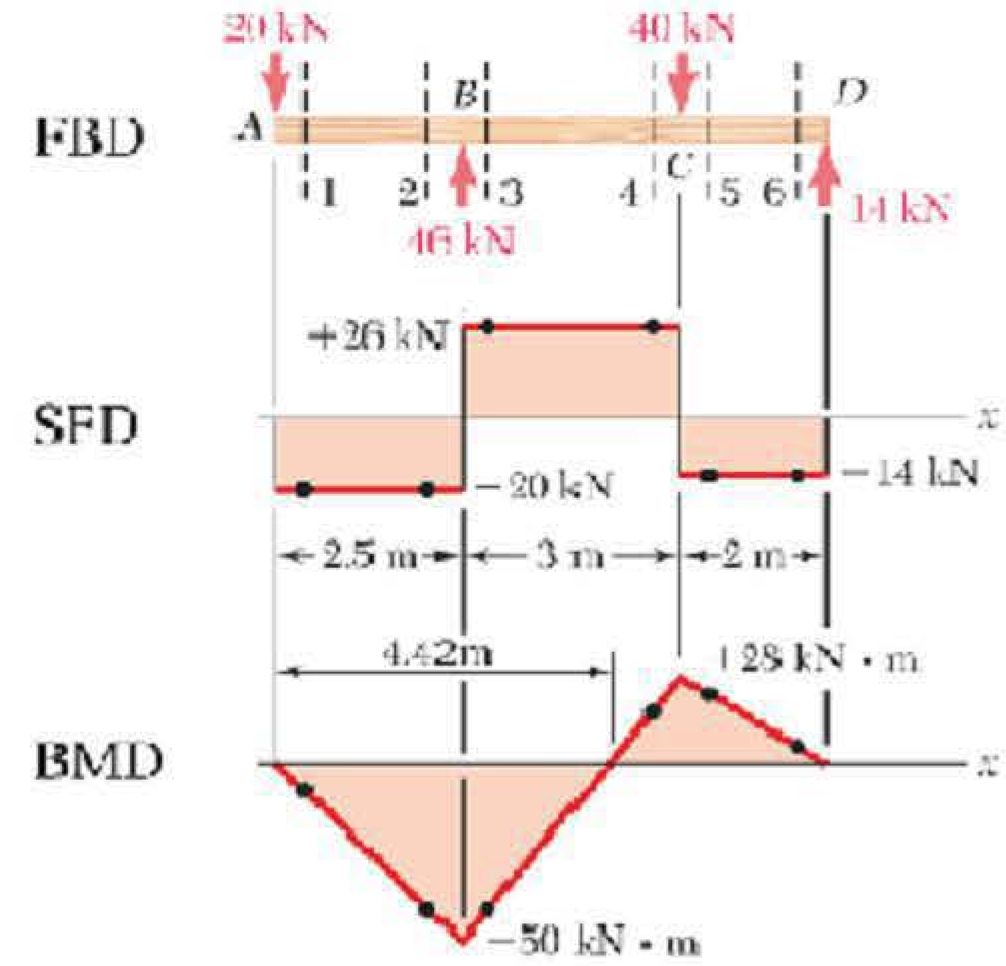
Nothing happens for the next few metres until we hit the 8 kN force, so draw a horizontal line. A step change occurs at each load—this time we go down by 8 kN (giving a shear force (SF) height of 2 kN).

Continue until you reach the right hand end—it should meet at zero shear force (free end).

Magnitudes can be shown along the lines for clarity.

< BACK GIVE FEEDBACK OK

Identify the following information from this shear force diagram.



Drag statements on the right to match the left.

The highest shear force in this beam is	20 kN
The highest force on this beam is	14 kN
The highest level of negative shear force in this beam is	46 kN
The continuous region of the smallest shear force is	26 kN

To construct a bending moment diagram we simply need to calculate the bending moment at each applied force, then plot these points onto a diagram and draw lines between them.

Constructing a bending moment diagram

We already know how to calculate bending moment at any point along the beam.

Now we will construct a bending moment diagram which shows the bending moment all along the beam.

Just as the shear force diagram has no change (horizontal line) between each force, the bending moment diagram will form sloping lines between the forces, joining the calculated magnitude of the bending moment at each load-bearing point.

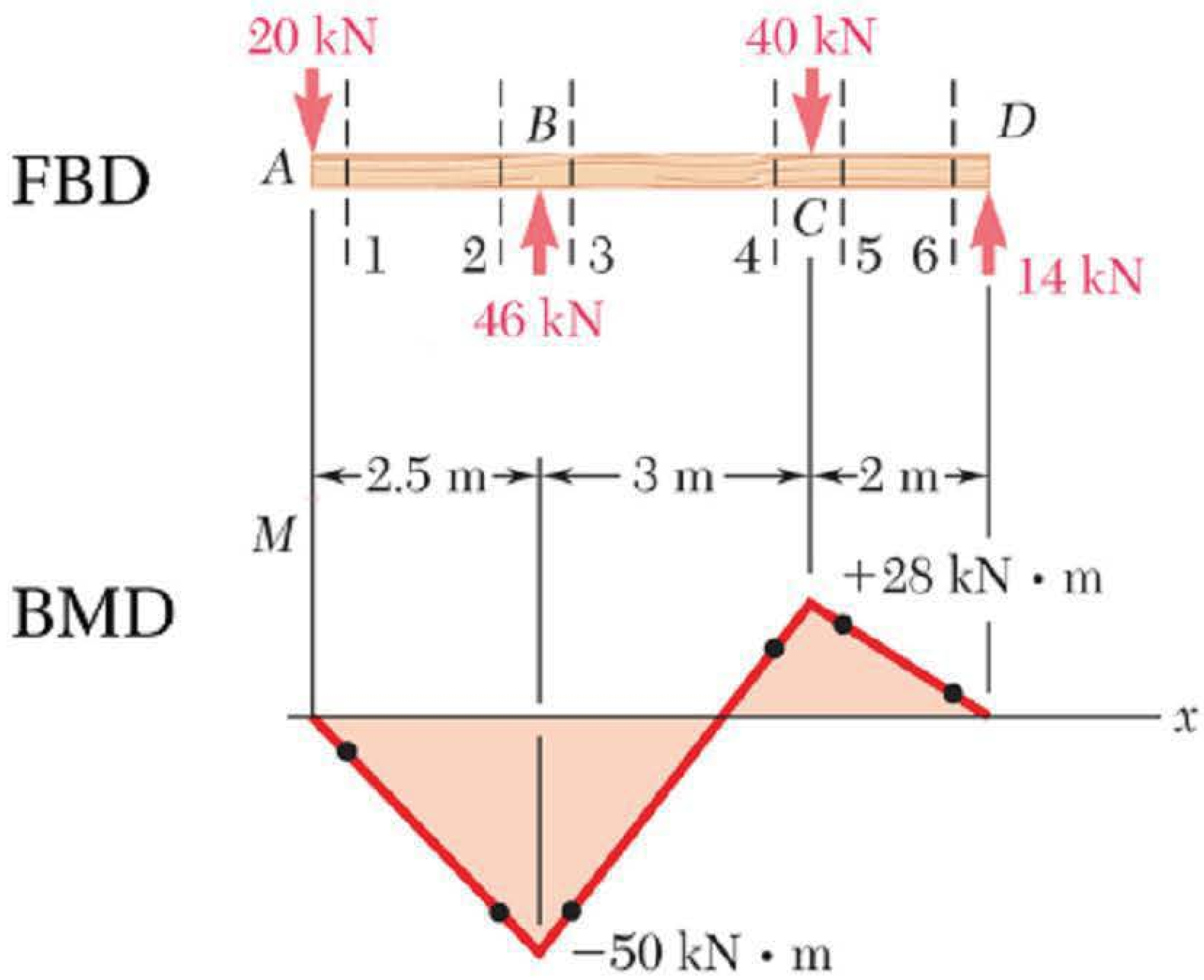
The steps involved are to:

1. Calculate the bending moment at each applied force (take moments of left-hand side)
2. Plot these points on a bending moment diagram and draw lines between them

GIVE FEEDBACK

OK

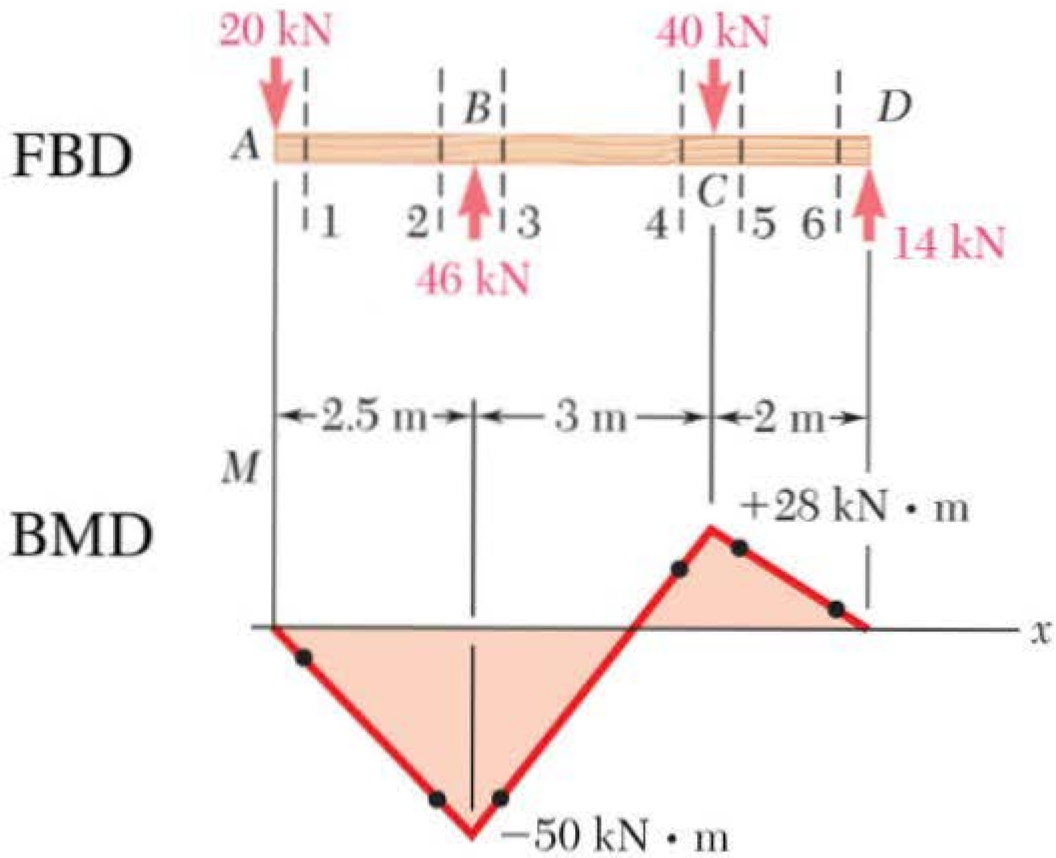
Which of the following statements are true about this particular bending moment diagram?



Check all that apply.

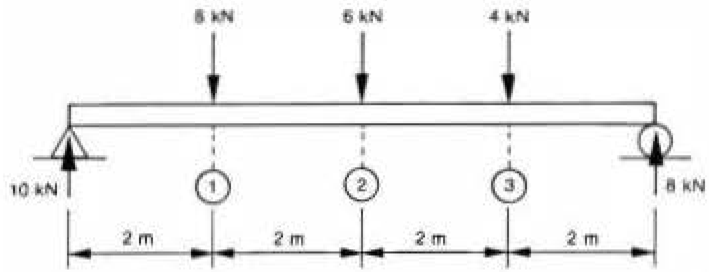
- [ ] The maximum sagging BM is 50 kNm
- [ ] The maximum hogging BM is 50 kNm
- [ ] This beam has higher hogging BM than sagging BM
- [ ] BM is 0 at three points on this beam

If this was a rectangular wooden beam, the most likely place to break would be (please select) ▾.



This method of constructing a bending moment diagram requires the bending moment to be calculated at each applied force. In this example we would need to calculate three bending moments for each point: -1, 2 and 3. These points are plotted on a bending moment diagram and then joined together with straight dotted lines.

Example



Draw the bending moment diagram using direct moments (sectioning of beam).

Method

1. Begin construction of the bending moment diagram directly below the free body diagram in order to match up the forces
2. Calculate the BM at each applied force and plot these points on the bending moment diagram (upwards for +BM)
3. Connect the dots with straight lines and include magnitudes of BM

GIVE FEEDBACK

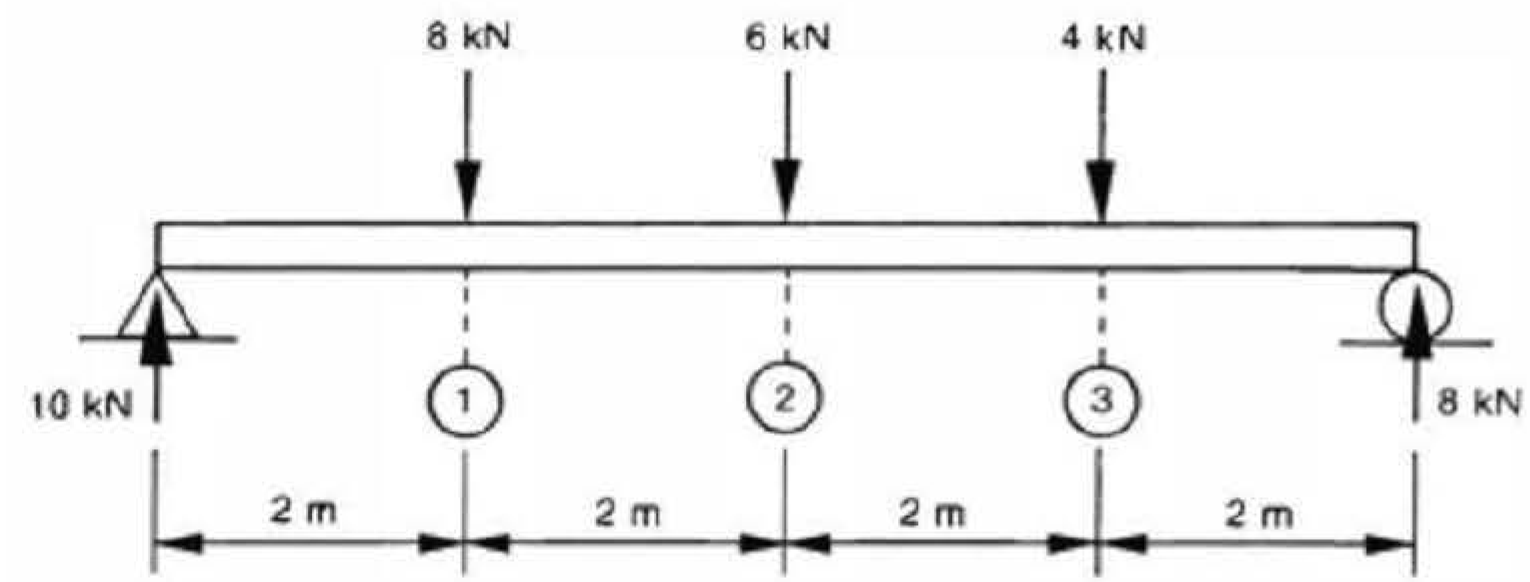
CONTINUE >

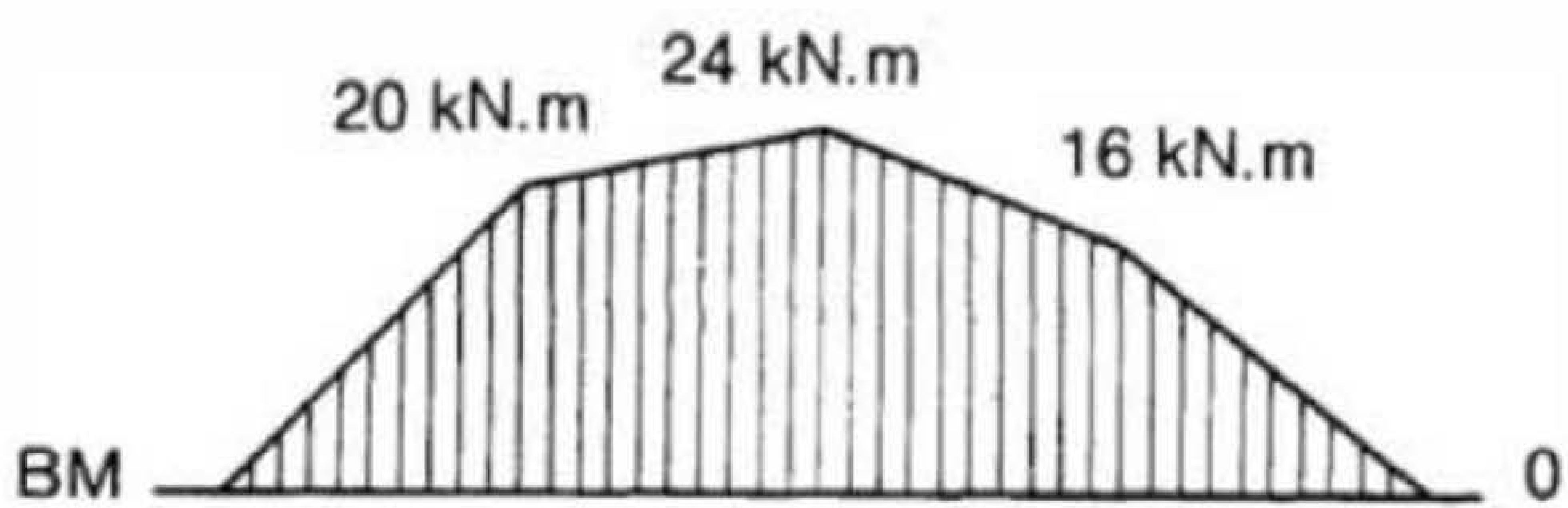
Solution

$BM_1 = \Sigma M = 10\ \text{kN} \times 2\ \text{m} = 20\ \text{kN.m}$
$BM_2 = \Sigma M = 10\ \text{kN} \times 6\ \text{m} - 8\ \text{kN} \times 4\ \text{m} = 24\ \text{kN.m}$
$BM_3 = \Sigma M = 10\ \text{kN} \times 6\ \text{m} - 8\ \text{kN} \times 4\ \text{m} - 6\ \text{kN} \times 2\ \text{m} = 16\ \text{kN.m}$

BM at each end is 0 because the ends are free to pivot. The bending moment diagrams appear on the following slide.

< BACK GIVE FEEDBACK CONTINUE >





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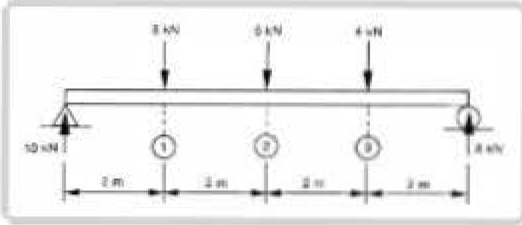
GIVE FEEDBACK
OK

Place these steps in the correct sequence for constructing a bending moment diagram directly using moments (i.e. without using a shear force diagram).



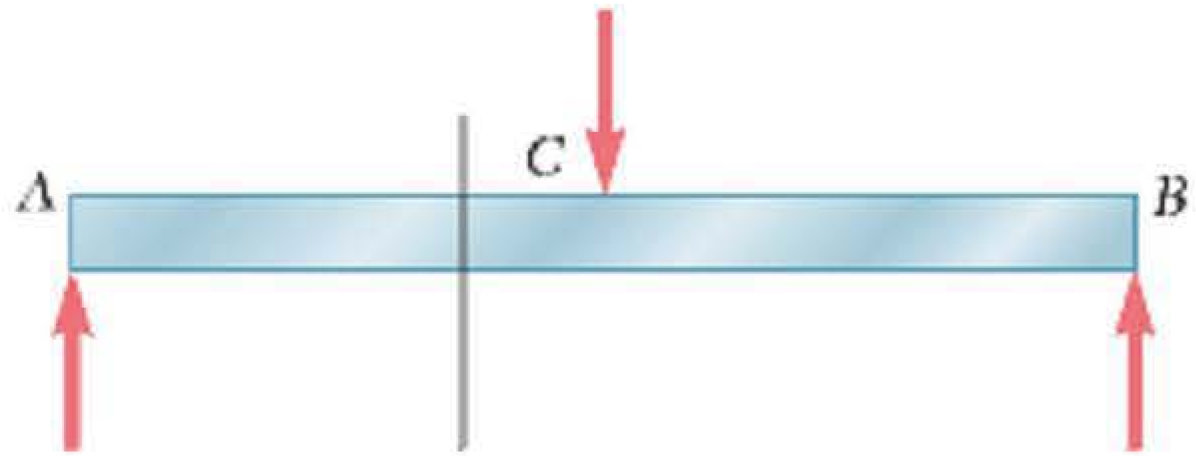
Use mouse to zoom. Click to keep enlarged.

↑↓ Place these in the proper order.

- Draw a free body diagram of the beam
- Prepare a bending moment diagram so that the forces are lined up with the forces in the free body diagram
- Calculate the bending moment at each applied force and plot these points on the bending moment diagram (upwards for +BM)
- Connect the dots with straight lines

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



A beam is 5.3 m long with a 54 kN force applied at the centre. Find the bending moment at the line shown (at a distance of 2.12 m from the left end).

(Minimum one decimal place. Include units. Include correct sign.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question

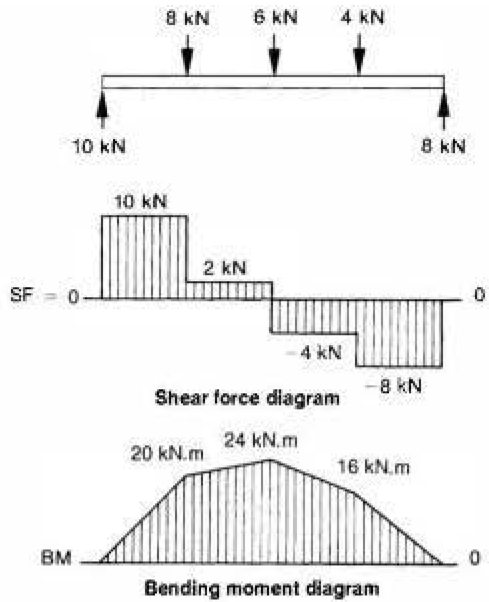
Starting from a free left end for any section of beam, the change in height of the bending moment diagram is equal to the change in area in the shear force diagram.

This alternative method to construct a bending moment diagram requires the shear force diagram to be done first.

Starting from a free left end for any section of beam, the change in height of the bending moment diagram equals the change in area under the shear force diagram.

GIVE FEEDBACK

CONTINUE >



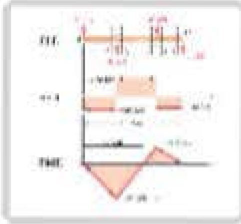
< BACK GIVE FEEDBACK CONTINUE >

- In the first 2 m, area under SFD = +10 kN x 2 m = +20 kNm. BM goes from 0 to 20 kNm
- In the second 2 m, area under SFD = +2 kN x 2 m = +4 kNm. BM goes from 20 kNm to 24 kNm
- In the third 2 m, area under SFD = –4 kN x 2 m = –8 kNm. BM goes from 24 kNm to 16 kNm
- In the last 2 m, area under SFD = –8 kN x 2 m = –16 kNm. BM goes from 16 kNm to 0 kNm, which is the free end

Notice that the area under the shear force graph is height x width, where height is kN and width is m, which gives kNm (moment).

< BACK GIVE FEEDBACK OK

What does the 4.42 m mean on this diagram?



Use mouse to zoom.
Click to keep enlarged.

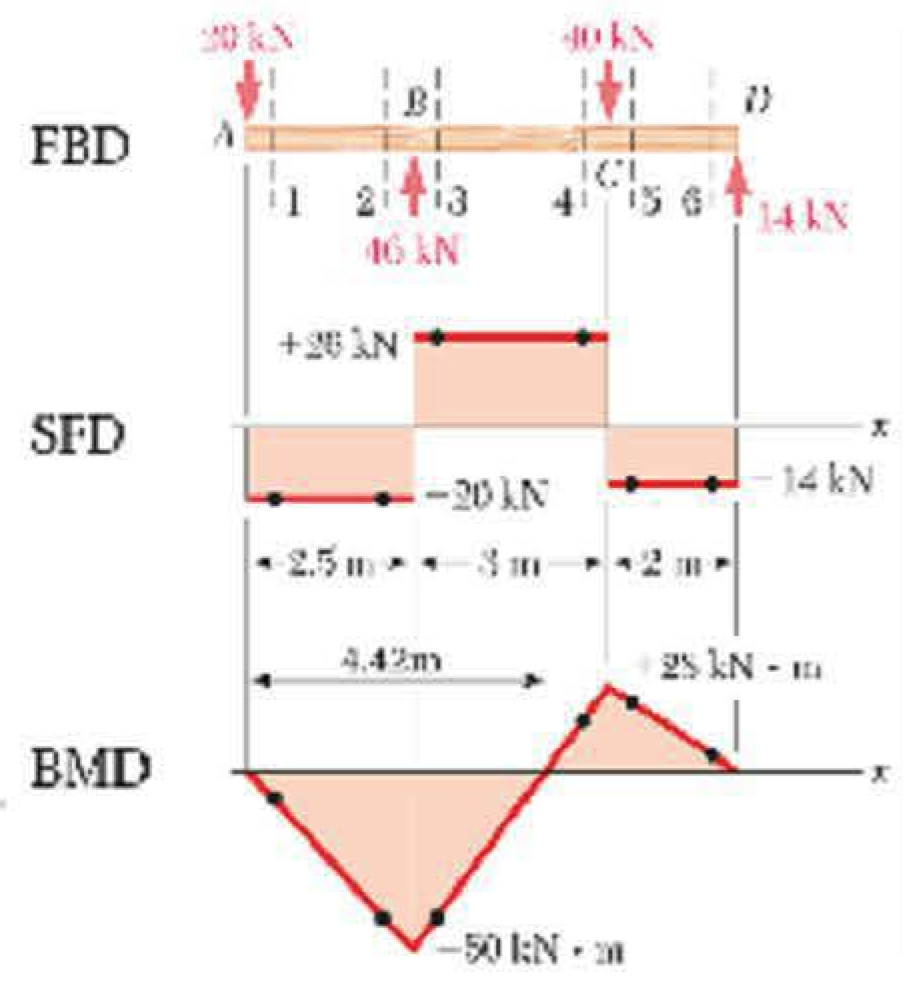
Check all that apply.

- [ ] This is where the beam goes from hogging to sagging
- [ ] This is where the maximum bending moment occurs
- [ ] This is where the negative shear force from AB is balanced by the positive shear force between B and C
- [ ] This is where the shear force is 0
- [ ] This is where the BM is 0

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Use the shear force diagram to correctly match the statements below as if you were creating a bending moment diagram.



Drag statements on the right to match the left.

Area within the shear force diagram from A to B =	78 kNm
Area within the shear force diagram from B to C =	−28 kNm
Area within the shear force diagram from C to D =	−50 kNm
Area within the shear force diagram from A to D =	28 kNm
Area within the shear force diagram from A to C =	0 kNm

Use shear force diagram to locate maximum bending moment

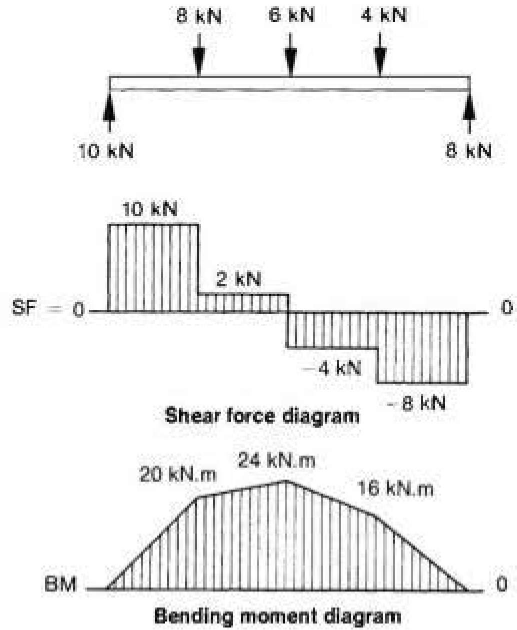
In engineering, the maximum bending moment is critical. However, if a beam has multiple forces, it is not always possible to tell at a glance where the maximum bending moment occurs.

It turns out that the peak bending moments coincide with the shear force diagram passing through the zero axis.

Example: This happens in the shear force diagram at the 6 kN force (4 m from left end).

At this point the BM is 24 kNm, which is the maximum for the whole beam.

This rule applies for every peak positive or negative BM, where there are multiple loads in many directions. One of these peaks is the maximum bending moment.



Maximum bending moment	Note

GIVE FEEDBACK

OK

Use shear force diagram to locate maximum bending moment

This is actually differential calculus, where the shear force (SF) is the derivative of bending moment:

$$\text{SF} = \frac{d\,(\text{BM})}{d\,x}$$

So if $\text{SF} = \frac{d\,(\text{BM})}{d\,x} = 0$

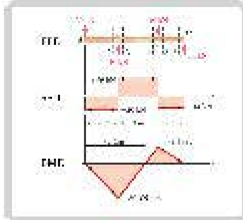
BM is at a maximum.

Maximum bending moment	Note

GIVE FEEDBACK

OK

Which of the following statements are true when finding the maximum bending moment (highest numerical value)?



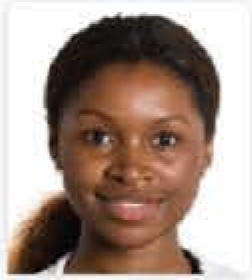
Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] The maximum BM always occurs where the shear force diagram crosses the zero axis
- [ ] Whenever the shear force diagram crosses the zero axis the maximum BM occurs
- [ ] There may be several peak BMs when the shear force diagram crosses the zero axis multiple times

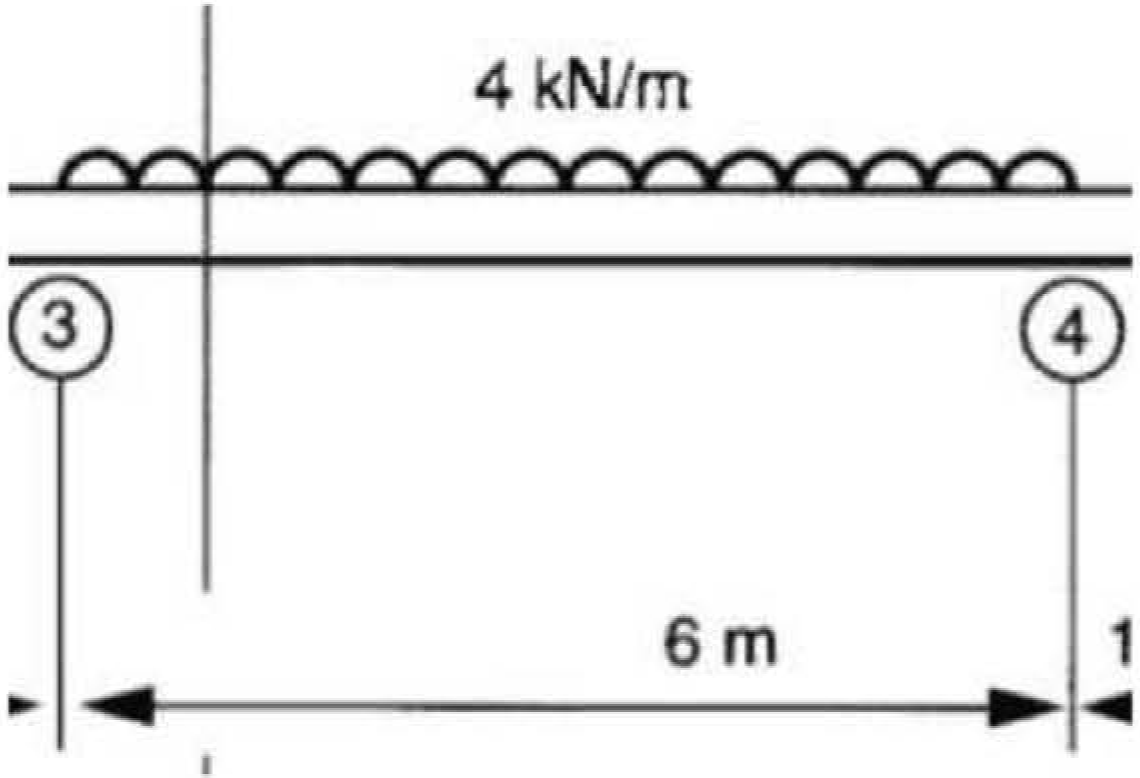
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA



A uniformly distributed load is a load that is spread out over a certain length of beam. Uniformly means the load is spread evenly, with a constant amount each metre of beam. Distributed means the load is spread out over a region instead of concentrated at one point. Typical uniformly distributed loads include: the weight of sand bags; a pile or wall of bricks; a water tank; snow loads on a roof; the weight of a concrete floor on a beam; air pressure against a vacuum chamber; and the weight of the beam itself.

Uniformly distributed loads can be represented like sandbags (below). In this example, 4 kN is spead over each metre of beam for a total length of 6 m.

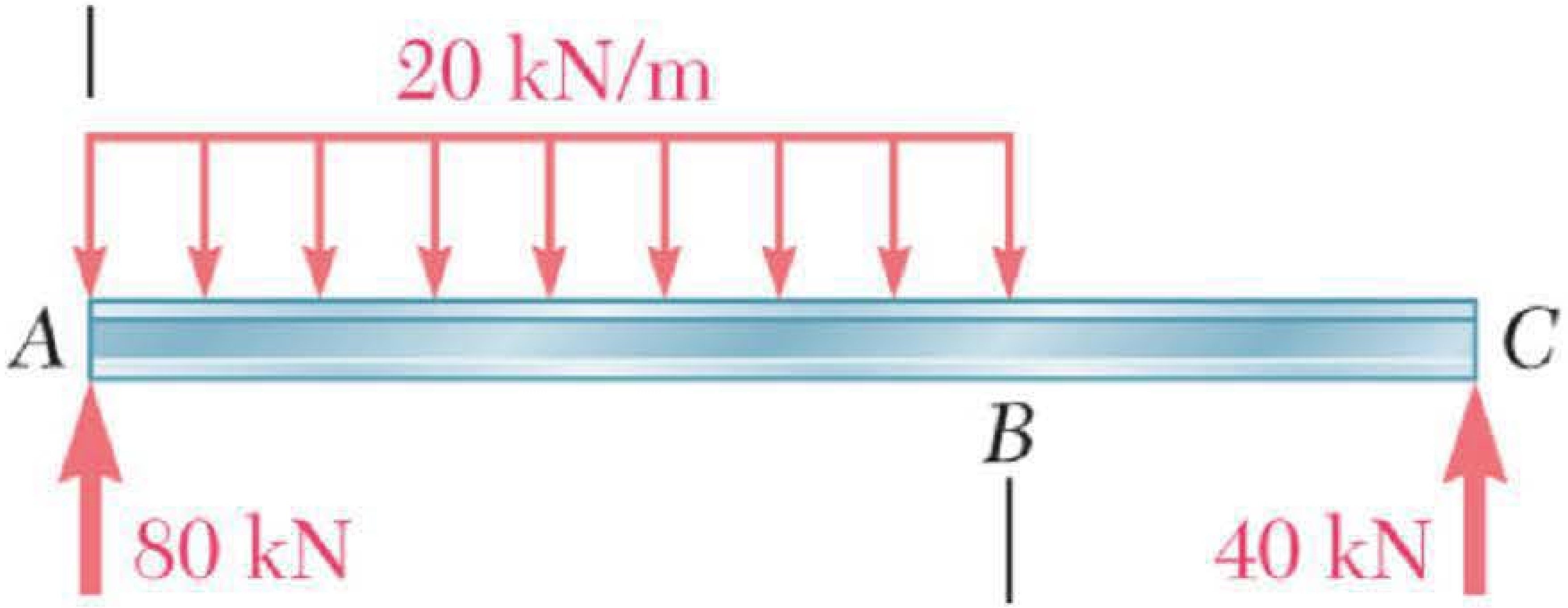


GIVE FEEDBACK

CONTINUE >

Another way to show distributed load is by using arrows (below).

Note: The arrows are not meant to be every metre, they just symbolise that the load is spread over that region.



< BACK GIVE FEEDBACK OK

Which of the following would normally be treated as a distributed load on a beam?

Check all that apply.

- [ ] The weight of the beam itself
- [ ] A load hanging from the beam by a cable
- [ ] The weight of a concrete floor supported by the beam
- [ ] The pin support reaction

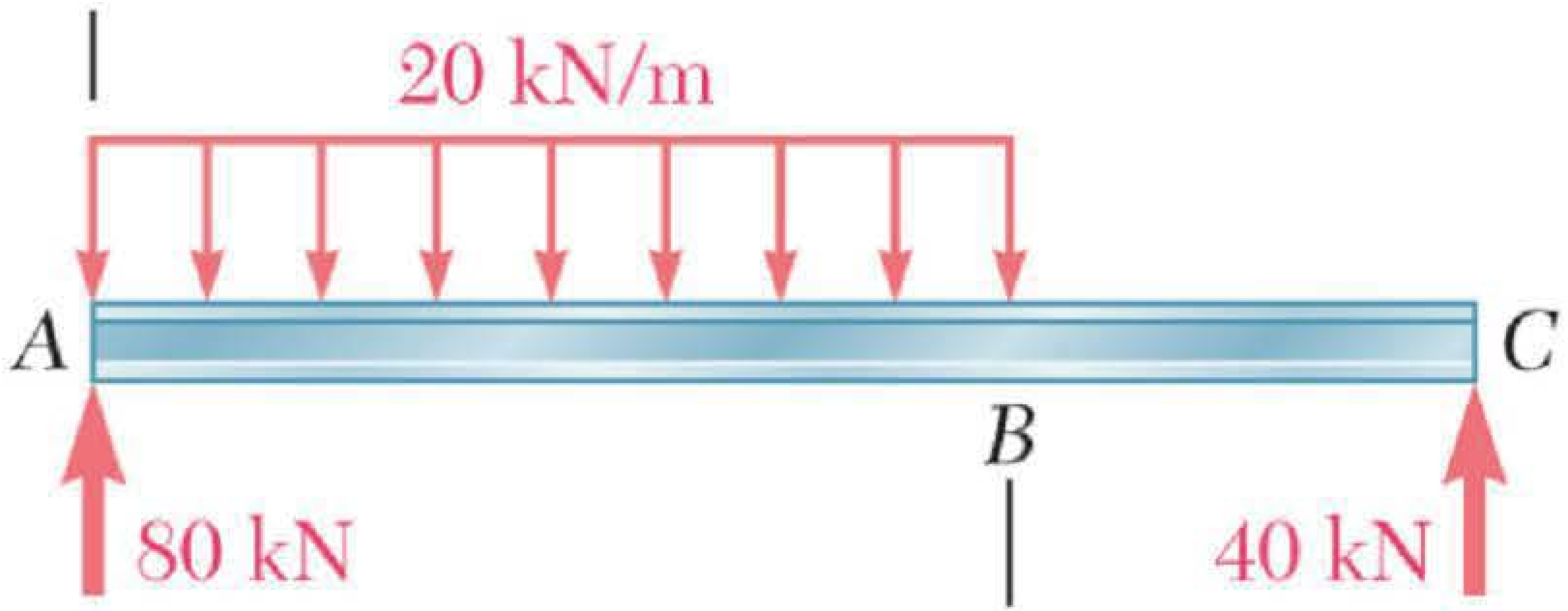
Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

A distributed load is converted to a point load by multiplying the loading (in kN/m) by the length of the region (in m) to get the total load (in kN). We then place this total load in the middle of the region.

Previously we converted uniformly distributed loads to point loads (equilibrium of non-concurrent forces).

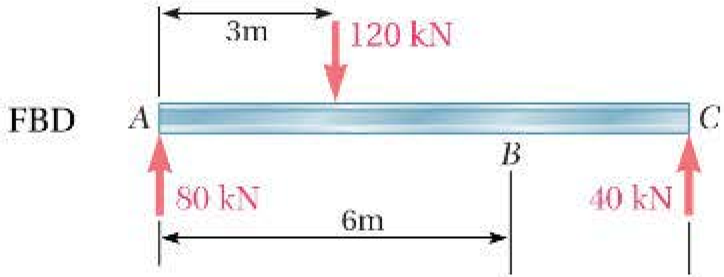
In the example below, the distributed load of 20 kN/m covers 6 m (120 kN in total).



GIVE FEEDBACK

CONTINUE >

To convert this to a point load, apply 120 kN midway in the region at 3 m.



To convert a distributed load to a point load, multiply the loading (kN/m) by the length of the region (m) to get the total load (kN). Place this total load in the middle of the region.

This works for external but not internal analysis. It is great for finding reactions but doesn't work for finding bending moment. Bending moment is higher for a single concentrated force than for a distributed force. Therefore we must work with distributed loads when calculating bending moment.

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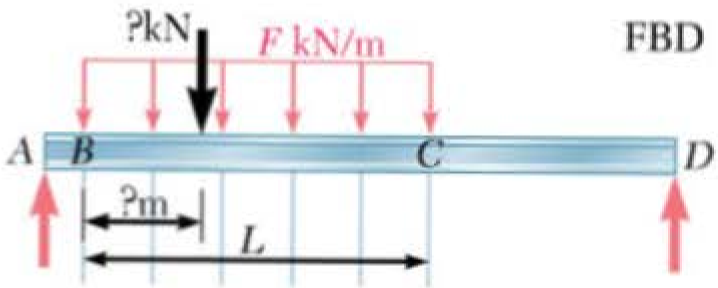


A uniformly distributed load of F = 18 kN/m is applied over a region L = 5.5 m long. If this loading is converted to a point force, where should that force be applied (measured in metres from point B)?

(Minimum two decimal places, include units.)

Clear

Clear line

Undo

Click and type your answer here

CHALLENGE

SUBMIT

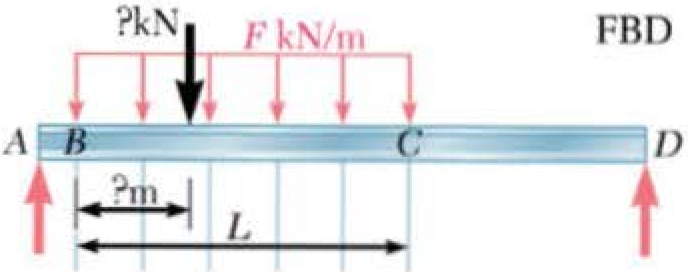
SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint

Each hint will reduce the credit received for this question



A uniformly distributed load of 18 kN/m is applied over a region L = 5.5 m long. If this loading is converted to a point force, what is the magnitude of the force in kN?
(Minimum one decimal place, include units.)

Clear
Clear line
Undo

Click and type your answer here

CHALLENGE SUBMIT SHOW ANSWER

INSTRUCTIONS

- No intermediate steps are required
- If you choose to show steps, write one on each line.
- Write your final answer on the last line.
- The computer will check all your work in detail when you click "Submit".

Hint   Each hint will reduce the credit received for this question

When working with external forces on distributed loads we can simply convert them to point forces and carry on. But we can't avoid them when working with internal forces. Swapping a distributed load for a point load will make changes to the inside, like bending moment and stresses.

We have seen distributed loads before, when finding reactions.

We simply converted the distributed load to a point load and used the point load instead.

This works for external equilibrium (reactions, force and moment equilibrium) but not internal analysis (shear force, bending moment and stresses).

Bending moment is higher for a point force than for a distributed force.

Therefore we must work with distributed loads when calculating bending moment.

GIVE FEEDBACK

CONTINUE >

Compare the effect of swapping distributed loads with point loads:

	Point load	Uniformly distributed load
Reactions	same	
Shear force	horizontal line	sloping line
Bending moment	sloping line	curve (parabola)
Beam stress	higher	lower

< BACK GIVE FEEDBACK OK

Match the statements to compare point loads to uniformly distributed loads.

Drag statements on the right to match the left.

Distributed loads are the same as point loads when analysing	reactions, force and moment equilibrium.
Distributed loads are different to point loads when analysing	shear force, bending moment and stresses.
A uniformly distributed load will produce a ______ line in the shear force diagram.	sloping
A point load will produce a ______ line in the shear force diagram.	horizontal

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Which of the following statements are true when comparing distributed loads to the equivalent point load where point load = distributed load x length of region?

Check all that apply.

- [ ] Distributed loads can be replaced by point loads when solving statics
- [ ] Distributed loads cause lower bending moment than point loads
- [ ] Distributed loads have twice the reaction force as point loads
- [ ] Distributed loads have the same reaction forces and bending moment as point loads

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

How a distributed load influences the shear force diagram

A distributed load behaves differently to a point load.

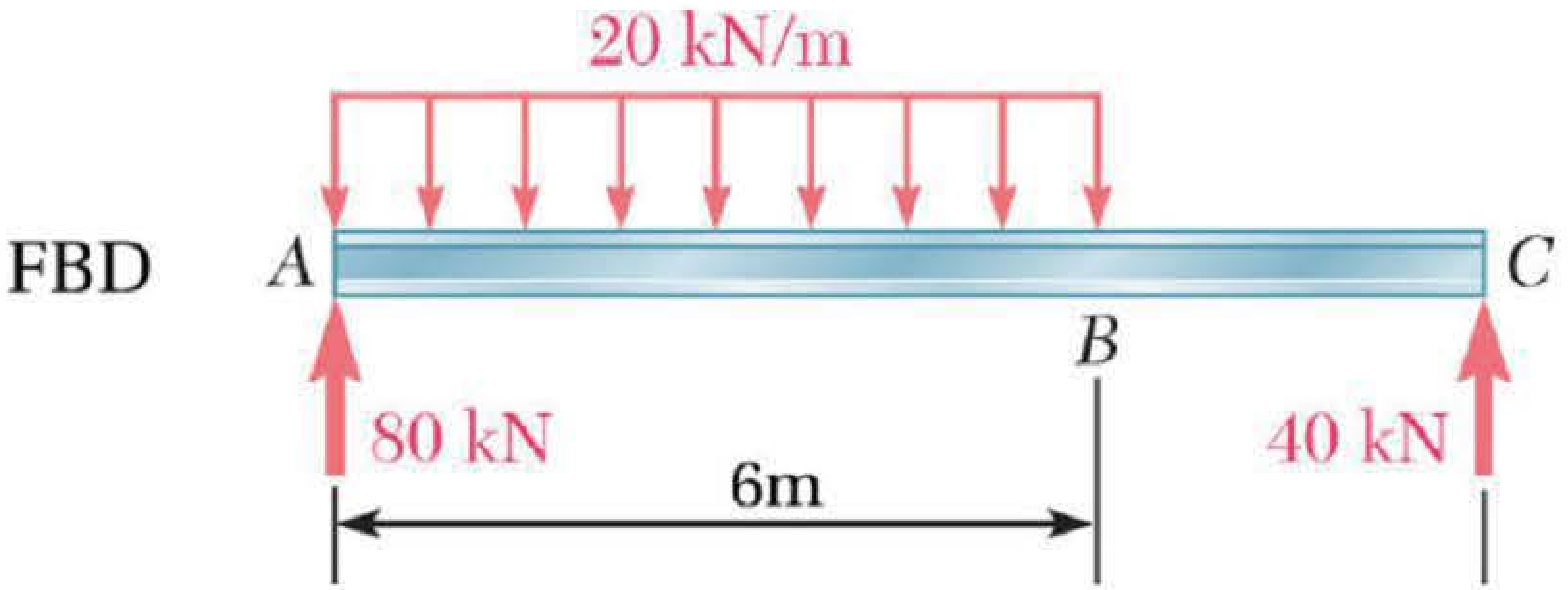
We can approximate a distributed load by splitting it up every metre. This results in a staircase appearance in the shear force diagram.

To make it more accurate, we should split the force up into finer and finer increments, until eventually the steps will be so small they resemble a sloping line.

So a uniformly distributed force creates a sloping line on the shear force diagram.

The following example examines the effect of a distributed load on the shear force diagram:

A uniformly distributed load of 20 kN/m is spread over a 6 m region of the beam.



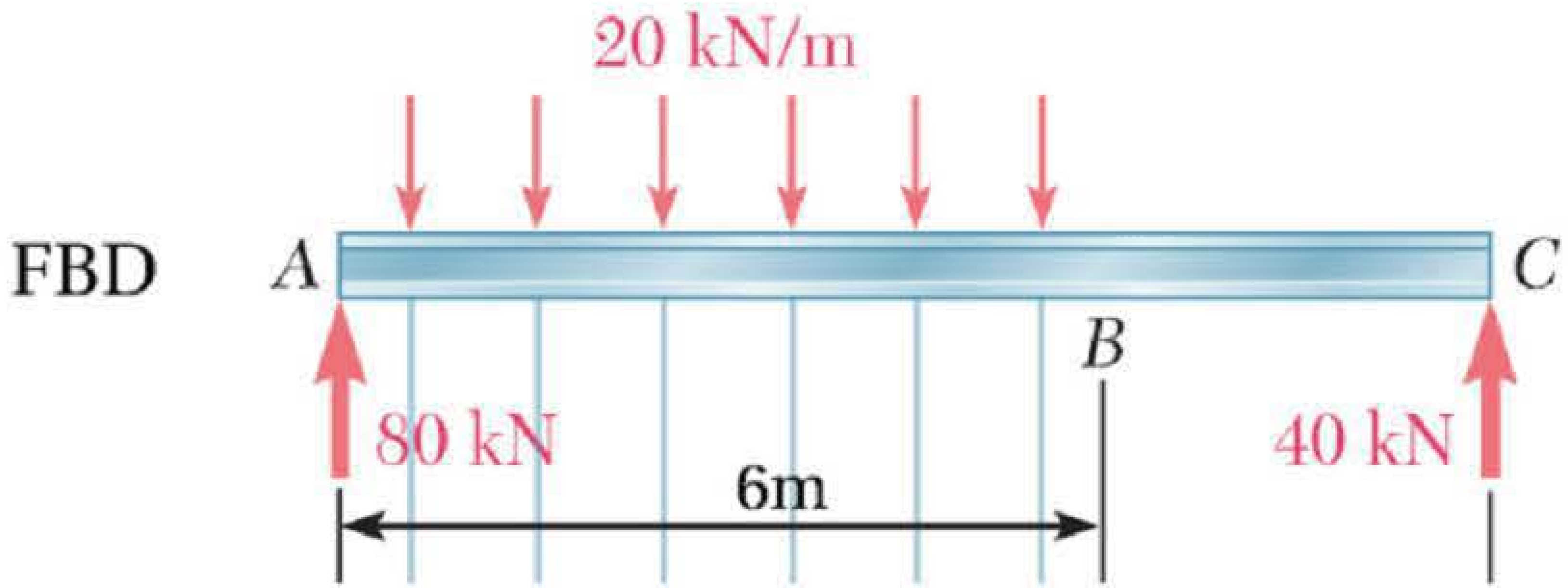
Distributed load	Approximate distributed load	Approximate diagram	Accurate diagram

GIVE FEEDBACK

OK

How a distributed load influences the shear force diagram

We could approximate the 20 kN/m loading by placing a 20 kN force in the middle of each metre of beam. In the example below we have six forces, each 20 kN, positioned at 0.5 m, 1.5 m, 2.5 m, 3.5 m, 4.5 m and 5.5 m.



We know enough to solve this right away.

Distributed load	Approximate distributed load	Approximate diagram	Accurate diagram

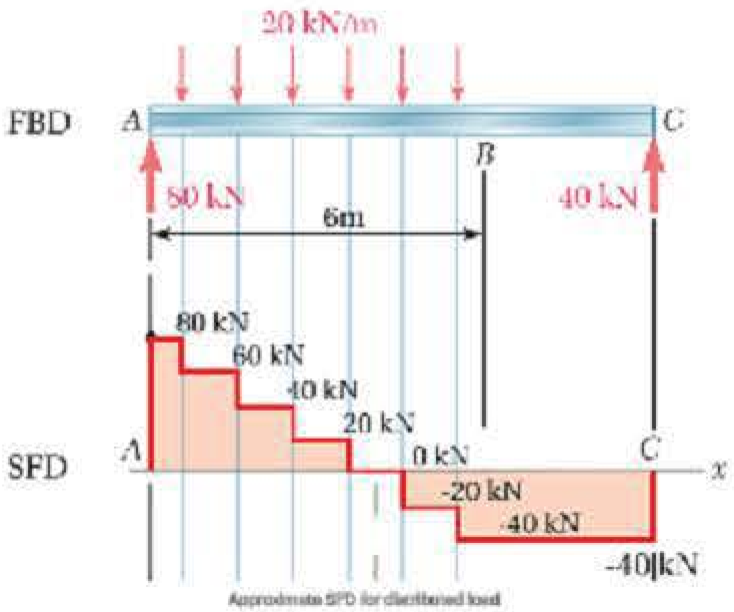
GIVE FEEDBACK

OK

How a distributed load influences the shear force diagram

Starting from the left end:

- The shear force for the first 0.5 m is +80 kN
- At the first 20 kN force, the shear force becomes +80 – 20 = +60 kN
- This repeats every metre, where shear force drops by 20 kN each time
- After the sixth force, the shear force is down to –40 kN



This is an approximation. There will be errors when we try to use this shear force diagram to construct the bending moment diagram.

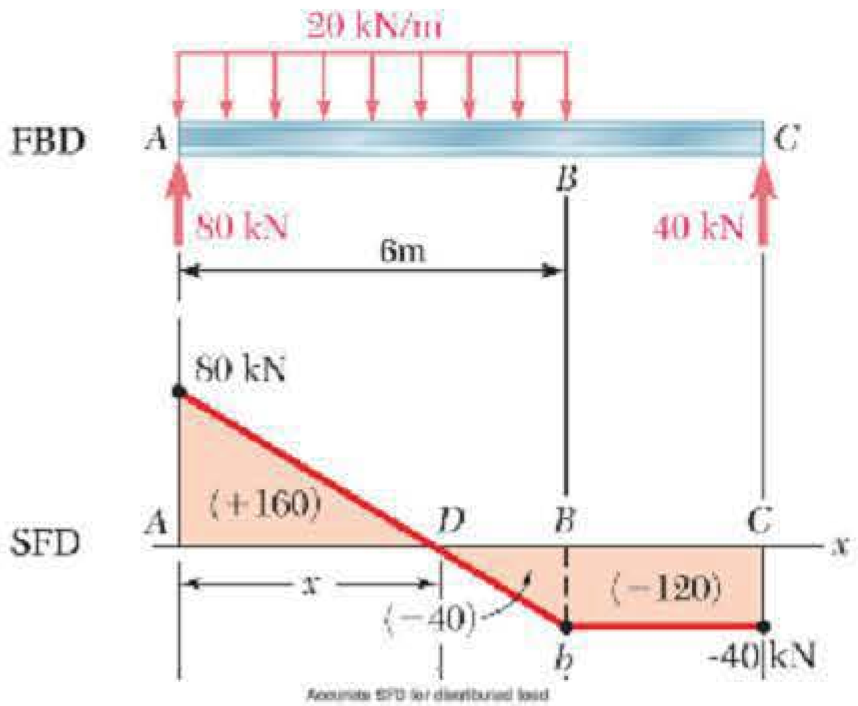
Distributed load	Approximate distributed load	Approximate diagram	Accurate diagram

GIVE FEEDBACK

OK

How a distributed load influences the shear force diagram

For an accurate diagram, start at the far left side, before the distributed load has a chance to start pushing down.
The shear force (which comes from the reaction force) is 80 kN.
With each metre, the shear force drops by 20 kN.
The total of the distributed load is 20 kN/m x 6 m = 120 kN.
So after 6 m the shear force is
$80\ \text{kN} - (6x^{20}\ \text{kN}) = -40\ \text{kN}.$
Draw a line between the two ends of the distributed load, left (80 kN) to right (–40 kN).
This –40 kN matches the shear force at the right end of the beam.

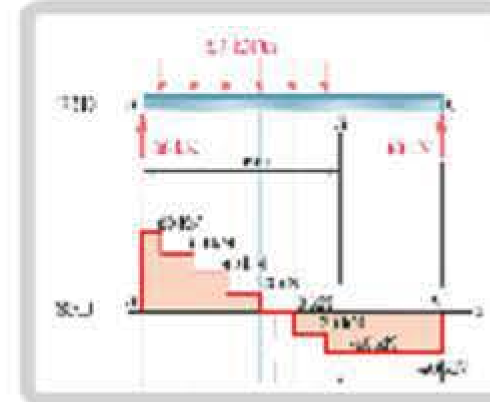


Distributed load	Approximate distributed load	Approximate diagram	Accurate diagram

GIVE FEEDBACK

OK

Look at the diagram and identify the statements that are true regarding distributed loads.



Use mouse to zoom. Click to keep enlarged.

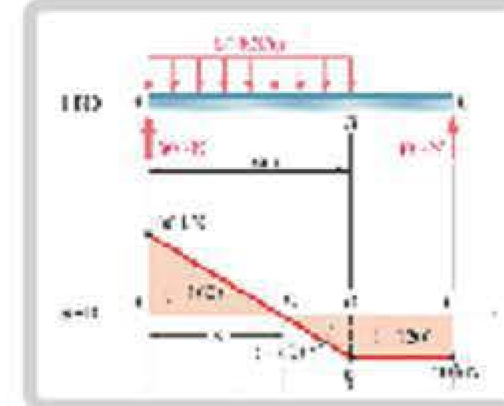
Check all that apply.

- [ ] It shows six distributed loads
- [ ] It shows that the total load is 20 kN/m for 6 m which is 120 kN
- [ ] It shows that a distributed load can be converted into a single point load with no change to the shear force diagram
- [ ] It is an approximation of a distributed load

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Put these steps in the correct order for constructing a uniformly distributed load in the shear force diagram.



Use mouse to zoom. Click to keep enlarged.

↑↓ Place these in the proper order.

Draw a line between the two ends of the distributed load, left (+80 kN) to right (–40 kN).

Start at the far left side. The shear force (which comes from the reaction force) is +80 kN. Plot this point on shear force diagram.

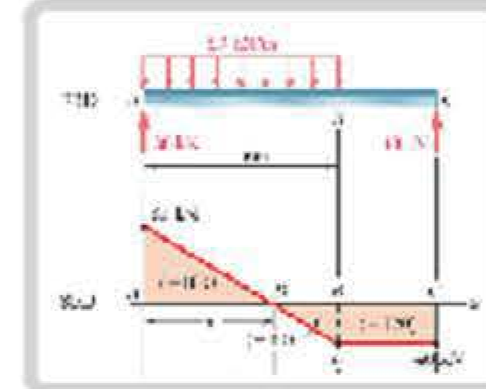
With each metre, the shear force drops by 20 kN. The total of the distributed load is 20 kN/m x 6 m = 120 kN.

After 6 m the shear force is +80 kN – (6 x 20 kN) = –40 kN. This is the shear force on the right side of the distributed load. Plot this point on the shear force diagram.

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Look at the shear force diagram and correctly sort the information below.



Use mouse to zoom. Click to keep enlarged.

Drag statements on the right to match the left.

The intensity of the distributed load	4 m
The total of the distributed load	80 kN
The length of the distributed load	120 kN
The maximum shear force	20 kN/m
The loaction of zero shear force	6 m

Do you know the answer?

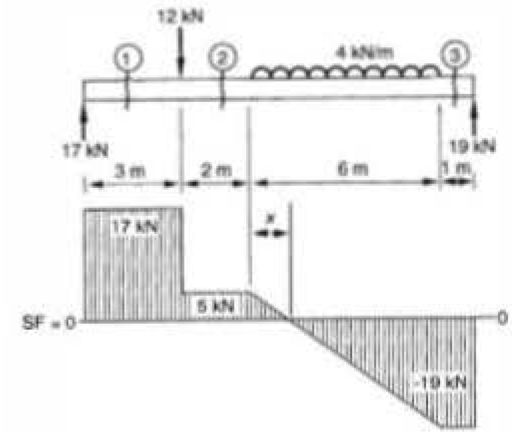
I KNOW IT | THINK SO | UNSURE | NO IDEA

Here is another example of the construction of a shear force diagram. As usual, the shear force diagram works from left to right. In this example, the shear force crosses the zero axis at a distance of 6.25 metres from the left end of the beam. This turns out to be an important number. The maximum bending moment occurs when the shear force crosses the line.

Example

When there is a uniformly distributed load applied, the shear force under the distributed load varies uniformly along the beam and can be illustrated on the shear force diagram by an inclined straight line.

Calculate the shear forces and draw the shear force diagram for the beam shown.



GIVE FEEDBACK

CONTINUE >

Solution

Shear forces are calculated for cross-sections 1, 2 and 3 as follows:

$$
\begin{aligned}
SF_1 &= 17\ \text{kN} \\
SF_2 &= 17\ \text{kN} - 12\ \text{kN} \\
&= 5\ \text{kN} \\
SF_3 &= 17\ \text{kN} - 12\ \text{kN} - (4\ \text{kN/m} \times 6\ \text{m}) \\
&= -19\ \text{kN}
\end{aligned}
$$

When constructing the shear force diagram, draw an inclined straight line under the distributed load, connecting the ordinates just to the left (SF_2= 5 kN) and just to the right (SF_3= –19 kN) of the load distribution.

< BACK GIVE FEEDBACK CONTINUE >

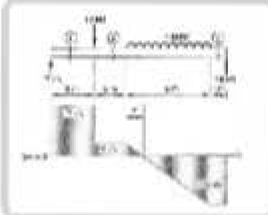
Notice where the shear force line intersects with the horizontal zero base line. This point has a special significance which will be discussed later in this chapter. Its precise location x with respect to point 2 can be calculated simply by dividing shear force at point 2 by the intensity of the distributed load:

$$x = \frac{5\,\text{kN}}{4\,\text{kN/m}}$$
$$= 1.25\,\text{m}$$

Therefore shear force is equal to zero at the point 6.25 m from the left-hand support.

< BACK GIVE FEEDBACK OK

Look at the shear force diagram and correctly sort the information below.



Use mouse to zoom. Click to keep enlarged.

Drag statements on the right to match the left.

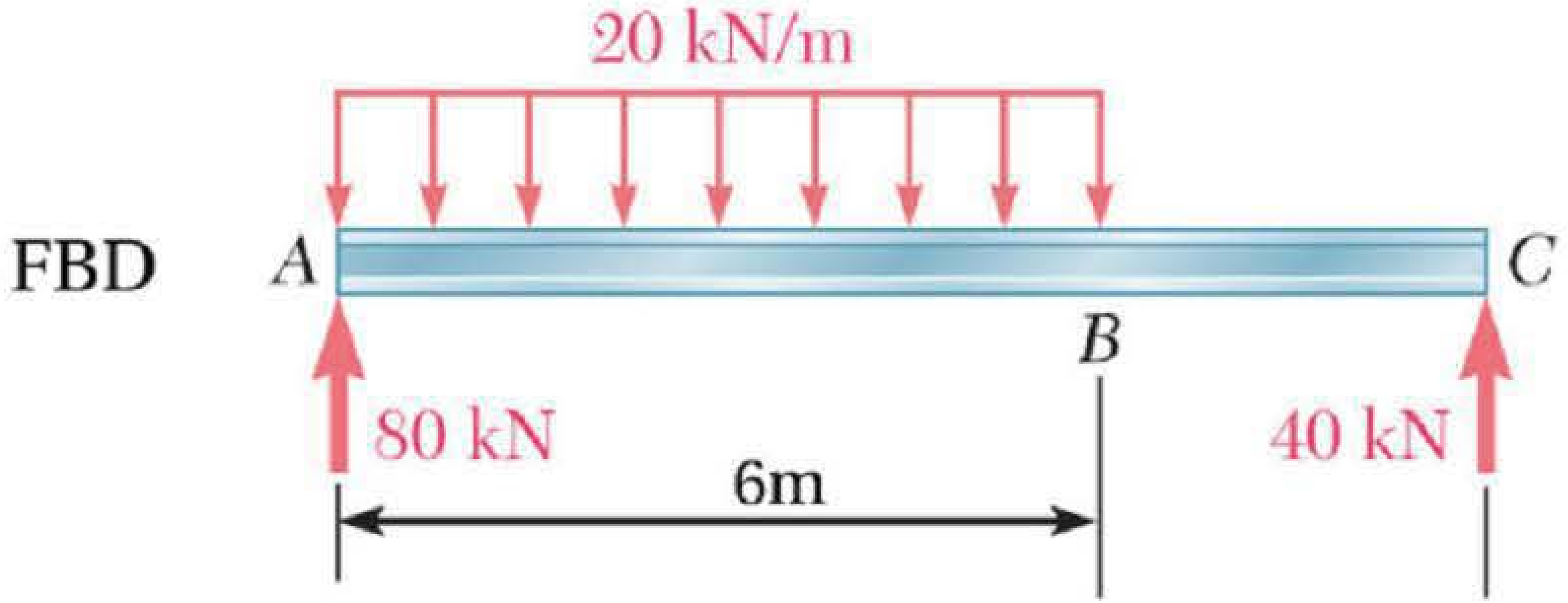
The intensity of the distributed load	4 kN/m
The total of the distributed load	24 kN
The length of the distributed load	6 m
The maximum shear force	19 kN
The location of zero shear force	6.25 m

Do you know the answer?

As we saw with the shear force diagram, a distributed load behaves differently to a point load. We can approximate the distributed load by splitting it up every metre. This results in a staircase appearance in the shear force diagram. As we add the area in each metre of the shear force diagram, we obtain the value of bending moment, and can plot it as a point on the bending moment diagram. To make it more accurate, we should split the force up into finer and finer increments, until eventually the steps will be so small that they resemble a sloping line.

The following example looks at the effect of a distributed load on the bending moment diagram.

A uniformly distributed load of 20 kN/m is spread over a 6 m region of the beam.



GIVE FEEDBACK

CONTINUE >

We saw this process for the shear force diagram earlier.

To approximate a distributed load for the bending moment diagram, construct the bending moment diagram using the area in the shear force diagram.

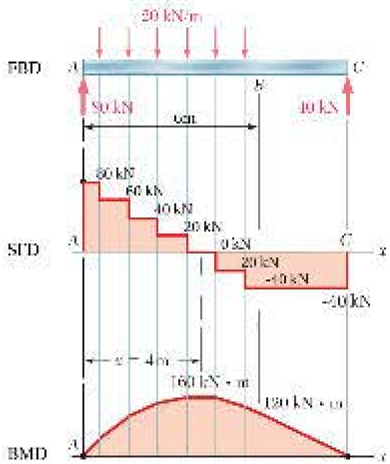
Add up the area in each column and add (or subtract) this from the bending moment.

The total positive area is 80 x 0.5 + 60 x 1 + 40 x 1 + 20 x 1 = 160 kNm.

So the maximum bending moment is 160 kNm.

This occurs when the shear force diagram crosses the zero axis (somewhere between 3.5 and 4.5 m).

< BACK GIVE FEEDBACK CONTINUE >



Approximate shear force diagram and bending moment diagram for distributed load

< BACK GIVE FEEDBACK CONTINUE >

To accurately solve a distributed load for the bending moment diagram, construct the bending moment diagram using the area in the shear force diagram.

We know two things:

1. The bending moment is zero on each end of the beam
2. The shear force diagram crosses the zero axis after dropping 80 kN, which takes 4 m

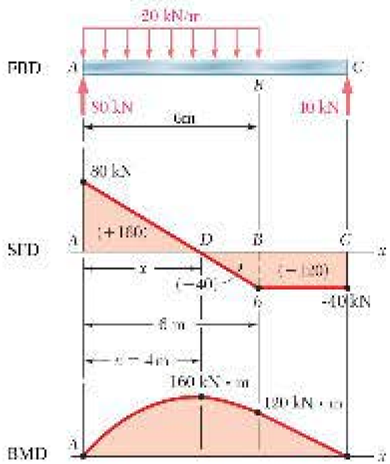
Adding shear force diagram area starting from the left end, the total positive area (of triangle) is 0.5 x 80 x 4 = 160 kNm (maximum bending moment).

This occurs when the shear force diagram crosses the zero axis (exactly 4 m).

This gives three points on the bending moment diagram through which to draw a parabola (from A to B).

The last part of the bending moment diagram is a straight line from B to C.

< BACK GIVE FEEDBACK CONTINUE >



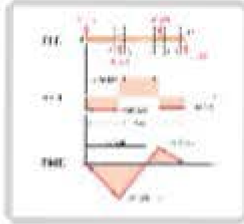
Accurate shear force diagram and bending moment diagram for distributed load

< BACK

GIVE FEEDBACK

OK

In which region(s) is this beam in pure sagging?



Use mouse to zoom.
Click to keep enlarged.

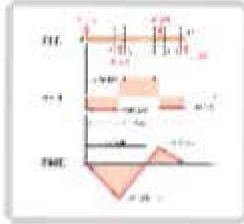
Check all that apply.

- [ ] From A for 4.42 m
- [ ] From A to B for 2.5 m
- [ ] From 4.42 m to D
- [ ] From B to C for 3 m
- [ ] From C to D for 2 m

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

In which region(s) is this beam in pure hogging?



Use mouse to zoom.
Click to keep enlarged.

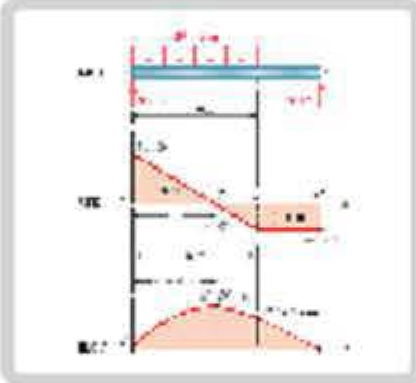
Click the correct answer.

- From A to D
- From D to B
- From B to C
- From A to C
- There is no sagging in this beam

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA

Look at the shear force and bending moment diagrams and correctly sort the information below.

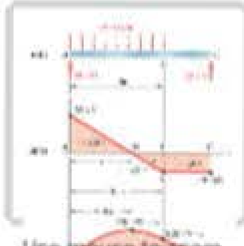


Use mouse to zoom.
Click to keep enlarged.

Drag statements on the right to match the left.

The intensity of the distributed load	20 kN/m
The maximum bending moment	4 m
Total area within the shear force diagram for entire beam	80 kN
The maximum shear force	0
The location of maximum bending moment	160 kNm

This simply supported beam is 9 m long. Constructing a bending moment diagram from the shear force diagram, where the beam includes a uniformly distributed load, which of the following statements are correct?



Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

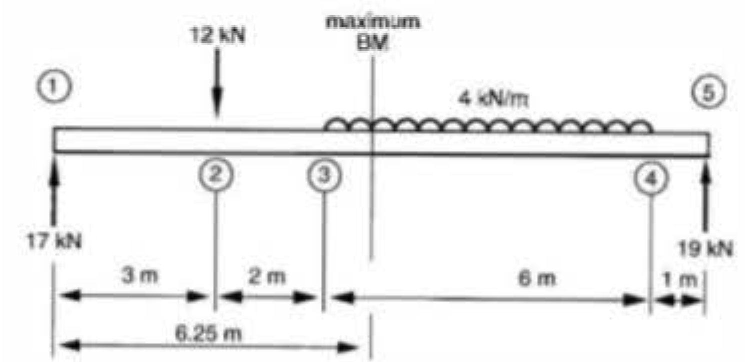
- [ ] The bending moment is zero at each end of the beam (points A and C)
- [ ] The bending moment at point B is +120 kNm
- [ ] The bending moment at point B is $\frac{(-40 \cdot 2)}{2} = -40\,\mathrm{kN\,m}$
- [ ] The bending moment at point D is 0 kNm
- [ ] The bending moment at point D is $\frac{(80 \cdot 4)}{2} = 160\,\mathrm{kN\,m}$
- [ ] The bending moment at point D is $\frac{(-40 \cdot 2)}{2} = -40\,\mathrm{kN\,m}$

A distributed load produces a sloping line in the shear force diagram and a parabola in the bending moment diagram. Lines are easy to draw but a parabola is more difficult. Instead of attempting to plot an accurate parabola, we usually only need three points and then draw a freehand curve through them. The three points of the distributed load parabola are usually left end, right end and maximum (or minimum). If the maximum does not occur on the parabola, it is not critical to draw it accurately.

When a region of a beam is subjected to a uniformly distributed load, the bending moment in that region is parabolic. It's not necessary to plot an accurate parabola but usually three points—left, right and the maximum bending moment—are found and plotted on the bending moment diagram.

Example

Calculate the bending moments and draw the bending moment diagram.



GIVE FEEDBACK

CONTINUE >

Solution by moments

Bending moments for points 1, 2 and 3 are obtained by simple calculations:

$$BM_1 = 0$$

$$BM_2 = 17\ \text{kN} \times 3\ \text{m} = 51\ \text{kN.m}$$

$$BM_3 = 17\ \text{kN} \times 5\ \text{m} - 12\ \text{kN} \times 2\ \text{m} = 61\ \text{kN.m}$$

When calculating the bending moment at point 4, we take into account the entire distributed load, which is equivalent to:

$$4\ \text{kN/m} \times 6\ \text{m} = 24\ \text{kN}$$

with the centre of its distribution 3 m to the left of point 4.

Hence:

$$\begin{aligned} BM_4 &= 17\ \text{kN} \times 11\ \text{m} - 12\ \text{kN} \times 8\ \text{m} - 24\ \text{kN} \times 3\ \text{m} \\ &= 19\ \text{kN.m} \end{aligned}$$

< BACK GIVE FEEDBACK CONTINUE >

A much easier way to calculate bending moment at point 4 is to consider the right side:

$$BM_4 = 19\ \text{kN} \times 1\ \text{m} = 19\ \text{kN m}$$ (positive because it is sagging).

Finally the right end is $BM_5 = 0$.

We know from previous discussion that maximum bending moment must occur where shear force = 0 (where the shear force diagram crosses the zero base line). Furthermore, we were able to establish that this point happens to be 6.25 m from the left-hand support. All that remains now is to calculate the bending moment at this point, and it will have to be the maximum value of the bending moment in the beam.

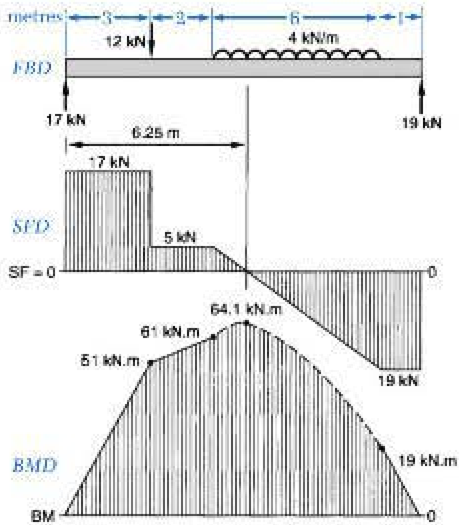
Taking moments of all forces which lie to the left of the 6.25 m point:

$$BM_{max} = 17\ \text{kN} \times 6.25\ \text{m} - 12\ \text{kN} \times 3.25\ \text{m} - (4\ \text{kN/m} \times 1.25\ \text{m}) \times \frac{1.25\ \text{m}}{2}$$

$$= 64.1\ \text{kN.m}$$

< BACK GIVE FEEDBACK CONTINUE >

This value should be shown on the bending moment diagram along with all other principal values of bending moment. The bending moment diagram is then completed by drawing a smooth curve to connect the maximum bending moment with the bending moments on either side of it, i.e. at points 3 and 4.



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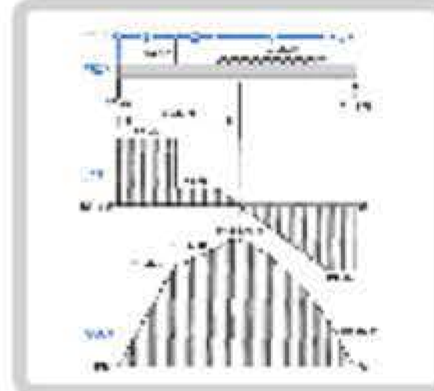
GIVE FEEDBACK
OK

Look at the shear force and bending moment diagram and correctly sort the information below.

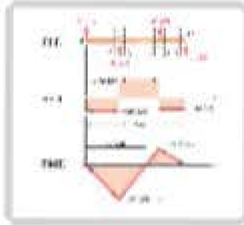


Use mouse to zoom.
Click to keep enlarged.

Drag statements on the right to match the left.

The intensity of the distributed load	4 kN/m
The maximum bending moment	0
Total area within the shear force diagram for entire beam	6.25 m
The maximum shear force	64.1 kNm
The location of maximum bending moment	19 kN

In which region(s) is this beam in pure hogging?



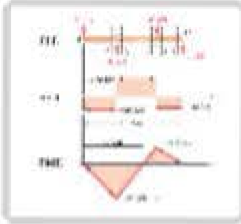
Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] From A to C
- [ ] From A to B m
- [ ] From C to D
- [ ] From B to C
- [ ] From A to 4.42 m
- [ ] From 4.42m to D

Do you know the answer?

In which region(s) is this beam in pure sagging?



Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] From A to C
- [ ] From A to B m
- [ ] From C to D
- [ ] From B to C
- [ ] From A to 4.42 m
- [ ] From 4.42m to D

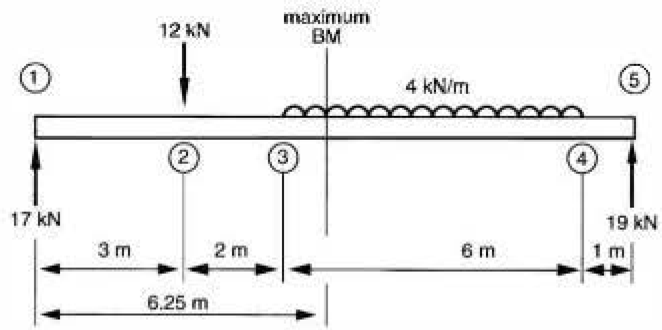
Do you know the answer?

Construct a bending moment diagram with a uniformly distributed load from the shear force diagram—Example 1/4

When a region of a beam is subjected to a uniformly distributed load, the bending moment in that region is parabolic. It is not necessary to plot an accurate parabola but usually three points—left, right and the maximum bending moment—are found and plotted on the bending moment diagram.

Example

Calculate the bending moments and draw the bending moment diagram.



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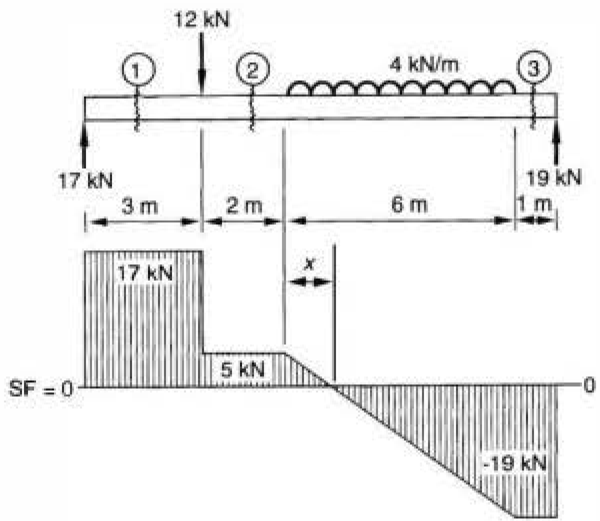
CONTINUE >

Construct a bending moment diagram with a uniformly distributed load from the shear force diagram—Example 2/4

Solution by area in the shear force diagram

Assume the shear force diagram has been completed.



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Construct a bending moment diagram with a uniformly distributed load from the shear force diagram—Example3/4

Bending moments for points 1, 2 and 3 are obtained by simple calculations:

$$BM_1 = 0$$
$$BM_2 = 17\ \text{kN} \times 3\ \text{m} = 51\ \text{kN.m}$$
$$BM_3 = 17\ \text{kN} \times 3\ \text{m} + 5\ \text{kN} \times 2\ \text{m} = 61\ \text{kN.m}$$

When calculating the bending moment at point 4, we need the area of the triangle:

$$0.5 \times 5\ \text{kN} \times 1.25\ \text{m} = 3.125\ \text{kN} \cdot \text{m}$$

So bending moment at point 4 is the total positive area in the shear force diagram (above the line):

$$BM_{max} = 51\ \text{kN m} + 10\ \text{kN m} + 3.125\ \text{kN m}$$
$$= 64.1\ \text{kN.m}$$

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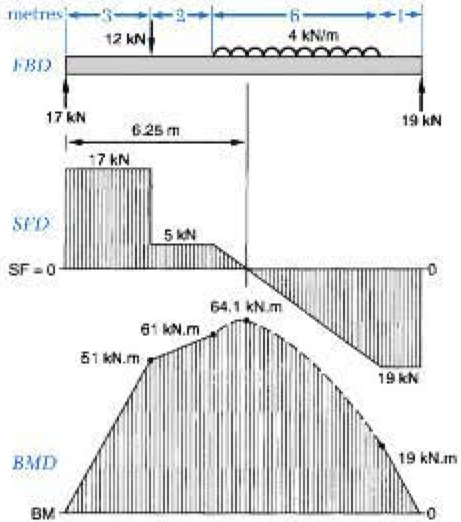
Construct a bending moment diagram with a uniformly distributed load from the shear force diagram—Example4/4

This value is shown on the bending moment diagram along with all other principal values of bending moment. The bending moment diagram is then completed by drawing a smooth curve to connect the maximum bending moment with the bending moments on either side of it, i.e. at points 3 and 4.



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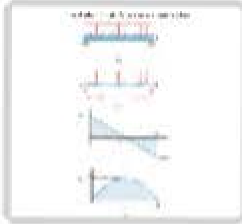
OK

Look at the diagrams and identify which statements are true (ω = distributed load in N/m).



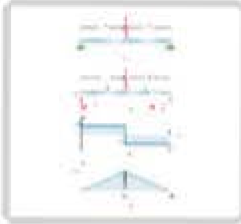
Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] The maximum bending moment is $\frac{\omega \cdot L^2}{8}$
- [ ] The maximum shear force is $\frac{\omega \cdot L}{2}$
- [ ] The total load is $\omega \cdot L$
- [ ] The entire beam is in hogging
- [ ] The beam is half hogging and half sagging

Do you know the answer?

Look at the diagram and identify which statements are true (P = point load in N).

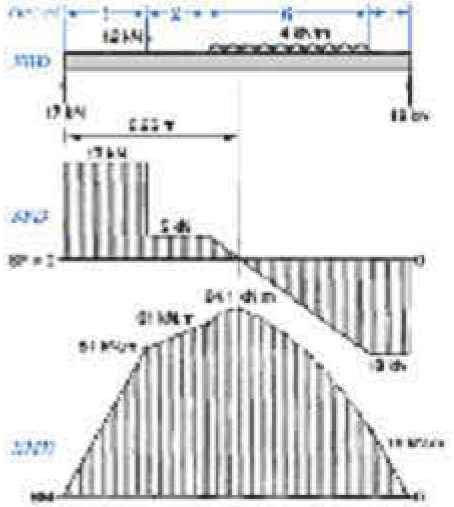


Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] The maximum bending moment is $\frac{P \cdot L}{2}$
- [ ] The maximum shear force is $\frac{P}{2}$
- [ ] The total load is P
- [ ] The entire beam is in sagging
- [ ] The beam is half hogging and half sagging

Do you know the answer?

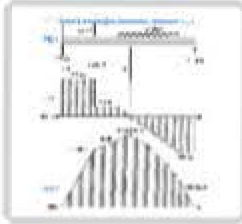


Using the shear force diagram to construct the bending moment diagram, correctly match the following information.

Drag statements on the right to match the left.

Area in shear force diagram from left end to 12 kN force	0.5 x 5 kN x 1.25 m = 3.125 kNm
Area in shear force diagram from 12 kN force to start of distributed load	5 kN x 2 m = 10 kNm
Area in shear force diagram from start of distributed load to 6.25 m mark	64.125 kNm
Total area in shear force diagram from left end to 6.25 m point	17 kN x 3 m = 51 kNm
Total area in shear force diagram for whole beam	0

In which region(s) is this beam in pure sagging?



Use mouse to zoom.
Click to keep enlarged.

Check all that apply.

- [ ] From left end to 12 kN force
- [ ] From 12 kN force to 6.25 m
- [ ] From 6.25 m to right end
- [ ] No sagging in this beam
- [ ] Whole beam is sagging

Do you know the answer?

I KNOW IT | THINK SO | UNSURE | NO IDEA